
Zugang Liu and Anna Nagurney
Department of Finance and Operations Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003
August 2007; revised April 2008

Abstract: In this paper, we develop a novel electric power supply chain network model with fuel supply markets that captures both the economic network transactions in energy supply chains and the physical network transmission constraints in the electric power network. The theoretical derivation and analyses are done using the theory of variational inequalities. We then apply the model to a specific case, the New England electric power supply chain, consisting of 6 states, 5 fuel types, 82 power generators, with a total of 573 generating units, and 10 demand market regions. The empirical case study demonstrates that the regional electric power prices simulated by the proposed model very well match the actual electricity prices in New England. We also compute the electric power prices under natural gas and oil price variations. The empirical examples illustrate that both the generating unit responsiveness and the electric power market responsiveness are crucial to the full understanding and determination of the impact of the residual fuel oil price on the natural gas price. Finally, we utilize the model to quantitatively investigate how changes in the demand for electricity influence the electric power and the fuel markets from a regional perspective.

The theoretical model can be applied to other regions and multiple electricity markets to quantify the interactions in electric power/energy supply chains and their effects on flows and prices in which deregulation is taking place.
1. Introduction

Electric power systems provide a critical infrastructure for the functioning of our modern economies and societies. Electric power lights (and cools) our homes, our commercial and industrial enterprises, powers our computers, and enables the production and dissemination of goods and services worldwide. It is undeniably an essential form of energy, whose absence and/or unavailability, can have profound and lasting impacts in both the developed and developing corners of the globe.

In order to understand the availability and, ultimately, reliability and vulnerability of electric power and the underlying systems, one must place the systems in the context of the industrial setting. For example, in the case of the United States, the electric power industry possesses more than half a trillion dollars of net assets, generates $220 billion annual sales, and consumes almost 40% of domestic primary energy [17, 18]. Currently, the electric power industry in the US is undergoing a deregulation process from once highly regulated, vertically integrated monopolistic utilities to emerging competitive markets [28, 32, 70]. This deregulation process has caused major changes to the electric power industry, and requires a deep and thorough identification of the structure of the emerging electricity supply chains, as well as new paradigms for the modeling, analysis, and computations for electric power markets.

Smeers [66] reviewed a wide range of models of energy markets with various market power assumptions (see also [1, 8, 9, 26, 48, 60, 65, 70, 71]). Hogan [31] proposed a market power model to study strategic interactions in an electricity transmission network. More recently, Chen and Hobbs [11] proposed an oligopolistic electricity market model with a nitrogen oxide permit market, and provided examples based on the PJM market (Pennsylvania, New Jersey, and Maryland). Nagurney and Matsypura [57], in turn, presented an electric power supply chain network model which provided an integrated perspective for electric power generation, supply, transmission, as well as consumption. Wu et al. [69] considered the generators’ generating unit portfolios and reformulated the electric power supply chain network model as a user-optimal transportation network model (see also [54]). Nagurney et al. [55] also established the connections between electric power supply chain networks and transportation networks, and developed a dynamic electric power supply chain network with time-varying
This paper focuses on the relationship and interaction between electric power supply chains and other energy markets. In the US, electric power generation accounts for 30% of the natural gas demand (over 50% in the summer), 90% of the coal demand, and over 45% of the residual fuel oil demand [19]. Moreover, in the US natural gas market, for the past four years, the demand from electric power generation has been significantly and steadily increasing while demands from all the other sectors (industrial, commercial, and residential) have been slightly decreasing [20].

For example, in New England (the northeastern region of the US consisting of the states of Connecticut, Massachusetts, Rhode Island, Vermont, New Hampshire, and Maine), since 1999, approximately 97 percent of all the newly installed generating capacity has depended partially or entirely on natural gas [33]. Hence, various energy markets are inevitably and constantly interacting with electric power supply chains. For instance, from December 1, 2005 to April 1, 2006, the wholesale electricity price in New England decreased by 38% mainly because the delivered natural gas price declined by 45% within the same period. For another example, in August, 2006, the natural gas price jumped 14% because hot weather across the US led to elevated demand for electricity. This high electricity demand also caused the crude oil price to rise by 1.6% [29]. Similarly, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures [62]. However, the quantitative connections between electric power supply chains and other fuel markets are not straightforward and depend on many factors, such as, the generating unit portfolios of power generating companies or generators (gencos), the technological characteristics of generating units as well as the underlying physical transportation/transmission networks (with their associated capacities).

Moreover, the availability and the reliability of diversified fuel supplies affect not only economic efficiency but also national security. For example, in January 2004, over 7000MW (megawatts or one million watts) of electric power generation, which accounts for almost one fourth of the total capacity of New England, was unavailable during the electric system peak due to the limited natural gas supply [37]. For another instance, the American Associ-
ation of Railroads has requested that the Federal Energy Regulatory Commission (FERC) investigate the reliability of the energy supply chain with a focus on electric power and coal transportation [6].

The relationships between electric power supply chains and other energy markets have drawn considerable attention from researchers in various fields. Emery and Liu [16] empirically estimated the cointegration of electric power futures and natural gas futures. Routledge, Seppi, and Spatt [63] focused on the connections between natural gas and electricity markets, and studied the equilibrium pricing of electricity contracts (see also [5]). Deng, Johnson, and Sogomonian [14] applied real option theory to develop models that utilize the relationship between fuel prices and electricity prices to value electricity generation and transmission assets. Huntington and Schuler [34] pointed out that the natural gas price was influenced by the residual fuel oil price because of the responsiveness of dual-fuel generating units in electric power networks (see also [2, 7]). The interesting interactions among oil, electric power, and natural gas markets will also be quantitatively investigated and generalized using our theoretical model and empirical examples. We note that Matsypura, Nagurney, and Liu [47] proposed the first network model that integrated fuel supplier networks and electric power supply chain networks. However, their model focused on the transactions of the electricity ownership and the economic decision-making processes of the market participants, and did not consider the physical constraints in electricity transmission networks and the electricity demand variations. Furthermore, no empirical results were presented.

In this paper, we develop a new electric power network model with fuel supply markets that considers both the economic network transactions in energy supply chains and the physical network transmission constraints in the electric power network which are critical to the understanding of the regional differences in electric power prices [9, 10, 30]. We then apply the model to a specific case, the New England electric power supply chain. In particular, we present an empirical case study that demonstrates that the regional electric power prices simulated by our model match the actual electricity prices in New England very well. We also compute the electric power prices under natural gas and oil price variations. The empirical examples illustrate that both the generating unit responsiveness and the electric power market responsiveness are crucial to the full understanding and estimation of the impact of the residual fuel oil price on the natural gas price. Finally, we utilize the model to quantita-
tively investigate how changes in the demand for electricity influence the electric power and the fuel markets.

This paper is organized as follows. In Section 2, we propose the integrated electric power supply chain and fuel supply network model. In Section 3, we provide some qualitative properties of the model and in Section 4, we discuss the computation of solutions to the model. In Section 5, we present a case study of the model applied to the New England electric power supply chain network. The empirical examples demonstrate how the theoretical model can be applied to investigate the interactions between electric power supply chains and fuel supply markets. The empirical application consists of 6 states, 5 fuel types, 82 power generators, with a total of 573 generating unit and generator combinations, and 10 demand market regions. Section 6 summarizes and concludes the paper and presents suggestions for future research.

2. The Integrated Electric Power Supply Chain and Fuel Supply Market Network Model

In this section, we develop the electric power supply chain network model which includes regional electricity markets and fuel supply markets.

2.1 A Brief Introduction

The electric power supply chain network model proposed in this paper includes four major components: the fuel supply markets, the power generators, the power buyers at demand markets, and the independent system operator (ISO).

The power generators or gencos purchase fuels from the supply markets and produce electric power at the generating units. Gencos can sell electric power to the power buyers at the demand markets directly through bilateral contracts or they can sell to the power pool which is managed by the ISO. Additionally, gencos can also sell their capacities in the regional operating reserve markets. We assume that each electric power generator seeks to determine the optimal production and allocation of the electric power in order to maximize his own profit.
The power buyers at the demand markets search for the lowest electricity price. They can purchase electric power either directly from the gencos or from the power pool. For example, in the New England electric power supply chain, about 75 – 80% of electricity is traded through bilateral contracts, while about 20 – 25% of electricity is traded through the power pool [43]. Thus, the power pools function as markets that balance the residuals of supply and demand, and clear the regional wholesale electricity markets [43].

In most deregulated electricity markets, there is a non-profit independent system operator (ISO) who manages the wholesale electricity market, maintains system reliability, and oversees the transmission network to ensure system security (e.g. New England ISO, www.iso-ne.com; PJM Interconnection, www.pjm.com; and Electric Reliability Council of Texas, www.ercot.com). One of the ISO’s objectives is to minimize the total cost and to achieve economic efficiency of the system by maintaining a competitive wholesale electricity market. The ISO undertakes two critical responsibilities in order to achieve a competitive and efficient wholesale market. First, the ISO dispatches the generating units that participate in the power pool, starting with the generating unit that offers the lowest supply price. In each regional power pool, all power suppliers will be paid at the same unit price which is equal to the market clearance price in that region. Secondly, the ISO schedules all transmission requests and monitors the entire transmission network; he also charges the network users congestion fees if certain transmission interface limits are reached [9, 10, 30]. Moreover, the ISO manages the operating reserve markets where the power generators can get paid for holding back their capacities to help to ensure system reliability. These major features of the ISO are fully reflected in our model.

We let $1, \ldots, a, \ldots, A$ denote the types of fuels. We assume that there are $M$ supply markets for each type of fuel. We assume that the electric power supply chain network includes $1, \ldots, r, \ldots, R$ regions which can be defined based on electricity transmission network interfaces. We let $1, \ldots, g, \ldots, G$ denote the gencos who may own and operate multiple generating units which may use different generating technologies and are located in various regions. For example, the genco, Con Edison Energy, owns one generating unit in New Hampshire, one generating unit in southeastern Massachusetts, and six generating units in western and central Massachusetts [39]. In particular, we let $N_{gr}$ denote the number of generating units owned by genco $g$ in region $r$. Note that in each region $r$, different gencos may
have a different number of generating units. If genco \( g \) has no generating unit in region \( r \), then \( N_{gr} = 0 \). We let \( N = \sum_{g=1}^{G} \sum_{r=1}^{R} N_{gr} \) denote the total number of generating units in the network. We assume that in each region there exist \( K \) demand market sectors which can be distinguished from one another by the types of associated consumers and the electricity consumption patterns.

The top-tiered nodes in the electric power supply chain network in Figure 1 represent the AM fuel supply markets. The nodes at the second tier in Figure 1 represent the generating units associated with the gencos and regions. The three indices of a generating unit indicate the genco that owns the unit, the region of the unit, and the sequential identifier of the unit, respectively. For example, in region 1, the node on the left denotes genco 1’s first generating unit in region 1 while the node on the right denotes genco \( G \)’s \( N_{G1}^{th} \) (last) generating unit in region 1. The bottom-tiered nodes in Figure 1 represent the \( RK \) region and demand market combinations.

Our model focuses on a single period, the length of which can be from several days to a month. We assume that the prices charged by the fuel suppliers in transacting with the gencos are relatively stable and do not change throughout the period. However, the electric power demands and prices may exhibit a strong periodic pattern within a day with the highest price typically being two to four times higher than the lowest price. Hence, we allow the electric power prices and demands to vary frequently within the study period. Note that there are two typical ways to represent electric power demand variations: load curves and load duration curves. A load curve plots electricity demands in temporal sequence while a load duration curve sorts and plots demand data according to the magnitude of the demands (e.g. [27, 46]). A point on the load duration curve represents the proportion of time that the demand is above a certain level.

