Management of Knowledge Intensive Systems as Supernetworks: Modeling, Analysis, Computations, and Applications

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Abstract

In this paper, we construct the foundation for a new theory for the management of knowledge intensive systems or organizations termed *knowledge supernetworks*. The framework allows for explicit multicriteria decision-making and determines the optimal flows. We develop the fundamental knowledge supernetwork model with fixed demands and present concrete applications to a news organization, an intelligence agency, and a global financial institution. We then propose model extensions with elastic demands. The formulations of the governing optimality/equilibrium conditions are given as variational inequality problems. Qualitative properties of the solution patterns are provided along with algorithms that exploit the network structure of the problem and enhance the operationalism of the framework. Finally, numerical examples are provided to illustrate both the generality of the knowledge supernetwork concept and theory as well as the computational procedures.

Key words: knowledge, systems, networks, multicriteria decision-making, optimization, variational inequalities

1. Introduction

Knowledge intensive organizations today, such as news organizations, intelligence agencies, and/or global financial institutions, are facing numerous challenges. Examples of some major challenges include: how to respond in a timely manner to new events, how to ensure the efficiency of their various production allocation processes, and how to best manage the scale and scope of their coverage and ultimate reach which is global in dimension. At the same time, there is a growing demand for such organizations to be responsive to the needs of their ultimate consumers in an environment of increasing uncertainty and risk.

Decision-makers in knowledge intensive organizations today, hence, can not operate in a vacuum but must be provided with informative management tools that can capture the complexity of the decision-making environment that they face. The decision-making environment itself is large-scale in nature and there may be several criteria that may need to be optimized; consequently, the decision-maker might have to consider such objective functions as: cost minimization, time minimization, and/or the minimization of risk, among others. Such criteria may be weighted differently depending upon the organization and the given scenario. At the same time, the decision-maker must adhere to the constraints on his resources.

In this paper, we lay down the foundations for a new theory to support the management of knowledge intensive systems. Our focus is on organizations in which the above-noted characteristics are of paramount importance. Our goal is to capture in a graphical format and with rigorous theory the alternatives available to the decision-maker, the complexity of the various underlying functions, as well as to determine the optimal allocation of resources, given multiple criteria and the constraints. The theory is based on supernetworks, and, for definiteness, we term it *knowledge supernetworks*.

Supernetworks, which consist of nodes, links (which may be physical or virtual and, hence, are necessarily abstract), as well as the associated flows, were originally proposed in the context of *transportation networks*. For the historical foundations, as well as numerous applications to the Information Age, including supply chains with electronic commerce, financial networks with intermediation, as well as telecommuting and teleshopping decision-making, see the papers by Nagurney et al. (2002a, b), Nagurney and Dong (2002), and the book by Nagurney and Dong (2002) and the references therein.

We note that there has also been research conducted in the modeling of knowledge networks from an economic perspective and, interestingly, by researchers in the transportation field (cf. Karlqvist and Lundqvist (1972), Andersson and Karlqvist (1976), Batten, Kobayashi, and Andersson (1989), Beckmann (1993, 1994), Kobayashi (1995), Nagurney (1999)). Our perspective, however, is distinct in that we consider supernetworks in which the links may correspond to different knowledge production links, including, but not limited to: information processing links, telecommunication and/or transportation links, interface links, consolidation links, etc. Moreover, we explicitly consider multicriteria decision-making. In addition, the demands for the knowledge products are placed within the context of the broader operations of the knowledge organizations. To enhance the operationalism of our theoretical framework for such knowledge organizations, we also provide algorithms, which are network-based, and can be used for resource allocation and for scheduling purposes.

The paper is organized as follows. In Section 2, we present the basic knowledge supernetwork model and discuss concrete applications to three knowledge organizations: a news organization, an intelligence agency, and a global financial institution. In Section 3, we then develop several model extensions with elastic demands. We demonstrate that all the supernetwork models can be uniformly formulated and analyzed as finite-dimensional variational inequality problems (cf. Nagurney (1999)). In Section 4, we provide some qualitative properties of the solution patterns and establish properties of the functions that enter the variational inequality formulations to guarantee convergence of the proposed computational procedure, which is the topic of Section 5. In Section 6, we present several numerical examples to illustrate both the modeling framework as well as the computational methodologies. In Section 7, we summarize the paper and present directions for future research.

2. The Basic Knowledge Supernetwork Model

The supernetwork model is first described in general terms in Section 2.1. We then apply it in Section 2.2 to several concrete knowledge organizations, in particular, to a news organization, to an intelligence agency, and to a global financial institution.

2.1 The Supernetwork Model with Fixed Demands

We consider a general network $G = [\mathcal{N}, \mathcal{L}]$, where $\mathcal{N}$ denotes the set of nodes in the network and $\mathcal{L}$ denotes the set of directed links. Let a denote a link of the network connecting a pair of nodes and let p denote a path, assumed to be acyclic, and consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. A link in the supernetwork framework need not correspond to a physical link but, rather, may be abstract and associated with a factor of production or activity required for knowledge production. For example, a link may correspond to an information processing link (either human-based or computer-based), to a transportation or communication link, to an interface link, whereby information from different sources needs to be synthesized, or to a necessary logistical link. A path then corresponds to a *production process* whereby the knowledge required by the organization is produced.

There are n links in the network and n_P paths. Let W denote the set of J O/D pairs. The set of paths connecting the O/D pair ω is denoted by P_ω and the entire set of paths in the network by P. An origin/destination pair of nodes connected by its paths, in turn, corresponds to the beginning and the end of knowledge production.

Conservation of Flow Equations

Assume that there are k distinct knowledge products that are produced by the knowledge organization with a typical product denoted by i. Let f_a^i denote the flow of knowledge product i on link a and let x_p^i denote the nonnegative flow of knowledge product i on path p. The relationship between the link flows by product and the path flows is given by:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, a, \tag{1}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. In other words, if the link may be used in producing the knowledge product i whether it is an information link, or a logistics link, or even a financial transaction link, then $\delta_{ap} = 1$; otherwise, $\delta_{ap} = 0$. We refer, henceforth, to the knowledge products as, simply, *products*. The flow of a product, thus, on a link is equal to the sum of the flows of the product on the paths that contain (that is, utilize) that link. We group the product link flows into the kn-dimensional vector f with components: $\{f_a^1, \ldots, f_n^1, \ldots, f_a^k, \ldots, f_n^k\}$. In addition, we group the product path flows into the kn_P-dimensional vector x with components: $\{x_{p_1}^1, \ldots, x_{p_{n_P}}^k\}$. We assume that all vectors are column vectors, except where noted otherwise.

The demand associated with origin/destination pair ω and product i is denoted by d_ω^i. Clearly, the demands must satisfy the following conservation of flow equations:

$$d_\omega^i = \sum_{p \in P_\omega} x_p^i, \quad \forall i, \omega, \tag{2}$$

that is, the demand for the product between an O/D pair is equal to the sum of the flows of the product on paths connecting the O/D pair. The feasible set is denoted by $\mathcal{K}$ and is defined as: $\mathcal{K} \equiv \{f | x \geq 0, \text{ and (1) and (2) hold}\}$.

