International Financial Networks with Intermediation and Electronic Transactions

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Abstract:

In this paper, we develop a multitiered international financial network model with intermediaries and with electronic transactions. The decision-makers associated with the tiers of the network are, respectively: source agents in different countries, financial intermediaries, which need not be country-specific, and consumers at the demand markets for the different financial products in distinct currencies and countries. Those with sources of funds can transact with the intermediaries either physically or electronically as well as directly with the consumers in an electronic manner. The intermediaries, in turn, can transact with the consumers either in a physical or an electronic fashion. We model the behavior of the decision-makers, which in the case of source agents and intermediaries is assumed to be net revenue-maximizing and risk minimizing, derive the equilibrium conditions, and establish the variational inequality formulation. We then utilize the variational inequality formulation to obtain qualitative properties of the equilibrium financial flow and price pattern. An algorithm is proposed, along with convergence results, and then applied to compute the equilibrium financial flows and prices in several numerical examples. This research extends the modeling of international financial networks with intermediation to incorporate electronic transactions.

1. Introduction

The landscape for financial decision-making has been transformed through advances in telecommunications and, in particular, the Internet. Indeed, the adoption of the Internet by businesses, consumers, as well as financial institutions from brokerages to banks has had an immense effect on financial services and the options as well as the means of transaction. New types of products and services have been introduced, new distribution channels made available, and the role of financial intermediaries transformed in this new networked economy.

Importantly, the growth of technology has allowed consumers and businesses to explore and conduct their financial transactions not only within the confines of national boundaries but outside as well. In addition, new currencies such as the euro have been introduced, and new financial products in different currencies and countries. The wealth of choices available for financial transactions in the international arena plus the number of different decision-makers involved, be they sources of financial funds, intermediaries, and/or, ultimately, the consumers of the various financial products, raise major challenges and opportunities for modeling, analysis, and computation.

In this paper, we focus on *financial networks* and develop an international financial network model which includes intermediaries and also allows for the decision-makers to transact either physically or electronically. The framework that we propose predicts the equilibrium financial flows between tiers of the international financial network as well as the equilibrium prices associated with the different tiers. It is sufficiently general to be able to handle as many countries, sources of funds, currencies, and financial products as mandated by the particular application. Although the topic of electronic finance, in general, has received much attention lately (cf. Economides (1996), McAndrews and Stefanidis (2000), Claessens, Glaessner, and Klingebiel (2000, 2001), Long (2000), Allen, Hawkins, and Sato (2001), Sato and Hawkins (2001), Banks (2001), and the references therein), there has been very little work in the modeling, analysis, and solution of such problems, except for that due to Nagurney and Ke (2002) who focused, however, on single country modeling. Fan, Stallaert, and Whinston (2000) have recognized and argued for the necessity of including financial intermediaries in any formal study of electronic finance.

Financial networks date to the work of Quesnay (1758) who introduced a graph to formulate the circular flow of financial funds in the economy. Since that conceptualization, financial flow of funds accounts (cf. Board of Governors (1980), Cohen (1987), and Nagurney and Hughes (1992)) have been utilized to statistically describe the flows of money and credit in an economy, albeit in matrix form. Thore, in (1980), recognized some of the shortcomings of such accounts and developed network models of linked portfolios with intermediation and proposed their solution using decentralization/decomposition theory. Nagurney, Dong, and Hughes (1992), in turn, developed a multi-sector, multi-instrument financial equilibrium model and recognized the network structure in both the formulation and computation of the equilibrium flows. Their approach was based on finite-dimensional variational inequality theory. Since that contribution both single and international financial network equilibrium models have been proposed (cf. Nagurney and Siokos (1997) and the references therein).

More recently, financial networks have been utilized to develop general models with multiple tiers of decision-makers by Nagurney and Ke (2001, 2002) and Nagurney and Cruz (2002). These models differ from the financial network models described in Nagurney and Siokos (1997) in that the behavior of the individual decision-makers associated with the distinct tiers of the network is explicitly captured and modeled. In particular, source agents as well as intermediaries are assumed to be both risk minimizers as well as net revenue maximizers. Consumers, on the other hand, seek to minimize costs, with all decision-makers being subject to transaction costs. Moreover, unlike the framework considered in Thore (1980), more general, including nonlinear and asymmetric functions can be handled.

The paper is organized as follows. In Section 2, we develop the international financial network model. It extends the network model of Nagurney and Cruz (2002) through the explicit incorporation of electronic transactions and that of Nagurney and Ke (2002) by considering an international economy. We describe the various decision-makers and their behaviors, and construct the equilibrium conditions, along with the variational inequality formulation. In Section 3, we derive qualitative properties of the equilibrium pattern, under appropriate assumptions, notably, the existence and uniqueness of a solution to the governing variational inequality. We also establish properties of the function that enters the variational inequality needed for proving convergence of the algorithmic scheme. In Section 4, we present the algorithm, which is then applied in Section 5 to several international financial network examples. We conclude the paper with a summary and discussion in Section 6.

2. The International Financial Network Model with Electronic Transactions

In this Section, we develop the international financial network model with electronic transactions. The model consists of L countries, with a typical country denoted by l or $\hat{l}$; I "source" agents in each country with sources of funds, with a typical source agent denoted by i, and J financial intermediaries with a typical financial intermediary denoted by j. Examples of source agents include households and businesses, whereas examples of financial intermediaries include banks, insurance companies, investment companies, brokers, including electronic brokers, etc.

We assume that each source agent can transact directly electronically with the consumers through the Internet and can also conduct his financial transactions with the intermediaries either physically or electronically in different currencies. There are H currencies in the international economy, with a typical currency being denoted by h. Also, we assume that there are K financial products which can be in distinct currencies and in different countries with a typical financial product (and associated with a demand market) being denoted by k. Hence, the financial intermediaries in the model, in addition to transacting with the source agents, also determine how to allocate the incoming financial resources among distinct uses, which are represented by the demand markets with a demand market corresponding to, for example, the market for real estate loans, household loans, or business loans, etc., which, as mentioned, can be associated with a distinct country and a distinct currency combination. We let m refer to a mode of transaction with $m = 1$ denoting a physical transaction and $m = 2$ denoting an electronic transaction via the Internet.

The international financial network with electronic transactions is now described and depicted graphically in Figure 1. The top tier of nodes consists of the agents in the different countries with sources of funds, with agent i in country l being referred to as agent il and associated with node il. There are, hence, IL top-tiered nodes in the network. The middle tier of nodes consists of the financial intermediaries (which need not be country-specific), with a typical intermediary j associated with node j in this (second) tier of nodes in the network. The bottom tier of nodes consists of the demand markets, with a typical demand market for product k in currency h and country $\hat{l}$ associated with node $kh\hat{l}$. There are, as depicted in Figure 1, J middle (or second) tiered nodes corresponding to the intermediaries

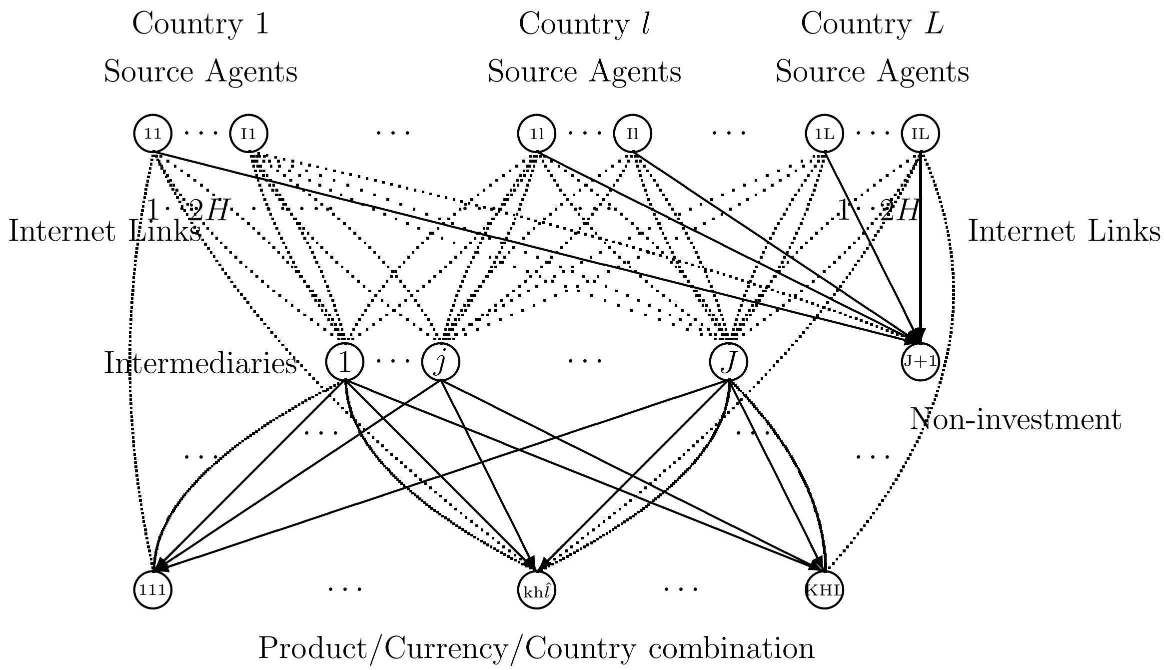

Figure 1: International Financial Network with Internediaries and with Electronic Transactions (Non-investment Allowed)

and KHL bottom (or third) tiered nodes in the financial network. In addition, we add a node $J+1$ to the middle tier of nodes in order to represent the possible non-investment (of a portion or all of the funds) by one or more of the source agents.

Now that we have identified the nodes in the international financial network we turn to the identification of the links joining the nodes in a given tier with those in the next tier. We also associate the financial flows with the appropriate links. We assume that each agent i in country l has an amount of funds S^{il} available in a base currency. Since there are assumed to be H currencies and 2 modes of transaction (physical or electronic), there are $2H$ links joining each top tier node il with each middle tier node j; $j = 1, \ldots, J$, with the first H links representing physical transactions between a source and intermediary, and with corresponding flow on such a link given, respectively, by x^{il}_{jh1}, and the subsequent H links representing electronic transactions with the corresponding flow given, respectively, by

x^{il}_{jh2}. Hence, x^{il}_{jh1} denotes the nonnegative amount invested (across all financial instruments) by source agent i in country l in currency h transacted through intermediary j using the physical mode whereas x^{il}_{jh2} denotes the analogue but for an electronic transaction. We group the financial flows for all source agents/intermediaries/modes into the column vector $x^1 \in R^{2ILJH}_+$. In addition, a source agent i in country l may transact directly with the consumers at demand market k in currency h and country $\hat{l}$ via an Internet link. The nonnegative flow on such a link joining node il with node $kh\hat{l}$ is denoted by $x^{il}_{kh\hat{l}}$. We group all such financial flows, in turn, into the column vector $x^2 \in R^{ILKHL}_+$. Also, we let x^{il} denote the $(2JH + KHL)$-dimensional column vector associated with source agent il with components:$\{x^{il}_{jhm}, x^{il}_{kh\hat{l}}; j = 1, \ldots, J; h = 1, \ldots, H; m = 1, 2; k = 1, \ldots, K; \hat{l} = 1, \ldots, L\}$. Furthermore, we construct a link from each top tiered node to the second tiered node $J + 1$ and associate a flow s^{il} on such a link emanating from node il to represent the possible nonnegative amount not invested by agent i in country l.

