Emission Paradoxes in Transportation Networks

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Introduction

In this lecture, I identify several distinct paradoxical phenomena that can occur in congested urban transportation networks as regards the total emissions generated.

In particular, I show that so-called "improvements" to the transportation network may actually have an opposite effect, that is, they may induce increases in the total emissions generated.

The emission paradoxes are presented through specific illustrative examples and reinforce the fundamental importance of the structure of the transportation networks themselves as well as that of the travelers' behavior on the networks in the study and analysis of environmental issues.

Hence, in order to be able to appropriately evaluate the effects of environmental policies aimed at pollution reduction one must consider such critical network parameters as: the network topology, the user travel cost structure, the travel demand structure, and the behavior of the travelers on the network, in addition to such environmental factors as emissions.

In particular, through specific examples, the following counterintuitive phenomena are identifies:

1. the addition of a road may result in an increase in total emissions with no change in travel demand;

2. a decrease in the travel demand may result in an increase in total emissions;

3. the improvement of a road in terms of travel cost reduction may result in an increase in total emissions without a change in the travel demand;

4. a transfer of travel demand from a mode with higher total emissions to a mode with lower total emissions on a network may result in an increase in the total emissions; and

5. making travel less attractive between an origin/destination pair as revealed through its travel disutility function may result in an increase in total emissions.

Summary of emission paradox examples

Paradox 1	
• Addition of road $\Rightarrow$ with no change in demand	total emissions can $\uparrow$
Paradox 2	
• A decrease in travel demand $\Rightarrow$	total emissions can $\uparrow$
Paradox 3	
• A road improvement$\Rightarrow$ with no change in demand	total emissions can $\uparrow$
Paradox 4	
• A mode transfer from a $\Rightarrow$ higher emitting mode to a lower one	total emissions can $\uparrow$
Paradox 5	
• Making travel less attractive $\Rightarrow$ for an O/D pair	total emissions can $\uparrow$

There is a growing body of literature based, in part, on models developed by the National Environmental Protection Agency (NEPA), that deals with the conversion of traffic data into an account of the pollutants emitted.

The key in the estimation of air pollution due to vehicular emissions is the relationship that the volume of emissions is equal to the product of an emission factor and the vehicle activity (or link load) (cf. DeCorla-Souza et al. 1995; Anderson et al. 1996; and Allen 1996).

Specifically, in this course, the emissions from vehicles traveling on different links on paths between origin/destination pairs are assumed to pollute at a certain receptor point.

Let h_a denote the emission factor on link $a \in L$, which is assumed to be given for all links; such emission factors are utilized in modeling efforts throughout this book.

For example, as noted in DeCorla-Souza et al. (1995), emission factors vary depending on several characteristic travel activities, such as: vehicle type and age mix, vehicle speed, trip length distribution, operating mode, and ambient temperature.

Variables affecting vehicular emissions

- Age distribution of vehicular fleet
- Number of miles each age cohort is driven
- Average emissions by cohort
- Rate of decay of emissions controls
- Degree of tampering
- Inspection and maintenance effectiveness
- Ambient temperature
- Driving speed
- Frequency of open-loop (off-cycle) operation
- Number of cold starts
- Idling time
- Type of fuel utilized

Source: Compiled by Hall (1995)

