A Knowledge Collaboration Network Model Across Disciplines

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Abstract. We propose a theoretical framework for the optimal collaboration among researchers in a knowledge network in which researchers are not limited to a single discipline and in which multiple modes of communication, including communication via the Internet, are available. We introduce a novel concept of distance to measure not only the communication distance but also the distance between disciplines. We formulate the knowledge network collaboration model as a variational inequality problem whose solution yields the optimal allocation of effort/time of the researchers as well as the associated opportunity costs.

Key words: Knowledge networks, Scientific collaboration

1 Introduction

Knowledge and the production of knowledge are driving forces in the modern economy. The study of knowledge production and the formation and evolution of knowledge networks, have been addressed by economists, sociologists, as well as management and organizational theorists (see, e.g., [2], [3]). Beckmann significantly advanced the modeling, analysis, and understanding of researchers' behavior in scientific collaboration including collaborations in which the system had an optimally efficient time allocation from an economics perspective. According to his models, which assumed face-to-face communication, the likelihood of two researchers collaborating decreases as the physical distance between them increases. However, by using today's highly developed communication technologies, researchers can now exchange ideas with one another with virtually no time or monetary costs. Moreover, given the availability of current advanced technologies, notably, the Internet, researchers have significantly more options when it comes to selecting modes of collaboration. In today's society, two collaborators may not even have to meet face-to-face. In addition, the effects of advances in research productivity due to advances in technology have also been profound. Gibbons et al. [5] and Hudson [6] emphasized that the emergence of critical technologies greatly impacts researchers' output. Rosenblat and Mobius [10] found that the number of co-authored papers in economics has increased thirty percent since the rise of the Internet.

Another assumption in the original knowledge network collaboration model that merits relaxation is that researchers collaborate exclusively within a field or discipline. Given today's complex world, we believe researchers will have to collaborate interdisciplinarily to solve problems more effectively. Börner et al. [4] indicated that there are many bibliometric studies that address the trends of co-authorship and interdisciplinary research in the literature of collaboration networks. These studies, however, are mainly descriptive in nature in that they focus on longitudinal publication data of certain journals. Although almost all of these studies reach similar conclusions regarding the rising trend of interdisciplinary co-authorship among more spatially separated co-authors, there is no theoretical model to support the empirical findings, which could also illuminate the economic decision-making behavior of the researchers.

2 The Knowledge Collaboration Network Model

Assume that there are N researchers, with a typical researcher denoted by i, j, etc. A researcher is an individual and may represent a particular discipline based on his education, research experiences, etc. Assume that pairs of researchers can collaborate with one another via O modes of communication (such as face-to-face, email, telephone, fax, etc.) with a typical mode of communication denoted by k. The following fundamental assumptions of the model are adopted from [2]:

1. researchers only collaborate in pairs, and
2. a pair of researchers has to mutually agree to collaborate with one another.

However, unlike Beckmann's assumption that the physical distance between two researchers determines the collaboration time, we assume that there are two factors affecting the collaboration time, namely, *virtual distance* and *communication distance* (both are measured in a time scale). According to the most commonly used definition, *interdisciplinarity* is used to refer to increasing levels of interaction among disciplines. The virtual distance is then used as a proxy to represent the time spent by two researchers on understanding each other's discipline. Bibliometric studies of interdisciplinarity can help in identifying a good measure of virtual distance. [8] classified journal publications by using subject categories in Science Citation Index, Social Sciences Citation Index, and the Arts & Humanities Citation Index from the Institute for Scientific Information. The bibliometric data was then employed to establish links between these categories. The strength of the links indicates how close disciplines are to one another. The study results were shown to agree with data in journal publications. We also use the strength of links as a proxy for the measure of virtual distance. In Table 1, the model primary notation appears.

