A Space-Time Network for Telecommuting versus Commuting Decision-Making

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Background

The topic of transportation and its relationships to telecommunications has been a subject of research interest for close to forty years (cf. Memmott (1963), Jones (1973), Khan (1976), Nilles, at el. (1976), Albertson (1977), and Harkness (1977)).

Commuting, in particular, as one of the most common uses of transportation, and, telecommuting, made possible by the advent of technologies, have garnered special attention.

Indeed, telecommuting has been explored in many studies in terms of its potential impact on reducing the negative effects of transportation such as congestion and environmental degradation due to pollution.

For conceptual studies on this topic, see Salomon (1986), and Mokhtarian (1990); for empirical studies, see Nilles (1988), Mokhtarian (1991), and Mokhtarian, Handy, and Salomon (1995).

Recently, Nagurney, Dong, and Mokhtarian (2000) proposed an *integrated multiclass, multicriteria network equilibrium framework* for telecommuting versus commuting. They demonstrated that, through the use of appropriate criteria, and the extension of the concept of a network to include not only links associated with **physical transportation** but also links associated with telecommunications and, hence, **virtual transportation**, one could predict the number of decision-makers of each class that would telecommute versus commute.

Importantly, they allowed each class of decision-maker to weight the criteria of travel time, travel cost, and opportunity cost in an individual fashion.

Here we address the more general question as to how many days (given, say, a weekly horizon), one can expect classes of individuals to telecommute or to commute.

The crucial concept that we utilize in this paper is that of a **space-time network** in order to abstract the decision-making not only over space, but also over time, where time here is considered to be a finite horizon of periods, such as a work week consisting of five days. This lecture is based mainly on the work of Nagurney, Dong, and Mokhtarian (2002), "A Space-time network for telecommuting versus commuting decision-making," forthcoming in *Papers in Regional Science*.

The framework allows one to **predict how many decision-makers of each class will choose to telecommute on any given day of the week.** The functions representing the criteria on the links of the network capture the dependence of the criteria on the flows over both space and over time.

Given recent legislation that permits federal employees to select the telecommuting option, as well as a resurgence of interest on this topic (see Hafner (2000)), a theoretical framework that can model telecommuting versus commuting behavior over a time horizon is clearly also of practical relevance.

The number of telecommuters in the USA (see Glater (2001)) has risen in the past decade from 4 million to 23.6 million.

However, how often individuals choose to telecommute versus commute (and, typically, on what days) is still an open question both theoretically and empirically.

Some recent summary survey results concerning telecommuting intensity can be found in International Telework Association & Council (2000).

See also Shore (2000) for an overview of teleworking, which highlights that different workers may choose to telecommute a different number of days.

We note that multicriteria traffic network models were introduced by Quandt (1967) and Schneider (1968) and further developed by Dial (1979) who proposed an uncongested model and Dafermos (1981) who introduced congestion and derived an infinite-dimensional variational inequality formulation. Subsequent contributions were made by Leurent (1993, 1996), Marcotte and Zhu (1994, 1997), Marcotte, Nguyen, and Tanguay (1996), Leurent (1996), Marcotte (1998), and Dial (1999), Nagurney and Dong (2000). A thorough discussion as to the particulars of the contributions can be found in Nagurney and Dong (2000).

Furthermore, in contrast to the previous authors, we allow the weights associated with the criteria to be not only **class** but also **link-dependent**. Hence, certain classes of decision-makers may associate a higher or a lower weight with a criterion depending on the period or day and/or whether the link is a telecommuting link or not.

The Multiperiod, Multiclass, Multicriteria Model

Let $\mathcal{T}$ denote the finite-time horizon with, typically, $\mathcal{T}$ being set equal to 5 working days of the week, and use the index $\tau = 1, 2, \ldots, \mathcal{T}$, to denote the time period or day.

The Space-Time Network

Assume that there are n locations with a subset of the locations corresponding to residential locations, employment locations, teleworking centers, as well as intermediate locations for transportation (or telecommunications) purposes, respectively.

The space-time network will consist of $\mathcal{T}$ subnetworks with each subnetwork τ corresponding to the choices available within time period τ.

Index the locations for a subnetwork τ of the space-time network corresponding to time period τ as follows: $(1, \tau), \ldots, (n, \tau)$; with τ ranging from 1 through $\mathcal{T}$.

Assume that the number of locations within each subnetwork is fixed at n, which is also the number of nodes in each subnetwork. Hence, the total number of nodes in the space-time network will be equal to $\mathcal{T}n$.

We now discuss the links on the space-time network.

The links will be links on each subnetwork plus connecting links which join two successive subnetworks.

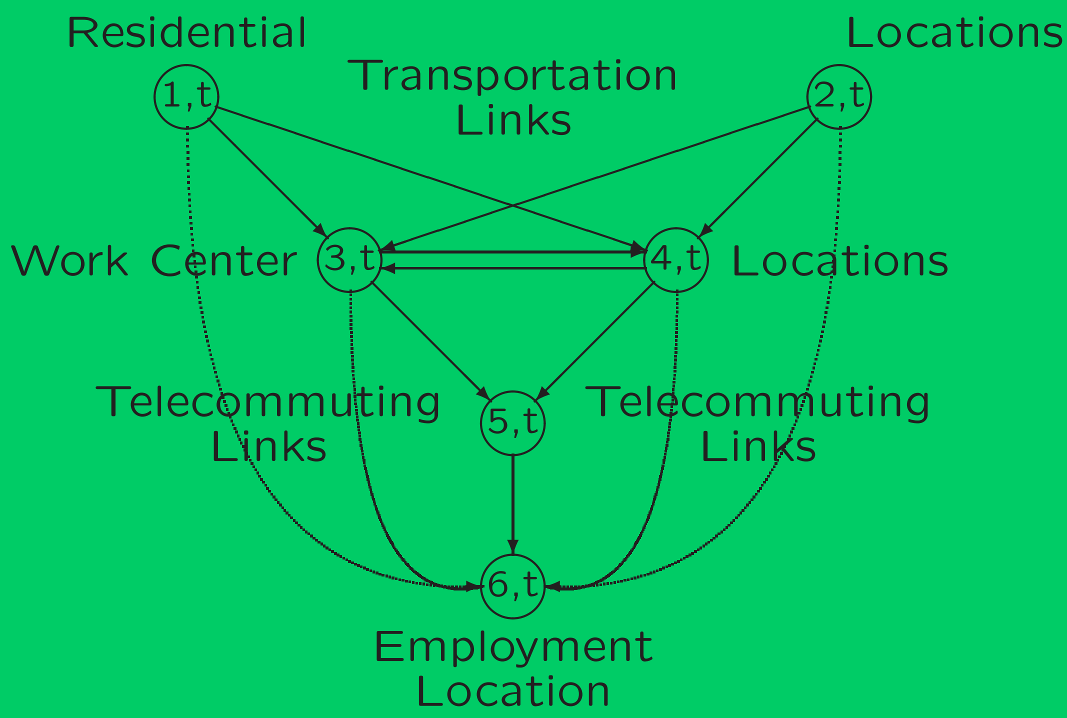
A link in our framework can represent either a **physical** link corresponding to a classical transportation link or a **virtual** link corresponding to a telecommunication link which decision-makers can select as a telecommuting option.