In this paper, we use the discretized load duration curve to represent the demand variations within a period. In particular, we divide the load duration curve of the study period into \( 1, \ldots, w, \ldots, W \) blocks with \( L_w \) denoting the time length of block \( w \). In our empirical examples, using the New England electric power supply chain network as a case study in Section 5, we assume that there are six demand levels, consisting of two peak demand levels, two intermediate demand levels, and two low demand levels; hence, \( W = 6 \).
Generating Units of Gencos in Regions (genco, region, unit)

Fuel Markets for Fuel Type 1

Fuel Markets for Fuel Type $a$

Fuel Markets for Fuel Type $A$

Figure 1: The Electric Power Supply Chain Network with Fuel Supply Markets
We now summarize the critical assumptions of the model:

1. The model focuses on a single period, the length of which can be from several days to a month.

2. The fuel prices are relatively stable and do not fluctuate within the study period while the regional electricity prices fluctuate significantly as the demands vary within the study period.

3. The regions can be defined based on the transmission network interfaces.

4. Each genco can own and operate multiple generating units which may use different technologies and are located in different regions. A genco is an economic entity and is not restricted to a specific region while a genco’s generating units are physically related to regions.

5. Each genco maximizes his own profit.

6. The decisions made by each genco include the quantities of fuels purchased, the production level of each of his generating units at each demand level, the sales of electricity to the power buyers and to the power pool at each demand level, and the amount of capacity sold at the operating reserve market.

7. Power buyers search for the lowest electric power price.

8. Power buyers can purchase electricity directly from the gencos through bilaterial contracts or from the power pool.

9. The ISO is a non-profit organization that maintains a competitive wholesale electricity market and ensures system security and reliability.

10. The ISO dispatches the generating units that participate in the power pool, starting with the generating unit that offers the lowest supply price.

11. The ISO schedules all transmission requests, monitors the transmission network, and charges congestion fees if certain transmission interface limits are reached.

12. The ISO manages operating reserve market to ensure system reliability.
2.2 The Model of the Integrated Electric Power Supply Chain Network with Fuel Supply Markets

We now present the electric power supply chain network model. We first describe the behavior of the fuel suppliers, the gencos, the ISO, and the power buyers at demand markets. We then state the equilibrium conditions for the electric power supply chain network and provide the variational inequality formulation.

Before we introduce the model, we would like to explain the use of $r_1$ and $r_2$ in the notation of the model. In general, a transaction of electricity is related to two regions (nodes): the region of injection where the generating unit is located and the region of withdrawal where the power buyer is located. In order to avoid confusion of the two regions, we use $r_1$ and $r_2$ to present the region of injection and the region of withdrawal, respectively.

In our notation system, the $m^{th}$ supply market of fuel $a$ is denoted by $am$; the $u^{th}$ generating unit owned by genco $g$ in region $r_1$ is denoted by $gr_1u$; and the $k^{th}$ demand market sector in region $r_2$ is denoted by $r_2k$.

For succinctness, we give the notation for the model in Tables 1, 2, 3, and 4. An equilibrium solution is denoted by “∗”. All vectors are assumed to be column vectors, except where noted. If a variable or cost function is related to the transaction of fuels, the superscript indicates the fuel market while the subscript indicates the generating unit and the demand level; if a variable or cost function is related to the bilateral transaction of electric power, the superscript indicates the generating unit while the subscript indicates the demand market and the demand level; and if a variable or cost function is related to electricity production or operating reserve, there is no superscript while the subscript indicates the generating unit as well as the demand level. The actual units for the prices, the demands, and the electric power flow transactions used in practice are explicated in Section 5, which contains the empirical case study and the associated examples.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$\bar{q}_{am}$</td>
<td>The aggregated demand from sectors except that of electric power generation at the $m^{th}$ supply market of fuel $a$.</td>
</tr>
<tr>
<td>$c_{amgr1uw}$</td>
<td>The unit transportation/transaction cost between the $m^{th}$ supply market of fuel $a$ and the $u^{th}$ generating unit of genco $g$ in region $r_1$ at demand level $w$.</td>
</tr>
<tr>
<td>$L_w$</td>
<td>The time duration of demand level $w$.</td>
</tr>
<tr>
<td>$TCap_b$</td>
<td>The interface (flowgate) limit of interface $b$.</td>
</tr>
<tr>
<td>$Cap_{gr1u}$</td>
<td>The generating capacity of the $u^{th}$ generating unit of genco $g$ in region $r_1$.</td>
</tr>
<tr>
<td>$OP_{gr1u}$</td>
<td>The maximum level of operating reserve that can be provided by the $u^{th}$ generating unit of genco $g$ in region $r_1$.</td>
</tr>
<tr>
<td>$OPR_{r1w}$</td>
<td>The operating reserve requirement of region $r_1$ at demand level $w$.</td>
</tr>
<tr>
<td>$\beta_{gr1ua}$ $(a \neq 0)$</td>
<td>The nonnegative conversion rate (the inverse of the heat rate) at the $u^{th}$ generating unit of genco $g$ in region $r_1$ if the generating unit utilizes fuel $a$; $\beta_{gr1ua}$ is equal to zero if the generating unit does not use fuel $a$. We assume that for units using renewable technologies, all $\beta_{gr1ua}$s are equal to zero.</td>
</tr>
<tr>
<td>$\beta_{gr1u0}$</td>
<td>The renewable unit indicator. $\beta_{gr1u0}$ is equal to one if the $u^{th}$ generating unit of genco $g$ in region $r_1$ utilizes renewable technologies and zero otherwise. For generating units using renewable technologies, all $\beta_{gr1ua}$s are equal to zero.</td>
</tr>
<tr>
<td>$\alpha_{r1r2b}$</td>
<td>The impact of transferring one unit electricity from region $r_1$ to region $r_2$ on interface $b$. $\alpha_{r1r2b}$ is equal to $PTDF_{r2b} - PTDF_{r1b}$ where $PTDF_{rb}$ denotes the power transmission distribution factor of region (node) $r$ for interface limit $b$. In particular, $PTDF_{rb}$ is defined as the quantity of power flow (MW) through the critical link of interface $b$ induced by a 1 MW injection at node $r$ [9, 10, 30].</td>
</tr>
<tr>
<td>$d_{r2kw}$</td>
<td>The fixed demand at demand market sector $k$ in region $r_2$ at demand level $w$.</td>
</tr>
<tr>
<td>$\kappa_{r2w}$</td>
<td>The transmission loss factor of region $r_2$ at demand level $w$.</td>
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Table 2: Decision Variables in the Electric Power Supply Chain Network Model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$h$</td>
<td>$AM$-dimensional vector of fuel supplies at the fuel markets with component $h_{am}$ denoting the total supply of fuel $a$ at the $m^{th}$ supply market of fuel $a$.</td>
</tr>
<tr>
<td>$Q^1$</td>
<td>$AMNW$-dimensional vector of fuel flows between fuel supply markets and generating units within the entire period with component $q_{amgr1uw}$ denoted by $q_{amgr1uw}$ and denoting the transactions between the $m^{th}$ supply market of fuel $a$ and the $u^{th}$ generating unit of genco $g$ in region $r_1$ at demand level $w$.</td>
</tr>
<tr>
<td>$q_w$</td>
<td>$N$-dimensional vector of the power generators’ electric power outputs at demand level $w$ with components $q_{gr1uw}$ denoting the power generation at the $u^{th}$ generating unit of genco $g$ in region $r_1$ at demand level $w$. We group the $q_w$ at all demand levels $w$ into vector $q$.</td>
</tr>
<tr>
<td>$Q^2_w$</td>
<td>$NRK$-dimensional vector of electric power flows between generating units and demand markets at demand level $w$ with component $q_{gr1ur2k}$ denoted by $q_{gr1ur2k}$ and denoting the transactions between the $u^{th}$ generating unit of genco $g$ in region $r_1$ and demand market sector $k$ in region $r_2$ at demand level $w$. We group the $Q^2_w$ at all demand levels $w$ into vector $Q^2$.</td>
</tr>
<tr>
<td>$Y^1_w$</td>
<td>$NR$-dimensional vector of electric power transactions between power generators and regional power pools at demand level $w$ with component $y_{gr1ur2}$ denoted by $y_{gr1ur2}$ and denoting the transactions between generating unit $u$ of generator $g$ in region $r_1$ and the regional power pool in region $r_2$ at demand level $w$. We group the $Y^1_w$ at all demand levels $w$ into vector $Y^1$.</td>
</tr>
<tr>
<td>$Y^2_w$</td>
<td>$R^2K$-dimensional vector of electric power transactions between demand markets and regional power pools at demand level $w$ with component $y_{r1r2k}$ denoted by $y_{r1r2k}$ and denoting the transactions between demand market sector $k$ in region $r_2$ and the regional power pool in region $r_1$ at demand level $w$. We group the $Y^2_w$ at all demand levels $w$ into vector $Y^2$.</td>
</tr>
<tr>
<td>$Z_w$</td>
<td>$N$-dimensional vector of regional operating reserves with component $z_{gr1u}$ denoted by $z_{gr1u}$ and denoting the operating reserve held by generating unit $u$ of genco $g$ in region $r_1$ at demand level $w$. We group the $Z_w$ at all demand levels $w$ into vector $Z$.</td>
</tr>
</tbody>
</table>
Table 3: Endogenous Prices and Shadow Prices of the Electric Power Supply Chain
Network Model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{gr1uw}^{am}$</td>
<td>The price offered by generating unit $u$ of genco $g$ in region $r_1$ at demand level $w$ in transacting with the $m^{th}$ supply market of fuel $a$. We group all $\rho_{gr1uw}^{am}$s into vector $\rho_1$.</td>
</tr>
<tr>
<td>$\rho_{r2kw}$</td>
<td>The unit price charged by generating unit $u$ of genco $g$ in region $r_1$ for the transaction with power buyers in demand market sector $k$ in region $r_2$ at demand level $w$. We group all $\rho_{r2kw}$s into vector $\rho_2$.</td>
</tr>
<tr>
<td>$\rho_{r2w}$</td>
<td>The unit electricity price at location $r_2$ on the electricity pool market at demand level $w$. We group all $\rho_{r2w}$s into vector $\rho_3$.</td>
</tr>
<tr>
<td>$\rho_{r2kw}$</td>
<td>The unit electric power price offered by the buyers at demand market sector $k$ in region $r_2$ at demand level $w$. We group all $\rho_{r2kw}$s into vector $\rho_4$.</td>
</tr>
<tr>
<td>$\varphi_{r1w}$</td>
<td>The unit price of capacity on the regional operating reserve market in region $r_1$ at demand level $w$. We group all $\varphi_{r1w}$s into vector $\varphi$.</td>
</tr>
<tr>
<td>$\mu_{bw}$</td>
<td>The unit congestion charge of interface (flowgate) $b$ at demand level $w$. We group all $\mu_{bw}$s into vector $\mu$.</td>
</tr>
<tr>
<td>$\eta_{gr1uw}$</td>
<td>The shadow price associated with the capacity constraint of generating unit $u$ of generator $g$ in region $r_1$ at demand level $w$. We group all $\eta_{gr1uw}$s into vector $\eta$.</td>
</tr>
<tr>
<td>$\lambda_{gr1uw}$</td>
<td>The shadow price associated with the maximum operation reserve constraint of generating unit $u$ of generator $g$ in region $r_1$ at demand level $w$. We group all $\lambda_{gr1uw}$s into vector $\lambda$.</td>
</tr>
</tbody>
</table>
Table 4: Cost and Price Functions of the Electric Power Supply Chain Network Model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{am}(h) )</td>
<td>The inverse supply function (price function) at the ( m^{th} ) supply market of fuel ( a ).</td>
</tr>
<tr>
<td>( f_{gr1uw}(q_{gr1uw}) )</td>
<td>The generating cost of generating unit ( u ) of genco ( g ) in region ( r_1 ) at demand level ( w ).</td>
</tr>
<tr>
<td>( c_{r2kw}(q_{r2kw}) )</td>
<td>The transaction/transmission cost incurred at generating unit ( u ) of genco ( g ) in region ( r_1 ) in transacting with demand market sector ( k ) in region ( r_2 ) at demand level ( w ).</td>
</tr>
<tr>
<td>( c_{gr1uw}(q_{gr1uw}) )</td>
<td>The transaction/transmission cost incurred at generating unit ( u ) of genco ( g ) in region ( r_1 ) in selling electricity to region ( r_2 ) through the power pool at demand level ( w ).</td>
</tr>
<tr>
<td>( c_{gr1uw}(z_{gr1uw}) )</td>
<td>The operating reserve cost at generating unit ( u ) of genco ( g ) in region ( r_1 ) at demand level ( w ).</td>
</tr>
<tr>
<td>( \hat{c}_{r2kw}(Q_w^2) )</td>
<td>The unit transaction/transmission cost incurred by power buyers in demand market sector ( k ) in region ( r_2 ) in transacting with generating unit ( u ) of genco ( g ) in region ( r_1 ) at demand level ( w ).</td>
</tr>
<tr>
<td>( \hat{c}_{r2kw}(Y_w^2) )</td>
<td>The unit transaction/transmission cost incurred by power buyers in demand market sector ( k ) in region ( r_2 ) when purchasing electricity from region ( r_1 ) through the power pool at demand level ( w ).</td>
</tr>
</tbody>
</table>

