Tradable Permits for System-Optimized Networks

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Introduction

In this lecture, I return to the policy mechanism of pollution permits which was discussed for user-optimized transportation networks in a preceding lecture. However, now I model explicitly distinct transportation jurisdictions, each with its own responsible authority for its transportation network.

Moreover, I assume that there is an environmental quality standard that cannot be exceeded by the total emissions generated by the flows in all the transportation networks under consideration.

Thus, the transportation networks may correspond to regions with the standard being imposed by, for example, a state government or the federal government to ensure that the environmental quality standard is upheld.

Moreover, one can also consider the individual entities to be nations, each responsible for its own transportation network and the total emissions generated by the vehicles on its network.

The Model with Tradable Pollution Permits

I now introduce the notation for the permit system model which assumes that there are I transportation jurisdictions, with a typical jurisdiction denoted by i.

For each jurisdiction i, consider a transportation network $G_i = [N_i, L_i]$ consisting of the set of nodes N_i and a set of directed links L_i. For a typical such network, let a, b, etc., denote the links and let p, q, etc., denote the paths, which are assumed to be acyclic.

Assume that there are J_i origin/destination (O/D) pairs in network i, with a typical O/D pair denoted by w, and the set of O/D pairs for network i is denoted by W_i.

Let P_w denote the set of paths connecting O/D pair w and let P_i denote the set of paths in the network i.

The flow on a link a is denoted by f_a, and the user cost associated with traveling on link a by c_a. Group the link loads for network i into a column vector $f_i \in R^{n_i}$, and the link user travel costs into a row vector $c_i \in R^{n_i}$, where n_i is the number of links in the network i.

Further group the link loads and the user travel costs into the respective column vectors: $f \in R^n_+$ and $c \in R^n$.

I am interested in system-optimized networks in which each jurisdiction is faced with its own objective function. I assume that, in general, the user link travel costs associated with a particular network can depend on the flow upon the entire link load pattern on their own network, and, thus, the total link cost for a link $a \in L_i$ can be expressed as follows:

$$\hat{c}_a = \hat{c}_a(f_i) \times f_a, \quad \forall a \in L_i. \tag{1}$$

The total travel cost on path p, in turn, denoted by $\hat{C}_p$, is given by:

$$\hat{C}_p(f) = \sum_{a \in L_i} \hat{c}_a(f_i)\delta_{ap}, \quad \forall p \in P_i, \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

I consider the case of fixed travel demands where the demand for O/D pair $w \in W_i$ is denoted by d_w. The nonnegative flow on path p is denoted by x_p, with the path flows grouped into a column vector $x_i \in R_+^{Q_i}$, where Q_i denotes the number of paths in the network i.

Further group the path flows for all the networks into the vector $x \in R_+^{n_P}$, where n_P denotes the number of paths in all the networks.

Conservation of Flow Equations

The following conservation of flow equations must be satisfied by the flows in each network i:

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w \in W_i, \tag{3}$$

and

$$f_a = \sum_{p \in P_i} x_p \delta_{ap}, \quad \forall a \in L_i. \tag{4}$$

The conservation of flow equation (3) states that the sum of the path flows on paths connecting an O/D pair must be equal to the travel demand for that O/D pair in network i. Equation (4), on the other hand, states that the flow on a link equals the sum of the path flows on paths that use that link. Let K_i denote the feasible set defined as follows:

$$K_i = \{f_i, \text{ such that there exists a vector } x \geq 0, \text{ satisfying (3) and (4)}\}.$$

Notation fo the Permits

Let l_a denote the number of license permits on link a that allows travelers to emit pollutants at a certain rate. Let l_a^0, in turn, denote the initial allocation of licenses on link a, which is assumed to be nonnegative. Group the licenses into the column vector $l_i \in R_+^{n_i}$ for each network i and further group all the licenses into the column vector $l \in R_+^n$. As in the previous chapters, let h_a denote the emission factor associated with link a.

Price and Cost Structure

The price and cost structure associated with the marketable pollution permits is now discussed.

Let ρ denote the price of a license in the transportation network and let τ_a denote the marginal cost of emission abatement on link a. Group the marginal costs of abatement for network i into the column vector $\tau_i \in R_+^{n_i}$. Further group the marginal costs of emission abatement for the links in all the networks into the column vector $\tau \in R_+^n$, where n is the total number of links.

I now present the system-optimization problem facing the central controller in each transportation network i. Specifically, assume that the objective function must include the total cost in the network which reflects the congestion, but now one needs to also incorporate into the objective function the net revenue due to trading in the permits, which is to be maximized.

