Dynamic Electric Power Supply Chains and Transportation Networks: An Evolutionary Variational Inequality Formulation

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Abstract: In this paper, we develop a static electric power supply chain network equilibrium model with known demands and establish the equivalence between the model and a transportation network equilibrium model with fixed demands over an appropriately constructed supernetwork. This equivalence yields a new interpretation of electric power supply chain network equilibria in path flows. We then exploit this equivalence to propose a dynamic electric power supply chain network model in which the demand varies over time using an evolutionary variational inequality formulation. Finally, we demonstrate how numerical dynamic electric power supply chain network problems can be solved utilizing recently obtained theoretical results in the unification of evolutionary variational inequalities and projected dynamical systems.

Keywords: Electric power; Supply chain networks; Dynamic transportation network equilibrium; Evolutionary variational inequalities

1. Introduction

Critical infrastructure networks including electric power supply chains, consisting of power generators, power suppliers, power transmitters, and the ultimate consumers, provide the foundations for the functioning of our modern economies and societies. Indeed, modern societies depend on electric power as an essential resource for communication, for transportation, for heating, lighting and cooling, as well as for the powering of computers and electronics. The dependence on electric power was vividly illustrated on August 14, 2003, when large portions of the Midwest, the Northeastern United States, and Ontario, Canada, experienced an electric power blackout that impacted not only the daily lives of an estimated 50 million people but also transportation systems and financial services. Estimates of the total associated costs due to the power losses in the United States alone ranged between $4 billion and $10 billion. Parts of Ontario suffered rolling blackouts for more than a week before full power restoration with the gross domestic product of Canada being down by 0.7% that August (cf. U.S.-Canada Power System Outage Task Force (2003)). In addition, two significant power outages occurred during the month of September 2003 – one in England and one, initiated in Switzerland, that cascaded over much of Italy. Such catastrophic events clearly indicate that recent changes in electric power markets, notably, deregulation, require deep and thorough analyses, coupled with new paradigms for modeling, analysis, and computations.

Given the importance of reliable electric power, there has been significant research conducted on this topic. There have been models proposed for simulating the interaction of competing generation companies by Kahn (1998), as well as those that simulate the exercising of market power (Day et al. (2002)). Schweppe et al. (1988), Hogan (1992), Chao and Peck (1996), Wu et al. (1996), and Willems (2002) have proposed a wide range of models to investigate different degrees of decentralization in electricity markets. Additional background on electric power systems can be found in the book by Casazza and Delea (2003) and in the edited volumes by Zaccour (1998) and Singh (1999).

Nagurney and Matsypura (2005) proposed a supply chain network perspective for electric power production, transmission, and consumption that captured the decentralized decision-making behavior of the various economic agents involved and demonstrated that the multi-tiered network equilibrium problem could be formulated and solved as a finite-dimensional variational inequality problem. The solution yielded the equilibrium nodal prices as well as the equilibrium electric power transaction flows. More recently, Nagurney and Liu (2005) took up a challenge posed in the fifth chapter of the classic book, **Studies in the Economics of Transportation**, by Beckmann, McGuire, and Winsten (1956), which not only laid down the mathematical foundations for the rigorous modeling and analysis of congested transportation networks (for the discussion of the impact of this book, see also Boyce, Mahmassani, and Nagurney (2005)), but also described some "unsolved problems" including a single commodity network equilibrium problem that the authors intuited could be generalized to capture electric power networks. As noted in that classic book on page 5.8, "The unsolved problems concern the application of this model to particular cases." "In particular, the problem of generation and distribution of electric energy in a network comes to mind." In particular, Nagurney and Liu (2005) proved that the model of Nagurney and Matsypura (2005), under the realistic assumption that the electric power could not be stored by the suppliers, could be reformulated as a transportation network equilibrium problem with known demand functions as proposed by Dafermos and Nagurney (1984 a,b)(see also Fisk and Boyce (1983)) over an appropriately constructed supernetwork. The equivalence was then exploited to provide a reinterpretation of the electric power supply chain network equilibrium conditions in terms of path costs and path flows and to apply existing algorithms developed for the solution of transportation network equilibrium problems to this new application domain. In addition, Nagurney and Liu (2005) also proposed an alternative electric power supply chain network model in which the inverse demand (demand market price) functions are given and established its equivalence with the well-known transportation network equilibrium model of Dafermos (1982).

In this paper, we demonstrate how electric power supply chain networks, consisting of power generators, power suppliers, transmission providers, as well as consumers at the demand markets, with known demands, can be reformulated and solved as fixed demand transportation network equilibrium problems as formulated by Smith (1979) and Dafermos (1980). We take, as the starting point, the model of Nagurney and Matsypura (2005), which provided a supply chain network perspective for electric power generation, supply, transmission, and consumption and whose governing equilibrium conditions were formulated as a finite-dimensional variational inequality problem. Here, however, in contrast to the model of Nagurney and Matsypura (2005), we assume, since electric power cannot be stored, that the power supplied by each supplier must be equal to the amount transmitted. We then use the supernetwork equivalence established in this paper, which provides Wardropian (1952) path flow information, to construct a dynamic electric power supply chain network model in which the demand varies over time by using theoretical results obtained recently by Cojocaru, Daniele, and Nagurney (2005a, b, c) in connecting evolutionary variational inequalities and projected dynamical systems. In particular, we propose an evolutionary (infinite-dimensional) variational inequality formulation of electric power supply chain networks in path flows, from which the corresponding link flows can then be recovered. We note that Nagurney (2005) proved that decentralized, multitiered supply chain network problems can be reformulated as transportation network equilibrium problems with elastic demands over appropriately constructed supernetworks.

This paper is organized as follows. In Section 2, we present the static electric power supply chain network model with known demands in which the behavior of the various economic decision-makers associated with the nodes of the network is made explicit. We also derive the governing variational inequality formulation. In Section 3, we then recall the fixed demand transportation network equilibrium model along with the path flow and link flow variational inequality formulations due to Smith (1979) and Dafermos (1980). In Section 4 we prove that the electric power supply chain network model with known demands of Section 2 can be reformulated, through a supernetwork equivalence, as a transportation network equilibrium model with fixed demands as described in Section 3. This equivalence allows us, as initiated by Nagurney and Liu (2005) for elastic demand electric power supply chain networks, to transfer the theory of fixed demand transportation network equilibrium modeling, analysis, and computations to the formulation and analysis of electric power supply chain networks with known demands.

In Section 5, we exploit the established equivalence to present a dynamic version of the static electric power supply chain network model in which the demands are known but allowed to vary over time. The formulation is in path flows. In Section 6, we utilize the results of Cojocaru, Daniele, and Nagurney (2005a, b, c), who showed that the solutions to numerical evolutionary variational inequality problems can be computed as series of projected dynamical systems, to solve numerical dynamic electric power supply chain network problems with time-varying demands.

The results in this paper further demonstrate, as originally speculated in Beckmann, McGuire, and Winsten (1956), that electric power network problems can be reformulated and, hence, solved, as transportation network equilibrium problems. Moreover, the connection is exploited further through the development of a dynamic electric power supply chain network equilibrium model. We summarize the results obtained in this paper in Section 7, in which we also present suggestions for future research.

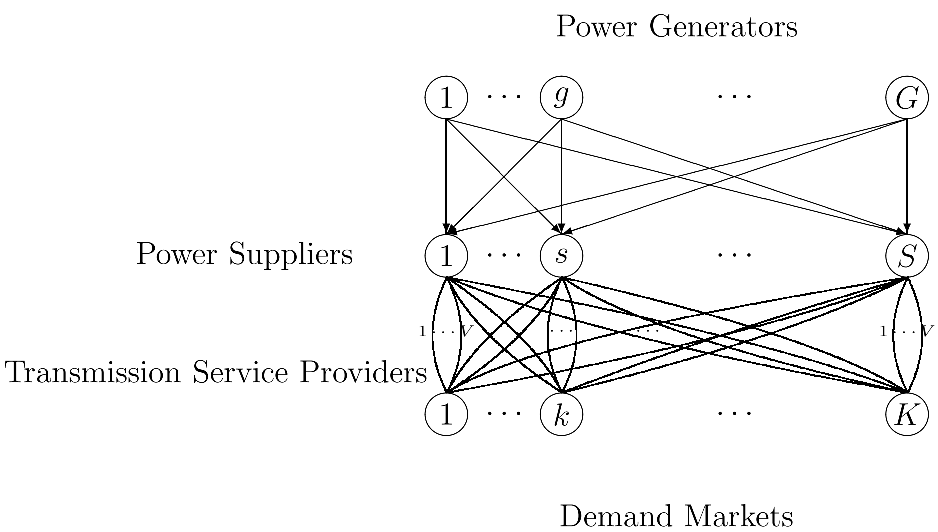

Figure 1: The Electric Power Supply Chain Network

2. The Electric Power Supply Chain Network Model with Fixed Demands

In this Section, we use, as the starting point of our electric power supply chain network model development, the electric power supply chain network equilibrium model proposed in Nagurney and Matsypura (2005). Here, however, we will assume that, since electric power cannot be stored, the electric power available at each power supplier is equal to the electric power transmitted. In addition, we will consider the case of known demands at the demand markets. We consider G power generators, S power suppliers, V transmission service providers, and K consumer markets, as depicted in Figure 1. The majority of the needed notation is given in Table 1. An equilibrium solution is denoted by "*". All vectors are assumed to be column vectors, except where noted otherwise.