Each intermediary node j; $j = 1, \ldots, J$, may transact with a demand market via a physical link, and/or electronically via an Internet link. Hence, from each intermediary node j, we construct two links to each node $kh\hat{l}$, with the first such link denoting a physical transaction and the second such link - an electronic transaction. The corresponding flow, in turn, which is nonnegative, is denoted by $y^j_{kh\hat{l}m}$; $m = 1, 2$, and corresponds to the amount of the financial product k in currency h and country l transacted from intermediary j via mode m. We group the financial flows between node j and the bottom tier nodes into the column vector $y^j \in R^{2KHL}_+$. All such financial flows for all the intermediaries are then further grouped into the column vector $y \in R^{2JKHL}_+$.

The notation for the prices is now given. There will be prices associated with each of the tiers of nodes in the international financial network. The prices are assumed to be in the base currency. Let ρ^{il}_{1jhm} denote a price (in the base currency) associated with the financial instrument in currency h transacted via mode m as quoted by intermediary j to agent il and group the top tier prices into the column vector $\rho_1 \in R^{2ILJH}_+$. In addition, let $\rho^{il}_{1kh\hat{l}}$ denote a price, also in the base currency, associated with the financial instrument as quoted by demand market $kh\hat{l}$ to agent il and group these top tier prices into the column vector $\rho_{12} \in R^{ILKHL}_+$. Let $\rho^j_{2kh\hat{l}m}$, in turn, denote the price associated with intermediary j for product k in currency h and country $\hat{l}$ transacted via mode m and group all such prices into the column vector $\rho_2 \in R_+^{2JKHL}$. Also, let $\rho_{3kh\hat{l}}$ denote the price of the financial product k in currency h and in country $\hat{l}$, and defined in the base currency, and group all such prices into the column vector $\rho_3 \in R_+^{KHL}$. Finally, let e_h denote the rate of appreciation of currency h against the base currency, which can be interpreted as the rate of return earned due to exchange rate fluctuations (see Nagurney and Siokos (1997)). These "exchange" rates are grouped into the column vector $e \in R_+^H$.

We now turn to describing the behavior of the various decision-makers represented by the three tiers of nodes in Figure 1. We first focus on the top-tier decision-makers. We then turn to the intermediaries and, subsequently, to the consumers at the demand markets.

The Behavior of the Agents with Sources of Funds and their Optimality Conditions

We denote the transaction cost associated with source agent il transacting with intermediary j for the instrument in currency h via mode m by c^{il}_{jhm} (and measured in the base currency) and assume that:

$$c^{il}_{jhm} = c^{il}_{jhm}(x^{il}_{jhm}), \quad \forall i, l, j, h, m, \tag{1a}$$

that is, the cost associated with source agent i in country l transacting with intermediary j for the instrument in currency h depends on the volume of the transaction. We denote the transaction cost associated with source agent il transacting with demand market k in country $\hat{l}$ for the instrument in currency h via the Internet link by $c^{il}_{kh\hat{l}}$ (and also measured in the base currency) and assume that:

$$c^{il}_{kh\hat{l}} = c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}}), \quad \forall i, l, k, h, \hat{l}, \tag{1b}$$

that is, the cost associated with source agent i in country l transacting with the consumers for product k in currency h and country $\hat{l}$. The transaction cost functions are assumed to be convex and continuously differentiable and depend on the volume of flow of the transaction.

The total transaction costs incurred by source agent il are equal to the sum of all of his transaction costs associated with dealing with the distinct intermediaries and demand markets in the different currencies. His revenue, in turn, is equal to the sum of the price (rate of return plus the rate of appreciation) that the agent can obtain for the financial instrument times the total quantity obtained/purchased of that instrument. Let now ρ^{il*}_{1jhm} denote the actual price charged agent il for the instrument in currency h by intermediary j by transacting via mode m and let $\rho^{il*}_{1kh\hat{l}}$, in turn, denote the price associated with source agent il transacting electronically with demand market $kh\hat{l}$. Similarly, let e^*_h denote the actual rate of appreciation in currency h. We later discuss how such prices are recovered.

We assume that each such source agent seeks to maximize his net return while, simultaneously, minimizing his risk, with source agent il's objective function denoted by U^{il}. In particular, we assume, as given, a risk function for sector il and denoted by r^{il}, such that

$$r^{il} = r^{il}(x^{il}), \quad \forall i, l, \tag{2}$$

where r^{il} is assumed to be strictly convex and continuously differentiable. Clearly, a possible risk function could be constructed as follows. Assume a variance-covariance matrix Q^{il} associated with agent il, which is of dimension $(2JH + KHL) \times (2JH + KHL)$, symmetric, and positive definite. Then a possible risk function for source agent i in country l would be given by:

$$r^{il}(x^{il}) = x^{il^T} Q^{il} x^{il}, \quad \forall i, l. \tag{3}$$

In such a case, one assumes that each source agent's uncertainty, or assessment of risk, is based on a variance-covariance matrix representing the source agent's assessment of the standard deviation of the prices of the financial instruments in the distinct currencies (see also Markowitz (1959)).

We now construct the optimization problem facing a sector i in country l. In particular, we can express the optimization problem facing agent il as:

$$\text{Maximize} \quad U^{il}(x^{il}) = \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{m=1}^{2} (\rho^{il*}_{1jhm} + e^*_h) x^{il}_{jhm} + \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} (\rho^{il*}_{1kh\hat{l}} + e^*_h) x^{il}_{kh\hat{l}}$$

$$- \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{m=1}^{2} c^{il}_{jhm}(x^{il}_{jhm}) - \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}}) - r^{il}(x^{il}), \tag{4}$$

subject to $x^{il}_{jh} \geq 0$, $x^{il}_{kh\hat{l}m} \geq 0$ for all $j, h, k, \hat{l}, m$, and to the constraint:

$$\sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{m=1}^{2} x^{il}_{jhm} + \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} x^{il}_{kh\hat{l}} \leq S^{il}, \tag{5}$$

that is, the allocations of source agent il's funds among those available from the different intermediaries in distinct currencies and demand market cannot exceed his holdings. The first two terms in the objective function (4) denote the revenue whereas the third and fourth terms denotes the transaction costs and the last term denotes the risk. Note that the objective function given in (4) is strictly concave in the x^{il} variables. Note also that constraint (5) allows a source agent not to invest a portion (or all) of his funds, with the "slack," that is, the funds not invested by agent i in country l being given by s^{il}.

Optimality Conditions for All Source Agents

The optimality conditions of all source agents i; $i = 1, \ldots, I$; in all countries: l; $l = 1, \ldots, L$ (see also Bazaraa, Sherali, and Shetty (1993) and Bertsekas and Tsitsiklis (1992)), under the above stated assumptions on the underlying functions, can be expressed as: determine $(x^{1*}, x^{2*}) \in \mathcal{K}^1$, satisfying

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}} - \rho^{il*}_{1jhm} \; - e^*_h\right] \times \left[x^{il}_{jhm} - x^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}} - \rho^{il*}_{1kh\hat{l}} \; - e^*_h\right] \times \left[x^{il}_{kh\hat{l}} - x^{il*}_{kh\hat{l}}\right] \geq 0, \; \forall (x^1, x^2) \in \mathcal{K}^1, \tag{6}$$

where the feasible set $\mathcal{K}^1 \equiv \{(x^1, x^2) | (x^1, x^2) \in R_+^{IL(2JH+KH\hat{L})}$ and satisfies (3), $\forall i, l\}$.

The Behavior of the Intermediaries and their Optimality Conditions

The intermediaries (cf. Figure 1), in turn, are involved in transactions both with the source agents in the different countries, as well as with the users of the funds, that is, with the ultimate consumers associated with the markets for the distinct types of loans/products in different currencies and countries and represented by the bottom tier of nodes of the network. Thus, an intermediary conducts transactions via a physical link, and/or electronically via an Internet link both with the "source" agents as well as with the consumers at the demand markets.

An intermediary j is faced with what we term a *handling/conversion* cost, which may include, for example, the cost of converting the incoming financial flows into the financial loans/products associated with the demand markets. We denote such a cost faced by intermediary j by c_j and, in the simplest case, we would have that c_j is a function of $\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} x_{jhm}^{il}$, that is, the holding/conversion cost of an intermediary is a function of how much he has obtained in the different currencies from the various source agents in the different countries. For the sake of generality, however, we allow the function to depend also on the amounts held by other intermediaries and, therefore, we may write:

$$c_j = c_j(x^1), \quad \forall j. \tag{7}$$

The intermediaries also have associated transaction costs in regards to transacting with the source agents, which can depend on the type of currency as well as the source agent. We denote the transaction cost associated with intermediary j transacting with agent il associated with currency h via mode m by $\hat{c}_{jhm}^{il}$ and we assume that it is of the form

$$\hat{c}_{jhm}^{il} = \hat{c}_{jhm}^{il}(x_{jhm}^{il}), \quad \forall i, l, j, h, m, \tag{8}$$

that is, such a transaction cost is allowed to depend on the amount allocated by the particular agent to the financial instrument in a currency and transacted with the particular intermediary via the particular mode. In addition, we assume that an intermediary j also incurs a transaction cost $c_{kh\hat{l}m}^{j}$ associated with transacting with demand market $kh\hat{l}$, where

$$c_{kh\hat{l}m}^{j} = c_{kh\hat{l}m}^{j}(y_{kh\hat{l}m}^{j}), \quad \forall j, k, h, \hat{l}, m. \tag{9}$$

Hence, the transaction costs given in (9) can vary according to the intermediary/product/currency/country combination and are a function of the volume of the product transacted. We assume that the cost functions (7) – (9) are convex and continuously differentiable and that the costs are measured in the base currency.

The actual price charged for the financial products by the intermediaries is denoted by $\rho_{2kh\hat{l}m}^{j*}$, for intermediary j and associated with transacting with consumers for product k via mode m in currency h and country $\hat{l}$. Subsequently, we discuss how such prices are arrived at. We assume that the intermediaries are also objective function optimizers with the objective functions for each being comprised of net revenue maximization as well as risk minimization.

