Environmental Networks

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Problems associated with environmental pollution are some of the most pressing ones facing policy-makers today. Concern with the deterioration, for example, of the quality of air and water resources is bringing new methodologies from a variety of disciplines to this problem domain.

In this lecture, we present environmental networks as a tool for the visualization and study of environmental problems. Environmental networks were introduced by Dhanda, Nagurney, and Ramanujam (1998) as a tool for economic decision-making and policy analysis.

In particular, we present a spatial oligopoly network model with ambient-based pollution permits on the production side. Pollution permits, whose formal study dates to Crocker (1966) and Dales (1968), are an economic-incentive based approach to pollution reduction. Their rigorous mathematical formulation is due to Montgomery (1972). Ambient-based pollution permits, as opposed to emission pollution permits, utilize dispersion information in order to model the spatial effects of trades on each receptor site.

Nagurney and Dhanda (1996) were the first to propose a variational inequality approach for marketable ambient-based pollution permits, which allowed for the modeling of alternative market behavior on the side of producers', specifically, that of oligopolistic behavior.

Nagurney, Dhanda, and Stranlund (1997) subsequently proposed a multiproduct, multipollutant perfectly competitive firm model with pollution permits.

Nagurney and Dhanda (1997a) developed a variational inequality framework for the modeling, qualitative analysis, and computation of equilibrium patterns in multiproduct, multipollutant oligopolistic markets with marketable pollution permits with opportunities for investment in production technology and/or emission/abatement technology.

Nagurney and Dhanda (1997b) studied issues of compliance versus noncompliance in both static and dynamic settings in oligopolistic markets with ambient-based pollution permits.

Nagurney and Dhanda (1997c) then modeled transaction costs within oligopolistic markets and ambient-based permit systems.

Nagurney, Thore, and Pan (1996) had earlier used the theory of variational inequalities in their analysis of spatial market policies with targets on the supply points, the demand points, and the transportation links within a taxation/subsidization setting.

The Spatial Oligopoly Model with Permits

We consider the spatial oligopoly model. We assume that there are m firms and n demand markets that are generally spatially separated and are involved, respectively, in the production and consumption of a homogeneous commodity in a noncooperative fashion. We let q_i denote the nonnegative commodity output produced by firm i. We group the production outputs into the column vector $q \in R^m_+$.

Let d_j denote the demand for the commodity at demand market j and group the demand into the column vector $d \in R^n_+$.

Let T_{ij} denote the nonnegative commodity shipment from supply market i to demand market j. We group the commodity shipments for firm i into the column vector $T_i \in R^n_+$ and the commodity shipments of all the firms into the column vector $T \in R^{mn}_+$.

The following conservation of flow equations must hold:

$$q_i = \sum_{j=1}^{n} T_{ij}, \quad i = 1, \ldots, m \tag{1}$$

$$d_j = \sum_{i=1}^{m} T_{ij}, \quad j = 1, \ldots, n, \tag{2}$$

where $T_{ij} \geq 0$, $\forall i, j$.

Equation (1) states that the production output of the commodity for each firm must be equal to the commodity shipments from that firm to all the demand markets.

Equation (2) says that the demand for the commodity at each demand market must be equal to the sum of the commodity shipments from all the firms to that demand market.

We associate with each firm i a production cost f_i, where

$$f_i = f_i(q). \tag{3a}$$

In view of (1), we may define the production cost function

$$\hat{f}(T) = f(q). \tag{3b}$$

We assume that each firm i is faced with a cost of emission G_i, where

$$G_i = G_i(e_i, q_i), \tag{4a}$$

where e_i denotes the rate at which firm i emits the pollutant associated with production. We may, in view of (1), define the emission cost function $\hat{G}_i$, where

$$\hat{G}_i \equiv \hat{G}_i(e_i, T_i) \tag{4b}$$

and, hence, the emission cost function is also a function of the commodity shipments.

We group the firms' emissions into the vector $e \in R^m_+$.

We allow the demand price for the commodity at a demand market to depend, in general, upon the entire consumption pattern, that is,

$$p_j = p_j(d). \tag{5a}$$

In view of the conservation of flow equation (2), we may define the demand price function $\hat{p}$ as

$$\hat{p}_j = \hat{p}_j(T) \tag{5b}$$

and, therefore, the demand price function can also be expressed in terms of the commodity shipments.