The net revenue term, assuming a given price for the license ρ^*, is then:

$$\sum_{a \in L_i} \rho^* (l_a^0 - l_a). \tag{5}$$

Moreover, one knows that the emissions on each link of the network cannot exceed the license holdings for that link which allow the travelers to emit at that rate. Combining the above, one obtains, hence, for each network $i \in I$, the following system-optimization problem:

$$\text{Minimize} \quad \sum_{p \in P_i} S_i(x_i) - \sum_{a \in L_i} \rho^*(l_a^0 - l_a)$$

$$= \text{Minimize} \quad \sum_{p \in P_i} \hat{C}_p - \sum_{a \in L_i} \rho^*(l_a^0 - l_a) \tag{6}$$

subject to:

$$h_a f_a \le l_a, \quad \forall a \in L_i, \tag{7}$$

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W_i \tag{8}$$

$$x_p \ge 0, \quad \forall p \in P_i, \tag{9}$$

$$l_a \ge 0, \quad \forall a \in L_i. \tag{10}$$

Since the feasible set is convex and, under the assumption that the user link travel cost functions are increasing functions of the flow, the total cost function is also convex, the Kuhn-Tucker optimality conditions are both necessary and sufficient and given by: For network i and all O/D pairs $w \in W_i$ and each path $p \in P_w$:

$$\hat{C}'_p(f_i^*, \tau^*) = \hat{C}'_p(x_i^*) + \sum_{a \in L_i} h_a \tau_a^* \delta_{ap} \begin{cases} = & \mu_w, \quad \text{if } x_p^* > 0 \\ \geq & \mu_w, \quad \text{if } x_p^* = 0, \end{cases} \tag{11}$$

where $\hat{C}'_p(x^*)$ denotes the marginal of the total cost on path p and is given by: $\hat{C}'_p(x) = \frac{\partial S_i(x_i)}{\partial x_p}$. Note that here the decoupled nature of the individual transportation networks is explicitly emphasized, since it is asssumed that each is under the jurisdiction of a distinct transportation authority.

Also, for each link $a \in L_i$, the equilibrium marginal cost of emission abatement τ_a^*, must satisfy:

$$h_a f_a^* \begin{cases} = & l_a^*, \quad \text{if } \tau_a^* > 0 \\ \leq & l_a^*, \quad \text{if } \tau_a^* = 0. \end{cases} \tag{12}$$

The following condition must also hold: For each link $a \in L_i$:

$$\tau_a^* \begin{cases} = & \rho^*, \quad \text{if } l_a^* > 0 \\ \leq & \rho^*, \quad \text{if } l_a^* = 0. \end{cases} \tag{13}$$

The above conditions are the optimality conditions for a system-optimal solution for transportation network i. Observe that τ_a^* is the marginal cost of emission abatement associated with constraint (7).

Market Equilibrium Condition for Licenses

I now state the market equilibrium conditions for licenses, which say that if the equilibrium price of a license is positive, then the market must clear for the licenses, that is, the total supply of the licenses, which is given by the sum of the initial license allocations, must be equal to the sum of the final (equilibrium license) holdings:

$$\sum_{i\in I}\sum_{a\in L_i}(l_a^0 - l_a^*)\begin{cases} = & 0, \quad \text{if } \rho^* > 0 \\ \geq & 0, \quad \text{if } \rho^* = 0. \end{cases} \tag{14}$$

Expression (14) corresponds to the well-known economic equilibrium conditions that state that, in equilibrium, if a price of a good (which in this case is the license) is positive, then the market for that good must clear, that is, the supply of the licenses, which is equal to $\sum_{i\in I}\sum_{a\in L_i} l_a^0$, must be equal to the demand for the licenses in equilibrium, which is given by $\sum_{i\in I}\sum_{a\in L_i} l_a^*$.

On the other hand, if the price of a license is zero, then one may have an excess supply of the licenses.

Trade in pollution permits

Jurisdiction 1	Jurisdiction 2 ...	Jurisdiction I
Network 1	Network 2 ...	Network I
$G_1 = [N_1, L_1]$	$G_2 = [N_2, L_2]$ ...	$G_I = [N_I, L_I]$
Initial: $\sum_{a \in L_1} l_a^0$	Initial: $\sum_{a \in L_2} l_a^0$ ...	Initial: $\sum_{a \in L_I} l_a^0$
	$\Downarrow$	
$\Rightarrow$	Trade	$\Leftarrow$
	$\Downarrow$	
Final: $\sum_{a \in L_1} l_a^*$	Final: $\sum_{a \in L_2} l_a^*$...	Final: $\sum_{a \in L_I} l_a^*$

Let $\mathcal{K}$ denote the feasible set such that $\mathcal{K} \equiv \prod_{i \in I} K_i \times R_+^{\sum_{i \in I} 2n_i \times R_+}$.

I am now ready to define the equilibrium state.

Definition 1 (Sustainable System-Optimum with Tradable Pollution Permits)

A vector $(f^*, \tau^*, l^*, \rho^*) \in \mathcal{K}$ is an equilibrium of the sustainable system-optimal tradable pollution permits model if and only if it satisfies the systems of equalities and inequalities (11)–(13) for all networks $i \in I$ and (14).

In the Table, the structure of the trade problem in licenses is given. Note that the licenses may also be interpreted as flows between the network jurisdictions whereas the transportation flows are internal to each network.

I now present the variational inequality formulation of the equilibrium conditions for the model. The proof is similar to that of other theorems in this course and, hence, is not given. A path flow version can also be readily obtained.

