Supply Chain Supernetworks and Environmental Criteria

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May 2002; revised November 2002

Appears in *Transportation Research D* **8** (2003), 185-213.

Abstract: In this paper, we develop a framework for the modeling and analysis of supply chain networks with electronic commerce in which the decision-makers are faced with multiple criteria, including environmental ones. We establish the optimality conditions for the manufacturers, retailers, and consumers, derive the equilibrium conditions, and provide the variational inequality formulation. We also propose a continuous time adjustment process for the study of the disequilibrium dynamics. Finally, we apply an algorithm for the determination of equilibrium prices and product shipments as well as the emissions generated in several supply chain examples.

Key words: supply chains, environment, multicriteria decision-making, network equilibrium, variational inequalities.

1. Introduction

The economic landscape has been transformed with the advent of the Internet Age and the growth of electronic commerce (e-commerce), which, in turn, has had a major effect on the manner in which manufacturers, retailers as well as consumers, order goods and have them transported. Estimates of business-to-business (B2B) commerce, which is the principal segment of e-commerce transactions, range from .1 trillion dollars to 1 trillion dollars in 1998, with forecasts reaching as high as \$4.8 trillion dollars in 2003 in the United States (see Federal Highway Administration 2000). The business-to-consumer (B2C) component, although a much smaller segment of US retail activity, is estimated to grow to \$80 billion per year (Southworth 2000). Both B2B and B2C commerce, in turn, have affected the structure and profitability of supply chain networks, where recall that a supply chain is a chain of relationships which synthesizes and integrates the movement of goods between suppliers, manufacturers, distributors, retailers, and consumers (see Handfield and Nichols 1999).

Supply chain networks with electronic commerce is a topic that has been receiving growing attention due to its practical importance. For example, recently, Nagurney et al. (2002a) proposed a rigorous framework for the formulation, qualitative analysis, and computation of equilibrium prices and product flows in supply chains with tiers of decision-makers in the presence of e-commerce (both B2B and B2C) transactions. Their approach utilized the supernetwork concept (cf. Nagurney and Dong (2002)) in which decision-making regarding telecommunication versus transportation tradeoffs could be handled within an appropriately constructed *abstract* network. Dong et al. (2002), in turn, recognized that decision-makers in supply chain networks may be faced with additional criteria to those of profit maximization (or cost minimization) and introduced multicriteria decision-making into supply chain networks but focused on supply chains without electronic commerce.

In this paper, we turn to the critical issue of supply chain networks with environmental concerns in the context of decision-making in the Information Age today. Environmental issues surrounding supply chains have only recently come to the fore, notably, in the context of conceptual and survey studies (cf. Hill 1997 and the references therein) as well as applied studies (see Hitchens et al. 2000). Here, in contrast, we develop a theoretical framework, which builds upon the work of Nagurney et al. (2002a,b), Nagurney and Dong (2002), and

Dong et al. (2002), and allows decision-makers in different tiers of the supply chain to be multicriteria ones with criteria including that of minimization of emissions. Moreover, we allow different decision-makers to weight the criteria (including the environmental ones) in distinct fashion. The framework, hence, allows one to simulate different scenarios depending on how environmentally conscious (or not) the decision-makers are and to explore the impact on the product prices, the product shipments, and the emissions generated.

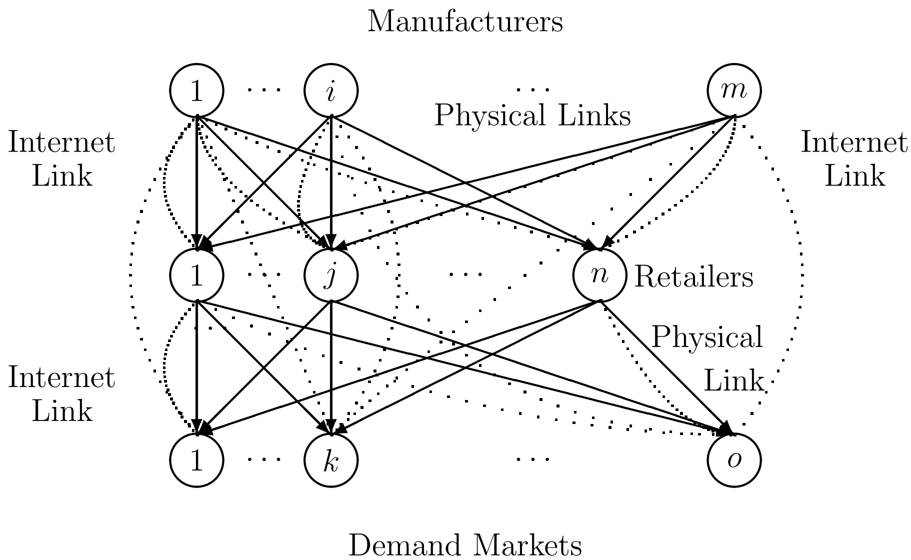

Figure 1: The Multitiered Supernetwork Structure of the Supply Chain Network with Multicriteria Decision-Makers and Electronic Commerce

2. The Supply Chain Supernetwork Model with Multicriteria Decision-Makers

In this Section, we develop the supply chain supernetwork model with manufacturers, retailers, and consumers in which the manufacturers can sell directly to the consumers at the demand markets through the Internet and can also conduct their business transactions with the retailers through the Internet. The consumers, in turn, can conduct their transactions with the retailers through the Internet, or in the standard manner, which we term *physical*. The supply chain network is as depicted in Figure 1.

We consider m manufacturers involved in the production of a homogeneous product which can then be purchased by n retailers and/or directly by the consumers located at the o demand markets. We denote a typical manufacturer by i, a typical retailer by j, and a typical demand market by k. The manufacturers are located at the top tier of nodes of the network, the retailers at the middle tier, and the demand markets at the third or bottom tier of nodes. The links in the supply chain in Figure 1 include classical physical links as well Internet links, denoted by dotted arcs, to allow for electronic commerce.

The Behavior of the Manufacturers

We assume that each manufacturer is faced with two criteria: the maximization of profit and the minimization of total emissions generated in production and in transaction (which is assumed here to include the delivery of the product). Let q_i denote the nonnegative production output by manufacturer i and group the production outputs of all manufacturers into the column vector $q \in R^m_+$. We assume that each manufacturer i is faced with a production cost function f_i, which can depend, in general, on the entire vector of production outputs, that is,

$$f_i = f_i(q), \quad \forall i. \tag{1}$$

A manufacturer (see Figure 1) may transact with a retailer via a physical link, and/or via an Internet link. We denote the transaction cost associated with manufacturer i transacting with retailer j via link (also referred to as *mode*) l, where $l = 1$ denotes a physical link and $l = 2$ denotes an Internet link, by c_{ijl}. We denote the product shipment associated with manufacturer i, retailer j, and mode of transaction l by q_{ijl}, and group these product shipments into the column vector $Q^1 \in R^{2mn}_+$. In addition, a manufacturer i may transact directly with consumers located at a demand market k with this transaction cost associated with the Internet transaction denoted by c_{ik} and the associated product shipment from manufacturer i to demand market k by q_{ik}. We group these product shipments into the column vector $Q^2 \in R^{mo}_+$.

We consider the situation in which the transaction cost between a manufacturer and retail pair as well as the transaction cost between a manufacturer and consumers at a demand market may depend upon the volume of transactions between each such pair, that is:

$$c_{ijl} = c_{ijl}(q_{ijl}), \quad \forall i, j, l, \tag{2a}$$

and

$$c_{ik} = c_{ik}(q_{ik}), \quad \forall i, k. \tag{2b}$$

The quantity of the product produced by manufacturer i must satisfy the following conservation of flow equation:

$$q_i = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{ijl} + \sum_{k=1}^{o} q_{ik}, \tag{3}$$

which states that the quantity produced by manufacturer i is equal to the sum of the quantities shipped from the manufacturer to all retailers and to all demand markets.

The total costs incurred by a manufacturer i, thus, are equal to the sum of the manufacturer's production cost plus the total transaction costs. His revenue, in turn, is equal to the price that the manufacturer charges for the product (and the consumers are willing to pay) times the total quantity obtained/purchased of the product from the manufacturer by all the retail outlets and consumers at all demand markets. Let ρ^*_{1ijl} denote the price charged for the product by manufacturer i to retailer j who has transacted using mode l, and let ρ^*_{1ik} denote the price charged by manufacturer i for the product to consumers at demand market k. We, later, discuss how these prices are arrived at.

Noting the conservation of flow equations (3), we can express the criterion of profit maximization for manufacturer i as:

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl} + \sum_{k=1}^{o}\rho^*_{1ik}q_{ik} - f_i(Q^1, Q^2) - \sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl}) - \sum_{k=1}^{o}c_{ik}(q_{ik}), \qquad (4)$$

subject to $q_{ijl} \geq 0$, for all j, l, and $q_{ik} \geq 0$, for all k.