Such a transformation of the production cost, emission cost, and demand price functions reduces the number of variables in the model without any loss in generality.

Let t_{ij} denote the transaction cost, which includes the transportation cost, associated with trading (shipping) the commodity between firm i and demand market j.

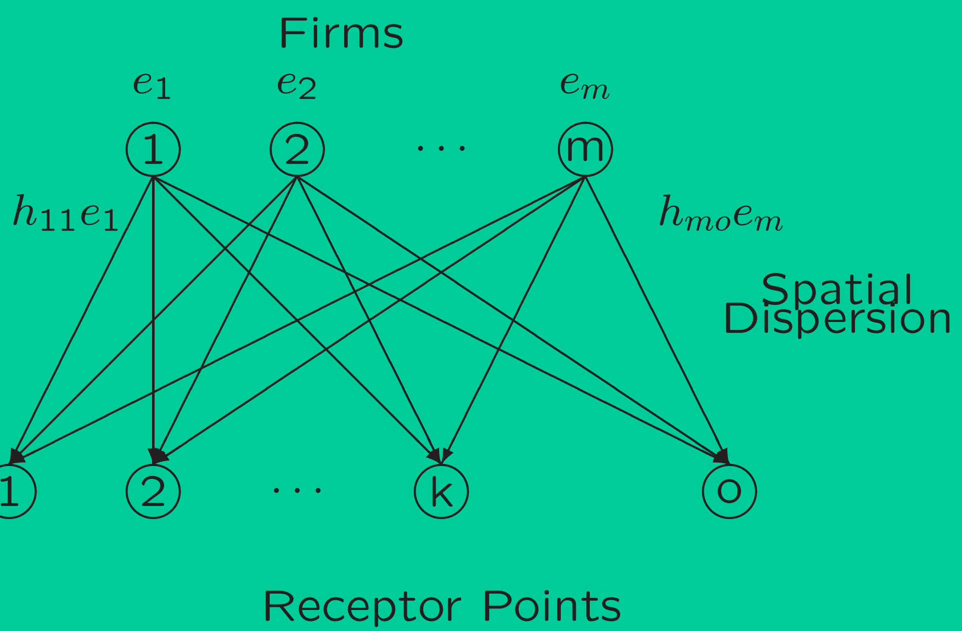
We allow the transaction cost to depend, in general, upon the entire shipment pattern, that is,

$$t_{ij} = t_{ij}(T). \tag{6}$$

We now introduce the notation for the ambient-based permit system (APS). We assume that there are o receptor points for the emissions generated in the production of the commodities by the firms, with a typical receptor point denoted by k. Let h_{ik} denote the contribution that one unit of emission generated by the production of the commodity by firm i makes to the average pollution concentration at receptor point k.

In Figure 1, we provide a network depiction of the spatial dispersion of pollution for this problem. The network representation emphasizes the spatial nature of pollution dispersion. Moreover, the receptor points for the pollution may not even be near the demand markets or the locations of the firms.

Assume that the firms are given an initial endowment of licenses given by: l^0_{ik}; $i = 1, \ldots, m; k = 1, \ldots, o$. Let p^*_k denote the price of a license to pollute at receptor point k. Then the value of a firm's initial endowment of licenses is given by: $\sum_{k=1}^{o} p^*_k l^0_{ik}$.



The network structure of spatial pollution dispersion for the spatial oligopoly problem

The profit u_i of firm i is then

$$u_i = \sum_{j=1}^{n} \widehat{p}_j(T)T_{ij} - \widehat{f}_i(T) - \sum_{j=1}^{n} t_{ij}(T)T_{ij} - \widehat{G}_i(e_i, T_i)$$

$$- \sum_{k=1}^{o} p^*_k(l_{ik} - l^0_{ik}). \qquad (7)$$

The first term to the right-hand side of the equality in (7) expresses the revenue of the firm, whereas the subsequent terms represent, respectively, the production cost, the total cost of shipping the commodity to the demand markets, the emission cost, and the cost of purchasing the permits associated with production.

One may write (7) as

$$u_i = u_i(T, e_i, l_i), \qquad (8)$$

where $l_i = \{l_{i1}, \ldots, l_{io}\}$ denotes the vector of licenses held by firm i associated with production emissions.

