Reverse Supply Chain Management and Electronic Waste Recycling: A Multitiered Network Equilibrium Framework for E-Cycling

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Abstract: In this paper, we develop an integrated framework for the modeling of reverse supply chain management of electronic waste, which includes recycling. We describe the behavior of the various decision-makers, consisting of the sources of electronic waste, the recyclers, the processors, as well as the consumers associated with the demand markets for the distinct products. We construct the multitiered e-cycling network equilibrium model, establish the variational inequality formulation, whose solution yields the material flows as well as the prices, and provide both qualitative properties of the equilibrium pattern as well as numerical examples that are solved using the proposed algorithm.

Key words: Reverse supply chain management; E-cycling; Recycling; Electronics; Multitiered networks; Environment; Waste management; Reverse logistics; Variational inequalities; Network equilibrium

1. Introduction

Recent legislation both in the United States as well as abroad and, in particular, in Europe and in Japan, has refocused attention on recycling for the management of wastes and, specifically, electronic wastes (see e.g. Appelbaum (2002a)). The impetus has come both from the recognition of the deleterious effects on the environment of dumping electronic wastes as well as the decreasing capacity of landfills. For example, Massachusetts in 2000 banned cathode-ray tubes (CRTs) from landfills whereas Japan in 2001 enacted a law that requires retailers and manufacturers to bear some collection and recycling costs of appliances (Appelbaum (2002b, c)).

Electronic waste recycling and the management thereof differs from household recycling in two significant ways: 1. electronic waste, unlike household waste, is usually a composition of several materials, including hazardous ones and 2. it may contain precious materials as well as materials that can be reused (such as computer components, for example) (cf. Sodhi and Reimer (2001)). Hence, it may be feasible and even profitable to collect and process electronic waste that may be obtained from sources that are dispersed in location. Furthermore, a variety of governmental mandates on recycling of electronic wastes or e-cycling is forcing decision-makers to explore their options.

Many researchers have studied recycling issues; see, for example, Hoshino, Yura, and Hitomi (1995), Inderfurth, de Kok, and Flapper (2001), Ron and Penev (1995), Stuart, Ammons, and Turbini (1999), and Uzsoy and Venkatachalam (1998). Hoshino, Yura, and Hitomi (1995) dealt with a recycling model that maximizes two measures of performance – total profit and recycling rate – by using linear programming. Inderfurth, de Kok, and Flapper (2001), in turn, formulated a stochastic remanufacturing system as a stochastic dynamic problem, and analyzed the structure of the optimal policy. Ron and Penev (1995) proposed an approach to determine the degree of disassembly at a single point in time. These previous studies, however, did not take into account supply chain management issues. For example, the price of a recycled material, which is in reality endogenously determined through the interactions between tiers and within tiers, is exogenously determined and fixed in the above-noted studies. Although Uzsoy and Venkatachalam (1998) considered a recycling problem in terms of a supply chain, they, nevertheless, limited the formulation to a linear programming world.

In this paper, we take a perspective of *supply chain* analysis and management for the collection, recycling, and processing of electronic waste, which may be ultimately converted into products demanded by the consumers. In particular, we propose an integrated reverse supply chain management framework that allows for the modeling, analysis, and computation of the material flows as well as the prices associated with the different tiers of the decision-makers in the multitiered electronic recycling or *e-cycling network*. Of specific interest is the synthesis of the behaviors of the various decision-makers involved in the recycling and the further processing of electronic waste beginning with the top tier of nodes in the network corresponding to the sources of the electronic waste to the bottom tier that reflects the demand markets for the recycled/reprocessed electronic products.

The foundations for our analytical framework are derived from recent contributions in supply chain analysis that focus on multiple tiers of decision-makers, be they manufacturers, distributors, retailers, and/or consumers, who may compete within a tier but cooperate between tiers, with the governing concept, assuming individual optimizing behavior, being that of a *network equilibrium* (see Nagurney, Dong, and Zhang (2002), Nagurney, et al. (2002a, b), Nagurney and Dong (2002), and Nagurney and Toyasaki (2003)). In addition, we build upon our tradition of a network perspective to environmental management as described in the book on environmental networks by Dhanda, Nagurney, and Ramanujam (1999).

However, unlike the supply chain network equilibrium models referenced above, the model developed in this paper assumes fixed amounts of electronic waste held by the various sources. In addition, the decision-makers, be they the sources of electronic waste, the recyclers, and/or the processors associated with the respective tier of nodes of the e-cycling network, have the option of not only transporting (or having transported) the material to the subsequent tier for recycling (or further processing) and, ultimately, consumption, but also have the option of shipping the waste to a landfill, with associated costs. Hence, the e-cycling network has characteristics of financial networks with intermediation (cf. Nagurney and Ke (2001, 2003)) but with specific prices and costs associated with opting out of further "processing." Notably, the model developed in this paper differs from the above multitiered supply chain network equilibrium models in that it allows for the transformation of the electronic waste into other products (or byproducts) as the waste "flows" down the e-cycling network.

The concept of an electronics recycling network was proposed by Sodhi and Reimer (2001), who also developed optimization models for the sources, the recyclers, and the processors. However, the scope of the integrated problem, in which the behavior of all the involved decision-makers is captured, along with the flows and prices, as well as the demand-side, was beyond the methodological tools available (see also Ammons et al. (1999)).

This paper is organized as follows. In Section 2, we develop the e-cycling network model for reverse supply chain management consisting of multiple tiers of decision-makers. We describe their optimizing behavior and establish the governing equilibrium conditions. We then derive the equivalent variational inequality formulation, whose solution yields the material flows between the tiers in the network as well as the equilibrium prices of the processed products.

In Section 3, we provide qualitative properties of the equilibrium pattern and also give conditions, under suitable assumptions, on the function that enters the variational inequality formulation in order to be able to establish convergence of the algorithmic scheme in Section 4. In Section 5, we apply the computational algorithm to several numerical examples to illustrate both the model as well as the algorithm. We summarize our results in Section 6, present our conclusions, and also suggest directions for future research.

2 The Multitiered E-Cycling Network Equilibrium Model for Reverse Supply Chain Management

In this section, we develop the multitiered e-cycling network model for reverse supply chain management depicted in graphical form in Figure 1. The network consists of four tiers of nodes. We consider r sources of the electronic waste as represented by the top tier of nodes in Figure 1; m recyclers of the electronic waste, as depicted by the second tier of nodes; n processors of the electronic waste obtained from the recyclers and associated with the third tier of nodes in the network, and o demand markets. We note that the demand markets can correspond, for example, to precious metal demand markets, refurbished product demand markets, and, finally, to demand markets for the components extracted from the electronic wastes. In addition, we let nodes: $m+1$, $n+1$, and $o+1$, denote, respectively, landfills associated with the option of transporting (or having transported) the associated waste from the preceding tier of nodes to such a destination.

For simplicity, we assume that the electronic waste associated with the top tier of nodes is homogeneous, in order to simplify the notation. Hence, one could consider CRTs, for example. In the case of multiple distinct types of electronic wastes, one could "layer" the different types of "commodities" as in the case of transportation and/or spatial price networks (cf. Nagurney (1999)) over appropriate links in the e-cycling network.

We denote a typical source of the electronic waste, which can include, for example, schools, universities, businesses, collectors of electronic waste, etc., at the same or at distinct locations, by h, and we associate such a source with top tier node h in the network. A typical recycler of the electronic waste, in turn, who may be involved in some value added functions such as dismantling or refurbishing or simply acting as an intermediary between the sources and processors, is denoted by i, and is associated with the second tier node i in the network.

A typical processor, on the other hand, is denoted by j and is associated with third tier node j in the network. Note that some processors may be smelters whereas others may conduct further dismantling of the electronic material for consumption purposes. Hence, processors may convert the recycled goods into metals, refurbished products, and/or disassemble the electronic waste into electronic component products such as mother boards, computer chips, and so on.

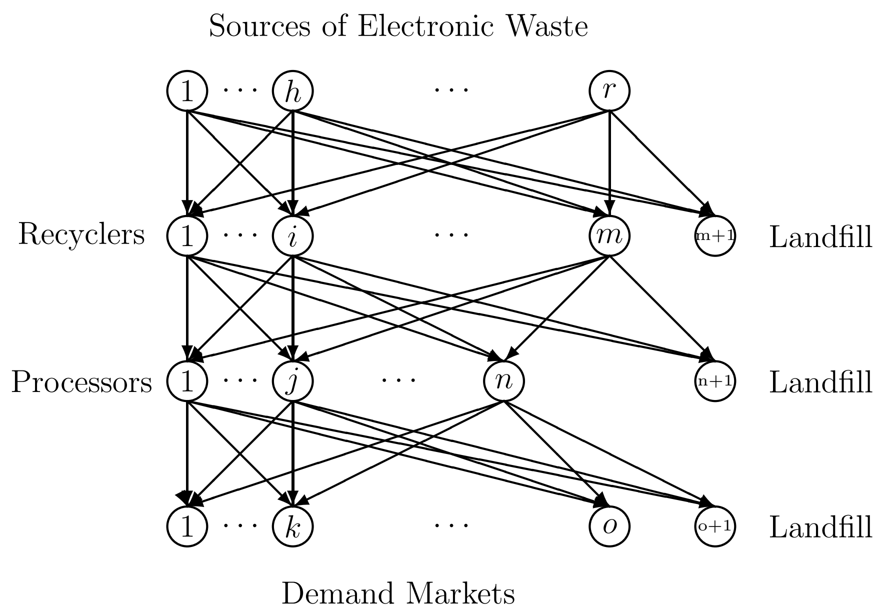

Figure 1: The 4-Tiered E-Cycling Network with Sources, Recyclers, Processors, and Demand Markets

The behavior of the e-cycling network decision-makers associated with the four tiers of the network is now described. We first describe the behavior of the sources of the electronic waste. We then discuss that of the recyclers and the processors. Finally, we turn to the consumers at the demand markets. Once the behavior of the various decision-makers is described, we then establish the governing equilibrium concept.

The Behavior of the Sources of the Electronic Waste

Let S^h denote the volume of the electronic waste possessed by decision-maker h and assume that this amount is given for all sources h; $h = 1, \ldots, r$. Let q_{hi} denote the nonnegative amount of the electronic waste that is allocated from source h to i, where the index i ranges from $1, \ldots, m$, if i is a recycler. We also let a source have the option of using a landfill for the electronic waste with node $m+1$ in the second tier of nodes representing a landfill. We group the material flows from the sources to the second tier of nodes into the column vector $Q^1 \in R_+^{r(m+1)}$.

Associated with the decisions are costs of transaction, which include, for example, the cost associated with traveling to the recycling center or landfill, respectively, or paying for the cost of transportation to the second tiered nodes. We denote the transaction cost faced by source h and option i by c_{hi} where we assume that

$$c_{hi} = c_{hi}(q_{hi}), \quad h = 1, \ldots, r; i = 1, \ldots, m+1, \tag{1}$$

that is, the cost associated with transacting depends upon the volume of the transaction.

In addition, we assume that each source pays a price per unit of electronic waste to the next tier of nodes associated with the recyclers with the value being endogenous to the model and, which we assume, for the time being, as being given by ρ^*_{1hi} for source h and receptor point i for $i = 1, \ldots, m$. In regards to the landfill option, in contrast, we assume that the price associated with depositing a unit of the electronic waste in the landfill is denoted by $\bar{\rho}_{1h(m+1)}$ for source h and this price is given and fixed.