A sequence of links from a residential location to an employment location within a time period is termed a "route" and is denoted by r.

Note that, as in Dial (1996), a route can represent a mode of transportation in this context (for example, public or private).

Furthermore, in our framework, since a mode of transportation includes telecommunications, a route can also represent a mode of telecommuting.

A space-time network for a time horizon $\mathcal{T}$ consists of $\mathcal{T}$ copies of a subnetwork τ with τ ranging from 1 through $\mathcal{T}$ to denote the subnetworks plus additional links to connect the subnetwork within a time period with the subsequent subnetwork.

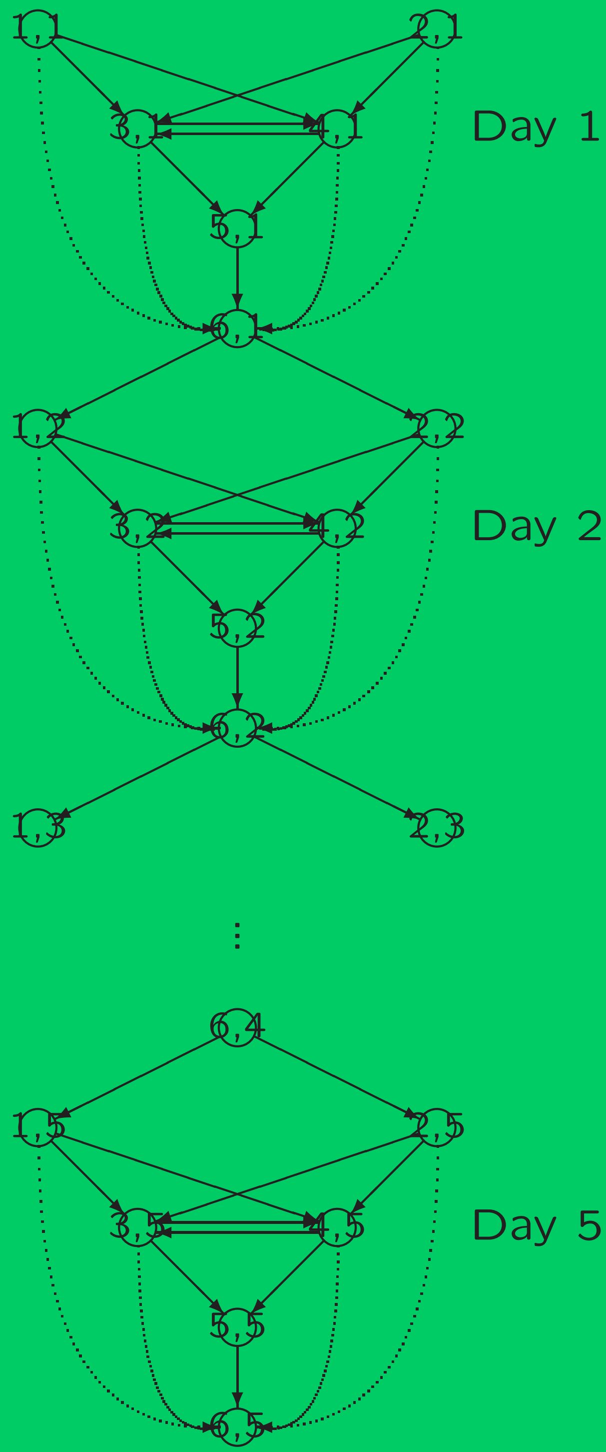


A Subnetwork for the Conceptualization of Commuting versus Telecommuting within Time Period t

A *path* is used to represent decisions over space and time and consists of a sequence of links (assumed acyclic) from a residential location node in period 1 to an employment location node in time period $\mathcal{T}$.

A residential location node in time period 1 is termed, henceforth, an *origin node* and the employment location node in time period $\mathcal{T}$ is a *destination node* with such a pair of nodes referred to as an *origin/destination* (O/D) pair.

A path consists of a sequence of routes, which a decision-maker selects joined by the "connecting" links between the successive subnetworks.