In addition to the criterion of profit maximization, we assume that each manufacturer also seeks to minimize the total emissions generated in the production of the product as well as its delivery to the next tier of decision-makers, whether retailers or consumers at the demand markets. We assume that the emissions generated by manufacturer i in producing the product are given by the function e_i, where

$$e_i = e_i(q_i), \quad \forall i, \qquad (5)$$

whereas the emissions generated in transacting with retailer j via mode l are given by a function e_{ijl}, such that

$$e_{ijl} = e_{ijl}(q_{ijl}), \quad \forall i, j, l, \qquad (6)$$

and, finally, the emissions generated with transacting (and delivery) of the product to demand market k is represented by a function e_{ik}, where

$$e_{ik} = e_{ik}(q_{ik}), \quad \forall i, k. \qquad (7)$$

Hence, the second criterion of each manufacturer can be expressed mathematically as:

$$\text{Minimize } e_i(q_i) + \sum_{j=1}^{n}\sum_{l=1}^{2} e_{ijl}(q_{ijl}) + \sum_{k=1}^{o} e_{ik}(q_{ik}) \tag{8}$$

subject to (3) and

$$q_{ijl} \geq 0, \quad \forall j, l; \quad q_{ik} \geq 0, \quad \forall k. \tag{9}$$

Here we consider emission functions of specific form (cf. (5), (6), and (7)) given by

$$e_i(q_i) = h_i q_i, \quad \forall i, \tag{10}$$

$$e_{ijl}(q_{ijl}) = h_{ijl} q_{ijl}, \quad \forall i, j, l, \tag{11}$$

and

$$e_{ik}(q_{ik}) = h_{ik} q_{ik}, \quad \forall i, k, \tag{12}$$

where the h_i, h_{ijl}, and h_{ik} terms are nonnegative and represent the amount of emissions generated per unit of product produced and transacted, respectively. Hence, we explicitly allow the emissions generated to be distinct according to whether the transaction was conducted electronically or not. Thus, the manufacturer's decision-making problem concerning the emissions generated, in view of (8), (3), and (10) – (12), can be expressed as:

$$\text{Minimize } \sum_{j=1}^{n}\sum_{l=1}^{2}(h_i + h_{ijl})q_{ijl} + \sum_{k=1}^{o}(h_i + h_{ik})q_{ik}, \tag{13}$$

subject to the nonnegativity assumption on the production and shipments quantities of the product given by (9).

A Manufacturer's Multicriteria Decision-Making Problem

We assume that manufacturer i assigns a nonnegative weight α_i with the emissions generated criterion with the weight associated with profit maximization serving as the numeraire and being set equal to 1. Thus, we can construct a value function for each manufacturer (cf. Keeney and Raiffa 1993) using a constant additive weight value function. Consequently, the multicriteria decision-making problem for manufacturer i is transformed into:

$$\text{Maximize } \sum_{j=1}^{n}\sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} + \sum_{k=1}^{o} \rho^*_{1ik} q_{ik} - f_i(Q^1, Q^2) - \sum_{j=1}^{n}\sum_{l=1}^{2} c_{ijl}(q_{ijl}) - \sum_{k=1}^{o} c_{ik}(q_{ik})$$

$$-\alpha_i(\sum_{j=1}^{n}\sum_{l=1}^{2}(h_i+h_{ijl})q_{ijl}+\sum_{k=1}^{o}(h_i+h_{ik})q_{ik}), \tag{14}$$

subject to (9).

The Optimality Conditions of the Manufacturers

The manufacturers compete in a noncooperative fashion following Nash (1950, 1951). We assume that the production cost functions and the transaction cost functions for each manufacturer are continuous and convex. Each manufacturer will determine his optimal production quantity and shipments, given the optimal ones of the competitors, with the optimality conditions for all manufacturers *simultaneously* expressed as the following inequality (cf. Bazaraa et al. 1993, Gabay and Moulin 1980; see also Nagurney 1999):

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+\alpha_i(h_i+h_{ijl})-\rho^*_{1ijl}\right]\times\left[q_{ijl}-q^*_{ijl}\right]$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}}+\alpha_i(h_i+h_{ik})-\rho^*_{1ik}\right]\times[q_{ik}-q^*_{ik}]\geq 0,$$
$$\forall Q^1\in R^{2mn}_+, \forall Q^2\in R^{mo}_+. \tag{15}$$

The inequality (15), which is a *variational inequality* (cf. Nagurney 1999) has a nice economic interpretation. In particular, from the first term we can infer that, if there is a positive shipment of the product transacted either in a classical manner or via the Internet from a manufacturer to a retailer, then the marginal cost of production plus the marginal cost of transacting plus what can be interpreted as the marginal cost of emissions: $\alpha_i(h_i+h_{ijl})$ must be equal to the price that the retailer is willing to pay for the product. If that sum, in turn, exceeds the price then there will be zero volume of flow of the product thus transacted. The second term in (15) has a similar interpretation; in particular, there will be a positive volume of flow of the product from a manufacturer to a demand market if the marginal cost of production of the manufacturer plus the marginal cost of transacting via the Internet for the manufacturer with the consumers and the marginal cost of emissions: $\alpha_i(h_i+h_{ik})$ is equal to the price the consumers are willing to pay for the product at the demand market.

The Behavior of the Retailers

The retailers, in turn, are involved in transactions both with the manufacturers since they wish to obtain the product for their retail outlets, as well as with the consumers, who are the ultimate purchasers of the product. Thus, as depicted in Figure 1, a retailer conducts transactions both with the manufacturers as well as with the consumers. The retailers are also assumed to be multicriteria decision-makers in that they seek to maximize profits and to minimize the emissions generated from the perspective of the amounts of the product that they purchase from the manufacturers and the manner in which the transactions occur and the products are shipped.

A retailer j is faced with what we term a *handling* cost, which may include, for example, the display and storage cost associated with the product. We denote this cost by c_j and, in the simplest case, we would have that c_j is a function of $\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}$, that is, the holding cost of a retailer is a function of how much of the product he has obtained from the various producers via the two different modes of transacting. However, for the sake of generality, and to enhance the modeling of competition, we allow the function to, in general, depend also on the amounts of the product held by other retailers and, therefore, we may write:

$$c_j = c_j(Q^1), \quad \forall j. \tag{16}$$

We denote the transaction cost associated with retailer j transacting with manufacturer i using mode l by $\hat{c}_{ijl}$ and we assume that the function can depend upon the manufacturer/retailer pair product shipment, that is,

$$\hat{c}_{ijl} = \hat{c}_{ijl}(q_{ijl}), \quad \forall i, j, l. \tag{17}$$

Let q_{jkl} denote the amount of the product purchased/consumed by consumers located at demand market k from retailer j and transacted via mode l and group these consumption quantities into the column vector $Q^3 \in R^{2no}_{+}$ and assume that the cost associated with transacting between retailer j, demand market k, and mode l, from the perspective of the retailer is given by the function c_{jkl}, where

$$c_{jkl} = c_{jkl}(q_{jkl}), \quad \forall j, k, l. \tag{18}$$

The retailers associate a price with the product at their retail outlet, which is denoted by ρ^*_{2j}, for retailer j. This price, as we will show, will also be endogenously determined in the model. The retailers are also profit-maximizers, with the criterion of profit maximization for retailer j given by:

$$\text{Maximize} \quad \rho_{2j}^* \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{2} \hat{c}_{ijl}(q_{ijl}) - \sum_{k=1}^{o} \sum_{l=1}^{2} c_{jkl}(q_{jkl}) - \sum_{i=1}^{m} \sum_{l=1}^{2} \rho_{1ijl}^* q_{ijl} \tag{19}$$

subject to:

$$\sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} \leq \sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}, \tag{20}$$

and the nonnegativity constraints: $q_{ijl} \geq 0$, and $q_{jkl} \geq 0$, for all i, l, k. Objective function (19) expresses that the difference between the revenues minus the handling cost plus the transaction costs and the payout to the manufacturers should be maximized. Constraint (20) simply expresses that consumers cannot purchase more from a retailer than is held in stock.

In addition, we assume that each retailer seeks to minimize the emissions associated with his transactions with the manufacturers, that is, he is also faced with the following criterion:

$$\text{Minimize} \sum_{i=1}^{m} \sum_{l=1}^{2} (h_i + h_{ijl}) q_{ijl} \tag{21}$$

subject to:

$$q_{ijl} \geq 0, \quad \forall i, l. \tag{22}$$

A Retailer's Multicriteria Decision-Making Problem

Retailer j associates a nonnegative weight β_j with the emission generation criterion (21) and a weight equal to 1 with profit maximization (19) (see also the discussion concerning manufacturers above), yielding the following multicriteria decision-making problem:

$$\text{Maximize} \quad \rho^*_{2j}\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} - c_j(Q^1) - \sum_{i=1}^{m}\sum_{l=1}^{2}\hat{c}_{ijl}(q_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} c_{jkl}(q_{jkl}) - \sum_{i=1}^{m}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl}$$
$$-\beta_j(\sum_{i=1}^{m}\sum_{l=1}^{2}(h_i + h_{ijl})q_{ijl}) \tag{23}$$

subject to: (20) and the nonnegativity assumption on the variables.

Optimality Conditions of the Retailers

We now consider the optimality conditions of the retailers assuming that each retailer is faced with the multicriteria decision-making problem (23), subject to (20), and the nonnegativity assumption on the variables. We assume, as we did for the manufacturers, that the retailers compete in a noncooperative manner, given the actions of the other retailers. Note that, at this point, we consider that retailers seek to determine not only the optimal amounts purchased by the consumers from their specific retail outlet but, also, the amount that they wish to obtain from the manufacturers. In equilibrium, all the shipments between the tiers of network decision-makers will have to coincide.

