Traffic Network Equilibrium and the Environment: A Multicriteria Decision-Making Perspective

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Abstract: This paper develops a traffic network equilibrium model in which the users or travelers on the network are assumed to be multicriteria decision-makers with an explicit environmental criterion. The members of a class of traveler perceive their generalized cost on a route as a weighting of travel time, travel cost, and the emissions generated. The model allows the weights to be not only class-dependent but also link-dependent. The multiclass, multicriteria network equilibrium conditions are shown to satisfy a finite-dimensional variational inequality problem. Qualitative properties of the solution are obtained. A special case of the model is then used to obtain sharper results and to illustrate the relationship between the weights and the attainment of a desired environmental quality standard. An algorithm is proposed for the computation of the equilibrium pattern, along with convergence results, and then applied to solve a numerical example. The multiclass, multicriteria network equilibrium model is the first to incorporate an environmental criterion.

1. Introduction

Congested urban transportation networks represent complex systems in which the behavior of the individual users or travelers has implications for the society as a whole. For example, the negative effects of vehicle use, notably, in terms of congestion and pollution, are now well established. Indeed, congestion in the United States results in \$100 billion in lost productivity annually with the figure being approximately \$150 billion in Europe. Moreover, cars and other motor vehicles are responsible for at least 50% of the air pollution in urban areas (see The Economist (1996, 1997)). The World Health Organization (WHO) (cf. Nagurney (2000a)) has estimated that only about 20% of the world's town residents enjoy good enough air quality as measured by the levels of emissions. Specifically, motor vehicles generate about 15% of the world's emissions of carbon dioxide, the principal global warming gas, 50% of the nitrogen oxide emissions, which in combination with other pollutants form nitric acid, which then falls to earth as acid rain, and 90% of the carbon monoxide (cf. Button (1990)).

Clearly, both congestion and environmental issues associated with vehicle use are problems of major concern for our societies today. In recent years, there has been a growing interest in the development of rigorous tools for both congestion and emission control management (see, e.g., Nagurney (2000a) and the references therein). The development has been driven, in part, by legislation. For example, in the United States, the 1990 Clean Air Act Amendments (cf. U. S. DOT (1992a)) and the 1991 Intermodal Surface Transportation Efficiency Act (U. S. DOT (1992b)), in particular, have stimulated a growing interest in transportation management policies, which can affect the total vehicle exhaust emissions, and, consequently, the levels of air pollution.

For any policy, however, to have an appropriate effect, it is imperative that the behavior of the individuals affected by the policy be taken into consideration. In the case of congested urban traffic networks, it has been historically assumed (cf. Beckmann. McGuire, and Winsten (1956)) that users behave in a *user-optimizing* fashion, as opposed to *system-optimizing*, and seek their optimal paths of travel from an origin to a destination so that their travel cost is minimized (see also Dafermos and Sparrow (1969)). The travelers adjust their paths until an equilibrium is reached governed by the equilibrium conditions which state that all used paths connecting each origin/destination pair have equal and minimal travel costs (Wardrop (1952)). For general background on the subject, see Nagurney (1999) and the references therein.

The first *multicriteria* traffic network models were introduced by Quandt (1967) and Schneider (1968) and explicitly considered that travelers may be faced with several criteria, notably, travel time and travel cost, in selecting their optimal routes of travel. Dial (1979), later, proposed an uncongested model. Dafermos (1981), on the other hand, introduced congestion effects and derived an infinite-dimensional variational inequality formulation of her multiclass, multicriteria traffic network equilibrium problem, along with some qualitative properties.

Recently, there has been renewed interest in the formulation, analysis, and computation of multicriteria traffic network equilibrium problems although this approach has not, heretofore, been utilized to examine environmental concerns or criteria (see, e.g., Leurent (1993a, b), (1996), (1998)). For example, Nagurney (2000b) developed a multiclass, multicriteria traffic network equilibrium model in which there were two criteria of travel time and travel cost and formulated the governing equilibrium conditions as a finite-dimensional variational inequality problem (see also Dial (1996)). Nagurney and Dong (2000), in turn, extended the fixed demand model to the case of elastic travel demands and also allowed for distinct weights associated not only with each class of traveler but also with each link. Nagurney, Dong, and Mokhtarian (2000a, b), subsequently, introduced multicriteria network equilibrium models in the contexts of telecommuting and teleshopping, respectively.

Interestingly, Tzeng and Chen (1993) proposed a multicriteria traffic network model with an explicit pollution minimization criterion. But their model assumed system-optimization in which a central controller could control the traffic and route the traffic in a way that was optimal from a system (or societal) perspective. Here, in contrast, we consider the more realistic situation that travelers behave in a user-optimizing manner. Moreover, the governing equilibrium conditions due to the generality of our functions can no longer be reformulated as the solution to an optimization problem and we need to utilize variational inequality theory instead (see Dafermos (1980)). Rilett and Benedek (1994) had considered an environmental criterion in the case of user-optimization, but assumed a single class of user and the criterion of environmental pollution minimization as an alternative one to travel time minimization. Moreover, they assumed that the equilibrium conditions could be reformulated as a solution to an optimization problem and their link "cost" functions were separable. In contrast, we assume that travelers can have several criteria that they take into consideration in their decision-making, notably, travel time, travel cost, and environmental pollution generated.

The paper is organized as follows. In Section 2, we develop the multicriteria traffic network equilibrium model with an explicit environmental criterion. In Section 3, we provide qualitative properties of the equilibrium pattern. In Section 4 we then consider a special case of the model for which we obtain quite sharp results. Moreover, we show that a desired environmental quality standard can be achieved solely through particular weights. The weights are related to an emission pricing policy. In Section 5, we give an algorithm for the computation of the multiclass, multicriteria traffic network equilibrium problem and provide conditions for convergence. In Section 6, we apply the algorithm to a numerical example.

2. The Traffic Network Equilibrium Model with an Environmental Criterion

In this section, we develop the multicriteria traffic network equilibrium model in which there is an explicit environmental criterion. The model is a fixed demand model and allows each class of traveler to perceive the travel cost, the travel time, as well as the pollution generated on each link in the network, in an individual manner. The equilibrium conditions are then shown to satisfy a finite-dimensional variational inequality problem.

We consider a general network $G = [\mathcal{N}, \mathcal{L}]$, where $\mathcal{N}$ denotes the set of nodes in the network and $\mathcal{L}$ the set of directed links. Let a denote a link of the network connecting a pair of nodes and let p denote a path, assumed to be acyclic, consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. There are n links in the network and n_P paths. Let W denote the set of J O/D pairs. The set of paths connecting the O/D pair w is denoted by P_w and the entire set of paths in the network by P.