Table 1. Parameters in the Knowledge Collaboration Network Model

Notation	**Definition**
a_{ijk}	coefficient denoting how often researchers i and j communicate via k
r_{1ij}	virtual distance between researchers i and j
r_{2ijk}	communication distance between i and j communicating via k
r_{ijk}	$=r_{1ij} + r_{2ijk}$ – total distance between researchers i and j via k
t_{ijk}	$=1+a_{ijk}r_{ijk}$ – actual time spent in mode k to achieve one time unit of effective collaboration between researchers i and j
T_i	time budget for researcher i
λ_i	opportunity cost for researcher i
λ	N-dimensional vector formed by grouping the λ_i over the i
x_{ijk}	effort of researcher i communicating with j via k in time units
x_{jik}	effort of researcher j communicating with i via k in time units
x	NNO-dimensional vector of the efforts x_{ijk} grouped over i, j, and k
$u_{ijk}(x_{ijk}, x_{jik})$	utility of researcher i collaborating with j via k
$u_{jik}(x_{jik}, x_{ijk})$	utility of researcher j collaborating with i via k

The knowledge collaboration network problem is now formulated as a system-optimization problem. Nodes correspond to researchers and links to different modes of communication. In particular, given a knowledge network system in which there are N researchers, together with their time budget constraints, we wish to determine a time (effort) allocation plan that maximizes the total utility of the knowledge collaboration network as represented by the sum of the utilities of the individual researchers involved in the collaboration network. For example, such an optimization problem may be faced by the manager of R&D in a knowledge organization or company, a director of a research organization, an academic dean of a school of science in a university, or a principal investigator of a major research project. The optimization problem can, hence, be expressed as:

$$\text{Maximize} \quad \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{O} u_{ijk}(x_{ijk}, x_{jik}) \tag{1}$$

subject to:

$$\sum_{j=1}^{N}\sum_{k=1}^{O} t_{ijk}x_{ijk} \le T_i, \qquad i = 1, \ldots, N, \tag{2}$$

$$x \in R_{+}^{N\times N\times O}. \tag{3}$$

We assume that the utility functions are continuously differentiable, concave, and strictly monotonically increasing. Since the time budget constraints are linear, according to the Karush-Kuhn-Tucker conditions (cf. [1]), an optimal solution x^* to (1), subject to (2) and (3), is guaranteed, with the optimality conditions given as follows: for all i, j, k:

$$\frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} + \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}} \begin{cases} = t_{ijk}\lambda^*_i, & \text{if} \quad x^*_{ijk} > 0, \\ \leq t_{ijk}\lambda^*_i, & \text{if} \quad x^*_{ijk} = 0, \end{cases} \tag{4}$$

$$T_i - \sum_{j=1}^{N} \sum_{k=1}^{O} t_{ijk} x^*_{ijk} \begin{cases} = 0, & \text{if} \;\; \lambda^*_i > 0, \\ \geq 0, & \text{if} \;\; \lambda^*_i = 0. \end{cases} \tag{5}$$

According to the definition of t_{ijk} and λ_i, t_{ijk} is the amount of actual time spent to achieve one unit of effective collaboration time while λ_i is the shadow price for each time unit. Hence, $t_{ijk}\lambda_i$ can be interpreted as the cost of time that researcher i is willing to spend on collaborating with researcher j in mode k. The optimality condition (4) can be interpreted as: researcher i will collaborate with researcher j via mode k given that the total marginal utility of the pair of collaborators i and j with respect to i's marginal contribution is equal to the cost of time that i is willing to spend on such collaboration. Researcher i will not collaborate with researcher j via k if the total marginal utility of the pair from i's marginal contribution cannot "cover" the cost of time that i is willing to spend on such collaboration. Since the model is a system-optimization model, the above interpretation is quite intuitive. The condition (5) states that a researcher has positive opportunity cost only if he uses up his time resources.

Theorem 1: Optimal Opportunity Costs
If in the knowledge collaboration network model presented in (1) – (3), the utility functions u_{ijk}, $\forall i, j, k$, are continuously differentiable and strictly monotonically increasing, then the optimal opportunity costs λ^*_i, $\forall i$, are positive.

Proof: Since u_{ijk} is assumed to be continuously differentiable and strictly monotonically increasing, its first-order derivative is positive. Hence, using also (4):

$$t_{ijk}\lambda^*_i \geq \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} + \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}} > 0. \tag{6}$$

Since $t_{ijk} > 0$ the conclusion follows. □

The variational inequality formulation (cf. [9]) of the optimality conditions (4) and (5) is given in the following theorem.