The Multicriteria Decision-Making Problem

As discussed in the Introduction, we expect that a knowledge organization operating in a knowledge intensive and dynamic environment will have to consider several objectives/criteria in regards to decision-making. We assume that there are H such objectives with a typical objective denoted by ϕ_h, where $\phi_h = \phi_h(f)$. Such functions we assume to be continuous, and, for simplicity, we assume that each of these objectives is to be minimized (otherwise, we can multiply through the objective function if it is a maximization one by minus one to put it into our format). Hence, the decision-maker seeks to:

$$\text{Minimize} \quad Z(\phi_1(f), \phi_2(f), \ldots \phi_H(f)) \tag{3}$$

subject to:

$$f \in \mathcal{K}. \tag{4}$$

Also, we assume that the decision-maker in the knowledge organization weights the various objective functions with weight w_h associated with objective function h with this term being nonnegative. Hence, we can construct a *weighted objective function* (cf. Yu (1985) and Keeney and Raiffa (1993)) given by: $Z(f) = \sum_{h=1}^{H} w_h \phi_h(f)$ and the optimization problem is transformed into:

$$\text{Minimize}_{f \in \mathcal{K}} \quad Z(f) = \sum_{h=1}^{H} w_h \phi_h(f). \tag{5}$$

We now discuss possible objective functions ϕ_h and how they are constructed on the knowledge supernetwork. In particular, the functions associated with the links are first described, from which we, subsequently, construct the respective objective functions, which are then weighted into a single objective function, as in (5).

Assume that c^i_{ha} denotes a *unit* function associated with criterion h, link a, and product i, where

$$c^i_{ha} = c^i_{ha}(f), \quad \forall i, h, a, \tag{6}$$

with the total function being denoted by $\hat{c}^i_{ha}$ and equal to:

$$\hat{c}^i_{ha} = c^i_{ha}(f) \times f^i_a, \quad \forall i, h, a. \tag{7}$$

These functions are assumed to be continuous and differentiable.

Then the first associated objective function ϕ_1 takes on the form: $\phi_1(f) = \sum_{i,a} \hat{c}^i_{1a}(f)$, the second objective function ϕ_2 takes on the form: $\phi_2(f) = \sum_{i,a} \hat{c}^i_{2a}(f)$, and, finally, the last objective function ϕ_H is expressed as: $\phi_H(f) = \sum_{i,a} \hat{c}^i_{Ha}(f)$ with the multicriteria optimization problem (cf. (5)) then being given by:

$$\text{Minimize}_{f \in \mathcal{K}} \quad Z(f) = \sum_{h=1}^{H} \sum_{i,a} w_h \hat{c}^i_{ha}(f). \tag{8}$$

Note that the objective function in (8) may also be interpreted as the total *generalized* cost on the supernetwork where the generalized total cost associated with a link and product is given by the expression: $\sum_{h=1}^{H} w_h \hat{c}^i_{ha}(f)$.

Minimization of Total (Production) Cost

We now give examples of appropriate such functions. For example, it is reasonable to expect that the decision-maker would wish to minimize the total cost of knowledge production in which case if we let the unit production cost be denoted by π_a^i for product i and link a, then

$$c_{1a}^i = \pi_a^i = \pi_a^i(f), \quad \forall i, a, \tag{9}$$

with the first objective to be minimized (and representing the total production cost on the supernetwork) being given by:

$$\text{Minimize}_{f \in \mathcal{K}} \quad \sum_{i,a} \pi_a^i(f) \times f_a^i. \tag{10}$$

Note that since π_a^i denotes the unit production cost associated with product i and link a that the *total* production cost for that product on that link corresponds to: $\hat{c}_{1a}^i = \pi_a^i(f) \times f_a^i$.

Minimization of Total (Production) Time

Similarly, we can expect that a decision-maker controlling a knowledge organization would also be interested in minimizing the total time associated with producing the knowledge products. This is especially important in the context of many applications in which the timely delivery of the knowledge products is of paramount importance. We let τ_a^i denote the unit time associated with link a and product i. We then have that

$$c_{2a}^i = \tau_a^i = \tau_a^i(f), \quad \forall i, a, \tag{11}$$

with the second objective expressing the total time to be minimized given by:

$$\text{Minimize}_{f \in \mathcal{K}} \quad \sum_{i,a} \tau_a^i(f) \times f_a^i. \tag{12}$$

Note that here we allow for the general situation in which the various link functions depend on the entire link flow pattern. Both in Dafermos (1981) and in Tzeng and Chen (1993) it was assumed that these functions were separable. Furthermore, in the latter paper, only three criteria were handled and also in an entirely different application context from the one considered here. Moreover, it was assumed that there was essentially only a single class of flow on the network. Nagurney and Dong (2002), in turn, focused on user-optimized networks rather than the system-optimized framework which is the focus here.

Minimization of Total Risk

In addition, since we expect risk to be an important criterion associated with the management of a knowledge intensive system/organization, we also introduce a continuous risk function ρ_a^i associated with each link and product in the supernetwork, where

$$c_{2a}^i = \rho_a^i = \rho_a^i(f), \quad \forall i, a, \tag{13}$$

with the total risk minimization problem being given by:

$$\text{Minimize}_{f \in \mathcal{K}} \quad \sum_{i,a} \rho_a^i(f) \times f_a^i. \tag{14}$$

Other reasonable criteria in a knowledge supernetwork context might be quality of service as well as safety (or security) (both of which should be maximized).

Recall that the central controller or decision-maker is faced with the decision as to how much of each of the knowledge products should be produced on which production processes, given the demand and, in the case of the multiple objectives (which themselves may be conflicting). Also, recall that we assume that the central controller has his own perception of the trade-offs among the production time, the cost, and the risk, which are represented, respectively, by the nonnegative weights w_1, w_2, and w_3.

A Specific Multicriteria Decision-Making Problem

In the context of the specific three-objective problem outlined above, and given, respectively, by (10), (12), and (14), we now put the specific problem into the format given by (5). Indeed, this multicriteria decision-making problem can be transformed into:

$$\text{Minimize}_{f \in \mathcal{K}} \quad w_1 \sum_{i,a} \pi_a^i(f) \times f_a^i + w_2 \sum_{i,a} \tau_a^i(f) \times f_a^i + w_3 \sum_{i,a} \rho_a^i(f) \times f_a^i. \tag{15}$$

Assuming that the objective function $Z(f)$ in (8) is convex, which follows under the assumption that the individual weighted functions (cf. (7) and (8)) are convex, since the constraint set $\mathcal{K}$ is also convex, we have, according to optimization theory (cf. Bertsekas and Tsitsiklis (1989) and Bazaraa, Sherali, and Shetty (1993); see also Kinderlehrer and Stampacchia (1980) and Nagurney (1999)), the following immediate result:

Theorem 1: Variational Inequality Formulation (Fixed Demand Model)

The optimal solution f^* to problem (8) (and to (15) with appropriate dimensions) is equivalent to the solution of the following variational inequality problem:

$$\langle \nabla Z(f^*), f - f^* \rangle \geq 0, \quad \forall f \in \mathcal{K}, \tag{16a}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space (with $N = kn$ here) and ∇Z is the gradient of the function Z with components: $\{\frac{\partial Z}{f_a^1}, \ldots, \frac{\partial Z}{\partial f_n^k}\}$ or, equivalently, in expanded notation, to:

$$\sum_{i,a} \sum_{h=1}^{H} \sum_{j=1}^{k} \sum_{b \in \mathcal{L}} w_h \frac{\partial \hat{c}_{hb}^j(f^*)}{\partial f_a^i} \times (f_a^i - f_a^{i*}) \geq 0, \quad \forall f \in \mathcal{K}. \tag{16b}$$

Variational inequality (16b) may be written in standard form (cf. Nagurney (1999))

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16c}$$

where $X \equiv f$ and $F(X)$ has components: $\{F_{1,a}, \ldots, F_{k,n}\}$ given, respectively, by the term preceding the multiplication sign in (16b) in the summand.