Theorem 1 (Variational Inequality Formulation of Tradable Pollution Permit System Traffic Network Equilibrium with System-Optimized Behavior)

A vector of link loads, marginal costs of emission abatement, licenses, and license price, $(f^*, \tau^*, l^*, \rho^*) \in \mathcal{K}$, *is an equilibrium of the tradable pollution permit market equilibrium model in the case of sustainable system-optimized networks if and only if it is a solution to the variational inequality problem:*

$$\sum_{i \in I} \sum_{a \in L_i} (\widehat{c}'_a(f_i^*) + h_a \tau_a^*) \times (f_a - f_a^*)$$

$$+ \sum_{i \in I} \sum_{a \in L_i} (l_a^* - h_a f_a^*) \times (\tau_a - \tau_a^*)$$

$$+ \sum_{i \in I} \sum_{a \in L_i} (\rho^* - \tau_a^*) \times (l_a - l_a^*) + \sum_{i \in I} \sum_{a \in L_i} (l_a^0 - l_a^*) \times (\rho - \rho^*) \geq 0,$$

$$\forall (f, \tau, l, \rho) \in \mathcal{K}. \qquad (15)$$

Variational inequality (15) is now put into standard form. Define column vector $X \equiv (f, \tau, l, \rho) \in \mathcal{K}$ and the column vector $F(X)$, where:

$$F(X) \equiv \left(\hat{C}'(X), T(X), L(X), P(X)\right).$$

$\hat{C}'(X)$, $L(X)$, $T(X)$ are each n-dimensional column vectors with component a given, respectively, as follows:

$$\hat{C}'_a(X) : \hat{c}'_a(f) + h_a \tau_a,$$

$$T_a(X) : l_a - h_a f_a,$$

$$L_a(X) : \rho - \tau_a,$$

whereas $P(X)$ is the one-dimensional vector with the single component:

$$P(X) : \sum_{a \in L_i} (l_a^0 - l_a).$$

Thus, variational inequality (15) can be expressed as:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{16}$$

I now turn to studying whether the equilibrium pattern is independent of the initial allocation of the licenses on the links and how to guarantee that the environmental emission standards imposed by the governing body are met in equilibrium. The question as to whether the initial allocation of licenses affects the equilibrium pattern is answered in the following corollary.

Corollary 1

If $l_a^0 \geq 0$ for all a, and $\sum_{i \in I} \sum_{a \in L_i} l_a^0 = \bar{Q}$, with $\bar{Q}$ fixed and positive, then the equilibrium pattern $(f^*, \tau^*, l^*, \rho^*)$ is independent of the initial allocation.

Proof:

The terms in the variational inequality (15) are either independent of l_a^0 or depend only on the sum, $\sum_{i \in I} \sum_{a \in L_i} l_a^0$. The conclusion follows.

In the following proposition, I show that the environmental standards are met by the equilibrium pattern, provided that the sum of the initial allocation of licenses is equal to the imposed environmental standard given by $\bar{Q}$.

Proposition 1

If $\sum_{i\in I}\sum_{a\in L_i} l_a^0 = \bar{Q}$, the equilibrium vector achieves the environmental quality standard, $\bar{Q}$, provided that the entire transportation network is viable.

Proof:

One has from equilibrium conditions (12) and (14) that:

$$\sum_{i\in I}\sum_{a\in L_i} h_a f_a^* \leq \sum_{i\in I}\sum_{a\in L_i} l_a^* \leq \sum_{i\in I}\sum_{a\in L_i} l_a^0 = \bar{Q}.$$

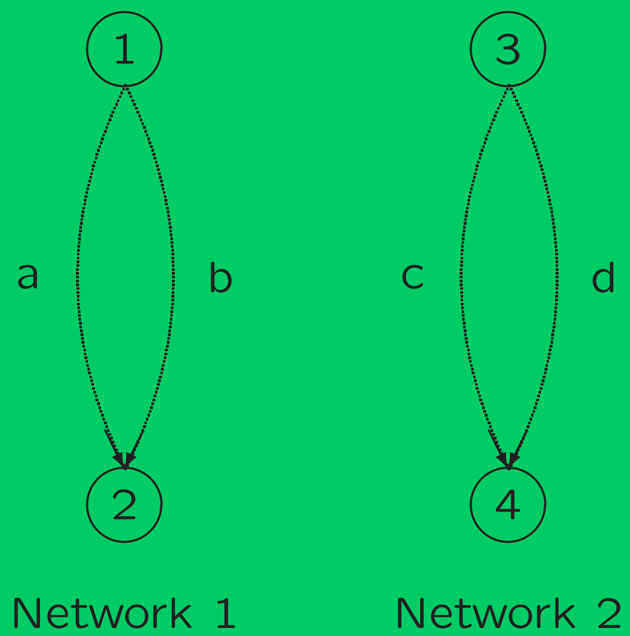
Hence, the environmental standards are met by the equilibrium pattern.

Proposition 1 says that the complete transportation network consisting of the I individual transportation networks is sustainable, that is, the environmental quality standard will not be exceeded, provided that the sum of the license allocations are set equal to the target.