We assume that the intermediaries have risk associated both with transacting with the various source agents in the different countries and with the consumers for the products in the different currencies and countries, and which can also depend on the mode of transaction. Hence, we assume for each intermediary j a risk function r^j, which is strictly convex in its variables and continuously differentiable, and of the form:

$$r^j = r^j(x^1, y), \quad \forall j. \tag{10}$$

For example, the risk for intermediary j could be represented by a variance-covariance matrix denoted by Q^j with this matrix being positive definite and of dimensions $(2IL + 2KHL) \times (2IL + 2KHL)$ for each intermediary j. Such a matrix would reflect the risk associated with transacting with the various source agents in the different countries and with the consumers at the demand markets for the products in different currencies and in different countries. If we let x_j, without any loss in generality, denote the $2ILH$-dimensional column vector with the $ilhm$-th component given by x^{il}_{jhm}. Indeed, then a possible risk function for intermediary j could be represented by the function:

$$r^j(x^1, y) = \begin{bmatrix} x_j \\ y^j \end{bmatrix}^T Q^j \begin{bmatrix} x_j \\ y^j \end{bmatrix}. \tag{11}$$

Note that, for the sake of modeling generality and flexibility, we allow the risk function for an intermediary to depend not only on the financial flows flowing "into" and "out of" that intermediary but on the other financial flows as well. The risk function given by (11) is actually a special case of the one in (10) in that it depends only on the financial volumes that the particular intermediary actually deals with.

The optimization problem for intermediary j, with his objective function expressed by U^j, and assuming net revenue maximization and risk minimization can, hence, be expressed as:

$$\text{Maximize} \quad U^j(x_j, y^j) = \sum_{k=1}^{k} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} (\rho^{j*}_{2kh\hat{l}m} + e^*_h) y^j_{kh\hat{l}m} - c_j(x^1) - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} \hat{c}^{il}_{jhm}(x^{il}_{jhm})$$

$$- \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}) - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} (\rho^{il*}_{1jhm} + e^*_h) x^{il}_{jhm} - r^j(x^1, y) \tag{12}$$

subject to: the nonnegativity constraints: $x^{il}_{jhm} \geq 0$, $y^j_{kh\hat{l}m} \geq 0$, for all $i, l, h, \hat{l}, m$, and

$$\sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} y^j_{kh\hat{l}m} \leq \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} x^{il}_{jhm}. \tag{13}$$

Objective function (12) expresses that the difference between the revenues (given by the first term) minus the handling cost, the two sets of transaction costs, and the payout to the source agents (given by the subsequent four terms, respectively) should be maximized, whereas the risk (see the last term in (12)) should be minimized. The objective function in (12) is concave in its variables under the above imposed assumptions.

Here we assume that the financial intermediaries can compete, with the governing optimality/equilibrium concept underlying noncooperative behavior being that of Nash (1950, 1951), which states that each decision-maker (intermediary) will determine his optimal strategies, given the optimal ones of his competitors. The optimality conditions for all financial intermediaries simultaneously, under the above stated assumptions, can be compactly expressed as (see also Gabay and Moulin (1980), Dafermos and Nagurney (1987), and Nagurney and Ke (2001)): determine $(x^{1*}, y^*, \gamma^*) \in R_+^{2ILJH+2JKH\hat{L}+J}$, such that

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^j(x^{1*},y^*)}{\partial x_{jhm}^{il}} + \frac{\partial c_j(x^{1*})}{\partial x_{jhm}^{il}} + \rho_{1jhm}^{il*} + e_h^* + \frac{\partial \hat{c}_{jhm}^{il}(x_{jhm}^{il*})}{\partial x_{jhm}^{il}} - \gamma_j^*\right] \times \left[x_{jhm}^{il} - x_{jhm}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial r^j(x^{1*},y^*)}{\partial y_{kh\hat{l}m}^{j}} + \frac{\partial c_{khl}^{j}(y_{kh\hat{l}m}^{j*})}{\partial y_{kh\hat{l}m}^{j}} - \rho_{2kh\hat{l}m}^{j*} - e_h^* + \gamma_j^*\right] \times \left[y_{kh\hat{l}m}^{j} - y_{kh\hat{l}m}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} x_{jhm}^{il*} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} y_{kh\hat{l}m}^{j*}\right] \times \left[\gamma_j - \gamma_j^*\right] \geq 0,$$

$$\forall (x^1, y, \gamma) \in R_+^{2ILJH+2JKHL+J}, \qquad (14)$$

where γ_j is the Lagrange multiplier associated with constraint (9) (see Bazaraa, Sherali, and Shetty (1993)), and γ is the J-dimensional column vector of Lagrange multiplers of all the intermediaries with γ^* denoting the vector of optimal multipliers.

The Consumers at the Demand Markets and the Equilibrium Conditions

We now describe the consumers located at the demand markets. The consumers take into account in making their consumption decisions not only the price charged for the financial product by the agents with source of funds and intermediaries but also their transaction costs associated with obtaining the product. Recall that there is a distinct product k in currency h and country l.

Let $\hat{c}^j_{kh\hat{l}m}$ denote the transaction cost associated with obtaining product k in currency h in country $\hat{l}$ via mode m from intermediary j and recall that $y^j_{kh\hat{l}m}$ is the amount of the financial product k in currency h flowing between intermediary j and consumers in country l via mode m. We assume that the transaction cost is measured in the base currency, is continuous, and of the general form:

$$\hat{c}^j_{kh\hat{l}m} = \hat{c}^j_{kh\hat{l}m}(y), \quad \forall j, k, h, \hat{l}, m. \tag{15a}$$

Furthermore, let $\hat{c}^{il}_{kh\hat{l}}$ denote the transaction cost associated with obtaining the financial at demand market k in currency h in country $\hat{l}$ electronically from source agent il, where we assume that the transaction cost is continuous and of the general form:

$$\hat{c}^{il}_{kh\hat{l}} = \hat{c}^{il}_{kh\hat{l}}(x^2), \quad \forall i, l, k, h, \hat{l}. \tag{15b}$$

Hence, the transaction cost associated with transacting directly with source agents is of a form of the same level of generality as the transaction costs associated with transacting with the financial intermediaries.

Denote the demand for product k in currency h in country $\hat{l}$ by $d_{kh\hat{l}}$ and assume, as given, the continuous demand functions:

$$d_{kh\hat{l}} = d_{kh\hat{l}}(\rho_3), \quad \forall k, h, \hat{l}. \tag{16}$$

Thus, according to (16), the demand of consumers for the financial product in a currency and country depends, in general, not only on the price of the product at that demand market (and currency and country) but also on the prices of the other products at the other demand markets (and in other countries and currencies). Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

The consumers take the price charged by the intermediary, which was denoted by $\rho^{j*}_{2kh\hat{l}m}$ for intermediary j, product k, currency h, and country $\hat{l}$ via mode m, the price charged by source agent il, which was denoted by $\rho^{il*}_{1kh\hat{l}}$, and the rate of appreciation in the currency, plus the transaction costs, in making their consumption decisions. The equilibrium conditions for the consumers at demand market $kh\hat{l}$, thus, take the form: for all intermediaries: $j = 1, \ldots, J$

and all mode m; $m = 1, 2$:

$$\rho^{j*}_{2kh\hat{l}m} + e^*_h + \hat{c}^j_{kh\hat{l}m}(y^*) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad y^{j*}_{kh\hat{l}m} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad y^{j*}_{kh\hat{l}m} = 0, \end{cases} \tag{17}$$

and for all source agents il;$i = 1, \ldots, I$ and $l = 1, \ldots, L$:

$$\rho^{il*}_{1kh\hat{l}} + e^*_h + \hat{c}^{il}_{kh\hat{l}}(x^{2*}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad x^{il*}_{kh\hat{l}} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad x^{il*}_{kh\hat{l}} = 0, \end{cases} \tag{18}$$

In addition, we must have that

$$d_{kh\hat{l}}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} > 0 \\ \leq \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} = 0. \end{cases} \tag{19}$$

Conditions (17) state that consumers at demand market $kh\hat{l}$ will purchase the product from intermediary j, if the price charged by the intermediary for the product and the appreciation rate for the currency plus the transaction cost (from the perspective of the consumer) does not exceed the price that the consumers are willing to pay for the product in that currency and country, i.e., $\rho^*_{3kh\hat{l}}$. Note that, according to (17), if the transaction costs are identically equal to zero, then the price faced by the consumers for a given product is the price charged by the intermediary for the particular product and currency in the country plus the rate of appreciation in the currency. Condition (18) state the analogue, but for the case of electronic transactions with the source agents.

Condition (19), on the other hand, states that, if the price the consumers are willing to pay for the financial product at a demand market is positive, then the quantity of at the demand market is precisely equal to the demand.

In equilibrium, conditions (17), (18), and (19) will have to hold for all demand markets and these, in turn, can be expressed also as an inequality analogous to those in (6) and (14) and given by: determine $(x^{2*}, y^*, \rho^*_3) \in R^{(IL+2J+1)KHL}_+$, such that

$$\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{\hat{L}} \sum_{m=1}^{2} \left[\rho^{j*}_{2kh\hat{l}m} + e^*_h + \hat{c}^j_{kh\hat{l}m}(y^*) - \rho^*_{3kh\hat{l}}\right] \times \left[y^j_{kh\hat{l}m} - y^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\rho_{1kh\hat{l}}^{il*}+e_h^*+\hat{c}_{kh\hat{l}}^{il}(x^{2*})-\rho_{3kh\hat{l}}^{*}\right]\times\left[x_{kh\hat{l}}^{il}-x_{kh\hat{l}}^{il*}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}y_{kh\hat{l}m}^{j*}+\sum_{i=1}^{I}\sum_{l=1}^{L}x_{kh\hat{l}}^{il*}-d_{kh\hat{l}}(\rho_3^*)\right]\times\left[\rho_{3kh\hat{l}}-\rho_{3kh\hat{l}}^{*}\right]\geq 0,$$

$$\forall(x^2,y,\rho_3)\in R_+^{(IL+2J+1)KHL}. \quad (20)$$

The Equilibrium Conditions for the International Financial Economy with Electronic Transactions

In equilibrium, the financial flows that the source agents in different countries transact with the intermediaries must coincide with those that the intermediaries actually accept from them. In addition, the amounts of the financial products that are obtained by the consumers in the different countries and currencies must be equal to the amounts that both the source agents and the intermediaries actually provide. Hence, although there may be competition between decision-makers at the same level of tier of nodes of the financial network there must be, in a sense, cooperation between decision-makers associated with pairs of nodes (through positive flows on the links joining them). Thus, in equilibrium, the prices and financial flows must satisfy the sum of the optimality conditions (6) and (14) and the equilibrium conditions (20). We make these relationships rigorous through the subsequent definition and variational inequality derivation below.

Definition 1: International Financial Network Equilibrium with Electronic Transactions

The equilibrium state of the international financial network with electronic transactions is one where the financial flows between the tiers of the network coincide and the financial flows and prices satisfy the sum of conditions (6), (14), and (20).