Table 1: Notation for the Electric Power Supply Chain Network Model

Notation	Definition
q	vector of the power generators' electric power outputs with components: $q_1, \ldots, q_G$
Q^1	GS-dimensional vector of electric power flows between power generators and power suppliers with component gs denoted by q_{gs}
Q^2	SVK-dimensional vector of power flows between suppliers and demand markets with component svk denoted by q^v_{sk} and denoting the flow between supplier s and demand market k via transmission provider v
ρ_3	K-dimensional vector of demand market prices with component k denoted by ρ_{3k}
d	K-dimensional vector of market demand with component k denoted by d_k
$f_g(q) \equiv f_g(Q^1)$	power generating cost function of power generator g with marginal power generating cost with respect to q_g denoted by $\frac{\partial f_g}{\partial q_g}$ and the marginal power generating cost with respect to q_{sg} denoted by $\frac{\partial f_g(Q^1)}{\partial q_{gs}}$
$c_{gs}(q_{gs})$	transaction cost incurred by power generator g in transacting with power supplier s with marginal transaction cost denoted by $\frac{\partial c_{gs}(q_{gs})}{\partial q_{gs}}$
h	S-dimensional vector of the power suppliers' supplies of the electric power with component s denoted by h_s, with $h_s \equiv \sum_{g=1}^{G} q_{gs}$
$c_s(h) \equiv c_s(Q^1)$	operating cost of power supplier s with marginal operating cost with respect to h_s denoted by $\frac{\partial c_s}{\partial h_s}$ and the marginal operating cost with respect to q_{gs} denoted by $\frac{\partial c_s(Q^1)}{\partial q_{gs}}$
$c^v_{sk}(q^v_{sk})$	transaction cost incurred by power supplier s in transacting with demand market k via transmission provider v with marginal transaction cost with respect to q^v_{sk} denoted by $\frac{\partial c^v_{sk}(q^v_{sk})}{\partial q^v_{sk}}$
$\hat{c}_{gs}(q_{gs})$	transaction cost incurred by power supplier s in transacting with power generator g with marginal transaction cost denoted by $\frac{\partial \hat{c}_{gs}(q_{gs})}{\partial q_{gs}}$
$\hat{c}^v_{sk}(Q^2)$	unit transaction cost incurred by consumers at demand market k in transacting with power supplier s via transmission provider v
d_k	demand at demand market k
ρ_{3k}	demand market price at demand market k

The top-tiered nodes in the electric power supply chain network in Figure 1 represent the G electric power generators, who are the decision-makers who own and operate the electric power generating facilities or power plants. They produce electric power and sell to the power suppliers in the second tier. We assume that each electric power generator seeks to determine the optimal production and allocation of the electric power in order to maximize his own profit.

Power suppliers, which are represented by the second-tiered nodes in Figure 1, function as intermediaries. They purchase electric power from the power generators and sell to the consumers at the different demand markets. We assume that the power suppliers compete with one another in a noncooperative manner. However, the suppliers do not physically possess electric power at any stage of the supplying process; they only hold and trade the right for the electric power. Therefore, the link connecting a power generator and power supplier pair represents the decision-making connectivity and the transaction of the right of electric power between the two entities.

The bottom-tiered nodes in Figure 1 represent the demand markets, which can be distinguished from one another by their geographic locations or the type of associated consumers such as whether they correspond, for example, to businesses or to households.

As noted in Nagurney and Matsypura (2005), a transmission service is necessary for the physical delivery of electric power from the power generators to the points of consumption. The transmission service providers are the entities who own and operate the electric power transmission and distribution systems, and distribute electric power from power generators to the consumption markets. However, since these transmission service providers do not make decisions such as to where or from whom the electric power will be delivered, they are not explicitly represented by nodes in this network model. We model them, instead, as different modes of transaction corresponding to the parallel links connecting a given supplier node to a given demand market node in Figure 1. An implicit assumption here is that the power suppliers need to cover the direct cost and decide which transmission service providers should be used and how much electric power should be delivered.

Now, for completeness and easy reference, we describe the behavior of the electric power generators, the suppliers, and the consumers at the demand markets. We then state the equilibrium conditions of the electric power supply chain network and provide the variational inequality formulation. We, subsequently, contrast the derived variational inequality with the one obtained by Nagurney and Matsypura (2005).

The Behavior of Power Generators and their Optimality Conditions

Since for each individual power generator the total amount of electric power sold cannot exceed the total production of electric power, the following conservation of flow equation must hold for each power generator:

$$\sum_{s=1}^{S} q_{gs} = q_g, \quad g = 1, \ldots, G. \tag{1}$$

Let ρ^*_{1gs} denote the unit price charged by power generator g for the transaction with power supplier s. ρ^*_{1gs} is an endogenous variable and can be determined once the complete network equilibrium model is solved. Since we have assumed that each individual power generator is a profit-maximizer, the optimization problem of power generator g can be expressed as follows:

$$\text{Maximize} \quad \sum_{s=1}^{S} \rho^*_{1gs} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(q_{gs}) \tag{2}$$

subject to:

$$q_{gs} \geq 0, \qquad s = 1, \ldots, S. \tag{3}$$

We assume that the generating cost and the transaction cost functions for each power generator are continuously differentiable and convex, and that the power generators compete in a noncooperative manner in the sense of Nash (1950, 1951). The optimality conditions for all power generators simultaneously, under the above assumptions (see also Gabay and Moulin (1980), Bazaraa, Sherali, and Shetty (1993), Bertsekas and Tsitsiklis (1989), and Nagurney (1993)), coincide with the solution of the following variational inequality: determine $Q^{1*} \in R^{GS}_+$ satisfying

$$\sum_{g=1}^{G} \sum_{s=1}^{S} \left[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(q^*_{gs})}{\partial q_{gs}} - \rho^*_{1gs} \right] \times [q_{gs} - q^*_{gs}] \geq 0, \quad \forall Q^1 \in R^{GS}_+. \tag{4}$$

As defined in Table 1, the power generating cost f_g is a function of the total electric power productions, that is:

$$f_g(q) \equiv f_g(Q^1). \tag{5}$$

Hence, the marginal power generating cost with respect to q_g is equal to the marginal generating cost with respect to q_{gs}:

$$\frac{\partial f_g(q)}{\partial q_g} \equiv \frac{\partial f_g(Q^1)}{\partial q_{gs}}, \quad g = 1, \ldots, G, \quad s = 1, \ldots, S. \tag{6}$$

Using (1) and (6), we can transform (4) into the following equivalent variational inequality: determine $(q^*, Q^{1*}) \in \mathcal{K}^1$ satisfying

$$\sum_{g=1}^{G} \frac{\partial f_g(q^*)}{\partial q_g} \times [q_g - q_g^*] + \sum_{g=1}^{G} \sum_{s=1}^{S} \left[\frac{\partial c_{gs}(q_{gs}^*)}{\partial q_{gs}} - \rho_{1gs}^* \right] \times [q_{gs} - q_{gs}^*] \geq 0, \quad \forall (q, Q^1) \in \mathcal{K}^1, \tag{7}$$

where $\mathcal{K}^1 \equiv \{(q, Q^1) | (q, Q^1) \in R_+^{G+GS}$ and (1) holds$\}$.

The Behavior of Power Suppliers and their Optimality Conditions

The power suppliers, such as the power marketers, traders, and brokers, in turn, are involved in transactions both with the power generators and with the consumers at demand markets through the transmission service providers.

Since electric power cannot be stored, it is reasonable to assume that the total amount of electricity sold by a power supplier is equal to the total electric power that he purchased from the generators. This assumption can be expressed as the following conservation of flow equations:

$$\sum_{k=1}^{K} \sum_{v=1}^{V} q_{sk}^v = \sum_{g=1}^{G} q_{gs}, \quad s = 1, \ldots, S. \tag{8}$$

In Nagurney and Matsypura (2005), in contrast, it was assumed that (8) was an inequality.

Let ρ_{2sk}^{v*} denote the price charged by power supplier s to demand market k via transmission service provider v. This price is determined endogenously in the model once the entire network equilibrium problem is solved. As noted above, it is assumed that each power supplier seeks to maximize his own profit. Hence the optimization problem faced by supplier s may be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{v=1}^{V}\rho_{2sk}^{v*}q_{sk}^{v} - c_s(Q^1) - \sum_{g=1}^{G}\rho_{1gs}^{*}q_{gs} - \sum_{g=1}^{G}\hat{c}_{gs}(q_{gs}) - \sum_{k=1}^{K}\sum_{v=1}^{V}c_{sk}^{v}(q_{sk}^{v}) \tag{9}$$

subject to:

$$\sum_{k=1}^{K}\sum_{v=1}^{V}q_{sk}^{v} = \sum_{g=1}^{G}q_{gs}$$

$$q_{gs} \geq 0, \quad g = 1, \ldots, G, \tag{10}$$

$$q_{sk}^{v} \geq 0, \quad k = 1, \ldots, K; v = 1, \ldots, V. \tag{11}$$

We assume that the transaction costs and the operating costs (cf. (9)) are all continuously differentiable and convex, and that the power suppliers compete in a noncooperative manner. Hence, the optimality conditions for all suppliers, simultaneously, under the above assumptions (see also Dafermos and Nagurney (1987), Nagurney, Dong, and Zhang (2002), Dong, Zhang, and Nagurney (2004), and Nagurney et al. (2005)), can be expressed as the following variational inequality: determine $(Q^{2*}, Q^{1*}) \in \mathcal{K}^2$ such that

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}} - \rho_{2sk}^{v*}\right] \times [q_{sk}^{v} - q_{sk}^{v*}]$$

$$+\sum_{g=1}^{G}\sum_{s=1}^{S}\left[\frac{\partial c_s(Q^{1*})}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}(q_{gs}^{*})}{\partial q_{gs}} + \rho_{1gs}^{*}\right] \times [q_{gs} - q_{gs}^{*}] \geq 0, \qquad \forall (Q^2, Q^1) \in \mathcal{K}^2, \tag{12}$$

where $\mathcal{K}^2 \equiv \{(Q^2, Q^1) | (Q^2, Q^1) \in R_{+}^{SVK+GS}$ and (8) holds$\}$.

In addition, for notational convenience, we let

$$h_s \equiv \sum_{g=1}^{G} q_{gs}, \quad s = 1, \ldots, S. \tag{13}$$

As defined in Table 1, the operating cost of power supplier s, c_s, is a function of the total electricity inflows to the power supplier, that is:

$$c_s(h) \equiv c_s(Q^1), \quad s = 1, \ldots, S. \tag{14}$$

Hence, his marginal cost with respect to h_s is equal to the marginal cost with respect to q_{gs}:

$$\frac{\partial c_s(h)}{\partial h_s} \equiv \frac{\partial c_s(Q^1)}{\partial q_{gs}}, \quad s = 1, \ldots, S. \tag{15}$$

After the substitution of (13) and (15) into (12) and algebraic simplification, we obtain a variational inequality, equivalent to (12), as follows: determine $(h^*, Q^{2*}, Q^{1*}) \in \mathcal{K}^3$ such that

$$\sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] + \sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V} \left[\frac{\partial c_{sk}^v(q_{sk}^{v*})}{\partial q_{sk}^v} - \rho_{2sk}^{v*}\right] \times [q_{sk}^v - q_{sk}^{v*}]$$

$$+\sum_{g=1}^{G}\sum_{s=1}^{S} \left[\frac{\partial \hat{c}_{gs}(q_{gs}^*)}{\partial q_{gs}} + \rho_{1gs}^*\right] \times [q_{gs} - q_{gs}^*] \geq 0, \qquad \forall (Q^1, Q^2, h) \in \mathcal{K}^3, \tag{16}$$

where $\mathcal{K}^3 \equiv \{(h, Q^2, Q^1) | (h, Q^2, Q^1) \in R_+^{S(1+VK+G)} \text{and (8) and (13) hold}\}$.