Note that the above variational inequality formulation (16b) (and (16c)) is in link flows.

We now provide an alternative but equivalent statement of the optimality conditions for the multicriteria optimization problem (8). The optimality conditions are now expressed in path flows. This assists us in deriving the subsequent equilibrium conditions for the elastic demand models in Section 3. In addition, the optimality conditions below have a nice economic interpretation.

The Knowledge Supernetwork Optimality Conditions

For each product i; $i = 1, \ldots, k$; for all O/D pairs $\omega \in W$, and for all paths $p \in P_\omega$, the flow pattern x^* is said to be the optimal solution of (8), if the following conditions hold:

$$\frac{\partial Z(f^*)}{\partial x_p^i} \equiv \hat{C}'^{\,i}_p(f^*) \begin{cases} = \lambda_\omega^i, & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_\omega^i, & \text{if} \quad x_p^{i*} = 0, \end{cases} \tag{17}$$

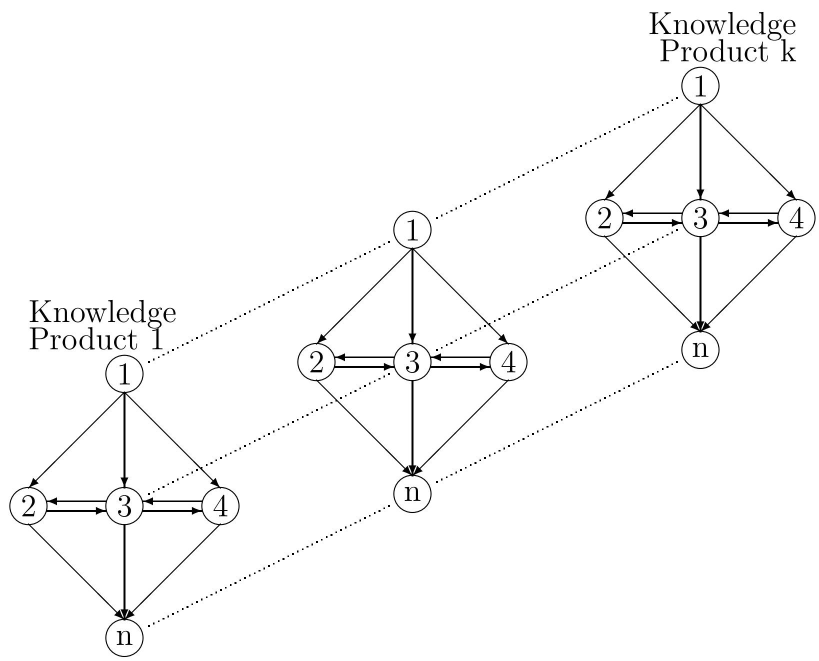

Figure 1: Example of a Knowledge Supernetwork

where

$$\hat{C}'^{\,i}_p(f) = \sum_{j=1}^{k} \sum_{h=1}^{H} \sum_{a,b \in \mathcal{L}} w_h \frac{\partial \hat{c}^j_{hb}(f)}{\partial f^i_a} \delta_{ap}. \tag{18}$$

We refer to $\hat{C}'^{\,i}_p$ as the *generalized marginal total cost* of product i on path p. Hence, according to (17), we have that, at optimality, all the *utilized* paths (that is, those with positive flow) for a product connecting an O/D pair have equal and minimal *generalized marginal total* costs. Note that the *costs* are termed generalized since they include the various criteria and the associated weights. We refer to the associated costs on the links (in view of (18)) as the generalized marginal total link costs.

In Figure 1, we depict an example of a knowledge supernetwork.

2.2 Specific Applications

In this subsection, we present three concrete applications of the above knowledge supernetwork framework. We first discuss an application to a news organization such as CNN. We then describe an application to an intelligence agency. We conclude with the mapping of a global financial institution (such as Citigroup) into a knowledge supernetwork. These applications demonstrate the generality of the knowledge supernetwork framework and also highlight the distinctive features of knowledge intensive dynamic organizations.

A News Organization

Clearly, a news organization is a knowledge organization and, as in the case of CNN, is involved in knowledge production and dissemination. Such an organization must respond in a timely manner and deals with multiple objectives including cost minimization, time minimization as well as risk minimization. Unlike classical manufacturing and production networks, the product is no longer a physical one but rather is in the form of processed information or knowledge.

Hence, questions arise as to what is precisely meant by an origin/destination pair in this application context; what is meant by a specific product, and what is meant by the demand for a product and O/D pair since the demand will not, as in the case of a physical product, correspond to the number of units required to be produced.

We now make the appropriate identifications and associations to map such a news organization into the knowledge supernetwork framework proposed in Section 2.1 above. An O/D pair ω in this context represents a *news program* (which may, for example, be a 30 minute program or a one hour program, or even a special news bulletin). Within a news program ω there may be several types of *news segments*, that is, knowledge products that need to be produced. For example, a knowledge product may correspond to a world report, a local report, a weather report, etc. In this section, we assume that the demand for each product (i.e., news segment in this case) is known and fixed (in the next section we relax this assumption). The demand, hence, would be given in terms of minutes which would need to be allocated among the various production processes to produce the total required for the news segment (and, summed up over all the products) would equal the time allotted for the news program).

The links of the knowledge supernetwork in this application would correspond to the activities (and associated with the necessary individuals) that are needed to produce the news segments. There may be alternative production processes available for any given news program but once the criteria and weights are constructed (cf. (8)) the solution of the multicriteria optimization problem would yield the optimal production processes, that is, the utilized ones, and how much of each product/news segment should be produced on which path. Clearly, the same link might be utilized for the production of the same or different products.

An Intelligence Agency

Another example *par excellence* of a knowledge organization is that of an intelligence agency which gathers information, processes that information, and then disseminates it to interested constituents. In this context, the processed information or knowledge can be expected to be in the form of *reports* with the mapping into the knowledge supernetwork framework as follows. The O/D pairs correspond to different reports/studies with the products corresponding to parts of the report, which could focus on military issues, cultural aspects, terrorist threats, etc. Note that not every product need be associated with an O/D pair. We have presented the framework above in its most general setting. The actual units for the demand for a particular product are given in this case in terms of *pages* (which, of course, in a sense, can also be mapped into bits of information), with a particular segment of a report having its own demand for the number of pages. Hence, an intelligence organization may need to produce several (or numerous) reports denoted by the O/D pairs. The segments of the reports may share certain links which correspond to information processing/transformation/acquisition links as well as to information synthesizing links. Note that to acquire the information there may be actual physical transportation involved as well as communication with intelligence gathering sources and the links would be defined and constructed accordingly.