We assume that each source h seeks to minimize the total cost associated with his management of electronic waste with the optimization problem given for source h by:

$$\text{Minimize} \sum_{i=1}^{m} \rho^*_{1hi} q_{hi} + \bar{\rho}_{1h(m+1)} q_{h(m+1)} + \sum_{i=1}^{m+1} c_{hi}(q_{hi}) \tag{2}$$

subject to:

$$\sum_{i=1}^{m+1} q_{hi} = S^h, \tag{3}$$

and the nonnegativity assumption that:

$$q_{hi} \geq 0, \quad i = 1, \ldots, m+1. \tag{4}$$

Clearly, from (3) it is apparent that we assume that all the electronic waste must be disposed of. One could also allow for such wastes to simply lie around or to be "dumped" in which case the expression in (3) would be an inequality rather than an equality. Also, note that the price may differ for the various receptor points, which includes the landfill.

Assuming that the transaction cost functions are continuously differentiable and convex for all the sources, $h = 1, \ldots, r$, the optimality conditions for all sources simultaneously can be expressed as the following inequality (see Bazaraa, Sherali, and Shetty (1993)): determine $Q^{1*} \in K$, such that:

$$\sum_{h=1}^{r} \sum_{i=1}^{m} \left[\frac{\partial c_{hi}(q^*_{hi})}{\partial q_{hi}} + \rho^*_{1hi} \right] \times [q_{hi} - q^*_{hi}]$$
$$+ \sum_{h=1}^{r} \left[\frac{\partial c_{h(m+1)}(q^*_{h(m+1)})}{\partial q_{h(m+1)}} + \bar{\rho}_{1h(m+1)} \right] \times \left[q_{h(m+1)} - q^*_{h(m+1)} \right] \geq 0, \quad \forall Q^1 \in K, \tag{5}$$

where the feasible set K is defined as $K \equiv \{Q^1 \text{ such that (3) and (4) hold for all } h\}$.

Observe that the behavior of the top tier of decision-makers in the e-cycling network is assumed to be that of cost-minimization in contrast to that of top tier decision-makers in other supply chain network equilibrium models (cf. Nagurney, Dong, and Zhang (2002) and Nagurney and Dong (2002)), which is profit-maximization.

The Behavior of the Recyclers and their Optimality Conditions

The recyclers, in turn, are involved in transactions both with the sources of the electronic waste as well as with the processors. We assume that the recyclers also have the option of having the waste transported to a landfill, with the landfill node being denoted by $n+1$. Let q_{ij} denote the flow of material from recycler i to processor j where $j = 1, \ldots, n$ denotes the respective processor and $j = n+1$ denotes the landfill with associated flow into it given by $q_{i(n+1)}$. We group the material flows from the recyclers (associated with the second tier of nodes in the e-cycling network in Figure 1) to the third tier of nodes into the column vector $Q^2 \in R_+^{m(n+1)}$.

We assume that each recycler receives revenue from selling the electronic waste that he collects and delivers to the different processors. We let ρ_{2ij}^* denote the price that recycler i charges processor j for a unit of the electronic waste (and that the processor is willing to pay) with this price being also endogenous to the model for all recycler and processor pairs. In addition, each recycler receives revenue from the sources for handling the electronic waste where recall that the price ρ_{1hi}^* denotes the price that source h pays recycler i for a unit of the waste. On the other hand, recycler i must pay for the use of the landfill with the fixed price being denoted by $\bar{\rho}_{2i(n+1)}$ for recycler i.

The transaction cost faced by recycler i transacting with receptor point j is denoted by c_{ij} and is assumed to be

$$c_{ij} = c_{ij}(q_{ij}), \quad i = 1, \ldots, m; j = 1, \ldots, n+1, \tag{6}$$

that is, the cost of transacting between recycler i and option j (from the perspective of recycler i) is a function of the volume of the transaction.

Since a recycler also transacts with the sources of electronic waste, we assume a transaction cost between each recycler i and source h pair, denoted by $\hat{c}_{hi}$, and assume that

$$\hat{c}_{hi} = \hat{c}_{hi}(q_{hi}), \quad h = 1, \ldots, r; i = 1, \ldots, m+1, \tag{7}$$

that is, this transaction cost is also a function of the volume of flow between the two decision-makers. This transaction cost is from the perspective of the recycler.

In addition, a recycler i is faced with what we term a recycling cost, which includes the storage cost and the cost to make, for example, the separated subassemblies which can then be sent to different processors (see also Sodhi and Reimer (2001)). The cost to recycle electronic waste at recycler i is denote by c_i and is assumed to be of the form

$$c_i = c_i(Q^2), \quad i = 1, \ldots, m. \tag{8}$$

In other words, the recycling cost may be recycler-specific and depends on all the material flows from recyclers to the processors and the landfill.

We are now ready to construct the optimization problem faced by a particular recycler.

In particular, assuming that a recycler is concerned with profit maximization, we have that recycler i's optimization problem can be formulated as:

$$\text{Maximize} \sum_{j=1}^{n} \rho^*_{2ij} q_{ij} + \sum_{h=1}^{r} \rho^*_{1hi} q_{hi} - \bar{\rho}_{2i(n+1)} q_{i(n+1)} - \sum_{j=1}^{n+1} c_{ij}(q_{ij}) - \sum_{h=1}^{r} \hat{c}_{hi}(q_{hi}) - c_i(Q^2) \tag{9}$$

subject to:

$$\sum_{j=1}^{n+1} \alpha_{ij} q_{ij} \leq \sum_{h=1}^{r} q_{hi} \tag{10}$$

and the nonnegativity assumption that:

$$q_{hi} \geq 0, \quad h = 1, \ldots, r; \quad q_{ij} \geq 0, \quad j = 1, \ldots, n+1. \tag{11}$$

Constraint (10) reflects the conversion of the input material flows into the recycled outputs with α_{ij} assumed to be positive for all i and j. Note that the flows out of a recycling node i may be distinct from the input flows. Hence, unlike other multitiered supply chain network equilibrium models developed to-date, here we allow for the transformation of the material flows as they flow down the e-cycling network. Furthermore, note that (10) captures the transformation of the product at a recycler into its component parts for further processing by the processors. If, for example, all the α_{ij}s are precisely equal to 1 then the ratio of input flows to output flows is equal to 1.

We assume that recyclers compete in a noncooperative way. Also, we assume that the recycling cost functions and the transaction cost functions are continuously differentiable and convex. Given the equilibrium concept of Nash (1950, 1951), the optimality conditions for all recyclers simultaneously can be expressed as the following inequality: determine $(q_{hi}^*, q_{ij}^*, \gamma_i^*)$ all greater than or equal to zero for all $h = 1, \ldots, r$; $i = 1, \ldots, m$; $j = 1, \ldots, n+1$, such that

$$\sum_{i=1}^{m}\sum_{h=1}^{r}\left[\frac{\partial \hat{c}_{hi}(q_{hi}^*)}{\partial q_{hi}} - \rho_{1hi}^* - \gamma_i^*\right] \times [q_{hi} - q_{hi}^*]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} - \rho_{2ij}^* + \alpha_{ij}\gamma_i^*\right] \times \left[q_{ij} - q_{ij}^*\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(q_{i(n+1)}^*)}{\partial q_{i(n+1)}} + \bar{\rho}_{2i(n+1)} + \alpha_{i(n+1)}\gamma_i^*\right] \times \left[q_{i(n+1)} - q_{i(n+1)}^*\right]$$

$$+\sum_{i=1}^{m}\left[\sum_{h=1}^{r} q_{hi}^* - \sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^*\right] \times [\gamma_i - \gamma_i^*] \geq 0,$$

$$\forall\, (q_{hi}, q_{ij}, \gamma_i) \geq 0, \;\; h = 1, \ldots, r; i = 1, \ldots, m; j = 1, \ldots, n+1. \qquad (12)$$

Note that γ_i is the Lagrange multiplier associated with constraint (10) for recycler i. We group these Lagrange multipliers for all the recyclers into the column vector $\gamma \in R_+^m$. Such a Lagrange multiplier also has an interpretation as a shadow price. For example, it is worth noting that from the last term in inequality (12), we observe that if $\gamma_i^* > 0$, then the total amount of the product shipped to recycler i, that is, $\sum_{h=1}^{r} q_{hi}^*$ is precisely equal to the flow out, that is, $\sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^*$. Hence, in a sense, we get a correlation between the α_{ij}s and the shadow price γ_i^*. On the other hand, if $\gamma_i^* = 0$, then there may be an excess supply of the product at that recycler. This situation will be illustrated subsequently in numerical examples in Section 5.

Note that both inequalities (5) and (12) are *variational inequalities* (cf. Nagurney (1999) and the references therein).

The Behavior of the Processors and their Optimality Conditions

A processor, in turn, receives items or the components of electronic products from the recyclers. The work of processors is to recover metals, to produce refurbished products, and/or to dispose the residuals of the process in the landfill.

Let q_{jk} denote the amount of "product" k that is allocated from processor j to demand market k, with the index k ranging from $1, \ldots, o$ to denote a particular demand market for a product and with $k = o + 1$ denoting a landfill. We group the material flows from the processors to the demand markets and the landfill as denoted by the bottom tiered nodes of the e-cycling network in Figure 1 into the column vector $Q^3 \in R_+^{n(o+1)}$.

We assume that each processor j charges a price for product k which is associated with demand market (and node) k and is denoted by ρ_{3jk}^* for $k = 1, \ldots, o$. This price is endogenous to the model and also reflects the price that the consumers at demand market k are willing to pay for that particular product which is obtained from processor j.

In addition, recall that a processor j pays recycler i a price ρ_{2ij}^* per unit of the recycled product. Finally, since each processor has also an option of using the landfill we let $\bar{\rho}_{3j(n+1)}$ denote the fixed price per unit of waste associated with processor j placed in landfill $n + 1$.

Associated with the decisions made by the third tier of decision-makers, the processors, are also costs of transaction, which can include, for example, transporting the products to the demand markets and/or to the landfill.

We denote the transaction cost faced by processor j and option k by c_{jk} where we assume that

$$c_{jk} = c_{jk}(q_{jk}), \quad j = 1, \ldots, n; k = 1, \ldots, o + 1, \tag{13}$$

that is, the cost associated with transacting depends upon the volume of the transaction.

We also assume that a processor j encumbers a transaction cost associated with dealing with each recycler. The transaction cost between recycler i and processor j from the perspective of processor j is denoted by

$$\hat{c}_{ij} = \hat{c}_{ij}(q_{ij}), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{14}$$

We assume that each processor has to bear the costs to process electronic wastes. The per unit processing cost that processor j bears is denoted by $c_j(Q^3)$ for all $j = 1, \ldots, n$.