Theorem 2: Variational Inequality Formulation
A solution to the knowledge network collaboration model is an optimal solution if and only if it satisfies the variational inequality problem: determine $(\lambda^*, x^*) \in \mathcal{K}$, where $\mathcal{K} \equiv \{(\lambda, x) \mid (\lambda, x) \in R_+^{N+N\times N\times O}\}$, such that

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{O} (t_{ijk}\lambda^*_i - \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} - \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}}) \times (x_{ijk} - x^*_{ijk})$$

$$+\sum_{i=1}^{N}(T_i - \sum_{j=1}^{N}\sum_{k=1}^{O} t_{ijk}x^*_{ijk}) \times (\lambda_i - \lambda^*_i) \geq 0, \quad \forall(\lambda, x) \in \mathcal{K}. \tag{7}$$

Proof: See [9].

For easy reference in the subsequent sections, variational inequality problem (7) can be rewritten in standard form (cf. [9]): determine $X^* \in \mathcal{K}$ satisfying:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K} \equiv R_+^{N+N\times N\times O}, \tag{8}$$

where $X \equiv (\lambda, x)$, $F(X) \equiv (F_{ijk}, F_i)_{i=1,\dots,N;j=1,\dots,N;k=1,\dots,O}$, with the specific components of F given by the functional terms preceding the multiplication signs in (7), respectively. Here $\langle\cdot,\cdot\rangle$ denotes the inner product in M–dimensional Euclidean space where $M = N + NNO$.

We now impose the following assumptions on the above model: every researcher has a utility function characterized by the property that a pair of collaborators shares the utility (which may also be interpreted as the research credit associated with co-authorship) evenly, that is, $u_{ijk}(x_{ijk}, x_{jik})$ is equal to $u_{jik}(x_{jik}, x_{ijk})$. This assumption is also a fundamental assumption in [2] but in the case of a single mode of collaboration (and single discipline).

With such a "symmetric" utility function for each pair of collaborators, $u_{ijk}(x_{ijk}, x_{jik})$ and $u_{jik}(x_{jik}, x_{ijk})$ are equal everywhere in the feasible set. Therefore, we have that $\frac{\partial u_{ijk}(x_{ijk},x_{jik})}{\partial x_{ijk}} = \frac{\partial u_{jik}(x_{jik},x_{ijk})}{\partial x_{ijk}}$.

The following expressions can, hence, be obtained for the symmetric utility function case from (4):

$$\text{if } x^*_{ijk} > 0, \text{ then } \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} = \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}} = \frac{1}{2}t_{ijk}\lambda^*_i; \tag{9}$$

$$\text{if } x^*_{jik} > 0, \text{ then } \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{jik}} = \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{jik}} = \frac{1}{2}t_{jik}\lambda^*_j. \tag{10}$$

Consequently, the optimality condition (4) can now be interpreted as follows: in the case of symmetric utility functions, researcher i; respectively, j, will collaborate with researcher j; respectively, i, via k only if i's marginal utility is equal to half of the total time cost that he is willing to pay. He will not collaborate if his marginal utility cannot "cover" the time cost he is willing to pay.

Further analysis of the optimality conditions (9) and (10) is also very interesting and worthy of interpretation. First, we discuss the relevant results for a particular collaboration mode. Let's assume that there is a pair of researchers i and j in the knowledge network. We know from Theorem 1 that the opportunity cost in the optimal solution is positive under the assumptions on the utility functions specified before. Therefore, the following results for researcher i can be obtained:

$$\text{if } x^*_{iik} > 0, \text{ then } \frac{\partial u_{iik}(x^*_{iik}, x^*_{iik})}{\partial x_{iik}} = \frac{1}{2}\lambda^*_i; \tag{11}$$

$$\text{if } x^*_{ijk} > 0, \text{ then } \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} = \frac{1}{2}t_{ijk}\lambda^*_i. \tag{12}$$

The analogous results hold for researcher j.

We now discuss three distinct cases for researcher i collaborating with researcher j: **Case i).** i works independently; **Case ii).** i works with j who shares the same discipline with i; **Case iii).** i works with j who is not in the same discipline as i.