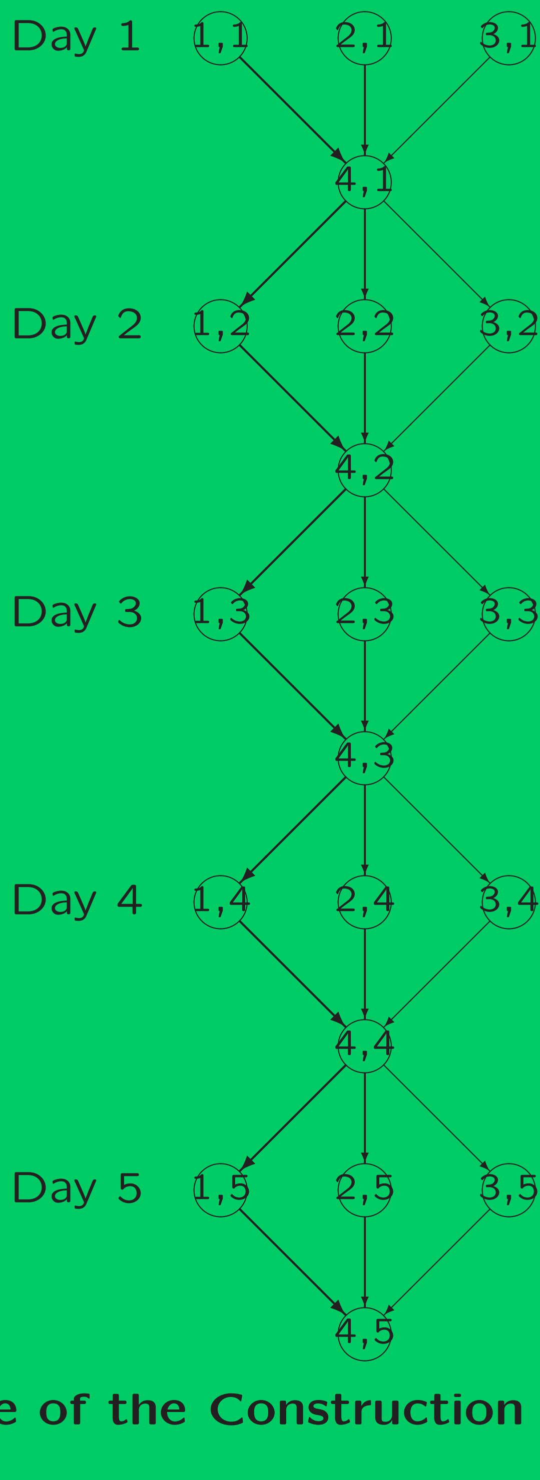
A Space-Time Supernetwork for an Example with a 5 Day Time Horizon

The space-time network, hence, is a general network $G = [\mathcal{N}, \mathcal{L}]$, (but of special, multiperiod structure), where $\mathcal{N}$ denotes the set of nodes in the network and $\mathcal{L}$ the set of directed links.

Let a denote a link of the network connecting a pair of nodes and let p denote a path, consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes.

There are N links in the space-time network and N_P paths.

Let Ω denote the set of J O/D pairs. The set of paths connecting the O/D pair ω is denoted by P_ω and the entire set of paths in the network by P.



Example of the Construction of a Path

The Conservation of Flow Equations

Assume that there are k classes of decision-makers in the network with a typical class denoted by i.

The Class Flows

Let f_a^i denote the flow of class i on link a and let x_p^i denote the nonnegative flow of class i on path p. The relationship between the link loads by class and the path flows is:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, \quad \forall a \in \mathcal{L},$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

The Total Flows

Let f_a denote the total flow on link a, where

$$f_a = \sum_{i=1}^{k} f_a^i, \quad \forall a \in \mathcal{L}.$$

Group the class link flows into the kN-dimensional column vector $\tilde{f}$ with components: $\{f_a^1, \ldots, f_n^1, \ldots, f_a^k, \ldots, f_N^k\}$ and the total link flows: $\{f_a, \ldots, f_N\}$ into the N-dimensional column vector f.

Also, group the class path flows into the kN_P-dimensional column vector $\tilde{x}$ with components: $\{x_{p_1}^1, \ldots, x_{p_{N_P}}^k\}$.

The travel demand associated with origin/destination (O/D) pair ω and class i will be denoted by d^i_ω. Group the travel demands into a column vector $d \in R^{kJ}$.

The Demand

The travel demands must satisfy the following conservation of flow equations:

$$d^i_\omega = \sum_{p \in P_\omega} x^i_p, \quad \forall i, \forall \omega.$$

The Generalized Cost Structure

The Travel Time Functions

Assume, as given, a travel time function t_a associated with each link a in the network, where

$$t_a = t_a(f), \quad \forall a \in \mathcal{L},$$

where the function represents the time that it takes to traverse link a.

The Travel Cost Functions

The travel cost function c_a associated with each link a, is assumed given by

$$c_a = c_a(f), \quad \forall a \in \mathcal{L},$$

with both these functions assumed to be continuous.

The Opportunity Cost Functions

In addition, in order to capture the opportunity costs associated with commuting versus telecommuting trade-offs, we also introduce an opportunity cost o_a associated with each link in the network, where

$$o_a = o_a(f), \quad \forall a \in \mathcal{L}.$$

We assume that each class of decision-maker i has his own perception of the trade-offs among travel time, travel cost, and opportunity cost associated with each link a, which are represented, respectively, by the non-negative weights w^i_{1a}, w^i_{2a}, and w^i_{3a}.

The Generalized Link Cost Function of a Class

The generalized cost function of class i associated with link a, denoted by u^i_a, is defined as:

$$u^i_a = w^i_{1a} t_a + w^i_{2a} c_a + w^i_{3a} o_a, \quad \forall i, \quad \forall a \in \mathcal{L}.$$

We may write

$$u^i_a = u^i_a(\tilde{f}), \quad \forall i, \quad \forall a \in \mathcal{L},$$

and group the generalized link costs into the kN-dimensional row vector u with components:
$\{u^1_a, \ldots, u^1_n, \ldots, u^k_a, \ldots, u^k_N\}$.

Generalized Path Cost of a Class

Let v^i_p denote the generalized cost of class i associated with traversing path p, where

$$v^i_p = \sum_{a \in \mathcal{L}} u^i_a(\tilde{f}) \delta_{ap}, \quad \forall i, \forall p.$$

The Behavioral Assumption

The behavioral assumption that we utilize here is based on the behavioral assumption underlying traffic network equilibrium problems, assuming user-optimizing behavior (see Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969)), in that we assume that each class of decision-maker in the space-time network selects (subject to constraints) his "travel" path so as to minimize the generalized cost on the path, given that all other decision-makers have made their choices.