The equilibrium state is equivalent to the following:

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the international financial network with electronic *transactions according to Definition 1 are equivalent to the solution of the variational inequality given by: determine $(x^{1*}, x^{2*}, y^*, \gamma^*, \rho_3^*) \in \mathcal{K}$, satisfying:*

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{jhm}}+\frac{\partial c^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}}+\frac{\partial r^{j}(x^{1*},y^*)}{\partial x^{il}_{jhm}}+\frac{\partial c_j(x^{1*})}{\partial x^{il}_{jhm}}+\frac{\partial \hat{c}^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}}-\gamma_j^*\right]$$

$$\times\left[x^{il}_{jhm}-x^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{kh\hat{l}}}+\frac{\partial c^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}}+\hat{c}^{il}_{kh\hat{l}}(x^{2*})-\rho^*_{3kh\hat{l}}\right]\times\left[x^{il}_{kh\hat{l}}-x^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial r^{j}(x^{1*},y^*)}{\partial y^{j}_{kh\hat{l}m}}+\frac{\partial c^{j}_{khl}(y^{j*}_{kh\hat{l}m})}{\partial y^{j}_{kh\hat{l}m}}+\hat{c}^{j}_{kh\hat{l}m}(y^*)-\rho^*_{3kh\hat{l}}+\gamma_j^*\right]\times\left[y^{j}_{kh\hat{l}m}-y^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}x^{il*}_{jhm}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}y^{j*}_{kh\hat{l}m}\right]\times\left[\gamma_j-\gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}y^{j*}_{kh\hat{l}m}+\sum_{i=1}^{I}\sum_{l=1}^{L}x^{il*}_{kh\hat{l}}-d_{kh\hat{l}}(\rho_3^*)\right]\times\left[\rho_{3kh\hat{l}}-\rho^*_{3kh\hat{l}}\right]\geq 0,$$

$$\forall (x^1, x^2, y, \gamma, \rho_3) \in \mathcal{K}, \tag{21}$$

where $\mathcal{K} \equiv \{\mathcal{K}^1 \times \mathcal{K}^2\}$, where $\mathcal{K}^2 \equiv \{(y, \gamma, \rho_3) | (y, \gamma, \rho_3) \in R_+^{2JKHL+J+KHL}\}$.

Proof:

We first establish that the equilibrium conditions imply variational inequality (21). Indeed, summation of inequalities (6), (14), and (20), after algebraic simplifications, yields variational inequality (21).

We now establish the converse, that is, that a solution to variational inequality (21) satisfies the sum of conditions (6), (14), and (20), and is, hence, an equilibrium.

To inequality (21), add the term: $-\rho^{il*}_{1jhm} - e_h^* + \rho^{il*}_{1jhm} + e_h^*$ to the term in the first set of brackets (preceding the first multiplication sign). Similarly, add the terms: $-\rho^{il*}_{1kh\hat{l}} - e_h^* + \rho^{il*}_{1kh\hat{l}} + e_h^*$ to the term in brackets preceding the second multiplication sign and $-\rho^{j*}_{2kh\hat{l}m} -$

$e_h^* + \rho_{2kh\hat{l}m}^{j*} + e_h^*$ to the term in brackets preceding the third multiplication sign in (21) . The addition of such terms does not change (21) since the value of these terms is zero and yields:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^{il}(x^{il*})}{\partial x_{jhm}^{il}} + \frac{\partial c_{jhm}^{il}(x_{jhm}^{il*})}{\partial x_{jhm}^{il}} + \frac{\partial r^{j}(x^{1*}, y^*)}{\partial x_{jhm}^{il}} + \frac{\partial c_j(x^*)}{\partial x_{jhm}^{il}} + \frac{\partial \hat{c}_{jhm}^{il}(x_{jhm}^{il*})}{\partial x_{jhm}^{il}}\right.$$

$$\left. -\gamma_j^* - \rho_{1jhm}^{il*} \; - e_h^* + \rho_{1jhm}^{il*} + e_h^*\right] \times \left[x_{jhm}^{il} - x_{jhm}^{il*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial r^{il}(x^{il*})}{\partial x_{kh\hat{l}}^{il}} + \frac{\partial c_{kh\hat{l}}^{il}(x_{kh\hat{l}}^{il*})}{\partial x_{kh\hat{l}}^{il}} + \hat{c}_{kh\hat{l}}^{il}(x^{2*}) - \rho_{3kh\hat{l}}^{*} - \rho_{1kh\hat{l}}^{il*} \; - e_h^* + \rho_{1kh\hat{l}}^{il*} + e_h^*\right]$$

$$\times \left[x_{kh\hat{l}}^{il} - x_{kh\hat{l}}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial r^{j}(x^{1*}, y^*)}{\partial y_{kh\hat{l}m}^{j}} + \frac{\partial c_{khl}^{j}(y_{kh\hat{l}m}^{j*})}{\partial y_{kh\hat{l}m}^{j}} + \gamma_j^* + \hat{c}_{kh\hat{l}m}^{j}(y^*) - \rho_{3kh\hat{l}}^{*}\right.$$

$$\left. -\rho_{2kh\hat{l}m}^{j*} - e_h^* + \rho_{2kh\hat{l}m}^{j*} + e_h^*\right]$$

$$\times \left[y_{kh\hat{l}m}^{j} - y_{kh\hat{l}m}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} x_{jhm}^{il*} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{\hat{L}}\sum_{m=1}^{2} y_{kh\hat{l}m}^{j*}\right] \times \left[\gamma_j - \gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2} y_{kh\hat{l}m}^{j*} + \sum_{i=1}^{I}\sum_{l=1}^{L} x_{kh\hat{l}}^{il*} - d_{kh\hat{l}}(\rho_3^*)\right] \times \left[\rho_{3kh\hat{l}} - \rho_{3kh\hat{l}}^{*}\right] \geq 0,$$

$$\forall (x^1, x^2, y, \gamma, \rho_3) \in \mathcal{K} \tag{22}$$

which, in turn, can be rewritten as:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^{il}(x^{il*})}{\partial x_{jhm}^{il}} + \frac{\partial c_{jhm}^{il}(x_{jhm}^{il*})}{\partial x_{jhm}^{il}} - \rho_{1jhm}^{il*} \; - e_h^*\right] \times \left[x_{jhm}^{il} - x_{jhm}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^{j}(x^{1*}, y^*)}{\partial x_{jhm}^{il}} + \frac{\partial c_j(x^*)}{\partial x_{jhm}^{il}} + \rho_{1jhm}^{il*} + e_h^* + \frac{\partial \hat{c}_{jhm}^{il}(x_{jhm}^{il*})}{\partial x_{jhm}^{il}} - \gamma_j^*\right] \times \left[x_{jhm}^{il} - x_{jhm}^{il*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial r^{il}(x^{il*})}{\partial x_{kh\hat{l}}^{il}} + \frac{\partial c_{kh\hat{l}}^{il}(x_{kh\hat{l}}^{il*})}{\partial x_{kh\hat{l}}^{il}} - \rho_{1kh\hat{l}}^{il*} \; - e_h^*\right] \times \left[x_{kh\hat{l}}^{il} - x_{kh\hat{l}}^{il*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\rho_{1kh\hat{l}}^{il*} + e_h^* + \hat{c}_{kh\hat{l}}^{il}(x^{2*}) - \rho_{3kh\hat{l}}^{*}\right] \times \left[x_{kh\hat{l}}^{il} - x_{kh\hat{l}}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial r^j(x^{1*},y^*)}{\partial y^j_{kh\hat{l}m}}+\frac{\partial c^j_{khl}(y^{j*}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}}-\rho^{j*}_{2kh\hat{l}m}-e^*_h+\gamma^*_j\right]\times\left[y^j_{kh\hat{l}m}-y^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\rho^{j*}_{2kh\hat{l}m}+e^*_h+\hat{c}^j_{kh\hat{l}m}(y^*)-\rho^*_{3kh\hat{l}}\right]\times\left[y^j_{kh\hat{l}m}-y^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}x^{il*}_{jhm}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}y^{j*}_{kh\hat{l}m}\right]\times\left[\gamma_j-\gamma^*_j\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}y^{j*}_{kh\hat{l}m}+\sum_{i=1}^{I}\sum_{l=1}^{L}x^{il*}_{kh\hat{l}}-d_{kh\hat{l}}(\rho^*_3)\right]\times\left[\rho_{3kh\hat{l}}-\rho^*_{3kh\hat{l}}\right]\geq 0. \quad (23)$$

But inequality (23) is equivalent to the sum of conditions (6), (14), and (20), and hence that financial flow and price pattern is an equilibrium according to Definition 1. □

We now put variational inequality (21) into standard form which will be utilized in the subsequent sections. For additional background on variational inequalities and their applications, see the book by Nagurney (1999). In paricular, we have that variational inequality (21) can be expressed as:

$$\langle F(X^*)^T, X-X^*\rangle \geq 0, \quad \forall X\in\mathcal{K}, \quad (24)$$

where $X\equiv(x^1,x^2,y,\gamma,\rho_3)$ and $F(X)\equiv(F_{iljhm},F_{ilkh\hat{l}},F_{jkh\hat{l}m},F_j,F_{kh\hat{l}})_{i=1,\ldots,I;\hat{l}=l=1,\ldots,L;j=1,\ldots,J;h=1,\ldots,H;m=1,2}$, and the specific components of F are given by the functional terms preceding the multiplication signs in (21), respectively. The term $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space.

We now describe how to recover the prices associated with the first two tiers of nodes in the international financial network. Clearly, the components of the vector ρ^*_3 are obtained directly from the solution of variational inequality (21) as will be demonstrated explicitly through several numerical examples in Section 5. In order to recover the second tier prices associated with the intermediaries and the exchange rates one can (after solving variational inequality (21) for the particular numerical problem) *either* (cf. (17)) set $\rho^{j*}_{2kh\hat{l}m}+e^*_h=\rho^*_{3kh\hat{l}}-\hat{c}_{kh\hat{l}}(y^*)$, for any $j,k,h,\hat{l},m$ such that $y^{j*}_{kh\hat{l}m}>0$, *or* (cf. (14)) for any $y^{j*}_{kh\hat{l}m}>0$, set $\rho^{j*}_{2kh\hat{l}m}+e^*_h=\frac{\partial r^j(x^{1*},y^*)}{\partial y^j_{kh\hat{l}m}}+\frac{\partial c^j_{kh\hat{l}m}(y^{j*}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}}-\gamma^*_j$.

Similarly, from (14) we can infer that the top tier prices comprising the vector ρ_1^* can be recovered (once the variational inequality (21) is solved with particular data) thus: for any i,l,j,h,m, such that $x_{jhm}^{il*} > 0$, set $\rho_{1jhm}^{il*} + e_h^* = \gamma_j^* - \frac{\partial r^j(x^{1*},y^*)}{\partial x_{jhm}^{il}} - \frac{\partial c_j(x^{1*})}{\partial x_{jhm}^{il}} - \frac{\partial \hat{c}_{jhm}^{il}(x_{jhm}^{il*})}{\partial x_{jhm}^{il}}$.