A Global Financial Institution

Another example of a knowledge organization which depends on the timely acquisition of information, its processing, and transformation into useful knowledge and dissemination to its clients is a global financial institution such as Citigroup. Clearly, one mapping of such an organization into a knowledge supernetwork could, at first glance, be of the form discussed above for an intelligence agency. However, since a primary goal of a global financial insitution is to provide not only information concerning financial products and investments but also to actually optimize clients' portfolios, we propose the following mapping. We associate particular clients (and their portfolios) with distinct O/D pairs. An individual client, in turn, has demands for the financial products (which in the fixed demand setting are assumed given; in the next section, we provide elastic demand extensions, which relax this assumption). Examples of links on the supernetwork can include transaction links associated with obtaining the product either physically or electronically. The costs described in Section 2.1 are also relevant and, in particular, the risk associated with a given product. Of course, expected returns should also be incorporated (with appropriate weight associated with the objective function which would be of maximization form). The demand for a financial product may be expressed in financial units. The network structure allows one to capture financial transactions that need not be limited to a single country. We note that the use of networks for the conceptualization of financial flows dates to Quesnay (1758) and many models have been constructed to predict optimal flows and to capture interactions among the decision-makers (cf. Nagurney and Siokos (1997) and the references therein). More recently, network models have been constructed to address financial intermediation in a global dimension (cf. Nagurney and Cruz (2003)) and to include electronic transactions. The knowledge supernetwork framework conceptualized and developed in this paper, however, allows for the interpretation of a financial organization's primary activites in terms of production processes and is not limited to a particular network structure but, rather, as discussed in Section 2.1, allows for a *general* network topology.

Obviously, in such a knowledge supernetwork representation particular links could correspond to information gleaned from reports.

3. Modeling Extensions

In this section, we provide several modeling extensions to the basic fixed demand knowledge supernetwork model constructed in Section 2.1. In Sections 3.2 and 3.3, we develop two elastic demand models under the assumptions, respectively, of known price functions or known demand functions and we provide the variational inequality formulations of the governing equilibrium conditions in both cases.

3.1 Elastic Demand Model with Known Price Functions

In this subsection, we consider the knowledge supernetwork model in which it is assumed that the price functions associated with the products and the O/D pairs are given. We then in Section 3.2 turn to the case in which we are provided with the demand functions directly.

Assume now that the demands (cf. (2)) are no longer fixed and known but, rather, are now *variables* and group the demands into the vector $d \in R^{kJ}_{+}$. We assume, as given, a price function associated with each product i and O/D pair ω and denoted by λ^i_ω which is assumed to be continuous and of the form:

$$\lambda^i_\omega = \lambda^i_\omega(d), \quad \forall i, \omega, \tag{19}$$

and we group the prices into the vector λ. We otherwise assume the same notation and multicriteria decision-making as outlined in Section 2.

Equilibrium Conditions in the Case of Known Price Functions

We can immediately extend optimality conditions (17), in the fixed demand case, as follows: for all products i; $i = 1, \ldots, k$; all O/D pairs $\omega \in W$, and all paths $p \in P_\omega$, a link flow and demand pattern (f^*, d^*) is said to be in equilibrium if the following holds:

$$\hat{C}'^{i}_{p}(f^*) \begin{cases} = \lambda^i_\omega(d^*), & \text{if} \quad x^{i*}_p > 0 \\ \geq \lambda^i_\omega(d^*), & \text{if} \quad x^{i*}_p = 0. \end{cases} \tag{20}$$

In other words, those paths or production processes will be used for a product and O/D pair such that the marginal generalized total cost associated with producing that product is equal to the price that the consumers (associated with that O/D pair) are willing to pay for that product. If the marginal total cost of a product on a path exceeds the price that the consumers are will to pay then that path will not be used for the production of that product. Note that, for example, in a news organization context, the price functions would express how attractive various programs and news segments would be to the audience.

In light of the similarity of conditions (20) to the traffic network equilibrium conditions with elastic demand (cf. Dafermos (1982) and Nagurney (1999)), we can immediately write down the variational inequality formulation of conditions (20). Indeed, we have the following:

Theorem 2: Variational Inequality Formulation (Price Functions Known)

The link flow and demand pattern $(f^*, d^*) \in \mathcal{K}$ satisfying equilibrium conditions (20) also satisfies the variational inequality problem:

$$\sum_{i,a}\sum_{j=1}^{k}\sum_{h=1}^{H}\sum_{b\in\mathcal{L}} w_h \frac{\partial \hat{c}^j_{hb}(f)}{\partial f^i_{ha}} \times (f^i_a - f^{i*}_a) - \sum_{i,\omega} \lambda^i_\omega(d^*) \times (d^i_\omega - d^{i*}_\omega) \geq 0, \quad \forall (f,d) \in \mathcal{K}, \qquad (21)$$

where now $\mathcal{K} \equiv \{(f,d) | \exists x \geq 0, \text{ such that} (1), (2) \text{ hold}\}$, and vice versa. Equivalently, in standard notation (cf. (16c)), we now may define: $X \equiv (f,d)$ and $F(X) \equiv (F_{1,1} \ldots, F_{k,n}, \lambda(d))$.

Note that variational inequality (21) has an additional term relative to the variational inequality (16b) formulation of the fixed demand optimality conditions. This is due to the fact that the demands for the products and O/D pairs are now variables. Hence, in the case of elastic demands it may so happen that the price associated with a product and O/D pair (and that the consumers are willing to pay) is not sufficiently high so that it is not economical to produce the product for that O/D pair (given the marginal total costs) and, thus, we may have that the associated demands and product flows are zero. This elastic demand extension of the basic knowledge supernetwork model with fixed demands provides added modeling flexibility and generality. Note that in presenting the price functions (19) we made no separability assumptions nor symmetry assumptions and presented the optimality conditions which we now term *equilibrium* conditions directly. Indeed, since there is no optimization reformulation of conditions (20) it is clear that we need to appeal to the theory of variational inequalities (cf. Nagurney (1999)) for formulation, analysis, and, ultimately, computational purposes.

3.2 Elastic Demand Model with Known Demand Functions

Of course, rather than having the price functions $\lambda(d)$ be available, it may be easier to estimate and to obtain the demand functions associated with the products and O/D pairs directly, in which case we would have, rather than the functions (19) being given, the demand functions d^i_ω, where

$$d^i_\omega = d^i_\omega(\lambda), \quad \forall i, \omega. \tag{22}$$

Here we only assume that the demand functions are continuous. Note that according to (22) we may have that the demand for a product and O/D pair may depend not only upon the price of that product at that O/D pair but also on the prices of the product at other O/D pairs and on other products, as well.

Equilibrium Conditions in the Case of Known Demand Functions

We now give the equilibrium conditions governing the knowledge supernetwork model in which the demand functions are known. In particular, we have that (cf. (17) and (20)): for all products i; $i = 1, \ldots, m$; all O/D pairs $\omega \in W$, and all paths $p \in P_\omega$, a pattern of link flows, prices, and demands is in equilibrium if it satisfies the conditions:

$$\hat{C}^{i'}_p(f^*) \begin{cases} = \lambda^{i*}_\omega, & \text{if} \quad x^{i*}_p > 0 \\ \geq \lambda^{i*}_\omega, & \text{if} \quad x^{i*}_p = 0 \end{cases} \tag{23}$$

and

$$d^i_\omega(\lambda^*) \begin{cases} = \sum_{p \in P_\omega} x^{i*}_p, & \text{if} \quad \lambda^{i*}_\omega > 0 \\ \leq \sum_{p \in P_\omega} x^{i*}_p, & \text{if} \quad \lambda^{i*}_\omega = 0. \end{cases} \tag{24}$$

Equilibrium conditions (23) correspond to those in (17) (and also to those in (20)) but now the prices are variables. Condition (24) is a market-type clearing condition which states that of the price the consumers are willing to pay for the product and O/D pair is positive then the market clears for that product and O/D pair; otherwise, there may be an excess supply associated with that product and O/D pair.

