Multicriteria Network Equilibrium Modeling for Decision-Making in the Information Age with Applications to Teleshopping and Telecommuting

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The advent of the Information Age with the increasing availability of new computer and communication technologies, along with the Internet, have transformed the ways in which many individuals work, travel, and conduct their daily activities today.

Moreover, the decision-making process itself has been altered through the addition of alternatives which were not, heretofore, possible or even feasible. Indeed, as stated in a recent issue of *The Economist* (2000), "The boundaries for employees are redrawn... as people work from home and shop from work."

Related transformations have occurred in history through such technological innovations as the telegraph and telephone, railroads, electricity, the mass production of the automobile, and/or the introduction of air travel, accompanied by the construction of the underlying infrastructure (cf. Friedlander (1995a, b, 1996)).

The Internet, however, may be viewed as being unique in the sense of its speed, very low cost, potential connectivity, and flexibility of usage.

Interestingly, all of the above noted technological innovations have been network in nature, with links corresponding, typically, to either transportation links, such as in the case of roads for automobiles and other vehicles, or tracks as in the case of railroads, or to communication links, such as telephone or fiberoptic cables or radio links. Flows on such networks would correspond, respectively, to vehicles, or to messages.

The operation and use of such network systems, however, is done by humans and it is their behavior which affects both the effectiveness and the efficiency of the systems. For example, it is now well-recognized that congestion on urban road networks is a serious problem resulting in $100 billion in lost productivity in the United States alone annually with the figure being approximately $150 billion in Europe (cf. Nagurney (2000a)).

Moreover, the emissions generated through increasing vehicular use have wide health as well as economic impacts (see Button (1993)).

Interestingly, congestion is also playing an increasingly prominent role in communication networks and recently discovered paradoxical phenomena therein are closely related to those occurring in transportation networks (cf. Korilis, Lazar, and Orda (1999) and Nagurney and Dong (2000a)).

Furthermore, it is the interaction between transportation and communication networks, and the individuals' use, thereof, which is of particular interest and relevance to the Information Age (see, e.g., Memmott (1963), Jones (1973), Khan (1976), Nilles, et al. (1976), Harkness (1977), Salomon (1986)). Indeed, as noted by Mokhtarian and Salomon (1997), in order to properly address transportation and telecommunication issues today one must ultimately include the transportation network and be able to forecast volumes of flow.

The Importance of Networks

• According to the US Department of Transportation, the significance of transportation in dollar value alone as spent by US customers, businesses, and governments was $950 billion in the 1998.

• As regards communications, corporate buyers alone spent $517.6 billion on such goods and service in 1999.

• In 1995, according to the US Department of Commerce, the enery expenditures in the United States were $515.8 billion.

In this lecture, we take on the challenge of developing a network equilibrium framework for decision-making in the Information Age.

The modeling approach captures choices made possible through transportation and telecommunication mode alternatives.

Moreover, it allows for the prediction of the volumes of flow in terms of decision-makers selecting particular choices and the effects of their choices on such possible criteria as time, cost, risk, and/or safety.

A network equilibrium framework is natural since not only are now many of the relevant decisions taking place on networks but also the concept of a network – as we demonstrate here – is sufficiently general in an abstract and mathematical setting to also capture many of the salient features comprising decision-making today.

The Multiclass, Multicriteria Network Equilibrium Models

• The multiclass, multicriteria network equilibrium models are with elastic demand and with fixed demand, respectively.

• Each class of decision-maker is allowed to have distinct weights associated with the criteria which are also permitted to be link-dependent for modeling flexibility purposes.

• The models are then applied to telecommuting versus commuting decision-making and to teleshopping versus shopping decision-making.

The Elastic and Fixed Demand Models

Consider a general network $G = [\mathcal{N}, \mathcal{L}]$, where $\mathcal{N}$ denotes the set of nodes in the network and $\mathcal{L}$ the set of directed links.

Let a denote a link of the network connecting a pair of nodes and let p denote a path, assumed to be acyclic, consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. There are n links in the network and n_P paths.

Let Ω denote the set of J O/D pairs. The set of paths connecting the O/D pair ω is denoted by P_ω and the entire set of paths in the network by P.

Note that in our framework a link may correspond to an actual physical link of transportation or an abstract or virtual link corresponding to telecommunications. A path abstracts a decision as a sequence of links or possible choices from an origin node, which represents the beginning of the decision, to the destination node, which represents its completion.

The Flows

There are k classes of decision-makers in the network with a typical class denoted by i. Let f_a^i denote the flow of class i on link a and let x_p^i denote the nonnegative flow of class i on path p. The relationship between the link loads by class and the path flows is:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, \quad \forall a \in \mathcal{L},$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

Lt f_a denote the total flow on link a, where

$$f_a = \sum_{i=1}^{k} f_a^i, \quad \forall a \in \mathcal{L}.$$

The class link flows are grouped into the kn-dimensional column vector $\tilde{f}$, and the total link flows into the n-dimensional column vector f. The class path flows are grouped into the kn_P-dimensional column vector $\tilde{x}$ with components: $\{x_{p_1}^1, \ldots, x_{p_{n_P}}^k\}$.

The Demands

The demand associated with origin/destination (O/D) pair ω and class i will be denoted by d^i_ω. Group the demands into a column vector $d \in \mathcal{R}^{kJ}$.

The demands must satisfy the following conservation of flow equations:

$$d^i_\omega = \sum_{p \in P_\omega} x^i_p, \quad \forall i, \forall \omega.$$

The Criteria Functions

Assume that there are H criteria which the decision-makers may utilize in their decision-making with a typical criterion denoted by h.

C_{ha} denotes criterion h associated with link a, where we have that

$$C_{ha} = C_{ha}(f), \quad \forall a \in \mathcal{L},$$

where C_{ha} is assumed to be a continuous function.

Time Criterion

For example, criterion 1 may be time, in which case we would have

$$C_{1a} = C_{1a}(f) = t_a(f), \quad \forall a \in \mathcal{L},$$

where $t_a(f)$ denotes the time associated with traversing link a.

Cost Criterion

Another relevant criterion may be cost, that is, we may have:

$$C_{2a} = C_{2a}(f) = c_a(f), \quad \forall a \in \mathcal{L}.$$

Opportunity Cost Criterion

Another relevant criterion for decision-making in the Information Age:

$$C_{3a} = C_{3a}(f) = o_a(f), \quad \forall a \in \mathcal{L},$$

with $o_a(f)$ denoting the opportunity cost associated with link a.