Assuming that the handling cost for each retailer is continuous and convex as are the transaction costs, the optimality conditions for all the retailers satisfy the variational inequality:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \beta_j(h_i + h_{ijl}) + \rho^*_{1ijl} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}} - \gamma^*_j\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}} - \rho^*_{2j} + \gamma^*_j\right]\times\left[q_{jkl} - q^*_{jkl}\right] + \sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{o}\sum_{l=1}^{2} q^*_{jkl}\right]\times\left[\gamma_j - \gamma^*_j\right] \geq 0,$$
$$\forall Q^1 \in R^{2mn}_+, \forall Q^3 \in R^{2no}_+, \forall \gamma \in R^n_+. \tag{24}$$

Note that γ_j is the Lagrange multiplier associated with constraint (20) for retailer j and γ the column vector of all the retailers' multipliers. In this derivation, as in the derivation of inequality (15), we have not had the prices charged by the manufacturers and by the retailers be variables. They become endogenous variables in the complete equilibrium model.

We now highlight the economic interpretation of the retailers' optimality conditions. From the second term in inequality (24), we have that, if consumers at demand market k purchase the product from a particular retailer j, transacted via mode l, that is, if the q^*_{jkl} is positive, then the price charged by retailer j, ρ^*_{2j}, is precisely equal to γ^*_j (which, from the third term in the inequality, serves as the price to clear the market from retailer j) plus the marginal cost associated with this transaction. Also, note that, from the second term, we see that if no product is sold by a particular retailer, then the price associated with holding the product can exceed the price charged to the consumers plus the marginal transaction cost. Furthermore, from the first term in inequality (24), we can infer that, if a manufacturer transacts with a retailer via a particular mode resulting in a positive flow of the product between the two, then the price γ^*_j is precisely equal to the retailer j's payment to the manufacturer, ρ^*_{1ijl}, plus his marginal cost of handling the product plus the retailer's marginal cost of transaction associated with transacting with the particular manufacturer and the marginal cost of emissions associated with this transaction.

The Consumers at the Demand Markets

The consumers take into account in making their consumption decisions not only the price charged for the product by the retailers and the manufacturers but also their transaction costs associated with obtaining the product as well as the emissions generated in their transactions. The consumers at the demand markets can transact either directly with the manufacturers through the Internet or physically with the retailers.

Let $\hat{c}_{jkl}$ denote the transaction cost associated with obtaining the product by consumers at demand market k from retailer j via mode l and recall that q_{jkl} is the amount of the product transacted between retailer j and consumers at demand market k via mode l. We assume that the transaction cost is continuous and of the general form:

$$\hat{c}_{jkl} = \hat{c}_{jkl}(Q^2, Q^3), \quad \forall j, k, l. \tag{25}$$

Also, let $\hat{c}_{ik}$ denote the transaction cost, from the perspective of the consumers at demand market k, associated with manufacturer i. Here we assume that

$$\hat{c}_{ik} = \hat{c}_{ik}(Q^2, Q^3), \quad \forall i, k, \tag{26}$$

Hence, the cost of conducting a transaction with a manufacturer via the Internet can depend, in general, upon the volume of the product obtained via the Internet as well as the amount purchased from the retailers.

Let ρ_{3k} denote the price of the product as perceived by the consumers at demand market k. Further, denote the demand for the product at demand market k by d_k and assume, as given, the continuous demand functions:

$$d_k = d_k(\rho_3), \quad \forall k, \tag{27}$$

where ρ_3 is the o-dimensional column vector of generalized prices. Thus, according to (27), the demand of consumers for the product at a demand market depends, in general, not only on the price of the product at that demand market but also on the prices of the product at the other demand markets. Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

The consumers take the price charged by the retailers for the product, which, recall was denoted by ρ^*_{2j} for retailer j, plus the transaction cost associated with obtaining the product, in making their consumption decisions. In addition, they take the price charged by a producer, ρ^*_{1ik}, plus that associated transaction cost into consideration. In addition, we assume that the consumers are also multicriteria decision-makers and weight the emissions associated with their transactions and purchases accordingly.

The Multicriteria Equilibrium Conditions for the Demand Markets

We assume that consumers at a demand market perceive the emissions generated through their transactions (and purchases) in an individual fashion with the nonnegative weight η_k associated with the total emissions generated through consumer transactions at demand market k. The equilibrium conditions for consumers at demand market k, thus, take the form:

for all retailers: $j; j = 1, \ldots, n$ and modes l; $l = 1, 2$:

$$\rho_{2j}^* + \hat{c}_{jkl}(Q^{2*}, Q^{3*}) + \eta_k h_{jkl} \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{jkl}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{jkl}^* = 0, \end{cases} \tag{28}$$

for all manufacturers i; $i = 1, \ldots, m$:

$$\rho_{1ik}^* + \hat{c}_{ik}(Q^{2*}, Q^{3*}) + \eta_k(h_i + h_{ik}) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{ik}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{ik}^* = 0, \end{cases} \tag{29}$$

and

$$d_k(\rho_3^*) \begin{cases} = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^* + \sum_{i=1}^{m} q_{ik}^*, & \text{if} \quad \rho_{3k}^* > 0 \\ \leq \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^* + \sum_{i=1}^{m} q_{ik}^*, & \text{if} \quad \rho_{3k}^* = 0. \end{cases} \tag{30}$$

Conditions (28) state that consumers at demand market k will purchase the product from retailer j, if the price charged by the retailer for the product plus the transaction cost (from the perspective of the consumers) plus the marginal cost of emissions associated with that transaction does not exceed the price that the consumers are willing to pay for the product. Conditions (29) state the analogue for the manufacturers and demand market. Condition (30), on the other hand, states that, if the price the consumers are will to pay for the product at a demand market is positive, then the quantity consumed by the consumers at the demand market is precisely equal to the demand. These conditions correspond to the well-known spatial price equilibrium conditions (cf. Takayama and Judge 1971, Nagurney 1999, and the references therein).

In equilibrium, conditions (28), (29), and (30) will have to hold for all demand markets k, and these, in turn, can also be expressed as a variational inequality problem akin to (15) and (24) and given by: determine $(Q^{2*}, Q^{3*}, \rho_3^*) \in R^{mo+2no+n}$, such that

$$\sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{2} \left[\rho_{2j}^* + \hat{c}_{jkl}(Q^{2*}, Q^{3*}) + \eta_k h_{jkl} - \rho_{3k}^* \right] \times \left[q_{jkl} - q_{jkl}^* \right]$$

$$+ \sum_{i=1}^{m} \sum_{k=1}^{o} \left[\rho_{1ik}^* + \hat{c}_{ik}(Q^{2*}, Q^{3*}) + \eta_k(h_i + h_{ik}) - \rho_{3k}^* \right] \times [q_{ik} - q_{ik}^*]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{*}+\sum_{i=1}^{m} q_{ik}^{*}-d_{k}(\rho_{3}^{*})\right]\times[\rho_{3k}-\rho_{3k}^{*}]\geq 0, \quad \forall(Q^{2}, Q^{3}, \rho_{3})\in R_{+}^{mo+2no+o}. \quad (31)$$

Note that, in the context of the consumption decisions, we have utilized demand functions, rather than utility functions, as was the case for the manufacturers and the retailers, who were assumed to be faced with profit functions, which correspond to utility functions. Of course, demand functions can be derived from utility functions. We expect the number of consumers to be much greater than that of the manufacturers and retailers and, hence, believe that the above formulation is the more natural and tractable one.

The Equilibrium Conditions of the Supply Chain

In equilibrium, the shipments of the product that the manufacturers ship to the retailers must be equal to the shipments that the retailers accept from the manufacturers. In addition the amounts of the product purchased by the consumers must be equal to the amounts sold by the retailers and directly to the consumers by the manufacturers. Furthermore, the equilibrium shipment and price pattern must satisfy the sum of the optimality conditions (15) and (24), and the conditions (31), in order to formalize the agreements between the tiers of the supply chain network (see also Nagurney et al. 2002a, b). We now state this formally.

Definition 1: Supply Chain Network Equilibrium with Multicriteria Decision-Makers and Electronic Commerce

The equilibrium state of the supply chain is one where the flows between the tiers of the supply chain network coincide and the product shipments, prices, and emissions satisfy the sum of the optimality conditions (15) and (24), and the equilibrium conditions (31).

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the supply chain supernetwork model are equivalent to the solution of the variational inequality problem given by: determine $(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^{*}, \rho_{3}^{*})$

$\in R_+^{2mn+mo+2no+n+o}$ *satisfying*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q_{ijl}^*)}{\partial q_{ijl}}+\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}}+\frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}}+(\alpha_i+\beta_j)(h_i+h_{ijl})-\gamma_j^*\right]$$

$$\times\left[q_{ijl}-q_{ijl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}}+\hat{c}_{ik}(Q^{2*},Q^{3*})+(\alpha_i+\eta_k)(h_i+h_{ik})-\rho_{3k}^*\right]\times[q_{ik}-q_{ik}^*]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q_{jkl}^*)}{\partial q_{jkl}}+\hat{c}_{jkl}(Q^{2*},Q^{3*})+\gamma_j^*+\eta_k h_{jkl}-\rho_{3k}^*\right]\times\left[q_{jkl}-q_{jkl}^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q_{ijl}^*-\sum_{k=1}^{o}\sum_{l=1}^{2}q_{jkl}^*\right]\times\left[\gamma_j-\gamma_j^*\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q_{jkl}^*+\sum_{i=1}^{m}q_{ik}^*-d_k(\rho_3^*)\right]\times[\rho_{3k}-\rho_{3k}^*]\geq 0,\ \forall(Q^1,Q^2,Q^3,\gamma,\rho_3)\in R_+^{2mn+mo+2no+n+o}. \tag{32}$$

Proof: We first establish that the equilibrium conditions imply variational inequality (32). Indeed, the summation of inequalities (15), (24), and (31) yields, after algebraic simplification, the variational inequality (32).