Equilibrium conditions are similar to those governing traffic network equilibrium problems with demand functions given (although in that context we deal with user travel costs on the paths rather than with marginal generalized total costs as is the case in the knowledge supernetwork context). These conditions (cf. Dafermos and Nagurney (1984) and Nagurney (1999)) have been formulated as a variational inequality problem and due to the similarity to the conditions (23) and (24) above we can immediately obtain:

Theorem 3: Variational Inequality Formulation (Demand Functions Known)

Let $\mathcal{K}$ now denote the feasible set defined by $\mathcal{K} \equiv \{(f,d,\lambda)|\lambda \geq 0, \exists x \geq 0$ *such that* $(1),(2)$ *hold*$\}$. *The vector* $(f^*, d^*, \lambda^*) \in \mathcal{K}$ *is an equilibrium according to (23) and (24) if and only if it satisfies the variational inequality problem:*

$$\sum_{i,a}\sum_{j=1}^{k}\sum_{h=1}^{H}\sum_{b\in\mathcal{L}} w_h \frac{\partial \hat{c}^j_{hb}(f^*)}{\partial f^i_a} \times (f^i_a - f^{i*}_a) - \sum_{i,\omega}\lambda^{i*}_\omega \times (d^i_\omega - d^{i*}_\omega)$$

$$+\sum_{i,\omega}(d^{i*}_\omega - d^i_\omega(\lambda^*)) \times (\lambda^i_\omega - \lambda^{i*}_\omega) \geq 0, \quad \forall (f,d,\lambda) \in \mathcal{K}. \tag{25}$$

Equivalently, we may put (25) in standard form by defining now $X \equiv (f,d,\lambda)$ *and* $F(X) \equiv (F_{1,1},\ldots,F_{k,n}, -\lambda, d - d(\lambda))$.

Clearly, the applications described in Section 2.2 can also, in appropriate circumstances and contexts, be modeled as elastic demand knowledge supernetworks. In particular, the relevance of elastic demands is especially appropriate in the setting of consumer-responsive and sensitive organizations since the price for an O/D pair and product can then vary according to the demand (and vice versa) and reflects the attractiveness of the knowledge product.

Finally, such prices provide, in a sense, a *benchmark* in terms of which knowledge products (if they are actually produced) yield higher or lower *values* with a price providing an economic measure of value. Furthermore, note that above it is the generalized marginal total cost on a path for a product that is equal to the price at optimality/equilibrium and, thus, that the generalized marginal path total cost captures criteria not limited to just cost, but, also, as discussed in Section 2.1, the relevant criteria such as time, risk, etc.

4. Qualitative Properties

In this section, we provide qualitative properties of the solutions to variational inequalities (16b) or (16c), (21), and (25), which formulate the optimality/equilibrium conditions of the fixed demand model, the elastic demand model with known price functions, and the elastic demand model with known demand functions, respectively. We refer, for simplicity, to the three variational inequality formulations as VI1, VI2, and VI3.

Theorem 4: Existence of a Solution to VI1

A solution f^* to VI1 *exists, since the feasible set $\mathcal{K}$ is compact (due to the fact that the demands are fixed and not infinite) and the marginal total costs on the links are continuous for all the products.*

Proof: Classical result from finite-dimensional variational inequality theory (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)). □

In the case of VI2 and VI3, however, the underlying feasible sets are no longer compact. Nevertheless, we can establish existence of the respective solution patterns under quite reasonable conditions. The conditions, in a sense, help to bound the region of feasible solutions.

Theorem 5: Existence of a Solution to VI2

A solution (f^*, d^*) to VI2 *is guaranteed to exist under the following assumptions: the generalized marginal total costs on the links (for each product and criterion) (cf. (7) and (18)) are continuous and there exist positive constants k_1 and k_2 such that*

$$w_h \frac{\partial \hat{c}^i_{hb}(f)}{\partial f^i_a} \geq k_1, \quad \forall i, a \textit{ and } f \in \mathcal{K}$$

and

$$\lambda^i_\omega(d) < k_1, \quad \forall i, \omega \textit{ and } d \textit{ with } d^i_\omega \geq k_2.$$

Proof: See Theorem 4.4 in Nagurney (1999). □

Finally, we provide an existence result for VI3.

Theorem 6: Existence of a Solution to VI3

Assume that the generalized link marginal total cost functions are continuous and that the demand functions are also continuous. If there exist positive constants k_3 and k_4 such that

$$w_h \frac{\partial \hat{c}^i_{hb}(f)}{\partial f^i_a} \geq k_3, \quad \forall i, a \textit{ and } f \in \mathcal{K},$$

where $\mathcal{K}$ is as defined for the elastic demand model with known demand functions (as in Section 3.2) and

$$d^i_\omega(\lambda) < k_4, \quad \forall \omega, i \textit{ and } \lambda, \textit{ with } \lambda^i_\omega \geq k_4,$$

then VI3 *has at least one solution (f^*, d^*, λ^*).*

Proof: See Theorem 4.3 in Nagurney (1999). □

We now provide the uniqueness results for all three variational inequalities in a succinct manner. In particular, we have that

Theorem 7: Uniqueness of a Solution

Uniqueness of a solution pattern to VI1, VI2*, or* VI3*, respectively, follows under the assumption that the corresponding function F that enters the variational inequality problem (cf. (16c), and following (21) and (25)) is strictly monotone, that is,*

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \quad X^1 \neq X^2, \tag{26}$$

with X and $F(X)$ defined accordingly for each problem.

Proof: The result is standard from the theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)). □

5. Computational Procedure

In this section, we describe the algorithm or computational procedure that can be applied to compute the solutions to the variational inequality problems: VI1, VI2, and VI3. The algorithm is the Euler method, which is a special case of the general iterative scheme of Dupuis and Nagurney (1993). It has to-date been applied to compute solutions to a plethora of variational inequality applications ranging from traffic network equilibrium problems and spatial equilibrium problems to a variety of financial and supply chain equilibrium problems (cf. Nagurney and Zhang (1996), Nagurney and Siokos (1996), Nagurney and Dong (2002), and the references therein). We propose this method, since in the context of the knowledge supernetwork models, the induced subproblems can be solved exactly and in closed form. Moreover, the Euler method suggests a natural underlying dynamics to these problems. Hence, the proposed scheme further lays the foundations for the ultimate development of dynamic versions of the models introduced in this paper.