Note that paths in the space-time framework correspond to decisions not only over space but also over time. Hence, the use of this behavioral assumption is reasonable and classical, in a sense, but, at the same time, novel.

Network Equilibrium Conditions

For each class i, for all O/D pairs $\omega \in \Omega$, and for all paths $p \in P_\omega$, the flow $\tilde{x}^*$ is said to be in equilibrium if the following condition holds:

$$v_p^i(\tilde{x}^*) \begin{cases} = \lambda_\omega^i, & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_\omega^i, & \text{if} \quad x_p^{i^*} = 0. \end{cases}$$

Theorem: Variational Inequality Formulations

A multicriteria, multiclass path flow pattern $\tilde{x}^* \in \mathcal{K}^1$ is a network equilibrium, that is, satisfies the equilibrium conditions, if and only if it satisfies the variational inequality problem:

Path Flow Formulation:

$$\sum_{i=1}^{k} \sum_{\omega \in \Omega} \sum_{p \in P_\omega} v_p^i(\tilde{x}^*) \times (x_p^i - x_p^{i\,*}) \geq 0, \quad \forall \tilde{x} \in \mathcal{K}^1,$$

where $\mathcal{K}^1 \equiv \{\tilde{x} | \tilde{x} \geq 0$, and satisfies the demand, or, in standard variational inequality form:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $F \equiv v$, $X \equiv \tilde{x}$, and $\mathcal{K} \equiv \mathcal{K}^1$, and $\langle \cdot, \cdot \rangle$ denotes the inner product in kN_P-dimensional space, or, equivalently, $\tilde{f}^* \in \mathcal{K}^2$ is an equilibrium link load pattern if and only if it satisfies the variational inequality problem:

Link Flow Formulation:

$$\langle u(\tilde{f}^*)^T, \tilde{f} - \tilde{f}^* \rangle \geq 0, \quad \forall \tilde{f} \in \mathcal{K}^2,$$

where $\mathcal{K}^2 \equiv \{\tilde{f} | \exists \tilde{x} \geq 0$, and satisfying the conservation of flow equations, or, in standard variational inequality form:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $F \equiv u$, $X \equiv \tilde{f}$, $\mathcal{K} \equiv \mathcal{K}^2$, and $\langle \cdot, \cdot \rangle$ denotes the inner product in kN-dimensional Euclidean space.

Qualitative Properties

We now derive some qualitative properties of the solutions to the variational inequalities. Note that the feasible set $\mathcal{K}^1$ underlying the variational inequality is a compact set because of the fixed travel demand as is the feasible set $\mathcal{K}^2$ underlying the link-based variational inequality. Moreover, the functions that enter the variational inequality problems and are continuous.

Theorem: Existence

Let t, c, and o be given continuous functions. Then both the variational inequalities have at least one solution.

Consider a generalized cost function of the form:

$$u_a^i = \psi_a^i t_a + \xi_a^i c_a + (1 - \psi_a^i - \xi_a^i) o_a, \quad \forall a, i,$$

where

$$t_a = g_a(f) + \alpha_a, \; c_a = g_a(f) + \beta_a, \; o_a = g_a(f) + \gamma_a, \; \forall a \in \mathcal{L}.$$

Assume now that t, c, and o are each strictly monotone in f, that is,

$$\langle (t(f^1) - t(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^2, \quad f^1 \neq f^2,$$

$$\langle (c(f^1) - c(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^2, \quad f^1 \neq f^2,$$

and

$$\langle (o(f^1) - o(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^2, \quad f^1 \neq f^2.$$

Then we have the following:

Theorem: Uniqueness of the Total Link Flow Pattern in a Special Case

The total link load pattern f^* induced by a solution $\tilde{f}^*$ to variational inequality in the case of generalized cost functions u of the form above, is guaranteed to be unique if the travel time, the travel cost, and the opportunity cost functions are each strictly monotone increasing in f.

Theorem: Monotonicity in a Special Case

Assume that the generalized cost functions u are as above with the travel time, the travel cost, and the opportunity cost functions differing on a given link only by the fixed cost terms as above. Assume also that these functions are monotone increasing in f. Then the function that enters the variational inequality problem governing the multiperiod, multiclass, multicriteria traffic network equilibrium model is monotone.

Theorem: Lipschitz Continuity

If the generalized cost functions u have bounded first-order derivatives, then the function, $F(X)$, that enters the variational inequality is Lipschitz continuous, that is, there exists a positive constant L, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}^2.$$

The Algorithm

The statement of the modified projection method is as follows, where $calk$ denotes an iteration counter:

Modified Projection Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\mathcal{I} = 1$ and let γ be a scalar such that $0 < \gamma \leq \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. Korpelevich (1977)).

Step 1: Computation

Compute $\bar{X}^{\mathcal{I}}$ by solving the variational inequality subproblem:

$$\langle (\bar{X}^{\mathcal{I}} + \gamma F(X^{\mathcal{I}-1})^T - X^{\mathcal{I}-1})^T, X - \bar{X}^{\mathcal{I}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 2: Adaptation

Compute $X^{\mathcal{I}}$ by solving the variational inequality subproblem:

$$\langle (X^{\mathcal{I}} + \gamma F(\bar{X}^{\mathcal{I}})^T - X^{\mathcal{I}-1})^T, X - X^{\mathcal{I}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 3: Convergence Verification

If max $|X_l^{\mathcal{I}} - X_l^{\mathcal{I}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{I} =: \mathcal{I} + 1$, and go to Step 1.

Modified Projection Method for the Solution of the Variational Inequality

Step 0: Initialization

Set $\tilde{f}^0 \in \mathcal{K}^2$. Let $\mathcal{I} = 1$ and set γ such that $0 < \gamma <\leq \frac{1}{L}$, where L is the Lipschitz constant for the problem.