Of course, the complete network must be viable in that there must exist a solution to the Linear System 1 in order to even be able to achieve sustainability.

Relationship between a viable system-optimized network, tradable permits, and sustainability

$$\boxed{\begin{array}{c} \textbf{System-Optimized Viable Network} \\ + \\ \textbf{License Allocation: } \sum_{i \in I} \sum_{a \in L_i} l_a^0 = \bar{Q} \end{array} \quad \Rightarrow \quad \textbf{Sustainability}}$$



Networks for Example 1

Example 1

I now present an example which illustrates the basic concepts. Consider the two networks depicted in the Figure, each of which is the responsibility of a separate transportation authority.

Each network consists of two links with the first network consisting of links a and b, whereas the second consists of links c and d.

The O/D pair for Network 1, denoted by w_1, is $(1,2)$, whereas the O/D pair for Network 2 is $(3,4)$.

The user link travel cost functions are:

$$c_a(f_a) = f_a + 5, \quad c_b(f_b) = f_b + 10,$$

$$c_c(f_c) = f_c + 5, \quad c_d(f_d) = f_d + 5.$$

The travel demands are: $d_{w_1} = 10$ and $d_{w_2} = 10$. Denote the paths as follows: $p_1 = a$, $p_2 = b$, $p_3 = c$, and $p_4 = d$.

Assume that the emission factors are: $h_a = h_b = 1$, and $h_c = h_d = 0.2$ with the initial license allocation given by: $l_a^0 = l_b^0 = l_c^0 = l_d^0 = 3$.

Hence, Network 1 has a total license allocation of 6 for its network links as does Network 2. Moreover, the environmental quality standard $\bar{Q} = 12$.

Note that Network 1 has links with higher emission factors than Network 2 and, in fact, with only an initial license allocation of 6 it cannot, without the trading of licenses, emit less than or equal to its allocation.

Indeed, in order to satisfy its fixed demand of 10, 10 units of pollutants will be generated due to its emission factors on its links. Network 2, on the other hand, has links which are characterized by lower emission factors.

It is easy to verify that it can satisfy its permission to emit at 6 units since its demand is 10 and the emissions generated on its network will be 2.

Therefore, it can sell several of its licenses to Network 1, which needs to purchase licenses since it cannot sustain its demand with the given allocation.

Since the network structure of both networks is simple, we can solve for the conditions (11)–(14) to obtain the following solution:

The flows are:

$$x_{p_1}^* = f_a^* = 6.35, \quad x_{p_2}^* = f_b^* = 3.75,$$

$$x_{p_3}^* = f_c^* = 5.00, \quad x_{p_4}^* = f_d^* = 5.00,$$

the marginal costs of emission abatement are:

$$\tau_a^* = \tau_b^* = \tau_c^* = \tau_d^* = 1.94,$$

which is also the price ρ^*;

the licenses are:

$$l_a^* = 6.25, \quad l_b^* = 3.75, \quad l_c^* = 1.00, \quad l_d^* = 1.00.$$

Hence, the market for licenses clears in this example, and Network 1 purchases four licenses from Network 2. Furthermore, the conditions (11)–(13) are precisely satisfied by the solution pattern.

Therefore, each network has minimized its objective function and the environmental quality standard has been achieved through the trading of pollution permits.

Algorithm and Numerical Examples

I now show the realization of the modified projection method for the solution of variational inequality problem (15).

Then, I state the algorithm and discuss it more fully from a computational perspective.

The algorithm is presented for solving variational inequality (15).

Modified Projection Method for the Tradable Permit Model for Sustainable System-Optimized Networks

Step 0: Initialization

Set $(f^0, \tau^0, l^0, \rho^0) \in \mathcal{K}$. Let $\mathcal{T} = 1$ and set α such that $0 < \alpha \leq \frac{1}{\bar{L}}$, where $\bar{L}$ is the Lipschitz constant for the problem.

Step 1: Computation

Compute $(\bar{f}^{\mathcal{T}}, \bar{\tau}^{\mathcal{T}}, \bar{l}^{\mathcal{T}}, \bar{\rho}^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i\in I}\sum_{a\in L_i}(\bar{f}_a^{\mathcal{T}} + \alpha(\hat{c}'_a(f_i^{\mathcal{T}-1}) + h_a\tau_a^{\mathcal{T}-1}) - f_a^{\mathcal{T}-1}) \times (f_a - \bar{f}_a^{\mathcal{T}})$$

$$+\sum_{i\in I}\sum_{a\in L_i}(\bar{l}_a^{\mathcal{T}} + \alpha(\rho^{\mathcal{T}-1} - \tau_a^{\mathcal{T}-1}) - l_a^{\mathcal{T}-1}) \times (l_a - \bar{l}_a^{\mathcal{T}})$$

$$+\sum_{i\in I}\sum_{a\in L_i}(\bar{\tau}_a^{\mathcal{T}}+\alpha(l_a^{\mathcal{T}-1}-h_a f_a^{\mathcal{T}-1})-\tau_a^{\mathcal{T}-1})\times(\tau_a-\bar{\tau}_a^{\mathcal{T}})$$