An oligopolistic firm's optimization problem is then expressed as:

$$\text{Maximize} \quad u_i(T, e_i, l_i) \tag{9}$$

subject to:

$$h_{ik} e_i \leq l_{ik}, \quad k = 1, \ldots, o, \tag{10}$$

and

$$e_i \geq 0, \quad l_{ik} \geq 0, \quad T_{ij} \geq 0, \quad k = 1, \ldots, o; \, j = 1, \ldots, n. \tag{11}$$

Inequality (10) states that each firm cannot emit above the rate that it holds licenses to emit at each receptor point whereas (11) guarantees that the variables are nonnegative.

Now consider the usual oligopolistic market mechanism, in which the m firms supply the commodity in a noncooperative fashion, each one trying to maximize its own profit. We seek to determine a nonnegative commodity shipment, emission, and license pattern, (T^*, e^*, l^*), for which the m firms will be in a state of Nash (1950) equilibrium as defined below.

Definition 1
(A Spatial Nash Equilibrium with Permits)

A commodity shipment, emission, and license pattern, (T^*, e^*, l^*) $\in R_+^{mn+m+mo}$*, is said to constitute a Nash equilibrium if for each firm* $i; i = 1, \ldots, m$*,*

$$u_i(T_i^*, \hat{T}_i^*, e_i^*, l_i^*) \geq u_i(T_i, \hat{T}_i^*, e_i, l_i),$$

$$\forall T_i \in R_+^n, \forall e_i \in R_+, \forall l_i \in R_+^o, \text{ satisfying } (10), \forall i, \quad (12)$$

where

$$T_i \equiv \{T_{i1}, \ldots, T_{in}\} \quad \text{and} \quad \hat{T}_i^* \equiv (T_1^*, \ldots, T_{i-1}^*, T_{i+1}^*, \ldots, T_m^*).$$

We let τ_{ik}, i; $k = 1, \ldots, o$, denote the marginal cost of abatement associated with constraint (10) (see, e. g., Montgomery (1972) and Nagurney and Dhanda (1996)).

We group the τ_{ik}'s into the vector $\tau_i \in R^o_+$ and then we further group all such vectors for all the firms into the vector $\tau \in R^{mo}_+$.

Observe that the profit function for each firm i is concave with respect to the license vector l_i, since the function is linear in the licenses.

Assuming that the profit function $u_i(T, e_i, l_i)$ is concave with respect to T_i and e_i, and that $u_i(T, e_i, l_i)$ is continuously differentiable, then the necessary and sufficient conditions for an optimal product, emission, license, and marginal abatement cost pattern (associated with constraint (10)), given the license price vector, $p^* = \{p^*_1, \ldots, p^*_o\}$, is that the pattern, $(T^*_i, e^*_i, l^*_i, \tau^*_i) \in R^{n+2o+1}_+$, and satisfies the inequality:

$$
\sum_{j=1}^{n}\left[\frac{\partial \hat{f}_i(T^*)}{\partial T_{ij}} + t_{ij}(T^*) - \hat{p}_j(T^*)\right.
$$

$$
+\sum_{g=1}^{n}\left[\frac{\partial t_{ig}(T^*)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^*)}{\partial T_{ij}}\right] T^*_{ig} - \left.\frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial T_{ij}}\right] \times \left[T_{ij} - T^*_{ij}\right]
$$

$$
+\left[\frac{\partial \hat{G}_i(e^*_i, Q^*_i)}{\partial e_i} + \sum_{k=1}^{o} \tau^*_{ik} h_{ik}\right] \times [e_i - e^*_i]
$$

$$
+\sum_{k=1}^{o} [p^*_k - \tau^*_{ik}] \times [l_{ik} - l^*_{ik}]
$$

$$
+\sum_{k=1}^{o} [l^*_{ik} - h_{ik} e^*_i] \times [\tau_{ik} - \tau^*_{ik}] \geq 0,
$$

$$
\forall T_i \in R^n_+, \ \forall e_i \in R_+, \ \forall l_i \in R^o_+, \ \forall \tau_i \in R^o_+. \qquad (13)
$$

In the case of Nash equilibrium, an inequality similar to (13) must hold for each of the oligopolistic firms i; $i = 1, \ldots, m$.

In Figure 2, we present the environmental network for the spatial oligopoly problem with emissions on the production side.

We now present the market equilibrium conditions for the licenses.