Assume that there are k classes of travelers in the network with a typical class denoted by i. Let f_a^i denote the flow of class i on link a and let x_p^i denote the nonnegative flow of class i on path p. The relationship between the link loads by class and the path flows is:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{1}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. Hence, the load of a class of traveler on a link is equal to the sum of the flows of the class on the paths that contain that link.

In addition, let f_a denote the total flow on link a, where

$$f_a = \sum_{i=1}^{k} f_a^i, \quad \forall a \in \mathcal{L}. \tag{2}$$

Hence, the total load on a link is equal to the sum of the loads of all classes on that link. Group the class link loads into the kn-dimensional column vector $\tilde{f}$ with components: $\{f_a^1, \ldots, f_n^1, \ldots, f_a^k, \ldots, f_n^k\}$ and the total link loads: $\{f_a, \ldots, f_n\}$ into the n-dimensional column vector f. Also, group the class path flows into the kn_P-dimensional column vector $\tilde{x}$ with components: $\{x_{p_1}^1, \ldots, x_{p_{n_P}}^k\}$.

We are now ready to describe the functions associated with the links. We assume, as given, a travel time function t_a associated with each link a in the network, where

$$t_a = t_a(f), \quad \forall a \in \mathcal{L}, \tag{3}$$

and a travel cost function c_a associated with each link a, that is,

$$c_a = c_a(f), \quad \forall a \in \mathcal{L}, \tag{4}$$

with both these functions assumed to be continuous. Note that here we allow for the general situation in which both the travel time and the travel cost can depend on the entire link load pattern, whereas in Dafermos (1981) it was assumed that these functions were separable.

In addition, in order to capture the environmental costs associated with traveling, we also introduce a continuous emission or pollution function e_a associated with each link in the network, where

$$e_a = e_a(f), \quad \forall a \in \mathcal{L}. \tag{5}$$

For simplicity of notation, we consider only a single pollutant here. Hence, e_a, for example, may correspond to the average emissions of carbon monoxide generated by a traveler on link a.

We assume that each class of traveler i has his own perception of the trade-offs among travel time, travel cost, and emissions generated, which are represented, respectively, by the nonnegative weights w^i_{1a}, w^i_{2a}, and w^i_{3a}. Here w^i_{1a} denotes the weight associated with class i's travel time on link a, w^i_{2a} denotes the weight associated with class i's travel cost on link a, and w^i_{3a} denotes the weight associated with class i's emissions on link a. The weights w^i_{1a}, w^i_{2a}, and w^i_{3a} are link-dependent and, hence, can incorporate such link-dependent factors as safety, comfort, view, sociability factors, as well as sensitivity to pollution in certain areas such as proximity to one's home, for example.

We then construct the *generalized* cost of class i associated with link a, and denoted by u^i_a, as:

$$u^i_a = w^i_{1a} t_a + w^i_{2a} c_a + w^i_{3a} e_a, \quad \forall i, \quad \forall a \in \mathcal{L}. \tag{6}$$

In view of (2) – (6), we may write

$$u_a^i = u_a^i(\tilde{f}), \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{7}$$

and group the generalized link costs into the kn-dimensional column vector u with components: $\{u_a^1, \ldots, u_n^1, \ldots, u_a^k, \ldots, u_n^k\}$.

Link-dependent weights were proposed in Nagurney and Dong (2000) and provide a greater level of generality and flexibility in modeling travel decision-making than weights that are identical for the travel time and for the travel cost on all links for a given class (see Nagurney (2000b)). Link-dependent weights were also used by Nagurney, Dong, and Mokhtarian (2000a,b) to model decision-making as regards telecommuting and teleshopping, respectively.

Let v_p^i denote the generalized travel cost of class i associated with traveling on path p, where

$$v_p^i = \sum_{a \in \mathcal{L}} u_a^i(\tilde{f}) \delta_{ap}, \quad \forall i, \forall p. \tag{8}$$

Hence, the generalized cost, as perceived by a class, associated with traveling on a path is its weighting of the travel times, the travel costs, and the pollution generated on links which comprise the path. Here, hence, we allow travelers to be environmentally conscious through the incorporation of an explicit environmental criterion which is weighted and part of the generalized cost on a path.

We denote the travel demand associated with origin/destination (O/D) pair w and class i by d_w^i. The travel demands must satisfy the following conservation of flow equations:

$$d_w^i = \sum_{p \in P_w} x_p^i, \quad \forall i, \forall w. \tag{9}$$

The behavioral assumption utilized here is similar to that underlying traffic assignment models (see also, Beckmann, McGuire, and Winsten (1956)) in that we assume that each class of user in the network selects (subject to constraints) his travel path so as to minimize the generalized cost on the path, given that all other users have made their choices.

In particular, we have the following traffic network equilibrium conditions for the problem outlined above:

Multicriteria Traffic Network Equilibrium Conditions

For each class i, for all O/D pairs $w \in W$, and for all paths $p \in P_w$, the flow pattern $\tilde{x}^*$ is said to be in equilibrium if the following conditions hold:

$$v_p^i(\tilde{f}^*) \begin{cases} = \lambda_w^i, & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_w^i, & \text{if} \quad x_p^{i^*} = 0. \end{cases} \tag{10}$$

In other words, all utilized paths by a class connecting an O/D pair have equal and minimal generalized costs.

We define the feasible set $\mathcal{K}$, underlying the respective problem as $\mathcal{K} \equiv \{\tilde{f} \,|\, \tilde{x} \geq 0$, and (1), (2), and (9) hold$\}$. We can write down immediately the variational inequality formulation of the equilibrium conditions (10) using Theorem 1 in Nagurney (2000b) and the proof therein.

Theorem 1: Variational Inequality Formulation

The variational inequality formulation of the multicriteria traffic network model satisfying equilibrium condition (10) is given by: Determine $\tilde{f} \in \mathcal{K}$, satisfying

$$\langle u(\tilde{f}^*)^T, \tilde{f} - \tilde{f}^* \rangle \geq 0, \quad \forall \tilde{f} \in \mathcal{K}, \tag{11a}$$

or, equivalently, in standard variational inequality form (cf. Nagurney (1999)):

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{11b}$$

where $F \equiv u$ and $X \equiv \tilde{f}$, and $\langle \cdot, \cdot \rangle$ denotes the inner product in R^n.