The general statement of the Euler method, which considers the solution of the variational inequality problem, in standard form, which recall is given by:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{27}$$

is as follows:

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)), \tag{28}$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \to 0$, as $T \to \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

$$y = P_{\mathcal{K}}X = \arg\min_{z \in \mathcal{K}} \|X - z\|. \tag{29}$$

Step 2: Convergence Verification

If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1,

Clearly, since each of the VI problems: VI1, VI2, and VI3, is characterized by a distinct vector function F, a distinct vector of variables X, and feasible set $\mathcal{K}$, the projection operation (28) at an iteration T will take on a distinct form. For example, in the case of VI1 the induced subproblem (at an iteration) is a quadratic programming problem (with separable functions) over a feasible set that is a network. Such problems can be solved using a variety of quadratic programming algorithms. However, in the next section, we utilize the equilibration algorithm of Dafermos and Sparrow (1969) for the solution of the embedded quadratic programming problems in the case of the fixed demand examples since the algorithm fully exploits the network structure. In addition, for the solution of elastic demand examples in the next section, we exploit the simplicity of the induced subproblems in the case of VI2. Additional background on problems of the structure of VI3 can be found in Nagurney and Zhang (1996).

Convergence results can be found in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996).

6. Numerical Examples

In this section, we present both fixed demand and elastic demand (with known price functions) numerical examples for which we then compute the equilibrium flow pattern using the Euler method described in the preceding section.

Specifically, we consider a knowledge supernetwork of an organization as given in Figure 2. The network consists of two origin nodes at the top; two destination nodes at the bottom, with two O/D pairs given by: $\omega_1 = (1, 11)$ and $\omega_2 = (2, 12)$.

There is a total of twenty links in the network and the links have been enumerated in Figure 2 for data presentation purposes. For simplicity, we assume that there is a single knowledge product being produced. According to the supernetwork depicted in Figure 2, some links are information acquisition links, while others correspond to information processing links (and these can be either physical or virtual), as well as information shipment links for ultimate delivery of the product to the consumers. Note that according to Figure 2, the information processing links are contained in several different paths, which means that this resource is shared among the various production processes

Both versions of the Euler method, that is, for the fixed demand problem and for the elastic demand with known price function case were coded in FORTRAN and the computer system used for the implementation and the execution of the code was the Unix located at the University of Massachusetts at Amherst. We report the number of iterations required for convergence.

We used the equilibration algorithm of Dafermos and Sparrow (1969) to solve the embedded quadratic programming problems corresponding to the variational inequality subproblem (28) for the fixed demand model and we used the Euler method for the subproblem in the elastic demand case.

The convergence criterion was that the absolute value of the path flows at two successive iterations was less than or equal to ϵ with ϵ set to 10^{-4}. The sequence used in the Euler method was: $.1\{1, \frac{1}{2}, \frac{1}{2}, \ldots\}$. For the fixed demand example, the demand for the product at each O/D pair was equally distributed among the paths connecting the O/D pair to construct the initial feasible path flow (and link flow) pattern. In the elastic demand example, on the other hand, we initialized the path flows (and hence also the link flows and the demands) to zero.

Example 1: Fixed Demands – Criteria Weighted Equally

In the first numerical example, we assumed that the demands for the knowledge product were fixed and known. Since there is only a single product in this and the subsequent examples, we suppress the superscript i associated with the functions and variables of Section 2. Denoting the O/D pairs by $\omega_1 = (1, 11)$ and $\omega_2 = (2, 12)$, the demands for the product, hence, were given by: $d_{w_1} = 80$, $d_{\omega_2} = 160$.

There were four paths connecting each O/D pair. The paths connecting O/D pair ω_1 were: $p_1 = (1, 9, 13)$, $p_2 = (2, 10, 15)$, $p_3 = (3, 11, 17)$, and $p_4 = (4, 12, 19)$, whereas the paths connecting O/D pair ω_2 were: $p_5 = (5, 9, 14)$, $p_6 = (6, 10, 16)$, $p_7 = (7, 11, 18)$, and $p_8 = (8, 12, 20)$. We assumed that there were three criteria associated with each link and consisting, respectively, of: ptoduction cost (criterion 1), production time (criterion 2), and the risk (criterion 3). Thus, following the notation in Section 2 (cf. (6) and (7)), we then have that the total cost on a link a is given by $\hat{c}_{1a}(f) = \pi_a(f) \times f_a$; the total time on a link a is given by $\hat{c}_{2a}(f) = t_a(f) \times f_a$, and the total risk on a link is given by: $\hat{c}_{3a}(f) = \rho_a(f) \times f_a$ for all links.

The weights were constructed as follows: we set $w_1 = 1$, $w_2 = 1$, and $w_2 = 1$. Hence, in this example the decision-makers weight the cost, the time, and the risk associated with each link equally. In Example 2, we alter the weights. The generalized marginal total link cost functions were constructed according to (15).

In Table 1 we provide the additional input data in a format accessible for reproducibility of the results. In particular, we provide the marginal total link criterion functions denoting the cost, time, and risk.

The Euler method converged in 33 iterations and yielded the link flow pattern given in Table 2 with the incurred path flow pattern reported in Table 3.