The Equilibrium Conditions for the Fuel Supply Markets

We first describe the equilibrium conditions for the fuel supply markets. The typical fuels used for electric power generation include coal, natural gas, residual fuel oil (RFO), distillate fuel oil (DFO), jet fuel, and uranium. We assume that the fuel suppliers take into account the prices offered by the power generators and the transportation/distribution costs in making their economic decisions (see also [24, 25, 64]).

We assume that the following conservation of flow equations must hold for all fuel supply markets \( a = 1, \ldots, A; \ m = 1, \ldots, M \):

\[
\sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{g_{r_1}}} q_{gr1uw}^{am} + \bar{q}_{am} = h_{am},
\]

where parameter \( \bar{q}_{am} \) denotes the aggregated demand from the other sectors. Equation (1) states that the total supply of fuel \( a \) at market \( am \) is equal to the sum of the demand from electric power generation and the aggregated demand from the other sectors.
We assume that the inverse supply function (price function), $\pi_{am}(h)$, is known for each fuel supply market $am$. We assume that the fuel price in a market depends not only on the supply of fuels in that market but also on the supplies of fuels at the other fuel markets. We also assume that $\pi_{am}(h)$ is a non-decreasing continuous function. A special case is where $\pi_{am}(h) = \bar{\pi}_{am} + 0 \times h = \bar{\pi}_{am}$ in which case the fuel price is fixed and equal to $\bar{\pi}_{am}$.

The (spatial price) equilibrium conditions (cf. [49]) for suppliers at fuel supply market $am$; $a = 1, \ldots, A$; $m = 1, \ldots, M$, take the form: for each generating unit $gr_{1w}$; $g = 1, \ldots, G$; $r_1 = 1, \ldots, R$; $u = 1, \ldots, N_{gr_1}$, and at each demand level $w$:

$$\pi_{am}(h^*) + c_{gr_{1w}}^a \begin{cases} = \rho_{gr_{1uw}}^{am*}, & \text{if } q_{gr_{1uw}}^{am*} > 0, \\ \geq \rho_{gr_{1uw}}^{am*}, & \text{if } q_{gr_{1uw}}^{am*} = 0; \end{cases}$$

(2)

equivalently, in view of (1),

$$\pi_{am}(Q^{1*}) + c_{gr_{1w}}^a \begin{cases} = \rho_{gr_{1uw}}^{am*}, & \text{if } q_{gr_{1uw}}^{am*} > 0, \\ \geq \rho_{gr_{1uw}}^{am*}, & \text{if } q_{gr_{1uw}}^{am*} = 0. \end{cases}$$

(3)

In equilibrium, conditions (3) must hold simultaneously for all the fuel supply market and generating unit pairs and at all demand levels. We can express these equilibrium conditions as the following variational inequality (see, e.g., [49]): determine $Q^{1*} \in \mathcal{K}^1$, such that

$$\sum_{w=1}^W \sum_{a=1}^A \sum_{m=1}^M \sum_{g=1}^G \sum_{r_1=1}^R \sum_{u=1}^{N_{gr_1}} \left( \pi_{am}(Q^{1*}) + c_{gr_{1uw}}^a - \rho_{gr_{1uw}}^{am*} \right) \times \left[ q_{gr_{1uw}}^{am*} - q_{gr_{1uw}}^{am} \right] \geq 0, \forall Q^1 \in \mathcal{K}^1,$n

(4)

where $\mathcal{K}^1 \equiv \{ Q^1 | Q^1 \in R_{+}^{AMNW} \}$.

The Behavior of the Power Generators and Their Optimality Conditions

Recall that the equilibrium prices are indicated by $^{*}$. Under the assumption that each individual genco is a profit-maximizer and may own multiple generating units in various regions, the optimization problem of genco $g$ can be expressed as follows:

Maximize $\sum_{w=1}^W L_w \sum_{r_1=1}^R \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^R \sum_{k=1}^K \rho_{gr_{1uw}}^{gr_{1u}^*} q_{gr_{1uw}}^{gr_{1u}^*}$

$\sum_{w=1}^W \sum_{r_1=1}^R \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^R \rho_{gr_{1uw}}^{gr_{1u}^*} q_{gr_{1uw}}^{gr_{1u}^*}$

$+ \sum_{w=1}^W \sum_{r_1=1}^R \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^R \rho_{gr_{1uw}}^{gr_{1u}^*} q_{gr_{1uw}}^{gr_{1u}^*}$

$+ \sum_{w=1}^W \sum_{r_1=1}^R \sum_{u=1}^{N_{gr_1}} \sum_{r_1=1}^R \rho_{gr_{1uw}}^{gr_{1u}^*} q_{gr_{1uw}}^{gr_{1u}^*}$

$- \sum_{w=1}^W \sum_{a=1}^A \sum_{m=1}^M \sum_{r_1=1}^R \sum_{u=1}^{N_{gr_1}} \rho_{gr_{1uw}}^{am} q_{gr_{1uw}}^{am}$

$15$
subject to:

\[- \sum_{u=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr1}} f_{gr1uw}(q_{gr1uw}) - \sum_{u=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{k=1}^{K} c_{r_2kw}^{gr1u}(q_{r_2kw}^{gr1u}) \]

\[- \sum_{u=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr1}} \sum_{r_2=1}^{K} c_{r_2uw}^{gr1u}(y_{r_2uw}) \]

\[- \sum_{u=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr1}} \sum_{r_2=1}^{K} \sum_{k=1}^{K} c_{gr1uw}(z_{gr1uw}) \]

subject to:

\[ R \sum_{r_2=1}^{R} \sum_{k=1}^{K} q_{r_2kw}^{gr1u} + R \sum_{r_2=1}^{R} y_{r_2uw}^{gr1u} = q_{gr1uw}, \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad w = 1, ..., W, \]  

(5)

\[ \sum_{a=1}^{A} \beta_{gr1uw} \sum_{m=1}^{M} q_{gr1uw}^{am} + L_w \beta_{gr1uw} q_{gr1uw} = L_w q_{gr1uw}, \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad w = 1, ..., W; \]  

(6)

\[ q_{gr1uw} + z_{gr1uw} \leq Cap_{gr1uw}, \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad w = 1, ..., W; \]  

(7)

\[ z_{gr1uw} \leq OP_{gr1uw}, \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad w = 1, ..., W; \]  

(8)

\[ q_{r_2kw}^{gr1u} \geq 0, \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad r_2 = 1, ..., R; \quad k = 1, ..., K; \quad w = 1, ..., W, \]  

(9)

\[ q_{gr1uw}^{am} \geq 0, \quad a = 1, ..., A; \quad m = 1, ..., M; \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad w = 1, ..., W; \]  

(10)

\[ y_{r_2uw}^{gr1u} \geq 0, \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad r_2 = 1, ..., R; \quad w = 1, ..., W; \]  

(11)

\[ z_{gr1uw} \geq 0, \quad r_1 = 1, ..., R; \quad u = N_{gr1}; \quad w = 1, ..., W; \]  

(12)

\[ a = 1, ..., A; \quad m = 1, ..., M; \quad r_1 = 1, ..., R; \quad u = 1, ..., N_{gr1}; \quad w = 1, ..., W; \]  

(13)

The first three terms in the objective function represent the revenues from bilateral transactions with the demand markets, the energy pool sales, and the regional operating reserve market, respectively. The fourth term is the total payout to the fuel suppliers. The fifth, sixth, and seventh terms represent the generating cost, the transaction costs of bilateral contracts with the demand markets, and the transaction costs of selling electric power to energy pools, respectively. The eighth term is the cost of providing operating reserve $z_{gr1uw}$. The last term of the objective function represents the cost of congestion charges where $\sum_{b=1}^{B} \mu_{bw}^{*} \alpha_{r_1r_2b}$ is equivalent to the congestion charge of transferring one unit electricity from region $r_1$ to region $r_2$. Note that $\alpha_{r_1r_2b}$ is equal to $PTDF_{r_2b} - PTDF_{r_1b}$ where $PTDF_{rb}$ denotes the power transmission distribution factor of region (node) $r$ for interface limit $b$. In particular,
$PTDF_{rb}$ is defined as the quantity of power flow (MW) through the critical link of interface $b$ induced by a 1 MW injection at node $r$ [9, 10, 30]. Here, we assume that the gencos have to pay the transmission right costs for bilateral transactions.

Next, we explain the constraints genco $g$ must satisfy when he maximizes the profit. Constraint (6) states that at each generating unit the total amount of electric power sold cannot exceed the total production of electric power.

Constraint (7) models the production of electricity at each generating unit. If a generating unit uses fossil fuel, at each demand level, the quantity of electricity produced is equal to the quantity of electricity converted from the fuels. In constraint (7), $\beta_{gr1,ua}$ is equal to the nonnegative conversion rate (the inverse of the heat rate) at generating generating unit $u$ of genco $g$ in region $r_1$ if the unit utilizes fuel $a$, and is equal to zero otherwise; $\beta_{gr1,u0}$ is equal to one if the generating unit utilizes renewable technologies and is zero otherwise. We assume that for units using renewable technologies, all $\beta_{gr1,ua}$s are equal to zero. Note that for renewable generating units, constraint (7) will automatically hold. In the electric power industry, generating units that burn the same type of fuel may have very different average heat rates depending on the technologies that the generating units use. For example, the heat rates of large natural gas generating units range from 5500 Btu/kWh to 20500 Btu/kWh while the heat rates of large oil generating units vary from 6000 Btu/kWh to 25000 Btu/kWh (e.g. [67]).