Safety Cost Criterion

Finally, a decision-maker may wish to associate a safety cost in which case the fourth criterion may be

$$C_{4a} = C_{4a}(f) = s_a(f), \quad \forall a \in \mathcal{L},$$

where $s_a(f)$ denotes a security or safety cost measure associated with link a.

The Weights

Assume that each class of decision-maker has a potentially different perception of the tradeoffs among the criteria, which are represented by the nonnegative weights: $w^i_{1a}, \ldots, w^i_{Ha}$.

Nagurney and Dong (2000) were the first to model link-dependent weights.

Nagurney, Dong, and Mokhtarian (2000), in turn, used fixed, link-dependent weights but assumed only three criteria, in particular, travel time, travel cost, and opportunity cost in their integrated multicriteria network equilibrium models for telecommuting versus commuting.

The Generalized Costs

Generalized cost functions (in the case of minimization) or value functions (in the case of maximization) have been studied extensively and used for decision problems with multiple criteria by numerous authors, including: Fishburn (1970), Chankong and Haimes (1983), Yu (1985), and Keeney and Raiffa (1993).

We propose a link generalized cost function as follows.

The generalized cost of class i associated with link a and denoted by C_a^i is given by:

$$C_a^i = \sum_{h=1}^{H} w_{ha}^i C_{ha}, \quad \forall i, \quad \forall a \in \mathcal{L}.$$

Generalized Cost on a Path of a Class

Let C_p^i denote the generalized cost of class i associated with path p in the network where

$$C_p^i = \sum_{a \in \mathcal{L}} C_a^i(\tilde{f})\delta_{ap}, \quad \forall i, \quad \forall p.$$

Hence, the generalized cost associated with a class and a path is that class's weighted combination of the various criteria on the links that comprise the path.

In the case of the elastic demand model, we assume, as given, the inverse demand functions λ^i_ω for all classes i and all O/D pairs ω, where:

$$\lambda^i_\omega = \lambda^i_\omega(d), \quad \forall i, \forall \omega,$$

where these functions are assumed to be smooth and continuous. We group the inverse demand functions into a column vector $\lambda \in \Re^{kJ}$.

The Behavioral Assumption

We assume that the decision-making involved in the problem is repetitive in nature such as, for example, in the case of commuting versus telecommuting, or shopping versus teleshopping. The behavioral assumption that we propose, hence, is that decision-makers select their paths so that their generalized costs are minimized (see, e.g., Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969), and Dafermos (1982)). Such an idea was also used in the context of multicriteria traffic networks by Dafermos (1981), Leurent (1993a), Dial (1996), Marcotte (1998), and Nagurney (2000), among others.

Multicriteria Network Equilibrium Conditions for the Elastic Demand Case

For each class i, for all O/D pairs $\omega \in \Omega$, and for all paths $p \in P_\omega$, the flow pattern $\tilde{x}^*$ is said to be *in equilibrium* if the following conditions hold:

$$C^i_p(\tilde{f}^*) \begin{cases} = \lambda^i_\omega(d^*), & \text{if} \quad x^{i*}_p > 0 \\ \geq \lambda^i_\omega(d^*), & \text{if} \quad x^{i^*}_p = 0. \end{cases}$$

In the case of the fixed demand model, in which the demands are now assumed known and fixed, the multicriteria network equilibrium conditions now take the form:

Multicriteria Network Equilibrium Conditions for the Fixed Demand Case

For each class i, for all O/D pairs $\omega \in \Omega$, and for all paths $p \in P_\omega$, the flow pattern $\tilde{x}^*$ is said to be *in equilibrium* if the following conditions hold:

$$C_p^i(\tilde{f}^*) \begin{cases} = \lambda_\omega^i, & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_\omega^i, & \text{if} \quad x_p^{i^*} = 0, \end{cases}$$

where now the λ_ω^i denotes simply an indicator representing the minimal incurred generalized path cost for class i and O/D pair ω.

Variational Inequality Formulations

Theorem: Variational Inequality Formulation of the Elastic Demand Model

The variational inequality formulation of the multicriteria network model with elastic demand satisfying the equilibrium conditions is given by: Determine $(\tilde{f}^*, d^*) \in \mathcal{K}^1$*, satisfying*

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}} C_a^i(\tilde{f}^*)\times(f_a^i - f_a^{i*}) - \sum_{i=1}^{k}\sum_{\omega\in\Omega}\lambda_\omega^i(d^*)\times(d_\omega^i - d_\omega^{i*}) \geq 0,$$

$$\forall(\tilde{f}, d)\in\mathcal{K}^1,$$

where $\mathcal{K}^1 \equiv$

$\{(\tilde{f}, d) | \tilde{x} \geq 0,$ *and the conservation of flow equations hold*$\}$*; equivalently, in standard variational inequality form:*

$$\langle F(X^*)^T, X - X^*\rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $F \equiv (C, \lambda)$*,* $X \equiv (\tilde{f}, d)$*, and* $\mathcal{K} \equiv \mathcal{K}^1$*.*

Theorem: Variational Inequality Formulation of the Fixed Demand Model

The variational inequality formulation of the fixed demand multiciteria network equilibrium model satisfying the equilibrium conditions is given by: Determine $\tilde{f}^* \in \mathcal{K}^2$, satisfying