$$+(\bar{\rho}^{\mathcal{T}}+\alpha(\sum_{i\in I}\sum_{a\in L_i}l_a^0-\sum_{i\in I}\sum_{a\in L_i}l_a^{\mathcal{T}-1})-\rho^{\mathcal{T}-1})\times(\rho-\bar{\rho}^{\mathcal{T}})\geq 0,$$

$$\forall(f,\tau,l,\rho)\in\mathcal{K}. \tag{17}$$

Step 2: Adaptation

Compute $(f^{\mathcal{T}},\tau^{\mathcal{T}},l^{\mathcal{T}},\rho^{\mathcal{T}})\in\mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i\in I}\sum_{a\in L_i}(f_a^{\mathcal{T}}+\alpha(\hat{c}'_a(\bar{f}_i^{\mathcal{T}})+h_a\bar{\tau}_a^{\mathcal{T}})-f_a^{\mathcal{T}-1})\times(f_a-f_a^{\mathcal{T}})$$

$$+\sum_{i\in I}\sum_{a\in L_i}(l_a^{\mathcal{T}}+\alpha(\bar{\rho}^{\mathcal{T}}-\bar{\tau}_a^{\mathcal{T}})-l_a^{\mathcal{T}-1})\times(l_a-l_a^{\mathcal{T}})$$

$$+\sum_{i\in I}\sum_{a\in L_i}(\tau_a^{\mathcal{T}}+\alpha(\bar{l}_a^{\mathcal{T}}-h_a\bar{f}_a^{\mathcal{T}})-\tau_a^{\mathcal{T}-1})\times(\tau_a-\tau_a^{\mathcal{T}})$$

$$+(\rho^{\mathcal{T}}+\alpha(\sum_{i\in I}\sum_{a\in L_i}l_a^0-\sum_{i\in I}\sum_{a\in L_i}\bar{l}_a^{\mathcal{T}})-\rho^{\mathcal{T}-1})\times(\rho-\rho^{\mathcal{T}})\geq 0,$$

$$\forall(f,d,l,\tau,\rho)\in\mathcal{K}. \tag{18}$$

Step 3: Convergence Verification

If $| f_a^{\mathcal{T}} - f_a^{\mathcal{T}-1} | \leq \epsilon$, $|l_a^{\mathcal{T}} - l_a^{\mathcal{T}-1} | \leq \epsilon$, $| \tau_a^{\mathcal{T}} - \tau_a^{\mathcal{T}-1} | \leq \epsilon$, for all $a \in L$, and $| \rho^{\mathcal{T}} - \rho^{\mathcal{T}-1} | \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

The decomposed subproblems can be computed efficiently.

Note that in the system-optimized problem over several transportation jurisdictions, the feasible set $\mathcal{K}$ is a Cartesian product, such that one can decompose both subproblems (17) and (18) for each network separately in terms of the flows (as well as the licenses and marginal costs of emission abatement).

The flow subproblems are, in fact, separable quadratic programming problems, each of which can be solved using the equilibration algorithm of Dafermos and Sparrow (1969). The subproblems in licenses, marginal costs of emission abatement and, finally, the license price, in turn, have an explicit form solution for (17) and analogously adapted for the induced subproblems in (18).

Numerical Examples

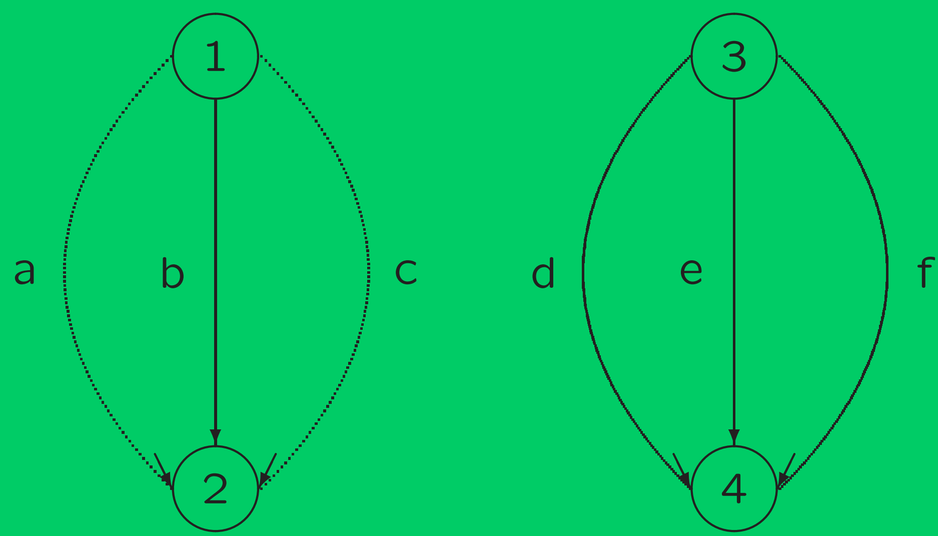
Numerical examples are now presented to illustrate both the pollution permit trading model and the algorithm.