In addition, in order to recover the first tier prices associated with the demand market and the exchange rates one can (after solving variational inequality (21) for the particular numerical problem) *either* (cf. (18)) set $\rho_{1kh\hat{l}}^{il*} + e_h^* = \rho_{3kh\hat{l}}^* - \hat{c}_{kh\hat{l}}^{il}(x^{2*})$, for any $i,l,k,h,\hat{l}$ such that $x_{kh\hat{l}}^{il*} > 0$, *or* (cf. (6)) for any $x_{kh\hat{l}}^{il*} > 0$, set $\rho_{1kh\hat{l}}^{il*} + e_h^* = \frac{\partial r^{il}(x^{il*})}{\partial x_{kh\hat{l}}^{il}} + \frac{\partial c_{kh\hat{l}}^{il}(x_{kh\hat{l}}^{il*})}{\partial x_{kh\hat{l}}^{il}}$

Note that in the absence of electronic transactions the above model collapses to the model developed by Nagurney and Cruz (2002).

3. Qualitative Properties

In this Section, we provide some qualitative properties of the solution to variational inequality (21). In particular, we derive existence and uniqueness results. We also investigate properties of the function F (cf. (24)) that enters the variational inequality of interest here.

Since the feasible set is not compact we cannot derive existence simply from the assumption of continuity of the functions. Nevertheless, we can impose a rather weak condition to guarantee existence of a solution pattern. Let

$$\mathcal{K}_b = \{(x^1, x^2, y, \gamma, \rho_3) | 0 \le x^1 \le b_1;\ 0 \le x^2 \le b_1; 0 \le y \le b_3; 0 \le \gamma \le b_4;\ 0 \le \rho_3 \le b_5\}, \tag{25}$$

where $b = (b_1, b_2, b_3, b_4, b_5) \ge 0$ and $x^1 \le b_1; x^2 \le b_2; y \le b_3; \gamma \le b_4; \rho_3 \le b_5$ means that $x_{jhm}^{il} \le b_1; x_{kh\hat{l}}^{il} \le b_2$; $y_{kh\hat{l}m}^{j} \le b_3$; $\gamma_j \le b_4$; and $\rho_{3kh\hat{l}} \le b_5$ for all $i,l,j,k,h,\hat{l},m$. Then $\mathcal{K}_b$ is a bounded closed convex subset of $R^{2ILJH+ILKHL+2JKHL+J+KHL}$. Thus, the following variational inequality

$$\langle F(X^b)^T, X - X^b \rangle \ge 0, \quad \forall X^b \in \mathcal{K}_b, \tag{26}$$

admits at least one solution $X^b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), we then have:

Theorem 2

Variational inequality (21) admits a solution if and only if there exists a $b > 0$, such that variational inequality (26) admits a solution in $\mathcal{K}_b$ with

$$x^{1b} < b_1, \quad x^{2b} < b_2, \quad y^b < b_3, \quad \gamma^b < b_4, \quad \rho_3^b < b_5. \tag{27}$$

Theorem 3: Existence

Suppose that there exist positive constants M, N, R, with $R > 0$, such that:

$$\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}} + \frac{\partial r^j(x^{1*}, y^*)}{\partial x^{il}_{jhm}} + \frac{\partial c_j(x^*)}{\partial x^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}} \geq M,$$

$$\forall x^1 \text{ with } x^{il}_{jhm} \geq N, \forall i, l, j, h, m, \tag{28}$$

$$\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}} + \hat{c}^{il}_{kh\hat{l}}(x^{2*}) \geq M, \forall x^2 \text{ with } x^{il}_{kh\hat{l}} \geq N, \forall i, l, j, h, \hat{l}, \tag{29}$$

$$\frac{\partial r^j(x^{1*}, y^*)}{\partial y^j_{kh\hat{l}m}} + \frac{\partial c^j_{khl}(y^{j*}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} + \hat{c}^j_{kh\hat{l}m}(y^*) \geq M, \quad \forall y \text{ with } y^j_{kh\hat{l}m} \geq N, \quad \forall j, k, h, \hat{l}, m, \tag{30}$$

$$d_{kh\hat{l}}(\rho_3^*) \leq N, \quad \forall \rho_3 \text{ with } \rho_{3kh\hat{l}} > R, \quad \forall k, h, \hat{l}. \tag{31}$$

Then variational inequality (21); equivalently, variational inequality (24), admits at least one solution.

Proof: Follows using analogous arguments as the proof of existence for Proposition 1 in Nagurney and Zhao (1993) (see also the existence proof in Nagurney and Ke (2001)). □

Assumptions (28), (29), and (30) are reasonable from an economics perspective, since when the financial flow between a source agent and intermediary or a demand market or between an intermediary and demand market is large, we can expect the corresponding sum of the associated marginal risks and marginal costs of transaction and handling to exceed a positive lower bound. Moreover, in the case where the demand price of the financial product in a currency and country as perceived by consumers at a demand market is high, we can expect that the demand for the financial product at the demand market to not exceed a positive bound.

We now establish additional qualitative properties both of the function F that enters the variational inequality problem (cf. (21) and (24)), as well as uniqueness of the equilibrium pattern. Monotonicity and Lipschitz continuity of F will be utilized in Section 4 to establish convergence of the proposed algorithmic scheme. Since the proofs of Theorems 4 and 5 below are similar to the analogous proofs in Nagurney and Ke (2001) they are omitted here. Additional background on the properties establish below can be found in the books by Nagurney and Siokos (1997) and Nagurney (1999).

Theorem 4: Monotonicity

Suppose that the risk function r^{il}; $i=1,\ldots,I$; $l=1,\ldots,L$, and r^j; $j=1,\ldots,J$, are strictly convex and that the c^{il}_{jhm}, $c^{il}_{kh\hat{l}}$, c_j, $\hat{c}^{il}_{jhm}$, and $c^j_{kh\hat{l}m}$ functions are convex; the $\hat{c}^j_{kh\hat{l}m}$ and $\hat{c}^{il}_{kh\hat{l}}$ functions are monotone increasing, and the $d_{kh\hat{l}}$ functions are monotone decreasing functions, for all $i,l,j,h,k,\hat{l},m$. Then the vector function F that enters the variational inequality (21) is monotone, that is,

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{32}$$

Monotonicity plays a role in the qualitative analysis of variational inequality problems similar to that played by convexity in the context of optimization problems.

Theorem 5: Strict Monotonicity

Assume all the conditions of Theorem 4. In addition, suppose that one of the families of convex functions c^{il}_{jhm}; $i=1,...,I$; $l=1,\ldots,L$; $j=1,...,J$; $h=1,\ldots,H$; $m=1,2$, $c^{il}_{kh\hat{l}}$; $i=1,...,I$; $l=1,\ldots,L$; $k=1,...,K$; $h=1,\ldots,H$; $\hat{l}=1,\ldots,L$, c_j; $j=1,...,J$; $\hat{c}^{il}_{jhm}$; $i=1,\ldots,I$; $l=1,\ldots,L$; $j=1,\ldots,J$; $h=1,\ldots,H$; $m=1,2$, and $c^j_{kh\hat{l}}$; $j=1,\ldots,J$; $k=1,\ldots,K$; $h=1,\ldots,H$, and $\hat{l}=1,\ldots,L$, is a family of strictly convex functions. Suppose also that $\hat{c}^j_{kh\hat{l}}$; $j=1,...,J$; $k=1,...,K$; $h=1,\ldots,H$; $\hat{l}=1,\ldots,L$, $\hat{c}^{il}_{kh\hat{l}}$; $i=1,...,I$;

$l = 1, \ldots, L$; $k = 1, ..., K$; $h = 1, \ldots, H$; $\hat{l} = 1, \ldots, L$ and -$d_{kh\hat{l}}$, $k = 1, ..., K$,$h = 1, \ldots, H$; $\hat{l} = 1, \ldots, L$, are strictly monotone. Then, the vector function F that enters the variational inequality (21) is strictly monotone, with respect to (x^1, x^2, y, ρ_3), that is, for any two X', X'' with $(x^{1'}, x^{2'}, y^{'}, \rho_3') \neq (x^{1''}, x^{2''}, y^{''}, \rho_3'')$

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0. \tag{33}$$

Theorem 6: Uniqueness

Assuming the conditions of Theorem 5, there must be a unique financial flow pattern (x^{1*}, x^{2*}, y^*), and a unique demand price price vector ρ_3^* satisfying the equilibrium conditions of the international financial network with intermediation. In other words, if the variational inequality (21) admits a solution, then that is the only solution in (x^1, x^2, y, ρ_3).

Proof: Under the strict monotonicity result of Theorem 5, uniqueness follows from the standard variational inequality theory (cf. Kinderlehrer and Stampacchia (1980)) □

Theorem 7: Lipschitz Continuity

The function that enters the variational inequality problem (21) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \text{ where } \mathcal{L} > 0, \tag{34}$$

under the following conditions:

(i). r^{il}, r^j, c^{il}_{jhm}, $c^{il}_{kh\hat{l}}$, c_j, $\hat{c}^{il}_{jhm}$, $c^j_{kh\hat{l}m}$ have bounded second-order derivatives, $\forall i, l, j, h, k, \hat{l}, m$;

(ii). $\hat{c}^j_{khl}$, $\hat{c}^{il}_{kh\hat{l}}$ and $d_{kh\hat{l}}$ have bounded first-order derivatives $\forall i, l, j, k, h, \hat{l}$.

Proof: The result is direct by applying a mid-value theorem from calculus to the vector function F that enters the variational inequality problem (21). □

It is worth noting that the risk functions of the form (3) and (11) have bounded second-order derivatives.

In the next Section, we will utilize the conditions of monotonicity and Lipschitz continuity in order to establish the convergence of the algorithm for the solution of the equilibrium financial flows and prices satisfying variational inequality (21).

4. The Algorithm

In this Section, we consider the computation of solutions to variational inequality (21). The algorithm that we propose is the modified projection method of Korpelevich (1977), which is guaranteed to solve any variational inequality problem in standard form (see (24)) provided that certain conditions are satisfied by the function F that enters the variational inequality problem and that a solution exists. The realization of the modified projection method for the variational inequality (21) (for further details see also Nagurney and Siokos (1997)) is as follows, where $\mathcal{T}$ denotes an iteration counter:

Modified Projection Method for the Solution of the Variational Inequality

Step 0: Initialization

Set $(x^{10}, x^{20}, y^0, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T} = 1$ and set α such that $0 < \alpha \leq \frac{1}{\mathcal{L}}$, where $\mathcal{L}$ is the Lipschitz constant for the problem (cf. (34)).