Paradoxes in Single Modal, Fixed Demand Networks

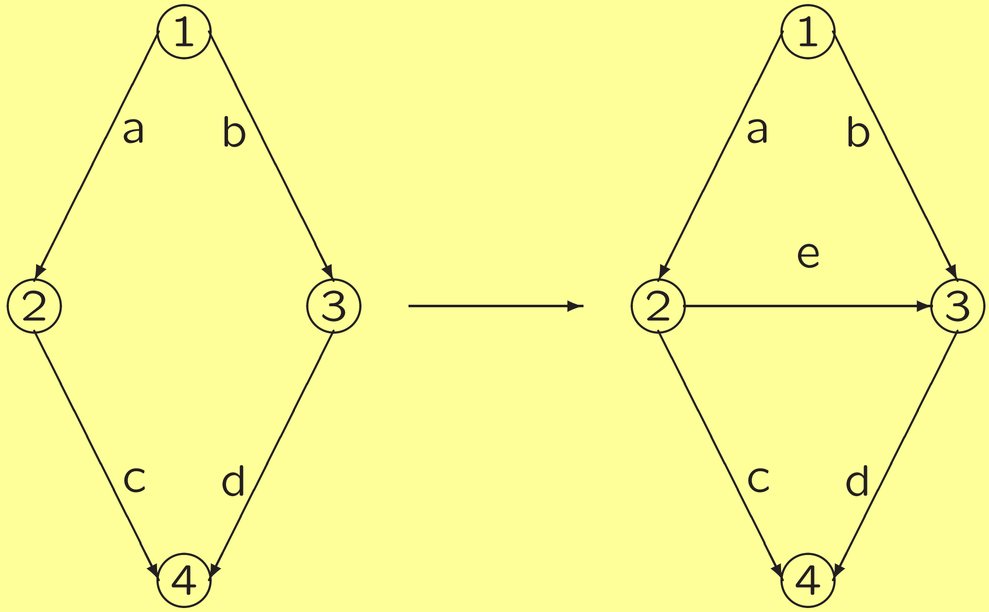
Here, I demonstrate, through three network examples, distinct counterintuitive phenomena which can occur as regards the total emissions generated. The models are single modal, fixed, travel demand models in which it is assumed that travelers act independently in a user-optimized fashion as described earlier. I assume here that the emission factors associated with the links are given.

Paradox 1

I identify, through a specific transportation network example, that the addition of a link to a network may result in an increase in the total emissions with no change in the travel demand. This example may be viewed as an analogue of the Braess (1968) Paradox in which the addition of a road makes all travelers worse off in terms of the travel cost associated with traveling from their origin to their destination.

In the case of emissions, however, I show that if one adds a road to a network then all the travelers may be worse off not only in terms of their travel cost but the total emissions may also increase without any change in the travel demand.

For completeness, I first recall the Braess example and then show that the paradox carries through also in terms of emissions. For easy reference, see the two networks depicted in the Figure.



Network topology for Paradox 1: Braess network

Assume a network as the first network depicted in the Figure in which there are four nodes: 1, 2, 3, 4; four links: a, b, c, d, and a single O/D pair $w_1 = (1, 4)$. There are, hence, two paths available to travelers between this O/D pair: $p_1 = (a, c)$ and $p_2 = (b, d)$.

The user link travel cost functions are:

$$c_a(f_a) = 10f_a,\ c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50,\ c_d(f_d) = 10f_d,$$

and the fixed travel demand $d_{w_1} = 6$.

It is easy to verify that the equilibrium path flows are:

$$x^*_{p_1} = 3, \quad x^*_{p_2} = 3,$$

the equilibrium link loads are:

$$f^*_a = 3, \quad f^*_b = 3, \quad f^*_c = 3, \quad f^*_d = 3,$$

with associated equilibrium user path travel costs:

$$C_{p_1} = 83, \quad C_{p_2} = 83.$$

Assume now that the emission factors on the links are: $h_a = h_b = h_c = h_d = 0.1$. The total emissions generated are, hence, equal to: $h_a f_a^* + h_b f_b^* + h_c f_c^* + h_d f_d^*$=0.3 + 0.3 + 0.3 + 0.3=1.2.

Suppose now that, as depicted in the Figure, a new link e, joining node 2 to node 3, is added to the original network, with user cost $c_e(f_e) = f_e + 10$. The addition of this link creates a new path $p_3 = (a, e, d)$ that is available to the travelers.

The travel demand d_{w_1} remains at 6 units of flow.

Note that the original flow distribution pattern with path flows given by: $x_{p_1} = 3$ and $x_{p_2} = 3$ is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

Hence, users from paths p_1 and p_2 would switch to path p_3.