Equilibrium Conditions for the Demand Markets

We now discuss the equilibrium conditions for the demand markets. The consumers take into account the prices charged by the power suppliers and the transaction costs in making their consumption decisions. In the static model, we assume that the demand for electric power at each demand market is fixed (later in this paper, we allow it to be time-varying). Hence, the following conservation equations must hold:

$$d_k = \sum_{s=1}^{S}\sum_{v=1}^{V} q_{sk}^v, \quad k = 1, \ldots, K. \tag{17}$$

We assume that the unit transaction cost functions $\hat{c}_{sk}^v$ are continuous for all s, k, v.

The equilibrium conditions for consumers at demand market k take the form: for each power supplier s; $s = 1, ..., S$ and transmission service provider v; $v = 1, ..., V$:

$$\rho_{2sk}^{v*} + \hat{c}_{sk}^v(Q^{2*}) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{sk}^{v*} > 0, \\ \geq \rho_{3k}^*, & \text{if} \quad q_{sk}^{v*} = 0, \end{cases} \tag{18}$$

where (17) is also satisfied for the equilibrium flows.

Conditions (18) state that, in equilibrium, if consumers at demand market k purchase the electricity from power supplier s transmitted via service provider v, then the price the consumers pay is exactly equal to the sum of the price charged by the power supplier and the unit transaction cost incurred by the consumers. However, if the price charged by the power supplier plus the transaction cost is greater than the price the consumers are willing to pay at the demand market, there will be no transaction between this power supplier/demand market pair via that transmission service provider.

In equilibrium, condition (18) must hold simultaneously for all demand markets. We can also express these equilibrium conditions using the following variational inequality: determine $Q^{2*} \in \mathcal{K}^4$, such that

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\rho_{2sk}^{v*} + \hat{c}_{sk}^{v}(Q^{2*})\right] \times [q_{sk}^{v} - q_{sk}^{v*}] \geq 0, \quad \forall Q^2 \in \mathcal{K}^4, \tag{19}$$

where $\mathcal{K}^4 \equiv \{Q^2 | Q^2 \in R_+^{K(SV)} \text{and (17) holds}\}$.

In Nagurney and Matsypura (2005), it was assumed that the demand functions associated with the demand markets were elastic and depended upon the prices of electric power at the demand markets. Nagurney and Liu (2005) demonstrated that that model in the case of constraints given by (8) could be transformed into an elastic demand transportation network equilibrium model formulated in Dafermos and Nagurney (1984a) and also considered the case in which the inverse demands or demand market price functions are given (cf. Dafermos (1982)).

The Equilibrium Conditions for the Electric Power Supply Chain Network

In equilibrium, the optimality conditions for all the power generators, the optimality conditions for all the power suppliers, and the equilibrium conditions for all the demand markets must be simultaneously satisfied so that no decision-maker has any incentive to alter his transactions. We now formally state the equilibrium conditions for the entire electric power supply chain network as follows.

Definition 1: Electric Power Supply Chain Network Equilibrium

The equilibrium state of the electric power supply chain network is one where the electric power flows between the tiers of the network coincide and the electric power flows satisfy the sum of conditions (7), (16), and (19).

We now state and prove:

Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}) \in \mathcal{K}^5$ satisfying:

$$\sum_{g=1}^{G} \frac{\partial f_g(q^*)}{\partial q_g} \times [q_g - q_g^*] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] + \sum_{g=1}^{G}\sum_{s=1}^{S} \left[\frac{\partial c_{gs}(q_{gs}^*)}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}(q_{gs}^*)}{\partial q_{gs}} \right] \times [q_{gs} - q_{gs}^*]$$

$$+ \sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V} \left[\frac{\partial c_{sk}^v(q_{sk}^{v*})}{\partial q_{sk}^v} + \hat{c}_{sk}^v(Q^{2*}) \right] \times [q_{sk}^v - q_{sk}^{v*}] \geq 0, \quad \forall (q, h, Q^1, Q^2) \in \mathcal{K}^5, \tag{20}$$

$$\textit{where } \mathcal{K}^5 \equiv \{(q,h,Q^1,Q^2) | (q,h,Q^1,Q^2) \in R_+^{G+S+GS+VSK}$$

$$\textit{and } (1),\ (8),\ (13),\ \textit{and } (17)\ \textit{hold}\}.$$

Proof: We first prove that an equilibrium according to Definition 1 coincides with the solution of variational inequality (20). Indeed, summation of (7), (16), and (19), after algebraic simplifications, yields (20).

We now prove the converse, that is, a solution to variational inequality (20) satisfies the sum of conditions (7), (16), and (19), and is, therefore, an electric power supply chain network equilibrium pattern according to Definition 1.

First, we add the term $\rho_{1gs}^* - \rho_{1gs}^*$ to the first term in the third summand expression in (20). Then, we add the term $\rho_{2sk}^{v*} - \rho_{2sk}^{v*}$ to the first term in the fourth summand expression in (20). Since these terms are all equal to zero, they do not change (20). Hence, we obtain the following inequality:

$$\sum_{g=1}^{G} \frac{\partial f_g(q^*)}{\partial q_g} \times [q_g - q_g^*] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+ \sum_{g=1}^{G}\sum_{s=1}^{S} \left[\frac{\partial c_{gs}(q_{gs}^*)}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}(q_{gs}^*)}{\partial q_{gs}} + \rho_{1gs}^* - \rho_{1gs}^* \right] \times [q_{gs} - q_{gs}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{gs}^{v*})}{\partial q_{sk}^{v}}+\hat{c}_{sk}^{v}(Q^{2*})+\rho_{2sk}^{v*}-\rho_{2sk}^{v*}\right]\times[q_{sk}^{v}-q_{sk}^{v*}]\geq 0,\quad\forall(q,h,Q^{1},Q^{2})\in\mathcal{K}^{5}, \tag{21}$$

which can be rewritten as:

$$\sum_{g=1}^{G}\frac{\partial f_{g}(q^{*})}{\partial q_{g}}\times[q_{g}-q_{g}^{*}]+\sum_{g=1}^{G}\sum_{s=1}^{S}\left[\frac{\partial c_{gs}(q_{gs}^{*})}{\partial q_{gs}}-\rho_{1gs}^{*}\right]\times[q_{gs}-q_{gs}^{*}]$$

$$+\sum_{s=1}^{S}\frac{\partial c_{s}(h^{*})}{\partial h_{s}}\times[h_{s}-h_{s}^{*}]+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}}-\rho_{2sk}^{v*}\right]\times[q_{sk}^{v}-q_{sk}^{v*}]$$

$$+\sum_{s=1}^{S}\sum_{g=1}^{G}\left[\frac{\partial \hat{c}_{gs}(q_{gs}^{*})}{\partial q_{gs}}+\rho_{1gs}^{*}\right]\times[q_{gs}-q_{gs}^{*}]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\rho_{2sk}^{v*}+\hat{c}_{sk}^{v}(Q^{2*})\right]\times[q_{sk}^{v}-q_{sk}^{v*}]\geq 0,\quad\forall(q,h,Q^{1},Q^{2})\in\mathcal{K}^{5}. \tag{22}$$

Clearly, (22) is the sum of the optimality conditions (7) and (16), and the equilibrium conditions (19) and is, hence, according to Definition 1 an electric power supply chain network equilibrium. □

Existence of a solution to variational inequality (20) is guaranteed from the standard theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980)) since the function that enters the variational inequality (20) (cf. Nagurney (1993)) is continuous under the above imposed assumptions on the various underlying functions, and the feasible set $\mathcal{K}^5$ is compact.

We will describe how to recover the nodal prices in the electric power supply chain network at the end of Section 4.

Nagurney and Matsypura (2005) derived a variational inequality formulation of electric power supply chain network equilibrium in the case of known demand functions and in the case where the conservation of flow expression (8) in their model was an inequality. The formulation also had Lagrange multipliers reflecting nodal prices associated with those inequalities as variables in their variational inequality. Moreover, the demand market prices were variables and appeared in the variational inequality derived therein.

3. The Transportation Network Equilibrium Model with Fixed Demands

In this Section, we recall the transportation network equilibrium model with fixed demands, due to Smith (1979) and Dafermos (1980).

Consider a network $\mathcal{G}$ with the set of links L with n_L elements, the set of paths P with n_P elements, and the set of origin/destination (O/D) pairs W with n_W elements. We denote the set of paths joining O/D pair w by P_w. Links are denoted by a, b, etc; paths by p, q, etc., and O/D pairs by w_1, w_2, etc.

Denote the flow on path p by x_p and the flow on link a by f_a. The user travel cost on a path p is denoted by C_p and the user travel cost on a link a by c_a. The user link cost functions are assumed to be continuous. Denote the travel demand associated with traveling between O/D pair w by d_w and the travel disutility by λ_w.

Since the travel demands are assumed fixed and known, the following conservation of flow equations must be satisfied:

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w, \tag{23}$$

that is, the travel demand associated with an O/D pair must be equal to the sum of the flows on the paths that connect that O/D pair.

The link flows are related to the path flows, in turn, through the following conservation of flow equations:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{24}$$

where $\delta_{ap} = 1$ if link a is contained in path p, and $\delta_{ap} = 0$, otherwise. Hence, the flow on a link is equal to the sum of the flows on paths that contain that link.

The user costs on paths are related to user costs on links through the following equations:

$$C_p = \sum_{a \in L} c_a \delta_{ap}, \quad \forall p \in P, \tag{25}$$

that is, the user cost on a path is equal to the sum of user costs on links that make up the path.

For the sake of generality, we allow the user cost on a link to depend upon the entire vector of link flows, denoted by f, so that

$$c_a = c_a(f), \quad \forall a \in L, \tag{26}$$

with the link cost functions assumed to be continuous.

Definition 2: Transportation Network Equilibrium

A path flow pattern $x^* \in \mathcal{K}^6$, where $\mathcal{K}^6 \equiv \{x | x \in R_+^{n_P}$ and (23) holds$\}$ is said to be a transportation network equilibrium (according to Wardrop's first principle (cf. Wardrop (1952) and Beckmann, McGuire, and Winsten (1956)), if, once established, no user has any incentive to alter his travel decisions. The state can be expressed by the following equilibrium conditions which must hold for every O/D pair $w \in W$ and every path $p \in P_w$:

$$C_p(x^*) - \lambda_w^* \begin{cases} = 0, & \text{if} \quad x_p^* > 0, \\ \geq 0, & \text{if} \quad x_p^* = 0. \end{cases} \tag{27}$$

Hence, conditions (27) state that all utilized paths connecting an O/D pair have equal and minimal user costs. As described in Dafermos (1980) (see also Smith (1979)) the transportation network equilibrium conditions (27) can be formulated as a finite-dimensional variational inequality in path flows.