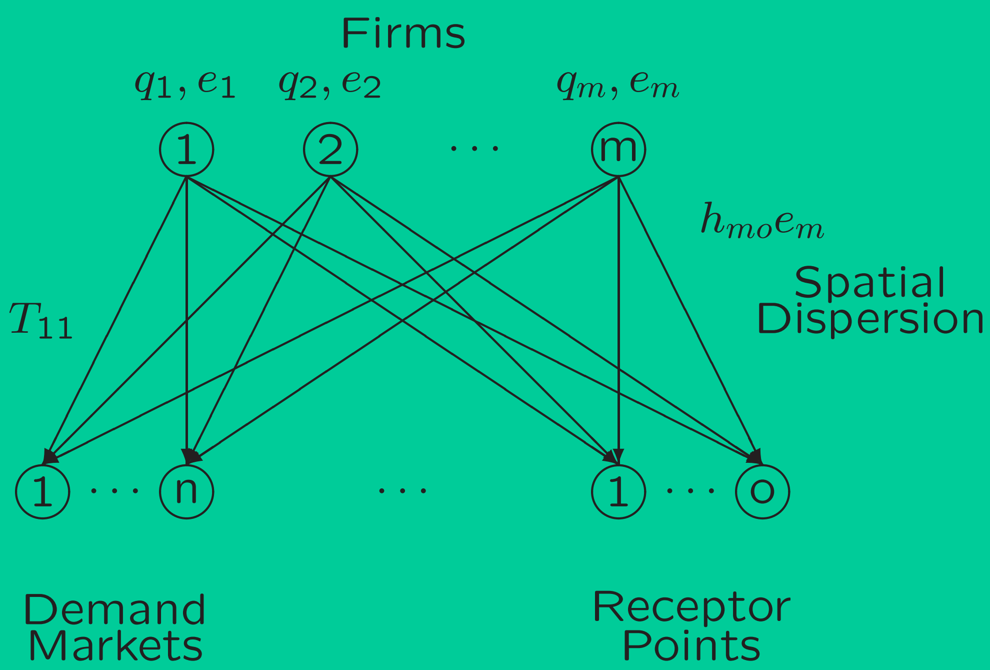
Market Equilibrium Conditions for Licenses

In terms of the market equilibrium conditions for the emission licenses associated with the production of the commodities by the firms, the equilibrium license prices p^* must satisfy the following conditions: For all receptor points: k; $k = 1, \ldots, o$:

$$\sum_{i=1}^{m} (l_{ik}^0 - l_{ik}^*) \begin{cases} = & 0, \quad \text{if } p_k^* > 0, \\ \geq & 0, \quad \text{if } p_k^* = 0. \end{cases} \tag{14}$$

Definition 2 (Ambient-Based Permit System Spatial Oligopolistic Equilibrium)

A vector of commodity shipments, emissions, licenses, marginal costs of abatement, and license prices, $(T^*, e^*, l^*, \tau^*, p^*) \in R_+^{mn+m+2mo+o}$*, is an equilibrium of the ambient-based permit system spatial oligopoly problem if it satisfies inequality (13) for all firms* i*;* $i = 1, \ldots, m$*, and the market equilibrium conditions (14) for all receptor points* k*;* $k = 1, \ldots, o$*.*



The environmental network for the spatial oligopoly

We now present the variational inequality formulation of the governing equilibrium conditions for this problem. It is presented without proof since the proof follows using similar arguments as in the variational inequality derivations provided in earlier lectures.

Theorem 1 (Variational Inequality Formulation of the APS Spatial Oligopoly Model)

A vector of commodity shipments, emissions, licenses, marginal costs of abatement, and license prices, $(T^*, e^*, l^*, \tau^*, p^*) \in R_+^{mn+m+2mo+o}$, is an equilibrium if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\Bigg[\frac{\partial \hat{f}_i(T^*)}{\partial T_{ij}} + t_{ij}(T^*) - \hat{p}_j(T^*)$$

$$+\sum_{g=1}^{n}\left[\frac{\partial t_{ig}(T^*)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^*)}{\partial T_{ij}}\right] T^*_{ig} - \frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial T_{ij}}\Bigg] \times \left[T_{ij} - T^*_{ij}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial e_i} + \sum_{k=1}^{o} \tau^*_{ik} h_{ik}\right] \times [e_i - e^*_i]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}[p^*_k - \tau^*_{ik}] \times [l_{ik} - l^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}[l^*_{ik} - h_{ik}e^*_i] \times [\tau_{ik} - \tau^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[l^0_{ik} - l^*_{ik}\right] \times [p_k - p^*_k] \geq 0,$$

