

Modeling of Covid-19 Trade Measures on Essential Products: A Multiproduct, Multicountry Spatial Price Equilibrium Framework

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Abstract

In this paper, we develop a unified variational inequality framework in the context of spatial price network equilibrium problems that handles multiple products with multiple demand and supply markets in multiple countries as well as multiple transportation routes. The model incorporates a plethora of distinct trade measures, which is particularly important in the pandemic, since PPEs and other essential products are in high demand, but short in supply globally. In the model, product flows as well as prices at the supply markets and the demand markets in different countries are variables which allows us to seamlessly introduce various trade measures, including tariffs, quotas, as well as price floors and ceilings. Qualitative properties are analyzed. Numerical examples are provided to illustrate the impacts of the trade measures on equilibrium product path and link flows, and on prices, and demand and supply quantities. Given the relevance of the trade measures in the world today and discussions concerning the impacts, the framework constructed in this paper is especially timely.

Keywords: Covid-19; essential supplies; trade measures; spatial price equilibrium; networks

1. Introduction

The World Health Organization (WHO) declared the Covid-19 pandemic on March 11, 2020 (WHO (2020a)). The ensuing global healthcare disaster has endangered and disrupted the lives of billions around the world, resulting in illnesses and deaths, and has also generated secondary crises. No-one knows with certainty when the pandemic will end. According to Johns Hopkins Coronavirus Resource Center (2021), as of March 8, 2021, more than 29,000,000 individuals in the United States have been infected and at least 525,000 have died. Worldwide, more than 117 million cases have been reported and

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more than 2.6 million deaths. In addition, the pandemic has resulted in severe economic damage and a wave of unemployment. The World Trade Organization forecasts that trade in 2020 will fall by 13% to 32%, a historically large plunge (WTO (2020a)). The International Monetary Fund (IMF), in the June 2020 World Economic Outlook, predicted that the global growth rate would be at -4.9 percent in 2020. The negative impact of the pandemic on low-income households is severe and poses serious challenges to the global fight against poverty that began in the 1990s (International Monetary Fund (2020)).

Since the coronavirus that causes Covid-19 can be transmitted from person to person through respiratory droplets, the Centers for Disease Control and Prevention (CDC) emphasizes that physical distance of at least 6 feet between persons is necessary to avoid infection in order to minimize contagion and to prevent healthcare systems from becoming overwhelmed (CDC (2020b), Greenstone and Nigam (2020)). Coupled with social distancing is the use of Personal Protective Equipment (PPE) (Ferguson et al. (2020)) to prevent individuals, including essential workers, from contracting the disease. Medical items, such as ventilators, have also been in great demand to treat some hospitalized Covid-19 patients (CDC (2020a)). The spread of the virus and the efforts of governments and of the public to combat it have led to a sharp increase in demand for medical supplies worldwide (Kamdar (2020)). WHO (2020b) estimates that 89 million masks and 76 million examination gloves are needed monthly to respond to the Covid-19 pandemic, noting that shortages of PPEs pose a serious threat to doctors, nurses, and other frontline workers. They warn that the production of these goods must increase by 40% to meet growing demand.

However, the impact of this pandemic on products is not limited to the surge in demand for medical products. The pandemic has disrupted air transportation systems and bankrupted many companies. It has also led to major changes in people's lifestyles, altering patterns of supply and demand, and even grocery shopping (Severson (2020)), and is threatening the economic stability and well-being of both people and governments (Suau-Sanchez, Voltes-Dorta, and Cuguer-Escofet (2020), Sheth (2020)). The pandemic has severely disrupted the supply chain networks of a plethora of essential products. For example, we have seen the negative impact of the Covid-19 pandemic on US meat supply chains as several processing plants were shut down due to infected workers (Nagurney (2021), Corkery and Yaffe-Bellany (2020)). China, where the coronavirus, which causes the disease Covid-19 originated, is also expected to face challenges in the agricultural sector. It is estimated that its agricultural sector's growth rate will decrease by 0.4% to 2% in 2020 (Zhang et al. (2020)). The Food and Agriculture Organization of the United Nations projects that the total meat production around the world will fall by 1.7% in 2020. It also points out that, although there is enough food globally, the dire economic situation, due to the Covid-19 pandemic, has made it difficult to access food, especially in countries already plagued by a hunger crisis (Food and Agriculture Organization (2020)).