The equilibrium flow pattern on the new network is:

$$x^*_{p_1} = 2, \quad x^*_{p_2} = 2, \quad x^*_{p_3} = 2,$$

with equilibrium link loads:

$$f^*_a = 4, \quad f^*_b = 2, \quad f^*_c = 2, \quad f^*_d = 4, \quad f^*_e = 2,$$

and with associated equilibrium path travel costs:

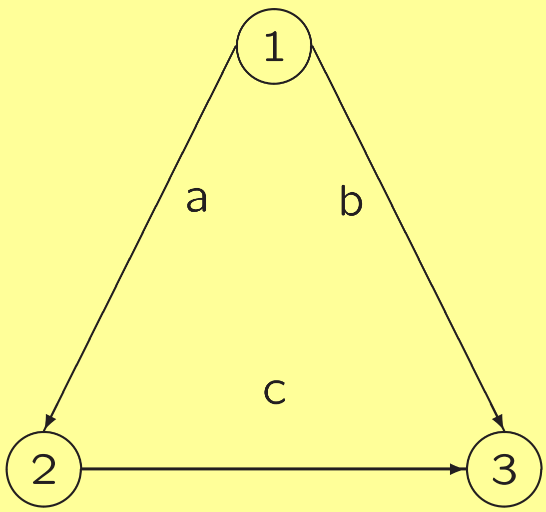
$$C_{p_1} = 92, \quad C_{p_2} = 92.$$

Note that the travel cost increased for every user of the network from 83 to 92!

Indeed, one can verify that *any* reallocation of the path flows would yield a higher travel cost on a path and, hence, this flow pattern is user-optimized.

Assume now that the emission factor on the new link e is $h_e = 0.1$. Indeed, the total emissions generated in the new network without any change in the travel demand are now equal to $h_a f_a^* + h_b f_b^* + h_c f_c^* + h_d f_d^* + h_e f_e^*$ $= 0.4 + 0.2 + 0.2 + 0.4 + 0.2 = 1.4$, which is greater than the total generated in the original network.

Hence, the addition of the new road makes everyone worse off in terms of both travel cost and the total emissions generated.



Network topology for Paradox 2

Paradox 2

I now demonstrate, through another transportation network example, that a decrease in travel demand associated with an origin/destination pair may result in an increase in emissions.

Consider the transportation network depicted in the next Figure, consisting of three nodes: 1, 2, 3, and three links: a, b, c.

There are two origin/destination pairs: $w_1 = (1, 2)$ and $w_2 = (1, 3)$. The path connecting O/D pair w_1, p_1, consists of the single link a. The paths connecting O/D pair w_2 are: $p_2 = (a, c)$ and $p_3 = b$.

The travel demands in the original problem are: $d_{w_1} = 1$ and $d_{w_2} = 2$.

The user link travel cost functions are:

$$c_a(f_a) = f_a + 1, \quad c_b(f_b) = f_b + 4, \quad c_c(f_c) = f_c + 1.$$

The emission factors on the links are: $h_a = 0.01$, $h_b = 0.01$, and $h_c = 0.5$.

The traffic equilibrium path flow pattern is:

$$x^*_{p_1} = 1; \quad x^*_{p_2} = 1, \quad x^*_{p_3} = 1,$$

with induced link load pattern:

$$f^*_a = 2, \quad f^*_b = f^*_c = 1.$$

The path travel costs are: For O/D pair w_1: $C_{p_1} = 3$, and for O/D pair w_2: $C_{p_2} = C_{p_3} = 5$.

The total emissions generated are equal to: $h_a f^*_a + h_b f^*_b + h_c f^*_c = .02 + .01 + .5 = .53$.

Consider now a decrease in travel demand associated with O/D pair w_1 with the new demand $d_{w_1} = 0.5$ and all other data remain the same.