$$\forall (T, e, l, \tau, p) \in R_+^{mn+m+2mo+o}. \qquad (16)$$

We now put variational inequality (16) into standard form. Define the column vector $x \equiv (T, e, l, \tau, p)$ and the column vector $F(x)$ consisting of the column vectors:

$$(C(x), E(x), L(x), T(x), P(x)),$$

where $C(x)$ is the mn-dimensional vector with component (i, j) given by:

$$C_{ij}(x) = \sum_{j=1}^{n} \left[\frac{\partial \hat{f}_i(T^*)}{\partial T_{ij}} + t_{ij}(T) - \hat{p}_j(T^*) \right.$$

$$\left. + \sum_{g=1}^{n} \left[\frac{\partial t_{ig}(T^*)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^*)}{\partial T_{ij}} \right] T^*_{ig} - \frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial T_{ij}} \right],$$

$E(x)$ is the m-dimensional vector with component i given by:

$$E_i(x) = \left[\frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial e_i} + \sum_{k=1}^{o} \tau^*_{ik} h_{ik} \right],$$

$L(x)$ is the mo-dimensional vector with component (i, k) given by:

$$L_{ik}(x) = [p_k - \tau_{ik}],$$

$T(x)$ is the mo-dimensional vector with component (i, k) given by:

$$T_{ik}(x) = [l_{ik} - h_{ik} e_i],$$

and $P(x)$ is the o-dimensional vector with component k; $k = 1, \ldots, o$, given by:

$$P_k(x) = \sum_{i=1}^{m} \left[l_{ik}^0 - l_{ik} \right].$$

Indeed, variational inequality (16) can now be expressed as:

Determine $x^* \in K$, such that

$$\langle F(x^*)^T, x - x^* \rangle \geq 0, \quad \forall x \in K, \tag{17}$$

where $K \equiv \{x = (T, e, l, \tau, p) \in R_+^{mn+m+2mo+o}\}$.

We establish, in the subsequent corollary, that the equilibrium pattern is independent of the initial license allocation, provided that the sum of licenses for each receptor point is fixed.

Corollary 1 (Equilibrium Pattern Independence from Initial License Allocation)

If $l^0_{ik} \geq 0$, for all $i = 1, \ldots, m$; $k = 1, \ldots, o$, and $\sum_{i=1}^m l^0_{ik} = \bar{E}_k$, for $k = 1, \ldots, o$, with each $\bar{E}_k$ fixed and positive, then the equilibrium pattern $(T^*, e^*, l^*, \tau^*, p^*)$ is independent of $\{l^0_{ik}\}$.

Proof: The first five terms of variational inequality (16) are independent of $\{l^0_{ik}\}$, whereas the last term depends only on the sum $\sum_{i=1}^m l^0_{ik}$.

In the next theorem, we provide a mechanism for determining the appropriate sums of the initial licenses.

Theorem 2
(Achievement of Environmental Standards)

An equilibrium pattern achieves environmental quality standards represented by the vector $\bar{E} = (\bar{E}_1, \ldots, \bar{E}_o)$ *provided that* $\sum_{i=1}^{m} l_{ik}^0 = \bar{E}_k$, *for* $k = 1, \ldots, o$.