Such challenges have led many governments to institute different trade policies and measures during the Covid-19 pandemic in order to reduce the risk of essential/vital product shortages. Countries, at different times, put specific products in the group of essential goods. In general, the commodities whose shortage endangers people's health and safety are in this category. India, for example, enacted the Essential Commodities Act in 1955 to protect the production and distribution of certain goods and to prevent shortages and sudden price increases. This act covers items such as food grains, fertilizers, edible oil, drugs, fuels, and petroleum, and, recently, masks and hand sanitizers have been added to it due to the Covid-19 pandemic (Gupta (2020)). In another example, the state of Florida defines essential commodity as a commodity that "its consumption or use is critical to the maintenance of the public health, safety or

welfare during the declared emergency” and puts commodities such as protective masks, sanitizing and disinfecting supplies, and all PPE on the list while mentioning that they continue to monitor the situation and revise the list as needed (Florida Attorney General (2020))

When the Covid-19 pandemic crisis hit one country after another, some countries, which were among the few exporters of PPEs, faced a very high demand within their own national boundaries and, therefore, prioritized meeting their needs first. Hence, they banned the export of medical products (Boykoff, Sebastian, and Di Donato (2020)). According to Global Trade Alert (GTA), as of 25 April 2020, 122 new export bans in more than 75 countries including the US, China, and the European Union (EU) were issued on medical supplies such as antibiotics, face masks, and ventilators. There is also a large number of countries that has reduced the tariffs on essential goods to accelerate the import of such products (Evenett (2020), Pelc (2020), Global Trade Alert (2020)). For example, Belarus has imposed temporary restrictions on exports of food products such as onions and garlic due to the pandemic crisis (WTO (2020b)). China has temporarily decreased import tariffs on several types of products such as medical supplies, raw materials, agricultural products, and meat (ITC MACMAP (2020)). The United States is temporarily excluding certain products from the additional duty of 25% on a list of 19 products from China and is putting restrictions on exports of five types of PPEs which are going to need explicit approval from FEMA before export (WTO (2020b)).

Governments have also taken steps to address concerns about the prices of essential goods during the pandemic crisis. China has warned that the price of essential goods should not increase, and has also announced how to enforce the law against the increase in the price of face masks. The European Competition Network issued a joint statement by the European Commission and the European Union’s national competition agencies highlighting that it is very important that the prices of the goods that are necessary for the health of the people, such as face masks and sanitizing gel, remain within the competitive range (OECD (2020)). The United States Senate has expressed concern that the prices of vaccines being developed with the help of the federal government be affordable (Owermohle (2020)). Also, although there are no federal price gouging laws in the United States, an executive order was signed by the President to prevent price gouging or the hoarding of essential goods with penalties of up to one year in prison and fines of up to \$10,000 (White House (2020)). The US Department of Justice, on the other hand, warns that criminal prosecution awaits those that fix prices or rig bids for PPEs such as sterile gloves and face masks (Department of Justice (2020), OECD (2020)). Although such measures are necessary to manage critical situations, governments must be careful in formulating trade measures and in implementing them to make sure that they are useful and that there are no adverse effects. For example, Dr. Anthony Fauci, the Director of the National Institute of Allergy and Infectious Diseases, points out that high medicine prices create problems in countries plagued by the Covid-19 pandemic. But he also mentions that, if you put a lot of pressure on a company and restrict it, the company may no longer work with you (Dearment (2020)). In India, the government tried to control the price of life-saving oxygen gas, but this policy was accompanied by shortcomings that led to the emergence of the oxygen black market (Biswas (2020)).