Step 1: Computation

Compute $(\bar{x}^{1\mathcal{T}}, \bar{x}^{2\mathcal{T}}, \bar{y}^{\mathcal{T}}, \bar{\gamma}^{\mathcal{T}}, \bar{\rho}_3{}^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\bar{x}_{jhm}^{il\mathcal{T}} + \alpha(\frac{\partial r^{il}(\bar{x}^{il\mathcal{T}-1})}{\partial x_{jhm}^{il}} + \frac{\partial r^{j}(\bar{x}^{1\mathcal{T}-1}, \bar{y}^{\mathcal{T}-1})}{\partial x_{jhm}^{il}} + \frac{\partial c_{jhm}^{il}(\bar{x}_{jhm}^{il\mathcal{T}-1})}{\partial x_{jhm}^{il}} + \frac{\partial c_{j}(\bar{x}^{1\mathcal{T}-1})}{\partial x_{jhm}^{il}}\right.$$
$$\left. + \frac{\partial \hat{c}_{jhm}^{il}(\bar{x}_{jhm}^{il\mathcal{T}-1})}{\partial x_{jhm}^{il}} - \bar{\gamma}_j{}^{\mathcal{T}-1}) - \bar{x}_{jhm}^{il\mathcal{T}-1}\right] \times \left[x_{jhm}^{il} - \bar{x}_{jhm}^{il\mathcal{T}}\right]$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\bar{x}_{kh\hat{l}}^{il\mathcal{T}} + \alpha(\frac{\partial r^{il}(\bar{x}^{il\mathcal{T}-1})}{\partial x_{kh\hat{l}}^{il}} + \frac{\partial c_{kh\hat{l}}^{il}(\bar{x}_{kh\hat{l}}^{il\mathcal{T}-1})}{\partial x_{kh\hat{l}}^{il}} + \hat{c}_{kh\hat{l}}^{il}(\bar{x}^{2\mathcal{T}-1}) - \bar{\rho}_{3kh\hat{l}}^{\mathcal{T}-1}) - \bar{x}_{kh\hat{l}}^{il\mathcal{T}-1}\right]$$
$$\times \left[x_{kh\hat{l}}^{il} - \bar{x}_{kh\hat{l}}^{il\mathcal{T}}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}+\alpha(\frac{\partial r^j(\bar{x}^{1\mathcal{T}-1},\bar{y}^{\mathcal{T}-1})}{\partial y^j_{kh\hat{l}m}}+\frac{\partial c^j_{kh\hat{l}m}(\bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}}+\bar{\gamma}_j^{\mathcal{T}-1}+\hat{c}^j_{kh\hat{l}m}(\bar{y}^{\mathcal{T}-1})\right.$$

$$\left.-\bar{\rho}^{\mathcal{T}-1}_{3kh\hat{l}})-\bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m}\right]\times\left[y^j_{kh\hat{l}m}-\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\bar{\gamma}_j^{\mathcal{T}}+\alpha(\sum_{m=1}^{2}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\bar{x}^{il\mathcal{T}-1}_{jhm}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m}\right])-\bar{\gamma}_j^{\mathcal{T}-1}\right]\times\left[\gamma_j-\bar{\gamma}_j^{\mathcal{T}}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}}+\alpha(\sum_{j=1}^{J}\sum_{m=1}^{2}\bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m}+\sum_{i=1}^{I}\sum_{\hat{l}=1}^{L}\bar{x}^{il\mathcal{T}-1}_{kh\hat{l}}-d_{kh\hat{l}}(\bar{\rho}_3^{\mathcal{T}-1}))-\bar{\rho}^{\mathcal{T}-1}_{3kh\hat{l}}\right]$$

$$\times\left[\rho_{3kh\hat{l}}-\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}}\right]\geq 0,\quad\forall(x^1,x^2,y,\gamma,\rho_3)\in\mathcal{K}. \tag{35}$$

Step 2: Adaptation

Compute $(x^{1\mathcal{T}},x^{2\mathcal{T}},y^{\mathcal{T}},\gamma^{\mathcal{T}},\rho_3^{\mathcal{T}})\in\mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\bar{x}^{il\mathcal{T}}_{jhm}+\alpha(\frac{\partial r^{il}(\bar{x}^{il\mathcal{T}})}{\partial x^{il}_{jhm}}+\frac{\partial r^j(\bar{x}^{1\mathcal{T}},\bar{y}^{\mathcal{T}})}{\partial x^{il}_{jhm}}+\frac{\partial c^{il}_{jhm}(\bar{x}^{il\mathcal{T}}_{jhm})}{\partial x^{il}_{jhm}}+\frac{\partial c_j(\bar{x}^{1\mathcal{T}})}{\partial x^{il}_{jhm}}\right.$$

$$\left.+\frac{\partial\hat{c}^{il}_{jhm}(\bar{x}^{il\mathcal{T}}_{jhm})}{\partial x^{il}_{jhm}}-\bar{\gamma}_j^{\mathcal{T}})-\bar{x}^{il\mathcal{T}-1}_{jhm}\right]\times\left[x^{il}_{jhm}-\bar{x}^{il\mathcal{T}}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\bar{x}^{il\mathcal{T}}_{kh\hat{l}}+\alpha(\frac{\partial r^{il}(\bar{x}^{il\mathcal{T}})}{\partial x^{il}_{kh\hat{l}}}+\frac{\partial c^{il}_{kh\hat{l}}(\bar{x}^{il\mathcal{T}}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}}+\hat{c}^{il}_{kh\hat{l}}(\bar{x}^{2\mathcal{T}})-\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}})-\bar{x}^{il\mathcal{T}-1}_{kh\hat{l}}\right]$$

$$\times\left[x^{il}_{kh\hat{l}}-\bar{x}^{il\mathcal{T}}_{kh\hat{l}}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}+\alpha(\frac{\partial r^j(\bar{x}^{1\mathcal{T}},\bar{y}^{\mathcal{T}})}{\partial y^j_{kh\hat{l}m}}+\frac{\partial c^j_{kh\hat{l}m}(\bar{y}^{j\mathcal{T}}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}}+\bar{\gamma}_j^{\mathcal{T}}+\hat{c}^j_{kh\hat{l}m}(\bar{y}^{\mathcal{T}})\right.$$

$$\left.-\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}})-\bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m}\right]\times\left[y^j_{kh\hat{l}m}-\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\bar{\gamma}_j^{\mathcal{T}}+\alpha(\sum_{m=1}^{2}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\bar{x}^{il\mathcal{T}}_{jhm}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}\right])-\bar{\gamma}_j^{\mathcal{T}-1}\right]\times\left[\gamma_j-\bar{\gamma}_j^{\mathcal{T}}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}}+\alpha(\sum_{j=1}^{J}\sum_{m=1}^{2}\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}+\sum_{i=1}^{I}\sum_{\hat{l}=1}^{L}\bar{x}^{il\mathcal{T}}_{kh\hat{l}}-d_{kh\hat{l}}(\bar{\rho}_3^{\mathcal{T}-1}))-\bar{\rho}^{\mathcal{T}-1}_{3kh\hat{l}}\right]$$

$$\times\left[\rho_{3kh\hat{l}}-\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}}\right]\geq 0,\quad\forall(x^1,x^2,y,\gamma,\rho_3)\in\mathcal{K}. \tag{36}$$

Step 3: Convergence Verification

If $|x^{il\mathcal{T}}_{jhm} - x^{il\mathcal{T}-1}_{jhm}| \leq \epsilon$, $|x^{il\mathcal{T}}_{kh\hat{l}} - x^{il\mathcal{T}-1}_{kh\hat{l}}| \leq \epsilon$, $|y^{j\mathcal{T}}_{kh\hat{l}m} - y^{j\mathcal{T}-1}_{kh\hat{l}m}| \leq \epsilon$, $|\gamma^{\mathcal{T}}_j - \gamma^{\mathcal{T}-1}_j| \leq \epsilon$, $|\rho^{\mathcal{T}}_{3kh\hat{l}} - \rho^{\mathcal{T}-1}_{3kh\hat{l}}| \leq \epsilon$, for all $i = 1, \cdots, I$; $l = 1, \ldots, L$; $\hat{l}; \hat{l} = 1, \ldots, \hat{L}$; $m = 1, 2$; $j = 1, \cdots, J$; $h = 1, \ldots, H$; $k = 1, \cdots, K$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Both variational inequality subproblems (35) and (36) can be solved explicitly and in closed form since they are actually quadratic programming problems and the feasible set is a Cartesian product consisting of the the product of $\mathcal{K}^1$ and $\mathcal{K}^2$ The former has a simple network structure, whereas the latter consists of the cross product of the nonnegative orthants: R^{2ILJH}_+, R^J_+, and R^{KHL}_+, and corresponding to the variables y, γ, and ρ_3, respectively. In fact, the subproblems in (35) and (36) corresponding to the x variables can be solved using exact equilibration (cf. Dafermos and Sparrow (1969) and Nagurney (1999)), whereas the remainder of the variables in (35) and (36) can be obtained by explicit formulae.

We now, for completeness, and also to illustrate the simplicity of the proposed computational procedure in the context of the international financial network model with electronic transactions, state the explicit formulae for the computation of the $\bar{y}^{\mathcal{T}}$, the $\bar{\gamma}^{\mathcal{T}}$, and the $\bar{\rho}^{\mathcal{T}}_3$ (cf. (35)). The $y^{\mathcal{T}}$, $\gamma^{\mathcal{T}}$, and $\rho^{\mathcal{T}}_3$ can then be computed for (36) in an analogous fashion.

Computation of the Financial Products from the Intermediaries

In particular, compute, at iteration $\mathcal{T}$, the $\bar{y}^{j\mathcal{T}}_{kh\hat{l}m}$s, according to:

$$\bar{y}^{j\mathcal{T}}_{kh\hat{l}m} = \max\{0, \bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m} - \alpha(\frac{\partial r^j(\bar{x}^{1\mathcal{T}-1}, \bar{y}^{\mathcal{T}-1})}{\partial y^j_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(\bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} + \bar{\gamma}^{\mathcal{T}-1}_j + \hat{c}^j_{kh\hat{l}m}(\bar{y}^{\mathcal{T}-1}) - \bar{\rho}^{\mathcal{T}-1}_{3kh\hat{l}})\},$$

$$\forall j, k, h, \hat{l}, m. \tag{37}$$

Computation of the Prices

At iteration $\mathcal{T}$, compute the $\bar{\gamma}^{\mathcal{T}}_j$s according to:

$$\bar{\gamma}^{\mathcal{T}}_j = \max\{0, \bar{\gamma}^{\mathcal{T}-1}_j - \alpha(\sum_{m=1}^{2} \left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} \bar{x}^{il\mathcal{T}-1}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{\hat{L}} \bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m}\right])\}, \quad \forall j, \tag{38}$$

whereas the $\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}}$s are computed explicitly and in closed form according to:

$$\bar{\rho}^{\mathcal{T}}_{3kh\hat{l}} = \max\{0, \bar{\rho}^{\mathcal{T}-1}_{3kh\hat{l}} - \alpha(\sum_{j=1}^{J}\sum_{m=1}^{2} \bar{y}^{j\mathcal{T}-1}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L} \bar{x}^{il\mathcal{T}-1}_{kh\hat{l}} - d_{kh\hat{l}}(\bar{\rho}_3^{\mathcal{T}-1}))\}, \quad \forall k, h, \hat{l}. \tag{39}$$

In the next Section, we apply the modified projection method to solve several international financial network examples.

We now state the convergence result for the modified projection method for this model.

Theorem 8: Convergence

Assume that the function that enters the variational inequality (21) (or (24)) has at least one solution and satisfies the conditions in Theorem 4 and in Theorem 7. Then the modified projection method described above converges to the solution of the variational inequality (21) or (24).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (21), provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 3, monotonicity follows Theorem 4, and Lipschitz continuity, in turn, follows from Theorem 7. The proof is complete. □

Of course, the algorithm may converge even if the conditions in Theorems 4 and 7 do not hold in which case the algorithm, nevertheless, converges to the equilibrium solution.