Theorem 2

A path flow pattern $x^* \in \mathcal{K}^6$ is a transportation network equilibrium according to Definition 2 if and only if it satisfies the variational inequality problem:

$$\sum_{w \in W} \sum_{p \in P_w} C_p(x^*) \times \left[x_p - x_p^*\right] \geq 0, \quad \forall x \in \mathcal{K}^6. \tag{28}$$

Note that (28) can be put into standard variational inequality form: determine $x^* \in \mathcal{K}$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in \mathcal{K},$$

if we define $F(x) \equiv C(x)$ and $\mathcal{K} \equiv \mathcal{K}^6$, where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional space where n here is equal to the dimension of path flows, that is, n_P. In Section 5, we describe a dynamic version of the transportation network equilibrium problem formulated as an evolutionary variational inequality and the above standard form will be helpful in connecting the static formulation with the dynamic one. The dynamic transportation network equilibrium model will also be the basis for the dynamic electric power supply chain network model with time-varying demands.

Now we also provide the equivalent variational inequality in link flows. For additional background, see the book by Nagurney (1993).

Theorem 3

A link flow pattern is a transportation network equilibrium according to Definition 2 if and only if it satisfies the variational inequality problem: determine $f^* \in \mathcal{K}^7$ satisfying

$$\sum_{a \in L} c_a(f^*) \times (f_a - f_a^*) \geq 0, \quad \forall f \in \mathcal{K}^7, \tag{29}$$

where $\mathcal{K}^7 \equiv \{f \in R_+^{n_L} \,|\, \text{there exists an } x \text{ satisfying} (23) \text{ and } (24)\}$.

Existence of solutions to both variational inequalities (28) and (29) is guaranteed since the feasible sets $\mathcal{K}^6$ and $\mathcal{K}^7$ are compact and the user link cost functions are assumed to be continuous.

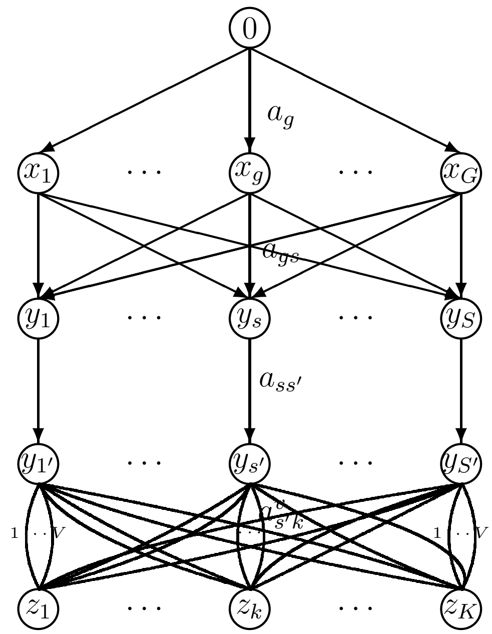

Figure 2: The $\mathcal{G}_{\mathcal{S}}$ Supernetwork Representation of Electric Power Supply Chain Network Equilibrium

4. Transportation Network Equilibrium Reformulation of the Electric Power Supply Chain Network Equilibrium Model with Known Demands

In this Section, we show that the electric power supply chain network equilibrium model presented in Section 2 is isomorphic to a properly configured transportation network equilibrium model as discussed in Section 3 through the establishment of a supernetwork (see also, e.g., Nagurney and Dong (2002)) equivalence of the former.

Supernetwork Equivalence of the Electric Power Supply Chain Network Model with Known Demands

We now establish the supernetwork equivalence of the electric power supply chain network equilibrium model to the transportation network equilibrium model with known demands over a particular network.

Consider an electric power supply chain network as discussed in Section 2 with given power generators: $g = 1, \ldots, G$; power suppliers: $s = 1, \ldots, S$; transmission service providers: $v = 1, \ldots, V$, and demand markets: $k = 1, \ldots, K$. The supernetwork, $\mathcal{G}_{\mathcal{S}}$, of the isomorphic transportation network equilibrium model is depicted in Figure 2 and is constructed as follows. It consists of five tiers of nodes with the origin node 0 at the top or first tier and the destination nodes at the fifth or bottom tier. Specifically, $\mathcal{G}_{\mathcal{S}}$ consists of a single origin node 0 at the first tier, and K destination nodes at the bottom tier, denoted, respectively, by: $z_1, \ldots, z_K$. There are K O/D pairs in $\mathcal{G}_{\mathcal{S}}$ denoted by $w_1 = (0, z_1)$, ..., $w_k = (0, z_k)$,..., $w_K = (0, z_K)$. Node 0 is connected to each second tiered node x_g; $g = 1, \ldots, G$ by a single link. Each second tiered node x_g, in turn, is connected to each third tiered node y_s; $s = 1, \ldots, S$ by a single link. Each third tiered node y_s is connected to the corresponding fourth tiered node $y_{s'}$ by a single link. Finally, each fourth tiered node $y_{s'}$ is connected to each destination node z_k; $k = 1, \ldots, K$ at the fifth tier by V parallel links.

Hence, in $\mathcal{G}_{\mathcal{S}}$, there are $G + 2S + K + 1$ nodes, $G + GS + S + SVK$ links, K O/D pairs, and $GSVK$ paths. We now define the link and link flow notation. Let a_g denote the link from node 0 to node x_g with associated link flow f_{a_g}, for $g = 1, \ldots, G$. Let a_{gs} denote the link from node x_g to node y_s with associated link flow $f_{a_{gs}}$ for $g = 1, \ldots, G$ and $s = 1, \ldots, S$. Also, let $a_{ss'}$ denote the link connecting node y_s with node $y_{s'}$ with associated link flow $f_{a_{ss'}}$ for $ss' = 11', \ldots, SS'$. Finally, let $a^v_{s'k}$ denote the v-th link joining node $y_{s'}$ with node z_k for $s' = 1', \ldots, S'$; $v = 1, \ldots, V$, and $k = 1, \ldots, K$ and with associated link flow $f_{a^v_{s'k}}$. We group the link flows into the vectors as follows: we group the $\{f_{a_g}\}$ into the vector f^1; the $\{f_{a_{gs}}\}$ into the vector f^2; the $\{f_{a_{ss'}}\}$ into the vector f^3, and the $\{f_{a^v_{s'k}}\}$ into the vector f^4.

Thus, a typical path connecting O/D pair $w_k = (0, z_k)$, is denoted by $p^v_{gss'k}$ and consists of four links: $a_g, a_{gs}, a_{ss'}$, and $a^v_{s'k}$. The associated flow on the path is denoted by $x_{p^v_{gss'k}}$. Finally, we let d_{w_k} be the demand associated with O/D pair w_k where λ_{w_k} denotes the travel disutility for w_k.

Note that the following conservation of flow equations must hold on the network $\mathcal{G}_{\mathcal{S}}$:

$$f_{a_g} = \sum_{s=1}^{S} \sum_{s'=1}^{S'} \sum_{k=1}^{K} \sum_{v=1}^{V} x_{p^v_{gss'k}}, \quad g = 1, \ldots, G, \tag{30}$$

$$f_{a_{gs}} = \sum_{s'=1}^{S'} \sum_{k=1}^{K} \sum_{v=1}^{V} x_{p^v_{gss'k}}, \quad g = 1, \ldots, G; s = 1, \ldots, S, \tag{31}$$

$$f_{a_{ss'}} = \sum_{g=1}^{G} \sum_{k=1}^{K} \sum_{v=1}^{V} x_{p^v_{gss'k}}, \quad ss' = 11', \ldots, SS', \tag{32}$$

$$f_{a^v_{s'k}} = \sum_{g=1}^{G} \sum_{s=1}^{S} x_{p^v_{gss'k}}, \quad s' = 1, \ldots, S'; v = 1, \ldots, V; k = 1, \ldots, K. \tag{33}$$

Also, we have that

$$d_{w_k} = \sum_{g=1}^{G} \sum_{s=1}^{S} \sum_{s'=1}^{S'} \sum_{v=1}^{V} x_{p^v_{gss'k}}, \quad k = 1, \ldots, K. \tag{34}$$

If all path flows are nonnegative and (30 – (34) are satisfied, the feasible path flow pattern induces a feasible link flow pattern.

We can construct a feasible link flow pattern for $\mathcal{G}_{\mathcal{S}}$ based on the corresponding feasible electric power flow pattern in the electric power supply chain network model, $(q, h, Q^1, Q^2) \in \mathcal{K}^5$, in the following way:

$$q_g \equiv f_{a_g}, \quad g = 1, \ldots, G, \tag{35}$$

$$q_{gs} \equiv f_{a_{gs}}, \quad g = 1, \ldots, G; s = 1, \ldots, S, \tag{36}$$

$$h_s \equiv f_{a_{ss'}}, \quad ss' = 11', \ldots, SS', \tag{37}$$

$$q^v_{sk} = f_{a^v_{s'k}}, \quad s = s' = 1', \ldots, S'; v = 1, \ldots, V; k = 1, \ldots, K, \tag{38}$$

$$d_k = \sum_{s=1}^{S} \sum_{v=1}^{V} q^v_{sk}, \quad k = 1, \ldots, K. \tag{39}$$

Note that if (q, Q^1, h, Q^2) is feasible then the link flow pattern constructed according to (35) – (39) is also feasible and the corresponding path flow pattern which induces this link flow pattern is also feasible.