We now establish the converse, that is, that a solution to variational inequality (32) satisfies the sum of conditions (15), (24), and (31) and is, hence, an equilibrium according to Definition 1. To inequality (32), add the term: $-\rho_{1ijl}^*+\rho_{1ijl}^*$ to the term in the first set of brackets, preceding the multiplication sign. Similarly, add the term: $-\rho_{1ik}^*+\rho_{1ik}^*$ to the term preceding the second multiplication sign, and, finally, add the term: $-\rho_{2j}^*+\rho_{2j}^*$ to the term preceding the third multiplication sign. Such "terms" do not change the value of the inequality since they are identically equal to zero, with the resulting inequality of the form:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q_{ijl}^*)}{\partial q_{ijl}}+\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}}+\frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}}+(\alpha_i+\beta_j)(h_i+h_{ijl})\right.$$

$$\left.-\gamma_j^*-\rho_{1ijl}^*+\rho_{1ijl}^*\right]\times\left[q_{ijl}-q_{ijl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}}+\hat{c}_{ik}(Q^{2*},Q^{3*})+(\alpha_i+\eta_k)(h_i+h_{ik})-\rho^*_{3k}-\rho^*_{1ik}+\rho^*_{1ik}\right]\times[q_{ik}-q^*_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}}+\hat{c}_{jkl}(Q^{2*},Q^{3*})+\gamma^*_j+\eta_k h_{jkl}-\rho^*_{3k}-\rho^*_{2j}+\rho^*_{2j}\right]\times\left[q_{jkl}-q^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q^*_{ijl}-\sum_{k=1}^{o}\sum_{l=1}^{2}q^*_{jkl}\right]\times\left[\gamma_j-\gamma^*_j\right]+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q^*_{jkl}+\sum_{i=1}^{m}q^*_{ik}-d_k(\rho^*_3)\right]\times[\rho_{3k}-\rho^*_{3k}]\geq 0,$$

$$\forall(Q^1,Q^2,Q^3,\gamma,\rho_3)\in R_+^{2mn+mo+2no+n+o}, \tag{33}$$

which, in turn, can be rewritten as:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+\alpha_i(h_i+h_{ijl})-\rho^*_{1ijl}\right]\times\left[q_{ijl}-q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}}+\alpha_i(h_i+h_{ik})-\rho^*_{1ik}\right]\times[q_{ik}-q^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}}+\rho^*_{1ijl}+\frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+\beta_j(h_i+h_{ijl})-\gamma^*_j\right]\times\left[q_{ijl}-q^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}}-\rho^*_{2j}+\gamma^*_j\right]\times\left[q_{jkl}-q^*_{jkl}\right]+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q^*_{ijl}-\sum_{k=1}^{o}\sum_{l=1}^{2}q^*_{jkl}\right]\times\left[\gamma_j-\gamma^*_j\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\rho^*_{2j}+\hat{c}_{jkl}(Q^{2*},Q^{3*})+\eta_k h_{jkl}-\rho^*_{3k}\right]\times\left[q_{jkl}-q^*_{jkl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\rho^*_{1ik}+\hat{c}_{ik}(Q^{2*},Q^{3*})+\eta_k(h_i+h_{ik})-\rho^*_{3k}\right]\times[q_{ik}-q^*_{ik}]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q^*_{jkl}+\sum_{i=1}^{m}q^*_{ik}-d_k(\rho^*_3)\right]\times[\rho_{3k}-\rho^*_{3k}]\geq 0,\ \forall(Q^1,Q^2,Q^3,\gamma,\rho_3)\in R_+^{2mn+mo+2no+n+o}. \tag{34}$$

But inequality (34) is equivalent to the price and product shipment pattern satisfying the sum of the conditions (15), (24), and (31). The proof is complete. □

For easy reference in the subsequent sections, variational inequality problem (32) can be rewritten in standard variational inequality form (cf. Nagurney 1999) as follows:

$$\langle F(X^*)^T, X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}\equiv R_+^{2mn+mo+2no+n+o}, \tag{35}$$

where $X \equiv (Q^1, Q^2, Q^3, \gamma, \rho_3)$, and $F(X) \equiv (F_{ijl}, F_{ik}, F_{jkl}, F_j, F_k)_{i=1,\ldots,m;j=1,\ldots,n;l=1,2;k=1,\ldots,o}$, and the specific components of F given by the functional terms preceding the multiplication signs in (32), respectively. The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

From (32) one can see the flexibility of the model in terms of the variety of scenarios that can be explored depending on the weights associated with the environmental criteria and the distinct tiers of decision-makers. Clearly, if the weights: α_i, β_j, and η_k are precisely equal to zero for all i, j, k, then this model collapses to generalizations of the supply chain network models with electronic commerce studied in Nagurney et al. (2002a) and Nagurney and Dong (2002). On the other hand, one can also evaluate from the solution of (32) not only the two extremes of all decision-makers being environmentally conscious (to varying degrees) and none of them being, but also intermediary scenarios in which, say, some of the manufacturing firms and/or some of the retailers, and/or some of the consumers at the demand markets are environmentally conscious.

Also, one can evaluate the case when all the decision-makers associated with a particular tier are environmentally conscious (say, the consumers at the demand markets or, perhaps, the retailers) and none of the other decision-makers are. Furthermore, it is important to note that, in terms of a transaction between a pair of decision-makers between tiers, that if one of the decision-makers has his weight associated with the environmental criterion equal to zero, the other one can still have a positive weight and since there must be agreement in terms of the shipments between tiers, there will still be an effect on the equilibrium product flows and prices and, consequently, on the emissions generated. Note that the total emissions generated in the supply chain can be computed from the sums of (10), (11), and (12), evaluated at the equilibrium product shipments.

We now discuss how to recover the prices ρ^*_{1ijl}, for all i, j, l, and ρ^*_{2j}, for all j, from the solution of variational inequality (32). (In Section 5 we describe an algorithm for computing the solution.) Recall that, in the preceding discussions, we have noted that if $q^*_{jkl} > 0$, for some k, j, and l, then ρ^*_{2j} is precisely equal to $\gamma^*_j + \frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}}$, with γ^*_j being obtained from the solution of (32). The prices ρ^*_{1ijl}, in turn (cf. also (15)), can be obtained by finding a $q^*_{ijl} > 0$, and then setting $\rho^*_{1ijl} = \left[\frac{\partial f(Q^{1*}, Q^{2*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}} + \alpha_i (h_i + h_{ik}) \right]$, or, equivalently, to

$\left[\gamma_j^* - \frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} - \beta_j(h_i + h_{ijl})\right]$, for all such i, j, l. The prices ρ_{1ik}^*, on the other hand, can be obtained by finding a $q_{ik}^* > 0$ and setting $\rho_{1ik}^* = \left[\frac{\partial f_i(Q^{1*}, Q^{2*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}} + \alpha_i(h_i + h_{ik})\right]$, or, equivalently, to $\left[\rho_{3k}^* - \hat{c}_{ik}(Q^{2*}, Q^{3*}) - \eta_k(h_i + h_{ik})\right]$, for all such i, k.

In Figure 1, we depict the structure of the supply chain network in equilibrium, consisting of all the manufacturers, all the retailers, and all the demand markets. Clearly, the special cases of our model in which there is only B2B commerce or only B2C commerce can be studied in our framework as well with a suitable reduction of the links and associated transaction costs and product shipments.

3. Qualitative Properties

In this Section, we provide some qualitative properties of the solution to variational inequality (32). In particular, we derive existence and uniqueness results. We also investigate properties of the function F (cf. (35)) that enters the variational inequality of interest here.

Since the feasible set is not compact we cannot derive existence simply from the assumption of continuity of the functions. Nevertheless, we can impose a rather weak condition to guarantee existence of a solution pattern. Let

$$\mathcal{K}_b = \{(Q^1, Q^2, Q^3, \gamma, \rho_3) | 0 \leq Q^1 \leq b_1;\ 0 \leq Q^2 \leq b_2; 0 \leq Q^3 \leq b_3;\ 0 \leq \gamma \leq b_4;\ 0 \leq \rho_3 \leq b_5\}, \tag{36}$$

where $b = (b_1, b_2, b_3, b_4, b_5) \geq 0$ and $Q^1 \leq b_1; Q^2 \leq b_2; Q^3 \leq b_3; \gamma \leq b_4; \rho_3 \leq b_5$ means that $q_{ijl} \leq b_1$; $q_{ik} \leq b_2$; $q_{jkl} \leq b_3$; $\gamma_j \leq b_4$; and $\rho_{3k} \leq b_5$ for all i, j, l, k. Then $\mathcal{K}_b$ is a bounded closed convex subset of $R^{2mn+mo+2no+n+o}$. Thus, the following variational inequality

$$\langle F(X^b)^T, X - X^b \rangle \geq 0, \quad \forall X^b \in \mathcal{K}_b, \tag{37}$$

admits at least one solution $X^b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney 1999), we then have:

Theorem 2

Variational inequality (32) admits a solution if and only if there exists a $b > 0$, such that variational inequality (37) admits a solution in $\mathcal{K}_b$ with

$$Q^{1b} < b_1, \quad Q^{2b} < b_2, \quad Q^{3b} < b_3, \quad \gamma^b < b_4, \quad \rho_3^b < b_5. \tag{38}$$

Theorem 3: Existence

Suppose that there exist positive constants M, N, R with $R > 0$, such that:

$$\frac{\partial f_i(Q^1, Q^2)}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}} + \frac{\partial c_j(Q^1)}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}} \geq M, \quad \forall Q^1 \text{ with } q_{ijl} \geq N, \quad \forall i, j, l, \tag{39}$$

$$\frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} + \hat{c}_{ik}(Q^2,Q^3) \geq M, \quad \forall Q^2 \textit{ with } q_{ik} \geq N, \quad \forall i,k,$$
$$\frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}} + \hat{c}_{jkl}(Q^2,Q^3) \geq M, \quad \forall Q^3 \textit{ with } q_{jkl} \geq N, \quad \forall j,k,l,$$
$$d_k(\rho^*_3) \leq N, \quad \forall \rho_3 \textit{ with } \rho_{3k} > R, \quad \forall k. \tag{40}$$

Then variational inequality (32); equivalently, variational inequality (35), admits at least one solution.