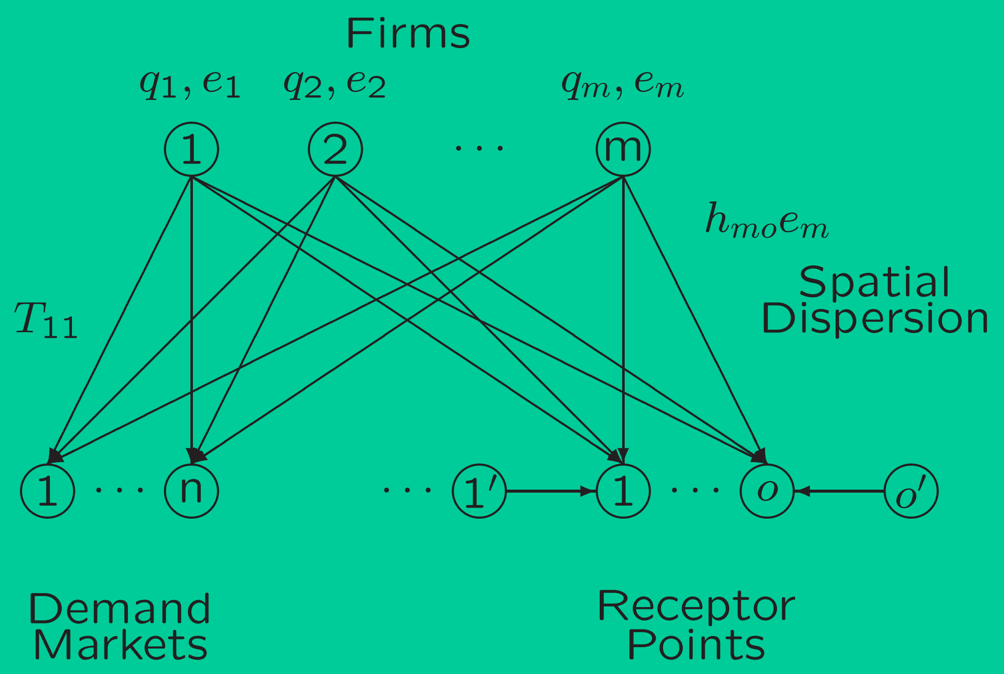
Proof: From inequality (10), we have that for each firm i; $i = 1, \ldots, m$:

$$h_{ik} e_i^* \leq l_{ik}^*, \quad k = 1, \ldots, o. \tag{18}$$

Further, it follows from equilibrium conditions (14) and the assumption on the initial license allocations that

$$\sum_{i=1}^{m} h_{ik} e_i^* \leq \sum_{i=1}^{m} l_{ik}^* \leq \sum_{i=1}^{m} l_{ik}^0 = \bar{E}_k, \quad \forall k. \tag{19}$$

In Figure 3, we depict the environmental standards network for the oligopoly problem. Note that the flows on the arcs $(1', 1), \ldots, (o', o)$, respectively, represent the amounts: $\bar{E}_1 - \sum_{i=1}^m h_{i1} e_i, \ldots, \bar{E}_o - \sum_{i=1}^m h_{io} e_i$. The licenses, in effect, serve as upperbounds on the pollution dispersion links.



The environmental standards network for the spatial oligopoly

Qualitative Properties

We now provide some qualitative properties of the variational inequality problem (16), equivalently, (17). We also establish the convergence of the modified projection method which can be applied to compute the equilibrium pattern for the spatial oligopoly problem with ambient-based pollution permits.

In particular, we first establish the existence of a solution pattern. We then show that the function $F(\cdot)$ that enters variational inequality (17) is monotone. We discuss Lipschitz continuity of the function. We present some sensitivity analysis results and conclude this section with the convergence result.

Theorem 3 (Existence)

If $(T^*, e^*, l^*, \tau^*, p^*) \in R_+^{mn+2mo+m+o}$ *satisfies variational inequality (17), the commodity shipment and emission pattern is a solution to the variational inequality problem:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial \hat{f}_i(T^*)}{\partial T_{ij}} + t_{ij}(T^*) - \hat{p}_j(T^*)\right.$$

$$\left. + \sum_{g=1}^{n}\left[\frac{\partial t_{ig}(T^*)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^*)}{\partial T_{ij}}\right] T^*_{ig} - \frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial T_{ij}}\right] \times \left[T_{ij} - T^*_{ij}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial \hat{G}_i(e_i^*,T_i^*)}{\partial e_i}\right]\times[e_i-e_i^*]\geq 0, \quad \forall (T,e)\in K^1, \quad (20)$$

where

$$K^1\equiv\Big\{(T,e)|(T,e)\in R_+^{mn+m};$$
$$\exists l_{ik}; l_{ik}\geq 0; h_{ik}e_i\leq l_{ik}, \forall i,k;$$
$$\sum_{i=1}^{m}(l_{ik}^0-l_{ik})\geq 0, \forall k\Big\}. \quad (21)$$

Moreover, a solution to (21) is guaranteed to exist provided that $-\nabla u(\cdot)$ is coercive. Furthermore, if (T^*,e^*) is a solution to (20), there exist $l^*\in R_+^{mo}$, $\tau^*\in R_+^{mo}$, and $p^*\in R_+^{o}$ with (T^*,e^*,l^*,τ^*,p^*) being a solution to variational inequality (17) and, hence, an equilibrium.