If the generalized cost on a link is composed of only the travel cost and the travel time on a link (with distinct weightings for each class but not by link), then the model collapses to the model of Nagurney (2000b).

3. Qualitative Properties

In this section, we present some qualitative properties of the solution to variational inequality (11a). We then, in Section 4, consider a special case of the model for which we give an interpretation of the weights with policy implications. We emphasize that in order to obtain certain qualitative properties such as, for example, uniqueness, one, typically, has to apply conditions which may be stronger than encountered in practice.

Note that the feasible set $\mathcal{K}$ underlying the variational inequality (11a) is a compact set since the travel demands are fixed and assumed to be bounded. Hence, we have immediately:

Theorem 2: Existence

A solution $\tilde{f}^*$ to variational inequality (11a) is guaranteed to exist.

Proof: The result follows for the standard theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)) since the generalized cost functions are continuous and the feasible set is compact. □

We now turn to examining the uniqueness of the equilibrium pattern. In particular, we consider a generalized class cost function on a link a for class i of the *special* form:

$$u_a^i = w_{1a}^i t_a + w_{2a}^i c_a + (1 - w_{1a}^i - w_{2a}^i) e_a, \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{12}$$

where

$$t_a = g_a(f) + \gamma_a, \quad \forall a \in \mathcal{L}, \tag{13a}$$

$$c_a = g_a(f) + \beta_a, \quad \forall a \in \mathcal{L}, \tag{13b}$$

$$e_a = g_a(f) + \kappa_a, \quad \forall a \in \mathcal{L}, \tag{13c}$$

that is, each criterion differs from any other on a link solely by its fixed term. Each of the weights preceding the respective criterion is assumed to be nonnegative and the generalized cost (12) is a weighted average of the three criteria.

Assume now that $g(f)$, where $g(f)$ is the column vector with components: $\{g_a, \ldots, g_n\}$, is strictly monotone, that is,

$$\langle (g(f^1) - g(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}, \quad f^1 \neq f^2. \tag{14}$$

Then we have the following:

Theorem 3: Uniqueness of the Equilibrium Total Link Load Pattern in a Special Case

The total link load pattern f^* induced by the solution $\tilde{f}^*$ to variational inequality (11a), in the case of generalized class link cost functions of the form (12) and (13), is guaranteed to be unique, provided that $g(f)$ is strictly monotone as in (14).

Proof: Assume that there are two solutions to variational inequality (11a), given, respectively, by $\tilde{f}'$ and $\tilde{f}''$ with total link loads corresponding to f' and f''. Then, we must have that $\tilde{f}'$ satisfies:

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\left[w^i_{1a}(g_a(f')+\gamma_a)+w^i_{2a}(g_a(f')+\beta_a)+(1-w^i_{1a}-w^i_{2a})(g_a(f')+\kappa_a)\right]$$
$$\times\left[f^i_a-f^{i\prime}_a\right]\geq 0,\quad \forall \tilde{f}\in\mathcal{K}. \tag{15}$$

Similarly, we must have that $\tilde{f}''$ satisfies:

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\left[w^i_{1a}(g_a(f'')+\gamma_a)+w^i_{2a}(g_a(f'')+\beta_a)+(1-w^i_{1a}-w^i_{2a})(g_a(f'')+\kappa_a)\right]$$
$$\times\left[f^i_a-f^{i\prime\prime}_a\right]\geq 0,\quad \forall \tilde{f}\in\mathcal{K}. \tag{16}$$

Let $\tilde{f}=\tilde{f}''$ and substitute into (15). Also, let $\tilde{f}=\tilde{f}'$ and substitute into (16). Adding the two resulting inequalities yields, after algebraic simplification:

$$\sum_{a\in\mathcal{L}}(g_a(f')-g_a(f''))\times(f'_a-f''_a)\leq 0. \tag{17}$$

But, (17) is in contradiction of the assumption of strict monotonicity of g. Hence, we must have that $f'=f''$. □

In the subsequent theorem, we establish monotonicity of the function F (see (11b)) in the case of generalized link cost functions of the special form (12) and (13).

Theorem 4: Monotonicity in a Special Case

Assume that the generalized link cost functions are of the form (12) and (13), where the monotonicity condition given by

$$\langle (g(f^1)-g(f^2))^T, f^1-f^2\rangle \geq 0, \quad \forall f^1, f^2 \in \mathcal{K}, \tag{18}$$

holds. Then, the function F that enters the variational inequality (11b) with such generalized link cost functions is monotone.

Proof: We must show that

$$\langle (F(X^1)-F(X^2))^T, X^1-X^2\rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{19}$$

where F, X, and $\mathcal{K}$ are as defined following (11b).

Note that

$$\langle (F(X^1)-F(X^2))^T, X^1-X^2\rangle$$
$$= \sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\Big[\Big[w^i_{1a}(g_a(f^1)+\gamma_a)+w^i_{2a}(g_a(f^1)+\beta_a)+(1-w^i_{1a}-w^i_{2a})(g_a(f^1)+\kappa_a)\Big]$$
$$-\Big[w^i_{1a}(g_a(f^2)+\gamma_a)+w^i_{2a}(g_a(f^2)+\beta_a)+(1-w^i_{1a}-w^i_{2a})(g_a(f^2)+\kappa_a)\Big]\Big]\times\Big[f^{i1}_a-f^{i2}_a\Big]$$
$$=\sum_{a\in\mathcal{L}}\Big[g_a(f^1)-g_a(f^2)\Big]\times\Big[f^1_a-f^2_a\Big]. \tag{20}$$

But, (20) must be greater than or equal to zero, under assumption (18), and, hence, the conclusion follows. □

Also, we have the following result:

Theorem 5: Lipschitz Continuity

If the generalized link cost functions have bounded first-order derivatives then the function F that enters variational inequality (11b) is Lipschitz continuous, that is, there exists a constant $L>0$, such that

$$\|F(X^1)-F(X^2)\| \leq L\|X^1-X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{21}$$

Proof: See the proof of Lemma 2 in Nagurney (2000b).

Note that Theorem 5 holds for *any* generalized link cost functions such that they have bounded first-order derivatives, whereas Theorem 4 establishes monotonicity in a special case of the functions. Nevertheless, F may be monotone in the case of other generalized link cost functions. Nagurney (2000b) presents an example, in the context of a multicriteria traffic network equilibrium problem with fixed travel demands, in which there are two classes of users of the network and two criteria, with a *monotone* F for that problem, and the link cost functions are not of the form (12) and (13).