Constraint (8) states that the sum of electric power generation and operating reserve cannot exceed generating unit capacity, $Cap_{gr1,u}$.

Constraint (9), in turn, states that the operating reserve provided by generating unit $u$ of genco $g$ in region $r_1$ cannot exceed the maximum level of operating reserve of that unit, $OP_{gr1,u}$.

We assume that the generating cost and the transaction cost functions for the generating units are continuously differentiable and convex, and that the gencos compete in a noncooperative manner in the sense of Cournot [13] and Nash [59, 60]. The optimality conditions for all power generators simultaneously, under the above assumptions (see also [3, 4, 23, 49]), coincide with the solution of the following variational inequality: determine
\((Q^1, q^*, Q^2, Y^1, Z, \eta^*, \lambda^*) \in \mathcal{K}^2\) satisfying

\[\begin{align*}
\sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} & \left[ \frac{\partial f_{gr1uw}(q_{gr1uw}^*)}{\partial q_{gr1uw}} + \eta_{gr1uw}^* \right] \times [q_{gr1uw}^* - q_{gr1uw}] \\
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} & \sum_{r_2=1}^{K} \left[ \frac{\partial c_{gr1uw}^{gr1uw}}{\partial q_{gr1uw}} + \sum_{b=1}^{B} \mu_{bgr1uw}^{gr1uw} \right] \times [q_{gr1uw}^* - q_{gr1uw}^{gr1uw}] \\
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} & \left[ \frac{\partial c_{gr1uw}(y_{gr1uw})}{\partial y_{gr1uw}} + \sum_{b=1}^{B} \mu_{bgr1uw}^{gr1uw} \right] \times [y_{gr1uw}^* - y_{gr1uw}^{gr1uw}] \\
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} & \left[ C_{apr1uw} - q_{gr1uw}^* - z_{gr1uw}^* \right] \times [\eta_{gr1uw}^* - \eta_{gr1uw}] \\
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} & [OP_{gr1uw} - z_{gr1uw}^*] \times [\lambda_{gr1uw}^* - \lambda_{gr1uw}] \geq 0, \forall (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in \mathcal{K}^2, (14) \end{align*}\]

where \(\mathcal{K}^2 = \{(Q^1, q, Q^2, Y^1, Z, \eta, \lambda) | (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in R^{AMNW+NRKW+NRW+4NW}_+, \text{ and (6) and (7) hold}\}.

The ISO’s Role

The ISO manages the wholesale electricity market, maintains system reliability, and oversees the transmission network to ensure system security.

The ISO dispatches the power generators that participate in the power pool, starting with the genco that offers the lowest supply price. In each regional power pool, all power suppliers will receive the same unit price which is equal to the market clearance price in that region. The ISO achieves this economic efficiency by providing and overseeing competitive power pools as well as maintaining market clearance at each power pool. In the previous section, we have assumed that at power pools the gencos compete with one another in a
noncooperative manner in the sense of Cournot [13] and Nash [59, 60], and have incorporated this competition in (14).

The competition among the generators and the maintenance of market clearance in the regional power pools lead to economic dispatches of electric power generation. The ISO ensures that the regional electricity markets \( r = 1, \ldots, R \) clear at each demand level \( w = 1, \ldots, W \), that is,

\[
G \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u_1=1}^{N_{gr_1}} y_{gr_1uw}^{r_1u} \left\{ \begin{array}{ll}
\sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r}, & \text{if } \rho_{rw} > 0, \\
\geq \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r}, & \text{if } \rho_{rw} = 0;
\end{array} \right. \quad (15)
\]
equivalently,

\[
L_w G \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u_1=1}^{N_{gr_1}} y_{gr_1uw}^{r_1u} \left\{ \begin{array}{ll}
L_w \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r}, & \text{if } \rho_{rw} > 0, \\
\geq L_w \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{r}, & \text{if } \rho_{rw} = 0.
\end{array} \right. \quad (15a)
\]

Here, the left-hand side of constraint (15) (or (15a)) represents the total quantity of electric power sold by power sellers at region \( r \) through the power pool, and the right-hand side represents the total amount of electric power purchased by power buyers from region \( r \) through the power pool.

We would like to note that (14) and (15) (or (15a)) together ensure that in equilibrium, at demand level \( w \), the generating units that participate in the power pool in region \( r \) and have lower marginal costs than \( \rho_{rw}^* \) will be paid at the same unit price which is equal to the market clearance price \( \rho_{rw}^* \).

Moreover, the ISO also manages operating reserve markets where the gencos can get paid for holding back their capacities to help to ensure system reliability. We have assumed that the generators compete in the operating reserve markets in a noncooperative manner. The ISO needs to ensure that the regional operating reserve markets \( r_1 = 1, \ldots, R \) clear at each demand level \( w = 1, \ldots, W \), that is,

\[
G \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr_1}} z_{gr_1uw}^{r_1u} \left\{ \begin{array}{ll}
OPR_{r_1w}, & \text{if } \varphi_{r_1w}^* > 0, \\
\geq OPR_{r_1w}, & \text{if } \varphi_{r_1w}^* = 0;
\end{array} \right. \quad (16)
\]
equivalently,

\[
L_w G \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr_1}} z_{gr_1uw}^{r_1u} \left\{ \begin{array}{ll}
L_w OPR_{r_1w}, & \text{if } \varphi_{r_1w}^* > 0, \\
\geq L_w OPR_{r_1w}, & \text{if } \varphi_{r_1w}^* = 0.
\end{array} \right. \quad (16a)
\]
The ISO also manages transmission congestion and impose congestion fees, which not only ensures system security, but also eliminates inter-region arbitrage opportunities and enhance economic efficiency of the system. We use a linearized direct current network to approximate the transmission network, and assume that the ISO charges network users congestion fees in order to ensure that the interfaces limits are not violated [9, 10, 30]. In our model, the following conditions must hold for each interface \( b \) and at each demand level \( w \), where \( b = 1, \ldots, B \); \( w = 1, \ldots, W \):

\[
\sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2kw}^{gr_1u} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2uw}^{gr_1u} + \sum_{k=1}^{K} y_{r_2kw}^{r_2w} \alpha_{r_1r_2b} \left\{ \begin{array}{ll}
= TCap_b, & \text{if } \mu_{bw} > 0, \\
\leq TCap_b, & \text{if } \mu_{bw} = 0;
\end{array} \right.
\]

(17)

equivalently,

\[
L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2kw}^{gr_1u} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2uw}^{gr_1u} + \sum_{k=1}^{K} y_{r_2kw}^{r_2w} \alpha_{r_1r_2b} \left\{ \begin{array}{ll}
= L_wTCap_b, & \text{if } \mu_{bw}^* > 0, \\
\leq L_wTCap_b, & \text{if } \mu_{bw}^* = 0.
\end{array} \right.
\]

(17a)

In equilibrium, conditions (15), (16), and (17) (equivalently, (15a), (16a) and (17a)) must hold simultaneously. We can express these equilibrium conditions using the following variational inequality: determine \((\mu^*, \rho_{3*}, \varphi^*) \in R_{+}^{WB+2WR}\), such that

\[
\sum_{w=1}^{W} L_w \sum_{b=1}^{B} [TCap_b - \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2kw}^{gr_1u} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2uw}^{gr_1u} + \sum_{k=1}^{K} y_{r_2kw}^{r_2w} \alpha_{r_1r_2b} \times [\mu_{bw} - \mu_{bw}^*] + \sum_{w=1}^{W} L_w \sum_{r=1}^{R} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} g_{r_2kw}^{gr_1u} - \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2kw}^{r_2w} \alpha_{r_1r_2b} \times [\rho_{r_2w} - \rho_{r_2w}^*] + \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} z_{r_1uw}^{gr_1u} - OPR_{r_1uw} \alpha_{r_1r_2b} \times [\varphi_{r_1w} - \varphi_{r_1w}^*] \geq 0, \quad \forall (\mu, \rho_3, \varphi) \in R_{+}^{WB+2RW}.
\]

(18)
Equilibrium Conditions for the Demand Markets

Next, we describe the equilibrium conditions at the demand markets. We assume that the consumers in the same demand market sector are homogenous and have the same consumption pattern. The consumers search for the lowest electricity cost which is equal to the sum of the electricity price and the transaction cost.

We assume that all demand market sectors in all regions have fixed and known demands, and the following conservation of flow equations, hence, must hold for all regions $r_2 = 1, \ldots, R$, all demand market sectors $k = 1, \ldots, K$, and at all demand levels $w = 1, \ldots, W$:

$$
\sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} q_{r_2kw}^{gr_1us} + \sum_{r_1=1}^{R} y_{r_2kw}^{r_1w} = (1 + \kappa_{r_2w}) d_{r_2kw},
$$

where $d_{r_2kw}$ denotes the demand at market sector $k$ in region $r_2$ at demand level $w$, and $\kappa_{r_2w}$ denotes the transmission loss factor which is usually between $1\% - 6\%$.

We also assume that all unit transaction cost functions $\hat{c}_{r_2kw}^{gr_1u}(Q_w)$ and $\hat{c}_{r_2kw}^{r_1w}(Y_w)$ are continuous and nondecreasing.

The equilibrium conditions for consumers at demand market sector $k$ in region $r_2$ take the form: for each generating unit $gr_1u; g = 1, \ldots, G; r_1 = 1, \ldots, R; u = 1, \ldots, N_{gr_1}$, and each demand level $w; w = 1, \ldots, W$:

$$
\rho_{r_2kw}^{gr_1us} + \hat{c}_{r_2kw}^{gr_1u}(Q_w^*) \begin{cases} = \rho_{r_2kw}^*, & \text{if } q_{r_2kw}^{gr_1us} > 0; \\ \geq \rho_{r_2kw}^*, & \text{if } q_{r_2kw}^{gr_1us} = 0; \end{cases}
$$

equivalently,

$$
L_w[\rho_{r_2kw}^{gr_1us} + \hat{c}_{r_2kw}^{gr_1u}(Q_w^*)] \begin{cases} = L_w \rho_{r_2kw}^*, & \text{if } q_{r_2kw}^{gr_1us} > 0; \\ \geq L_w \rho_{r_2kw}^*, & \text{if } q_{r_2kw}^{gr_1us} = 0; \end{cases}
$$

and

$$
\rho_{r_1w}^* + \sum_{b=1}^{B} \mu_{bw}^* \alpha_{r_1r_2b} + \hat{c}_{r_2kw}^{r_1w}(Y_w^*) \begin{cases} = \rho_{r_2kw}^*, & \text{if } y_{r_2kw}^{r_1w} > 0; \\ \geq \rho_{r_2kw}^*, & \text{if } y_{r_2kw}^{r_1w} = 0; \end{cases}
$$

equivalently,

$$
L_w[\rho_{r_1w}^* + \sum_{b=1}^{B} \mu_{bw}^* \alpha_{r_1r_2b} + \hat{c}_{r_2kw}^{r_1w}(Y_w^*)] \begin{cases} = L_w \rho_{r_2kw}^*, & \text{if } y_{r_2kw}^{r_1w} > 0; \\ \geq L_w \rho_{r_2kw}^*, & \text{if } y_{r_2kw}^{r_1w} = 0. \end{cases}
$$
Conditions (20) and (20a) state that, in equilibrium, if power buyers at demand market sector $k$ in region $r_2$ purchase electricity from generating unit $u$ of genco $g$ in region $r_1$, then the price the consumers pay is exactly equal to the sum of the electricity price and the unit transaction cost. However, if the electricity price plus the transaction cost is greater than the price the buyers are willing to pay at the demand market, there will be no transaction between this generating unit / demand market pair. Conditions (21) and (21a) state that power buyers in demand markets need to also consider congestion fees when they purchase electric power from other regions through the power pool.