We now assign user (travel) costs on the links of the network $\mathcal{G}_{\mathcal{S}}$ as follows: with each link a_g we assign a user cost c_{a_g} defined by

$$c_{a_g} \equiv \frac{\partial f_g}{\partial q_g}, \quad g = 1, \ldots, G, \tag{40}$$

with each link a_{gs} we assign a user cost $c_{a_{gs}}$ defined by:

$$c_{a_{gs}} \equiv \frac{\partial c_{gs}}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}}{\partial q_{gs}}, \quad g = 1, \dots, G; s = 1, \dots, S, \tag{41}$$

with each link ss' we assign a user cost defined by

$$c_{a_{ss'}} \equiv \frac{\partial c_s}{\partial h_s}, \quad ss' = 11', \dots, SS'. \tag{42}$$

Finally, for each link $a^v_{s'k}$ we assign a user cost defined by

$$c_{a^v_{s'k}} \equiv \frac{\partial c^v_{sk}}{\partial q^v_{sk}} + \hat{c}^v_{sk}, \quad s' = s = 1, \dots, S; v = 1, \dots, V; k = 1, \dots, K. \tag{43}$$

Then a user of path $p^v_{gss'k}$, for $g = 1, \dots, G$; $s = 1, \dots, S$; $s' = 1', \dots, S'$; $v = 1, \dots, V$; $k = 1, \dots, K$, on network $\mathcal{G}_{\mathcal{S}}$ in Figure 2 experiences a path cost $C_{p^v_{gss'k}}$ given by

$$C_{p^v_{gss'k}} = c_{a_g} + c_{a_{gs}} + c_{a_{ss'}} + c_{a^v_{s'k}} = \frac{\partial f_g}{\partial q_g} + \frac{\partial c_{gs}}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}}{\partial q_{gs}} + \frac{\partial c_s}{\partial h_s} + \frac{\partial c^v_{sk}}{\partial q^v_{sk}} + \hat{c}^v_{sk}. \tag{44}$$

Also, we assign the (travel) demands associated with the O/D pairs as follows:

$$d_{w_k} \equiv d_k, \quad k = 1, \dots, K, \tag{45}$$

and the (travel) disutilities:

$$\lambda_{w_k} \equiv \rho_{3k}, \quad k = 1, \dots, K. \tag{46}$$

Consequently, the equilibrium conditions (27) for the transportation network equilibrium model on the network $\mathcal{G}_{\mathcal{S}}$ state that for every O/D pair w_k and every path connecting the O/D pair w_k:

$$C_{p^v_{gss'k}} - \lambda^*_{w_k} = \frac{\partial f_g}{\partial q_g} + \frac{\partial c_{gs}}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}}{\partial q_{gs}} + \frac{\partial c_s}{\partial h_s} + \frac{\partial c^v_{sk}}{\partial q^v_{sk}} + \hat{c}^v_{sk} - \rho^*_{3k} \begin{cases} = 0, & \text{if} \quad x^*_{p^v_{gss'k}} > 0, \\ \geq 0, & \text{if} \quad x^*_{p^v_{gss'k}} = 0. \end{cases} \tag{47}$$

We now show that the variational inequality formulation of the equilibrium conditions (47) in link form as in (29) is equivalent to the variational inequality (20) governing the electric power supply chain network equilibrium. For the transportation network equilibrium problem on $\mathcal{G}_{\mathcal{S}}$, according to Theorem 3, we have that a link flow pattern $f^* \in \mathcal{K}^7$ is an equilibrium (according to (47)), if and only if it satisfies the variational inequality:

$$\sum_{g=1}^{G} c_{a_g}(f^{1*}) \times (f_{a_g} - f^*_{a_g}) + \sum_{g=1}^{G}\sum_{s=1}^{S} c_{a_{gs}}(f^{2*}) \times (f_{a_{gs}} - f^*_{a_{gs}})$$

$$+ \sum_{ss'=11'}^{SS'} c_{a_{ss'}}(f^{3*}) \times (f_{a_{ss'}} - f^*_{a_{ss'}}) + \sum_{s'=1}^{S'}\sum_{k=1}^{K}\sum_{v=1}^{V} c_{a^v_{s'k}}(f^{4*}) \times (f_{a^v_{s'k}} - f^*_{a^v_{s'k}}) \geq 0, \quad \forall f \in \mathcal{K}^7. \quad (48)$$

After the substitution of (35) – (39) and (40) – (43), we have the following variational inequality: determine $(q^*, h^*, Q^{1*}, Q^{2*}) \in \mathcal{K}^5$ satisfying:

$$\sum_{g=1}^{G} \frac{\partial f_g(q^*)}{\partial q_g} \times [q_g - q^*_g] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h^*_s] + \sum_{g=1}^{G}\sum_{s=1}^{S} \left[\frac{\partial c_{gs}(q^*_{gs})}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}(q^*_{gs})}{\partial q_{gs}}\right] \times [q_{gs} - q^*_{gs}]$$

$$+ \sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V} \left[\frac{\partial c^v_{sk}(q^{v*}_{sk})}{\partial q^v_{sk}} + \hat{c}^v_{sk}(Q^{2*})\right] \times [q^v_{sk} - q^{v*}_{sk}] \geq 0, \quad \forall (q, h, Q^1, Q^2) \in \mathcal{K}^5. \quad (49)$$

Variational inequality (49) is precisely variational inequality (20) governing the electric power supply chain network equilibrium. Hence, we have the following result:

Theorem 4

A solution $(q^*, h^*, Q^{1^*}, Q^{2^*}) \in \mathcal{K}^5$ of the variational inequality (20) governing the electric power supply chain network equilibrium coincides with (via (35) – (43)) the feasible link flow pattern for the supernetwork $\mathcal{G}_{\mathcal{S}}$ constructed above and satisfies variational inequality (20). Hence, it is a transportation network equilibrium according to Theorem 3.

Note that equilibrium conditions (47) are in path flows over the network in Figure 2. These conditions define the electric power supply chain network equilibrium in terms of paths and path flows, which, as shown above, coincide with Wardrop's (1952) first principle of travel behavior, now commonly referred to as, see, e. g., Dafermos and Sparow (1969), user-optimization in the context of transportation networks. Hence, we now have an entirely new interpretation of electric power supply chain network equilibrium in the case of known demands which is economic in nature and which states that only minimal cost paths will be used from the super source node 0 to any destination node. Moreover, the cost on the utilized paths for a particular O/D pair is equal to the disutility (or the demand market price) that the consumers pay. We further emphasize that the demand for electric power may not be as price-sensitive as the demand for other types of "supply chain-type" products and, hence, the fixed demand versions developed in this paper are realistic from a practical perspective.

Now, for completeness, we provide the path flow version of variational inequality (48) which provides us with the counterpart of Theorem 2 specialized for the electric power supply chain network equilibrium problem. Indeed, the following result is immediate due to Theorems 2 and 4.

Theorem 5

A path flow pattern on the supernetwork in Figure 2 is a transportation network equilibrium if and only if it satisfies the variational inequality problem: determine $x^*_{p^v_{gss'k}} \geq 0$*, for all* g, s, s', v, k *and satisfying (34) such that*

$$\sum_{g=1}^{G}\sum_{s=1}^{S}\sum_{s'=1'}^{S'}\sum_{v=1}^{V}\sum_{k=1}^{K} C_{p^v_{gss'k}}(x^*) \times \left[x_{p^v_{gss'k}} - x^*_{p^v_{gss'k}}\right] \geq 0,$$

$$\forall x_{p^v_{gss'k}} \geq 0, \forall g, s, s', v, k \textit{ and satisfying} (34). \tag{50}$$

Hence, to obtain the solution to the electric power supply chain network equilibrium problem one can solve variational inequality (20) or, equivalently, variational inequality (48) in link flows or variational inequality (50) in path flows. The solution of variational inequality (50) yields information that was not available, however, from the solution of variational inequality (20) since we now have equilibrium path flows. Of course, the equilibrium link flows can be recovered simply through equations (30) – (33) after (50) is solved. It is also important to emphasize that the connection formalized above between electric power supply chain networks and transportation networks with fixed demands unveils new opportunities for further modeling enhancements. In Section 5, we exploit this equivalence to develop a dynamic electric power supply chain network equilibrium model in path flows.

We now describe how to recover the prices in the electric power supply chain network. The vector of prices ρ_3^* associated with electric power at the demand markets can be obtained by setting (cf. (47)) $C_{p^v_{gss'k}} = \rho^*_{3k}$ for demand market k such that $x^*_{p^v_{gss'k}} > 0$. The prices ρ_2^* associated with the power suppliers, in turn, can be obtained by setting (cf. (18)) $\rho^{v*}_{2sk} = \rho^*_{3k} - \hat{c}^v_{sk}(Q^{2*})$ for any s, v, k such that $q^{v*}_{sk} > 0$. The prices ρ_1^* can be recovered by setting (cf. (4)) $\rho^*_{1gs} = \frac{\partial f_g(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(q^*_{gs})}{\partial q_{gs}}$ for any g, s such that $q^*_{gs} > 0$.

5. Dynamic Transportation Network Equilibrium Reformulation of Electric Power Supply Chains

In this Section, we utilize the results obtained in Section 4 to construct a dynamic electric power supply chain network equilibrium model through its dynamic transportation network equilibrium representation and formulation as an evolutionary variational inequality problem. Daniele, Maugeri, and Oettli (1998, 1999) formulated time-dependent transportation network equilibria as evolutionary variational inequalities. Cojocaru, Daniele, and Nagurney (2005a) showed that these, as well as related dynamic spatial price equilibrium problems and financial equilibrium problems, could be formulated into a unified definition.

Specifically, we consider the nonempty, convex, closed, bounded subset of the Hilbert space $L^2([0,T], R^{n_P})$ (where T denotes the time interval under consideration and $\mu = constant$ and very large) given by

$$\hat{\mathcal{K}} = \left\{ x \in L^2([0,T], R^{n_P}) : 0 \le x(t) \le \mu \text{ a.e. in } [0,T]; \sum_{p \in P_w} x_p(t) = d_w(t), \forall w, \text{ a.e. in } [0,T] \right\}. \tag{51}$$

Hence, for definiteness, and greater ease in relating the discussion to the existing literature, we, without any loss of generality, consider the vector of path flows on the network at time t to be denoted by $x(t)$ with an individual element by $x_p(t)$ and with $d_w(t)$ denoting the demand associated with O/D pair w and time t.

Hence, we assume now that the demands are dynamic, that is, they vary over time. Consequently, the path flows will also vary over time. Then, setting

$$\langle\langle \Phi, x \rangle\rangle = \int_0^T \langle \Phi(t), x(t) \rangle dt \tag{52}$$

where $\Phi \in L^2([0,T], R^{n_P})^*$ and $x \in L^2([0,T], R^{n_P})$, if F is given such that $F : \hat{\mathcal{K}} \to L^2([0,T], R^{n_P})$ we consider now the standardized form of the infinite-dimensional evolutionary variational inequality (cf. Cojocaru, Daniele, and Nagurney (2005a, b)):

$$\text{determine } x \in \hat{\mathcal{K}} : \langle\langle F(x), z - x \rangle\rangle \ge 0, \quad \forall z \in \hat{\mathcal{K}}. \tag{53}$$

In Daniele, Maugeri, and Oettli (1999) sufficient conditions (including monotonicity-type conditions) that ensure the existence of a solution to (53) are given.