Step 1: Computation

Compute $\bar{\tilde{f}}^{\mathcal{I}} \in \mathcal{K}^2$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}(\bar{f}_a^{i\mathcal{I}}+\gamma(u_a^i(\tilde{f}^{\mathcal{I}-1}))-f_a^{i\mathcal{I}-1})\times(f_a^i-\bar{f}_a^{i\mathcal{I}}) \geq 0, \forall \tilde{f} \in \mathcal{K}^2.$$

Step 2: Adaptation

Compute $\tilde{f}^{\mathcal{I}} \in \mathcal{K}^2$ by solving the variational inequality subproblem:

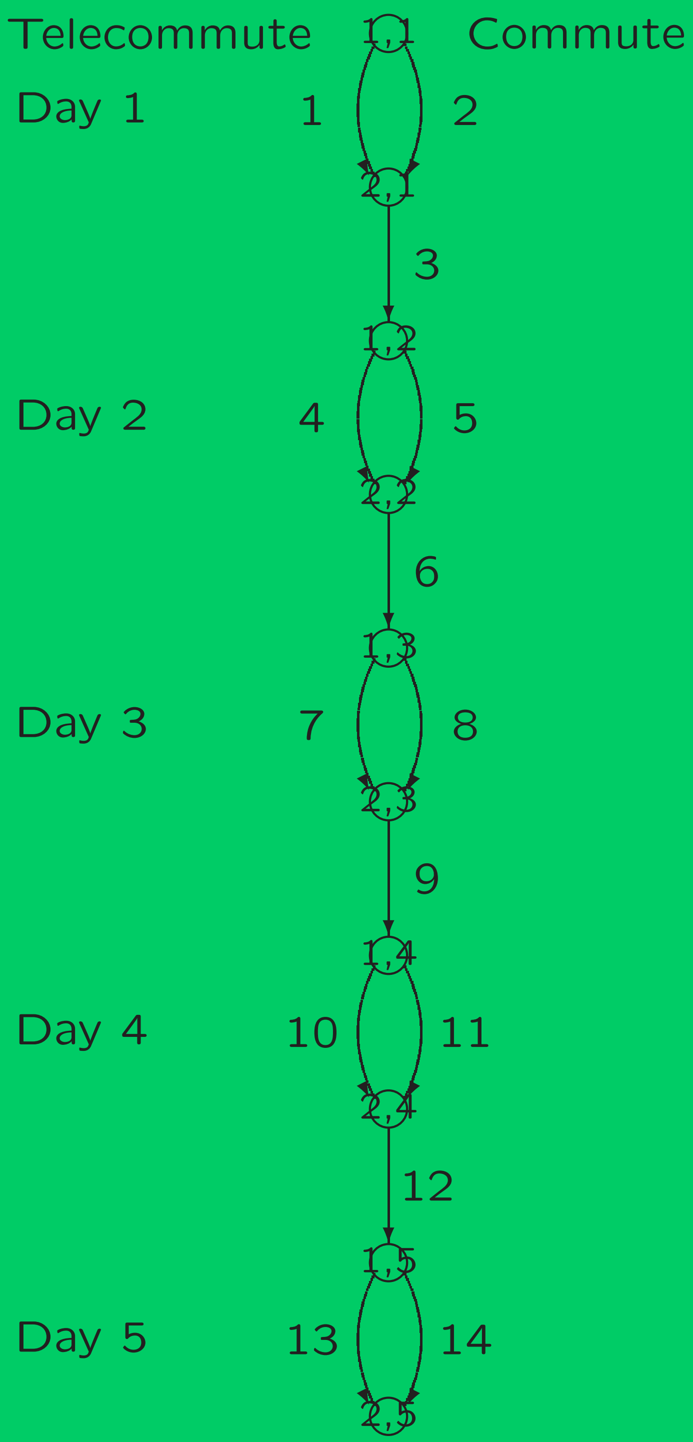
$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}(f_a^{i\mathcal{I}}+\gamma(u_a^i(\bar{\tilde{f}}^{\mathcal{I}})-f_a^{i\mathcal{I}-1})\times(f_a^i-f_a^{i\mathcal{I}}) \geq 0, \forall \tilde{f} \in \mathcal{K}^2.$$

Step 3: Convergence Verification

If $|f_a^{i\mathcal{I}} - f_a^{i\mathcal{I}-1}| \leq \epsilon$, for all $i = 1, \cdots, k$, and all $a \in \mathcal{L}$,with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{I} := \mathcal{I} + 1$, and go to Step 1.

Theorem: Convergence

Assume that u takes the previous form and is monotone increasing. Also, assume that u has bounded first-order derivatives. Then the modified projection method described above converges to the solution of the variational inequality.



Space-Time Network for the Examples

Numerical Examples

For the solution of the variational inequality subproblems we utilized the equilibration algorithm of Dafermos and Sparrow (1969).

The γ parameter in the modified projection method was set to .01, except where noted. The convergence criterion was that the absolute value of the flow for each class of decision-maker at two successive iterations was less than or equal to ϵ, with $\epsilon = .0001$.

Example 1

The first numerical example is simple but serves to illustrate interesting features. It consists of a single class of decision-maker with a single residential location and a single employment location. The time horizon $\mathcal{T} = 5$. Also, we assume that the choices available to the members of the class of decision-maker are expressed simply as whether to telecommute or to commute.

There are a total of 2^5 or 32 paths connecting the O/D pair $\omega = ((1,1),(2,5))$, and these are:

$p_1 = (1,3,4,6,7,9,10,12,13)$, $p_2 = (1,3,4,6,7,9,10,12,14)$,

$p_3 = (1,3,4,6,7,9,11,12,13)$, $p_4 = (1,3,4,6,7,9,11,12,14)$,

$p_5 = (1,3,4,6,8,9,10,12,13)$, $p_6 = (1,3,4,6,8,9,10,12,14)$,

$p_7 = (1,3,4,6,8,9,11,12,13)$, $p_8 = (1,3,4,6,8,9,11,12,14)$,

$p_9 = (1,3,5,6,7,9,10,12,13)$, $p_{10} = (1,3,5,6,7,9,10,12,14)$,

$p_{11} = (1,3,5,6,7,9,11,12,13)$, $p_{12} = (1,3,5,6,7,9,11,12,14)$,

$p_{13} = (1,3,5,6,8,9,10,12,13)$, $p_{14} = (1,3,5,6,8,9,10,12,14)$,

$p_{15} = (1,3,5,6,8,9,11,12,13)$, $p_{16} = (1,3,5,6,8,9,11,12,14)$,

$p_{17} = (2,3,4,6,7,9,10,12,13)$, $p_{18} = (2,3,4,6,7,9,10,12,14)$,

$p_{19} = (2,3,4,6,7,9,11,12,13)$, $p_{20} = (2,3,4,6,7,9,11,12,14)$,

$p_{21} = (2,3,4,6,8,9,10,12,13)$. $p_{22} = (2,3,4,6,8,9,10,12,13)$,

$p_{23} = (2,3,4,6,8,9,11,12,13)$, $p_{24} = (2,3,4,6,8,9,11,12,14)$,

$p_{25} = (2,3,5,6,7,9,10,12,13)$, $p_{26} = (2,3,5,6,7,9,10,12,14)$,

$p_{27} = (2,3,5,6,7,9,11,12,13)$, $p_{28} = (2,3,5,6,7,9,11,12,14)$,

$p_{29} = (2,3,5,6,8,9,10,12,13)$, $p_{30} = (2,3,5,6,8,9,10,12,14)$,

$p_{31} = (2,3,5,6,8,9,11,12,13)$, $p_{32} = (2,3,5,6,8,9,11,12,14)$.

The weights were constructed as follows: For class 1, the weights were: $w^1_{1,1} = .25$, $w^1_{2,1} = .25$, $w^1_{3,1} = 1.$, $w^1_{1,2} = .25$, $w_{2,2} = .25$, $w^1_{3,2} = 1.$, $w^1_{1,3} = .4$, $w^1_{2,3} = .4$, $w^1_{3,3} = 1.$, $w^1_{1,4} = .5$, $w^1_{2,4} = .5$, $w^1_{3,4} = 2.$, $w^1_{1,5} = .4$, $w^1_{2,5} = .5$, $w^1_{3,5} = 1.$, $w^1_{1,6} = .5$, $w^1_{2,6} = .3$, $w^1_{3,6} = 2.$, $w^1_{1,7} = .2$, $w^1_{2,7} = .4$, $w^1_{3,7} = 1.$, $w^1_{1,8} = .3$, $w^1_{2,8} = .5$, $w^1_{3,8} = 1.$, $w^1_{1,9} = .6$, $w^1_{2,9} = .2$, $w^1_{3,9} = 2.$, $w^1_{1,10} = .3$, $w^1_{2,10} = .4$, $w^1_{3,10} = 1.$, $w^1_{1,11} = .2$, $w^1_{2,11} = .7$, $w^1_{3,11} = 1.$, $w^1_{1,12} = .3$, $w^1_{2,12} = .4$, $w^1_{3,12} = 1.$, $w^1_{1,13} = .2$, $w^1_{2,13} = .3$, $w^1_{3,13} = 2.$, $w^1_{1,14} = .5$, $w^1_{2,14} = .2$, $w^1_{3,14} = .1$,

The travel time, travel cost, and opportunity cost functions on the links were as reported in Table 1.

The demand $d^1_\omega = 100$.

The Travel Time, Travel Cost, and Opportunity Cost Functions for the Links

Link a	$t_a(f)$	$c_a(f)$	$o_a(f)$
1	$.00005f_1^4+.5f_1+.1f_2+.2$	$.00005f_1^4+f_1+.5f_2+1$	$.4f_1+.2f_4+.2$
2	$.00005f_2^4+2f_2+f_1+1$	$.00005f_2^4+5f_2+2f_1+2$	$.2f_2+.1f_5+1$
3	0.0000	0.0000	0.0000
4	$.00005f_4^4+.5f_4+.1f_5+.2$	$.00005f_4^4+f_4+.5f_5+1$	$.3f_4+.2f_1+1$
5	$.00005f_5^4+2f_5+f_4+1$	$.00005f_5^4+2f_4+5f_5+2$	f_5+f_2+1
6	0.0000	0.0000	0.0000
7	$.00005f_7^4+.5f_7+.1f_8+.2$	$.00005f_7^4+f_7++.5f_8+1$	$.5f_7+.1f_4+.2$
8	$.00005f_8^4+2f_8+f_7+1$	$.00005f_8^4+5f_8+2f_7+2$	$2f_8+f_5+1$
9	0.0000	0.0000	0.0000
10	$.00005f_{10}^4+.5f_{10}+.1f_{11}+.2$	$.00005f_{10}^4+f_{10}+.5f_{11}+1$	$.5f_{10}+.1f_7+.2$
11	$.00005f_{11}^4+2f_{11}+f_{10}+1$	$.00005f_{11}^4+5f_{11}+2f_{10}+2$	$f_{11}+.4f_8+1$
12	0.0000	0.0000	0.0000
13	$.00005f_{13}^4+.5f_{13}+.1f_{14}+.2$	$.00005f_{13}^4+f_{13}+.5f_{14}+1$	$.4f_{13}+.1f_{10}+.2$
14	$.00005f_{14}^4+2f_{14}+f_{13}+1$	$.00005f_{14}^4+5f_{14}+2f_{13}+2$	$.2f_{14}+.1f_{11}+1$