In the next section, we investigate another special case of the general model developed here for which we provide not only qualitative properties but also describe what weights will guarantee that a desired environmental quality standard, in terms of the total emissions generated, will not be exceeded. The result suggests that environmental standards may be achieved through behavior modification (in addition to pricing).

4. A Bicriteria Model with Policy Implications

We now consider the following special case of the general model described in the preceding section. The travelers consider two criteria: travel cost and pollution generated and seek to minimize both. (Analogously, we could consider travel time and emissions generated and the same results would follow through.) With increasing concern of the degradation of the environment due to pollution caused by vehicle use, it is not unreasonable to assume that certain classes of travelers include an environmental criterion into their decision-making. Indeed, with the growing research of Intelligent Transportation Systems, we can expect not only travel times/costs to be broadcast to travelers on the networks but emissions, as well.

Specifically, we assume that the generalized cost on a link for a class (cf. (6)) has the following weights: $w^i_{1a} = 0$, $w^i_{2a} = 1$, and $w^i_{3a} = w^i$, with $w^i > 0$, for all links a and classes i. Hence, we assume that the generalized cost on a link for a class depends only on the travel time and the emissions generated, that is, the generalized cost on a link for a class is now given by:

$$u^i_a = c_a + w^i e_a, \quad \forall i, \quad \forall a \in \mathcal{L}. \tag{22}$$

Clearly, the generalized costs of the form (22) are distinct from those given by (12).

Furthermore, we assume that the emission function e_a is precisely equal to the emission factor on a link a, h_a, which, as discussed in Nagurney (2000a) (see, also, DeCorla-Souza et al. (1995), Anderson et al. (1996), and Allen (1996)), denotes the emission generated by a single traveler traveling on link a. Hence, the generalized cost on link a for class i is now assumed to be:

$$u^i_a = c_a + w^i h_a, \quad \forall i, \quad \forall a \in \mathcal{L}. \tag{23}$$

The generalized cost on a path p for class i is still given by the expression (8). For convenience, we now rewrite the network equilibrium conditions (10) explicitly for this model, where we denote the emissions generated by a traveler on path p by $h_p = \sum_{a \in L} h_a \delta_{ap}$.

Network Equilibrium Conditions for a Bicriteria Model

Hence, the multicriteria network equilibrium conditions with the generalized path costs explicitly stated for this specialized bicriterion model, take the form: For each class i, for all

O/D pairs $w \in W$, and each path $p \in P_w$:

$$C_p(\tilde{f}^*) + w^i h_p \begin{cases} = & \bar{\lambda}_w, \quad \text{if } x_p^{i*} > 0 \\ \geq & \bar{\lambda}_w, \quad \text{if } x_p^{i*} = 0. \end{cases} \tag{24}$$

We may immediately write down variational inequality (11b) for this problem, where we seek to determine $\tilde{f}^* \in \mathcal{K}$ such that

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} \left[\tilde{c}_a(\tilde{f}^*) + w^i h_a \right] \times \left[f_a^i - f_a^{i*} \right] \geq 0, \quad \forall \tilde{f} \in \mathcal{K}, \tag{25}$$

where (for consistency of standard variational inequality notation) we have, using (1), (2), and (4), defined $\tilde{c}(\tilde{f}) \equiv c(f)$.

Clearly, it follows from Theorem 2 that a solution to (25) is guaranteed to exist. We now establish uniqueness of the solution to (25), under the assumption that $c(f)$ is strictly monotone, that is,

$$\langle (c(f^1) - c(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}, \quad f^1 \neq f^2. \tag{26a}$$

Using similar arguments as in the proofs of Theorems 3 and 4, respectively, we can establish the following two theorems.

Theorem 6: Uniqueness of the Equilibrium in the Bicriteria Model

The total link load pattern f^* induced by the solution $\tilde{f}^*$ to variational inequality (25), in the case of generalized user link cost functions of the form (23), is guaranteed to be unique, provided that the travel cost function satisfies the strict monotonicity condition (26a).

Theorem 7: Monotonicity in the Bicriteria Model

Assume that the generalized link cost functions are of the form (23), and that the monotonicity condition given by

$$\langle (c(f^1) - c(f^2))^T, f^1 - f^2 \rangle \geq 0, \quad \forall f^1, f^2 \in \mathcal{K} \tag{26b}$$

holds. Then, the function F that enters the variational inequality (11b) with such generalized link cost functions is monotone.

4.2 Relationship to an Emission Pricing Policy Model

We now relate the above bicriteria model to an emission pricing policy model developed in Nagurney (2000a). That model, through either a path-based, or, equivalently, link-based, emission pricing policy guarantees that the environmental quality standard given by $\bar{Q}$, which reflects the total emissions generated in the network, is achieved. Specifically, the emission pricing policies guarantee that the total emissions do not exceed the desired environmental quality standard given by $\bar{Q}$ through the imposition of an appropriate emission price.

Network Equilibrium Conditions in the Presence of an Emission Pricing Policy

The equilibrium conditions in the presence of the policy take, assuming only travel cost on a link, as a criterion, associated weights equal to 1, and a single class of traveler (for details, see Nagurney (2000a)):

For each O/D pair $w \in W$ and each path $p \in P_w$:

$$\hat{C}_p(f^*, \tau^*) = C_p(f^*) + \tau^* \sum_{a \in L} h_a \delta_{ap} \begin{cases} = & \hat{\lambda}_w, & \text{if } x_p^* > 0 \\ \geq & \hat{\lambda}_w, & \text{if } x_p^* = 0. \end{cases} \tag{27}$$

Condition (27) states that all used paths connecting an O/D pair have equal and minimal *generalized* costs on their paths where the generalized cost on a path p is denoted by $\hat{C}_p$. The generalized user cost on a path now includes both the user cost on a path plus the term $\tau^* \sum_{a \in L} h_a \delta_{ap}$. If we interpret τ^* as the marginal cost of emission abatement, then the term $\tau^* \sum_{a \in L} h_a \delta_{ap}$ corresponds to the true cost associated with emission generation by a user of path p.

In addition, according to Nagurney (2000a), one must also have that:

$$\bar{Q} - \sum_{a \in L} h_a f_a^* \begin{cases} = & 0, & \text{if } \tau^* > 0 \\ \geq & 0, & \text{if } \tau^* = 0. \end{cases} \tag{28}$$

We now recall the following emission pricing policies, due to Nagurney (2000a), which are equivalent, but which are, respectively, path-based and link-based. Letting $h_p = \sum_{a \in L} h_a \delta_{ap}$ denote the emissions generated on path p by a user, then setting a price of $\tau^* h_p$ for each user of a path p and on all paths $p \in P$, where τ^* satisfies (27) and (28) guarantees that the users of the network will select their paths according to (27) and (28) and will not exceed the emission bound $\bar{Q}$.