The new traffic equilibrium path flow pattern is:

$$x^*_{p_1} = 0.5, \quad x^*_{p_2} = 1.6666..., \quad x^*_{p_3} = 0.833...,$$

with induced equilibrium link load pattern:

$$f^*_a = 1.5666..., \quad f^*_b = 0.833..., \quad f^*_c = 1.166...$$

The new path travel costs are: For O/D pair w_1: $C_{p_1} = 2.666...$, and for O/D pair w_2: $C_{p_2} = C_{p_3} = 4.833....$

The total emissions now generated by the new equilibrium pattern are equal to: $h_a f^*_a + h_b f^*_b + h_c f^*_c = 0.0166... + 0.00833... + 0.583 = 0.6079....$

Hence, the total emissions have increased from 0.53 to 0.6079... even though the travel demand has decreased. Note that the travel cost, however, associated with traveling between each O/D pair has decreased.

Paradox 3

It is now shown that an improvement in the cost structure in the network, through the reduction of the user travel cost on a link, may result in an increase in total emissions. Consider now the transportation network depicted in the subsequent Figure consisting of two nodes: 1, 2, and two links: a, b.

The user link travel cost functions are given by:

$$c_a(f_a) = f_a + 5, \quad c_b(f_b) = f_b + 9.$$

There is a single origin/destination pair $w_1 = (1, 2)$ with an associated travel demand $d_{w_1} = 6$. There are two paths: $p_1 = a$ and $p_2 = b$.

The emission factors are: $h_a = 0.5$ and $h_b = 0.4$.

The traffic equilibrium path flow pattern is:

$$x^*_{p_1} = 5, \quad x^*_{p_2} = 1,$$

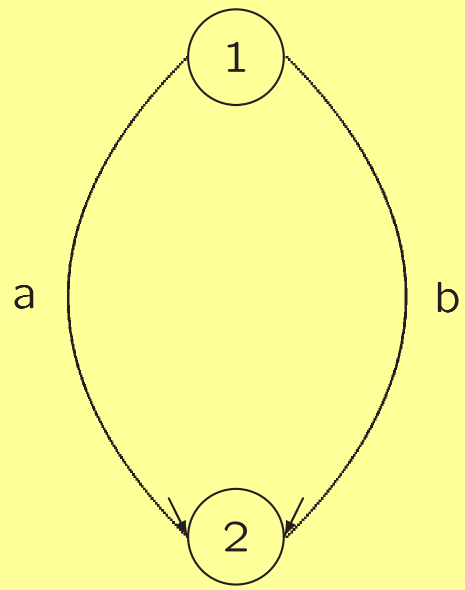
which induces the equilibrium link load pattern:

$$f^*_a = 5, \quad f^*_b = 1,$$

and travel costs on the paths:

$$C_{p_1} = C_{p_2} = 10.$$

The total emissions generated are, hence, equal to: $h_a f^*_a + h_b f^*_b = 2.5 + 0.4 = 2.9.$



Network topology for Paradox 3

Consider now the following improvement in the cost structure, which may represent, for example, a repaving of the road or link a. Let the new cost function on link a be given by:

$$c_a(f_a) = f_a + 4.$$

Assume that all the other data are as previously described for this example.

The new traffic equilibrium path flow pattern is:

$$x^*_{p_1} = 5.5, \quad x^*_{p_2} = 0.5,$$

with induced equilibrium link load pattern:

$$f^*_a = 5.5, \quad f^*_b = 0.5.$$

The new user travel costs on the paths are:

$$C_{p_1} = C_{p_2} = 9.5.$$

The total emissions generated by the new equilibrium flow pattern are equal to: $h_a f^*_a + h_b f^*_b = 2.75 + 0.20 = 2.95$.

Hence, with no change in travel demand, an improvement in a road resulted in an increase in total emissions from 2.9 to 2.95!

In this network what occurred was that the travelers switched to the reduced cost path which had a higher emission factor, resulting in an overall increase in emissions.

A Paradox in Multimodal, Fixed Demand Networks

I now turn to the presentation of a paradox which may occur in multimodal traffic networks with fixed travel demands. Multimodal networks may include such modes as private cars as well as public transit, for example. As discussed earlier, I assume that the travelers behave in a user-optimized fashion.