We assume that each processor seeks to maximize the profit associated with his management of electronic wastes with the optimization problem given for processor j being given by:

$$\text{Maximize} \sum_{k=1}^{o} \rho_{3jk}^* q_{jk} - c_j(Q^3) - \bar{\rho}_{3j(o+1)} q_{j(o+1)} - \sum_{i=1}^{m} \rho_{2ij}^* q_{ij} - \sum_{k=1}^{o+1} c_{jk}(q_{jk}) - \sum_{i=1}^{m} \hat{c}_{ij}(q_{ij}) \quad (15)$$

subject to:

$$\sum_{k=1}^{o+1} \beta_{jk} q_{jk} \leq \sum_{i=1}^{m} q_{ij} \quad (16)$$

and the nonnegativity assumptions:

$$q_{ij} \geq 0, \quad i = 1, \ldots, m; \quad q_{jk} \geq 0, \quad k = 1, \ldots, o+1. \quad (17)$$

We assume that the processors also compete in a noncooperative way, given the action of the other processors. Also, we assume that the processing cost functions and the transaction cost functions for each processor are continuously differentiable and convex. Hence, the optimality conditions for all processors simultaneously can be expressed as the following inequality: determine $(q_{ij}^*, q_{jk}^*, \eta_j^*)$ all greater than or equal to zero for $i = 1, \ldots, m$; $j = 1, \ldots, n$; $k = 1, \ldots, o+1$, such that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \left[\frac{\partial \hat{c}_{ij}(q_{ij}^*)}{\partial q_{ij}} + \rho_{2ij}^* - \eta_j^* \right] \times \left[q_{ij} - q_{ij}^* \right]$$

$$+ \sum_{j=1}^{n} \sum_{k=1}^{o} \left[\frac{\partial c_j(Q^{3*})}{\partial q_{jk}} + \frac{\partial c_{jk}(q_{jk}^*)}{\partial q_{jk}} + \beta_{jk} \eta_j^* - \rho_{3jk}^* \right] \times \left[q_{jk} - q_{jk}^* \right]$$

$$+ \sum_{j=1}^{n} \left[\frac{\partial c_j(Q^{3*})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(q_{j(o+1)}^*)}{\partial q_{j(o+1)}} + \beta_{j(o+1)} \eta_j^* + \bar{\rho}_{3j(o+1)} \right] \times \left[q_{j(o+1)} - q_{j(o+1)}^* \right]$$

$$+ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o+1} \beta_{jk} q_{jk}^* \right] \times \left[\eta_j - \eta_j^* \right] \geq 0,$$

$$\forall (q_{ij}, q_{jk}, \eta_j) \geq 0, \; i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o+1. \quad (18)$$

Constraint (16) guarantees that the conversion of recycled material into processed products satisfies the material flow with the conversion parameters β_{jk} being positive for all j and k. Note that η_j is the Lagrange multiplier associated with constraint (16) for processor j and also has an interpretation as a shadow price. We group these Lagrange multipliers into the n-dimensional column vector $\eta \in R^n_+$. Also, observe from the last term in inequality (18) that if the shadow price $\eta^*_j > 0$, then the total volume of flow into processor j given by $\sum_{i=1}^m q^*_{ij}$ is precisely equal to $\sum_{k=1}^{o+1} \beta_{jk} q^*_{jk}$. Here, we also get an interpretation of a correlation between the β_{jk}s and the shadow price η^*_j. On the other hand, if there is excess supply at the processor then the shadow price at the processor will be zero.

Observe that inequality (18) is a variational inequality as are the inequalities (5) and (12), which formulate the optimality conditions, respectively, for the sources of electronic wastes and the recyclers.

The Demand Markets and the Equilibrium Conditions

We now turn to the bottom tier of nodes in the e-cycling network. We assume that the consumers associated with the products at the demand markets incur a transaction cost with the transaction cost associated with processor j and demand market k (from the perspective of the consumers) being denoted by $\hat{c}_{jk}$ for the pair (j, k). Here we assume that this transaction cost is of the form

$$\hat{c}_{jk} = \hat{c}_{jk}(q_{jk}), \quad j = 1, \ldots, n; k = 1, \ldots, o, \tag{19}$$

that is, we allow the transaction cost between a processor and demand market pair to depend on the volume of shipment/transaction between the processor and the demand market pair. We assume here, for the sake of generality, that the demands associated with the demand markets can depend upon, in general, the entire vector of demand market prices, that is, that $d_k = d_k(\rho_4)$, for $k = 1, \ldots, o$, where ρ_4 is the o-dimensional column vector of demand market prices with k-th component given by ρ_{4k}.

The equilibrium conditions for consumers at demand market k, take the form: For all processors: $j; j = 1, \ldots, n$, in equilibrium, we must have that:

$$\rho^*_{3jk} + \hat{c}_{jk}(q^*_{jk}) \begin{cases} = \rho^*_{4k}, & \text{if} \quad q^*_{jk} > 0 \\ \geq \rho^*_{4k}, & \text{if} \quad q^*_{jk} = 0, \end{cases} \tag{20}$$

$$d_k(\rho^*_4) \begin{cases} = \sum_{j=1}^n q^*_{jk}, & \text{if} \quad \rho^*_{4k} > 0 \\ \leq \sum_{j=1}^n q^*_{jk}, & \text{if} \quad \rho^*_{4k} = 0. \end{cases} \tag{21}$$

Conditions (20) state that consumers at demand market k will purchase product k from processor j, if the price charged by the processor for the product plus the transaction cost (from the perspective of the consumers) does not exceed the price that the consumers are willing to pay for the recycled/reprocessed product. Condition (21), on the other hand, states that, if the price the consumers are willing to pay for the product at a demand market is positive, then the quantity consumed at the demand market is precisely equal to the demand. Otherwise, the availability of product k may exceed its demand.

These conditions correspond to the well-known spatial price equilibrium conditions (cf. Samuelson (1952), Takayama and Judge (1971), Nagurney (1999), and the references therein) and have also been utilized in a variety of supply chain network equilibrium problems (see, e.g., Nagurney, Dong, and Zhang, (2002), Nagurney et al. (2002a,b), and Nagurney and Dong (2002)). Clearly, (see e.g., Nagurney and Dong (2002)) conditions (20) and (21) must hold for all demand markets k; $k = 1, \ldots, o$, and can be formulated as a variational inequality problem, given by: determine (q^*_{jk}, ρ^*_{4k}) greater than or equal to zero for $j = 1, \ldots, n$; $k = 1, \ldots, o$, such that

$$\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\rho^*_{3jk} + \hat{c}_{jk}(q^*_{jk}) - \rho^*_{4k}\right] \times \left[q_{jk} - q^*_{jk}\right] + \sum_{k=1}^{o}\left[\sum_{j=1}^{n} q^*_{jk} - d_k(\rho^*_4)\right] \times \left[\rho_{4k} - \rho^*_{4k}\right] \geq 0,$$

$$\forall (q_{jk}, \rho_{4k}) \geq 0, \;\; j = 1, \ldots, n; k = 1, \ldots, o. \quad (22)$$

The Equilibrium Conditions of the Multitiered E-Cycling Network for Reverse Supply Chain Management

In equilibrium, the shipments of the electronic wastes the sources transport to the recyclers must be equal to the shipments that the recyclers accept. Moreover, the amounts of the recycled electronic wastes that the recyclers transport to the processors must be equal to the amounts that the processors accept. Furthermore, the equilibrium material flow and price pattern must satisfy the sum of the optimality conditions (5), (12), (18), and the equilibrium conditions (22) in order to formalize the agreements between the tiers of the e-cycling network (see also Nagurney et al. (2002a, b)).

We now make the above statement formal.

Definition 1: An E-Cycling Network Equilibrium

The equilibrium state of the e-cycling network is one where the material flows between the tiers of the e-cycling network coincide and the material flows and prices satisfy the sum of the optimality conditions (5), (12), (18), and the equilibrium conditions (22).

We now establish the following:

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the e-cycling network model for reverse supply chain management are equivalent to the solution of the variational inequality problem given by: determine $(Q^{1*}, Q^{2*}, \gamma^*, Q^{3*}, \eta^*, \rho_4^*) \in \mathcal{K}$ *satisfying*

$$\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\frac{\partial c_{hi}(q_{hi}^*)}{\partial q_{hi}}+\frac{\partial \hat{c}_{hi}(q_{hi}^*)}{\partial q_{hi}}-\gamma_i^*\right]\times[q_{hi}-q_{hi}^*]$$

$$+\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q_{h(m+1)}^*)}{\partial q_{h(m+1)}}+\bar{\rho}_{1h(m+1)}\right]\times\left[q_{h(m+1)}-q_{h(m+1)}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}}+\frac{\partial \hat{c}_{ij}(q_{ij}^*)}{\partial q_{ij}}+\alpha_{ij}\gamma_i^*-\eta_j^*\right]\times\left[q_{ij}-q_{ij}^*\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{i(n+1)}}+\frac{\partial c_{i(n+1)}(q_{i(n+1)}^*)}{\partial q_{i(n+1)}}+\bar{\rho}_{2i(n+1)}+\alpha_{i(n+1)}\gamma_i^*\right]\times\left[q_{i(n+1)}-q_{i(n+1)}^*\right]$$

$$+\sum_{i=1}^{m}\left[\sum_{h=1}^{r}q_{hi}^*-\sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^*\right]\times[\gamma_i-\gamma_i^*]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{jk}}+\frac{\partial c_{jk}(q_{jk}^*)}{\partial q_{jk}}+\hat{c}_{jk}(q_{jk}^*)+\beta_{jk}\eta_j^*-\rho_{4k}^*\right]\times\left[q_{jk}-q_{jk}^*\right]$$

$$+\sum_{j=1}^{m}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{j(o+1)}}+\frac{\partial c_{j(o+1)}(q_{j(o+1)}^*)}{\partial q_{j(o+1)}}+\beta_{j(o+1)}\eta_j^*+\bar{\rho}_{3j(o+1)}\right]\times\left[q_{j(o+1)}-q_{j(o+1)}^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}q_{ij}^*-\sum_{k=1}^{o+1}\beta_{jk}q_{jk}^*\right]\times\left[\eta_j-\eta_j^*\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}q_{jk}^*-d_k(\rho_4^*)\right]\times[\rho_{4k}-\rho_{4k}^*]\geq 0,\quad \forall(Q^{1*}, Q^{2*}, \gamma^*, Q^{3*}, \eta^*, \rho_4^*)\in\mathcal{K}, \tag{23}$$

where $\mathcal{K} \equiv \{Q^1, Q^2, \gamma, Q^3, \eta, \rho_4)|(Q^1, Q^2, \gamma, Q^3, \eta, \rho_4) \geq 0,$ *and* Q^1 *satisfies* (3) *for all* $h\}$.

Proof: We first establish that the equilibrium conditions imply variational inequality (23). Indeed, the summation of inequalities (5), (12), (18), and (22) yields, after algebraic simplification, the variational inequality (23).

We now establish the converse, that is, that a solution to variational inequality (23) satisfies the sum of conditions (5), (12), (18), and (22) and is, hence, an equilibrium according to Definition 1.