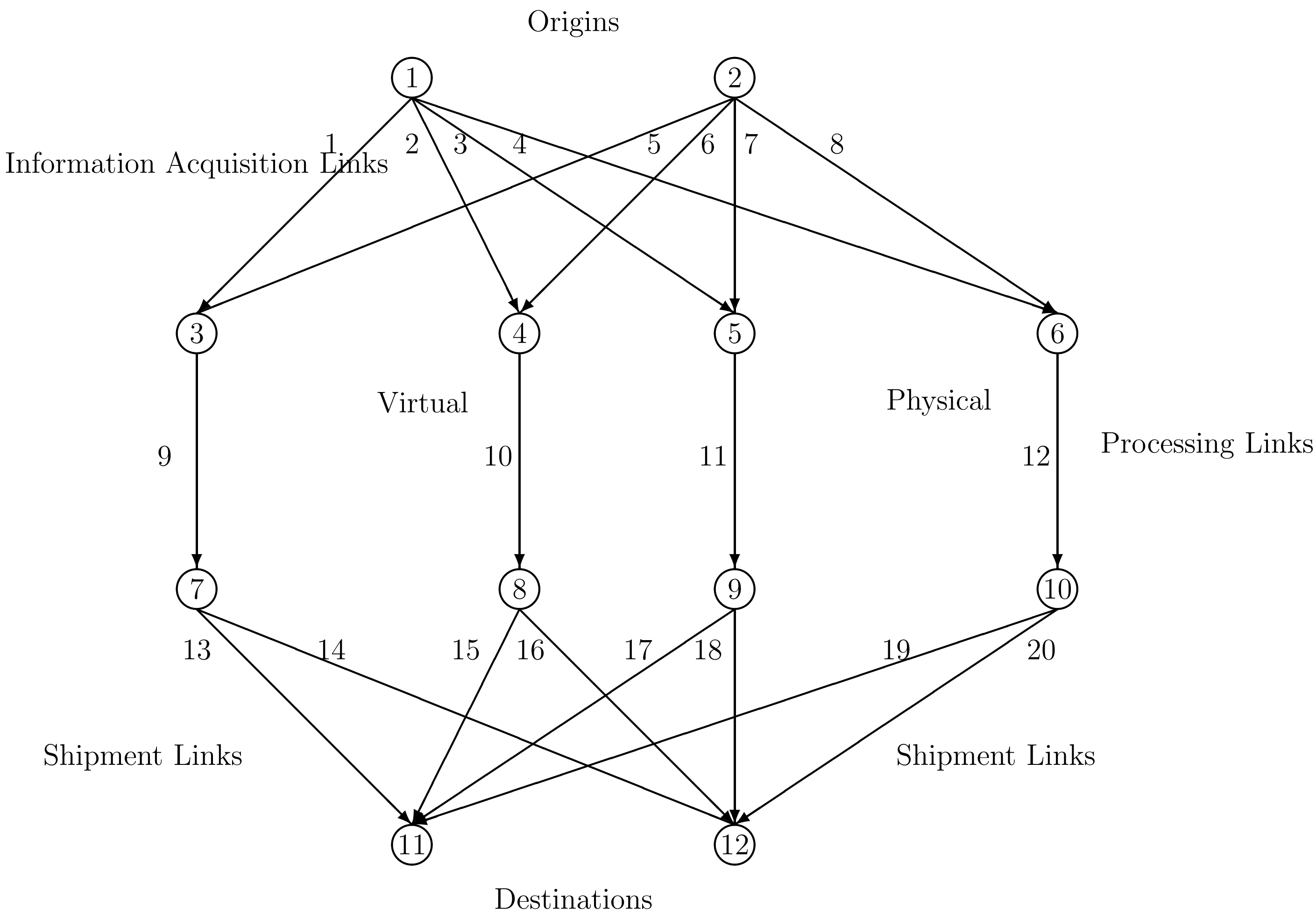

Figure 2: The Knowledge Supernetwork for the Numerical Examples

Table 1: The Marginal Total Link Criterion Functions Representing: Cost, Time, and Risk for the Numerical Examples

Link a	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{c}_{1b}(f)}{\partial f_a}$	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{c}_{2b}(f)}{\partial f_a}$	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{c}_{3b}(f)}{\partial f_a}$
1	$.00005f_1^4 + f_1 + f_2 + 2$	$.00005f_1^4 + 2f_1 + f_2 + 2$	$2f_1 + 4$
2	$.00003f_2^4 + f_2 + .5f_5 + 1$	$.00003f_2^4 + 2f_2 + f_1 + 1$	$3f_2 + 2$
3	$.00005f_3^4 + 4f_3 + f_4 + 1$	$.00005f_3^4 + 3f_3 + .5f_4 + 3$	$f_3 + 1$
4	$.00003f_4^4 + 6f_4 + 2f_5 + 4$	$.00003f_4^4 + 7f_4 + 3f_3 + 1$	$f_4 + 1$
5	$f_5 + 1$	$f_5 + 2$	$2f_5 + 5$
6	$.00007f_6^4 + f_6 + .5f_2 + 1$	$.00007f_6^4 + 2f_6 + f_5 + 1$	$3f_6 + 6$
7	$8f_7 + 7$	$4f_7 + 6$	$f_7 + 1$
8	$.00001f_8^4 + 7f_8 + 3f_5 + 6$	$.00001f_8^4 + 4f_8 + 2f_7 + 1$	$f_8 + 1$
9	$2f_9 + 1$	$2f_9 + 1$	$5f_9 + 12$
10	$.00003f_{10}^4 + 2f_{10} + f_9 + 1$	$.00003f_{10}^4 + 2f_{10} + f_9 + 1$	$11f_{10} + 11$
11	$.00004f_{11}^4 + 2f_{11} + f_{10} + 4$	$.00004f_{11}^4 + 4f_{11} + 2f_{12} + 2$	$f_{11} + 1$
12	$.00002f_{12}f^4 + 2f_{12} + f_{11} + 2$	$.00002f_{12}^4 + 4f_{12} + 2f_{11} + 1$	$f_{12} + 1$
13	$.00003f_{13}^4 + 9f_{13} + 3f_{14} + 3$	$.00003f_{13}^4 + 3f_{13} + f_{14} + 2$	$f_{13} + 11$
14	$5f_{14} + 3$	$4f_{14} + 2$	$6f_{14} + 21$
15	$6f_{15} + 4$	$4f_{15} + 1$	$7f_{15} + 14$
16	$10f_{16} + 10$	$2f_{16} + 10$	$5f_{16} + 10$
17	$5f_{17} + 10$	$5f_{17} + 10$	$f_{17} + 2$
18	$f_{18} + 20$	$6f_{18} + 20$	$2f_{18} + 1$
19	$6f_{19} + 20$	$5f_{19} + 10$	$f_{19} + 1$
20	$10f_{20} + 15$	$4f_{20} + 10$	$f_{20} + 1$

Table 2: The Equilibrium Link Flows for Example 1

Link a	f_a^*
1	23.31
2	16.21
3	21.05
4	19.43
5	65.03
6	30.11
7	31.16
8	33.70
9	88.34
10	46.32
11	52.21
12	53.13
13	23.31
14	65.03
15	16.21
16	30.11
17	21.05
18	31.16
19	19.43
20	33.70

Table 3: The Equilibrium Path Flows for Example 1

Path p	x_p^*
p_1	23.31
p_2	16.21
p_3	21.05
p_4	19.43
p_5	65.03
p_6	30.11
p_7	31.16
p_8	33.70

The incurred generalized path marginal total costs (cf. (18)) were:

O/D pair ω_1:

$$\hat{C}'_{p_1} = 1595.36, \quad \hat{C}'_{p_2} = 1595.41, \quad \hat{C}'_{p_3} = 1595.42, \quad \hat{C}'_{p_4} = 1595.42,$$

O/D pair ω_2:

$$\hat{C}'_{p_5} = 2078.50, \quad \hat{C}'_{p_6} = 2078.42, \quad \hat{C}'_{p_7} = 2078.42, \quad \hat{C}'_{p_8} = 2078.43.$$

Note that the Euler method, embedded with the equilibration algorithm of Dafermos and Sparrow (1969), which exploits the network structure of the problem yielded quite accurate solutions as reflected by the equalization of the marginal generalized total costs on the used paths for each O/D pair. Indeed, optimality conditions (17) were satisfied with excellent accuracy.

Example 2 – Fixed Demands with Unequal Weights

In the second numerical example, we kept the data as in Example 1, but we made the following change. Now, rather than having all the weights be identically equal and equal to one we had that $w_1 = 1$, $w_2 = .5$, and $w_3 = 1$. Hence, in this example, the decision-maker now weights time less than the two other criteria of cost and risk.

The Euler method converged in 29 iterations and yielded the new equilibrium link flow and path flow patterns reported in Tables 4 and 5, respectively.

The incurred generalized marginal toal costs on the paths were now:

O/D pair ω_1:

$$\hat{C}'_{p_1} = 1312.15, \quad \hat{C}'_{p_2} = 1312.09, \quad \hat{C}_{p_3} = 1312.09, \quad \hat{C}'_{p_4} = 1312.10,$$

O/D pair ω_2:

$$\hat{C}'_{p_5} = 1749.99, \quad \hat{C}'_{p_6} = 1749.93, \quad \hat{C}'_{p_7} = 1749.98, \quad \hat{C}'_{p_8} = 1749.92.$$

Table 4: The Equilibrium Link Flows for Example 2

link a	f_a^*
1	21.53
2	15.21
3	21.88
4	21.38
5	62.54
6	30.08
7	32.86
8	34.52
9	84.07
10	45.29
11	54.74
12	55.91
13	21.52
14	62.54
15	51.21
16	30.08
17	21.88
18	32.86
19	21.38
20	34.52

Table 5: The Equilibrium Path Flows for Example 2

Path p	x_p^*
p_1	21.52
p_2	15.21
p_3	21.88
p_4	21.38
p_5	62.54
p_6	30.08
p_7	32.86
p_8	34.52

Since the weight associated with the time criterion was now halved, the incurred marginal total costs on the used paths were lower than the analogous ones obtained for Example 1. One can also see that those links with higher marginal total time functions in Example 1 (such as link 4) and the paths containing such links, now due to the lower weight associated with time, had increased flow.