In equilibrium, conditions (20) and (21) (equivalently, (20a) and (21a)) must hold simultaneously for all demand markets in all regions. We can express these equilibrium conditions using the following variational inequality: determine $(Q^{2*}, Y^{2*}) \in K^3$, such that

$$
\sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[ \rho_{gr_1u}^{r_2kw} + c_{gr_1u}^{r_2kw}(Q^{2*}_w) \right] \times \left[ q_{gr_1u}^{r_2kw} - q_{gr_1u}^{r_1kw} \right] \\
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[ \rho_{r_2w} + \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} + c_{r_2kw}(Y^{2*}_w) \right] \times \left[ y_{r_2kw} - y_{r_1kw} \right] \geq 0
$$

where $K^3 \equiv \{(Q^2, Y^2) | (Q^2, Y^2) \in R^{N_{RW}+R^2KW} \text{ and } (19) \text{ holds}\}$.

The Equilibrium Conditions for the Electric Power Supply Chain Network

In equilibrium, the optimality conditions for all gencos, the equilibrium conditions for all fuel supply markets, all demand market sectors, and the independent system operator must be simultaneously satisfied so that no decision-maker has any incentive to alter his/her transactions. We now formally state the equilibrium conditions for the electric power supply chain with fuel supply markets as follows.

**Definition 1: Electric Power Supply Chain Network Equilibrium**

The equilibrium state of the electric power supply chain network with fuel supply markets is one where the fuel and electric power flows and the prices satisfy the sum of conditions (4), (14), (18), and (22).
We now state and prove:

**Theorem 1:** Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium Model with Fuel Suppliers

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine \((Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho^*, \varphi^*) \in K^1\) satisfying

\[
\sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[ \pi_{am}(Q^{1*}) + c_{gr_1uw}^m \right] \times [q_{gr_1uw}^m - q_{gr_1uw}^{m*}]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ \frac{\partial f_{gr_1uw}(q_{gr_1uw}^*)}{\partial q_{gr_1uw}} + \eta_{gr_1uw}^* \right] \times [q_{gr_1uw}^* - q_{gr_1uw}]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{k=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ \frac{\partial g_{r_2kw}(q_{gr_1uw}^*)}{\partial q_{gr_1uw}} + \mu_{bw}^* \alpha_{r_1r_2b} + \epsilon_{r_2kw}(Q_{w}^{2*}) \right] \times [q_{r_2kw} - q_{r_2kw}^*]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ \frac{\partial h_{r_2kw}(y_{r_2kw}^*)}{\partial y_{r_2kw}} + \mu_{bw}^* \alpha_{r_1r_2b} - \rho_{r_2w}^* \right] \times [y_{r_2kw}^* - y_{r_2kw}]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ \frac{\partial i_{r_2kw}(z_{gr_1uw}^*)}{\partial z_{gr_1uw}} + \lambda_{gr_1uw}^* + \eta_{gr_1uw}^* - \varphi_{r_1w}^* \right] \times [z_{gr_1uw} - z_{gr_1uw}^*]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{k=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ \rho_{r_1w}^* + \epsilon_{r_2kw}(Y_{w}^{2*}) + \mu_{bw}^* \alpha_{r_1r_2b} \right] \times [y_{r_2kw}^* - y_{r_2kw}]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{k=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ Cap_{gr_1u}^* - q_{gr_1uw}^* - z_{gr_1uw}^* \right] \times [\eta_{gr_1uw} - \eta_{gr_1uw}^*]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{k=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ OP_{gr_1u}^* - z_{gr_1uw}^* \right] \times [\lambda_{gr_1uw} - \lambda_{gr_1uw}^*]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{k=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ TC_{Cap_b} - \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} y_{r_2kw}^* \alpha_{r_1r_2b} \right] \times [\mu_{bw} - \mu_{bw}^*]
\]

\[+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{K} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} \sum_{u=1}^{N_{gr_1}} \left[ y_{r_2kw}^* - y_{r_2kw} \right] \times [\delta_{r_1w} - \delta_{r_1w}^*]
\]
\[ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \left( \sum_{g=1}^{G} \sum_{u=1}^{U} z_{gr_1uw} - OPR_{r_1} \right) \times [\varphi_{r_1w} - \varphi^*_{r_1w}] \geq 0, \]

\[ \forall (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in \mathcal{K}^4, \] (23)

where \( \mathcal{K}^4 \equiv \{(Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) | (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in R^{AMNW + NRKW + NRW + 4NW + R^2KW + BW + 2RW} \) and (6), (7), and (19) hold.}

**Proof:** We first establish that Definition 1 implies variational inequality (23). Indeed, summation of (4), (14), (18), and (22), after algebraic simplifications, yields (23).

Now we prove the converse, that is, that a solution to (23) satisfies the sum of (4), (14), (18), and (22), and is, hence, an equilibrium.

To variational inequality (23), add \( \rho_{gr_1uw}^a - \rho_{gr_1uw}^m \) to the term in the first brackets preceding the first multiplication sign, and add \( \rho_{r_2kw}^u - \rho_{r_2kw}^u \) in the brackets preceding the third multiplication sign. The addition of such terms does not change (23) since the value of these terms is zero, and yields the sum of (4), (14), (18), and (22) after simple algebraic simplifications. Q.E.D.

The variational inequality problem (23) can be rewritten in standard variational inequality form (cf. [49]) as follows: determine \( X^* \in \mathcal{K} \) satisfying

\[ \langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \] (24)

where \( X \equiv (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi)^T, \mathcal{K} \equiv \mathcal{K}^4 \), and

\[ F(X) \equiv (F_{gr_1uw}^a, F_{w}^{gr_1u}, F_{r_2kw}^{gr_1u}, F_{w}^{gr_1u}, F_{gr_1uw}^{gr_1u}, F_{w}^{gr_1u}, F_{r_1r_2kw}, F_{\lambda gr_1u}, F_{\eta gr_1u}, F_{bw}, F_{rw}, F_{r_1w}), \]

with indices \( a = 1, \ldots, A; m = 1, \ldots, M; w = 1, \ldots, W; r_1 = 1, \ldots, R; r_2 = 1, \ldots, R; r = 1, \ldots, R; g = 1, \ldots, G; u = 1, \ldots, N_{gr_1}; k = 1, \ldots, K; b = 1, \ldots, B \), and the functional terms preceding the multiplication signs in (23), respectively. Here \( \langle \cdot, \cdot \rangle \) denotes the inner product in \( \Omega \)-dimensional Euclidian space where \( \Omega = AMNW + NRKW + NRW + 4NW + R^2KW + BW + 2RW \).
3. Qualitative Properties

In this section, we provide some qualitative properties of the solution to variational inequality (24); equivalently, (23). We can derive existence of a solution $X^*$ to (24) simply from the assumption of continuity of functions that enter $F(X)$, which is the case in this model [45, 49]. We now state the following theorems.

**Theorem 2: Existence**

If $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho^*_3, \varphi^*)$ satisfies variational inequality (23) then $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*)$ is a solution to the variational inequality problem: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*) \in \mathcal{K}^5$ satisfying

\[
\sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[ \tau_{am}(Q^{1*}) + \epsilon_{am}^{\ast} \right] \times [q_{gr_1uw}^{am} - q_{gr_1uw}^{am*}] + \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[ \partial f_{gr_1uw}(q_{gr_1uw}^{1}) \right] \times [q_{gr_1uw}^{1} - q_{gr_1uw}^{1*}] + \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[ \partial c_{gr_1uw}(Q^{2*}_w) \right] \times [q_{r_2kw}^{1} - q_{r_2kw}^{1*}] + \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[ \partial z_{gr_1uw}^{1} \right] \times [z_{r_2kw}^{1} - z_{r_2kw}^{1*}] + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \left[ \partial c_{r_2kw}^{1} Y^{2*}_w \right] \times [y_{r_2kw}^{1} - y_{r_2kw}^{1*}] \geq 0, \ \forall(Q^{1}, q, Q^{2}, Y^{1}, Y^{2}, Z) \in \mathcal{K}^5, \ \ \ \ (25)
\]

where $\mathcal{K}^5 \equiv \{ (Q^{1}, q, Q^{2}, Y^{1}, Y^{2}, Z) | (Q^{1}, q, Q^{2}, Y^{1}, Y^{2}, Z) \in R^{AMNW+NRKW+NRW+2NW+R^3KW}$

and (6), (7), (8), (9), (19), and

\[
L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} y_{r_2kw}^{1} \geq L_w \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2kw}^{1}, \ \forall r_1; \forall w, \ \ \ \ (26)
\]

\[
L_w \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} z_{gr_1uw} \geq L_w OPR_{r_1w}, \ \forall r_1; \forall w, \ \ \ \ (27)
\]

25
and \( L_{w} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} q_{r_2kw}^{r_1u} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2kw}^{r_1u} + \sum_{k=1}^{K} y_{r_2kw}^{r_1u} \alpha_{r_1r_2b} \leq L_{w}TCap_{b}, \forall b; \forall w \) (28) are satisfied}. 

A solution to (25) is guaranteed to exist provided that \( \mathcal{K}^5 \) is nonempty. Moreover, if \((Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*)\) is a solution to (25), there exist \((\eta^*, \lambda^*, \mu^*, \rho^*_3, \varphi^*)\) \(\in \mathbb{R}^{2NW+BW+2RW} \) with \((Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho^*_3, \varphi^*)\) being a solution to variational inequality (23).

**Proof:** The proof is an analog of the proof of Theorem 3 in [53].

Since all the constraints of \( \mathcal{K}^5 \) are linear, it is easy to verify the existence of a feasible point in \( \mathcal{K}^5 \). In our case study, because \( \mathcal{K}^5 \) is nonempty for the New England electric power supply chain, the existence of a solution is guaranteed for each empirical example in Section 5.

We now recall the concept of monotonicity and state an additional theorem.

**Theorem 3: Monotonicity**

Suppose that all cost functions in the model are continuously differentiable and convex; all unit cost functions are monotonically increasing, and the inverse price functions at the fuel supply markets are monotonically increasing. Then the vector \( F \) that enters the variational inequality (23) as expressed in (24) is monotone, that is,

\[
\langle (F(X') - F(X''))^T, X' - X'' \rangle \geq 0, \ \forall X', X'' \in \mathcal{K}, X' \neq X''.
\] (29)

**Proof:** See the Appendix.

In our case study for New England, \( F(X) \) is monotone and the Jacobian of \( F(X) \) is uniformly positive semidefinite. Moreover, \( F(X) \) is linear and, hence, Lipschitz continuous (see [49]).
4. Computational Method

In this section, we consider the computation of solutions to variational inequality (23). In particular, we recall the modified projection method [49]. The method converges to a solution of the model provided that $F(X)$ is monotone and Lipschitz continuous, and a solution exists, which is the case for our empirical application. For problems with special structure or special cost function specifications, other (decomposition-type) algorithms may be exploited. Next, we present the modified projection method.

