

# Preface to “On a Paradox of Traffic Planning”

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This article is a preface to the translation by Braess, Nagurney, and Wakolbinger of the 1968 paper by Braess, “Über ein Paradoxon aus der Verkehrsplanung” (*Unternehmensforschung* 12 258–268).

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In 1968, Braess published the paper “Über ein Paradoxon aus der Verkehrsplanung” in the journal *Unternehmensforschung*. The paper was inspired by a seminar given by W. Knoedel in Muenster in 1967, when Braess was 29 years old. In the paper, Braess set out to clarify distinct concepts of travel behavior and presented what has since become the renowned Braess paradox. The paper was accepted by the editor of the journal although Braess at that time was unaware of the transportation literature to that date, including the paper of Wardrop (1952) that stated two widely quoted criteria of traffic network utilization that have come to be termed user optimal and system optimal, respectively, based on similar terms introduced by Dafermos and Sparrow (1969). He also was unaware of the book by Beckmann, McGuire, and Winsten (1956), which provided the first rigorous mathematical formulation of the conditions described by Wardrop’s two criteria (pp. 60–67 and 87–94) that allowed for the ultimate solution of the traffic network equilibrium problem in the framework of certain link cost functions that are increasing functions of the flows on the links (Braess 2005). As formulated by Beckmann et al., both problems posited that demand was a function of origin-destination cost, rather than assuming fixed origin-destination flow, as considered by Braess.

The Braess paradox was brought to the attention of the English-speaking community through a short communication by Murchland (1970), who also presented a discussion. The Braess paradox has since captured the imagination and interest of generations of scholars and practitioners, not only in transportation science, but also, more recently, in computer science (cf. Boyce, Mahmassani, and Nagurney 2005 and the references therein). A recent search through the

Web of Science found 95 papers with citations of the original paper by Braess, including 11 in this journal.

Motivated by requests for a translation of the original German article, Dietrich Braess, Anna Nagurney, and Tina Wakolbinger translated the article into English, which follows immediately in this issue.

In this preface, we clarify some of the concepts and terms in the translation, which have been kept as close to the original as possible.

## 1. Remarks and Commentary

The 1968 paper of Braess contained a summary in both German and English; “running times” in the summary corresponds to what is known today as “travel times.” The *optimal* and *critical flows* in the paper are currently called *system optimal* and *user optimal* or *user equilibrium*, respectively. The *total flow* in the paper corresponds to the origin-destination travel demand.

The Braess paper describes two different concepts of traffic network utilization that correspond, respectively, to analogues of system optimization and user optimization. In addition, the paper introduced the example that has come to be known as the Braess paradox.

Section 1 of the paper provides an introduction that emphasizes that realistic traffic network models must take into account that the travel time depends strongly on traffic flow. Moreover, the introduction emphasizes that with flow-dependent travel times, new effects will be encountered and more precise formulations of problems will be required that “distinguish between the flow that will be optimal for all vehicles” and “flow that is achieved if each user attempts to optimize his own route.” The possibility of the paradox in which the extension of the road

network by an additional road causing a redistribution of the flow with concomitant increased travel time is noted.

Section 2 outlines the notation and gives the conservation of flow equations along with the definitions of the link and path travel times and the properties of the link travel time functions.

Section 3 provides analogues of system optimization. As noted, the total flow, or travel demand, is assumed fixed. Braess focuses on a single origin-destination pair and notes that "optimality" could easily be defined for traffic networks with several origins and destinations. He first provides a definition of optimality in which the maximum travel time on any used path is minimal. He notes that according to this definition, the travel time of all drivers is taken into consideration. He then proposes a measure in which the mean value of the travel time (similar to the well-known total cost in path flows for system optimization, but weighted by the inverse of the total flow) should be minimized. He notes that it should be up to the traffic planners as to which specification is more appropriate and that these concepts and the resulting optimal flows "do not differ substantially" from one another. In this section, Braess introduces the Braess network with all five links present and shows how the optimal solution varies as the total flow (demand) increases.