Proof: Assume, on the contrary, that is, for some $(T, e) \in K^1$, we have that

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left[\frac{\partial \hat{f}_i(T^*)}{\partial T_{ij}} + t_{ij}(T^*) - \hat{p}_j(T^*) \right.$$

$$+ \sum_{g=1}^{n} \left[\frac{\partial t_{ig}(T^*)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^*)}{\partial T_{ij}} \right] T^*_{ig}$$

$$\left. - \frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial T_{ij}} \right] \times \left[T_{ij} - T^*_{ij} \right]$$

$$+ \sum_{i=1}^{m} \sum_{k=1}^{o} \left[\frac{\partial \hat{G}_i(e^*_i, T^*_i)}{\partial e_i} \right] \times [e_i - e^*_i] < 0. \qquad (22)$$

But, according to variational inequality (17), it then follows that

$$\sum_{i=1}^{m} [\tau_{ik}^* h_{ik}] \times [e_i - e_i^*]$$

$$+ \sum_{i=1}^{m} \sum_{k=1}^{o} [p_k^* - \tau_{ik}^*] \times [l_{ik} - l_{ik}^*]$$

$$+ \sum_{i=1}^{m} \sum_{k=1}^{o} [l_{ik}^* - h_{ik} e_i^*] \times [\tau_{ik} - \tau_{ik}^*]$$

$$+ \sum_{k=1}^{o} \sum_{i=1}^{m} \left[l_{ik}^0 - l_{ik}^*\right] \times [p_k - p_k^*]$$

$$\geq - \sum_{i=1}^{m} \sum_{j=1}^{n} \left[\frac{\partial \hat{f}_i(T^*)}{\partial T_{ij}} + t_{ij}(T^*) - \hat{p}_j(T^*) \right.$$

$$+ \sum_{g=1}^{n} \left[\frac{\partial t_{ig}(T^*)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^*)}{\partial T_{ij}} \right] T_{ig}^*$$

$$\left. - \frac{\partial \hat{G}_i(e_i^*, T_i^*)}{\partial T_{ij}} \right] \times \left[T_{ij} - T_{ij}^*\right]$$

$$- \sum_{i=1}^{m} \left[\frac{\partial \hat{G}_i(e_i^*, T_i^*)}{\partial e_i} \right] \times [e_i - e_i^*] > 0. \qquad (23)$$

Letting now $\tau_{ik} = 0$, for all i, k, and $p_k = 0$ for all k, and substituting these values into the first expression in (23) yields, after algebraic simplifications:

$$\sum_{i=1}^{m} \sum_{k=1}^{o} \tau_{ik}^{*} \left[h_{ik} e_i - l_{ik} \right] + \sum_{i=1}^{m} \sum_{k=1}^{o} \left[l_{ik}^{0} - l_{ik} \right] \left[-p_k^{*} \right]. \qquad (24)$$

Clearly, in view of the feasible set K^1, the expression in (24) is negative and, hence, we have obtained a contradiction to (22). It thus follows that variational inequality (20) must be satisfied.

Furthermore, under the coercivity condition assumption on $-\nabla u$, the existence of a solution to (20) is guaranteed from the standard theory of variational inequalities. Moreover, since $\sum_{i=1}^{m} l_{ik}^{0}$ for all k is finite, l and e must lie in a compact set and, hence, the existence of vectors: l^* and e^* is guaranteed.

Finally, according to the Lagrange Multiplier Theorem, we are guaranteed the existence of the nonnegative multipliers τ^* and p^* associated, respectively, with the inequality constraints corresponding to the emissions and the licenses that comprise the feasible set K^1 and these must satisfy (17).

Lemma 1 (Monotonicity)

If the utility functions are concave for each firm i, then $F(x)$ is monotone.