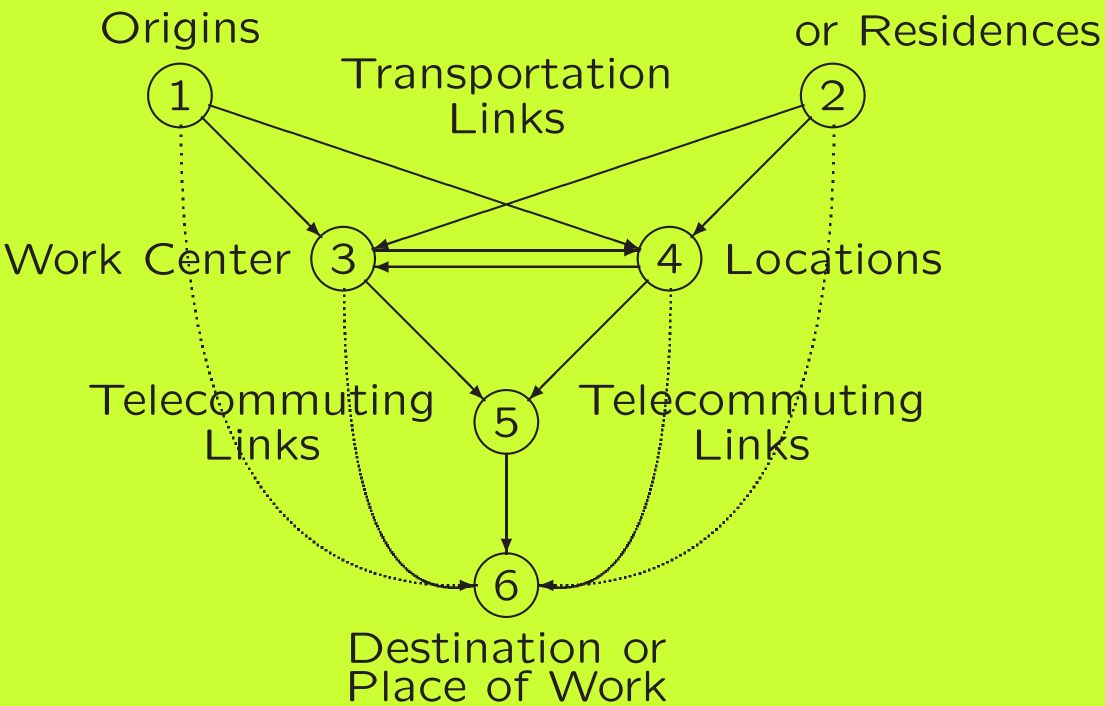
$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} C_a^i(\tilde{f}^*) \times (f_a^i - f_a^{i*}) \geq 0, \qquad \forall \tilde{f} \in \mathcal{K}^2,$$

where $\mathcal{K}^2 \equiv$

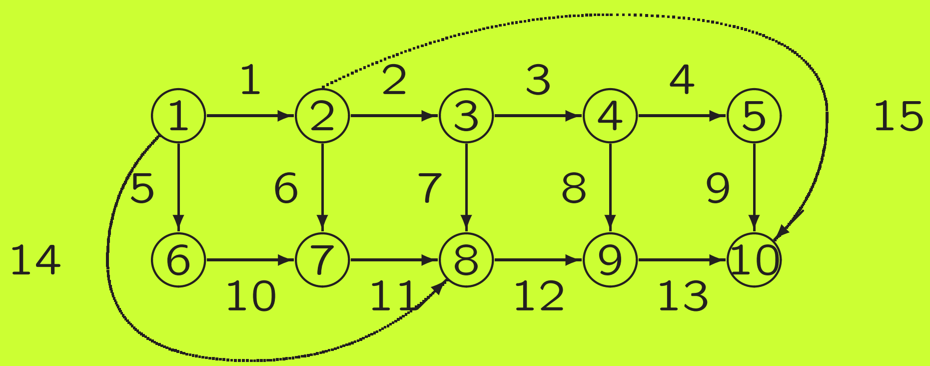
$\{\tilde{f} | \exists \tilde{x} \geq 0,$ *and satisfying the conservation of flow equations with d known*$\}$*; equivalently, in standard variational inequality form:*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $F \equiv C$, $X \equiv \tilde{f}$, *and* $\mathcal{K} \equiv \mathcal{K}^2$.



A network conceptualization of commuting versus telecommuting



Network Topology for Telecommuting versus Commuting Numerical Example

A Telecommuting versus Commuting Numerical Example

The numerical example consisted of two classes of decision-makers, each of which is faced with four criteria and had fixed demands.

The modified projection method was coded in FORTRAN and implemented on the DEC Alpha system at the University of Massachusetts at Amherst.

The example had the topology depicted in the Figure. Links 1 through 13 are transportation links whereas links 14 and 15 are telecommunication links. The network consisted of ten nodes, fifteen links, and two O/D pairs where $\omega_1 = (1, 8)$ and $\omega_2 = (2, 10)$

The travel demands by class were given by: $d^1_{\omega_1} = 10$, $d^1_{\omega_2} = 20$, $d^2_{\omega_1} = 10$, and $d^2_{\omega_2} = 30$.

The paths connecting the O/D pairs were: for O/D pair ω_1: $p_1 = (1, 2, 7)$, $p_2 = (1, 6, 11)$, $p_3 = (5, 10, 11)$, $p_4 = (14)$, and for O/D pair ω_2: $p_5 = (2, 3, 4, 9)$, $p_6 = (2, 3, 8, 13)$, $p_7 = (2, 7, 12, 13)$, $p_8 = (6, 11, 12, 13)$, and $p_9 = (15)$.

The Weights

The weights were: For class 1, the weights were: $w^1_{1,1} = .25$, $w^1_{2,1} = .25$, $w^1_{3,1} = 1.$, $w^1_{1,2} = .25$, $w_{2,2} = .25$, $w^1_{3,2} = 1.$, $w^1_{1,3} = .4$, $w^1_{2,3} = .4$, $w^1_{3,3} = 1.$, $w^1_{1,4} = .5$, $w^1_{2,4} = .5$, $w^1_{3,4} = 2.$, $w^1_{1,5} = .4$, $w^1_{2,5} = .5$, $w^1_{3,5} = 1.$, $w^1_{1,6} = .5$, $w^1_{2,6} = .3$, $w^1_{3,6} = 2.$, $w^1_{1,7} = .2$, $w^1_{2,7} = .4$, $w^1_{3,7} = 1.$, $w^1_{1,8} = .3$, $w^1_{2,8} = .5$, $w^1_{3,8} = 1.$, $w^1_{1,9} = .6$, $w^1_{2,9} = .2$, $w^1_{3,9} = 2.$, $w^1_{1,10} = .3$, $w^1_{2,10} = .4$, $w^1_{3,10} = 1.$, $w^1_{1,11} = .2$, $w^1_{2,11} = .7$, $w^1_{3,11} = 1.$, $w^1_{1,12} = .3$, $w^1_{2,12} = .4$, $w^1_{3,12} = 1.$, $w^1_{1,13} = .2$, $w^1_{2,13} = .3$, $w^1_{3,13} = 2.$, $w^1_{1,14} = .5$, $w^1_{2,14} = .2$, $w^1_{3,14} = .1$, $w^1_{1,15} = .5$, $w^1_{2,15} = .3$, $w^1_{3,15} = .1$.
All the weights $w^1_{4,a} = .2$ for all links a.