To inequality (23), add the term, $-\rho^*_{1hi} + \rho^*_{1hi}$, to the the term in the first set of brackets preceding the first multiplication sign. Similarly, add the term, $-\rho^*_{2ij} + \rho^*_{2ij}$, to the term in brackets preceding the third multiplication sign. Finally, add the term, $-\rho^*_{3jk} + \rho^*_{3jk}$, to the term preceding the sixth multiplication sign in (23). As the result of the above, inequality (23) becomes:

$$\sum_{h=1}^{r}\sum_{i=1}^{m}\left[-\rho^*_{1hi} + \rho^*_{1hi} + \frac{\partial c_{hi}(q^*_{hi})}{\partial q_{hi}} + \frac{\partial \hat{c}_{hi}(q^*_{hi})}{\partial q_{hi}} - \gamma^*_i\right] \times [q_{hi} - q^*_{hi}]$$

$$+\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q^*_{h(m+1)})}{\partial q_{h(m+1)}} + \bar{\rho}_{1h(m+1)}\right] \times \left[q_{h(m+1)} - q^*_{h(m+1)}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[-\rho^*_{2ij} + \rho^*_{2ij} + \frac{\partial c_i(Q^{2*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{ij}} + \frac{\partial \hat{c}_{ij}(q^*_{ij})}{\partial q_{ij}} + \alpha_{ij}\gamma^*_i - \eta^*_j\right] \times \left[q_{ij} - q^*_{ij}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(q^*_{i(n+1)})}{\partial q_{i(n+1)}} + \bar{\rho}_{2i(n+1)} + \alpha_{i(n+1)}\gamma^*_i\right] \times \left[q_{i(n+1)} - q^*_{i(n+1)}\right]$$

$$+\sum_{i=1}^{m}\left[\sum_{h=1}^{r} q^*_{hi} - \sum_{j=1}^{n+1}\alpha_{ij}q^*_{ij}\right] \times [\gamma_i - \gamma^*_i]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[-\rho^*_{3jk} + \rho^*_{3jk} + \frac{\partial c_j(Q^{3*})}{\partial q_{jk}} + \frac{\partial c_{jk}(q^*_{jk})}{\partial q_{jk}} + \hat{c}_{jk}(q^*_{jk}) + \beta_{jk}\eta^*_j - \rho^*_{4k}\right] \times \left[q_{jk} - q^*_{jk}\right]$$

$$+\sum_{j=1}^{m}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(q^*_{j(o+1)})}{\partial q_{j(o+1)}} + \beta_{j(o+1)}\eta^*_j + \bar{\rho}_{3j(o+1)}\right] \times \left[q_{j(o+1)} - q^*_{j(o+1)}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q^*_{ij} - \sum_{k=1}^{o+1}\beta_{jk}q^*_{jk}\right] \times \left[\eta_j - \eta^*_j\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n} q^*_{jk} - d_k(\rho^*_4)\right] \times [\rho_{4k} - \rho^*_{4k}] \geq 0, \quad \forall (Q^1, Q^2, \gamma, Q^3, \eta, \rho_4) \in \mathcal{K}, \qquad (24a)$$

which, in turn, can be rewritten as:

$$\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\frac{\partial c_{hi}(q^*_{hi})}{\partial q_{hi}} + \rho^*_{1hi}\right] \times [q_{hi} - q^*_{hi}]$$

$$+\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q^*_{h(m+1)})}{\partial q_{h(m+1)}} + \bar{\rho}_{1h(m+1)}\right] \times \left[q_{h(m+1)} - q^*_{h(m+1)}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{ij}} - \rho^*_{2ij} + \alpha_{ij}\gamma^*_i\right] \times \left[q_{ij} - q^*_{ij}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(q^*_{i(n+1)})}{\partial q_{i(n+1)}} + \bar{\rho}_{2i(n+1)} + \alpha_{i(n+1)}\gamma^*_i\right] \times \left[q_{i(n+1)} - q^*_{i(n+1)}\right]$$

$$+\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\frac{\partial \hat{c}_{hi}(q^*_{hi})}{\partial q_{hi}} - \rho^*_{1hi} - \gamma^*_i\right] \times [q_{hi} - q^*_{hi}] + \sum_{i=1}^{m}\left[\sum_{h=1}^{r} q^*_{hi} - \sum_{j=1}^{n+1}\alpha_{ij}q^*_{ij}\right] \times [\gamma_i - \gamma^*_i]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial \hat{c}_{ij}(q^*_{ij})}{\partial q_{ij}} + \rho^*_{2ij} - \eta^*_j\right] \times \left[q_{ij} - q^*_{ij}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{jk}} + \frac{\partial c_{jk}(q^*_{jk})}{\partial q_{jk}} + \beta_{jk}\eta^*_j - \rho^*_{3jk}\right] \times \left[q_{jk} - q^*_{jk}\right]$$

$$+\sum_{j=1}^{m}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(q^*_{j(o+1)})}{\partial q_{j(o+1)}} + \beta_{j(o+1)}\eta^*_j + \bar{\rho}_{3j(o+1)}\right] \times \left[q_{j(o+1)} - q^*_{j(o+1)}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q^*_{ij} - \sum_{k=1}^{o+1}\beta_{jk}q^*_{jk}\right] \times \left[\eta_j - \eta^*_j\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\rho^*_{3jk} + \hat{c}_{jk}(q^*_{jk}) - \rho^*_{4k}\right] \times \left[q_{jk} - q^*_{jk}\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n} q^*_{jk} - d_k(\rho^*_4)\right] \times [\rho_{4k} - \rho^*_{4k}] \geq 0, \quad \forall (Q^{1*}, Q^{2*}, \gamma^*, Q^{3*}, \eta^*, \rho^*_4) \in \mathcal{K}. \qquad (24b)$$

But inequality (24b) is equivalent to the price and material flow pattern satisfying the sum of the conditions (5), (12), (18), and (22), which is precisely the definition of an e-cycling network equilibrium according to Definition 1. The proof is complete. □

For easy reference in the subsequent sections, variational inequality problem (23) can be rewritten in standard variational inequality form (cf. Nagurney (1999)) as follows: determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{25}$$

where $X \equiv (Q^1, Q^2, \gamma, Q^3, \eta, \rho_4)$ and

$$F(X) \equiv (F_{hi}, F_{ij}, F_i, F_{jk}, F_{\hat{j}}, F_{\hat{k}})_{h=1,\ldots,r;i=1,\ldots,m+1;j=1,\ldots,n+1;k=1,\ldots,o+1;\hat{j}=1,\ldots,n;\hat{k}=1,\ldots,o},$$

with the specific components of $F(X)$ being given by the respective functional terms preceding the multiplication signs in (23). The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

In Section 4, we describe a computational procedure which can be applied to determine the equilibrium material flows: (Q^{1*}, Q^{2*}, Q^{3*}) as well as the equilibrium prices: $(\gamma^*, \eta^*, \rho_4^*)$. Recall that, as discussed earlier, the equilibrium prices associated with the sources of electronic waste, the recyclers, and the processors are endogenous to the model but can be recovered as follows, once the solution to the variational inequality (23) (for the particular problem) has been obtained.

We now discuss how to recover the prices ρ^*_{1hi}, for $h = 1, \ldots, r$ and $i = 1, \ldots, m$ from the solution of variational inequality (23). Note that from (12) we have that for such a $q^*_{hi} > 0$, then $\rho^*_{1hi} = \frac{\partial \hat{c}_{hi}(q^*_{hi})}{\partial q_{hi}} - \gamma^*_i$.

The prices ρ^*_{2ij}, in turn, can be obtained for an i, j with $i = 1, \ldots, m$; $j = 1, \ldots, n$ such that $q^*_{ij} > 0$ by setting (cf. (12)) $\rho^*_{2ij} = \left[\frac{\partial c_j(Q^{2*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^*_{ij})}{\partial q_{ij}} + \alpha_{ij}\gamma^*_i\right]$, or, equivalently (cf. (18)), to $\eta^*_j - \left[\frac{\partial \hat{c}_{ij}(q^*_{ij})}{\partial q_{ij}}\right]$.

Finally, the prices ρ^*_{3jk} for $j = 1, \ldots, n$ and $k = 1, \ldots, o$ with $q^*_{jk} > 0$ can be obtained by setting (cf. (22)) $\rho^*_{3jk} = \rho^*_{4k} - \hat{c}_{jk}(q^*_{jk})$.

In this section, we have proposed an equilibrium framework for the formulation of e-cycling network problems for the purpose of reverse supply chain management since we believe that the concept of equilibrium provides a valuable benchmark against which existing material flows between tiers and the prices at different tiers of the e-cycling network can be compared. Moreover, the equilibrium concept provides a generalization of an optimization one and allows for not only the capture of cooperation between tiers of decision-makers but also the possibility of competition within a tier. Of course, depending upon the particular application under study, the e-cycling network depicted in Figure 1 may be simplified accordingly.

3. Qualitative Properties

In this section, we provide some qualitative properties of the function F that enters variational inequality (25), which are needed to establish convergence of the algorithmic scheme in Section 4. In addition, we derive some theoretical properties of the solution of (25).

We note that the material flows in the e-cycling network are bounded since we assume a fixed (and finite) amount of electronic wastes at each of the sources. The prices, however, do not lie in a compact set. Hence, according to the standard theory of variational inequalities (cf. Nagurney (1999)) one could impose either a coercivity condition on the demand functions or a boundedness condition to guarantee the existence of the demand prices.

We now establish qualitative properties of the function $F(X)$ that enters the variational inequality problem (cf. (25)), as well as uniqueness of the equilibrium pattern.

Theorem 2: Monotonicity

Assume that the recycling cost functions c_i; $i = 1, ..., m$, and the processing cost functions c_j; $j = 1, ..., n$ are convex functions; the transaction cost functions c_{hi}; $h = 1, ..., r$; $i = 1, ..., m+1$, c_{ij}; $i = 1, ..., m$, $j = 1, ..., n+1$, c_{jk}; $j = 1, ..., n$;$k = 1, ..., o$, and $\hat{c}_{hi}$; $h = 1, ..., r; i = 1, \ldots, m$ functions are convex, and that the $\hat{c}_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, $\hat{c}_{jk}$; $j = 1, ..., n$; $k = 1, ..., o+1$ functions are monotone increasing, whereas the demand functions d_k; $k = 1, ..., o$ functions are monotone decreasing functions of the prices, for all h, i, j, and k. Then the vector function $F(X)$ that enters the variational inequality (25) is monotone, that is,

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{26}$$

Proof: From the definition of $F(X)$, the left-hand side of inequality (26) is:

$$\sum_{h=1}^{r} \sum_{i=1}^{m} \left[\frac{\partial c_{hi}(q_{hi}^{'})}{\partial q_{hi}} + \frac{\partial \hat{c}_{hi}(q_{hi}^{'})}{\partial q_{hi}} - \gamma_i^{'} - (\frac{\partial c_{hi}(q_{hi}^{''})}{\partial q_{hi}} + \frac{\partial \hat{c}_{hi}(q_{hi}^{''})}{\partial q_{hi}} - \gamma_i^{''}) \right] \times \left[q_{hi}^{'} - q_{hi}^{''} \right]$$

$$\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q_{h(m+1)}^{'})}{\partial q_{h(m+1)}}+\bar{\rho}_{1h(m+1)}-(\frac{\partial c_{h(m+1)}(q_{h(m+1)}^{''})}{\partial q_{h(m+1)}}+\bar{\rho}_{1h(m+1)})\right]\times\left[q_{h(m+1)}^{'}-q_{h(m+1)}^{''}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_i(Q^{2'})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^{'})}{\partial q_{ij}}+\frac{\partial \hat{c}_{ij}(q_{ij}^{'})}{\partial q_{ij}}+\alpha_{ij}\gamma_i^{'}-\eta_j^{'}\right.$$

$$\left.-(\frac{\partial c_i(Q^{2''})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^{''})}{\partial q_{ij}}+\frac{\partial \hat{c}_{ij}(q_{ij}^{''})}{\partial q_{ij}}+\alpha_{ij}\gamma_i^{''}-\eta_j^{''})\right]\times\left[q_{ij}^{'}-q_{ij}^{''}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2'})}{\partial q_{i(n+1)}}+\frac{\partial c_{i(n+1)}(q_{i(n+1)}^{'})}{\partial q_{i(n+1)}}+\bar{\rho}_{2i(n+1)}+\alpha_{i(n+1)}\gamma_i^{'}\right.$$

$$\left.-(\frac{\partial c_i(Q^{2''})}{\partial q_{i(n+1)}}+\frac{\partial c_{i(n+1)}(q_{i(n+1)}^{''})}{\partial q_{i(n+1)}}+\bar{\rho}_{2i(n+1)}+\alpha_{i(n+1)}\gamma_i^{''})\right]\times\left[q_{i(n+1)}^{'}-q_{i(n+1)}^{''}\right]$$