We denote the t_{ijk}s in the above cases as: t^1_{iik}, t^2_{ijk}, and t^3_{ijk}, respectively. According to the definition of t_{ijk}, we have that $t^3_{ijk} > t^2_{ijk} > t^1_{iik} = 1$. We also let $u^1_{iik}(x_{iik}, x_{iik})$, $u^2_{ijk}(x_{ijk}, x_{jik})$, and $u^3_{ijk}(x_{ijk}, x_{jik})$ denote researcher i's utility functions corresponding, respectively, to the above three cases.

From (11), (12), and that $t^3_{ijk} > t^2_{ijk} > t^1_{iik} = 1$, we conclude:

$$\frac{\partial u^3_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} > \frac{\partial u^2_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} > \frac{\partial u^1_{iik}(x^*_{iik}, x^*_{iik})}{\partial x_{iik}}. \tag{13}$$

The same relationships as in (13) also hold for researcher j.

From the above discussion, we can see that interdisciplinary collaboration will only occur if the marginal utility of such a collaboration is higher than that of the intradisciplinary collaboration while intradisciplinary collaboration will occur only if the associated marginal utility is higher than that of a researcher working independently. Hence, as we conjectured earlier in this paper, researchers will not collaborate with one another across disciplines unless such collaboration brings them higher "benefit." This phenomenon has been witnessed in many empirical studies.

Moreover, for a pair of researchers i and j, if there exists a collaboration between them via communication mode k, that is, $x^*_{ijk} > 0$ and $x^*_{jik} > 0$, from (12) and the corresponding condition for researcher j, we have that: according to the definitions of t_{ijk} and t_{jik}: $\frac{1}{2}t_{ijk} = \frac{1}{2}t_{jik} = \frac{\frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}}}{\lambda^*_i} = \frac{\frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{jik}}}{\lambda^*_j}$, and, therefore,

$$\frac{\frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}}}{\frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{jik}}} = \frac{\lambda^*_i}{\lambda^*_j}. \tag{14}$$

From (14), we can see that in order to achieve optimality, a researcher with higher opportunity cost must have higher marginal utility and his collaborator who has lower opportunity cost must have a lower marginal utility. Furthermore, given the definition of productivity (the amount of output based on the unit input), if two researchers share the credit evenly, the researcher with lower productivity has to contribute more time to the collaboration in order to compensate for his counterpart's effort. This finding coincides with Beckmann's [2] conclusion.

Having discussed comunication/collaboration via a particular mode, we now analyze the collaboration of a certain pair of researchers across different communication modes. Let's assume that in the optimal solution, researcher i has a collaboration with researcher j and that there are two communication modes available to them, namely, f and h. Let's further assume that the communication

distance r_{2ijf} is larger than r_{2ijh}. According to the definition of t_{ijk}, we know that $t_{ijf} > t_{ijh}$. We now discuss the following three possibilities regarding x^*_{ijf} and x^*_{ijh}:
Case iv). $x^*_{ijf} > 0$ and $x^*_{ijh} > 0$. According to (12), we have that

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{t_{ijf}} = \frac{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}}{t_{ijh}} = \frac{1}{2}\lambda^*_i. \tag{15}$$

By a small perturbation of (15), we obtain

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}} = \frac{t_{ijf}}{t_{ijh}}. \tag{16}$$

We have a "constant elasticity of substitution" between the two communication modes, that is, mode f which consumes more communication time yields a higher marginal utility while mode h which consumes less communication time yields a lower marginal utility. However, a researcher does not care which mode to use in order to collaborate in this case.
Case v). $x^*_{ijf} > 0$ and $x^*_{ijh} = 0$. Via a similar derivation to that constructed for Case iv, we obtain

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}} \geq \frac{t_{ijf}}{t_{ijh}}. \tag{17}$$

In this scenario, a pair of researchers selects mode f for collaboration. Although mode f consumes more communication time, it yields a larger marginal utility that compensates for the additional time. This result is interesting and intuitive. For instance, some collaboration work cannot be completed without collaborators having a meeting in person although other time-efficient communication modes are available. Such cases occur often in physics, for example, when an important experiment has to be conducted and analyzed by both of the collaborators. Although traveling is time-consuming, it may be necessary.
Case vi). $x^*_{ijf} = 0$ and $x^*_{ijh} > 0$. Similarly, we have that, in this case

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}} \leq \frac{t_{ijf}}{t_{ijh}}. \tag{18}$$

Here, the marginal utility associated with mode f cannot compensate for the additional time. Hence, mode f will not be used as a method of communication.