For class 2: $w^2_{1,1} = .5$, $w^2_{2,1} = .5$, $w^2_{3,1} = .5$, $w^2_{1,2} = .5$, $w^2_{2,2} = .4$, $w^2_{3,2} = .4$, $w^2_{1,3} = .4$, $w^2_{2,3} = .3$, $w^2_{3,3} = .7$, $w^2_{1,4} = .3$, $w^2_{2,4} = .2$, $w^2_{3,4} = .6$, $w^2_{1,5} = .5$, $w^2_{2,5} = .4$, $w^2_{3,5} = .5$, $w^2_{1,6} = .7$, $w^2_{2,6} = .6$, $w^2_{3,6} = .7$, $w^2_{1,7} = .4$, $w^2_{2,7} = .3$, $w^2_{3,7} = .8$, $w^2_{1,8} = .3$, $w^2_{2,8} = .2$, $w^2_{3,8} = .6$, $w^2_{1,9} = .2$, $w^2_{2,9} = .3$, $w^2_{3,9} = .9$, $w^2_{1,10} = .1$, $w^2_{2,10} = .4$, $w^2_{3,10} = .8$, $w^2_{1,11} = .4$, $w^2_{2,11} = .5$, $w^2_{3,11} = .9$, $w^2_{1,12} = .5$, $w^2_{2,12} = .5$, $w^2_{3,12} = .7$, $w^2_{1,13} = .4$, $w^2_{2,13} = .6$, $w^2_{3,13} = .9$, $w^2_{1,14} = .3$, $w^2_{2,14} = .4$, $w^2_{3,14} = 1.$, $w^2_{1,15} = .2$, $w^2_{2,15} = .3$, $w^2_{3,15} = .2$.
All the weights $w^2_{4,a} = .1$ for all links a.

The travel time functions and the travel cost functions for this example are reported in the Table as are the opportunity cost functions and the safety cost functions for the links. The generalized link cost functions were constructed as described earlier.

The Opportunity Cost and Safety Cost Functions for the Links for the Telecommuting Example

Link a	$o_a(f)$	$s_a(f)$
1	$2f_1 + 4$	$f_1 + 1$
2	$3f_2 + 2$	$f_2 + 2$
3	$f_3 + 4$	$f_3 + 1$
4	$f_4 + 2$	$f_4 + 2$
5	$2f_5 + 1$	$2f_5 + 2$
6	$f_6 + 2$	$f_6 + 1$
7	$f_7 + 3$	$f_7 + 1$
8	$2f_8 + 1$	$2f_8 + 2$
9	$3f_9 + 2$	$3f_9 + 3$
10	$f_{10} + 1$	$f_{10} + 2$
11	$4f_{11} + 3$	$2f_{11} + 3$
12	$3f_{12} + 2$	$3f_{12} + 3$
13	$f_{13} + 1$	$f_{13} + 2$
14	$6f_{14} + 1$	$.5f_{14} + .1$
15	$7f_{15} + 4$	$.4f_{15} + .1$

Note that the opportunity costs associated with links 14 and 15 were high since these are telecommunication links and users by choosing these links forego the opportunities associated with working and associating with colleagues from a face to face perspective. Observe, however, that the weights for class 1 associated with the opportunity costs on the telecommunication links are low (relative to those of class 2). This has the interpretation that class 1 does not weight such opportunity costs highly and may, for example, prefer to be working from the home for a variety, including familial, reasons. Also, note that class 1 weights the travel time on the telecommunication links more highly than class 2 does. Furthermore, observe that class 1 weights the safety or security cost higher than class 2.

The convergence criterion was that the maximum of the absolute value of the path flows at two successive iterations was .0001. The modified projection method was initialized by equally distributing the demand for each class and each O/D pair among the paths for that O/D pair. The parameter α in the modified projection method was set to .001 for this example. The modified projection method was embedded with the equilibration algorithm of Dafermos and Sparrow (1969) for the solution of the variational inequality subproblems, which are equivalent to quadratic programming problems over a feasible set with network structure.

The modified projection method required 12 iterations for convergence. It yielded the equilibrium multiclass link flow and total link flow patterns reported in the Table below, which were induced by the equilibrium multiclass path flow pattern given below.

The Equilibrium Link Flows for the Telecommuting Example

Link a	Class 1 f_a^{1*}	Class 2 f_a^{2*}	Total flow f_a^*
1	0.000	0.0000	0.0000
2	0.0000	24.0109	24.0109
3	0.0000	22.7600	22.7600
4	0.0000	17.3356	17.3356
5	0.0000	4.6901	4.6901
6	0.0000	5.9891	5.9891
7	0.0000	1.2509	1.2509
8	0.0000	5.4244	5.4244
9	0.0000	17.3556	17.3556
10	0.0000	4.6901	4.6901
11	0.0000	10.6792	10.6792
12	0.0000	7.2400	7.2400
13	0.0000	12.6644	12.6644
14	10.0000	5.3090	15.3099
15	20.0000	0.0000	20.0000

The Equilibrium Path Flows for the Telecommuting Example

Path p	Class 1 - x_p^{1*}	Class 2 - x_p^{2*}
p_1	0.0000	0.0000
p_2	0.0000	0.0000
p_3	0.0000	4.6901
p_4	10.0000	5.3099
p_5	0.0000	17.3357
p_6	0.0000	5.4244
p_7	0.0000	1.2509
p_8	0.0000	5.9892
p_9	20.0000	0.0000