Cojocaru, Daniele, and Nagurney (2005b) have shown that for the case of Hilbert spaces (namely $L^2([0,T],R^{n_P})$) the following infinite-dimensional PDS can be associated to the EVI (53) as follows:

$$\frac{dx(t,\tau)}{d\tau} = \Pi_{\hat{\mathcal{K}}}(x(t,\tau), -F(x(t,\tau))), \quad x(t,0) \in \hat{\mathcal{K}}, \tag{54}$$

where

$$\Pi_{\hat{\mathcal{K}}}(y, -F(y)) = \lim_{\delta \to 0^+} \frac{P_{\hat{\mathcal{K}}}((y - \delta F(y)) - y)}{\delta}, \quad \forall y \in \hat{\mathcal{K}} \tag{55}$$

with the projection operator $P_{\hat{\mathcal{K}}} : H \to \hat{\mathcal{K}}$ given by

$$\|P_{\hat{\mathcal{K}}}(z) - z\| = \inf_{y \in \hat{\mathcal{K}}} \|y - z\|, \tag{56}$$

Following Dupuis and Nagurney (1993), in finite dimensions, and Cojocaru and Jonker (2004), in infinite-dimensional Hilbert spaces, Cojocaru, Daniele, and Nagurney (2005b) have shown the following:

Theorem 6

Assume that $\hat{\mathcal{K}} \subseteq H$ is non-empty, closed and convex and $F : \hat{\mathcal{K}} \to H$ is a pseudo-monotone Lipschitz continuous vector field, where H is a Hilbert space. Then the solutions of EVI (53) are the same as the critical points of the projected differential equation (54) that is, they are the functions $x \in \hat{\mathcal{K}}$ such that

$$\Pi_{\hat{\mathcal{K}}}(x(t), -F(x(t))) = 0, \tag{57}$$

and vice versa.

According to Theorem 6, the solutions to the evolutionary variational inequality:

$$\text{determine} \quad x \in \hat{\mathcal{K}} : \quad \int_0^T \langle F(x(t)), z(t) - x(t) \rangle dt \geq 0, \quad \forall z \in \hat{\mathcal{K}}, \tag{58}$$

are the same as the critical points of the equation:

$$\frac{dx(t,\tau)}{d\tau} = \Pi_{\hat{\mathcal{K}}}(x(t,\tau), -F(x(t,\tau))), \tag{59}$$

that is, the points such that

$$\Pi_{\hat{\mathcal{K}}}(x(t,\tau), -F(x(t,\tau))) \equiv 0 \text{ a.e. in } [0,T], \tag{60}$$

which are obviously stationary with respect to τ.

Cojocaru, Daniele, and Nagurney (2005c) discuss the meaning of the two "times" in (59). In particular, they note that, intuitively, at each instant $t \in [0,T]$, the solution of the evolutionary variational inequality (53) represents a static state of the underlying system. As t varies over $[0,T]$, the static states describe one (or more) curves of the equilibria. In contrast, τ here is the time that describes the dynamics of the system until it reaches one of the equilibria of the curve. We can expect the specific interpretation of time τ to be application-dependent.

The dynamic, evolutionary variational inequality analogue of the static, finite-dimensional variational inequality (28) (and following) is now immediate and substitution of the vector of path costs and path flows into (53) yields the evolutionary variational inequality for time-dependent transportation network equilibria given by:

$$\text{determine} \quad x \in \hat{\mathcal{K}} : \langle\langle C(x), z - x \rangle\rangle \geq 0, \quad \forall z \in \hat{\mathcal{K}}, \tag{61}$$

where C is the vector of path costs.

From Theorem 5, in turn, we know that the electric power supply chain network equilibrium problem with fixed demands can be reformulated as a fixed demand transportation network equilibrium problem in path flows over the supernetwork $\mathcal{G}_{\mathcal{S}}$ given in Figure 2. Evolutionary variational inequality (61), in turn, provides us now with a dynamic version of the electric supply chain network problem in which the demands vary over time, where the path costs are given by (44) but these are functions of path flows that now vary with time. Indeed, evolutionary variational inequality (61) is a dynamic (and infinite-dimensional) version of variational inequality (50) with the paths as defined prior to (30) and the path costs defined in (44). We know, in turn, from the theory established in Section 2 and Section 4, that at this solution at each point in time, each of the decision-makers has achieved his optimal solution.

The economic interpretation of evolutionary variational inequality (61) in the context of the dynamic electric power supply chain model is now given. In particular, we have, at each instant in time, only the most "efficient" cost paths connecting each O/D pair are used in a Wardropian sense. Hence, at each instance in time, the path costs given by (44) on used paths connecting each origin/destination node pair are equal and minimal where the O/D pairs are as defined in Section 4 for the supernetwork.

6. Dynamic Numerical Electric Supply Chain Network Examples with Computations

In this Section, we provide numerical examples in order to demonstrate how the theoretical results in this paper can be applied in practice. In particular, we consider numerical electric power supply chain network examples with time-varying demands.

To solve the associated evolutionary variational inequality, we utilize the approach set forth in Cojocaru, Daniele, and Nagurney (2005 a, b, c), in which the time horizon T is discretized and at each fixed time we then solve the associated projected dynamical system (cf. also Nagurney and Zhang (1996)). We have chosen the examples so that the corresponding vector field F satisfies the requirements in Theorem 6 (see also Nagurney, Dong, and Zhang (2002)), which we expect to be readily fulfilled in practice.

The algorithmic procedure is now described. We selected discrete points in time for each example over the interval $[0, T]$. We then applied the Euler method at each discrete time point over the time interval T. The Euler method is induced by the general iterative scheme of Dupuis and Nagurney (1993) and has been applied by Nagurney and Zhang (1996) and Zhang and Nagurney (1997) to solve the variational inequality problem (28) in path flows. Obviously, this procedure is correct if the continuity of the solution is guaranteed. Continuity results for solutions to evolutionary variational inequalities, in the case where $F(x(t)) = A(t)x(t) + B(t)$ is a linear operator, $A(t)$ is a continuous and positive definite matrix in $[0, T]$, and $B(t)$ is a continuous vector can be found in Barbagallo (2005). Of course, the examples could also be computed via the computational procedure given in Daniele, Maugeri, and Oettli (1999) but here we utilize a time-discretization approach which also has intuitive appeal.

The Euler method was implemented in FORTRAN and the computer system used was a Sun system at the University of Massachusetts at Amherst. The convergence criterion utilized was that the absolute value of the path flows between two successive iterations differed by no more than 10^{-5}. The sequence $\{\alpha_\tau\}$ in the Euler method (cf. Nagurney and Zhang (1996)) was set to: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. The Euler method was initialized by distributing the demand for each O/D pair equally among the paths connecting the respective O/D pair for each discretized point in time. We embedded the Euler method with the exact

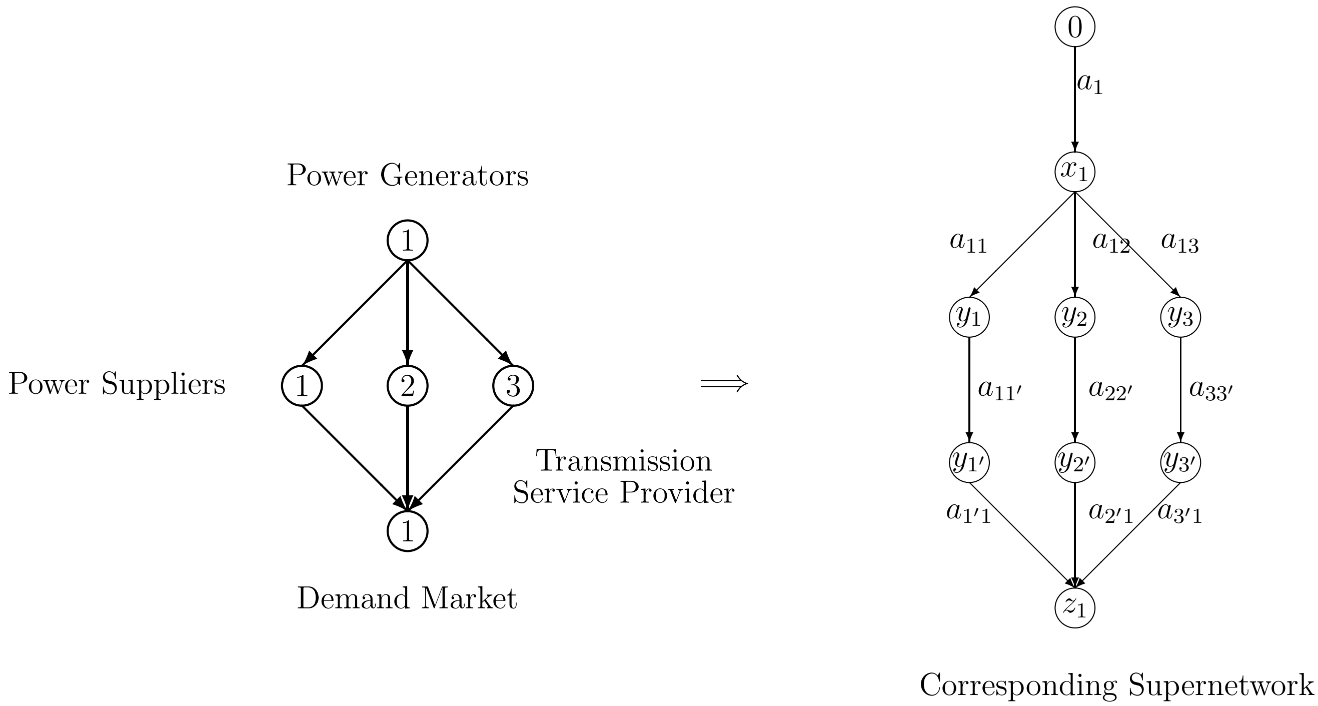

Figure 3: Electric Power Supply Chain Network and Corresponding Supernetwork $\mathcal{G}_{\mathcal{S}}$ for Numerical Example 1

equilibration algorithm of Dafermos and Sparrow (1969) to solve the resulting quadratic programming problems of special network structure in closed form (see also Nagurney and Zhang (1996)).

Example 1

In the first numerical example, the electric power supply chain network consisted of one power generator, three power suppliers, one transmission provider, and one demand market as depicted in Figure 3. The supernetwork representation which allows for the transformation (as proved in Section 4) to a transportation network equilibrium problem is given also in Figure 3. Hence, in the first numerical example (see also Figure 2) we had that: $G = 1$, $S = 3$, $S' = 3'$, $V = 1$, and $K = 1$.

The notation is presented here in the form of the electric power supply chain network models as delineated in Table 1 in Section 2 but now we make the time dimension explicit and, hence, the functions depend on time, as in the preceding section. We then provide the complete supernetwork representation in terms of O/D pairs, paths, etc. The translations of the equilibrium path flows, link flows, and travel disutilities into the equilibrium electric power flows and prices is then given, for completeness and easy reference.