Section 4 provides the definition of a *critical flow* (that is, user-optimal flow) and also shows, through the renowned example, that the critical flow does not always coincide with the optimal flow and that the elimination of a link in a network may improve the distribution of traffic flow. Braess explicitly states that *in unfavorable situations an extension of the road network may lead to increased travel times*. He postulates that each driver attempts to find for himself the most favorable path and obtains the information that is necessary for determining the path. According to Braess (1968), the critical flow satisfies the condition that all destinations connecting an origin-destination pair with nonvanishing (that is, positive) flow would be reached at the same time. This is equivalent to the statement that all utilized paths connecting each origin-destination pair have equal and minimal travel times, as per Wardrop's first criterion. Braess emphasized that optimal paths are determined by their respective travel costs where the costs depend on the length of road, travel time, and other costs. For the sake of clarity, he identified the costs with travel time.

Braess goes on in §5 to establish that in the case of continuous and nondecreasing (user) link cost functions the critical (user-optimal) flows could be obtained as the solution to a convex minimization problem, the result obtained by Beckmann, McGuire,

and Winsten (1956) and later independently in an unpublished master's thesis by Jorgensen (1963). The formulations in subsequent papers by Dafermos and Sparrow (1969) and Bruynooghe, Gilbert, and Sakarovitch (1969) appear to have been initially motivated by Jorgensen's thesis. Braess also provides an existence result in the same section and uses strict monotonicity of the user link cost functions to obtain uniqueness, a condition subsequently utilized in the context of variational inequality formulations of traffic network equilibria; see Smith (1979) and Dafermos (1980).

What is fascinating is that in §5 Braess emphasizes that the characterization of the critical (user-optimal) flows as the solution of a minimization problem is connected with a symmetry in the model. In other words, according to Braess, "roughly speaking, each driver induces the same delay for the other drivers as the other one does for him." He also recognized that travel time would depend on the class or type of vehicle with the "most significant difference" expected to be between cars and trucks. Hence, he anticipated the importance of multimodal/multiclass traffic network equilibrium models. In fact, in his 1968 paper, he introduced multimodal user link cost functions in which the travel time on a link as perceived by members of a "group" would be a function of the flows on links, in general, of all the groups (or classes/modes). Interestingly, he also provided constructs in which the total flow on a link is equal to the sum of the flows of the distinct groups on the links (also, subsequently used in multicriteria traffic network equilibrium models). Further, he notes that the defining critical flow conditions could be extended by introducing indices referring to "groups." However, he recognizes that the symmetry condition may be violated and the governing conditions "can no longer be related to a variational principle."

In the final section of the paper, §6, Braess notes that well-known algorithms can be used for the computation of the critical flows through the convex program reformulation of the conditions defining a critical flow (that is, the user-optimal flow). He also proposes a computational procedure for the determination of the optimal flows as defined in §3, which makes use of the shortest paths and allocates portions of the total flow (demand) accordingly. The procedure suggested by Braess, known as *incremental assignment* in North American and British travel forecasting practice, was investigated by Martin (1964) (cf. Martin and Manheim 1965); however, Martin's implementation was a heuristic procedure, which was not informed by the user-optimal formulation.

## 2. Discussion and Conclusions

It is remarkable that the paper of Braess (1968) not only provided the paradox for which the paper is

often acknowledged and cited, but also provided rigorous formulations of analogues of system optimization and user optimization that yielded, respectively, *optimal flows* and *critical flows*. In addition, game-theoretic concepts were invoked in the case of system optimization that, in the first formulation, utilized a min-max construct. This is to be contrasted with the perspective of Dafermos and Sparrow (1969), who invoked Nash equilibria for the motivation of user optimization.

The level of rigor of the paper is notable as well as the depth and breadth of contributions, which were obtained without any knowledge of the state of the art in transportation science at that time. The Braess (1968) paper, accompanied by the citations in this preface as well as other related works, are generating a resurgence of interest from the computer science community who have discovered that traffic network models are relevant to telecommunication networks and the Internet, which speaks to the legacy of ground-breaking research in traffic network modeling, analysis, and computations.

Braess, subsequently, with a student published a second paper on traffic problems that appeared in this journal (cf. Braess and Koch 1979). He continues to be very interested in counterintuitive phenomena and maintains the web page (<http://homepage.ruhr-uni-bochum.de/Dietrich.Braess/#paradox>) that contains additional relevant citations to the Braess paradox.

### Postscript

The journal *Unternehmensforschung* was taken over by *Zeitschrift für Operations Research* in 1975. The latter journal has since ceased publication.

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