From the above three cases, it is clear that for a communication mode that consumes more communication time to be selected, it must yield a sufficiently large marginal utility to compensate for its additional communication time. Through the inclusion of communication and virtual distance, the model provides insightful interpretations into the manner in which researchers make their decisions in terms of optimal collaboration.

3 Qualitative Properties

Note that the collaboration times, x_{ijk} and x_{jik}, are bounded by each researcher's time budget. The opportunity costs, however, do not lie in a compact set. Hence, according to the standard theory of variational inequalities (cf. [9]), one can impose either a coercivity condition on the vector function $F(X)$ (see (8)) or a boundedness condition in order to guarantee the existence of the opportunity costs. Here, we apply a more direct approach. We note that, from the standard theory of variational inequality theory, optimization problem (1), subject to (2) and (3), under the assumption that the utility functions are concave and continuously differentiable, can also be formulated as the variational inequality problem: determine $x^* \in \mathcal{K}^2$, where $\mathcal{K}^2 \equiv \{x|x \in R_+^{N\times N\times O}$ and satisfies (2), (3)$\}$ such that

$$-\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{O}(\frac{\partial u_{ijk}(x^*_{ijk},x^*_{jik})}{\partial x_{ijk}}+\frac{\partial u_{jik}(x^*_{jik},x^*_{ijk})}{\partial x_{ijk}})\times(x_{ijk}-x^*_{ijk})\geq 0, \quad \forall x\in\mathcal{K}^2. \tag{19}$$

Theorem 3: Existence
Existence of a solution $x^* \in \mathcal{K}^2$ to variational inequality (19) is guaranteed.
Proof: Follows from the standard theory of variational inequalities since the marginal utility functions are continuous and the feasible set $\mathcal{K}^2$ is compact.
Theorem 4: Uniqueness
Assume that the utility functions u_{ijk}, $\forall i,j,k$, are strictly concave functions. Then the optimal effort (time) allocation pattern satisfying variational inequality (19) is unique.
Proof: Under the assumption of strictly concave utility functions, the vector function F^2 with components: F^2_{ijk} given by $-\frac{\partial u_{ijk}(\cdot)}{\partial x_{ijk}} - \frac{\partial u_{jik}(\cdot)}{\partial x_{ijk}}$ is strictly monotone and the conclusion follows (cf. [9]). □

Clearly, under the same assumptions as in Theorem 4, variational inequality (19) also admits a unique optimal effort (time) allocation pattern, from which one can then by using (4) and (5) recover the unique opportunity cost pattern.

4 Computational Procedure and Numerical Examples

We recall the modified projection method [7], which converges if the function F that enters the variational inequality is monotone and Lipschitz continuous and a solution exists, which all hold under our assumptions.

Step 0: Initialization: Set $X^0 \in \mathcal{K}$. Let $\mathcal{T}=1$ and set α so that $0<\alpha\leq\frac{1}{L}$, where L is the Lipschitz continuity constant.
Step 1: Computation: Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle(\bar{X}^{\mathcal{T}}+\alpha F(X^{\mathcal{T}-1})-X^{\mathcal{T}-1})^T, X-\bar{X}^{\mathcal{T}}\rangle\geq 0, \quad \forall X\in\mathcal{K}. \tag{20}$$

Step 2: Adaptation: Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (X^{\mathcal{T}} + \alpha F(\bar{X}^{\mathcal{T}}) - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{21}$$

Step 3: Convergence Verification: If max $|X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

The above computational procedure for the solution of the knowledge collaboration network model yields closed form expressions for (20) and (21) for the collaboration times and the opportunity costs.