The power generating cost function for the power generator was given by:

$$f_1(q_1(t)) = 2.5(q_1(t))^2 + 2q_1(t).$$

The transaction cost functions faced by the power generator and associated with transacting with the power suppliers were given by:

$$c_{11}(q_{11}(t)) = .5(q_{11}(t))^2 + 3.5q_{11}(t), \quad c_{12}(q_{12}(t)) = .5(q_{12}(t))^2 + 2.5q_{12}(t),$$

$$c_{13}(q_{13}(t)) = .5(q_{13}(t))^2 + 1.5q_{13}(t).$$

The operating costs of the power suppliers, in turn, were given by:

$$c_1(Q^1(t)) = .5(q_{11}(t))^2, \quad c_2(Q^1(t)) = .5(q_{12}(t))^2, \quad c_3(Q^1(t)) = .5(q_{13}(t))^2.$$

The unit transaction costs associated with transacting between the power suppliers and the demand market were:

$$\hat{c}^1_{11}(Q^2(t)) = q^1_{11}(t) + 1, \quad \hat{c}^1_{21}(Q^2(t)) = q^1_{21}(t) + 5, \quad \hat{c}^1_{31}(Q^2(t)) = q^1_{31}(t) + 10.$$

All other costs were set equal to zero.

We utilized the supernetwork representation of this example depicted in Figure 3 with the links enumerated as in Figure 3 in order to solve the problem. Note that there are 9 nodes and 10 links in the supernetwork in Figure 3. Using the procedure outlined in Section 4, we defined O/D pair $w_1 = (0, z_1)$ with the user link travel cost functions as given in (40) – (43) and the path costs as in (44).

There were three paths in P_{w_1} denoted by: p_1, p_2, p_3. The paths were comprised of the following links:

$$p_1 = (a_1, a_{11}, a_{11'}, a_{1'1}), \quad p_2 = (a_1, a_{12}, a_{22'}, a_{2'1}), \quad p_3 = (a_1, a_{13}, a_{33'}, a_{3'1}).$$

The time horizon $T = 1$. The time-varying demand function was given by:

$$d_{w_1}(t) = d_1(t) = 41 + 10t.$$

We discretized the time horizon T as follows: $t_0 = 0$, $t_1 = \frac{1}{2}$, and $t_2 = T = 1$. We report the solutions obtained by the Euler method at each discrete time step, for which we had, respectively, demands: $d_1(t_0) = 41$; $d_1(t_1) = 46$, and $d_1(T) = 51$.

In reporting the equilibrium solutions at a particular discrete point in time we suppress the time index (and we do the same for Example 2), for simplicity.

Example 1 Solution at time $t = t_0 = 0$:

The Euler method converged and yielded the following equilibrium path flow pattern:

$$x^*_{p_1}(t_0) = 14.78, \quad x^*_{p_2}(t_0) = 13.78, \quad x^*_{p_3}(t_0) = 12.45,$$

with the incurred travel costs on the paths at $t = t_0$ being equal to: $C_{p_1} = C_{p_2} = C_{p_3} = \lambda^*_{w_1} = 255.83$.

The corresponding equilibrium link flows (cf. also the supernetwork in Figure 3) at $t = 0$ were:

$$f^*_{a_1}(t_0) = 41.00,$$

$$f^*_{a_{11}}(t_0) = 14.78, \quad f^*_{a_{12}}(t_0) = 13.78, \quad f^*_{a_{13}}(t_0) = 12.45,$$

$$f^*_{a_{11'}}(t_0) = 14.78, \quad f^*_{a_{22'}}(t_0) = 13.78, \quad f^*_{a_{33'}}(t_0) = 12.45,$$

$$f^*_{a_{1'1}}(t_0) = 14.78, \quad f^*_{a_{2'1}}(t_0) = 13.78, \quad f^*_{a_{3'1}}(t_0) = 12.45.$$

We now provide the translations of the above equilibrium flows into the electric power supply chain network flow and price notation using (40) – (43) and (45) – (49).

The power flows at $t = t_0 = 0$ were:

$$q^*_{11}(t_0) = 14.78, \quad q^*_{12}(t_0) = 13.78, \quad q^*_{13}(t_0) = 12.45,$$

$$q^{1*}_{11}(t_0) = 13.78, \quad q^{1*}_{21}(t_0) = 12.45, \quad q^{1*}_{31}(t_0) = 14.78.$$

The demand price at the demand market was: $\rho^*_{31} = 255.83$, which corresponds to the travel costs on the paths (all are used) connecting the O/D pair.

It is easy to verify that the equilibrium conditions were satisfied with excellent accuracy.

Example 1 Solution at time $t = t_1 = \frac{1}{2}$:

The Euler method converged and yielded the following equilibrium path flow pattern:

$$x^*_{p_1}(t_1) = 16.44, \quad x^*_{p_2}(t_1) = 15.44, \quad x^*_{p_3}(t_1) = 14.11.$$

The path costs were now: $C_{p_1} = C_{p_2} = C_{p_3} = \lambda^*_{w_1} = 285.83$.

The corresponding equilibrium link flows at $t = t_1 = \frac{1}{2}$ (cf. also the supernetwork in Figure 3) were:

$$f^*_{a_1}(t_1) = 46.00,$$

$$f^*_{a_{11}}(t_1) = 16.44, \quad f^*_{a_{12}}(t_1) = 15.44, \quad f^*_{a_{13}}(t_1) = 14.11,$$

$$f^*_{a_{11'}}(t_1) = 16.44, \quad f^*_{a_{22'}}(t_1) = 15.44, \quad f^*_{a_{33'}}(t_1) = 14.11,$$

$$f^*_{a_{1'1}}(t_1) = 16.44, \quad f^*_{a_{2'1}}(t_1) = 15.44, \quad f^*_{a_{3'1}}(t_1) = 14.11.$$

The translations into the corresponding equilibrium power flows can be easily done as above for time t_0.

The demand price at the demand market at $t = t_1 = \frac{1}{2}$ was: $\rho^*_{31} = 285.83$, which corresponds to the travel costs on the paths (all paths are again used) connecting the O/D pair.

It is easy to verify that the equilibrium conditions were again satisfied with excellent accuracy.

Example 1 Solution at time $t = T = 1$:

We applied the Euler method to the end of the time horizon where $T = 1$. The Euler method now yielded the following equilibrium path flow pattern:

$$x^*_{p_1}(T) = 18.11, \quad x^*_{p_2}(T) = 17.11, \quad x^*_{p_3}(T) = 15.78,$$

with the associated path costs: $C_{p_1} = C_{p_2} = C_{p_3} = 315.84$.

The corresponding equilibrium link flows at $T = 1$ (cf. also the supernetwork in Figure 3) were:

$$f^*_{a_1}(T) = 51.00,$$

$$f^*_{a_{11}}(T) = 18.11, \quad f^*_{a_{12}}(T) = 17.11, \quad f^*_{a_{13}}(T) = 15.78,$$

$$f^*_{a_{11'}}(T) = 18.11, \quad f^*_{a_{22'}}(T) = 17.11, \quad f^*_{a_{33'}}(T) = 15.78,$$

$$f^*_{a_{1'1}}(T) = 18.11, \quad f^*_{a_{2'1}}(T) = 17.11, \quad f^*_{a_{3'1}}(T) = 15.78.$$

The translations into the corresponding equilibrium power flows can be easily done as above for time t_0.

The demand price at time $t = T = 1$ the demand market was now: $\rho^*_{31} = 315.83$, which corresponds to the travel costs on the paths (again all are used) connecting the O/D pair.

Explicit Formulae

We now note that, due to the linearity of F in this example, as well as the separability of the components of F, and the special nature of the underlying network topology of the supernetwork in Figure 3, we can write down explicit formulae for the path flows over time $[0, T]$. See also, Dafermos and Sparrow (1969) who made the same observation in the context of transportation network equilibrium problems on networks in which all paths connecting an O/D pair consisted of single links, and the user link cost functions were linear and separable. Cojocaru, Daniele, and Nagurney (2005a, b) provided explicit formulae for solutions to dynamic transportation networks over networks of such special structure and costs for specific examples.

In particular, we obtain the following formulae for the equilibrium path flows for Example 1 at each point t:

$$x^*_{p_1}(t) = 3.33t + 14.78,$$

$$x^*_{p_2}(t) = 3.33t + 13.78,$$

$$x^*_{p_3}(t) = 3.33t + 12.45,$$

and these formulae are valid even for $T > 1$, that is, outside the range $[0, 1]$, which is of concern here. We also have an explicit formulae for the travel disutility where:

$$\lambda^*_{w_1}(t) = 60t + 255.83, \quad \text{for} \quad t \in [0, T].$$

Example 2

In the second numerical example, the electric power supply chain network consisted of two power generators, one power supplier, one transmitter, and two demand markets. Hence, we now had that $G = 2$, $S = 1$, $V = 1$, and $K = 2$ as depicted in Figure 4.

The data were now as follows: The power generating cost functions for the power generators were given by:

$$f_1(q(t)) = 2.5(q_1(t))^2 + q_1(t)q_2(t) + 2q_1(t), \quad f_2(q(t)) = 2.5(q_2(t))^2 + q_2(t)q_1(t) + 2q_2(t).$$

The transaction cost functions faced by the power generator and associated with transacting with the power suppliers were given by:

$$c_{11}(q_{11}(t)) = .5(q_{11}(t))^2 + 3.5q_{11}(t), \quad c_{21}(q_{21}(t)) = .5(q_{21}(t))^2 + 1.5q_{21}(t).$$