The Computational Procedure

Step 0: Initialization

Start with an $X^0 \in \mathcal{K}$ and select $\omega$, such that $0 < \omega \leq \frac{1}{L}$, where $L$ is the Lipschitz constant for function $F(X)$. Let $T = 1$.

Step 1: Construction and Computation

Compute $\bar{X}^{T-1}$ solving the variational inequality subproblem:
\[
\left\langle (\bar{X}^{T-1} + (\omega F(\bar{X}^{T-1}) - X^{T-1}))^T, X' - \bar{X}^{T-1} \right\rangle \geq 0, \quad \forall X' \in \mathcal{K}.
\] (30a)

Step 2: Adaptation

Compute $X^T$ solving the variational inequality subproblem:
\[
\left\langle (X^T + (\omega F(\bar{X}^{T-1}) - X^{T-1}))^T, X' - X^T \right\rangle \geq 0, \quad \forall X' \in \mathcal{K}.
\] (30b)

Step 2: Convergence Verification

If $\|X^T - X^{T-1}\|_{\infty} \leq \epsilon$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $T := T + 1$ and go to Step 1. (We set the parameter $\omega = 0.05$ and the tolerance $\epsilon = 0.001$ for all computations of the numerical examples in Section 5.)

Note that the subproblems in Steps 1 and 2 above are separable quadratic programming problems and, hence, there are numerous algorithms that can be used to solve these embedded subproblems.
5. Empirical Case Study and Examples

In this section, we present the results for empirical examples based on the New England electric power market and fuel markets data. We show that the regional electric power prices simulated by our theoretical model very well match the actual electricity prices in New England. We also conduct sensitivity analysis for peak-hour electricity prices under natural gas and oil price variations. In addition, we provide numerical examples in which the natural gas price is influenced by the oil price through electric power markets. In particular, we present examples that illustrate that both the generator responsiveness, and the electric power market responsiveness, are crucial to the understanding of and the determination of the impact of the residual fuel oil price on the natural gas price. Finally, we apply our model to investigate how the changes in the electricity demands for electricity affect the electric power and fuel supply markets. Throughout this section, we use the demand market prices, $\rho^r_{r_2} w = 1, \ldots, W; r_2 = 1, \ldots, R; k = 1, \ldots, K$, as the simulated regional electric power prices. Note that the model developed in this paper can be easily expanded to include multiple electricity markets and can be applied to larger areas where deregulation is taking place.

Data

The data that we used for the $d_{r_2}$s (see (19)) were New England day-ahead hourly zonal demands. We downloaded the data from the ISO New England hourly demand and price datasets [40] and the Connecticut Valley Electric Exchange [12]. The regional demands were adjusted based on the imported and the exported electric power between New England and the surrounding regions [41].

There are 82 gencos who own and operate 573 generating units ($G=82, N=573$). We obtained the electric generating unit data including the heat rates (the inverse of the $\beta_{gr_1, u}$s), the generating costs, the $f_{gr_1, u}$s, the fuel types, the capacities, the $\text{Cap}_{gr_1, u}$, and the locations from various sources: (1) the national electric energy data system [67], (2) the ISO New England seasonal claimed capacity reports [39], and (3) the New England FERC natural gas infrastructure report [22]. We adjusted the generation capacities based on the average forced outage rate data obtained from the ISO New England website [35]. The transac-
Figure 2: The Ten Regions of the New England Electric Power Supply Chain

1. Maine
2. New Hampshire
3. Vermont
4. Connecticut (excluding Southwestern Connecticut)
5. Southwestern Connecticut (excluding the Norwalk-Stamford area)
6. Norwalk-Stamford area
7. Rhode Island
8. Southeastern Massachusetts
9. Western and Central Massachusetts
10. Boston/Northeastern Massachusetts
tion/transmission costs: the $c_{r_{2k,w}}$, the $c_{r_{1w}}$, the $c_{r_{2k,w}}$, and the $c_{r_{1k}}$, were also obtained from the ISO New England website [44].

We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal ($A=5$). We downloaded the monthly delivered fuel price data for each state of New England from the Energy Information Administration website [21]. Hence, $M=$number of states$=6; a = 1, \ldots, 5$. Note that in the case study we used actual delivered regional fuel price data to set the fuel prices and the constructed price functions, the $\pi_{am}$, for each market (these prices already included the transportation costs, the $c_{am}$, from other areas to New England). For more fuel transportation and transmission rate data see the website of the Federal Regulation and Oversight of Energy (www.ferc.gov).

We approximated the physical transmission constraints (the $TCap_b$ and the $a_{r_{1r_{2b}}}$) using the interface constraints provided in the ISO New England regional system plan [38] and the financial transmission right and day-ahead limit data [42]. Based on the interface constraints and the demand/price data, we divided the whole area into ten regions ($R=10$): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut (excluding Southwest Connecticut), 5. Southwestern Connecticut (excluding the Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeastern Massachusetts, 9. Western and Central Massachusetts, 10. Boston/Northeast Massachusetts, as shown in Figure 2. We assumed that there is one aggregated demand market for each region ($K=1$).

Based on the demand/price data, we assumed that the transmission loss factor, the $\kappa_{r_{2w}}$, is 4% for the highest demand level and 3% for the other demand levels. We did not include the regional operating reserve markets in the case study version of the model because before 2007 New England did not have such markets. The New England ISO, instead, designated several generating units as second-contingency units to support system reliability. We obtained this information from the ISO New England website [36].

We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, \ldots, 6$) and calculated the average regional demands and the average weighted regional prices for each block. In our model, all cost functions
and fuel price functions are assumed linear based on the data and the literature [22, 67, 68]. We then implemented the model and the modified projection method in Matlab (see www.mathworks.com). Moreover, in Steps 1 and 2 of the modified projection method, due to the special structure of the underlying feasible set, the subproblems are completely separable and can be solved as $W$ transportation network problems with the prices in each subproblem solvable in closed form (see, e.g., [50, 51, 54, 55, 56, 69]). In particular, in Steps 1 and 2, we applied the general equilibration algorithm (cf. [49]) to fully exploit the structure of the network subproblems [49]. The demand market prices $\rho^*_{r_2k_w}$ for all $r_2, k, w$ can be recovered from the path costs of the active paths in the reformulated path flow formulation, or from conditions (20) or (21). Each example in this section was solved within 150 minutes on a Lenovo laptop with 2.1GHz Core(TM) 2 CPUs.

**Example 1: Regional Electric Power Supply Chain Simulation**

We first set the fuel prices of each regional market equal to the actual regional delivered fuel prices which were obtained from the Energy Information Administration website [21]. We downloaded the hourly locational marginal price (LMP) data from the ISO New England website [40]. For each block, we used the average regional demand data as model input to compute the regional electricity prices. The average regional demands for each block are shown in Table 5. Tables 6 and 7, and Figure 3 compare the simulated prices and the actual weighted average LMPs in the ISO day-ahead market. In Tables 6 and 7, the simulated prices of Connecticut were the weighted average of the prices of regions 4, 5, and 6.

In Figure 3, each individual point represents a simulated and actual price pair for one region at one demand level. The diagonal line in Figure 3 indicates perfect matches. Tables 6 and 7, and Figure 3 show that the simulated average regional prices match the actual prices very well at all demand levels except that at the lowest demand level the simulated prices are higher than the actual prices. This may be due to the following reason: in this specific example, the prices of electricity at the lowest demand level are determined by those natural gas generating units that have lowest generating costs (lowest heat rates). Note that a generating unit’s generating cost is approximately equal to the product of the fuel cost and the average heat rate of that generating unit. Due to the limitation of the data, we used the average natural gas price at the regional markets to estimate the fuel costs for those
generating units. However, in reality, some generating units may be able to purchase natural gas at lower costs. Therefore, in our model, the lowest generating costs of those natural gas generating units may be overestimated.

Additionally, both actual and simulated electricity prices have significant geographic differences due to the limited interfaces’ capacities in the physical transmission network. Note that if the physical transmission limits are not considered, all regions will always pay the identical electric power prices which will result in significant errors in the predictions. We can also see that the simulated prices have smaller regional differences compared to the actual prices. This is because the complete and detailed interface limits data are not available to the public and we used approximated data in our model [38].

Table 5: Average Regional Demands for Each Demand Level (Mwh)

<table>
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<tr>
<th>Region</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
<th>Block 5</th>
<th>Block 6</th>
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<td>1384</td>
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<td>1481</td>
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<td>717</td>
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<td>560</td>
<td>500</td>
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<td>1706</td>
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Table 6: Actual Regional Prices ($/Mwh)

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<th>Block 4</th>
<th>Block 5</th>
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Table 7: Simulated Regional Prices ($/Mwh)

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<th>Block 4</th>
<th>Block 5</th>
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<td>58.71</td>
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<td>64.82</td>
<td>58.71</td>
<td>50.31</td>
<td>48.00</td>
</tr>
<tr>
<td>All Regions’ Average</td>
<td>108.28</td>
<td>86.34</td>
<td>67.01</td>
<td>60.56</td>
<td>50.31</td>
<td>48.00</td>
</tr>
<tr>
<td>All Regions’ Average (*)</td>
<td>100.28</td>
<td>76.19</td>
<td>65.07</td>
<td>58.71</td>
<td>50.31</td>
<td>48.00</td>
</tr>
</tbody>
</table>

(*) is the predicted weighted average electricity price without the consideration of physical transmission constraints

Example 2: Peak Electric Power Prices under Fuel Price Variations

We then conducted sensitivity analysis for the electricity prices and the fuel prices. In New England, natural gas and oil are the most important fuels for electric power generation. Natural gas units and oil units generate 38% and 24% of electric power in New England, respectively [33]. Moreover, generating units that burn gas or oil set electric power market price 85% of the time [39].
Figure 3: Actual Prices Vs. Predicted Prices ($/Mwh)
We used the same demand data, but varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price. Table 8 and Figure 4 present the average electricity price for the two peak blocks under oil/gas price variations. The surface in Figure 4 represents the average peak electricity prices under different natural gas and oil price combinations. Note that, if the price of one type of fuel is fixed, the electricity price changes less percentage-wise than the other fuel price does. For example, when the residual fuel price is fixed and equal to 9$/MMBtu, if the natural gas price increases 100% from 4$/MMBtu to 8$/MMBtu the electric power price increases 7.73$/MMBtu to 9.97$/MMBtu which is about 29%. This is mainly because fuel diversity can mitigate fuel price shocks.