Proof: Follows using analogous arguments as the proof of existence for Proposition 1 in Nagurney and Zhao (1993) (see also existence proof in Nagurney et al. 2002a). □

Assumptions (39) and (40) are reasonable from an economics perspective, since when the product shipment between a manufacturer and demand market pair or a manufacturer and retailer is large, we can expect the corresponding sum of the associated marginal costs of production, handling, and transaction from either the manufacturer's or the retailer's perspectives as well as the transaction cost associated with the consumers, to exceed a positive lower bound. Moreover, in the case where the price of the product as perceived by consumers at a demand market is high, we can expect that the demand for the product at the demand market to not exceed a positive bound.

We now recall the definition of an additive production cost introduced in Zhang and Nagurney (1996) and also utilized in Nagurney et al. (2002a), which we will use as an assumption for establishing additional qualitative properties.

Definition 2: Additive Production Cost

Suppose that for each manufacturer i, the production cost f_i is additive, that is,

$$f_i(q) = f^1_i(q_i) + f^2_i(\bar{q}_i), \tag{41}$$

where $f^1_i(q_i)$ is the internal production cost that depends solely on the manufacturer's own output level q_i, which may include the production operation and the facility maintenance, etc., and $f^2_i(\bar{q}_i)$ is the interdependent part of the production cost that is a function of all the other manufacturers' output levels $\bar{q}_i = (q_1, \cdots, q_{i-1}, q_{i+1}, \cdots, q_m)$ and reflects the impact of *the other manufacturers' production patterns on manufacturer i's cost. This interdependent part of the production cost may describe the competition for the resources, consumption of the homogeneous raw materials, etc.*

We now establish additional qualitative properties both of the function F that enters the variational inequality problem (cf. (35) and (32)), as well as uniqueness of the equilibrium pattern. Since the proofs of Theorems 4 and 5 are similar to the analogous proofs in Nagurney et al. (2002b), they are omitted here.

Theorem 4: Monotonicity

Suppose that the production cost functions $f_i; i = 1, ..., m$, *are additive, as defined in Definition 2, and that the* $f_i^1; i = 1, ..., m$, *are convex functions. If the* c_{ijl}, c_j, *and* $\hat{c}_{ijl}$, *and* c_{ik} *functions are convex; the* $\hat{c}_{jkl}$ *and the* $\hat{c}_{ik}$ *functions are monotone increasing, and the* d_k *functions are monotone decreasing functions of the generalized prices, for all* i, l, j, k, *then the vector function* F *that enters the variational inequality (35) is monotone, that is,*

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle \geq 0, \quad \forall X', X'' \in R_+^{2mn+mo+2no+n+o}. \tag{42}$$

Theorem 5: Strict Monotonicity

Assume all the conditions of Theorem 4. In addition, suppose that one of the families of convex functions $f_i^1; i = 1, ..., m$, $c_{ijl}; i = 1, ..., m; j = 1, ..., n; l = 1, 2; c_j; j = 1, ..., n;$ $\hat{c}_{ijl}; i = 1, \ldots, m; j = 1, \ldots, n; l = 1, 2;$ *and* $c_{ik}; i = 1, \ldots, m; k = 1, \ldots, o$, *is a family of strictly convex functions. Suppose also that* $\hat{c}_{ik}; i = 1, \ldots, m; k = 1, \ldots, o; \hat{c}_{jkl}; j = 1, ..., n; k = 1, ..., o; l = 1, 2$, *and* -$d_k; k = 1, ..., o$, *are strictly monotone. Then, the vector function* F *that enters the variational inequality (35) is strictly monotone, with respect to* (Q^1, Q^2, Q^3, ρ_3), *that is, for any two* X', X'' *with* $(Q^{1'}, Q^{2'}, Q^{3'}, \rho_3') \neq (Q^{1''}, Q^{2''}, Q^{3''}, \rho_3'')$

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0. \tag{43}$$

Theorem 6: Uniqueness

Assuming the conditions of Theorem 5, there must be a unique shipment pattern (Q^{1*}, Q^{2*}, Q^{3*})*, and a unique price vector* ρ_3^* *satisfying the equilibrium conditions of the supply chain. In other words, if the variational inequality (35) admits a solution, then that is the only solution in* (Q^1, Q^2, Q^3, ρ_3)*.*

Proof: Under the strict monotonicity result of Theorem 5, uniqueness follows from the standard variational inequality theory (cf. Kinderlehrer and Stampacchia 1980) □

Theorem 7: Lipschitz Continuity

The function that enters the variational inequality problem (35) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \textit{ where } L > 0, \tag{44}$$

under the following conditions:

(i). Each f_i*;* $i = 1, ..., m$*, is additive and has a bounded second-order derivative;*

(ii). The c_{ijl}*,* c_j*,* $\hat{c}_{ijl}$*,* c_{ik}*, and* c_{jkl} *have bounded second-order derivatives, for all* i, j, l, k*;*

(iii). The $\hat{c}_{ik}$*,* $\hat{c}_{jkl}$*, and* d_k *have bounded first-order derivatives.*

Proof: The result is direct by applying a mid-value theorem from calculus to the vector function F that enters the variational inequality problem (35). □

In the next Section, we utilize the Lipschitz continuity property in order to guarantee that the dynamic trajectories associated with the proposed continuous time adjustment process are well-defined. The monotonicity property of the function F, in turn, is utilized in the next Section in order to establish a stability result.

4. The Dynamics

In this Section, we propose a dynamic adjustment process, formulated as a projected dynamical system. We then establish that the set of stationary points of the projected dynamical system coincides with the set of solutions of variational inequality (35), equivalently, variational inequality (32).

In particular, we now turn to describing the dynamics by which the manufacturers adjust their product shipments over time, the consumers adjust their consumption amounts based on the prices of the product at the demand markets, and the retailers operate between the two, except in the case of electronic commerce when the consumers at the demand markets can deal with the manufacturers directly. We also describe the dynamics by which the prices adjust over time. The dynamics are derived from the bottom tier of nodes on up since it is the demand for the product (and the corresponding prices) that actually drives the supply chain dynamics.

The Demand Market Price Dynamics

We begin by describing the dynamics underlying the prices of the product associated with the demand markets. Assume that the rate of change of the price ρ_{3k}, denoted by $\dot{\rho}_{3k}$, is equal to the difference between the demand at the demand market k, as a function of the demand market prices, and the amount available from the retailers and the manufacturers at the demand market. Hence, if the demand for the product at the demand market (at an instant in time) exceeds the amount available, the price at that demand market will increase; if the amount available exceeds the demand at the price, then the price at the demand market will decrease. Furthermore, it is guaranteed that the prices do not become negative. Consequently, the dynamics of the price ρ_{3k} associated with the product at demand market k can be expressed as:

$$\dot{\rho}_{3k} = \begin{cases} d_k(\rho_3) - \sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m} q_{ik}, & \text{if} \quad \rho_{3k} > 0 \\ \max\{0, d_k(\rho_3) - \sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m} q_{ik}\}, & \text{if} \quad \rho_{3k} = 0. \end{cases} \qquad (45)$$

The Dynamics of the Product Shipments between the Retailers and the Demand Markets

The dynamics of the product shipments over the links joining the retailers to the demand markets are now described. The rate of change of the product shipment q_{jkl} is assumed to be equal to the difference between the price the consumers are willing to pay for the product at demand market k minus the sum of the various costs which include: the unit transaction cost, the marginal transaction cost, the price charged for the product at the retail outlet, and the (weighted) marginal emissions. Note that here, without loss of generality, we refer to γ_j as a "price" associated with retailer j since in variational inequality (32) it is the γs that appear as the variables.

Of course, one also must guarantee that these product shipments do not become negative. Hence, one may write:

$$\dot{q}_{jkl} = \begin{cases} \rho_{3k} - \hat{c}_{jkl}(Q^2, Q^3) - \frac{\partial c_{jkl}(q_{jkl})}{\partial q_{jkl}} - \eta_k h_{jkl} - \gamma_j, & \text{if} \quad q_{jkl} > 0 \\ \max\{0, \rho_{3k} - \hat{c}_{jkl}(Q^2, Q^3) - \frac{\partial c_{jkl}(q_{jkl})}{\partial q_{jkl}} - \eta_k h_{jkl} - \gamma_j\}, & \text{if} \quad q_{jkl} = 0, \end{cases} \tag{46}$$

where $\dot{q}_{jkl}$ denotes the rate of change of the product shipment q_{jkl}.