The generalized path costs were: for Class 1, O/D pair ω_1:

$$C^1_{p_1} = 13478.4365,\ C^1_{p_2} = 11001.0342$$

$$C^1_{p_3} = 8354.5420,\ C^1_{p_4} = 1025.4167,$$

for Class 1, O/D pair ω_2:

$$C^1_{p_5} = 45099.8047,\ C^1_{p_6} = 27941.5918,$$

$$C^1_{p_7} = 25109.3223,\ C^1_{p_8} = 22631.9199,$$

$$C^1_{p_9} = 2314.7222;$$

for Class 2, O/D pair ω_1:

$$C^2_{p_1} = 15427.5996,\ C^2_{p_2} = 15427.2021,$$

$$C^2_{p_3} = 8721.8945,\ C^2_{p_4} = 8721.3721,$$

and for Class 2, O/D pair ω_2:

$$C^2_{p_5} = 34924.6602,\ C^2_{p_6} = 34924.6094,$$

$$C^2_{p_7} = 34925.3789,\ C^2_{p_8} = 34924.9805,$$

$$C^2_{p_9} = 41574.2617.$$

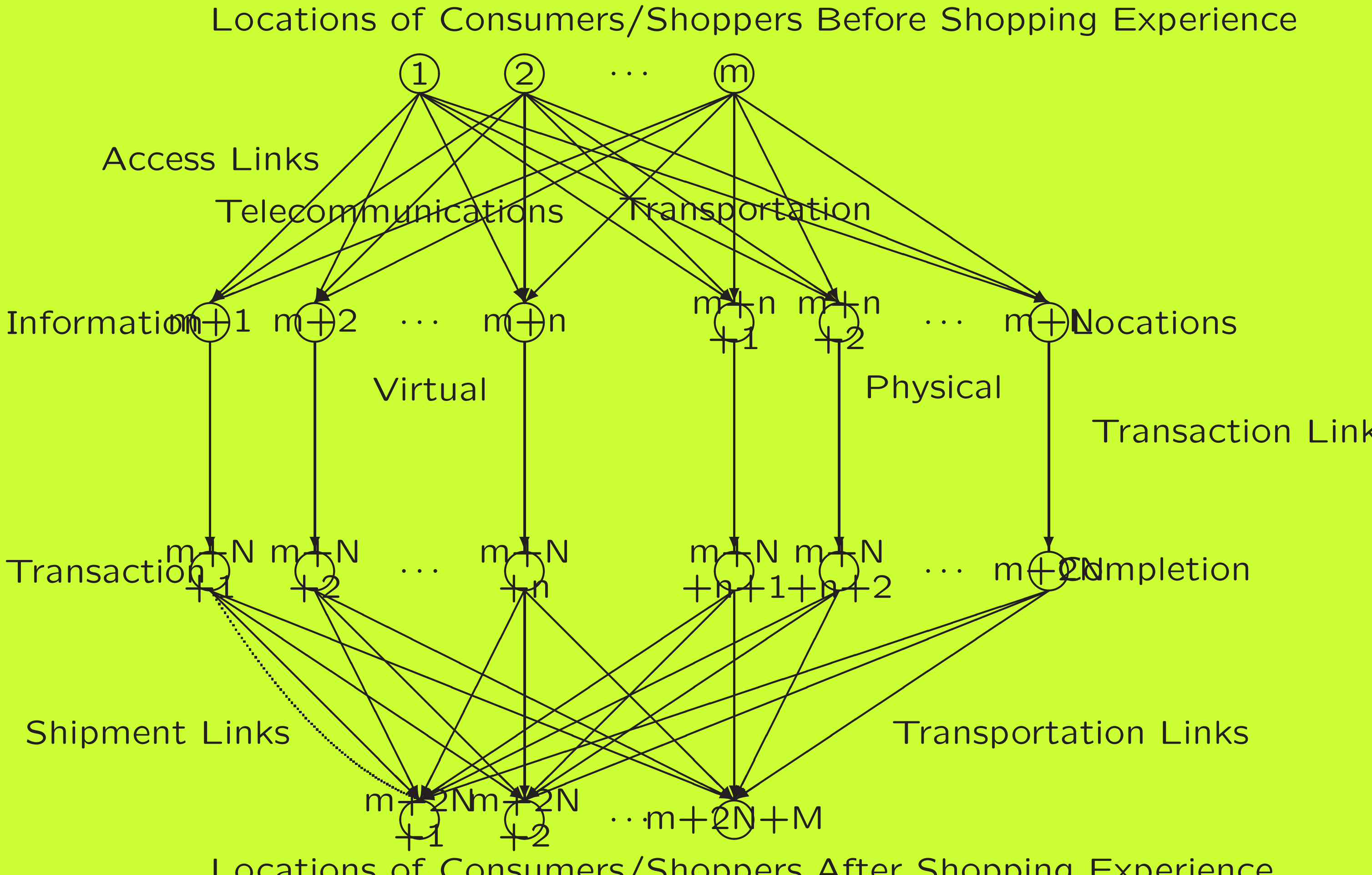
It is interesting to see the separation by classes in the equilibrium solution. Note that all members of class 1, whether residing at node 1 or node 2, were telecommuters, whereas all members of class 2 chose to commute to work. This outcome is realistic, given the weight assignments of the two classes on the opportunity costs associated with the links (as well as the weight assignments associated with the travel times). Of course, different criteria functions, as well as their numerical forms and associated weights, will lead to different equilibrium patterns.

This example demonstrates the flexibility of the modeling approach.