$$+\sum_{i=1}^{m}\left[\sum_{h=1}^{r}q_{hi}^{'}-\sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^{'}-(\sum_{h=1}^{r}q_{hi}^{''}-\sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^{''})\right]\times\left[\gamma_i^{'}-\gamma_i^{''}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_j(Q^{3'})}{\partial q_{jk}}+\frac{\partial c_{jk}(q_{jk}^{'})}{\partial q_{jk}}+\hat{c}_{jk}(q_{jk}^{'})+\beta_{jk}\eta_j^{'}-\rho_{4k}^{'}\right.$$

$$\left.-(\frac{\partial c_j(Q^{3''})}{\partial q_{jk}}+\frac{\partial c_{jk}(q_{jk}^{''})}{\partial q_{jk}}+\hat{c}_{jk}(q_{jk}^{''})+\beta_{jk}\eta_j^{''}-\rho_{4k}^{''})\right]\times\left[q_{jk}^{'}-q_{jk}^{''}\right]$$

$$+\sum_{j=1}^{m}\left[\frac{\partial c_j(Q^{3'})}{\partial q_{j(o+1)}}+\frac{\partial c_{j(o+1)}(q_{j(o+1)}^{'})}{\partial q_{j(o+1)}}+\beta_{j(o+1)}\eta_j^{'}+\bar{\rho}_{3j(o+1)}\right.$$

$$\left.-(\frac{\partial c_j(Q^{3''})}{\partial q_{j(o+1)}}+\frac{\partial c_{j(o+1)}(q_{j(o+1)}^{''})}{\partial q_{j(o+1)}}+\beta_{j(o+1)}\eta_j^{''}+\bar{\rho}_{3j(o+1)})\right]\times\left[q_{j(o+1)}^{'}-q_{j(o+1)}^{''}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}q_{ij}^{'}-\sum_{k=1}^{o+1}\beta_{jk}q_{jk}^{'}-(\sum_{i=1}^{m}q_{ij}^{''}-\sum_{k=1}^{o+1}\beta_{jk}q_{jk}^{''})\right]\times\left[\eta_j^{'}-\eta_j^{''}\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}q_{jk}^{'}-d_k(\rho_4^{'})-(\sum_{j=1}^{n}q_{jk}^{''}-d_k(\rho_4^{''}))\right]\times\left[\rho_{4k}^{'}-\rho_{4k}^{''}\right]. \tag{27}$$

After simplifying (27), we obtain

$$\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\frac{\partial c_{hi}(q_{hi}^{'})}{\partial q_{hi}}+\frac{\partial \hat{c}_{hi}(q_{hi}^{'})}{\partial q_{hi}}-(\frac{\partial c_{hi}(q_{hi}^{''})}{\partial q_{hi}}+\frac{\partial \hat{c}_{hi}(q_{hi}^{''})}{\partial q_{hi}})\right]\times\left[q_{hi}^{'}-q_{hi}^{''}\right]$$

$$+\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q^{'}_{h(m+1)})}{\partial q_{h(m+1)}}-\frac{\partial c_{h(m+1)}(q^{''}_{h(m+1)})}{\partial q_{h(m+1)}}\right]\times\left[q^{'}_{h(m+1)}-q^{''}_{h(m+1)}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_i(Q^{2'})}{\partial q_{ij}}+\frac{\partial c_{ij}(q^{'}_{ij})}{\partial q_{ij}}+\frac{\partial \hat{c}_{ij}(q^{'}_{ij})}{\partial q_{ij}}-(\frac{\partial c_i(Q^{2''})}{\partial q_{ij}}+\frac{\partial c_{ij}(q^{''}_{ij})}{\partial q_{ij}}+\frac{\partial \hat{c}_{ij}(q^{''}_{ij})}{\partial q_{ij}})\right]\times\left[q^{'}_{ij}-q^{''}_{ij}\right]$$

$$+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2'})}{\partial q_{i(n+1)}}+\frac{\partial c_{i(n+1)}(q^{'}_{i(n+1)})}{\partial q_{i(n+1)}}-(\frac{\partial c_i(Q^{2''})}{\partial q_{i(n+1)}}+\frac{\partial c_{i(n+1)}(q^{''}_{i(n+1)})}{\partial q_{i(n+1)}})\right]\times\left[q^{'}_{i(n+1)}-q^{''}_{i(n+1)}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_j(Q^{3'})}{\partial q_{jk}}+\frac{\partial c_{jk}(q^{'}_{jk})}{\partial q_{jk}}+\hat{c}_{jk}(q^{'}_{k})-(\frac{\partial c_j(Q^{3''})}{\partial q_{jk}}+\frac{\partial c_{jk}(q^{''}_{jk})}{\partial q_{jk}}+\hat{c}_{jk}(q^{''}_{jk}))\right]\times\left[q^{'}_{jk}-q^{''}_{jk}\right]$$

$$+\sum_{j=1}^{m}\left[\frac{\partial c_j(Q^{3'})}{\partial q_{j(o+1)}}+\frac{\partial c_{j(o+1)}(q^{'}_{j(o+1)})}{\partial q_{j(o+1)}}-(\frac{\partial c_j(Q^{3''})}{\partial q_{j(o+1)}}+\frac{\partial c_{j(o+1)}(q^{''}_{j(o+1)})}{\partial q_{j(o+1)}})\right]\times\left[q^{'}_{j(o+1)}-q^{''}_{j(o+1)}\right]$$

$$+\sum_{k=1}^{o}\left[-d_k(\rho^{'}_4)+d_k(\rho^{''}_4))\right]\times\left[\rho^{'}_{4k}-\rho^{''}_{4k}\right]. \quad (28)$$

Since the c_i, c_j, c_{hi}, c_{ij}, c_{jk}, and $\hat{c}_{hi}$ functions are assumed to be convex, the second derivatives of those functions are positive semidefinite (see also Bazaraa, Sherali, and Shetty (1993)). Thus, these functions are monotone (see Nagurney (1999) Theorem 1.7). In addition, since $\hat{c}_{ij}$ and $\hat{c}_{jk}$ are monotone increasing functions, they are also monotone. Hence, the F_{hi}, F_{ij}, and F_{jk} are monotone. Finally, since -d_k are monotone increasing functions, we have that the F_i, F_j, and F_k are also monotone. Hence, we can conclude that the left hand-side of (28) is greater than or equal to zero. □

Theorem 3: Strict Monotonicity

Assume all the conditions of Theorem 2. In addition, suppose that one of the families of convex functions c_i; $i = 1, ..., m$; c_j; $j = 1, ..., n$; c_{hi}; $h = 1, ..., r$; $i = 1, ..., m+1$; c_{ij}; $i = 1, ..., m$; $j = 1, ..., n+1$; c_{jk}; $j = 1, ..., n$; $k = 1, ..., o+1$, and $\hat{c}_{hi}$; $h = 1, ..., r$; $i = 1, \ldots, m$, is a family of strictly convex functions. Suppose also that $\hat{c}_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, $\hat{c}_{jk}$; $j = 1, ..., n$; $k = 1, ..., o$, and -d_k; $k = 1, ..., o$ are strictly monotone. Then, the vector function $F(X)$ that enters the variational inequality (25) is strictly monotone for any X', X'' such that $X' \neq X''$, that is,

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0, \quad \forall X', X'' \in \mathcal{K}, \ X' \neq X''. \quad (29)$$

Theorem 4: Uniqueness

Assuming the conditions of Theorem 3, there must be a unique shipment pattern (Q^{1*}, Q^{2*}, Q^{3*})*, and a unique price vector* ρ_4^* *satisfying the equilibrium conditions of the multitiered e-cycling network. In other words, if the variational inequality (25) admits a solution, then that is the only solution in* $(Q^{1*}, Q^{2*}, Q^{3*}, \rho_4^*)$.

Proof: Under the strict monotonicity result of Theorem 3, uniqueness follows from the standard variational inequality theory (cf. Kinderlehrer and Stampacchia (1980)). □

Theorem 5: Lipschitz Continuity

The function that enters the variational inequality problem (25) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \textit{ where } L > 0, \tag{30}$$

under the following conditions: (i). Each c_i*;* $i = 1, ..., m$*,* c_j*;* $j = 1, ..., n$ *has a bounded second-order derivative; (ii). The* c_{hi}*,* c_{ij}*,* c_{jk}*, and* $\hat{c}_{hi}$ *functions have bounded second-order derivatives, for all* h*,* i*,* j*, and* k*; (iii). The* $\hat{c}_{ij}$*,* $\hat{c}_{jk}$ *and -*d_k *functions have bounded first-order derivatives for all* i*,* j*, and* k*.*

Proof: The result is directed by applying a mid-value theorem from calculus to the vector function $F(X)$ that enters the variational inequality problem (25). □

We emphasize that the conditions that guarantee monotonicity of the function F are not unreasonable since, for example, convex functions include linear functions, as a special case, and such functions have been used in e-cycling applications by, amongst others, Sodhi and Reimer (2001). Moreover, it is commonly assumed that demand functions are monotonically decreasing functions of the prices. Finally, assuming that the transaction cost functions, from the perspective of the consumers, are monotone increasing functions of the volumes of the flow has been assumed in many spatial price equilibrium problems (see, e.g., Nagurney (1999)).

4. The Algorithm

In this section, we propose the algorithm to compute the variational inequality (25). The algorithm we use is the modified projection method of Korpelevich (1977). This algorithm requires only monotonicity of $F(X)$ and with Lipschitz continuity condition holding. The statement of algorithm in the context of our model is as follows, where τ denotes an iteration counter:

Step 0: Initialization Step

Set $(Q^{10}, Q^{20}, \gamma^0, Q^{30}, \eta^0, \rho_4^0) \in \mathcal{K}$. Let $\tau = 1$ and set δ such that $0 < \delta \le \frac{1}{L}$, where L is the Lipschitz constant for the problem (cf. (30)).

Step 1: Computation

Compute $(\tilde{Q}^{1\tau}, \tilde{Q}^{2\tau}, \tilde{\gamma}^{\tau}, \tilde{Q}^{3\tau}, \tilde{\eta}^{\tau}, \tilde{\rho}_4^{\tau}) \in \mathcal{K}$ by solving the variational inequality problem:

$$\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\tilde{q}_{hi}^{\tau} + \delta(\frac{\partial c_{hi}(q_{hi}^{\tau-1})}{\partial q_{hi}} + \frac{\partial \hat{c}_{hi}(q_{hi}^{\tau-1})}{\partial q_{hi}} - \gamma_i^{\tau-1}) - q_{hi}^{\tau-1}\right] \times [q_{hi} - \tilde{q}_{hi}^{\tau}]$$

$$+\sum_{h=1}^{r}\left[\tilde{q}_{h(m+1)}^{\tau} + \delta(\frac{\partial c_{h(m+1)}(q_{h(m+1)}^{\tau-1})}{\partial q_{h(m+1)}} + \bar{\rho}_{1h(m+1)}) - q_{h(m+1)}^{\tau-1}\right] \times \left[q_{h(m+1)} - \tilde{q}_{h(m+1)}^{\tau}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\tilde{q}_{ij}^{\tau} + \delta(\frac{\partial c_i(Q^{2\tau-1})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^{\tau-1})}{\partial q_{ij}} + \frac{\partial \hat{c}_{ij}(q_{ij}^{\tau-1})}{\partial q_{ij}} + \alpha_{ij}\gamma_i^{\tau-1} - \eta_j^{\tau-1}) - q_{ij}^{\tau-1}\right] \times \left[q_{ij} - \tilde{q}_{ij}^{\tau}\right]$$