5. Numerical Examples

In this Section, we apply the modified projection method to several international financial network examples. The modified projection method was implemented in FORTRAN and the computer system used was a DEC Alpha system located at the University of Massachusetts at Amherst. For the solution of the induced network subproblems in the (x^1, x^2) variables we utilized the exact equilibration algorithm (see Dafermos and Sparrow (1969), Nagurney (1999), and the references therein). The other variables were determined in the computation and adaptation steps of the modified projection method explicitly and in closed form as described in the preceding section.

The convergence criterion used was that the absolute value of the flows and prices between two successive iterations differed by no more than 10^{-4}. The parameter α was set to .1 in the algorithm for all the examples. We assumed in all the examples that the risk functions were of the form (3) and (11), that is, that risk was represented through variance-covariance matrices for both the source agents in the countries and for the intermediaries. We initialized the modified projection method as follows: We set $x^{il}_{jh1} = \frac{S^{il}}{JH}$ for each source agent i and country l and for all j and h. All the other variables were initialized to zero.

We solved two sets of numerical examples, with two examples each. Detailed descriptions are given below. The first example in each set was an international financial network with no electronic transactions in order to create a baseline. Additional numerical examples for international financial networks without electronic transactions can be found in Nagurney and Cruz (2002).

Example 1

The first set of numerical examples consisted of one country, two source agents, two currencies, two intermediaries, and two financial products. Hence, $L = 1$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$, for this and the subsequent two numerical examples. The international financial network for the first example is depicted in Figure 2. Note that in the first example no electronic transactions are allowed.

The data for the first example were constructed for easy interpretation purposes. The financial holdings of the two source agents were: $S^{11} = 20$ and $S^{21} = 20$. The variance-covariance matrices Q^{il} and Q^{j} were equal to the identity matrices (appropriately dimensioned) for all source agents and all intermediaries, respectively. Note that since only physical transactions are allowed, we have that only mode $m = 1$ can be used.

The transaction cost functions faced by the source agents associated with transacting with the intermediaries (cf. (1a)) were given by:

$$c^{il}_{jhm}(x^{il}_{jhm}) = .5(x^{il}_{jhm})^2 + 3.5x^{il}_{jhm}, \quad \text{for } i = 1,2; l = 1; j = 1,2; h = 1,2; m = 1.$$

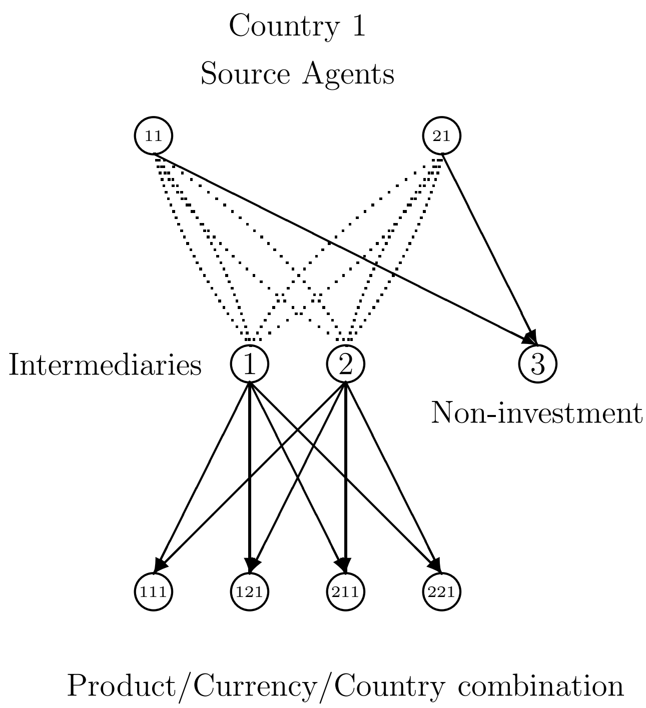

Figure 2: International Financial Network for Example 1

The handling costs of the intermediaries, in turn (see (7)), were given by:

$$c_j(x^1) = .5(\sum_{i=1}^{2}\sum_{h=1}^{2} x_{jh1}^{i1})^2, \quad \text{for } j = 1, 2.$$

The transaction costs of the intermediaries associated with transacting with the source agents were (cf. (8)) given by:

$$\hat{c}_{jhm}^{il}(x_{jhm}^{il}) = 1.5{x_{jhm}^{il}}^2 + 3x_{jhm}^{il}, \quad \text{for } i = 1, 2; l = 1; j = 1, 2; h = 1, 2; m = 1.$$

The demand functions at the demand markets (refer to (16)) were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000.$$

and the transaction costs between the intermediaries and the consumers at the demand markets (see (15a)) were given by:

$$\hat{c}_{kh\hat{l}m}^{j}(y) = y_{kh\hat{l}m}^{j} + 5, \quad \text{for } k = 1, 2; h = 1, 2; \hat{l} = 1; m = 1.$$

We assumed for this and the subsequent examples that the transaction costs as perceived by the intermediaries and associated with transacting with the demand markets (cf. (9)) were all zero, that is, $c_{kh\hat{l}m}^{j}(y_{kh\hat{l}m}^{j}) = 0$, for all $j, k, h, \hat{l}, m$.

The modified projection method converged in 94 iterations and yielded the following equilibrium financial flow pattern:

$$x^{1*} := x_{111}^{11*} = x_{121}^{11*} = x_{211}^{11*} = x_{221}^{11*} = x_{11*}^{211} = x_{22*}^{211} = x_{211}^{21*} = x_{221}^{21*} = 5.0000;$$

$$y^* := y_{1111}^{1*} = y_{1211}^{1*} = y_{2111}^{1*} = y_{2211}^{1*} = y_{1111}^{2*} = y_{1211}^{2*} = y_{2111}^{2*} = y_{2211}^{2*} = 5.0000.$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 262.8566$, and the computed demand prices at the demand markets were: $\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = 282.856$.

We also, for completeness, recover the equilibrium prices associated with the source agents according to the discussion following (24). In particular, we had that all components of the vector ρ_1^* were identically equal to 214.8566.

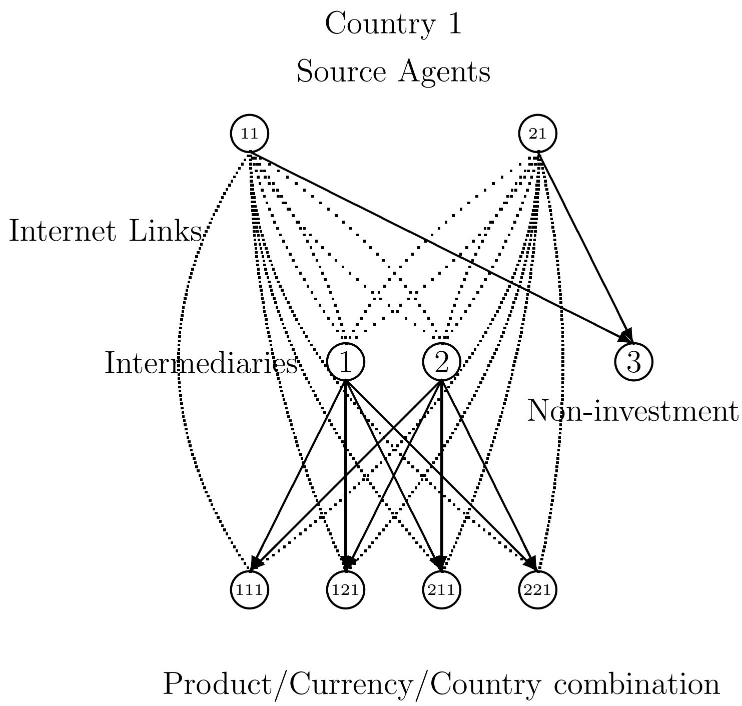

Figure 3: International Financial Network for Example 2

It is easy to verify that the optimality/equilibrium conditions were satisfied with good accuracy. Note that in this example, constraint (5) was tight for both source agents, that is, there was zero flow on the links connecting node 3 with top tier nodes. Thus, it was optimal for both source agents to invest their entire financial holdings in the instrument made available by each of the two intermediaries in each of the two currencies. Clearly, due to the input data in this highly stylized example, the equilibrium financial flow pattern could have been "predicted" even without any computations; however, the same does not hold (even in this quite "symmetric") example for the prices.

Example 2

In the second example, we kept the data as in Example 1 except that now we allowed for electronic transactions between the source agents and the consumers at the demand markets. Hence, the international financial network was now as depicted in Figure 3.

The additional data was as follows. The transaction costs (cf. (1b)) were given by:

$$c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}}) = .5(x^{il}_{kh\hat{l}})^2 + x^{il}_{khl}, \quad \forall i, l, \hat{l}, k, h,$$

whereas the transaction costs (cf. (15b)) were given by:

$$\hat{c}^{il}_{kh\hat{l}}(x^2) = .1x^{il}_{kh\hat{l}} + 1, \quad \forall i, l, \hat{l}, k, h.$$

The variance-covariance matrices were redimensioned and, again, were equal to the identity matrices.

The modified projection method converged in 88 iterations and yielded the following equilibrium financial flow pattern:

$$x^{1*} := x^{11\ *}_{111} = x^{11*}_{121} = x^{11*}_{211} = x^{11*}_{221} = x^{211}_{11*} = x^{211}_{22*} = x^{21*}_{211} = x^{21*}_{221} = .372;$$

$$x^{2*} := x^{11*}_{111} = x^{11*}_{121} = x^{11*}_{211} = x^{11*}_{221} = x^{21*}_{111} = x^{21*}_{121} = x^{21*}_{211} = x^{21*}_{221} = 4.627;$$

$$y^* := y^{1*}_{1111} = y^{1*}_{1211} = y^{1*}_{2111} = y^{1*}_{2211} = .372 = y^{2*}_{1111} = y^{2*}_{1211} = y^{2*}_{2111} = y^{2*}_{2211} = .372.$$

As was the case in Example 1, both source agents allocated the entirety of their funds to the instrument in the two currencies; thus, there was no non-investment.

The vector γ^* had components: $\gamma^*_1 = \gamma^*_2 = 276.738$, and the computed demand prices at the demand markets were: $\rho^*_{3111} = \rho^*_{3121} = \rho^*_{3211} = \rho^*_{3221} = 282.857$.

We also computed (as discussed in Example 1 and following (24)) the new equilibrium prices associated with the top tier of nodes in the international financial network and now the new equilibrium vector ρ^*_1 had all of its components equal to 270.385.

Note that due to the lower transaction costs associated with electronic transactions directly between the source agents and the demand markets a sizeable portion of the financial funds vis a vis those in Example 1 were transacted in this manner.