Table 8: Average Peak Electricity Prices under Fuel Price Variations

<table>
<thead>
<tr>
<th>Electricity Price (cents/kwh)</th>
<th>Residual Fuel Oil Prices ($/MMBtu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>5.00  7.00  9.00  11.00  13.00</td>
</tr>
<tr>
<td>5.00</td>
<td>5.76  6.74  7.73  8.70  9.45</td>
</tr>
<tr>
<td>Natural Gas ($/MMBtu) 6.00</td>
<td>6.06  7.24  8.22  9.20  9.95</td>
</tr>
<tr>
<td>7.00</td>
<td>6.45  7.82  8.81  9.79 10.54</td>
</tr>
<tr>
<td>8.00</td>
<td>6.67  8.19  9.39 10.36 11.12</td>
</tr>
</tbody>
</table>

Example 3: The Interactions Among Electric Power, Natural Gas, and Oil Markets

Next, we utilized the model as well as the New England electricity and fuel market data to explore the interactions between electric power markets and the natural gas and oil markets. In particular, we present examples where the natural gas price can be influenced by the residual fuel oil price through electric power markets. Huntington and Schuler [34] pointed out that the responsiveness of dual-fuel (natural gas and residual fuel oil) generating units and industrial users is critical to the natural gas market price because the less expensive fuel to generate electric power would be chosen. Moreover, in a competitive power generation market, the market mechanism will automatically select electricity from generating units that have lower costs and that use less expensive fuels. Therefore, if the residual fuel oil price decreases, the demand for residual fuel oil will increase, which will reduce the natural gas demand and put downward pressure on the natural gas price. We present two examples
Figure 4: The Average Peak-Hour Electric Power Price under Fuel Price Variations
to demonstrate how our theoretical model can be used to quantify this impact. In Example 3.1, we assume that the dual-fuel generating units have to use their primary fuels, and cannot switch to the alternative fuels, while in Example 3.2, we allow the dual-fuel generating units to freely switch between their primary and alternative fuels. In both examples, we assume that the natural gas market has elastic price functions and the residual oil price is given. We also assume that the prices of the other fuels (coal, distillate fuel oil, and jet fuel) do not change.

**Example 3.1**

We now compared two cases: a high residual fuel oil price (base case) and a low residual fuel oil price.

We first constructed the base case in which the residual fuel oil price is high and equal to $7/\text{MMBtu}$: $\pi_{RFOm} = 7; \ m = 1, \ldots, 6$.

Next, we constructed the inverse supply functions for the natural gas supply in which the price elasticity is consistent with the literature.

Recall that $h_{GASm} = \sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_{1}=1}^{R} \sum_{u=1}^{\bar{N}_{gr}} q_{gr1uw} + \bar{q}_{GASm}$. We assumed that the natural gas price function (unit: $/\text{MMBtu}$) takes the form:

$$\pi_{GASm}(h) = 1 + \frac{6}{d_{GAS0} + d_{GAS0}} \sum_{m=1}^{6} h_{GASm}; \quad m = 1, \ldots, 6; \quad (31)$$

where $d_{GAS0}$ denotes the total natural gas demand from the power generating sector in the base case; $\bar{d}_{GAS0} = \sum_{m=1}^{6} \bar{q}_{GASm}$ denotes the total demand from the other sectors. Note that (31) can be rewritten as:

$$\pi_{GASm}(h) = 7 + \frac{6}{d_{GAS0} + d_{GAS0}} \sum_{w=1}^{6} \sum_{m=1}^{6} \sum_{g=1}^{G} \sum_{r_{1}=1}^{R} \sum_{u=1}^{\bar{N}_{gr}} q_{gr1uw} - d_{GAS0}; \quad m = 1, \ldots, 6; \quad (32)$$

where

$$\frac{\sum_{w=1}^{6} \sum_{m=1}^{6} \sum_{g=1}^{G} \sum_{r_{1}=1}^{R} \sum_{u=1}^{\bar{N}_{gr}} q_{gr1uw} - d_{GAS0}}{d_{GAS0} + d_{GAS0}}$$

is the percentage change of the natural gas demand relative to the base case. Based on the natural gas demand data on the Energy Information Administration website [19], we assumed
also that the demand from electric generating units accounts for 50% of total natural gas demand, i.e., \( d_{GAS0} = \bar{d}_{GAS0} = \sum_{m=1}^{6} \bar{q}_{GASM} \).

We determined the value of \( d_{GAS0} \) in the following way. We first assumed that the gas price in the base case was equal to 7 $/MMBtu, and we computed the equilibrium solution to the model, \( SOL_0 \). We obtained that the total natural gas demand in \( SOL_0 \) was equal to 35953910754 MMBtu. We then set \( d_{GAS0} = 35953910754 \) MMBtu and \( \bar{d}_{GAS0} = 35953910754 \) MMBtu, and obtained the complete natural gas price function which will also be used in the low RFO price case. It is easy to verify that \( SOL_0 \) is a solution to the base case.

We can now rewrite (32) as:

\[
\frac{\pi_{GASM}(h) - 7}{7} = \frac{6 \sum_{g=1}^{6} \sum_{r=1}^{R} \sum_{n=1}^{N_{gr}} \sum_{u=1}^{U} q^{GASM}_{gr} u - d_{GAS0}}{d_{GAS0} + \bar{d}_{GAS0}}; \quad m = 1, \ldots, 6. \tag{33}
\]

Equation (33) implies that one percent change in the natural gas demand will result in a \( \frac{6}{7} \) percent change in the natural gas supply price. This price elasticity is consistent with the literature which says price elasticity of natural gas ranges between 0.8 to 2 in most natural gas supply models [68].

We then let \( \pi_{RFOm} = 4.4$/MMBtu; \( m = 1, \ldots, 6 \) and solved for the low RFO price case. Table 9 shows the prices of electric power and natural gas before and after the decrease in the residual oil price.

**Example 3.2**

In Example 3.2, we allowed the dual-fuel power generating units to freely switch between their primary and alternative fuels. We also considered the high RFO price case (base case) and the low RFO price case.

We assumed that in the base case the residual fuel oil price is still equal to 7$/MMBtu: \( \pi_{RFOm} = 7; \quad m = 1, \ldots, 6 \).

We constructed the natural gas inverse supply function and determined the value of \( d_{GAS0} \) in the same way as in Example 3.1. We obtained that \( d_{GAS0} = 41947834761 \) MMBtu.

We also obtained the solution to the base case. Finally, we set \( \pi_{RFOm} = 4.4, \quad m = \)
1, . . . , 6, and solved for the low RFO price case. Table 9 shows the prices of electric power and natural gas before and after the decrease in the residual oil price.

If we allow the dual-fuel generating units to freely switch between natural gas and residual fuel oil, the natural gas price will fall over 10% due to the decrease in natural gas demand. This is consistent with the theory which says the residual fuel oil price and the responsiveness of generating units are critical to the understanding of and the estimation of the natural gas price [2, 7, 34]. Moreover, even if we assume that the dual-fuel generating units have to always use their primary fuels, the decrease in natural gas price is still 6% which implies that the electric power market automatically chooses electricity from generating units using less expensive fuels.

Table 9: The Price Changes of Natural Gas and Electric Power Under Residual Fuel Oil Price Variation

<table>
<thead>
<tr>
<th></th>
<th>Example 3.1</th>
<th>Example 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High RFO</td>
<td>Low RFO</td>
</tr>
<tr>
<td>RFO Price ($/MMBtu)</td>
<td>7.00</td>
<td>4.40</td>
</tr>
<tr>
<td>GAS Demand (Billion MMBtu)</td>
<td>35.95</td>
<td>30.99</td>
</tr>
<tr>
<td>GAS Price ($/MMBtu)</td>
<td>7.00</td>
<td>6.58</td>
</tr>
<tr>
<td>GAS Price Percentage Change</td>
<td>-6.0%</td>
<td></td>
</tr>
<tr>
<td>EP Ave. Price Blocks 1 and 2 (c/kwh)</td>
<td>8.28</td>
<td>5.94</td>
</tr>
<tr>
<td>EP Ave. Price Blocks 3 and 4 (c/kwh)</td>
<td>6.54</td>
<td>5.37</td>
</tr>
<tr>
<td>EP Ave. Price Blocks 5 and 6 (c/kwh)</td>
<td>4.99</td>
<td>4.55</td>
</tr>
</tbody>
</table>

GAS=Natural Gas, RFO=Residual Fuel Oil, EP=Electric Power

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

We now present an example that illustrates the impact of electricity demand changes on the electric power and natural gas markets. When electricity demands increase (or decrease), the electric power prices will increase (or decrease) due to two reasons: 1. generating units with higher generating costs (e.g. heat rates) have to operate more (or less) frequently; 2. the demands for various fuels will also rise which may result in higher (or lower) fuel prices/costs. The second factor is important for the prediction of electric power and fuel prices. For
example, in August, 2006, the natural gas price soared by 14% because hot weather across the US led to high electricity demand [29]. Similarly, in July 2007, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures [62]. The rising fuel costs due to the increase of electricity demands will then result in higher electric power generating costs as well as electric power prices. The magnitude of fuel price changes may depend on the elasticities of fuel prices as well as the fuel competition in power generation which we have demonstrated in Examples 3.1 and 3.2. Next, we apply our theoretical model to show how these interactions between electric power and fuel markets lead to an equilibrium of the entire power supply chain network. Similar to Examples 3.1 and 3.2, we assume that the natural gas market has an elastic price function and the other fuel prices are fixed.

We used the first case of Example 3.1 as the base case and assumed that the demand in each block increases by 10%. We also assumed that the residual oil price is relatively insensitive and still equal to $7/MMBtu. Tables 10 and 11 show the electric power prices, the natural gas prices, and the demands before and after the increases in the electric power demands.

Table 10: Prices Before the Demand Increase ($/Mwh)

<table>
<thead>
<tr>
<th>Region</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
<th>Block 5</th>
<th>Block 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maine</td>
<td>78.73</td>
<td>76.36</td>
<td>67.69</td>
<td>61.56</td>
<td>50.14</td>
<td>49.18</td>
</tr>
<tr>
<td>Hew Hampshire</td>
<td>84.82</td>
<td>76.36</td>
<td>67.69</td>
<td>61.56</td>
<td>50.14</td>
<td>49.18</td>
</tr>
<tr>
<td>Vermont</td>
<td>84.82</td>
<td>76.36</td>
<td>67.69</td>
<td>61.56</td>
<td>50.14</td>
<td>49.18</td>
</tr>
<tr>
<td>Connecticut</td>
<td>101.81</td>
<td>97.45</td>
<td>71.27</td>
<td>62.22</td>
<td>51.46</td>
<td>49.18</td>
</tr>
<tr>
<td>Rhode Island</td>
<td>84.82</td>
<td>76.36</td>
<td>67.69</td>
<td>62.22</td>
<td>51.46</td>
<td>49.18</td>
</tr>
<tr>
<td>Southeast MA</td>
<td>84.82</td>
<td>76.36</td>
<td>67.69</td>
<td>62.22</td>
<td>51.46</td>
<td>49.18</td>
</tr>
<tr>
<td>Western and Central MA</td>
<td>84.82</td>
<td>76.36</td>
<td>67.69</td>
<td>62.22</td>
<td>51.46</td>
<td>49.18</td>
</tr>
<tr>
<td>Northeastern MA and Boston</td>
<td>91.30</td>
<td>76.36</td>
<td>67.69</td>
<td>62.22</td>
<td>51.46</td>
<td>49.18</td>
</tr>
<tr>
<td>All Regions’ Average</td>
<td>90.23</td>
<td>81.76</td>
<td>68.61</td>
<td>62.08</td>
<td>51.20</td>
<td>49.18</td>
</tr>
<tr>
<td>Natural Gas Demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>35.95 Billion MMBtu</td>
</tr>
<tr>
<td>Natural Gas Price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.00 $/MMBtu</td>
</tr>
</tbody>
</table>
Table 11: Prices after the Demand Increase ($/Mwh)