The modified projection method converged in 28 iterations. It yielded the following equilibrium single class link load (and total, since there is only 1 class) pattern:

$$f_1^* = 53.1127,\ f_2^* = 46.8873,\ f_3^* = 100.0000,$$

$$f_4^* = 53.7822,\ f_5^* = 46.2178,\ f_6^* = 100.0000,$$

$$f_7^* = 59.2427,\ f_8^* = 40.7573,\ f_9^* = 100.0000,$$

$$f_{10}^* = 57.6488,\ f_{11}^* = 42.3512,\ f_{12}^* = 100.0000,$$

$$f_{13}^* = 54.4498,\ f_{14}^* = 45.5502,$$

which was induced by the equilibrium single-class path flow pattern:

$$x^{1}_{p_1}{}^{*} = 8.8423,\ x^{1}_{p_2}{}^{*} = 3.1286,\ x^{1}_{p_3}{}^{*} = 3.2258,$$

$$x_{p_4}{}^{*} = 3.1458,\ x^{1}_{p_5}{}^{*} = 3.1435,\ x^{1}_{p_6}{}^{*} = 3.0630,$$

$$x^{1}_{p_7}{}^{*} = 1.9958,\ x^{1}_{p_8}{}^{*} = 3.1984,\ x^{1}_{p_9}{}^{*} = 3.1874,$$

$$x^{1}_{p_{10}}{}^{*} = 3.6961,\ x^{1}_{p_{11}}{}^{*} = 3.2472,\ x^{1}_{p_{12}}{}^{*} = 3.1638,$$

$$x_{p_{13}}{}^{*} = 2.8312,\ x^{1}_{p_{14}}{}^{*} = 2.9486,\ x^{1}_{p_{15}}{}^{*} = 2.7673,$$

$$x^{1}_{p_{16}}{}^{*} = 1.580,\ x^{1}_{p_{17}}{}^{*} = 3.8039,\ x^{1}_{p_{18}}{}^{*} = 3.4827,$$

$$x^{1}_{p_{19}}{}^{*} = 3.3014,\ x^{1}_{p_{20}}{}^{*} = 3.4188,\ x^{1}_{p_{21}}{}^{*} = 3.0862,$$

$$x_{p_{22}}{}^{*} = 3.0025,\ x^{1}_{p_{23}}{}^{*} = 2.5539,\ x^{1}_{p_{24}}{}^{*} = 1.3897,$$

$$x^{1}_{p_{25}}{}^{*} = 3.0516,\ x^{1}_{p_{26}}{}^{*} = 4.2542,\ x^{1}_{p_{27}}{}^{*} = 3.1870,$$

$$x^{1}_{p_{28}}{}^{*} = 3.1062,\ x^{1}_{p_{29}}{}^{*} = 3.1035,\ x^{1}_{p_{30}}{}^{*} = 3.0237,$$

$$x_{p_{31}}{}^{*} = 3.1221,\ x^{1}_{p_{32}}{}^{*} = 0.0000.$$

All the path generalized costs were equal to 1999.4 (approximately to 4 decimal places).

We now discuss the results. Note that only one path, and that is path p_{32}, which represents commuting all 5 days of the week was not used.

This means that *none* of the decision-makers in this example opt to commute five days of the week.

Path p_1, on the other hand, represents the option of telecommuting 5 days of the week and only 8.8423 (see $x^*_{p_1}$) elect this option.

This is, nevertheless, under the demand $d_\omega = 100$, the most popular choice since the largest number of decision-makers make this choice over the 5 day horizon.

The next least popular choice (outside of path p_{32} which is not used) is represented by path p_{16} which has the flow ${x^1_{p_{16}}}^* = 1.580$. This represents the following decision: to telecommute on the first day of the week, and to commute on the remaining four days. This may have the interpretation that this class of decision-maker likes to work at home at the beginning of the week (following the weekend, say).

Example 2: Example 1 Subject to an Increase in Demand

We then proceeded to make the following change. We increased the demand d_ω from 100 to 300. The modified projection method converged in 5 iterations and yielded the following equilibrium path flow pattern: only path p_{24} was used and it, hence, had all the demand assigned to it, that is, ${x^1_{p_{24}}}^* = 300$, with all other path flows being, thus, equal to zero.

Path p_{24} represents the following decision(s): to commute on days 1, 3, 4, and 5, and to telecommute on day 2.

Path p_{24} had a generalized path cost of 1915.2499, and all other paths (which were unused) had (substantially) higher generalized path costs.

Example 3

In the third numerical example, we added another class of decision-maker, denoted by class 2, whose weights were as follows: $w^2_{1,1} = .5$, $w^2_{2,1} = .3$, $w^2_{3,1} = .1$, $w^2_{1,2} = 1.$, $w^2_{2,2} = 1.$, $w^2_{3,2} = 1.$, $w^2_{1,3} = 1.$, $w^2_{2,3} = 1.$, $w^2_{3,3} = 1.$, $w^2_{1,4} = 1.$, $w^2_{2,4} = 1.$, $w^2_{3,4} = 1.$, $w^2_{1,5} = 1.$, $w^2_{2,5} = 1.$, $w^2_{3,5} = 1.$, $w^2_{1,6} = .5$, $w^2_{2,6} = .5$, $w^2_{3,6} = .5$, $w^2_{1,7} = .5$, $w^2_{2,7} = .4$, $w^2_{3,7} = .4$, $w^2_{1,8} = .4$, $w^2_{2,8} = .3$, $w^2_{3,8} = .2$, $w^2_{1,9} = .3$, $w^2_{2,9} = .2$, $w^2_{3,9} = .6$, $w^2_{1,10} = .5$, $w^2_{2,10} = .4$, $w^2_{3,10} = .5$, $w^2_{1,11} = .7$, $w^2_{2,11} = .6$, $w^2_{3,11} = .7$, $w^2_{1,12} = .4$, $w^2_{2,12} = .3$, $w^2_{3,12} = .8$, $w^2_{1,13} = .3$, $w^2_{2,13} = .2$, $w^2_{3,13} = .6$, $w^2_{1,14} = .2$, $w^2_{2,14} = .3$, $w^2_{3,14} = .9$.

We kept the remainder of the data as in Example 1. We set the demand for class 2 as $d^2_\omega = 100$.

We now present and discuss the results obtained by an application of the modified project method which converged in 7 iterations.

Recall that in Example 1, when there was only a single class of decision-maker, then path p_{32} was the only path that was not used by class 1.

Now, however, with the introduction of a new class of decision-maker, the following equilibrium pattern was obtained: For class 1, *only* path p_{32} was used and, hence, $x^{1*}_{p_{32}} = 100$, with an associated generalized path cost given by $v^1_{p_{32}} = 1200.0499$; all other generalized path costs were higher for this class since those paths were not used.