$$+\sum_{i=1}^{m}\left[\tilde{q}_{i(n+1)}^{\tau} + \delta(\frac{\partial c_i(Q^{2\tau-1})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(q_{i(n+1)}^{\tau-1})}{\partial q_{i(n+1)}} + \alpha_{i(n+1)}\gamma_i^{\tau-1} + \bar{\rho}_{2i(n+1)}) - q_{i(n+1)}^{\tau-1}\right]$$

$$\times \left[q_{i(n+1)} - \tilde{q}_{i(n+1)}^{\tau}\right] + \sum_{i=1}^{m}\left[\tilde{\gamma}_i{}^{\tau} + \delta(\sum_{h=1}^{r} q_{hi}^{\tau-1} - \sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^{\tau-1}) - \gamma_i^{\tau-1}\right] \times [\gamma_i - \tilde{\gamma}_i^{\tau}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\tilde{q}_{jk}^{\tau} + \delta(\frac{\partial c_j(Q^{3\tau-1})}{\partial q_{jk}} + \frac{\partial c_{jk}(q_{jk}^{\tau-1})}{\partial q_{jk}} + \hat{c}_{jk}(q_{jk}^{3\tau-1}) + \beta_{jk}\eta_j^{\tau-1} - \rho_{4k}^{\tau-1}) - q_{jk}^{\tau-1}\right] \times \left[q_{jk} - \tilde{q}_{jk}^{\tau}\right]$$

$$+\sum_{j=1}^{n}\left[\tilde{q}_{j(o+1)}^{\tau} + \delta(\frac{\partial c_j(Q^{3\tau-1})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(q_{j(o+1)}^{\tau-1})}{\partial q_{j(o+1)}} + \beta_{j(o+1)}\eta_j^{\tau-1} + \bar{\rho}_{3j(o+1)}) - q_{j(o+1)}^{\tau-1}\right]$$

$$\times \left[q_{j(o+1)} - \tilde{q}^{\tau}_{j(o+1)}\right] + \sum_{j=1}^{n} \left[\tilde{\eta}^{\tau}_{j} + \delta(\sum_{i=1}^{m} q^{\tau-1}_{ij} - \sum_{k=1}^{o+1} \beta_{jk} q^{\tau-1}_{jk}) - \eta^{\tau-1}_{j}\right] \times \left[\eta_j - \tilde{\eta}^{\tau}_{j}\right]$$

$$+ \sum_{k=1}^{o} \left[\tilde{\rho}^{\tau}_{4k} + \delta(\sum_{j=1}^{n} q^{\tau-1}_{jk} - d_k(\rho^{\tau}_{4})) - \rho^{\tau-1}_{4k}\right] \times [\rho_{4k} - \tilde{\rho}^{\tau}_{4k}] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, Q^3, \eta, \rho_4) \in \mathcal{K}. \tag{31}$$

Step 2: Adaptation

Compute $(Q^{1\tau}, Q^{2\tau}, \gamma^{\tau}, Q^{3\tau}, \eta^{\tau}, \rho^{\tau}_{4}) \in \mathcal{K}$ by solving the variational inequality problem:

$$\sum_{h=1}^{r} \sum_{i=1}^{m} \left[q^{\tau}_{hi} + \delta(\frac{\partial c_{hi}(\tilde{q}^{\tau-1}_{hi})}{\partial q_{hi}} + \frac{\partial \hat{c}_{hi}(\tilde{q}^{\tau-1}_{hi})}{\partial q_{hi}} - \tilde{\gamma}^{\tau-1}_{i}) - q^{\tau-1}_{hi}\right] \times [q_{hi} - q^{\tau}_{hi}]$$

$$+ \sum_{h=1}^{r} \left[q^{\tau}_{h(m+1)} + \delta(\frac{\partial c_{h(m+1)}(\tilde{q}^{\tau-1}_{h(m+1)})}{\partial q_{h(m+1)}} + \bar{\rho}_{1h(m+1)}) - q^{\tau-1}_{h(m+1)}\right] \times \left[q_{h(m+1)} - q^{\tau}_{h(m+1)}\right]$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \left[q^{\tau}_{ij} + \delta(\frac{\partial c_i(\tilde{Q}^{2\tau-1})}{\partial q_{ij}} + \frac{\partial c_{ij}(\tilde{q}^{\tau-1}_{ij})}{\partial q_{ij}} + \alpha_{ij} \tilde{\gamma}^{\tau-1}_{i} + \frac{\partial \hat{c}_{ij}(\tilde{q}^{\tau-1}_{ij})}{\partial q_{ij}} - \tilde{\eta}^{\tau-1}_{j}) - q^{\tau-1}_{ij}\right] \times \left[q_{ij} - q^{\tau}_{ij}\right]$$

$$+ \sum_{i=1}^{m} \left[q^{\tau}_{i(n+1)} + \delta(\frac{\partial c_i(\tilde{Q}^{2\tau-1})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(\tilde{q}^{\tau-1}_{i(n+1)})}{\partial q_{i(n+1)}} + \alpha_{ij} \tilde{\gamma}^{\tau-1}_{i} + \bar{\rho}_{2i(n+1)}) - q^{\tau-1}_{i(n+1)}\right]$$

$$\times \left[q_{i(n+1)} - q^{\tau}_{i(n+1)}\right] + \sum_{i=1}^{m} \left[{\gamma_i}^{\tau} + \delta(\sum_{h=1}^{r} \tilde{q}^{\tau-1}_{hi} - \sum_{j=1}^{n+1} \alpha_{ij} \tilde{q}^{\tau-1}_{ij}) - \gamma^{\tau-1}_{i}\right] \times [\gamma_i - \gamma^{\tau}_{i}]$$

$$+ \sum_{j=1}^{n} \sum_{k=1}^{o} \left[q^{\tau}_{jk} + \delta(\frac{\partial c_j(\tilde{Q}^{3\tau-1})}{\partial q_{jk}} + \frac{\partial c_{jk}(\tilde{q}^{\tau-1}_{jk})}{\partial q_{jk}} + \beta_{jk} \tilde{\eta}^{\tau-1}_{j} + \hat{c}_{jk}(\tilde{q_{jk}}^{\tau-1}) - \tilde{\rho}^{\tau-1}_{4k}) - q^{\tau-1}_{jk}\right] \times \left[q_{jk} - q^{\tau}_{jk}\right]$$

$$+ \sum_{j=1}^{n} \left[q^{\tau}_{j(o+1)} + \delta(\frac{\partial c_j(\tilde{Q}^{3\tau-1})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(\tilde{q}^{\tau-1}_{j(o+1)})}{\partial q_{j(o+1)}} + \beta_{j(o+1)} \tilde{\eta}^{\tau-1}_{j} + \bar{\rho}_{3j(o+1)}) - q^{\tau-1}_{j(o+1)}\right]$$

$$\times \left[q_{j(o+1)} - q^{\tau}_{j(o+1)}\right] + \sum_{j=1}^{n} \left[\eta^{\tau}_{j} + \delta(\sum_{i=1}^{m} \tilde{q}^{\tau-1}_{ij} - \sum_{k=1}^{o+1} \beta_{jk} \tilde{q}^{\tau-1}_{jk}) - \eta^{\tau-1}_{j}\right] \times \left[\eta_j - \eta^{\tau}_{j}\right]$$

$$+ \sum_{k=1}^{o} \left[\rho^{\tau}_{4k} + \delta(\sum_{j=1}^{n} \tilde{q}^{\tau-1}_{jk} - d_k(\tilde{\rho}^{\tau}_{4})) - \rho^{\tau-1}_{4k}\right] \times [\rho_{4k} - \rho^{\tau}_{4k}] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, Q^3, \eta, \rho_4) \in \mathcal{K}. \tag{32}$$

Step 3: Convergence Verification

If $|q_{hi}^{\tau}-q_{hi}^{\tau-1}| \leq \epsilon$, $|q_{ij}^{\tau}-q_{ij}^{\tau-1}| \leq \epsilon$, $|q_{i(n+1)}^{\tau}-q_{i(n+1)}^{\tau-1}| \leq \epsilon$, $|q_{jk}^{\tau}-q_{jk}^{\tau-1}| \leq \epsilon$, $|q_{j(o+1)}^{\tau}-q_{j(o+1)}^{\tau-1}| \leq \epsilon$, $|\gamma_i^{\tau}-\gamma_i^{\tau-1}| \leq \epsilon$, $|\eta_j^{\tau}-\eta_j^{\tau-1}| \leq \epsilon$, and $|\rho_{4k}^{\tau}-\rho_{4k}^{\tau-1}| \leq \epsilon$ for all $h = 1, \cdots, r$; $i = 1, \cdots, m+1$; $j = 1, \cdots, n$; $k = 1, \cdots, o$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

We now state the convergence result for the modified projection method for this model.

Theorem 6: Lipschitz Continuity

Assume that the function that enters the variational inequality (25) satisfies the conditions in Theorem 2 and in Theorem 5 and that a solution exists. Then the modified projection method descrived above converges to the solution of the variational inequality (25).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (25), provided that the function $F(X)$ is monotone and Lipschitz continuous and that a solution exists. Monotonicity follows from Theorem 2. Lipschitz continuous follows from Theorem 5. □

We now, for completeness, discuss the simplicity of the above computational procedure in the context of the model. Notably, the algorithm takes advantage of the network structure of the problem and allows for the explicit and exact computation of the material flows (except for the top tiered ones) as well as the prices. The top-tiered material flows are computed explicitly but not with specific formulae, since these flows must lie in the feasible set K, whereas the others, through our formulation, are only subject to nonnegativity constraints.

Indeed, the $\tilde{q}_{hi}^{\tau}$; $i = 1, \ldots, m$, at a given iteration τ, and satisfying (31) for a fixed source of electronic waste h, are simply the solution to the following quadratic programming problem:

$$\text{Minimize} \quad \sum_{i=1}^{m+1} \tilde{q}_{hi}^2 + \tilde{h}_{hi}, \tag{33}$$

subject to constraints (3) and (4), where $\tilde{h}_{hi}$ denotes the fixed term following the plus sign in the first term in (31) for $i = 1, \ldots, m$ and $\tilde{h}_{h(m+1)}$ denotes the corresponding term in the second line in (31). In Section 5, we discuss further the solution of this problem in the context of numerical examples.

For definiteness, we now demonstrate how the remainder of the material flows and prices satisfying (31) can be computed exactly and in closed form; similar expressions can be derived for the flows and prices satisfying (32).

Computation of the Material Flows

The precise formulae for the computation of the material flows following (31) are now given.