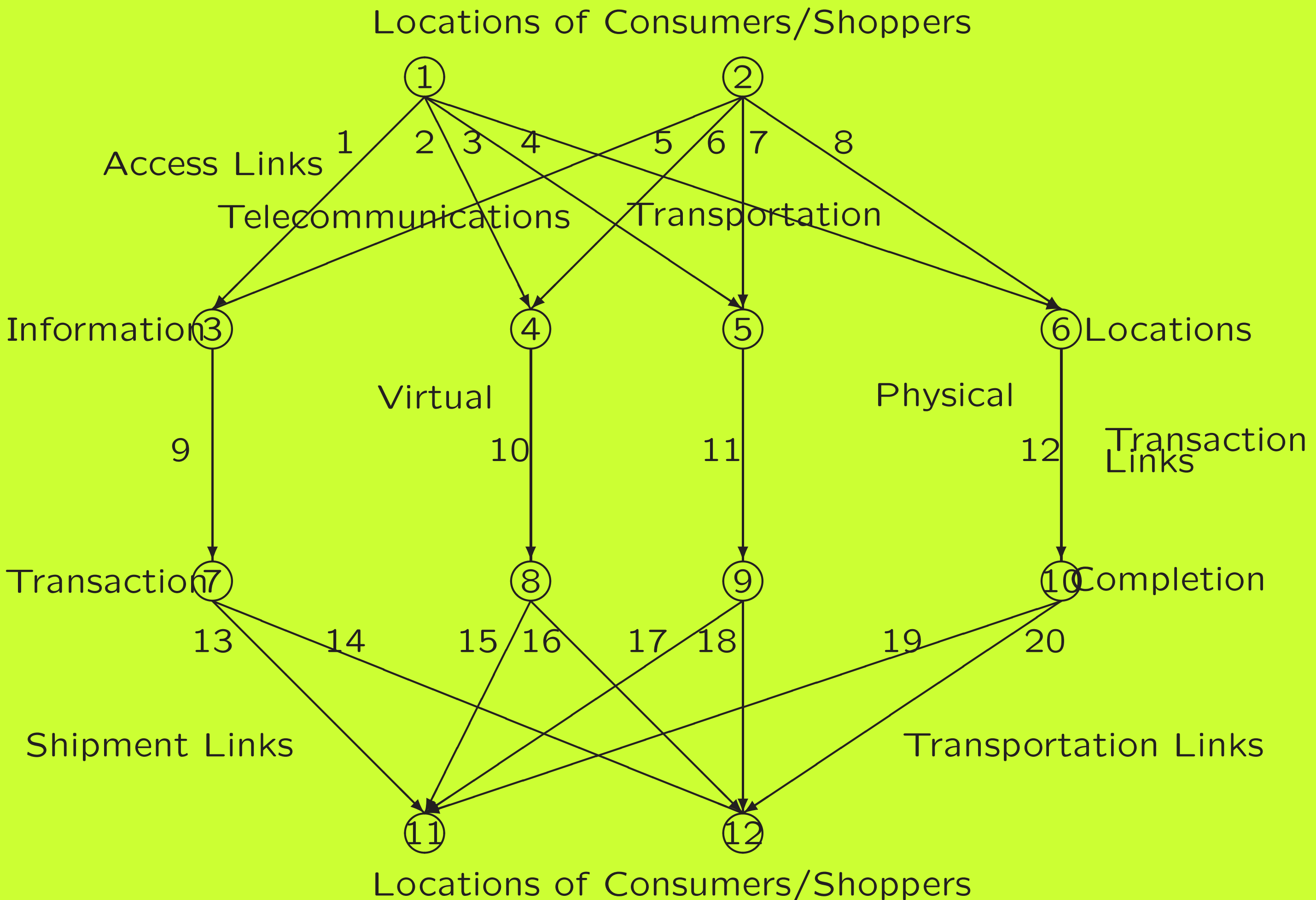
Moreover, it allows one to conduct a variety of "what if" simulations in that, one can modify the functions and the associated weights to reflect the particular telecommuting versus commuting scenario.

For example, during a downturn in the economy, the opportunity costs associated with the telecommuting links may be high, and, also, different classes may weight this criteria on such links higher, resulting in a new solution. On the other hand, highly skilled employees who are in demand may have lower weights associated with such links in regards to the opportunity costs.

This framework is, hence, sufficiently general to capture a variety of realistic situations while, at the same time, allowing decision-makers to identify their specific values and preferences.



A Network Framework for Teleshopping versus Shopping



The Network for the Teleshopping versus Shopping Numerical Example

A Teleshopping versus Shopping Example

We considered a situation in which there are consumers located at two locations with the possibility of shopping virtually through telecommunications at two sites and physically at two other sites. Assume that there are two classes of shoppers. The shopping network for the problem is given below. The network consists of two origin nodes at the top; two destination nodes at the bottom, with two O/D pairs given by: $\omega_1 = (1, 11)$ and $\omega_2 = (2, 12)$.

There is a total of twenty links in the network, where eight links (in the first set) are access links, four links (in the second set) are transaction links, and eight links (in the final set of links) are shipment/transportation links. In the first set of links, four links represent access links to the virtual sites through telecommunications and the remaining four links correspond to access links to the physical sites through transportation. The links have been enumerated in the Figure for data presentation purposes.

The modified projection method for the solution of variational inequality was implemented in FORTRAN and the Euler method was embedded for the solution of variational inequality subproblems.

The convergence criterion was that the absolute value of the path flows at two successive iterations was less than or equal to ϵ with ϵ set to .0001. The α parameter in the modified projection method was set to .001. The demand for the product for each class of consumer was initialized to zero.

Denoting the O/D pairs by
$\omega_1 = (1, 11)$ and $\omega_2 = (2, 12)$, the inverse demand functions of the two classes were:

$$\lambda^1_{\omega_1}(d) = -.5d^1_{\omega_1} + 956, \quad \lambda^1_{\omega_2}(d) = -.1d^1_{\omega_2} + 920,$$

and

$$\lambda^2_{\omega_1}(d) = -.2d^2_{\omega_1} + 580, \quad \lambda^2_{\omega_2}(d) = -.1d^2_{\omega_2} + 1050.$$

There were four paths connecting each O/D pair. The paths connecting O/D pair ω_1 were: $p_1 = (1, 9, 13)$, $p_2 = (2, 10, 15)$, $p_3 = (3, 11, 17)$, and $p_4 = (4, 12, 19)$, whereas the paths connecting O/D pair ω_2 were: $p_5 = (5, 9, 14)$, $p_6 = (6, 10, 16)$, $p_7 = (7, 11, 18)$, and $p_8 = (8, 12, 20)$. It was assumed that there were four criteria associated with each link and consisting, respectively, of: time (criterion 1), monetary cost (criterion 2), an *opportunity* cost (criterion 3), and a safety or security cost (criterion 4).

The generalized link cost functions were constructed as previously using the weights given below, the time functions and the cost functions reported in the Table, and the opportunity and safety cost functions reported in the subsequent Table.