<table>
<thead>
<tr>
<th>Region</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
<th>Block 5</th>
<th>Block 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maine</td>
<td>78.73</td>
<td>83.45</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.46</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>93.68</td>
<td>84.82</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.46</td>
</tr>
<tr>
<td>Vermont</td>
<td>93.68</td>
<td>84.82</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.46</td>
</tr>
<tr>
<td>Connecticut</td>
<td>109.09</td>
<td>104.20</td>
<td>100.84</td>
<td>75.74</td>
<td>69.23</td>
<td>53.73</td>
</tr>
<tr>
<td>Rhode Island</td>
<td>93.68</td>
<td>84.82</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.73</td>
</tr>
<tr>
<td>Southeastern MA</td>
<td>93.68</td>
<td>84.82</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.73</td>
</tr>
<tr>
<td>Western and Central MA</td>
<td>93.68</td>
<td>84.82</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.73</td>
</tr>
<tr>
<td>Northeastern MA and Boston</td>
<td>165.16</td>
<td>91.30</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
<td>53.73</td>
</tr>
<tr>
<td>All Regions’ Average</td>
<td>111.48</td>
<td>91.04</td>
<td>86.49</td>
<td>73.95</td>
<td>66.16</td>
<td>53.68</td>
</tr>
<tr>
<td>Natural Gas Demand</td>
<td>43.62 Billion MMBtu</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural Gas Price</td>
<td>7.64 $/MMBtu</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

6. Summary, Conclusions, and Future Research

In this paper, we proposed a new model of electric power supply chain networks with fuel markets, which considers both economic transaction networks and physical transmission networks. We derived the optimality conditions of the decision-makers and proved that the governing equilibrium conditions satisfy a variational inequality problem. We also provided some qualitative properties of the model as well as a computational method. We then conducted a case study where our theoretical model was applied to the New England electric power market and fuel supply markets. The model provided a good simulation of the actual regional electric power prices in New England. We also conducted sensitivity analysis in order to investigate the electric power prices under fuel price variations. Additionally, we utilized the model to show how natural gas prices can be significantly influenced by oil prices through electric power networks and markets. In particular, we showed that not only the market responsiveness, but also the electric power market responsiveness, are both crucial to the understanding and determination of the impact of the residual fuel oil price on the natural gas price. Finally, we applied our model to quantitatively demonstrate how changes in the demand for electricity influence the electric power and fuel markets.

The model and results presented in this paper are useful in determining and quantifying the interactions between electric power flows and prices and the various fuel supply markets.
Such information is important to policy-makers who need to ensure system reliability as well as for the energy asset owners and investors who need to manage risk and evaluate their assets.

For future research, several extensions can be developed based on this model. One can add maximum amounts of fuel supplies of the fuel markets to the model to study the system reliability under limited fuel supplies; secondly, one can expand the model to include multiple electric power markets, and to consider broader areas, such as a country or multiple countries. In addition, one can incorporate the price relationship results obtained using this model to other risk management and asset pricing models.

Acknowledgments: The authors are indebted to the two anonymous reviewers and to the Associate Editor, whose many helpful suggestions greatly improved the original version of the paper.

This research was supported by a National Science Foundation Grant No.: IIS-0002647 and by the John F. Smith Memorial Fund at the Isenberg School of Management. This support is gratefully appreciated.

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http://tonto.eia.doe.gov/dnav/pet/pet_pri_resid_a_EPPRL_PTA_cpgal_m.htm
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Appendix

Proof of Theorem 3:

First, we show that if $\pi_{am}(h)$, $a = 1, \ldots, A$; $m = 1, \ldots, M$ is monotone then $\pi_{am}(Q^1)$, $a = 1, \ldots, A$; $m = 1, \ldots, M$, is also monotone. In view of (1), $\pi_{am}(h)$ is equivalent to $\pi_{am}(Q^1)$, $a = 1, \ldots, A$; $m = 1, \ldots, M$. We let $\pi(\cdot)$ denote the vector of all $\pi_{am}(\cdot)$s. We now expand

$$\left\langle (\pi(Q^1) - \pi(Q^{1''}))^T, Q^1 - Q^{1''} \right\rangle$$

(A.1)

$$= \sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} [\pi_{am}(h') - \pi_{am}(h'')] \times [q^{am'}_{gr_1uw} - q^{am''}_{gr_1uw}]$$

$$= \sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} [\pi_{am}(h') - \pi_{am}(h'')] \times [q^{am'}_{gr_1uw} - q^{am''}_{gr_1uw}]$$

$$= \sum_{a=1}^{A} \sum_{m=1}^{M} [\pi_{am}(h') - \pi_{am}(h'')] \times \left[ \sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} (q^{am'}_{gr_1uw} - q^{am''}_{gr_1uw}) \right]$$

which, after the substitution of (1), yields

$$= \sum_{a=1}^{A} \sum_{m=1}^{M} [\pi_{am}(h') - \pi_{am}(h'')] \times [h'_am - \bar{q}_am - (h''_am - \bar{q}_am)]$$

$$= \sum_{a=1}^{A} \sum_{m=1}^{M} [\pi_{am}(h') - \pi_{am}(h'')] \times [h'_am - h''_am].$$

(A.2)

Therefore, if $\pi(h)$ is monotone, then (A.2), equivalently, (A.1) are both greater than or equal to zero, and, therefore, $\pi(Q^1)$ is monotone.

Next, we prove Theorem 3 by expanding

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle$$

(A.3)

$$= \sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} [\pi_{am}(Q^1') + c^{am}_{gr_1uw} - \pi_{am}(Q^{1''}) - c^{am}_{gr_1uw}] \times [q^{am'}_{gr_1uw} - q^{am''}_{gr_1uw}]$$

49
+ \sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \frac{\partial f_{gr_1uw} (q_{gr_1uw}')} {\partial q_{gr_1uw}} - \frac{\partial f_{gr_1uw} (q_{gr_1uw}'')} {\partial q_{gr_1uw}} \times [q_{gr_1uw}' - q_{gr_1uw}'']

+ \sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \frac{\partial c_{rkw}^{gr_1u} (q_{rkw}^{gr_1u}')} {\partial q_{rkw}^{gr_1u}} \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} + c_{rkw}^{gr_1u} (q_{rkw}^{gr_1u}'') \times [q_{rkw}^{gr_1u}' - q_{rkw}^{gr_1u}'']

+ \sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \frac{\partial f_{gr_1uw} (q_{gr_1uw}')} {\partial q_{gr_1uw}} + \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} + \frac{\partial f_{gr_1uw} (q_{gr_1uw}'')} {\partial q_{gr_1uw}} + \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} - \rho_{r_2w} - \frac{\partial c_{rkw}^{gr_1u} (q_{rkw}^{gr_1u}'')} {\partial q_{rkw}^{gr_1u}} \times [y_{rkw}^{gr_1u}' - y_{rkw}^{gr_1u}'']

+ \sum_{w=1}^{W} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} \times [y_{rkw}^{gr_1u}' - y_{rkw}^{gr_1u}'']

+ \sum_{w=1}^{W} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} \times [y_{rkw}^{gr_1u}' - y_{rkw}^{gr_1u}'']

+ \sum_{w=1}^{W} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} \sum_{b=1}^{B} \mu_{bw} \alpha_{r_1r_2b} \times [y_{rkw}^{gr_1u}' - y_{rkw}^{gr_1u}'']

which, after algebraic simplification, yields

\sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \left[ \pi_{am} (Q^{1v}) + \zeta_{gr_1uw}' - \pi_{am} (Q_{1w}') - \zeta_{gr_1uw}'' \right] \times [q_{gr_1uw}' - q_{gr_1uw}'']

50
\[
\sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{u_1=1}^{N_{gr1}} [\partial f_{gr1uw}(q_{gr1uw}) - \partial f_{gr1uw}(q_{gr1uw}')] \times [q_{gr1uw}' - q_{gr1uw}']
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} [\partial c_{gr1uw}(q_{gr1uw}') - \partial c_{gr1uw}(q_{gr1uw}')] - \partial c_{gr1uw}(q_{gr1uw}') \times [q_{gr1uw}' - q_{gr1uw}']
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} [\partial c_{gr1uw}(q_{gr1uw}') - \partial c_{gr1uw}(q_{gr1uw}')] \times [q_{gr1uw}' - q_{gr1uw}']
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} [\partial c_{gr1uw}(q_{gr1uw}') - \partial c_{gr1uw}(q_{gr1uw}')] \times [q_{gr1uw}' - q_{gr1uw}']
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{b=1}^{B} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} \alpha_{r1r2b}(q_{gr1uw}' - q_{gr1uw}'') \times [\mu_{bw}' - \mu_{bw}'']
\]

\[
- \sum_{w=1}^{W} L_w \sum_{b=1}^{B} \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} \alpha_{r1r2b}(q_{gr1uw}' - q_{gr1uw}'') \times [\mu_{bw}' - \mu_{bw}]
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} \alpha_{r1r2b}(q_{gr1uw}' - q_{gr1uw}'') \times [\mu_{bw}' - \mu_{bw}]
\]

\[
- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} \alpha_{r1r2b}(q_{gr1uw}' - q_{gr1uw}'') \times [\mu_{bw}' - \mu_{bw}]
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} \alpha_{r1r2b}(q_{gr1uw}' - q_{gr1uw}'') \times [\mu_{bw}' - \mu_{bw}]
\]

\[
- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u_1=1}^{N_{gr1}} \sum_{k=1}^{K} \alpha_{r1r2b}(q_{gr1uw}' - q_{gr1uw}'') \times [\mu_{bw}' - \mu_{bw}]
\]
\[ -\sum_{w=1}^{W} L_w \sum_{r=1}^{R} \sum_{r_1=1}^{R} \sum_{k=1}^{K} [y_{r_2kw} - y_{r_2kw}^\prime] \times [\rho_{rw}^\prime - \rho_{rw}^\prime] + \sum_{w=1}^{W} L_w \sum_{r=1}^{R} \sum_{g=1}^{G} \sum_{l=1}^{N_{gr_1}} [z_{gr_1uw} - z_{gr_1uw}^\prime] \times [\varphi_{r_1w}^\prime - \varphi_{r_1w}^\prime] - \sum_{w=1}^{W} L_w \sum_{r=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} [z_{gr_1uw} - z_{gr_1uw}^\prime] \times [\varphi_{r_1w}^\prime - \varphi_{r_1w}^\prime] \] (A.5)

which is equal to

\[ \sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{l=1}^{N_{gr_1}} \sum_{u=1}^{N_{gr_1}} \left[ \pi_{am}(Q^{1\prime}) - \pi_{am}(Q^{1\prime\prime}) \right] \times [q_{gr_1uw}^\prime - q_{gr_1uw}^\prime] + \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{l=1}^{N_{gr_1}} \left[ \frac{\partial f_{gr_1uw}(q_{gr_1uw})}{\partial q_{gr_1uw}} - \frac{\partial f_{gr_1uw}(q_{gr_1uw})}{\partial q_{gr_1uw}} \right] \times [q_{gr_1uw}^\prime - q_{gr_1uw}^\prime] + \sum_{w=1}^{W} L_w \sum_{g=1}^{G} \sum_{l=1}^{N_{gr_1}} \sum_{m=1}^{M} \sum_{l_1=1}^{L} \sum_{k=1}^{K} \left[ \frac{\partial c_{gr_1uw}^u(y_{r_2kw})}{\partial y_{r_2kw}} - \frac{\partial c_{gr_1uw}^u(y_{r_2kw})}{\partial y_{r_2kw}} \right] \times [q_{gr_1uw}^\prime - q_{gr_1uw}^\prime] \] (A.6)

Hence, if all cost functions in the model are continuously differentiable and convex; all unit cost functions are nondecreasing, and the inverse supply functions are nondecreasing, then (A.6), equivalently, (A.3) are both greater than or equal to zero, and, therefore, \( F(X) \) is monotone. Q.E.D.