The operating cost of the power supplier, in turn, was given by:

$$c_1(Q^1(t)) = .5(q_{11}(t))^2.$$

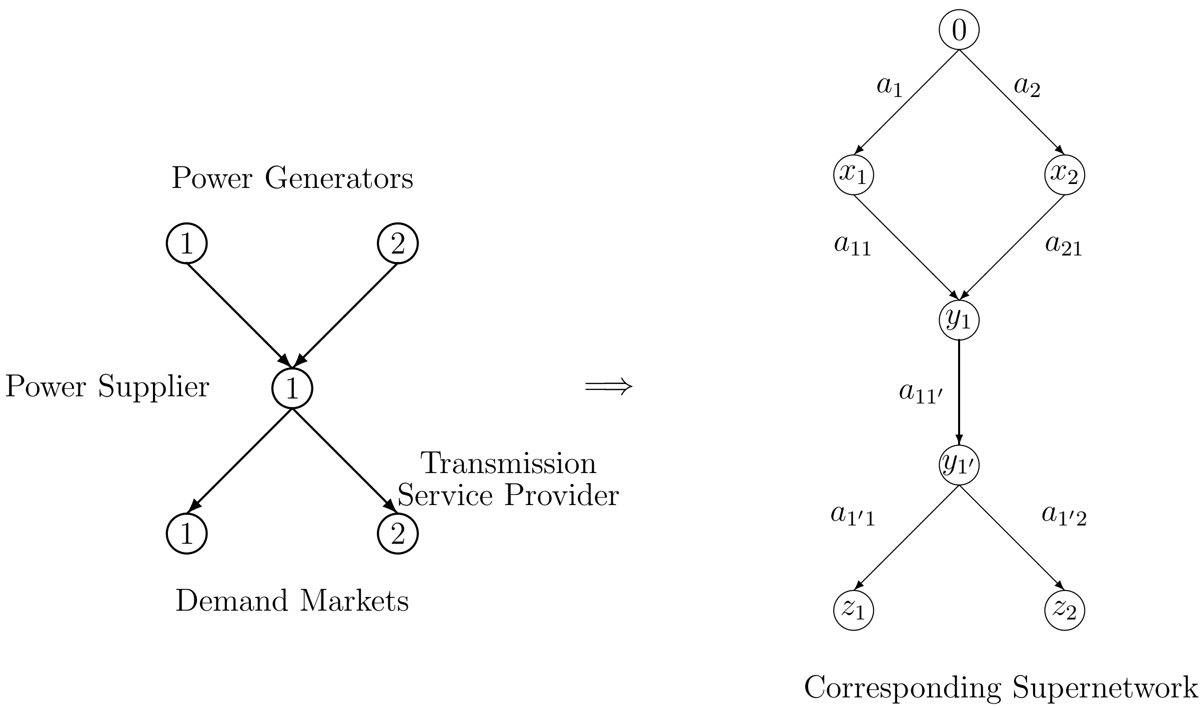

Figure 4: Electric Power Supply Chain Network and Corresponding Supernetwork $\mathcal{G}_{\mathcal{S}}$ for Numerical Example 2

The unit transaction costs associated with transacting between the power suppliers and the demand market were:

$$\hat{c}^1_{11}(Q^2(t)) = q^1_{11}(t) + 1, \quad \hat{c}^1_{12}(Q^2(t)) = q^1_{12}(t) + 1,$$

with all other cost functions being set to zero, with the t in the superscript denoting transmitter t.

We utilized the supernetwork representation of this example depicted in Figure 4 with the links enumerated as in Figure 4 in order to solve the problem. Note that there are 7 nodes and 7 links in the supernetwork in Figure 4. Using the procedure outlined in Section 4, we defined O/D pair $w_1 = (0, z_1)$ and O/D pair $w_2 = (0, z_2)$ with the user link travel cost functions as given in (40) – (43) and the path costs as in (44).

There were two paths in P_{w_1} denoted by: p_1, p_2 and two paths in P_{w_2} denoted by: p_3 and p_4, respectively. The paths were comprised of the following links:

$$p_1 = (a_1, a_{11}, a_{11'}, a_{1'1}),\ p_2 = (a_2, a_{21}, a_{11'}, a_{1'1}),\ p_3 = (a_1, a_{11}, a_{11'}, a_{1'2}),\ p_4 = (a_2, a_{21}, a_{11'}, a_{1'2}).$$

The time horizon $T = 1$. The time-varying demand functions were given by:

$$d_{w_1}(t) = d_1(t) = 100 + 5t, \quad d_{w_2}(t) = d_2(t) = 80 + 4t.$$

We discretized the time horizon T as follows: $t_0 = 0$, $t_1 = \frac{1}{2}$, and $t_2 = T = 1$. We report the solutions obtained by the Euler method at each discrete time step, for which we had, respectively, demands: $d_1(t_0) = 100$, $d_1(t_1) = 102.5$, and $d_1(T) = 105$, and $d_2(t_0) = 80$, $d_2(t_1) = 82$, and $d_2(T) = 84$.

Example 2 Solution at time $t = t_0 = 0$:

We applied the Euler method to the beginning of the time horizon where $t = t_0 = 0$. The Euler method now yielded the following equilibrium path flow pattern:

$$x^*_{p_1}(t_0) = 49.90, \quad x^*_{p_2}(t_0) = 50.10, \quad x^*_{p_3}(t_0) = 39.90, \quad x^*_{p_4}(t_0) = 40.10.$$

The path costs were: $C_{p_1} = C_{p_2} = 815.50 = \lambda^*_{w_1}, \quad C_{p_3} = C_{p_4} = 815.50 = \lambda^*_{w_2}$.

The corresponding equilibrium link flows at time $t = t_0 = 0$ (cf. also the supernetwork in Figure 4) were:

$$f^*_{a_1}(t_0) = 89.80, \quad f^*_{a_2}(t_0) = 90.20$$

$$f^*_{a_{11}}(t_0) = 89.80, \quad f^*_{a_{21}}(t_0) = 90.20,$$

$$f^*_{a_{11'}}(t_0) = 180.00,$$

$$f^*_{a_{1'1}}(t_0) = 100.00, \quad f^*_{a_{1'2}}(t_0) = 80.00.$$

The translations into the corresponding equilibrium power flows are now given:

$$q^*_{11}(t_0) = 89.80, \quad q^*_{21}(t_0) = 92.90,$$

$$q^{1*}_{11}(t_0) = 100.00, \quad q^{1*}_{12}(t_0) = 80.00.$$

The demand prices at the demand markets at $t = t_0 = 0$ were: $\rho^*_{31} = 815.50$, $\rho^*_{32} = 815.50$, which correspond to the travel costs on the paths (all are used) connecting the respective O/D pair.

Example 2 Solution at time $t = t_1 = \frac{1}{2}$:

We applied the Euler method to time $t = t_1 = \frac{1}{2}$. The Euler method now yielded the following equilibrium path flow pattern:

$$x^*_{p_1}(t_1) = 51.15, \quad x^*_{p_2}(t_1) = 51.35, \quad x^*_{p_3}(t_1) = 40.90, \quad x^*_{p_4}(t_1) = 41.10.$$

The incurred path costs were now: $C_{p_1} = C_{p_2} = 835.75 = \lambda^*_{w_1}$, $C_{p_3} = C_{p_4} = \lambda^*_{w_2}$.

The corresponding equilibrium link flows at $t = t_1 = \frac{1}{2}$ (cf. also the supernetwork in Figure 3) were:

$$f^*_{a_1}(t_1) = 92.05, \quad f^*_{a_2}(t_1) = 92.45$$

$$f^*_{a_{11}}(t_1) = 92.05, \quad f^*_{a_{21}}(t_1) = 92.45,$$

$$f^*_{a_{11'}}(t_1) = 184.50,$$

$$f^*_{a_{1'1}}(t_1) = 102.50, \quad f^*_{a_{1'2}}(t_1) = 82.00.$$

The translations into the corresponding equilibrium power flows at time $t = t_1 = \frac{1}{2}$ are now given:

$$q_{11}^*(t_1) = 92.05, \quad q_{21}^*(t_1) = 92.45,$$

$$q_{11}^{1*}(t_1) = 102.50, \quad q_{12}^{1*}(t_1) = 82.00.$$

The demand prices at the demand markets at time $t = t_1 = \frac{1}{2}$ were: $\rho_{31}^* = 835.75$, $\rho_{31}^* = 835.75$, which correspond to the travel costs on the paths (all are used) connecting the respective O/D pair.

Example 2 Solution at time $t = T = 1$:

Finally, we applied the Euler method to the end of the time horizon where $t = T = 1$. The Euler method now yielded the following equilibrium path flow pattern:

$$x_{p_1}^*(T) = 52.40, \quad x_{p_2}^*(T) = 52.60, \quad x_{p_3}^*(T) = 41.90, \quad x_{p_4}^*(T) = 42.10.$$

The path costs were: $C_{p_1} = C_{p_2} = 856.00 = \lambda_{w_1}^*$, $C_{p_3} = C_{p_4} = 856.00 = \lambda_{w_2}^*$.

The corresponding equilibrium link flows at time $t = T = 1$ (cf. Figure 4) were:

$$f_{a_1}^*(T) = 94.30, \quad f_{a_2}^*(T) = 94.70,$$

$$f_{a_{11}}^*(T) = 94.30, \quad f_{a_{21}}^*(T) = 94.70,$$

$$f_{a_{11'}}^*(T) = 189.00,$$

$$f_{a_{1'1}}^*(T) = 105.00, \quad f_{a_{1'2}}^*(T) = 84.00.$$

The translations into the corresponding equilibrium power flows at $t = T = 1$ were:

$$q_{11}^*(T) = 94.30, \quad q_{21}^*(T) = 94.70,$$

$$q_{11}^{1*}(T) = 105.00, \quad q_{12}^{1*}(T) = 84.00.$$

The demand prices at the demand markets at $t = T = 1$ were: $\rho_{31}^* = 856.00$, $\rho_{31}^* = 856.00$, which correspond to the travel costs on the paths (all are used) connecting the respective O/D pair.

These numerical examples demonstrate the types of simulations that can be carried out. For example, one can easily investigate the trends in market prices given different demand patterns over time.

7. Summary and Conclusions

In this paper, we have focused on critical infrastructure networks in the form of electric power supply chains and we have demonstrated that in the case of known demands such problems can be reformulated as transportation network equilibrium problems with fixed demands. We then used the supernetwork equivalence to formulate a dynamic electric power supply chain network model as an evolutionary variational inequality problem in order to model the dynamics as the demands vary over time. We exploited the recent theoretical results obtained by Cojocaru, Daniele, and Nagurney (2005a, b, c) in the unification of projected dynamical systems and evolutionary variational inequalities. We also demonstrated how the theoretical connections can be used to compute solutions to time-varying electric power supply chain networks.

The results in this paper further reinforce the connections between different critical infrastructure networks, notably, electric power networks and transportation networks in terms of common theoretical (see also, e.g., Nagurney (2005) and Nagurney and Liu (2005)) frameworks and substantiate further, as first posed in Beckmann, McGuire, and Winsten (1956) in Chapter 5 of their book on "some unsolved problems," the relationship and application of transportation network equilibrium models to electric power networks (see also McGuire (1997, 1999)). As noted in that classic book on page 5.8, "The unsolved problems concern the application of this model to particular cases." "In particular, the problem of generation and distribution of electric energy in a network comes to mind."

The results in this paper suggest several directions for future research, including, as noted earlier, the incorporation of complete electric power grids into the supernetwork akin to the work of Dafermos and Nagurney (1984b) in spatial price equilibrium, as well as the development of empirical versions of the real-time dynamic electric power supply chain network model proposed in this paper. Of course, one may also want to investigate specific functional forms of setup costs (which in our present framework are embedded in the power generating cost functions) and we leave such an investigation for future research.

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