The modified projection method was implemented in FORTRAN and the numerical experiments were conducted on the IBM SP2 located in the Computer Science Department at the University of Massachusetts at Amherst.

For the solution of the standard traffic network equilibrium problem encountered in both the computation and adaptation steps (cf. (17) and (18)) I utilized the equilibration method (cf. Dafermos and Sparrow 1969).

The convergence criterion was given by: $|x_p^{\mathcal{T}} - x_p^{\mathcal{T}-1}| \leq \epsilon$, for all $p \in P$, $|\tau_a^{\mathcal{T}} - \tau_a^{\mathcal{T}-1}| \leq \epsilon$, for all $a \in L$, $|l_a^{\mathcal{T}} - l_a^{\mathcal{T}-1}| \leq \epsilon$, for all $a \in L$, and $|\rho^{\mathcal{T}} - \rho^{\mathcal{T}-1}| \leq \epsilon$.

The modified projection method was initialized by setting the flow on a path equal to the travel demand for the O/D pair that the path belongs to, divided by the number of paths. All other variables were initialized to zero.



Network topology for Example 2

Example 2

The first numerical example in this subsection consisted of the two networks depicted in the Figure, each of which had three links and a single O/D pair, where $w_1 = (1, 2)$ and $w_2 = (3, 4)$.

The user link travel cost functions were:

$$c_a(f_a) = 2f_a + 1, \quad c_b(f_b) = f_b + 5, \quad c_c(f_c) = 2f_c + 3,$$

$$c_d(f_d) = 1.5f_d + 1, \quad c_e(f_e) = 3f_e + 2, \quad c_f(f_f) = f_e + 5.$$

The travel demands were:

$$d_{w_1} = 20, \quad d_{w_2} = 80.$$

The emission factors were:

$$h_a = 3, \quad h_b = 0.2, \quad h_c = 1,$$

$$h_d = 1, \quad h_e = 0.5, \quad h_f = 0.1.$$

The initial license allocations were:

$$l_a^0 = l_b^0 = l_c^0 = l_d^0 = l_e^0 = l_f^0 = 2,$$

and, hence, the environmental quality standard $\bar{Q} = 12$.

I set $\alpha = 0.2$ in the modified projection method. The modified projection method converged in 10,351 iterations and required 1.25 CPU seconds for convergence. It computed the following solution:

The flow pattern was:

$$x^*_{p_1} = f^*_a = 0.00, \quad x^*_{p_2} = f^*_b = 20.00, \quad x^*_{p_3} = f^*_c = 0.00,$$

$$x^*_{p_4} = f^*_d = 0.00, \quad x^*_{p_5} = f^*_e = 0.00, \quad x^*_{p_6} = f^*_f = 80.00.$$

The marginal costs of emission abatement were:

$$\tau^*_a = 41.8327, \quad \tau^*_b = 407.4937, \quad \tau^*_c = 123.5008,$$

$$\tau^*_d = 204.7472, \quad \tau^*_d = 407.4937, \quad \tau^*_f = 407.936.$$

The licenses were:

$$l^*_a = l^*_c = l^*_d = l^*_e = 0.00,$$

$$l^*_b = 4.00, \quad l^*_f = 8.00.$$

The price of the license was:

$$\rho^* = 407.937.$$

The market cleared for the licenses and, hence, the license price was positive. Furthermore, note that for those links with positive license holdings in equilibrium, the marginal cost of emission abatement was positive and equal to the license price and conditions (14) were met.

Note that the transportation authority for Network 1, which had six initial licenses, sold two of those licenses to Network 2.

Example 3

This example was identical to Example 2, except that now the initial license allocations were increased so that:

$$l_a^0 = l_b^0 = l_c^0 = l_d^0 = l_e^0 = l_f^0 = 3,$$

which corresponds to a loosening of the environmental quality standard from 12 to $\bar{Q} = 18$.

The modified projection method converged in 1907 iterations and required 0.29 CPU seconds for convergence. It yielded the following solution:

The new flow pattern was:

$$x_{p_1}^* = f_a^* = 0.00, \quad x_{p_2}^* = f_b^* = 20.00, \quad x_{p_3}^* = f_c^* = 0.00,$$

$$x_{p_4}^* = f_d^* = 1.02, \quad x_{p_5}^* = f_e^* = 12.70, \quad x_{p_6}^* = f_f^* = 66.28.$$

The new marginal costs of emission abatement were:

$$\tau_a^* = 24.5554, \quad \tau_b^* = 148.3249, \quad \tau_c^* = 71.6638,$$

$$\tau_d^* = \tau_e^* = \tau_f^* = 148.3249.$$

The new licenses were:

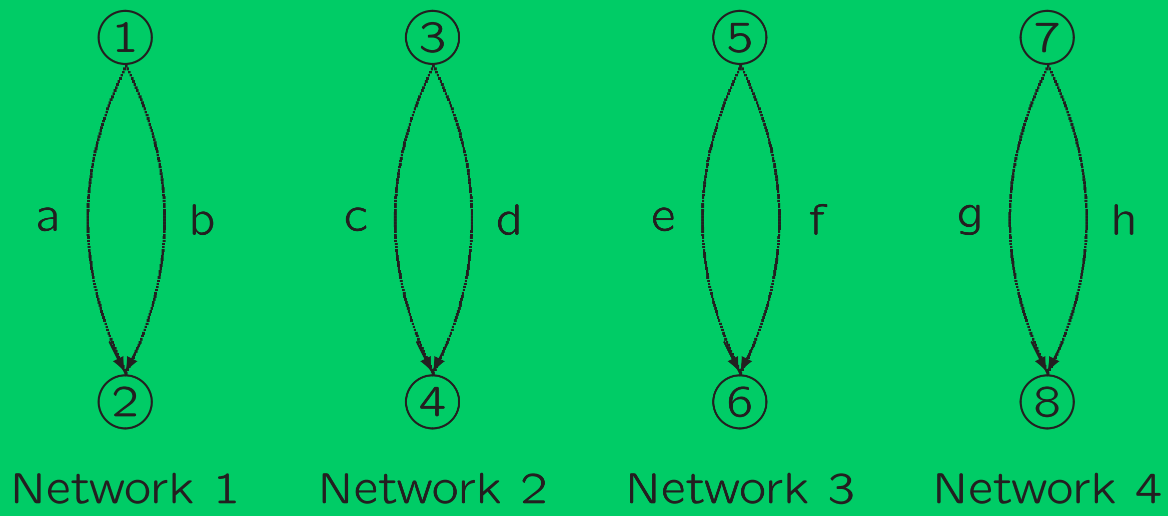
$$l_a^* = l_c^* = 0.00,$$

$$l_b^* = 4.00, \quad l_d^* = 1.0204, \quad l_e^* = 6.3520, \quad l_f^* = 6.6275.$$

The new price of the license was:

$$\rho^* = 148.3249.$$

The market for licenses cleared in this example and the license price was positive. Note now that since the supply of initial licenses is greater than that in Example 2, the equilibrium license price decreased. The authority responsible for Network 2 purchased five licenses from Network 1.



Networks for Example 4

Example 4

I then considered the four networks depicted in the Figure, each of which was under the jurisdiction of a separate transportation authority.

Each network consisted of a single O/D pair and two links, where $w_1 = (1,2)$, $w_2 = (3,4)$, $w_3 = (5,6)$, and $w_4 = (7,8)$. The paths were: $p_1 = a$, $p_2 = b$, $p_3 = c$, $p_4 = d$, $p_5 = e$, $p_6 = f$, $p_7 = g$ and $p_8 = h$.

The user link travel cost functions were:

$$c_a(f_a) = f_a + 1, \quad c_b(f_b) = 2f_b + 5,$$

$$c_c(f_c) = f_c + 2, \quad c_d(f_d) = 3f_d + 4,$$

$$c_e(f_e) = 2f_e + 1, \quad c_f(f_f) = f_f + 3,$$

$$c_g(f_g) = 3f_g + 5, \quad c_h(f_h) = 2f_h + 2.$$

The travel demands were:

$$d_{w_1} = 20, \quad d_{w_2} = 20, \quad d_{w_3} = 10, \quad d_{w_4} = 10.$$

The emission factors were:

$$h_a = 0.1, \quad h_b = 0.2, \quad h_c = 1, \quad h_d = 0.5,$$

$$h_e = 1, \quad h_f = 0.5, \quad h_g = 0.1, \quad h_h = 1.$$

The environmental quality standard was $\bar{Q} = 20$ and the initial license allocations were evenly distributed so that

$$l_a^0 = l_b^0 = l_c^0 = l_d^0 = l_e^0 = l_f^0 = l_g^0 = l_h^0 = 2.5.$$

I set $\alpha = 0.2$ in the modified projection method. The algorithm converged in 12,022 iterations and 1.99 seconds of CPU time.

The flow pattern was:

$$x_{p_1}^* = f_a^* = 17.15, \quad x_{p_2}^* = f_b^* = 2.85, \quad x_{p_3}^* = f_c^* = 3.43,$$

$$x_{p_4}^* = f_d^* = 16.57,$$

$$x_{p_5}^* = f_e^* = 0.00, \quad x_{p_6}^* = f_f^* = 10.00, \quad x_{p_7}^* = f_g^* = 10.00,$$

$$x_{p_8}^* = f_h^* = 0.00.$$

The marginal costs of emission abatement were:

$$\tau_a^* = \tau_b^* = \tau_c^* = \tau_d^* = \tau_f^* = \tau_g^* = 189.1181,$$

$$\tau_e^* = 116.5568, \quad \tau_h^* = 81.9086.$$

The licenses were:

$$l_a^* = 1.7151, \quad l_b^* = 0.5697, \quad l_c^* = 3.4302, \quad l_d^* = 8.2849,$$

$$l_e^* = 0.0000, \quad l_f^* = 5.0000, \quad l_g^* = 1.0000, \quad l_h^* = 0.0000.$$

The price of the license was:

$$\rho^* = 189.1181.$$

The market for licenses cleared in this example and the license price was positive.