The modified projection method described above was implemented in FORTRAN and the computer used was a Sun at the University of Massachusetts Amherst. The convergence criterion was as above with ϵ set to .0001 and we set $\alpha = .01$. We assume that the utilities are directly proportional to the efforts spent and we consider Cobb-Douglas production-type functions, which were also utilized in [2], of the form: $u_{ijk}(x_{ijk}, x_{jik}) = b_{ijk}(x_{ijk})^{1/2}(x_{jik})^{1/2}, \quad \forall i, j, k$. We have that $b_{ijk} = b_{jik}$, which is adopted from Beckmann's notation. We identify the optimal effort allocation plan (and associated opportunity costs) for the knowledge collaboration network model with symmetric utility functions.

In the examples, there are two computer scientists, one operations researcher, and one economist working in a knowledge organization. There are two communication modes with the communication mode represented by the Internet denoted by mode 1 and with the face-to-face communication mode denoted by mode 2. We indexed the four researchers as: 1, 2, 3, and 4, where researchers 1 and 2 are the computer scientists; researcher 3 is the operations researcher, and researcher 4 is the economist. Hence, in the numerical examples, $N = 4$, $O = 2$, and $M = 36$. There are 32 elements in the time/effort allocation vector x and 4 elements in the opportunity cost vector λ. There are 36 variables that we need to compute to determine the optimal solution. In Table 2 we list the data for the relevant parameters: All other t_{ijk} and b_{ijk} terms not reported in Table 2 can be identified from the relationships: $t_{jik} = t_{ijk}$ and $b_{jik} = b_{ijk}$. In addition, recall that $t_{iik} = 1$ for all i, k and we set $b_{iik} = 1$ for all i, i, k.

Example 1: In Example 1, the data were as given above with the time budgets for each researcher $i = 1, \ldots, 4$ being set equal to 100. The modified projection method yielded the following optimal solution: the optimal efforts were given by: $x^*_{121} = x^*_{211} = 100.00$, $x^*_{331} = x^*_{332} = 50.00$, and $x^*_{441} = x^*_{442} = 50$, with all other x_{ijk}s =0.00; the optimal opportunity costs are: $\lambda^*_1 = \lambda^*_2 = 2.00$ and $\lambda^*_3 = \lambda^*_4 = 1.00$. In this example, the computer scientists collaborated only with one another and both the operations researcher and the economist worked alone. Note that all scientists in this knowledge collaboration network used up all the time available in their time budgets. Hence, in this example, there was no interdisciplinary collaboration. The computer scientists collaborated exclusively using communication mode 1.

Example 2: Example 2 was constructed from Example 1 as follows: the data were identical to that in Example 1, except that the time budgets were increased for all researchers so that now we had that: $T_i = 120$, for $i = 1, \ldots, 4$. The

Table 2. Data for Example 1

Collaborators i, j	**Mode** k	$\mathbf{r_{1ij}}$	$\mathbf{r_{2ijk}}$	$\mathbf{r_{ijk}}$	$\mathbf{a_{ijk}}$	$\mathbf{t_{ijk}}$	$\mathbf{b_{ijk}}$
1, 2	1	0	0	0	5	1	2
1, 3	1	3	0	3	5	16	5
1, 4	1	5	0	5	5	26	10
2, 3	1	3	0	3	5	16	5
2, 4	1	5	0	5	5	26	10
3, 4	1	5	0	5	5	26	9
1, 2	2	0	1	1	5	6	1
1, 3	2	3	1	4	5	21	4
1, 4	2	5	1	6	5	31	8
2, 3	2	3	1	4	5	21	6
2, 4	2	5	1	6	5	31	8
3, 4	2	5	1	6	5	31	12

modified projection method yielded the new optimal solution: the optimal efforts were now given by: $x^*_{121} = x^*_{211} = 120.00$, $x^*_{331} = x^*_{332} = 60.00$, and $x^*_{441} = x^*_{442} = 60$, with all other x_{ijk}s =0.00; the optimal opportunity costs were: $\lambda^*_1 = \lambda^*_2 = 2.00$ and $\lambda^*_3 = \lambda^*_4 = 1.00$. Qualitatively, we have the same result as in Example 1, in that an increase in the time budgets still resulted in the computer scientists collaborating with one another, and both the operations researcher and the economist working individually.

Numerous sensitivity analysis exercises and simulations are possible with the above framework as well as game theoretic extensions to capture competition between researchers.

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