Proof: In view of the definition of $F(x)$ in this model, the monotonicity condition here takes the form:

$$\langle (F(x^1) - F(x^2))^T, x^1 - x^2 \rangle \geq 0, \quad \forall x^1, x^2 \in K, \qquad (25)$$

where the left-hand side of (25) here, after algebraic simplifications, reduces to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\Bigg[\Bigg[\frac{\partial \hat{f}_i(T^1)}{\partial T_{ij}} + t_{ij}(T^1) - \hat{p}_j(T^1)$$

$$+\sum_{g=1}^{n}\left[\frac{\partial t_{ig}(T^1)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^1)}{\partial T_{ij}}\right] T^1_{ig} - \frac{\partial \hat{G}_i(e^1_i, T^1_i)}{\partial T_{ij}}\Bigg]$$

$$-\Bigg[\frac{\partial \hat{f}_i(T^2)}{\partial T_{ij}} + t_{ij}(T^2) - \hat{p}_j(T^2)$$

$$+\sum_{g=1}^{n}\left[\frac{\partial t_{ig}(T^2)}{\partial T_{ij}} - \frac{\partial \hat{p}_j(T^2)}{\partial T_{ij}}\right] T^2_{ig}$$

$$-\frac{\partial \hat{G}_i(e^2_i, T^2_i)}{\partial T_{ij}}\Bigg]\Bigg] \times \left[T^1_{ij} - T^2_{ij}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial \hat{G}_i(e^1_i, T^1_i)}{\partial e_i} - \frac{\partial \hat{G}_i(e^2_i, T^2_i)}{\partial e_i}\right] \times \left[e^1_i - e^2_i\right]. \qquad (26)$$

However, the expression in (26) is nonnegative under the assumption that the utility function is concave and, hence, the conclusion follows.

The subsequent lemma is presented without proof since its proof is similar to the Lipschitz continuity lemma proofs in Dhanda, Nagurney, and Ramanujam (1998).

Lemma 2 (Lipschitz Continuity)

The function $F(x)$ is Lipschitz continuous under the assumption that the utility functions have bounded second order derivatives.

We now provide some sensitivity analysis results surrounding changes in the initial license allocations and the effect on the equilibrium license prices. In particular, we address the effect of a change in the initial license allocation for the production emission licenses in Corollary 1.

Corollary 1

Consider the fixed change Δ^0_{ik}, where $-l^0_{ik} < \Delta^0_{ik} < \infty$, for firm i and receptor point k. Let Δp_k denote the resulting change in equilibrium price of the emission license for k. Then

$$-\Delta^0_{ik} \times \Delta p_k \geq 0. \tag{27}$$

Proof: Let $F(x)$ and x^* denote, respectively, the function in (17) and the solution of the variational inequality problem (17) before the perturbation. Let $\widehat{F}(x)$ and $\widehat{x}$ denote the perturbed function and the corresponding solution to the resulting variational inequality problem. We, thus, have that

$$\langle F(x^*)^T, x - x^* \rangle \geq 0, \quad \forall x \in K \tag{28}$$

and

$$\langle \widehat{F}(\widehat{x})^T, x - \widehat{x} \rangle \geq 0, \quad \forall x \in K. \tag{29}$$

Substituting $\widehat{x}$ for x in (28) and x^* for x in (29) and adding the resulting inequalities, yields:

$$\langle (F(x^*) - \widehat{F}(\widehat{x}))^T, \widehat{x} - x^* \rangle \geq 0, \tag{30}$$

and, with the use of Lemma 1:

$$-\Delta^0_{ik} \times \Delta p_k \geq \langle (F(x^*) - F(\widehat{x}))^T, x^* - \widehat{x} \rangle \geq 0. \tag{31}$$

Note that from Corollary 1, we have that if the initial license for firm i and receptor point k is increased (decreased) then the equilibrium price for a license at the receptor point decreases (increases).

As for the numerical computation of the equilibrium pattern, we know that the modified projection method is guaranteed to converge (Korpelevich (1977)), provided that the function $F(\cdot)$ enters the variational inequality problem is monotone and Lipschitz continuous. We thus have the following:

Theorem 4 (Convergence)

The modified projection method is guaranteed to converge to the solution of the variational inequality problem (17).

Proof: Follows from Lemmas 1 and 2.

In this lecture, we presented a spatial oligopoly network model with ambient-based pollution permits that specifically takes into consideration that demand markets may be spatially separated. The model allows for the spatial dispersion of emissions generated by the firms in the production of the commodity. The formulation and qualitative analysis of the model was done using the theory of variational inequalities.

For this problem, we presented the environmental network and established that the environmental standards are met, provided that the initial allocation of the production licenses are set accordingly.

For background on environmental economics, see Baumol and Oates (1988), the survey by Cropper and Oates (1992), and the books by Pearce and Turner (1990) and Tietenberg (1996). Tietenberg (1980, 1985) provide additional material on marketable pollution permits.

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