Thus, according to (46), if the price the consumers are willing to pay for the product at a demand market exceeds the price the retailers charge for the product at the outlet plus the various costs (at an instant in time), then the volume of the product between that retail and demand market pair will increase; if the price charged by the retailer plus the various costs associated with this transaction exceeds the price the consumers are willing to pay, then the volume of flow of the product between that pair will decrease.

The Dynamics of the Product Shipments between the Manufacturers and the Demand Markets

In Section 2, it was assumed that each manufacturer i is faced with a production cost f_i, which can depend, in general, upon all the product shipments from all the manufacturers to the retailers and demand markets. In addition, recall that c_{ik} is the transaction cost associated with manufacturer i transacting with demand market k, with the function being given by (2b). The consumers at the demand markets, in turn, are also faced with a transaction cost associated with transacting with a manufacturer directly. For manufacturer/demand market pair (i,k), this function is denoted by $\hat{c}_{ik}$ and, as in (26), can depend, in general, upon all the product shipments to all the demand markets from all the manufacturers or retailers.

Since each manufacturer is assumed to be a multicriteria decision-maker according to (14) and the consumers at a demand market are also assumed to be multicriteria decision-makers, we propose the following rate of change for the product shipments between the top tier of nodes and the bottom tier of nodes:

$$\dot{q}_{ik} = \begin{cases} \rho_{3k} - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} - \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} - \hat{c}_{ik}(Q^2,Q^3) - (\alpha_i+\eta_k)(h_i+h_k), & \text{if} \quad q_{ik}>0 \\ \max\{0, \rho_{3k} - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} - \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} - \hat{c}_{ik}(Q^2,Q^3) - (\alpha_i+\eta_k)(h_i+h_k)\}, & \text{if} \quad q_{ik}=0, \end{cases} \tag{47}$$

where $\dot{q}_{ik}$ denotes the rate of change of the product shipment q_{ik}.

Hence, according to (47), if the demand price at a demand market exceeds the various costs (including the weighted marginal emissions) associated with transacting via the Internet directly, then the volume of the product transacted via the Internet between the manufacturer/demand market pair will increase; if the demand price at the demand market is less than the above described marginal and unit costs, then the volume of product shipment between the pair will decrease.

The Dynamics of the Prices at the Retail Outlets

The prices for the product at the retail outlets, in turn, must reflect supply and demand conditions as well. In particular, assume that the price for the product associated with retail outlet j, γ_j, evolves over time according to:

$$\dot{\gamma}_j = \begin{cases} \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}, & \text{if} \quad \gamma_j>0 \\ \max\{0, \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}\}, & \text{if} \quad \gamma_j=0, \end{cases} \tag{48}$$

where $\dot{\gamma}_j$ denotes the rate of change of the price γ_j. Hence, if the amount of the product desired to be transacted by the consumers (at an instant in time) exceeds that available at the retail outlet, then the price at the retail outlet will increase; if the amount available is greater than that desired by the consumers, then the price at the retail outlet will decrease.

The Dynamics of Product Shipments between Manufacturers and Retailers

Since the product shipments sent from the manufacturers must be accepted by the retailers in order for the transactions to take place in the supply chain, we propose the following rate of change for the product shipments between the top tier of nodes and the middle tier:

$$\dot{q}_{ijl} = \begin{cases} \gamma_j - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ijl}} - \frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}} - \frac{\partial c_j(Q^1)}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}} - (\alpha_i + \beta_j)(h_i + h_{ijl}), \text{ if } q_{ijl} > 0 \\ \max\{0, \\ \gamma_j - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ijl}} - \frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}} - \frac{\partial c_j(Q^1)}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}} - (\alpha_i + \beta_j)(h_i + h_{ijl})\}, \text{ if } q_{ijl} = 0, \end{cases} \tag{49}$$

where $\dot{q}_{ijl}$ denote the rate of change of the product shipment between manufacturer i and retailer j transacted via mode l.

Following the above discussion, (49) states that the product shipment between a manufacturer/retailer pair via a transaction mode evolves according to the difference between the price charged for the product by the retailer and sum of the various costs associated with the transaction. Here it is also guaranteed that the product shipments do not become negative as they evolve over time.

The Projected Dynamical System

Consider now the dynamic model in which the demand prices evolve according to (45) for all demand market prices k, the retail/demand market product shipments evolve according to (46) for all retailers/demand markets/modes j, k, l, and the product shipments between the manufacturers and the demand markets evolve according to (47). The prices associated with the retailers, in turn, evolve according to (48) for all retailers j, and the product shipments between the manufacturers and retailers evolve over time according to (49) for all manufacturer/retailer/mode combinations i, j, l.

Let X and $F(X)$ be defined as following (35). Then the dynamic model described by (49), (47), (46), (48), and (45) for all i, j, l, k can be rewritten as the projected dynamical system (PDS) (cf. Nagurney and Zhang (1996)) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{50}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = ({Q^1}^0, {Q^2}^0, {Q^3}^0, \gamma^0, \rho_3^0)$

is the initial point corresponding to the initial product shipments between the manufacturers and the retailers and the demand markets; the initial product shipments between the retailers and the demand markets; and the initial retailers' prices and the demand prices. Since the feasible set $\mathcal{K}$ underlying the dynamic supply chain is simply the nonnegative orthant, the projection operation is very simple. Indeed, it simply guarantees, through the use of the "max" term (cf. (45)–(48)), that the dynamic trajectory never yields negative values for the product flows and prices.

The dynamical system (50) is non-classical in that the right-hand side is discontinuous in order to guarantee that the constraints, which in the context of the above model are nonnegativity constraints on the variables, are not violated. Such dynamical systems were introduced by Dupuis and Nagurney (1993) and to date have been used to model a variety of applications ranging from dynamic traffic network problems to dynamic oligopoly problems (cf. Nagurney and Zhang 1996 and the references therein) as well as to supply chain network problems but without multicriteria decision-makers (see Nagurney and Dong 2002).

Stationary/Equilibrium Points

The following theorem states that the projected dynamical system evolves until it reaches a stationary point, that is, $\dot{X} = 0$, in which there is no change in the product shipments and prices, and that the stationary point coincides with the equilibrium point of the supply chain network model according to Definition 1. The notation "*" is utilized here to denote an equilibrium point, as was also done in Section 2, as well as a stationary point, since these are shown to be equivalent in Theorem 8 below.

Theorem 8: The Set of Stationary Points Coincides with the Set of Equilibrium Points

The set of stationary points of the projected dynamical system (50) coincides with the set of equilibrium points defined by Definition 1.

Proof: According to Dupuis and Nagurney (1993), the necessary and sufficient condition for X^* to be a stationary point of the PDS (50), that is, to satisfy:

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)), \tag{51}$$

is that $X^* \in \mathcal{K}$ solves the variational inequality problem:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{52}$$

where, in our problem, $F(X)$, X, and $\mathcal{K}$ are as defined following (35). Writing out (52) explicitly, we have that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}}+\frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+(\alpha_i+\beta_j)(h_i+h_{ijl})-\gamma^*_j\right]$$

$$\times\left[q_{ijl}-q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}}+\hat{c}_{ik}(Q^{2*},Q^{3*})+(\alpha_i+\eta_k)(h_i+h_{ik})-\rho^*_{3k}\right]\times[q_{ik}-q^*_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}}+\hat{c}_{jkl}(Q^{2*},Q^{3*})+\gamma^*_j+\eta_k h_{jkl}-\rho^*_{3k}\right]\times\left[q_{jkl}-q^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q^*_{ijl}-\sum_{k=1}^{o}\sum_{l=1}^{2}q^*_{jkl}\right]\times\left[\gamma_j-\gamma^*_j\right]+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q^*_{jkl}+\sum_{i=1}^{m}q^*_{ik}-d_k(\rho^*_3)\right]\times[\rho_{3k}-\rho^*_{3k}]\geq 0, \tag{53}$$

$$\forall (Q^1, Q^2, Q^3, \gamma, \rho_3) \in \mathcal{K} = R_+^{2mn+mo+2no+n+o}.$$

But variational inequality (53) is precisely the variational inequality (32) (and their corresponding $F(\cdot)$s, Xs, and $\mathcal{K}$s are one and the same), which, in turn, according to Theorem 1 coincides with $(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \rho_3^*)$ being an equilibrium pattern according to Definition 1. The proof is complete. □

Hence, Theorem 8 establishes the linkage between the solution to the variational inequality problem (32) governing the static supply chain network model with multicriteria decision-makers described in Section 2, and the stationary points of the dynamic supply chain model described by the projected dynamical system (50). Indeed, it shows that they are one and the same. Thus, once a stationary point of the dynamic supply chain model has been achieved, that point satisfies the equilibrium conditions, at which the manufacturers, retailers, and consumers have formalized their agreements and the shipments between the tiers coincide.

Theorem 9: Existence and Uniqueness of a Solution to the Initial Value Problem

Assume the conditions of Theorem 7. Then, for any $X_0 \in \mathcal{K}$, there exists a unique solution $X_0(t)$ to the initial value problem (50).

Proof: Lipschitz continuity of the function F is sufficient for the conclusion based on Theorem 2.5 in Nagurney and Zhang (1996) $\square$.

Theorem 9 guarantees that, if the Lipschitz property is satisfied, then the disequilibrium dynamics associated with the proposed projected dynamical system model of the supply chain are well-defined. In other words, given an initial product shipment and price pattern, there exists a unique trajectory associated with (50). Note that this existence and uniqueness result is not the same as those given in Theorems 3 and 6, respectively, since the latter results are for the equilibrium or stationary point, rather than for the dynamic trajectories.