Pricing in the Case of User-Optimized Behavior

Consider now that the transportation authorities are still faced with determining the system-optimal solution, as was done in the preceding section, but now the travelers on the networks behave in a *user-optimized* manner.

Hence, the different transportation authorities will need to implement policies to guarantee that the users behave also in a system-optimized fashion in the presence of tradable pollution permits.

Note that for the system-optimal solution with tradable pollution permits to also be user-optimized, it must satisfy the conditions: For each $i \in I$, and for O/D pair $w \in W_i$, and each path $p \in P_w$:

$$C_p(x_i^*) + t_p \begin{cases} = & \lambda_w, \quad \text{if } x_p^* > 0 \\ \geq & \lambda_w, \quad \text{if } x_p^* = 0, \end{cases} \tag{19}$$

where t_p here denoted a path toll policy on path p.

Furthermore, the system-optimized flow pattern already the conditions: For each $i \in I$, and O/D pair $w \in W_i$, and each path $p \in P_w$:

$$\hat{C}'_p(x_i^*) + \tau_a^* \sum_{a \in L} h_a \delta_{ap} \begin{cases} = & \mu_w, \quad \text{if } x_p^* > 0 \\ \geq & \mu_w, \quad \text{if } x_p^* = 0, \end{cases} \tag{20}$$

For a solution to (20) to coincide with that of (19) implies that, for each path $p \in P$, we must have that:

$$t_p = \widehat{C}'_p(x^*) - C_p(x^*) + \tau_a^* \sum_{a \in L} h_a \delta_{ap}. \tag{21}$$

Hence, (21) consists of a procedure in which to construct a path-toll policy, that is, solve the problem (6)–(10) and determine then for each path p in the network the path-toll policy accoding to the equation (21).

A link-toll policy, in turn, can also be determined according to:

$$t_a = \widehat{c}'_a(f^*) - c_a(f^*) + \tau_a^* h_a, \quad \forall a \in L. \tag{22}$$

In the Table, I depict the relationships between the behavioral principles and the environmental policy instruments just described.

I now present several numerical examples. In particular, I construct tolls for the networks in Examples 2–4 to guarantee that the system-optimized pattern for a sustainable transportation network is also user-optimized (after the imposition of the tolls).

Relationship between a viable system-optimized network, user-optimized behavior, tradable permits, tolls, and sustainability

System-Optimized Viable Network $+$ License Allocation: $\sum_{i\in I}\sum_{a\in L_i} l_a^0 = \bar{Q} \quad \Rightarrow \quad$ Sustainability $+$ User-Optimized Behavior $+$ Tolls

Example 5: Example 2 Revisited

I considered Example 2 in which the paths consist of single links (as in the subsequent examples). I utilized the computed sustainable S-O solution for Example 2 in expression (22) to obtain the following toll policy:

$$t_a = 125.4981, \quad t_b = 101.4988, \quad t_c = 123.5008,$$

$$t_d = 204.7472, \quad t_e = 203.7481, \quad t_e = 120.7488.$$

Under this link-toll policy, the user costs, after the imposition of these tolls were: for O/D pair w_1 126.4981 and for O/D pair w_2 205.7482.

Example 6: Example 3 Revisited

I returned to Example 3 and constructed a link-toll policy using formula (22) and the computed sustainable S-O flow pattern for Example 3. Recall that Example 3 was constructed from Example 2 by increasing the initial license allocation. Note that in this example the paths consist of single links.

Solving for the link tolls as in (22) yields the following toll policy:

$$t_a = 73.6661, \quad t_b = 49.6649, \quad t_c = 71.6638,$$

$$t_d = 149.8554, \quad t_e = 112.2744, \quad t_e = 81.1081.$$

Under this link-toll policy, one has that the user costs, after the imposition of tolls are: for O/D pair w_1, the user cost plus the toll on the path for each of the three paths connecting O/D pair w_1 is equal to 74.66; whereas for O/D pair w_2 the user cost plus the toll on each path is equal to 152.38. Hence, the computed sustainable S-O pattern is also U-O, after the imposition of tolls as described.

Example 7: Example 4 Revisited

I then considered Example 4, in which there are four networks, again, in which each path consists of an individual link. An application of formula (22) into which the sustainable S-O solution was substituted, yielded the following link-toll policy:

$$t_a = 36.0632, \quad t_b = 43.5209,$$

$$t_c = 192.5484, \quad t_d = 144.2683,$$

$$t_e = 116.5568, \quad t_f = 104.5591,$$

$$t_g = 48.9118, \quad t_h = 81.9086.$$

The incurred user travel costs with this toll policy were: for O/D pair w_1 54.21; for O/D pair w_2 197.97; for O/D pair w_3 117.55, and for O/D pair w_4 83.91.

References

The references for this lecture are the textbook, *Sustainable Transportation Networks*, Anna Nagurney (2000), Edward Elgar Publishers, Chelthenham, England and the following citations.

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