The Time and Cost Functions for the Links for the Teleshopping Example

Link a	$t_a(f)$	$c_a(f)$
1	$.00005f_1^4+f_1+f_2+5$	$.00005f_1^4+2f_1+f_2+1$
2	$.00003f_2^4+f_2+.5f_5+1$	$.00003f_2^4+2f_2+f_1+1$
3	$.00005f_3^4+4f_3+f_4+1$	$.00005f_3^4+3f_3+.5f_4+3$
4	$.00003f_4^4+6f_4+2f_5+4$	$.00003f_4^4+7f_4+3f_3+1$
5	f_5+1	f_5+2
6	$.00007f_6^4+f_6+.5f_2+1$	$.00007f_6^4+2f_6+f_5+1$
7	$8f_7+7$	$4f_7+6$
8	$.00001f_8^4+7f_8+3f_5+6$	$.00001f_8^4+4f_8+2f_7+1$
9	$2f_9+1$	$2f_9+1$
10	$.00003f_{10}^4+2f_{10}+f_9+1$	$.00003f_{10}^4+2f_{10}+f_9+1$
11	$.00004f_{11}^4+2f_{11}+f_{10}+4$	$.00004f_{11}^4+4f_{11}+2f_{12}+2$
12	$.00002f_{12}f^4+2f_{12}+f_{11}+2$	$.00002f_{12}^4+4f_{12}+2f_{11}+1$
13	$.00003f_{13}^4+9f_{13}+3f_{14}+3$	$.00003f_{13}^4+3f_{13}+f_{14}+2$
14	$5f_{14}+3$	$4f_{14}+2$
15	$6f_{15}+4$	$4f_{15}+1$
16	$10f_{16}+10$	$2f_{16}+10$
17	$5f_{17}+10$	$5f_{17}+10$
18	$f_{18}+20$	$6f_{18}+20$
19	$6f_{19}+20$	$5f_{19}+10$
20	$10f_{20}+15$	$4f_{20}+10$

The weights were as follows: For class 1, the weights were: $w^1_{1,1} = .25$, $w^1_{2,1} = .25$, $w^1_{3,1} = 1.$, $w^1_{4,1} = .2$ $w^1_{1,2} = .25$, $w^1_{2,2} = .25$, $w^1_{3,2} = 1.$, $w^1_{4,2} = .2$, $w^1_{1,3} = .4$, $w^1_{2,3} = .4$, $w^1_{3,3} = 1.$, $w^1_{4,3} = .1$, $w^1_{1,4} = .5$, $w^1_{2,4} = .5$, $w^1_{3,4} = 2.$, $w^1_{4,4} = .1$, $w^1_{1,5} = .4$, $w^1_{2,5} = .5$, $w^1_{3,5} = 1.$, $w^1_{4,5} = .1$, $w^1_{1,6} = .5$, $w^1_{2,6} = .3$, $w^1_{3,6} = 2.$, $w^1_{4,6} = .1$, $w^1_{1,7} = .2$, $w^1_{2,7} = .4$, $w^1_{3,7} = 1.$, $w^1_{4,7} = .1$, $w^1_{1,8} = .3$, $w^1_{2,8} = .5$, $w^1_{3,8} = 1.$, $w^1_{4,8} = .2$, $w^1_{1,9} = .6$, $w^1_{2,9} = .2$, $w^1_{3,9} = 2.$, $w^1_{4,9} = .2$, $w^1_{1,10} = .3$, $w^1_{2,10} = .4$, $w^1_{3,10} = 1.$, $w^1_{4,10} = .3$, $w^1_{1,11} = .2$, $w^1_{2,11} = .7$, $w^1_{3,11} = 1.$, $w^1_{4,11} = .3$, $w^1_{1,12} = .3$, $w^1_{2,12} = .4$, $w^1_{3,12} = 1.$, $w^1_{4,12} = .1$, $w^1_{1,13} = .2$, $w^1_{2,13} = .3$, $w^1_{3,13} = 2.$, $w^1_{4,13} = .1$, $w^1_{1,14} = .5$, $w^1_{2,14} = .2$, $w^1_{3,14} = .1$, $w^1_{4,14} = .1$, $w^1_{1,15} = .5$, $w^1_{2,15} = .3$, $w^1_{3,15} = .1$, $w^1_{4,15} = .1$, $w^1_{1,16} = 1.$, $w^1_{2,16} = 1.$, $w^1_{3,16} = 1.$, $w^1_{4,16} = .2$, $w^1_{1,17} = 1.$, $w^1_{2,17} = 1.$, $w^1_{3,17} = 1.$, $w^1_{4,17} = .2$, $w^1_{1,18} = 1.$, $w^1_{2,18} = 1.$, $w^1_{3,18} = 1.$, $w^1_{4,18} = .2$, $w^1_{1,19} = 1.$, $w^1_{2,19} = 1.$, $w^1_{3,19} = 1.$, $w^1_{4,19} = .3$, $w^1_{1,20} = 1.$, $w^1_{2,20} = 1.$, $w^1_{3,20} = 1.$, $w^1_{4,20} = .2$, For class 2, the weights were: $w^2_{1,1} = .5$, $w^2_{2,1} = .5$, $w^2_{3,1} = .5$, $w^2_{4,1} = .1$, $w^2_{1,2} = .5$, $w^2_{2,2} = .4$, $w^2_{3,2} = .4$, $w^2_{4,2} = .1$, $w^2_{1,3} = .4$, $w^2_{2,3} = .3$, $w^2_{3,3} = .7$, $w^2_{4,3} = .1$, $w^2_{1,4} = .3$, $w^2_{2,4} = .2$, $w^2_{3,4} = .6$, $w^2_{4,4} = .1$, $w^2_{1,5} = .5$, $w^2_{2,5} = .4$, $w^2_{3,5} = .5$, $w^2_{4,5} = .2$, $w^2_{1,6} = .7$, $w^2_{2,6} = .6$, $w^2_{3,6} = .7$, $w^2_{4,6} = .2$, $w^2_{1,7} = .4$, $w^2_{2,7} = .3$, $w^2_{3,7} = .8$, $w^2_{4,7} = .1$, $w^2_{1,8} = .3$, $w^2_{2,8} = .2$, $w^2_{3,8} = .6$, $w^2_{4,8} = .1$, $w^2_{1,9} = .2$, $w^2_{2,9} = .3$, $w^2_{3,9} = .9$, $w^2_{4,9} = .4$, $w^2_{1,10} = .1$, $w^2_{2,10} = .4$, $w^2_{3,10} = .8$, $w^2_{4,10} = .4$, $w^2_{1,11} = .4$, $w^2_{2,11} = .5$, $w^2_{3,11} = .9$, $w^2_{4,11} = .3$, $w^2_{1,12} = .5$, $w^2_{2,12} = .5$, $w^2_{3,12} = .7$, $w^2_{4,12} = .3$, $w^2_{1,13} = .4$, $w^2_{2,13} = .6$, $w^2_{3,13} = .9$, $w^2_{4,13} = .1$, $w^2_{1,14} = .3$, $w^2_{2,14} = .4$, $w^2_{3,14} = 1.$, $w^2_{4,14} = .1$, $w^2_{1,15} = .2$, $w^2_{2,15} = .3$, $w^2_{3,15} = .2$, $w^2_{4,15} = .2$, $w^2_{1,16} = 1.$, $w^2_{2,16} = 1.$, $w^2_{3,16} = 1.$, $w^2_{4,16} = .2$, $w^2_{1,17} = 1.$, $w^2_{2,17} = 1.$, $w^2_{3,17} = 1.$, $w^2_{4,17} = .1$, $w^2_{1,18} = 1.$, $w^2_{2,18} = 1.$, $w^2_{3,18} = 1.$, $w^2_{4,18} = .1$, $w^2_{1,19} = 1.$, $w^2_{2,19} = 1.$, $w^2_{3,19} = 1.$, $w^2_{4,19} = .1$, $w^2_{1,20} = 1.$, $w^2_{2,20} = 1.$, $w^2_{3,20} = 1.$, $w^2_{4,20} = .1$.