We now turn to addressing the stability (see also Zhang and Nagurney 1995) of the supply chain network system through the initial value problem (50). First, we recall the following:

Definition 3: Stability of the System

The system defined by (50) is stable if, for every X_0 and every equilibrium point X^*, the Euclidean distance $\|X^* - X_0(t)\|$ is a monotone nonincreasing function of time t.

A *global* stability result is stated in the next theorem.

Theorem 10: Stability of the Dynamic Supply Chain Network System

Assume the conditions of Theorem 4. Then the dynamical system (50) underlying the supply chain with multicriteria decision-makers is stable.

Proof: Under the assumptions of Theorem 4, $F(X)$ is monotone and, hence, the conclusion follows directly from Theorem 4.1 of Zhang and Nagurney (1995). $\square$

The stability of the supply chain network system is an important result and demonstrates the validity of the equilibrium concept. In particular, stability of the system says that for any initial product shipment and price pattern, an equilibrium product shipment and price pattern is eventually attained.

5. The Algorithm

In this Section, we propose a discrete-time adjustment process that provides a time discretization of the projected dynamical system (50) and also yields a stationary point of the system; equivalently, a solution to variational inequality (32). The discrete-time algorithm is the Euler method, which is a special case of the general iterative scheme proposed by Dupuis and Nagurney (1993) for tracking the trajectories of projected dynamical systems. Convergence results can also be found therein. The statement of the algorithm in the context of our model is as follows:

Step 0: Initialization Step

Set $(Q^{10}, Q^{20}, Q^{30}, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\tau = 1$ and set the sequence $\{a_\tau\}$ so that $\sum_{\tau=1}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Such a sequence is required for convergence.

Step 1: Computation

Compute $(Q^{1^\tau}, Q^{2^\tau}, Q^{3^\tau}, \gamma^\tau, \rho_3^\tau) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\Bigg[q_{ijl}^{\tau} + a_\tau\Big(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}}$$

$$+(\alpha_i + \beta_j)(h_i + h_{ijl}) - \gamma_j^{\tau-1}\Big) - q_{ijl}^{\tau-1}\Bigg] \times \Big[q_{ijl} - q_{ijl}^{\tau}\Big]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{2}\Bigg[q_{ik}^{\tau} + a_\tau\Big(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^{\tau-1})}{\partial q_{ik}} + \hat{c}_{ik}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}})$$

$$+(\alpha_i + \eta_k)(h_i + h_{ik}) - \rho_{3k}^{\tau-1}\Big) - q_{ik}^{\tau-1}\Bigg] \times [q_{ik} - q_{ik}^{\tau}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\Bigg[q_{jkl}^{\tau} + a_\tau\Big(\frac{\partial c_{jkl}(q_{jkl}^{\tau-1})}{\partial q_{jkl}} + \hat{c}_{jkl}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}}) + \gamma_j^{\tau-1} + \eta_k h_{jkl} - \rho_{3k}^{\tau-1}\Big) - q_{jkl}^{\tau-1}\Bigg]$$

$$\times \Big[q_{jkl} - q_{jkl}^{\tau}\Big]$$

$$+\sum_{j=1}^{n}\Bigg[\gamma_j^{\tau} + a_\tau\Big(\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{\tau-1} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{\tau-1}\Big) - \gamma_j^{\tau-1}\Bigg] \times \Big[\gamma_j - \gamma_j^{\tau}\Big]$$

$$+\sum_{k=1}^{o}\Bigg[\rho_{3k}^{\tau} + a_\tau\Big(\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{\tau-1} + \sum_{i=1}^{m} q_{ik}^{\tau-1} - d_k(\rho_3^{\tau-1})\Big) - \rho_{3k}^{\tau-1}\Bigg] \times [\rho_{3k} - \rho_{3k}^{\tau}] \geq 0,$$

$$\forall (Q^1, Q^2, Q^3, \gamma, \rho_3) \in \mathcal{K}. \tag{54}$$

Step 2: Convergence Verification

If $|q_{ijl}^\tau - q_{ijl}^{\tau-1}| \leq \epsilon$, $|q_{ik}^\tau - q_{ik}^{\tau-1}| \leq \epsilon$, $|q_{jkl}^\tau - q_{jkl}^{\tau-1}| \leq \epsilon$, $|\gamma_j^\tau - \gamma_j^{\tau-1}| \leq \epsilon$, $|\rho_{3k}^\tau - \rho_{3k}^{\tau-1}| \leq \epsilon$, for all $i = 1, \cdots, m$; $j = 1, \cdots, n$; $l = 1, 2$; $k = 1, \cdots, o$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

Since $\mathcal{K}$ is the nonnegative orthant the solution of (54) is accomplished exactly and in closed form. Furthermore, the sequence $\{a_\tau\}$, has an interpretation that "learning" is taking place from time period to time period in that, at first, the value is larger, and then as time proceeds, it gets smaller and smaller. For completeness and easy reference, we show how problem (54) can be solved exactly and in closed form below:

Computation of the Product Shipments

At iteration τ compute the q_{ijl}^τs according to:

$$q_{ijl}^\tau = \max\left\{0, q_{ijl}^{\tau-1} - a_\tau\left(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}}\right.\right.$$
$$\left.\left. + (\alpha_i + \beta_j)(h_i + h_{ijl}) - \gamma_j^{\tau-1}\right)\right\}, \forall i, j, l. \tag{55}$$

In addition, at iteration τ, compute the q_{ik}^τs according to:

$$q_{ik}^\tau = \max\left\{0, q_{ik}^{\tau-1} - a_\tau\left(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^{\tau-1})}{\partial q_{ik}} + \hat{c}_{ik}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}})\right.\right.$$
$$\left.\left. + (\alpha_i + \eta_k)(h_i + h_{ik}) - \rho_{3k}^{\tau-1}\right)\right\}, \quad \forall i, k. \tag{56}$$

Also, at iteration τ compute the q_{jkl}^τs according to:

$$q_{jkl}^\tau = \max\{0, q_{jkl}^{\tau-1} - a_\tau\left(\frac{\partial c_{jkl}(q_{jkl}^{\tau-1})}{\partial q_{jkl}} + \hat{c}_{jkl}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}}) + \gamma_j^{\tau-1} + \eta_k h_{jkl} - \rho_{3k}^{\tau-1}\right)\}, \quad \forall j, k, l. \tag{57}$$

Computation of the Prices

The prices, γ_j^τ, in turn, are computed at iteration τ explicitly according to:

$$\gamma_j^\tau = \max\{0, \gamma_j^{\tau-1} - a_\tau(\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{\tau-1} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{\tau-1})\}, \quad \forall j, \tag{58}$$

whereas the demand market prices, ρ_{3k}, are computed according to:

$$\rho_{3k}^\tau = \max\{0, \rho_{3k}^{\tau-1} - a_\tau(\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{\tau-1} + \sum_{i=1}^{m} q_{ik}^{\tau-1} - d_k(\rho_3^{\tau-1}))\}, \quad \forall k. \tag{59}$$

According to the discrete-time adjustment process described above, the process is initialized with a vector of product shipments and prices. Of course, the vector components can all be set to zero, which signifies that "at the beginning" there is no production and shipment and zero prices for the product. The system will then evolve, simulated, in a sense, by the discrete-time adjustment process (55) through (59) until a stationary/equilibrium point is achieved. It is easy to see from (55)–(59) that once the convergence tolerance has been reached (and, hence, these differences are approximately zero) then the equilibrium conditions according to Definition 1 are satisfied; equivalently, a stationary point of the projected dynamical system (50) is attained, and also a solution to variational inequality (32) (or (35)).

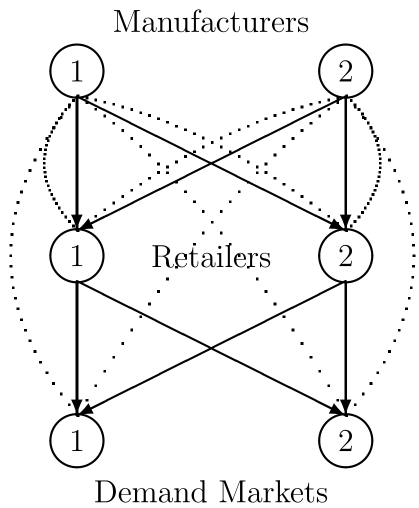

Figure 2: Supply Chain Supernetwork Structure for the Numerical Examples

6. Numerical Examples

In this Section, the discrete-time algorithm, the Euler method, described in Section 5, is applied to several numerical examples. The algorithm was implemented in FORTRAN and the computer system used was a DEC Alpha system located at the University of Massachusetts at Amherst. The convergence criterion utilized was that the absolute value of the product flows and prices between two successive iterations differed by no more than 10^{-4}. The sequence a_τ was set to $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$ for all the examples. The numerical examples had the network structure depicted in Figure 2 and consisted of two manufacturers, two retailers, and two demand markets, with both B2B and B2C transactions permitted, with the B2C transactions being between the manufacturers and the demand markets, for simplicity.

The Euler method required no more than 369 iterations for convergence in any of the seven examples reported below.

Example 1

The data for the first example were constructed for easy interpretation purposes and to serve as a baseline for the subsequent examples.