Computation of Material Flows from Recyclers to the Processors and Landfill

In particular, compute, at iteration τ, the $\tilde{q}^{\tau}_{ij}$s according to:

$$\tilde{q}^{\tau}_{ij} = \max\{0, q^{\tau-1}_{ij} - \delta(\frac{\partial c_i(Q^{2\tau-1})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^{\tau-1}_{ij})}{\partial q_{ij}} + \alpha_{ij}\gamma^{\tau-1}_i + \frac{\partial \hat{c}_{ij}(q^{\tau-1}_{ij})}{\partial q_{ij}} - \eta^{\tau-1}_j)\}, \quad \forall i, j, \tag{34}$$

and the $\tilde{q}^{\tau}_{i(n+1)}$s according to:

$$\tilde{q}^{\tau}_{i(n+1)} = \max\{0, q^{\tau-1}_{i(n+1)} - \delta(\frac{\partial c_i(Q^{2\tau-1})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(q^{\tau-1}_{i(n+1)})}{\partial q_{i(n+1)}} + \alpha_{i(n+1)}\gamma^{\tau-1}_i + \bar{\rho}_{2i(n+1)})\}, \quad \forall i. \tag{35}$$

Computation of Material Flows from Processors to Demand Markets and Landfill

Compute, at iteration τ, the $\tilde{q}^{\tau}_{jk}$s according to:

$$\tilde{q}^{\tau}_{jk} = \max\{0, q^{\tau-1}_{jk} - \delta(\frac{\partial c_j(Q^{3\tau-1})}{\partial q_{jk}} + \frac{\partial c_{jk}(q^{\tau-1}_{jk})}{\partial q_{jk}} + \beta_{jk}\eta^{\tau-1}_j + \hat{c}_{jk}(q^{\tau-1}_{jk}) - \rho^{\tau-1}_{4k})\}, \quad \forall j, k, \tag{36}$$

and the $\tilde{q}^{\tau}_{j(o+1)}$s according to:

$$\tilde{q}^{\tau}_{j(o+1)} = \max\{0, q^{\tau-1}_{j(o+1)} - \delta(\frac{\partial c_j(Q^{3\tau-1})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(q^{\tau-1}_{j(o+1)})}{\partial q_{j(o+1)}} + \beta_{j(o+1)}\eta^{\tau-1}_j + \bar{\rho}_{3j(o+1)})\}, \quad \forall j, k. \tag{37}$$

Computation of the Prices

The prices, in turn, can also be computed (cf. (31)) as follows:

Computation of the Shadow Prices at the Recyclers

At iteration τ, compute the $\tilde{\gamma}_i^\tau$s according to:

$$\tilde{\gamma}_i^\tau = \max\{0, \gamma_i^{\tau-1} - \delta(\sum_{h=1}^{r} q_{hi}^{\tau-1} - \sum_{j=1}^{n+1} \alpha_{ij} q_{ij}^{\tau-1})\}, \quad \forall i. \tag{38}$$

Computation of the Shadow Prices at the Processors

Similarly, we can compute the $\tilde{\eta}_j^\tau$s according to:

$$\tilde{\eta}_j^\tau = \max\{0, \eta_j^{\tau-1} - \delta(\sum_{i=1}^{m} q_{ij}^{\tau-1} - \sum_{k=1}^{o+1} \beta_{jk} q_{jk}^{\tau-1})\}, \quad \forall j. \tag{39}$$

Computation of the Demand Market Prices at the Demand Markets

Finally, we can compute at iteration τ the $\tilde{\rho}_{4k}^\tau$s according to:

$$\tilde{\rho}_{4k}^\tau = \max\{0, \rho_{4k}^{\tau-1} - \delta(\sum_{j=1}^{n} q_{jk}^{\tau-1} - d_k(\rho_4^{\tau-1}))\}, \quad \forall k. \tag{40}$$

In the next section, we apply the modified projection method to three sets of examples to compute the equilibrium material flows between tiers as well as the demand market prices and shadow prices.

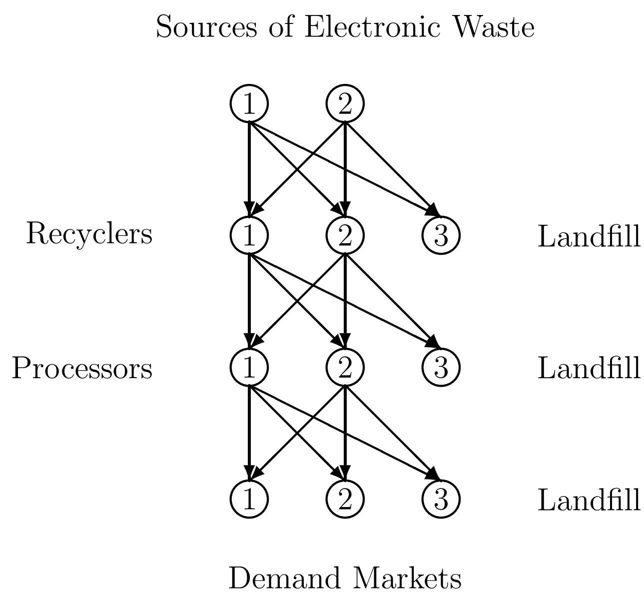

Figure 2: E-Cycling Network for the Numerical Examples

5. Numerical Examples and Discussion

In this section, we present numerical examples which are solved using the modified projection method of the preceding section, along with a discussion.

The modified projection method outlined in Section 4 was implemented in FORTRAN and the computer system used was a Sun system located at the University of Massachusetts at Amherst. The convergence criterion utilized was that the absolute value of the material flows and prices between two successive iterations differed by no more than 10^{-4}. The structure of the e-cycling network for all the examples is depicted in Figure 2. In particular, there were two sources of electronic waste, two recyclers, two processors, and two demand markets, with the option of using a landfill available to the sources of electronic waste, to the recyclers, as well as to the processors.

For all the numerical examples, we set the parameter δ in the modified projection method (cf. Section 4) equal to .1. Clearly, according to Step 0 of the algorithm, the contraction parameter δ should lie in the range $0 < \delta \leq \frac{1}{L}$ and although $\frac{1}{L}$ can be computed it is not always straightforward, so we recommend a "small" δ. Of course, based on increasing experience with particular examples, one may try to increase this parameter, which will typically result in fewer iterations.

The material flows and prices were initialized as follows for all the examples: the (Q^{20}, Q^{30}) and the $(\gamma^0, \eta^0, \rho_4^0)$ were all set to zero. The Q^{10}, in turn, to ensure that constraint (3) was satisfied for all h, was set to $q_{hi} = \frac{S^h}{m}$, for $i = 1, \ldots, m$ and $q_{h(m+1)} = 0$ for each $h = 1, \ldots, r$. In all the examples, we had $r = 2$, $m = 2$, $n = 2$, and $o = 2$.

We solved three sets of examples, with three examples in each set and numbered as Example 1.1, Example 1.2, Example 1.3, and so on, through Example 3.3.

It is important to emphasize the simplicity of the computational procedure in the form of the modified projection method which takes advantage of the network structure of the problem. In particular, in view of the feasible set, which, except for the first tier of material flows, consists of the nonnegative orthant, the materials flows and the prices are computed using explicit simple formulas (as discussed in Section 4). The material flows from the sources, in turn, which must satisfy constraint (3), are computed in our implementation using the exact equilibriation algorithm of Dafermos and Sparrow (1969) which has also been utilized in the context of multitiered financial network problems with intermediation (see, e.g., Nagurney and Ke (2001, 2003)).

The data (both input and output) are reported for each of the examples below.

Set 1

In the first set of examples, the common input data are given in Table 1.

Table 1: **Common Input Data for Set 1 Examples**

Volumes of electronic waste	$S^1 = S^2 = 20$
Transaction cost functions between sources and recyclers (cf. (1))	$c_{hi} = .5q_{hi}^2 + 3.5, h = 1,2; i = 1,2$
Transaction cost functions between sources and the landfill (cf. (1))	$c_{h3} = .5q_{h3}^2 + 2, h = 1,2$
Second-tier landfill fixed prices	$\bar{\rho}_{1h3} = 1, h = 1,2$
Transaction cost functions between recyclers and processors (cf. (6))	$c_{ij} = .5q_{ij}^2 + 5, i = 1,2; j = 1,2$
Transaction cost functions between recyclers and the landfill (cf. (6))	$c_{i3} = .5q_{i3}^2 + 3, i = 1,2$
Third-tier landfill fixed prices	$\bar{\rho}_{2i3} = 1, i = 1,2$
Transaction cost functions between recyclers and sources (cf. (7))	$\hat{c}_{hi} = 1.5q_{hi}^2 + 3, h = 1,2; i = 1,2$
Recycling cost functions (cf. (8))	$c_i = \sum_{j=1}^{3} q_{ij}, i = 1,2$
Processing cost functions (cf. after (14))	$c_j = 2\sum_{k=1}^{3} q_{jk}, j = 1,2$
Transaction cost functions between the processors and the demand markets (cf. (13))	$c_{jk} = .5q_{jk}^2 + 1, j = 1,2; k = 1,2$
Transaction cost functions between processors and demand market pair (from the perspective of the consumers at the demand markets) (cf. (19))	$\hat{c}_{jk} = q_{jk} + 1, j = 1,2; k = 1,2$
Fourth-tier landfill fixed prices	$\bar{\rho}_{3j3} = 1, j = 1,2$
Demand functions (cf. after (19))	$d_1 = -2\rho_{41} - 1.5\rho_{42} + 1000$ $d_2 = -2\rho_{42} - 1.5\rho_{41} + 1000$

Example 1.1

Example 1.1 had the input data as described in Table 1 and, although stylized, serves as a baseline from which we can conduct other simulations. In Example 1.1, we assumed that (cf. (10) and (16)) the α_{ij}s and the β_{jk}s were all equal to 1. Thus, the ratio of output material flows to input flows was equal to 1 for both the recyclers and the processors.

The modified projection method converged in 321 iterations and yielded the equilibrium solution reported in the second column Table 2.

Hence, as expected, the solution in material flows and prices was symmetric in that the data for the particular tiered nodes (and corresponding links) was identical. Note that the prices increase as the material flows propagate through the e-cycling network.

Example 1.2

Example 1.2 was constructed from Example 1.1 as follows. The data were identical to the data in Example 1 except now the α_{ij}s were no longer identically equal to 1 but, rather, we had that $\alpha_{ij} = .5$ for $i = 1, 2$; $j = 1, 2, 3$. This means that as the electronic material arrives at the recyclers it is then converted (in a proportionate manner) into recycled materials for further processing. Hence, in this example, in contrast to Example 1.1, the ratio of output material flows to input material flows was 2 at each recycler but 1 at each processor.

The modified projection method converged in 850 iterations and yielded the equilibrium pattern given in the third column in Table 2. Note that in this example, as in Example 1.1, none of the material flows were sent to the landfills. Also, note that since there is more material flow at each processor, the shadow price at each processor is lower than the shadow price at each processor in Example 1.1.

Example 1.3

Example 1.3, which is the final example in this set, was constructed from Example 1.2 and had the identical data except that now the parameters associated with the flows leaving the processor nodes were no longer equal to one but, instead, we had that $\beta_{jk} = .25$ for $j = 1, 2$ and $k = 1, 2, 3$. Hence, the ratio of material output flows to input flows at the recyclers remained 1 but now the ratio of output flows to input flows at the processors was 2 for each processor (as compared to Example 1.2).

The modified projection method converged in 241 iterations and yielded the equilibrium material flows and prices reported in the fourth column in Table 2.

Note that now, in contrast to the two preceding examples, there is a positive flow of electronic waste material from the sources of electronic waste to the landfill. This is due, in part, to the production requirements for the recycled and processed waste. Also, in referring back to variational inequality (23) we note that since the q^*_{ij}s in Example 1.2 and Example 1.3 did not significantly differ, hence, the corresponding (cf. (25)) F_{ij} terms also did not differ significantly. We can thus infer that the $\alpha_{ij}\gamma^*_i - \eta^*_j$s would have to be approximately equal in both of these examples, which, indeed, they are, with a value of approximately -26. In particular, for Example 1.2, we have that $\alpha_{ij}\gamma^*_i - \eta^*_j$=.5(372.47) $-$ 212.24 for all i, j whereas for Example 1.3, we have that $\alpha_{ij}\gamma^*_i - \eta^*_j$=.5(40.67) $-$ 45.40.