Of course, the paradoxes that have been presented for single modal transportation networks are also relevant in the case of multimodal traffic networks.

Paradox 4

Specifically, I present an example which illustrates the following phenomenon: the reallocation of travelers from a mode with higher total emissions to that of a mode with lower total emissions may result in an increase in the total emissions.

The network topology in the example is identical to that of the network depicted in the preceding Figure but now there are two modes traveling on the network, rather than a single mode.

Let the superscript of 1 on the demands, flows, costs, and emission factors refer to mode 1 and the superscript of 2 to mode 2. Denote the single origin/destination pair $w_1 = (1,2)$ and let path $p_1 = a$ and path $p_2 = b$. Assume that the travel demands for the modes are given by:

$$d^1_{w_1} = 10, \quad d^2_{w_1} = 5.$$

The user link travel cost functions are:

$$c^1_a(f^1_a) = f^1_a + 5, \quad c^1_b(f^1_b) = f^1_b + 5,$$

$$c^2_a(f^2_a) = f^2_a + 10, \quad c^2_b(f^2_b) = f^2_b + 5.$$

The emission factors associated with the links and modes are:

$$h^1_a = 0.2, \quad h^1_b = 0.2, \quad h^2_a = 0.4, \quad h^2_b = 0.1.$$

The multimodal traffic network equilibrium conditions, which are a generalization of the single modal ones, simply require that the conditions also hold for each mode separately where the conservation of flow equations are also satisfied for each mode individually.

It is easy to verify that the traffic equilibrium pattern is given by:

$$x^{1\,*}_{p_1} = 5, \quad x^{1\,*}_{p_2} = 5, \quad x^{2\,*}_{p_1} = 0, \quad x^{2\,*}_{p_2} = 5,$$

with associated equilibrium link load pattern:

$$f^{1*}_a = 5, \quad f^{1*}_b = 5, \quad f^{2*}_a = 0, \quad f^{2*}_b = 5,$$

and user path travel costs:

$$C^1_{p_1} = C^1_{p_2} = 10, \quad C^2_{p_1} = C^2_{p_2} = 10.$$

The total emissions due to travelers using mode 1 are: $h^1_a f^{1*}_a + h^1_b f^{1*}_b = 1 + 1 = 2$, whereas the total emissions due to travelers using mode 2 are: $h^2_a f^{2*}_a + h^2_b f^{2*}_b = 0 + 0.5 = 0.5$ with the total emissions generated equal to 2.5.

Consider now the transfer of 2.5 units of travel demand from mode 1 to mode 2 (which may correspond, for example, to a reallocation of a subset of travelers by car to public transit) so that the new travel demands are given by:

$$d^1_{w_1} = 7.5, \quad d^2_{w_1} = 7.5.$$

The new traffic equilibrium pattern is then:

$$x^{1\,*}_{p_1} = 3.75, \quad x^{1\,*}_{p_2} = 3.75, \quad x^{2\,*}_{p_1} = 1.25, \quad x^{2\,*}_{p_2} = 6.25,$$

with associated link load pattern:

$$f^{1*}_a = 3.75, \quad f^{1*}_b = 3.75, \quad f^{2*}_a = 1.25, \quad f^{2*}_b = 6.25,$$

and user path travel costs:

$$C^1_{p_1} = C^1_{p_2} = 8.75, \quad C^2_{p_1} = C^2_{p_2} = 11.25.$$

The total emissions are now equal to 2.625, which exceed those prior to the mode transfer.

A Paradox in Elastic Demand Networks

I now turn to elastic demand traffic networks, but in the case of a single mode of transportation, and demonstrate, through an example, that decreasing the attractiveness associated with traveling between an origin/destination pair may result in an increase in total emissions.

It is assumed that travelers behave in a user-optimized manner.