We assumed that the decision-makers assigned weights equal to zero for the environmental criteria, that is: α_i; $i = 1, 2$; β_j; $j = 1, 2$, and η_k; $k = 1, 2$, were all set to zero. Hence, none of the decision-makers took environmental emissions into consideration in their decision-making. The emission parameters were as follows: $h_i = 1$; $i = 1, 2$; $h_{ijl} = 1$, for $i = 1, 2$; $j = 1, 2$; $l = 1$; and $h_{ijl} = .2$, for $i = 1, 2$; $j = 1, 2$, and $l = 2$; $h_{ik} = .1$ for $i = 1, 2$; $k = 1, 2$, and $h_{jkl} = .7$ for $j = 1, 2$; $k = 1, 2$ and $l = 1$.

The production cost functions for the manufacturers were given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers using the physical link, that is, mode 1, and denoted, typically, by c_{ij1}, were given by:

$$c_{ij1}(q_{ij1}) = .5q_{ij1}^2 + 3.5q_{ij1}, \quad \text{for } i = 1, 2; j = 1, 2,$$

whereas the analogous transaction costs, but for mode 2, and, denoted, typically, by c_{ij2}, were given by:

$$c_{ij2}(q_{ij2}) = 1.5q_{ij2}^2 + 3q_{ij2}, \quad \text{for } i = 1, 2; j = 1, 2.$$

The transaction costs of the manufacturers associated with dealing with the consumers at the demand markets via the Internet and denoted for a pair i, k by c_{ik}, were given by:

$$c_{ik}(q_{ik}) = q_{ik}^2 + 2q_{ik}, \quad \text{for } i = 1, 2; k = 1, 2.$$

The handling costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i1l})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i2l})^2.$$

The transaction costs of the retailers associated with transacting with the manufacturers via mode 1 and mode 2, denoted by $\hat{c}_{ijl}$, were, respectively, given by:

$$\hat{c}_{ijl}(q_{ijl}) = 1.5q_{ijl}^2 + 3q_{ijl}, \quad \text{for } i = 1,2; j = 1,2; l = 1,2.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets (denoted for a typical pair by $\hat{c}_{jkl}$ with the associated shipment by q_{jkl} with $l = 1$) were given by:

$$\hat{c}_{jk1}(Q^2, Q^3) = q_{jk1} + 5, \quad \text{for } j = 1,2; k = 1,2,$$

whereas the transaction costs associated with transacting with the manufacturers via the Internet for the consumers at the demand markets (denoted for a typical such pair by $\hat{c}_{ik}$ with the associated shipment of q_{ik}) were given by:

$$\hat{c}_{ik}(Q^2, Q^3) = q_{ik} + 1, \quad \text{for } i = 1,2; k = 1,2.$$

The Euler method yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were: $Q^{1^*}: q_{111}^* = q_{121}^* = q_{211}^* = q_{221}^* = 3.4611$; $q_{112}^* = q_{122}^* = q_{212}^* = q_{222}^* = 2.3907$. The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were: $Q^{2^*}: q_{11}^* = q_{12}^* = q_{21}^* = q_{22}^* = 13.3033$. The product shipments (consumption volumes) between the two retailers and the two demand markets were: $Q^{3^*}: q_{111}^* = q_{121}^* = q_{211}^* = q_{221}^* = 5.8513$.

The vector γ^*, which was equal to the prices charged by the retailers, had components: $\gamma_{21}^* = \gamma_{22}^* = 263.9088$, and the demand prices at the demand markets were: $\rho_{31}^* = \rho_{32}^* = 274.7701$.

The total emissions generated are given by the expression

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{l=1}^{2}(h_i + h_{ijl})q_{ijl}^* + \sum_{i=1}^{2}\sum_{k=1}^{2}(h_i + h_{ik})q_{ik}^* + \sum_{j=1}^{2}\sum_{k=1}^{2} h_{jk1}q_{jk1}^*$$

with the value for this example being equal to: 114.0895.

It is easy to verify that the optimality/equilibrium conditions were satisfied with good accuracy.

Example 2

Example 2 was constructed as follows: we kept the data as in Example 1 except that we now had the manufacturers weight the environmental criteria with the weights set at $\alpha_1 = 1$ and $\alpha_2 = 1$.

The Euler method yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were: $Q^{1*}: q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.3214$; $q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.4309$. The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were: $Q^{2*}: q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.3127$. The product shipments (consumption volumes) between the two retailers and the two demand markets were: $Q^{3*}: q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 5.7509$.

The vector γ^*, which was equal to the prices charged by the retailers, had components: $\gamma^*_{21} = \gamma^*_{22} = 264.0714$, and the demand prices at the demand markets were: $\rho^*_{31} = \rho^*_{32} = 274.8201$.

The total emissions generated were now equal to: 112.9185. Hence, as expected, environmentally conscious manufacturers can reduce the total emissions generated. Observe that the volumes of flow on the Internet links from the manufacturers now increased since in this set of examples the emission factors associated with the Internet link have been assigned lower values than those associated with the physical links.

Example 3

We then proceeded to construct Example 3, which had the same data as in Example 2, but now the retailers also had positive weights associated with the environmental criteria with $\beta_1 = \beta_2 = 1$. The Euler method yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were: Q^{1^*} : $q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.1136$; $q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.4250$. The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were: Q^{2^*} : $q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.4861$. The product shipments (consumption volumes) between the two retailers and the two demand markets were: Q^{3^*} : $q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 5.5362$.

The vector γ^*, which was equal to the prices charged by the retailers, had components: $\gamma^*_{21} = \gamma^*_{22} = 264.3097$, and the demand prices at the demand markets were: $\rho^*_{31} = \rho^*_{32} = 274.8436$.

The total emissions generated were now further reduced and were equal to: 111.3810. Again, the volumes of products transacted over the Internet links increased.

Example 4

Example 4, in turn, was constructed from Example 3 with now the consumers at the demand markets also having positive weights associated with the emissions generated with $\eta_1 = \eta_2 = 1$. Hence, in this example, all the decision-makers in the three tiers of the supply chain were now environmentally conscious as regards their decision-making.

The Euler method converged and yielded the following new equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were: Q^{1^*} : $q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.1270$; $q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.4347$. The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were: Q^{2^*} : $q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.3960$. The product shipments (consumption volumes) between the two retailers and the two demand markets were: Q^{3^*} : $q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 5.5603$.

The vector γ^*, which was equal to the prices charged by the retailers, had components: $\gamma^*_{21} = \gamma^*_{22} = 263.6234$, and the demand prices at the demand markets were: $\rho^*_{31} = \rho^*_{32} = 274.8813$.

The total emissions generated were now further reduced and were equal to: 111.2138.

Example 5

We then proceeded to increase the weights associated with the environmental criteria and the demand markets. Hence, Example 5 had the same data as in Example 4 but now we increased the weights associated with the environmental criteria at the demand markets so that $\eta_1 = \eta_2 = 5$. In this and the subsequent two examples, we just report, for the sake of brevity, the total emissions generated. In Example 5 the total emissions generated were equal to 110.5442.

Example 6

In the next simulation exercise, we constructed Example 6 from Example 5 by having the weights associated with the manufacturers' environmental criteria β_1 and β_2 being set equal to 5 for each such firm. The result was that the total emissions now decreased further to: 105.8604.

Example 7

In our final example, we utilized the data for Example 6 but set also the retailers' weights $\beta_1 = \beta_2 = 5$. The total emissions generated were now equal to: 99.7104.

7. Summary and Conclusions

In this paper, we have proposed a framework for the formulation, analysis, and computation of solutions to supply chain problems with multicriteria decision-makers and environmental concerns in the presence of electronic commerce in the form of B2B and B2C transactions. Specifically, we have proposed a supernetwork framework and developed both a static and a dynamic model. For the latter, we established that its set of stationary points coincides with the set of solutions to the variational inequality formulation of the equilibrium conditions governing the static model.

We provided qualitative properties of the equilibrium product shipment and price pattern in the form of existence and uniqueness results and also showed that the proposed projected dynamical system for the supply chain network is stable under reasonable conditions. In addition, we provided a discrete-time algorithm which was then applied to several numerical examples in which we evaluated the equilibrium solutions as we changed the weights associated with the environmental criteria of the manufacturers, the retailers, and the consumers at the demand markets.

The results in this paper demonstrate that supply chain network models with numerous decision-makers and associated environmental concerns can be formulated, analyzed qualitatively, and solved. Of course, the supernetwork model constructed in this paper is stylized but it contains important features such as the behavior of distinct decision-makers, which is optimizing but with multiple criteria, including environmental ones; explicit transaction costs; and transportation versus telecommunication tradeoffs in decision-making (as applied to logistics). Moreover, once solved, it provides the equilibrium product shipments between tiers of the supply chain as well as the demand market and retailer prices.

Of course, there are many other interesting questions that arise in regards to the study of the dynamics of supply chain interactions and environmental issues. In particular, we note that the model developed here considered the environmental criterion of the minimization of total emissions generated and there are other criteria that can also be included (such as the minimization of waste generation in production). Nevertheless, we hope that the work in this paper provides part of the theoretical foundations for this subject area.

Acknowledgments

The research of the authors was supported, in part, by an AT&T Foundation Industrial Ecology Fellowship. The research of the first author was also supported, in part, by NSF Grant No.: IIS-0002647, NSF Grant No.: CMS-0085720, and NSF Grant No.: INT-0000309. This support is gratefully acknowledged.

The authors are grateful to the two anonymous referees and to the editor, Professor Ken Button, for many helpful comments and suggestions.

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