Table 2: **Equilibrium Solutions to Set 1 Examples (Baseline Set)**

Set 1 Examples			
Equilibrium Solution	**Example 1.1** $\alpha_{ij} = 1, \beta_{jk} = 1,$ $\forall i, j, k$	**Example 1.2** $\alpha_{ij} = .5, \beta_{jk} = 1,$ $\forall i, j, k$	**Example 1.3** $\alpha_{ij} = .5$, $\beta_{jk} = .25,$ $\forall i, j, k$
Material Flows from Sources to Second Tier (Recyclers and Landfill)			
$q^*_{hi}, h = 1, 2; i = 1, 2$	10.00	10.00	9.53
$q^*_{h3}, h = 1, 2$	0.00	0.00	.95
Material Flows from Recyclers to Third Tier (Processors and Landfill)			
$q^*_{ij}, i = 1, 2; j = 1, 2$	10.00	20.00	19.06
$q^*_{i3}, i = 1, 2$	0.00	0.00	0.00
Material Flows from Processors to Bottom Tier (Demand Markets and Landfill)			
$q^*_{jk}, j = 1, 2; k = 1, 2$	10.00	20.00	76.26
$q^*_{j3}, j = 1, 2$	0.00	0.00	0.00
Shadow Prices at the Recyclers			
$\gamma^*_i, i = 1, 2$	231.97	372.47	40.67
Shadow Prices at the Processors			
$\eta^*_j, j = 1, 2$	247.97	212.24	45.40
Demand Market Prices at the Demand Markets			
$\rho^*_{4k}, k = 1, 2$	279.99	274.28	242.14

Set 2

We now describe the examples in the second set. The equilibrium solutions for all examples in this set are given in Table 3.

Example 2.1

The first example in the second set, Example 2.1, was constructed as follows. The data were identical to that in Example 1.1, but now we decreased the demand for the products at the demand markets so that the new demand functions were:

$$d_1 = -2\rho_{41} - 1.5\rho_{42} + 100, \quad d_2 = -2\rho_{42} - 1.5\rho_{41} + 100.$$

The modified projection method converged in 117 iterations and yielded the solution given in column 2 of Table 3. Hence, due to the decreased demand for the recycled/processed products, there was a sizeable amount of electronic waste transported to the landfill from the sources of electronic waste.

Note that now, unlike the situation in Example 1.1, in which the demand at each of the demand markets was 20. The demand for each of the products was now only 7.

Example 2.2

The second example in Set 2, Example 2.2, was constructed from Example 2.1 and had the same data except (as we did for Example 1.2) that we now set the α_{ij}s equal to .5 for $i = 1, 2$ and $j = 1, 2, 3$. Hence, as in Example 1.2, the ratio of output material flows to input material flows at each recycler was 2. The modified projection method converged in 86 iterations and yielded the equilibrium solution given in column 3 of Table 3. Observe that now, due to the decreased demand for the products (unlike in Example 1.2), a sizeable amount of the electronic waste from the sources is now deposited in the landfill. The demand for the final products is 9 for both demand markets. Note also that the shadow prices associated with the recyclers are now equal to zero and this is due to the fact (see also discussion following (12)) that constraint (10) holds as a strict inequality for both recyclers. In other words, there is an excess supply of the product at each recycler.

Table 3: **Equilibrium Solutions to Set 2 Examples (with Demands Functions Distinct from Set 1's Examples)**

Set 2 Examples			
Equilibrium Solution	**Example 2.1** $\alpha_{ij} = 1, \beta_{jk} = 1$, $\forall i, j, k$	**Example 2.2** $\alpha_{ij} = .5, \beta_{jk} = 1$, $\forall i, j, k$	**Example 2.3** $\alpha_{ij} = .5, \beta_{jk} = .25$, $\forall i, j, k$
Material Flows from Sources to Second Tier (Recyclers and Landfill)			
$q^*_{hi}, h = 1, 2; i = 1, 2$	3.50	2.75	2.75
$q^*_{h3}, h = 1, 2$	13.00	14.50	14.50
Material Flows from Recyclers to Third Tier (Processors and Landfill)			
$q^*_{ij}, i = 1, 2; j = 1, 2$	3.50	4.50	1.72
$q^*_{i3}, i = 1, 2$	0.00	0.00	0.00
Material Flows from Processors to Bottom Tier (Demand Markets and Landfill)			
$q^*_{jk}, j = 1, 2; k = 1, 2$	3.50	4.50	6.90
$q^*_{j3}, j = 1, 2$	0.00	0.00	0.00
Shadow Prices at the Recyclers			
$\gamma^*_i, i = 1, 2$	4.52	0.00	0.00
Shadow Prices at the Processors			
$\eta^*_j, j = 1, 2$	14.03	10.49	7.71
Demand Market Prices at the Demand Markets			
$\rho^*_{4k}, k = 1, 2$	26.56	26.00	24.63

Example 2.3

The third and final example in Set 2, Example 2.3, had data identical to that in Example 2.2 except now (as we did for Example 1.3) we altered the β_{jk}s so that $\beta_{jk} = .25$ for $j = 1, 2$ and $k = 1, 2, 3$.

The modified projection method converged in 112 iterations and yielded the equilibrium solution given in column 4 of Table 3. Observe that, as was the case in Example 2.2, the shadow prices associated with the two recyclers are again zero.

Set 3

In the third set of examples, we also solved three problems using the modified projection method. The input data for these examples is described for each particular case below. The computed solutions are given in Table 4.

Example 3.1

In the first example in Set 3, Example 3.1, we used data identical to that of Example 2.1, except that the demand functions were as follows:

$$d_1 = -2\rho_{41} - 1.5\rho_{42} + 150, \quad d_2 = -2\rho_{42} - 1.5\rho_{41} + 100.$$

Hence, unlike the preceding examples, the demand functions associated with the recycled/processed products were now distinct.

The modified projection method converged in 235 iterations and yielded the equilibrium solution reported in column 2 of Table 4. Due to lower demand (relative to that in Example 1.1), electronic waste was shipped to the landfill from both sources of waste.

Example 3.2

Example 3.2 had data identical to that in Example 3.1, except that now we increased the fixed unit prices associated with the sources using the landfill. In particular, we now set $\bar{\rho}_{h3} = 10$ for $h = 1, 2$. Hence, we increased these prices tenfold relative to their values in Example 3.1.

The modified projection method required 215 iterations for convergence and yielded the equilibrium material flow and price pattern reported in column 3 of Table 4. Thus, with the increased unit prices associated with the use of the landfill by the sources, less electronic waste was transported to the landfills (but not significantly less since the demand functions have not changed).

Example 3.3

In the final example in our numerical experiments, Example 3.3, we used the data of Example

Table 4: **Equilibrium Solutions to Set 3 Examples (Demand Functions Distinct at each Market)**

Set 3 Examples			
Equilibrium Solution	**Example 3.1** $\alpha_{ij}=1, \beta_{jk}=1$, $\forall i, j, k$	**Example 3.2** $\alpha_{ij}=1, \beta_{jk}=1$, $\forall i, j, k$	**Example 3.3** $\alpha_{ij}=.5, \beta_{jk}=.25$, $\forall i, j, k$
Material Flows from Sources to Second Tier (Recyclers and Landfill)			
$q^*_{hi}, h=1,2, i=1,2$	5.36	5.87	4.25
$q^*_{h3}, h=1,2$	9.29	8.27	11.50
Material Flows from Recyclers to Third Tier (Processors and Landfill)			
$q^*_{ij}, i=1,2; j=1,2$	5.36	5.87	2.21
$q^*_{i3}, i=1,2$	0.00	0.00	0.00
Material Flows from Processors to Bottom Tier (Demand Markets and Landfill)			
$q^*_{jk}, j=1,2; k=1,2$	10.73	11.75	$q^*_{j1}=16.00; j=1,2$ $q^*_{j2}=1.72, j=1,2$
$q^*_{j3}, j=1,2$	0.00	0.00	0.00
Shadow Prices at the Recyclers			
$\gamma^*_i, i=1,2$	15.64	9.71	0.00
Shadow Prices at the Processors			
$\eta^*_j, j=1,2$	27.00	21.58	8.20
Demand Market Prices at the Demand Markets			
$\rho^*_{4k}, k=1,2$	4.11	5.87	9.22

3.2, but as we had done in Examples 1.3 and 2.3, we set α_{ij}s =.5 for $i=1,2$; $j=1,2,3$ and β_{jk}s=.25 for $j=1,2$ and $k=1,2,3$.

The modified projection method converged in 110 iterations with the equilibrium pattern given in column 4 of Table 4. Observe that the shadow prices at both the recyclers are zero for this example and this is due to the fact that there remains an excess supply of the product at both of the recyclers. In other words, constraint (10) holds as a strict inequality in each case.

Also, note that there was an increase in the use of the landfill at the second tier level relative to that encountered in Example 3.2 with the change in the α_{ij}s and β_{jk} and this is due to the fact that with the new values for these parameters there was an increase in material flow from both the second tier and the third tier relative to the flows in Example 3.2 and with no alteration in the demand functions the excess was transported to the landfill.

These numerical results illustrate the variety of problems that can be solved using the integrated e-cycling framework developed here based on the theory of variational inequalities. In particular, one can evaluate, numerically, the effects of numerous changes to the data of a specific example, in terms of changes in the demand functions, fixed prices, production/recycling coefficients, transaction cost functions, etc. Of course, the examples are stylized but, nevertheless, show the spectrum of problems that can be solved and the robustness of the computational procedure. Moreover, the results and the analysis also emphasize the elegance of the variational inequality formulation (23) from which we were able to obtain additional illumination of the obtained equilibrium patterns. It would be very interesting to further the research through the use of the variational inequality formulation to obtain other sensitivity analysis results as has been done for other network equilibrium problems (see, e.g., Nagurney (1999)). Finally, it would be worthwhile to investigate theoretically how the parameter δ in the modified projection method can be effectively determined a priori to allow for, if possible, the determination of an expected number of iterations for convergence for different ranges of numerical data.

6. Summary and Conclusions

In this paper, we proposed a framework for the formulation, analysis, and computation of solutions to reverse supply chain network problems of electronic waste management and recycling. We formulated the multitiered e-cycling network model consisting of sources of electronic waste, recyclers, processors, and consumers associated with the demand markets for the distinct products. We derived the decision-makers' optimality conditions and provided the governing equilibrium conditions, along with the variational inequality formulation. The compact variational inequality formulation allows one to formulate the complex reverse supply chain network of electronic waste management and recycling to obtain the endogenous equilibrium prices and material flows between tiers. In addition, we provided qualitative properties of the equilibrium electronic waste material flow and price pattern. Finally, we provided numerical results for a spectrum of problems which were solved utilizing the proposed algorithmic scheme for which convergence was also established.

The model presented in this paper can be applied to address a variety of recycling issues associated with recent recycling policy instruments, such as the home appliances recycling law in Japan, or the electronic equipment waste directive in the European Union (cf. Applebaum (2002a, b, c)). The model can also be further extended in several directions. For example, in practice, recyclers need to confront the uncertainty in terms of both the quantities (as well as the quality) of electronic waste that they actually obtain. Moreover, they must adjust their decisions through some form of dynamic process (although our proposed algorithm may be interpreted as a type of adjustment process with an iteration corresponding to a time period). In view of this situation, we can extend the framework presented in this research to a reverse supply chain network model with random supply of electronic wastes associated with the sources (as well as random demands associated with the products at the bottom tier of the network).

Another extension of the research in this paper is to integrate the production and the distribution systems of electronic products. Since many of the recycling policy instruments require that the manufacturers recycle electronic wastes, it is conceivable that the recycling activities in the reverse supply chain network system must be closely related with those in the classical supply chain network. Finally, we emphasize the importance of empirical research. We leave such work for future research.

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