Similarly, since the cost on a path is equal to the sum of the costs on its links, a link emission pricing policy, satisfying (27) and (28) would charge $\tau^* h_a$ for each user of link a in the network and for all links $a \in L$.

The imposition of either the above path or link emission pricing policy would guarantee that the desired emission bound would not be exceeded. The users of the network, assuming as we have throughout, would behave in a user-optimizing fashion and would then select their paths according to (27).

Moreover, conditions (27) and (28) can also be formulated as a variational inequality problem. Indeed, we have the following:

Theorem 8: Variational Inequality Formulation of Network Equilibrium in the Presence of an Emission Pricing Policy (Nagurney (2000a))

A path flow and price pattern $(x^*, \tau^*) \in \mathcal{K}^1$ is an equilibrium of the network equilibrium model with price policy described above if and only if it is a solution to the variational inequality problem:

Path Flow Formulation:

$$\sum_{w \in W} \sum_{p \in P_w} \left[C_p(x^*) + \tau^* \sum_{a \in L} h_a \delta_{ap} \right] \times \left[x_p - x_p^* \right]$$

$$+ \left[\bar{Q} - \sum_{a \in \mathcal{L}} h_a \sum_{p \in P} x_p^* \delta_{ap} \right] \times [\tau - \tau^*] \geq 0, \ \forall (x, \tau) \in \mathcal{K}^1, \tag{29}$$

where $\mathcal{K}^1 \equiv \bar{K}^1 \times R^1_+$ and $\bar{K}^1 \equiv \{x | x \geq 0$ and satisfies $(9)\}$, or, equivalently, $(f^*, \tau^*) \in \mathcal{K}^2$ is an equilibrium link load and price pattern if and only if it satisfies the variational inequality problem:

Link Load Formulation:

$$\sum_{a \in \mathcal{L}} [c_a(f^*) + \tau^* h_a] \times [f_a - f_a^*] + \left[\bar{Q} - \sum_{a \in \mathcal{L}} h_a f_a^* \right] \times [\tau - \tau^*] \geq 0, \quad \forall (f, \tau) \in \mathcal{K}^2, \tag{30}$$

where $\mathcal{K}^2 \equiv \bar{K}^2 \times R^1_+$, *and* $\bar{K}^2 \equiv \{f | \, there \; exists \; an \, x \geq 0 \; satisfying \, (1), (9)\}$.

Proof: See proof of Theorem 5.1 in Nagurney (2000a).

Relationship Between the Bicriteria Model and the Emission Pricing Policy

Assume now that each user of the network has a weight τ associated with the emissions that he generates on a path. Hence, his multicriteria disutility associated with a path p is given by:

$$C_p(x) + \tau h_p, \quad \forall p \in P, \tag{31}$$

where we have used, as previously, the relationship $h_p = \sum_{a \in L} h_a \delta_{ap}$ to denote the emissions generated by an individual on path p. In this multicriteria decision-making setting, the network equilibrium conditions (5) will now take the form:

For each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) + \tau h_p \begin{cases} = & \bar{\lambda}_w, & \text{if } x_p^* > 0 \\ \geq & \bar{\lambda}_w, & \text{if } x_p^* = 0. \end{cases} \tag{32}$$

Clearly, if the weight τ in (32) is set precisely to τ^*, where τ^* corresponds to the marginal cost of emission abatement as in (27) and (28), then the multicriteria network equilibrium pattern satisfying (32) will coincide with the network equilibrium pattern under the emission pricing policy satisfying (27) and (28). Hence, the two flow patterns will coincide.

Thus, if there is no emission pricing policy in place, but, rather, users of the network are now multicriteria decision-makers and explicitly take into consideration not only the cost on a path but also the emissions generated, the desired outcome, that is, not exceeding the emission bound $\bar{Q}$ can be attained as well. Note that it is imperative, however, that the weight τ that the users associate with their emission generation coincides with τ^*. Hence, this value gives us a measure of how environmentally conscious the users need to be in order to have the "standard" achieved independently by the users through their behavior and without any policy imposition.

We now ask the following questions: Can we generalize the above observation to the case of multiple classes and can we identify other weights w^i for the classes i such that the environmental quality standard is not exceeded? We provide the answer through the subsequent theorem and corollary.

Theorem 9

Assume that $w_{\max} = \max_i w^i$ *and* $w_{\min} = \min_i w^i$*, for all* $i = 1, \ldots, k$*, where* τ^* *solves, along with* f^**, variational inequality (30). Also assume that all link cost functions are monotone. Let* f' *denote the total link load pattern induced by* $\tilde{f}'$*, a solution to variational inequality (25). Then, we have the following results regarding to the bound on the total emissions generated by* f'*:*

(i). If $w_{min} > \tau^*$*, then*

$$\frac{(w_{\max} - \tau^*)}{(w_{\min} - \tau^*)} \bar{Q} \geq \sum_{a \in \mathcal{L}} h_a f'_a, \tag{33a}$$

(ii). If $\tau^* > w_{max}$*, then*

$$\frac{(w_{\max} - \tau^*)}{(w_{\min} - \tau^*)} \bar{Q} \leq \sum_{a \in \mathcal{L}} h_a f'_a, \tag{33b}$$

that is, if the weights of the classes are such that they exceed τ^**, then the total emissions generated are bounded above as in (33a), while if the weights of the classes are such that they are less than* τ^**, then the total emissions generated are exceed the bound as in (33b).*

Proof: Let f^* denote the solution to variational inequality (30), let $\tau = \tau^*$, where we write:

$$\sum_{a \in \mathcal{L}} [c_a(f^*) + \tau^* h_a] \times [f_a - f_a^*] \geq 0, \tag{34}$$

or, equivalently,

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} [c_a(f^*) + \tau^* h_a] \times \left[f_a^i - f_a^{i*}\right] \geq 0. \tag{35}$$

Also, let f' denote the solution to variational inequality (25), that is,

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} \left[c_a(f') + w^i h_a\right] \times \left[f_a^i - f_a^{i'}\right] \geq 0. \tag{36}$$