The modified projection method converged in 93 iterations. It yielded the multiclass link flow and total flow pattern reported below and the equilibrium path flows.

The Opportunity Cost and Safety Cost Functions for the Links for the Teleshopping Example

Link a	$o_a(f)$	$s_a(f)$
1	$2f_1 + 4$	$f_1 + 1$
2	$3f_2 + 2$	$f_2 + 2$
3	$f_3 + 1$	$f_3 + 1$
4	$f_4 + 1$	$f_4 + 1$
5	$2f_+5$	$f_5 + 1$
6	$3f_6 + 6$	$2f_6 + 1$
7	$f_7 + 1$	$f_7 + 1$
8	$f_8 + 1$	$f_8 + 1$
9	$5f_9 + 12$	$f_9 + 1$
10	$11f_{10} + 11$	$f_{10} + 11$
11	$f_{11} + 1$	$f_{11} + 1$
12	$f_{12} + 1$	$f_{12} + 1$
13	$f_{13} + 11$	$f_{13} + .5$
14	$6f_{14} + 21$	$2f_{14} + 1$
15	$7f_{15} + 14$	$f_{15} + .5$
16	$5f_{16} + 10$	$f_{16} + 1$
17	$f_{17} + 2$	$f_{17} + 1$
18	$2f_{18} + 1$	$f_{18} + 1$
19	$f_{19} + 1$	$f_{19} + 1$
20	$f_{20} + 1$	$f_{20} + 1$

The Equilibrium Link Flows for the Teleshopping Example

Link a	Class 1 - f_a^{1*}	Class 2 - f_a^{2*}	Total flow - f_a^*
1	0.000	0.4662	0.4662
2	1.2921	0.0000	1.2921
3	1.8871	0.0000	1.8871
4	0.6965	0.0000	0.6965
5	1.0004	0.0000	1.0004
6	0.0000	0.1612	0.1612
7	0.0000	0.1573	0.1573
8	0.0000	1.0884	1.0884
9	1.0040	0.4462	1.4666
10	1.2921	0.1612	1.4533
11	1.8871	0.1573	2.0444
12	0.6965	1.0884	1.7848
13	0.0000	0.4662	0.4662
14	1.0004	0.0000	1.0004
15	1.2921	0.0000	1.2921
16	0.0000	0.1612	.01612
17	1.8871	0.0000	1.8871
18	0.0000	0.1573	0.1573
19	0.6965	0.0000	0.6965
20	0.0000	1.0884	1.0884

The incurred generalized path costs were:

for Class 1, O/D pair ω_1:

$$C^1_{p_1} = 1099.8754, \quad C^1_{p_2} = 950.3034,$$

$$C^1_{p_3} = 963.5551, \quad C^1_{p_4} = 975.1231,$$

for Class 1, O/D pair ω_2:

$$C^1_{p_5} = 920.8164, \quad C^1_{p_6} = 1216.0173,$$

$$C^1_{p_7} = 1047.4919, \quad C^1_{p_8} = 1114.4659,$$

for Class 2, O/D pair ω_1:

$$C^2_{p_1} = 579.2358, \quad C^2_{p_2} = 1795.4930,$$

$$C^2_{p_3} = 970.5146, \quad C^2_{p_4} = 947.5757,$$

and for Class 2, O/D pair ω_2:

$$C^2_{p_5} = 1144.5128, \quad C^2_{p_6} = 1043.4633,$$

$$C^2_{p_7} = 1067.9226, \quad C^2_{p_8} = 1063.9229.$$

The incurred inverse demands were:

$$\lambda^1_{\omega_1} = 954.0622, \quad \lambda^1_{\omega_2} = 919.9000,$$

$$\lambda^2_{\omega_1} = 579.9067, \quad \lambda^2_{\omega_2} = 1049.8593.$$

Here, note the separation of classes of consumers. Observe that, in the case of the first O/D pair, consumers of class 1 only utilized paths 2, 3, and 4, whereas consumers of class 2 only utilized path 1. Hence, for this O/D pair, consumers of class 2 all shopped on the Internet, whereas only some of the consumers of class 1 did, with others electing to select and purchase the product at physical locations. However, in the case of the second O/D pair, class 1 consumers only utilized path 5, whereas consumers of class 2 utilized paths 6, 7, and 8, only. Hence, in regards to the second O/D pair, consumers of class 1 now elected to all shop virtually. Thus, for the second O/D pair, consumers of class 2 now shopped both virtually and physically for the product.

Summary and Conclusions

• We have developed a multiclass, multicriteria network equilibrium framework for decision-making in the Information Age and applied it to telecommuting and teleshopping decision-making.

• The framework can consider either elastic or fixed demands, and handles distinct classes of decision-makers, each of whom weights a finite number of criteria distinctly. The weights are not only class-depenent but also link-dependent.

• The models are formulated as finite-dimensional variational inequality problems and these formulations are then utilized to establish both qualitative properties of the equilibrium patterns as well as a computational procedure, along with convergence results.

Additional information as well as reports on these and such subjects including:

- **supply chain networks and electronic commerce**

- **dynamic financial networks with intermediation**

- **supernetworks and the environment**

can be found at the Virtual Center for Supernetworks at http://supernet.som.umass.edu

The book **Supernetworks: Decision-Making for the Information Age** co-authored by Nagurney and Dong and published by Edward Elgar Publishers in the series *New Dimensions in Networks* will be avilable in February 2002.

Here we list the references cited in the lecture as well as additional ones on the topic.

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