Class 2 also only utilized path p_{32} and, hence, the flow for class 2 on that path was $x^{2*}_{p_{32}} = 100$. Its generalized cost was $v^2_{p_{32}} = 1953.9998$, with the other unused paths having higher generalized costs for this class.

Interestingly, with the addition of a new class of decision-maker the equilibrium pattern for class 1 changed entirely. Also, interestingly, despite different weights associated with the criteria both decision-makers of class 1 and of class 2 chose to commute 5 days a week!

Example 4: Example 3 Subject to an Increase in Demand for Class 1

We then made the following perturbation to the data. We increased the demand for class 1 to 300, that is, $d_\omega^1 = 300$, but kept the demand for class 2 as in Example 3, that is, $d_\omega^2 = 100$. The new equilibrium pattern was computed by the modified projection method in 2 iterations.

Now both class 1 and class 2 used solely path p_{24}, where recall that this path corresponds to telecommuting on the second day the week and commuting on the remaining 4 days.

Interestingly, both classes of decision-makers selected the same option, again.

Example 5: Example 3 Subject to an Increase in Demand for Both Class 1 and Class 2

We then proceeded to increase the demand for class 2 to 300, that is, $d^2_\omega = 300$, with the other data as in the example immediately preceding. The modified projection method converged in 4 iterations and the solution stayed the same as in the preceding example.

Example 6: Example 3 Subject to a Decrease in Demand for Both Class 1 and Class 2

We next decreased the demands for both classes so that $d^1_\omega = 30$ and $d^2_\omega = 30$, and kept all other data as in Example 3. Interestingly, except for 1 path, which was used by both classes, the other paths used were distinct for each class. The modified projection method converged in 243 iterations and yielded the following path flow pattern:

For class 1:

$$x^{1\,*}_{p_1} = .3359,\ x^{1\,*}_{p_2} = 1.9953,\ x^{1\,*}_{p_9} = 1.4688,$$

$$x^{1\,*}_{p_{10}} = 2.7508,\ x^{1\,*}_{p_{17}} = 1.8524,\ x^{1\,*}_{p_{18}} = 4.1009,$$

$$x^{1\,*}_{p_{25}} = 2.4664,\ x^{1\,*}_{p_{26}} = 15.0296,$$

with all other path flows for this class being equal to zero. The generalized path costs on the used paths was approximately 644.95 for all such paths.

For class 2:

$$x^{2\,*}_{p_1} = 4.3939,\ x^{2\,*}_{p_3} = 4.3290,\ x^{2\,*}_{p_5} = 7.740,$$

$$x^{2\,*}_{p_7} = 13.5531,$$

with all other path flows for this class being equal to zero. The generalized path costs on the used paths was 809.95, approximately.

Note that both classes used path 1, that is, there were members of each class who sought to telecommute all five days. However, all other paths used by class 2 were distinct from those chosen by class 1.

For completeness, we also report the computed equilibrium multiclass link flow pattern and the total link flow pattern:

For class 1:

$$f_1^{1^*} = 6.5508,\ f_2^{1^*} = 23.4492,\ f_3^{1^*} = 30.0000,$$

$$f_4^{1^*} = 8.2845,\ f_5^{1^*} = 21.7155,\ f_6^{1^*} = 30.0000,$$

$$f_7^{1^*} = 30.0000,\ f_8^{1^*} = 0.0000,\ f_9^{1^*} = 30.0000,$$

$$f_{10}^{1\ *} = 30.0000,\ f_{11}^{1\ *} = 0.0000,\ f_{12}^{1\ *} = 30.0000,$$

$$f_{13}^{1\ *} = 6.1235,\ f_{14}^{1\ *} = 23.8765;$$

For class 2:

$$f_1^{2^*} = 30.0000,\ f_2^{2^*} = 0.0000,\ f_3^{2^*} = 30.0000,$$

$$f_4^{2^*} = 30.0000,\ f_5^{2^*} = 0.0000,\ f_6^{2^*} = 30.0000,$$

$$f_7^{2^*} = 8.7229,\ f_8^{2^*} = 21.2771,\ f_9^{2^*} = 30.0000,$$

$$f_{10}^{2\ *} = 12.1179,\ f_{11}^{2\ *} = 17.8821,\ f_{12}^{2\ *} = 30.0000,$$

$$f_{13}^{2\ *} = 30.0000,\ f_{14}^{2\ *} = 0.0000,$$

and

$$f_1{}^* = 36.5508,\ f_2{}^* = 23.4492,\ f_3{}^* = 60.0000,$$

$$f_4{}^* = 38.2845,\ f_5{}^* = 21.7155,\ f_6{}^* = 60.0000,$$

$$f_7{}^* = 38.7229,\ f_8{}^* = 21.2771,\ f_9{}^* = 60.0000,$$

$$f_{10}{}^* = 42.1179,\ f_{11}{}^* = 17.8821,\ f_{12}{}^* = 60.0000,$$

$$f_{13}{}^* = 36.1235,\ f_{14}{}^* = 23.8765.$$

As can be seen from the total link flows, over two thirds of the decision-makers chose to telecommute on the fourth day, with telecommuting being selected by more than half of the decision-makers on any given day.

Hence, the number of commuters is lowest on day four and highest on day 5.

Summary and Conclusions

• We have developed a multiperiod, multiclass, multicriteria network equilibrium model for the determination of telecommuting versus commuting decision-making over space and time.

• The model considers distinct classes of decision-makers, each of whom weights the three criteria of travel time, travel cost, and opportunity costs distinctly. The weights are not only class-dependent but also link-dependent.

• We conceptualized the problem, which assumes a finite time horizon, typically, a five day work week, through the use of a space-time network which abstracts the decision-making process as the selection of paths over the network.

• The paths consist of routes that are taken between residential and employment locations within each time period and also include connecting links which join subnetworks between successive periods.

- We derived the equilibrium conditions and provide the finite-dimensional variational inequality formulation.

- We also provided qualitative properties of the solution pattern.

- We proposed an algorithm for computational purposes and provide convergence results. Finally, we presented numerical examples which illustrate the model.

Below are referencess cited in the lecture as well as additional ones relevant to the topic.

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