Let $\tilde{f} = f'$ and substitute into (35). Also, let $\tilde{f} = \tilde{f}^*$ and substitute into (36). Adding the two inequalities, applying the monotonicity assumption of the link cost functions, yields:

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} (w^i - \tau^*) h_a f_a^{i*} \geq \sum_{i=1}^{k} \sum_{a \in \mathcal{L}} (w^i - \tau^*) h_a f_a^{i'}. \tag{37}$$

Letting now $w_{\max} = \max_i w^i$ and $w_{\min} = \min_i w^i$, we obtain using (37):

$$(w_{\max} - \tau^*)\bar{Q} \geq \sum_{i=1}^{k} \sum_{a \in \mathcal{L}} (w^i - \tau^*) h_a f_a^{i*} \geq \sum_{i=1}^{k} \sum_{a \in \mathcal{L}} (w^i - \tau^*) h_a f_a^{i'} \geq (w_{\min} - \tau^*) \sum_{a \in \mathcal{L}} h_a f'_a. \tag{38}$$

The conclusion follows. □

Corollary 1

Assume that $w_{\max} = w_{\min} > \tau^*$, *then*

$$\bar{Q} \geq \sum_{a \in \mathcal{L}} h_a f'_a, \tag{39a}$$

that is, the total emissions generated do not exceed the desired environmental quality standard if travelers' weights pollution emission exceed τ^*. *Assume that* $w_{\max} = w_{\min} < \tau^*$, *then*

$$\bar{Q} \leq \sum_{a \in \mathcal{L}} h_a f'_a, \tag{39b}$$

that is, the total emissions generated will exceed the desired environmental quality standard if travelers' weights for pollution emission are less than τ^*.

Proof: Referring to inequality (38), the conclusion is reached. □

Note that the corollary says that if the classes of travelers assign sufficiently high enough weights to their emission generation criterion then the environmental quality standard may be attained without any emission pricing policy. Hence, changing the behavior of travelers in their weighting of the environmental criterion may have the same effect as an emission pricing policy.

5. The Algorithm

In this section, an algorithm is presented which can be applied to solve any variational inequality problem in standard form (see (11b)), that is, Determine $X^* \in \mathcal{K}$, satisfying:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

The algorithm is guaranteed to converge provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists). The algorithm is the modified projection method of Korpelevich (1977).

The statement of the modified projection method is as follows, where $\mathcal{T}$ denotes an iteration counter:

Modified Projection Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and let γ be a scalar such that $0 < \gamma < \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. Korpelevich (1977)) (see (21)).

Step 1: Computation

Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (\bar{X}^{\mathcal{T}} + \gamma F(X^{\mathcal{T}-1})^T - X^{\mathcal{T}-1})^T, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{40}$$

Step 2: Adaptation

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (X^{\mathcal{T}} + \gamma F(\bar{X}^{\mathcal{T}})^T - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{41}$$

Step 3: Convergence Verification

If max $|X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} =: \mathcal{T} + 1$, and go to Step 1.

We now give an explicit statement of the modified projection method for the solution of variational inequality problem (11a) for the multicriteria traffic network equilibrium model with an environmental criterion.

Modified Projection Method for the Solution of Variational Inequality (11a)

Step 0: Initialization

Set $\tilde{f}^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and set γ such that $0 < \gamma < \frac{1}{L}$, where L is the Lipschitz constant for the problem.

Step 1: Computation

Compute $\bar{\tilde{f}}^{\mathcal{T}} \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} (\bar{f}_a^{i\mathcal{T}} + \gamma(u_a^i(\tilde{f}^{\mathcal{T}-1})) - f_a^{i\mathcal{T}-1}) \times (f_a^i - \bar{f}_a^{i\mathcal{T}}) \geq 0, \quad \forall \tilde{f} \in \mathcal{K}. \tag{42}$$

Step 2: Adaptation

Compute $\tilde{f}^{\mathcal{T}} \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} (f_a^{i\mathcal{T}} + \gamma(u_a^i(\bar{\tilde{f}}^{\mathcal{T}}) - f_a^{i\mathcal{T}-1}) \times (f_a^i - f_a^{i\mathcal{T}}) \geq 0, \quad \forall \tilde{f} \in \mathcal{K}. \tag{43}$$

Step 3: Convergence Verification

If $|f_a^{i\mathcal{T}} - f_a^{i\mathcal{T}-1}| \leq \epsilon$, for all $i = 1, \cdots, k$, and all $a \in \mathcal{L}$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

We now state the convergence result for the modified projection method for this model. Note that the algorithm may converge even for functions of more general form provided that they are monotone and Lipschitz continuous. Theorem 6 identifies specific functions for which monotonicity can be readily established.

Theorem 10: Convergence

Assume that the generalized link cost functions u take the form (12) and (13) and are monotone increasing. Also assume that u has bounded first-order derivatives. Then the modified projection method described above converges to the solution of the variational inequality (11a) (and (11b)).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (11b), provided that the function that enters the variational inequality, F is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 2. Lipschitz continuity, in turn, follows from Theorem 5 under the assumption that the generalized cost functions have bounded first-order derivatives.

The conclusion follows. $\square$

6. Numerical Example

In this section, we present a numerical example which illustrates the general model developed in Section 2. The example is solved using the modified projection method for the solution of variational inequality (11a) discussed in the preceding section in order to compute the multiclass, multicriteria network equilibrium pattern. To solve the embedded quadratic programming problem (cf. (42) and (43)), which is actually a traffic assignment problem, we use the equilibration algorithm of Dafermos and Sparrow (1969).

The topology of the network problem is given in Figure 1. The multicriteria network example consists of two classes of users. The convergence criterion was that the absolute value of the path flows at two successive iterations was less than or equal to ϵ with ϵ set to 10^{-3}. The γ parameter used in the modified projection method (see Section 5) was set to .01. The O/D pairs were: $w_1 = (1,8)$ and $w_2 = (2,10)$ with demands for the two classes given by: $d^1_{w_1} = 50$, $d^1_{w_2} = 80$, and $d_{w_1} = 40$, $d_{w_2} = 30$. The demands were equally distributed, for each class, among all the paths connecting the O/D pair to construct the initial path flow pattern.

The algorithm was coded in FORTRAN and the system used was the Dec Alpha system at the University of Massachusetts at Amherst.