Example 3

In the first example in the second set, the international financial network was as given in Figure 4. The two examples in this set consisted of two countries with two source agents in

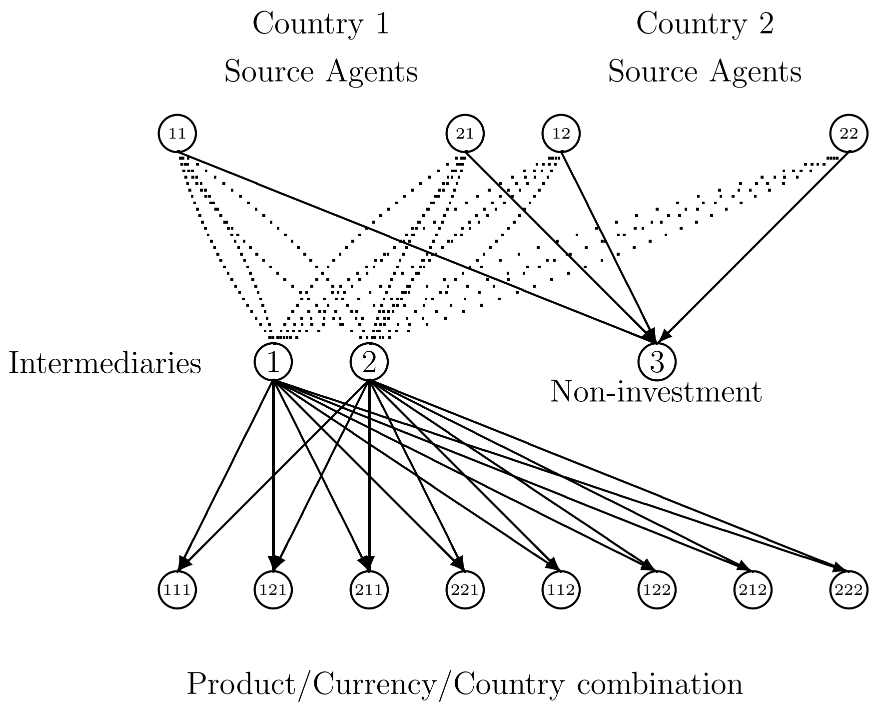

Figure 4: International Financial Network for Example 3

each country; two currencies, two intermediaries, and two financial products. Hence, $L = 2$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$. In the first example, however, we assumed that there were no electronic transactions and, hence, the only mode for transacting was given by $m = 1$.

The data for the first example in this set was constructed for easy interpretation purposes and to create a baseline from which the simulations could be conducted. In fact, we essentially "replicated" the data for the first country as it appeared in Example 1 in order to construct the data for the second country.

Specifically, the financial holdings of the source agents were: $S^{11} = 20$, $S^{21} = 20$, $S^{12} = 20$, and $S^{22} = 20$. The variance-covariance matrices Q^{il} and Q^j were equal to the identity matrices (appropriately dimensioned) for all source agents in each country and for all intermediaries, respectively.

The transaction cost functions faced by the source agents associated with transacting with the intermediaries were given by:

$$c^{il}_{jhm}(x^{il}_{jhm}) = .5(x^{il}_{jhm})^2 + 3.5x^{il}_{jhm}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2; m = 1.$$

The handling costs of the intermediaries (since the number of intermediaries in this set is still equal to two) remained as in Example 1, that is, they were given by:

$$c_j(x^1) = .5(\sum_{i=1}^{2}\sum_{h=1}^{2} x^{i1}_{jh1})^2, \quad \text{for } j = 1,2.$$

The transaction costs of the intermediaries associated with transacting with the source agents in the two countries were given by:

$$\hat{c}^{il}_{jhm}(x^{il}_{jhm}) = 1.5{x^{il}_{jhm}}^2 + 3x^{il}_{jhm}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2; m = 1.$$

The demand functions at the demand markets were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000,$$

$$d_{112}(\rho_3) = -2\rho_{3112} - 1.5\rho_{3122} + 1000, \quad d_{122}(\rho_3) = -2\rho_{3122} - 1.5\rho_{3112} + 1000,$$

$$d_{212}(\rho_3) = -2\rho_{3212} - 1.5\rho_{3222} + 1000, \quad d_{222}(\rho_3) = -2\rho_{3222} - 1.5\rho_{3212} + 1000,$$

and the transaction costs between the intermediaries and the consumers at the demand markets were given by:

$$\hat{c}^{j}_{kh\hat{l}}(y) = y^{j}_{kh\hat{l}} + 5, \quad \text{for } j = 1,2; k = 1,2; h = 1,2; \hat{l} = 1,2; m = 1.$$

The modified projection method converged in 64 iterations and yielded the following equilibrium financial flow pattern:

$$x^{1*} := x^{11*}_{111} = x^{11*}_{121} = x^{11*}_{211} = x^{11*}_{221} = x^{211}_{11*} = x^{21*}_{221} = x^{21*}_{211} = x^{21*}_{221} = 5.0000,$$

$$x^{12*}_{111} = x^{12*}_{121} = x^{12*}_{211} = x^{12*}_{221} = x^{221}_{11*} = x^{221}_{22*} = x^{21*}_{221} = x^{22*}_{221} = 5.0000;$$

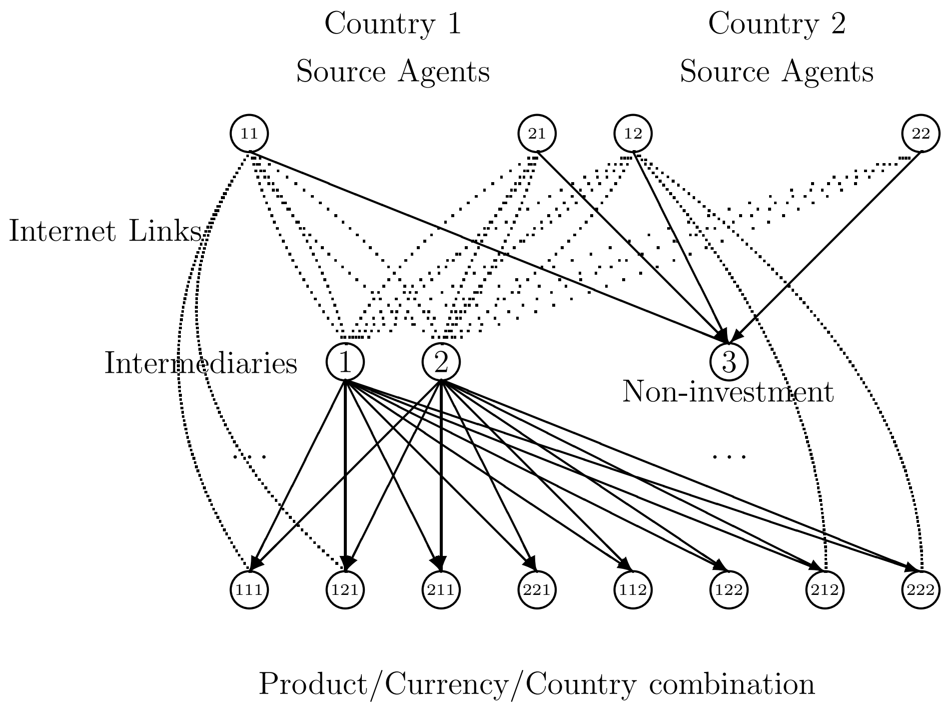

Figure 5: International Financial Network for Example 4

$$y^* := y^{1*}_{1111} = y^{1*}_{1211} = y^{1*}_{2111} = y^{1*}_{2211} = y^{2*}_{1111} = y^{2*}_{1211} = y^{2*}_{2111} = y^{2*}_{2211} = 5.0000,$$

$$y^{1*}_{1121} = y^{1*}_{1221} = y^{1*}_{2121} = y^{1*}_{2221} = y^{2*}_{1121} = y^{2*}_{1221} = y^{2*}_{2121} = y^{2*}_{2221} = 5.0000.$$

The vector γ^* had components: $\gamma^*_1 = \gamma^*_2 = 262.8486$, and the computed demand prices at the demand markets were: $\rho^*_{3111} = \rho^*_{3121} = \rho^*_{3211} = \rho^*_{3221}$ $\rho^*_{3112} = \rho^*_{3122} = \rho^*_{3212} = \rho^*_{3222} =$ 282.8591. The components of the vector ρ^*_1 were identically equal to 194.8486.

Example 4

Example 4 was constructed from the preceding example as follows. We kept the data as in Example 3 except that now we added links from the source agents to the demand markets to represent electronic transactions as depicted in Figure 5. The additional data for the Internet links were replications of the analogous functions in Example 2.

The variance-covariance matrices were redimensioned and were equal to the identity matrices.

The modified projection method converged in 59 iterations and yielded the following equilibrium financial flow pattern: only the electronic links had positive flows with all other flows being identically equal to 0.000. In particular, the financial holdings of the source agents in the different countries were equally allocated via electronic transactions directly to the demand markets with $x^{il*}_{kh\hat{l}} = 2.5000$ for all $i, l, k, h, \hat{l}$.

The vector γ^* had components: $\gamma^*_1 = \gamma^*_2 = 278.0899$, and the computed demand prices at the demand markets were: $\rho^*_{3kh\hat{l}} = 282.8568$, $\forall k, h, \hat{l}$. Note that the demand market prices were essentially unchanged from those in Example 3 whereas the γ^* vector components did change. In this example, all the financial transactions were conducted electronically.

These examples have been presented to show both the model and the computational procedure. Obviously, different input data and dimensions of the problems solved will affect the equilibrium financial flow and price patterns. One now has a powerful tool with which to explore the effects of perturbations to the data as well as the effects of changes in the number of source agents, countries, currencies, and/or products, as well as the effects of the introduction of electronic transactions.

6. Summary and Conclusions

In this paper, we developed a framework for the modeling, analysis, and computation of solutions to international financial problems with intermediaries in the presence of electronic transactions. We proposed an international financial network model consisting of three tiers of decision-makers: the source agents, the financial intermediaries, and the consumers associated with the demand markets for distinct financial products in distinct currencies and countries. We modeled the behavior of the decision-makers, derived the optimality conditions as well as the governing equilibrium conditions which reflect (possible) competition among decision-makers at the same tier of nodes but cooperation between tiers of nodes. The framework allows for the handling of as many countries, as many source agents in each country, as many currencies in which the financial products can be obtained, and as many financial intermediaries, as mandated by the specific application. Moreover, it incorporates the possibility of electronic transactions between distinct tiers of the international financial network.

The formulation of the equilibrium conditions was shown to be equivalent to a finite-dimensional variational inequality problem. The variational inequality problem was then utilized to obtain qualitative properties of the equilibrium financial flow and price pattern as well as to propose a computational procedure for the numerical determination of the equilibrium flows in particular examples. The algorithm was subsequently applied to several international financial network examples to illustrate both the model as well as the computational procedure.

This framework generalizes the recent work of Nagurney and Cruz (2002) in international financial networks with intermediation to include explicit electronic transactions. Further research will focus on the exploration of the dynamics of international financial networks (see, e.g., in a single country setting, Nagurney and Dong (2002)) as well as the incorporation of policy instruments such as taxes.

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