We now present two elastic demand examples.

Example 3: Elastic Demands – Criteria Weighted Equally

In the third and fourth examples, we considered problems with variational inequality formulation given by (21). Again, the knowledge supernetwork was as depicted in Figure 2. The data were also as in Example 1 except that now rather than having the demands being fixed, we utilized the following demand price functions:

$$\lambda_{\omega_1}(d) = -.5d_{\omega_1} + 1200, \quad \lambda_{\omega_2}(d) = -.1d_{\omega_2} + 900.$$

In addition to the marginal total cost criterion given in Table 1, we added a new criterion to represent the cost associated with security on the links with the data given in Table 6. The weights for the criterion were equal and given by: $w_1 = w_2 = w_3 = w_4 = 1$. In Example 4, we alter the weightings.

The Euler method converged in 1554 iterations and yielded the equilibrium link flow pattern given in Table 7 and induced by the equilibrium path flow pattern in Table 8.

Table 6: Additional Criterion Data – Security Cost

link a	$\sum_{b\in\mathcal{L}} \frac{\partial \hat{c}_{4b}(f)}{\partial f_a}$
1	$f_1 + 1$
2	$f_2 + 2$
3	$f_3 + 1$
4	$f_4 + 1$
5	$f_5 + 1$
6	$2f_6 + 1$
7	$f_7 + 1$
8	$f_8 + 1$
9	$f_9 + 1$
10	$f_{10} + 11$
11	$f_{11} + 1$
12	$f_{12} + 1$
13	$f_{13} + .5$
14	$2f_{14} + 1$
15	$f_{15} + .5$
16	$f_{16} + 1$
17	$f_{17} + 1$
18	$f_{18} + 1$
19	$f_{19} + 1$
20	$f_{20} + 1$

Table 7: The Equilibrium Link Flows for Example 3

link a	f_a^*
1	8.28
2	5.66
3	8.78
4	5.43
5	0.00
6	0.00
7	0.00
8	0.00
9	8.28
10	5.66
11	8.78
12	5.43
13	8.28
14	0.00
15	5.66
16	0.00
17	8.78
18	0.00
19	5.43
20	0.00

Table 8: The Equilibrium Path Flows for Example 3

Path p	x_p^*
p_1	8.28
p_2	5.66
p_3	8.78
p_4	5.43
p_5	0.00
p_6	0.00
p_7	0.00
p_8	0.00

The incurred generalized marginal total costs on the paths were:

O/D pair ω_1:

$$\hat{C}'_{p_1} = 11985.93, \quad \hat{C}'_{p_2} = 11985.75, \quad \hat{C}'_{p_3} = 11985.97, \quad \hat{C}'_{p_4} = 11986.03,$$

O/D pair ω_2:

$$\hat{C}'_{p_5} = 3951.88, \quad \hat{C}'_{p_6} = 6553.36, \quad C'_{p_7} = 4059.84, \quad \hat{C}'_{p_8} = 2499.29.$$

The incurred prices were $\lambda_{\omega_1}(d^*) = 11985.93$ and $\lambda_{\omega_2}(d^*) = 11985.93$. Note that equilibrium conditions (20) were satisfied very accurately. Interestingly, since the price the consumers were willing to pay for the knowledge product at the second O/D pair was lower than the generalized marginal total costs on paths connecting that O/D pair, there was zero volume of the product produced between that O/D pair and, hence, zero flows on paths connecting that O/D pair.

Example 4: Elastic Demands – Unequal Weights

In the fourth and final example, the data were as in Example 3, except that we now increased the weight associated with the risk criterion. In other words, in this situation the decision-maker weights the risk twice as high as the other three criteria so that we had that $w_1 = w_2 = w_4 = 1$ whereas $w_3 = 2$.

The Euler method converged in 2418 iterations and yielded the new equilibrium link flow and path flow patterns given, respectively, in Tables 9 and 10.

The generalized marginal total costs on the paths were now:

O/D pair ω_1:

$$\hat{C}'_{p_1} = 11987.41, \quad \hat{C}'_{p_2} = 11987.80, \quad \hat{C}'_{p_3} = 11987.41, \quad \hat{C}'_{p_4} = 11987.39,$$

O/D pair ω_2:

$$\hat{C}'_{p_5} = 5918.69, \quad \hat{C}'_{p_6} = 7082.38, \quad \hat{C}'_{p_7} = 4014.71, \quad \hat{C}'_{p_6} = 2513.26.$$

Table 9: The Equilibrium Link Flows for Example 4

link a	f_a^*
1	7.09
2	4.02
3	8.68
3	5.41
5	0.00
6	0.00
7	0.00
8	0.00
9	7.09
10	4.02
11	8.68
12	5.41
13	7.09
14	0.00
15	4.02
16	0.00
17	8.68
18	0.00
19	5.41
20	0.00

Table 10: The Equilibrium Path Flows for Example 4

Path p	x_p^*
p_1	7.09
p_2	4.02
p_3	8.68
p_4	5.41
p_5	0.00
p_6	0.00
p_7	0.00
p_8	0.00

The incurred demand prices were now: $\lambda_{\omega_1}(d^*) = 11987.40$ and $\lambda_{\omega_2}(d^*) = 900$. Hence, as in Example 3, the equilibrium conditions (20) were again satisfied very accurately. In addition, there was again no production of the knowledge product associated with O/D pair ω_2.

7. Summary and Directions for Future Research

In this paper, we have proposed a conceptual and theoretical framework for the study of knowledge organizations. In particular, we refer to the framework as *knowledge supernetworks* since it allows for the abstraction of decision-making involving knowledge organizations. We developed both fixed demand and elastic demand versions of the knowledge supernetwork and showed how such knowledge organizations as a news organization, an intelligence agency, and a global financial institution can be formalized as a supernetwork. The decision-maker is assumed to be a multiple-criteria decision-maker and to weight the various criteria.

We derived the optimality/equilibrium conditions and showed that they can all be uniformly formulated and studied as variational inequality problems. We provided qualitative properties of the flow patterns on the knowledge supernetworks in terms of existence and uniqueness results. In addition, we proposed computational schemes and implemented both fixed demand and elastic demand versions to solve several numerical knowledge supernetwork examples. Although the examples are stylized, they illustrate both the generality and the flexibility of the proposed theoretical approach.

Clearly, there are numerous directions in which the foundations set forth in this paper can be extended. Some directions include: incorporating competition with the introduction of several more knowledge organizations who may share a subset of the links; introducing uncertainty into the framework especially regarding the demands associated with the knowledge products; developing dynamic models, and conducting empirical tests on different knowledge organizations, which are large-scale in practice.

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