The Example

As illustrated in Figure 1, the network consisted of ten nodes, fifteen links, and two O/D pairs where $w_1 = (1,8)$ and $w_2 = (2,10)$ with the demands as given above. The paths connecting the O/D pairs were: for O/D pair w_1: $p_1 = (1,2,7)$, $p_2 = (1,6,11)$, $p_3 = (5,10,11)$, $p_4 = (14)$, and for O/D pair w_2: $p_5 = (2,3,4,9)$, $p_6 = (2,3,8,13)$, $p_7 = (2,7,12,13)$, $p_8 = (6,11,12,13)$, and $p_9 = (15)$. We assumed that members of class 1 were more environmentally conscious and, hence, their weights associated with the environmental pollution criterion (criterion 3) were higher than those for class 1. The weights were constructed as follows: For class 1, the weights were: $w^1_{1,1} = .25$, $w^1_{2,1} = .25$, $w^1_{3,1} = 1.$, $w^1_{1,2} = .25$, $w^1_{2,2} = .25$, $w^1_{3,2} = 1.$, $w^1_{1,3} = .4$, $w^1_{2,3} = .4$, $w^1_{3,3} = 1.$, $w^1_{1,4} = .5$, $w^1_{2,4} = .5$, $w^1_{3,4} = 2.$, $w^1_{1,5} = .4$, $w^1_{2,5} = .5$, $w^1_{3,5} = 1.$, $w^1_{1,6} = .5$, $w^1_{2,6} = .3$, $w^1_{3,6} = 2.$, $w^1_{1,7} = .2$, $w^1_{2,7} = .4$, $w^1_{3,7} = 1.$, $w^1_{1,8} = .3$, $w^1_{2,8} = .5$, $w^1_{3,8} = 1.$, $w^1_{1,9} = .6$, $w^1_{2,9} = .2$, $w^1_{3,9} = 2.$, $w^1_{1,10} = .3$, $w^1_{2,10} = .4$, $w^1_{3,10} = 1.$, $w^1_{1,11} = .2$,

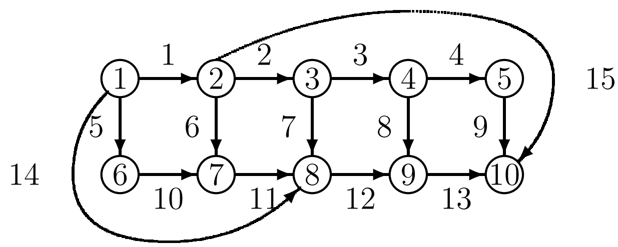

Figure 1: Network Topology for Example 1 – Situation/Problem 1

$w^1_{2,11} = .7$, $w^1_{3,11} = 1.$, $w^1_{1,12} = .3$, $w^1_{2,12} = .4$, $w^1_{3,12} = 1.$, $w^1_{1,13} = .2$, $w^1_{2,13} = .3$, $w^1_{3,13} = 2.$, $w^1_{1,14} = .5$, $w^1_{2,14} = .2$, $w^1_{3,14} = 1.$, $w^1_{1,15} = .5$, $w^1_{2,15} = .3$, $w^1_{3,15} = 1.$,

For class 2: $w^2_{1,1} = .5$, $w^2_{2,1} = .5$, $w^2_{3,1} = .5$, $w^2_{1,2} = .5$, $w^2_{2,2} = .4$, $w^2_{3,2} = .4$, $w^2_{1,3} = .4$, $w^2_{2,3} = .3$, $w^2_{3,3} = .7$, $w^2_{1,4} = .3$, $w^2_{2,4} = .2$, $w^2_{3,4} = .6$, $w^2_{1,5} = .5$, $w^2_{2,5} = .4$, $w^2_{3,5} = .5$, $w^2_{1,6} = .7$, $w^2_{2,6} = .6$, $w^2_{3,6} = .7$, $w^2_{1,7} = .4$, $w^2_{2,7} = .3$, $w^2_{3,7} = .8$, $w^2_{1,8} = .3$, $w^2_{2,8} = .2$, $w^2_{3,8} = .6$, $w^2_{1,9} = .2$, $w^2_{2,9} = .3$, $w^2_{3,9} = .9$, $w^2_{1,10} = .1$, $w^2_{2,10} = .4$, $w^2_{3,10} = .8$, $w^2_{1,11} = .4$, $w^2_{2,11} = .5$, $w^2_{3,11} = .9$, $w^2_{1,12} = .5$, $w^2_{2,12} = .5$, $w^2_{3,12} = .7$, $w^2_{1,13} = .4$, $w^2_{2,13} = .6$, $w^2_{3,13} = .9$, $w^2_{1,14} = .3$, $w^2_{2,14} = .4$, $w^2_{3,14} = 1.$, $w^2_{1,15} = .2$, $w^2_{2,15} = .3$, $w^2_{3,15} = .2$,

The generalized link cost functions were constructed according to (6).

The travel time functions and the travel cost functions were given by:

$$t_1(f) = .00005f_1^4+4f_1+2f_3+2,\ t_2(f) = .00003f_2+2f_2+f_5+1,\ t_3(f) = .00005f_3^4+f_3+.5f_2+3,$$

$$t_4(f) = .00003f_4^4 + 7f_4 + 3f_1 + 1, \quad t_5(f) = 5f_5 + 2, \quad t_6(f) = .00007f_6^4 + 3f_6 + f_9 + 4,$$

$$t_7(f) = 4f_7 + 6, \quad t_8(f) = .00001f_8^4 + 4f_8 + 2f_{10} + 1, \quad t_9(f) = 2f_9 + 8,$$

$$t_{10}(f) = .00003f_{10}^4 + 4f_{10} + f_{12} + 7, \quad t_{11}(f) = .00004f_{11}^4 + 6f_{11} + 2f_{13} + 2,$$

$$t_{12}(f) = .00002 f_{12} f^4 + 4f_{12} + 2f_5 + 1, \quad t_{13}(f) = .00003 f_{13}^4 + 7f_{13} + 4f_{10} + 8,$$

$$t_{14}(f) = f_{14} + 2, \quad t_{15}(f) = f_{15} + 1,$$

and

$$c_1(f) = .00005 f_1^4 + 5f_1 + 1, \quad c_2(f) = .00003 f_2^4 + 4f_2 + 2f_3 + 2, \quad c_3(f) = .00005 f_3^4 + 3f_3 + f_1 + 1,$$

$$c_4(f) = .00003 f_4^4 + 6f_4 + 2f_6 + 4, \quad c_5(f) = 4f_5 + 8, \quad c_6(f) = .00007 f_6^4 + 7f_6 + 2f_2 + 6,$$