Paradox 5

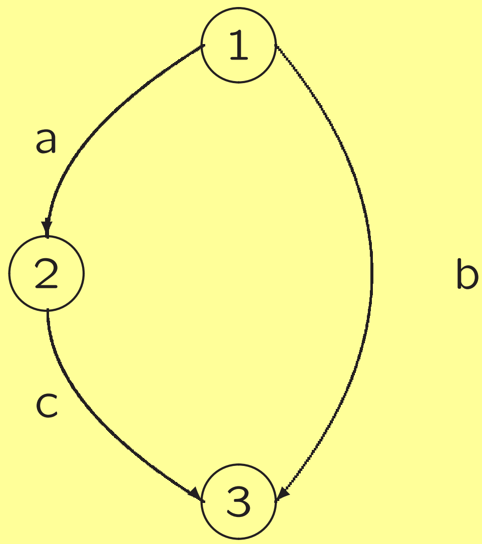
Consider the transportation network depicted in the next Figure in which there are three nodes: 1, 2, 3, and three links: a, b, c. Assume two origin/destination pairs of travel: $w_1 = (1, 2)$ and $w_2 = (1, 3)$. The travel disutility functions associated with traveling between the origin/destination pairs are:

$$\lambda_{w_1}(d_{w_1}) = -d_{w_1} + 4, \quad \lambda_{w_2}(d_{w_2}) = -d_{w_2} + 7.$$

The paths are: For O/D pair w_1: $p_1 = a$, and for O/D pair w_2: $p_2 = (a, c)$, and $p_3 = b$.

The user link travel cost functions are:

$$c_a(f_a) = f_a + 1, \quad c_b(f_b) = f_b + 4, \quad c_c(f_c) = f_c + 1.$$



Network topology for Paradox 5

The emission factors are: $h_a = 0.02$, $h_b = 0.02$, and $h_c = 0.6$.

The traffic network equilibrium path flow and demand pattern is:

$$x^*_{p_1} = 1, \quad x^*_{p_2} = 1, \quad x^*_{p_3} = 1,$$

and

$$d^*_{w_1} = 1, \quad d^*_{w_2} = 2,$$

with induced link load pattern:

$$f^*_a = 2, \quad f^*_b = 1, \quad f^*_c = 1.$$

The path travel cost and travel disutility for O/D pair w_1 are:

$$C_{p_1} = \lambda_{w_1} = 3,$$

and for O/D pair w_2:

$$C_{p_2} = C_{p_3} = \lambda_{w_2} = 5.$$

The total emissions generated are:

$$h_a f^*_a + h_b f^*_b + h_c f^*_c = 0.04 + 0.02 + 0.6 = .66.$$

Now, suppose that the travel disutility associated with O/D pair w_1 is modified as follows:

$$\lambda_{w_1}(d_{w_1}) = -d_{w_1} + 3.5$$

with all other data remaining as above.

The new traffic network equilibrium path flow and demand pattern is then:

$$x^*_{p_1} = 0.6875, \quad x^*_{p_2} = 1.1250, \quad x^*_{p_3} = 0.9375,$$

with

$$d^*_{w_1} = 0.6875, \quad d^*_{w_2} = 2.0625,$$

and new link loads given by:

$$f^*_a = 1.8125, \quad f^*_b = 0.9375, \quad f^*_c = 1.1250.$$

The path travel costs and travel disutilities are: For O/D pair w_1:

$$C_{p_1} = \lambda_{w_1} = 2.8125,$$

and for O/D pair w_2:

$$C_{p_2} = C_{p_3} = \lambda_{w_2} = 4.9375.$$

The total emissions now generated at the new equilibrium flow pattern are equal to 0.73, which exceed the emissions in the network prior to the travel disutility modification.

Hence, in this lecture, it has been demonstrated, through specific numerical examples, that so-called “improvements” to a transportation network may actually result in increases in the total emissions generated.

Such counterintuitive phenomena suggest that policy makers must use caution and careful quantitative analysis before any policy implementations for environmental emissions reductions in transportation networks.

Indeed, the network topology, the demand structure, the link travel cost structure, as well as the behavior of the travelers on the network must all be incorporated into any environmental modeling schema for transportation network environmental policy analysis.

The references below, in addition to the text, are relevant to this lecture.

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