$$c_7(f) = 8f_7 + 7, \quad c_8(f) = .00001 f_8^4 + 7f_8 + 3f_5 + 6, \quad c_9(f) = 8f_9 + 5,$$

$$c_{10}(f) = .00003 f_{10}^4 + 6f_{10} + 2f_8 + 3, \quad c_{11}(f) = .00004 f_{11} + 4f_{11} + 3f_{10} + 4,$$

$$c_{12}(f) = .00002 f_{12} + 6f_{12} + 2f_9 + 5,$$

$$c_{13}(f) = .00003 f_1^4 + 9f_{13} + 3f_8 + 3,$$

$$c_{14}(f) = .1 f_{14} + 1, \quad c_{15}(f) = .2 f_{15} + 1.$$

The pollution (emission) functions on the links were as follows:

$$e_1(f) = 2f_1 + 4, \quad e_2(f) = 3f_2 + 2, \quad e_3(f) = f_3 + 4,$$

$$e_4(f) = f_4 + 2, \quad e_5(f) = 2f_5 + 1, \quad e_6(f) = f_6 + 2,$$

$$e_7(f) = f_7 + 3, \quad e_8(f) = 2f_8 + 1, \quad e_9(f) = 3f_9 + 2,$$

$$e_{10}(f) = f_{10} + 1, \quad e_{11}(f) = 4f_{11} + 3, \quad e_{12}(f) = 3f_{12} + 2,$$

$$e_{13}(f) = f_{13} + 1, \quad e_{14}(f) = 6f_{14} + 1, \quad e_{15}(f) = 7f_{15} + 4.$$

The modified projection method converged in 83 iterations. It yielded the following equilibrium multiclass link load pattern:

$$f_1^{1*} = 0.0000, \quad f_2^{1*} = 0.0000, \quad f_3^{1*} = 0.0000, \quad f_4^{1*} = 0.0000, \quad f_5^{1*} = 0.0000,$$

$$f_6^{1*} = 0.0000, \quad f_7^{1*} = 0.0000 \quad f_8^{1*} = 0.0000, \quad f_9^{1*} = 0.0000, \quad f_{10}^{1\,*} = 0.0000,$$

$$f_{11}^{1\,*} = 0.0000, \quad f_{12}^{1\,*} = 0.0000, \quad f_{13}^{1\,*} = 0.0000,$$

$$f_{14}^{1\,*} = 50.0000, \quad f_{15}^{1\,*} = 80.0000,$$

$$f_1^{2*} = 9.2915, \quad f_2^{2*} = 37.6045, \quad f_3^{2*} = 25.9776, \quad f_4^{2*} = 19.1542, \quad f_5^{2*} = 18.9190,$$

$$f_6^{2*} = 1.6870, \quad f_7^{2*} = 11.6269, \quad f_8^{2*} = 6.8233, \quad f_9^{2*} = 19.1542, \quad f_{10}^{2\ *} = 18.9190,$$

$$f_{11}^{2\ *} = 20.6061, \quad f_{12}^{2\ *} = 4.0224, \quad f_{13}^{2\ *} = 10.8458,$$

$$f_{14}^{2\ *} = 11.7895, \quad f_{15}^{2\ *} = 0.0000.$$

with total link loads given by:

$$f_1^* = 9.2915, \quad f_2^* = 37.6045, \quad f_3^* = 25.9776, \quad f_4^* = 19.1542, \quad f_5^* = 18.9190,$$

$$f_6^* = 1.6870, \quad f_7^* = 11.6269, \quad f_8^* = 6.8233, \quad f_9^* = 19.1542, \quad f_{10}^* = 18.9190,$$

$$f_{11}^* = 20.6061, \quad f_{12}^* = 4.0224, \quad f_{13}^* = 10.8458,$$

$$f_{14}^* = 61.7895, \quad f_{15}^* = 80.0000,$$

and induced by the equilibrium multiclass path flow pattern:

$$x_{p_1}^{1\ *} = 0.0000, \quad x_{p_2}^{1\ *} = 0.0000, \quad x_{p_3}^{1\ *} = 0.0000, \quad x_{p_4}{}^* = 50.0000,$$

$$x_{p_5}^{1\ *} = 0.0000, \quad x_{p_6}^{1\ *} = 0.0000, \quad x_{p_7}^{1\ *} = 0.0000, \quad x_{p_8}^{1\ *} = 0.0000,$$

$$x_{p_9}^{1\ *} = 80.0000.$$

$$x_{p_1}^{2\ *} = 9.2915, \quad x_{p_2}^{2\ *} = 0.0000, \quad x_{p_3}^{2\ *} = 18.9190, \quad x_{p_4}^{2\ *} = 11.7895,$$

$$x_{p_5}^{2\ *} = 19.1542, \quad x_{p_6}^{2\ *} = 6.8233, \quad x_{p_7}^{2\ *} = 2.3354, \quad x_{p_8}^{2\ *} = 1.6870,$$

$$x_{p_9}^{2\ *} = 0.0000.$$

The generalized path costs were:

for Class 1, O/D pair w_1:

$$v_{p_1}^1 = 352.6341, \quad v_{p_2}^1 = 338.8106, \quad v_{p_3}^1 = 441.6051, \quad v_{p_4}^1 = 70.5042,$$

for Class 1, O/D pair w_2:

$$v_{p_5}^1 = 690.4105, \quad v_{p_6}^1 = 504.9864, \quad v_{p_7}^1 = 434.4886, \quad v_{p_8}^1 = 420.6651,$$

$$v^1_{p_9} = 102.0000.$$

for Class 2, O/D pair w_1:

$$v^2_{p_1} = 393.6954, \quad v^2_{p_2} = 393.7939, \quad v^2_{p_3} = 393.7939, \quad v^2_{p_4} = 393.7455,$$

and for Class 2, O/D pair w_2:

$$v^2_{p_5} = 510.6390, \quad v^2_{p_6} = 510.6396, \quad v^2_{p_7} = 510.6401, \quad v^2_{p_8} = 510.6401,$$

$$v^2_{p_9} = 1149.30000.$$

The combination of the modified projection method embedded with the equilibration algorithm for the solution of the network subproblems of Steps 1 and 2 of the modified projection method yielded accurate solutions. Indeed, the equilibrium conditions (10) were satisfied with good accuracy.

Note that members of class 1 utilized a single path for each O/D pair and those paths had minimal generalized costs. Members of class 2, however, utilized several of the paths connecting each O/D pair. The generalized costs on the used paths for each O/D pair were minimal and approximately equal (to the desired convergence tolerance accuracy).

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