Trade measures, including tariffs, quotas, as well as associated price floors and ceilings, have advantages and disadvantages and are used in trade policy by governments. They need not be limited to the time horizon of the pandemic; they may be applied for various reasons and altered based on the realities of the associated supply chain networks and issues of governmental concern (Cassing and Hillman (1985), Pelcovits (1976), Nagurney, Besik, and Li (2019)). Trade policy measures such as tariffs, quotas,

and two-tiered tariffs have been investigated in the context of different trade networks of various products ranging from food to consumer durables. The impacts on the product flows have also been examined (cf. Nagurney, Besik, and Dong (2019) and the references therein).

In this paper, we construct a multiproduct, multicountry spatial price equilibrium model that integrates a plethora of trade measures that different countries can impose (and have been imposing) on essential products such as medical supplies, certain raw materials, and agricultural goods, as they continue to deal with the pandemic and work for the interests of their nations. In the model there are multiple supply markets and multiple demand markets in each country.

2. Literature Review and Our Contributions

Since the Covid-19 pandemic is unprecedented in terms of both scale and scope and quite different as compared to other disasters, which are limited in both geography and time, researchers have turned to investigating various aspects of the pandemic, including the impacts on product flows, and gradually publishing the results. Queiroz et al. (2020) provided a research agenda by constructing a structured literature review of recent studies on the Covid-19 pandemic and the impacts of previous epidemic outbreaks on supply chains. Nagurney et al. (2021) constructed the first Generalized Nash Equilibrium (GNE) model with stochastic demands to investigate the competition among healthcare organizations for medical supplies in the pandemic. Ivanov (2020) documented a simulation study on the impacts of the Covid-19 pandemic on global supply chains. That study also provides an analysis of predicting short-term and long-term effects of epidemic outbreaks on the supply chains. Nagurney (2021) developed a supply chain network optimization model for perishable food in the Covid-19 pandemic, which included the critical labor resource. The model can be used to investigate the impacts of labor disruptions, due to illnesses, death, etc., on prices and product flows.

2.1. Trade Measures

The coronavirus, which causes Covid-19, spread rapidly worldwide, and pushed governments and policymakers to adopt new trade policies to pave the way to provide essential supplies including medical ones for their nations. Researchers and experts have varying views on the methods and effects of these trade policies and are publishing research results while we are still in the midst of the pandemic. Baldwin and Evenett (2020) suggest that the governments should not turn inward in response to the Covid-19 pandemic because national trade obstacles would make the production of medical supplies harder for countries. But the protectionist trade policies that we are observing during the Covid-19 pandemic are mostly anti-export. Smith and Glauber (2020) report that although protectionist policies in food international trade have not worked well in past experiences such as the 2007-08 crisis, policymakers are still keen to use these strategies. Indeed, in the current pandemic, certain governments have imposed restrictions such as export bans and export quotas on the trade in food, including on wheat and rice. Stellingner, Berglund, and Isakson (2020) mention that we are dealing with a dual crisis, a healthcare crisis, and an economic one. They argue that some trade measures, such as import bans and buy-national, are unnecessary and, while they may be useful in protecting the domestic manufacturing, they do not

have a role in protecting patients. Bown (2020), pointing out that the European Union imports 90% of its PPEs, suggests that even temporary export bans might cause mistrust and retaliations that would harm such countries. Also, the European Union export restrictions put Eastern Europe, northern Africa, and sub-Saharan Africa countries at risk. The above cited papers, however, do not add to the literature of mathematical modeling, as our paper here does.

Shingal (2020) says that, although the pandemic is a health crisis, with proper trade measures, secondary financial crises can be prevented, and that the new trade barriers can replace traditional trade instruments and even new jobs and skills may be created. Pauwelyn (2020) and Lawrence (2020) argue that establishing trade measures that restrict the export of essential medical supplies and food is not legal under normal circumstances under WTO and EU law, but is not subject to the law in critical situations such as the current pandemic in which people's lives are in danger. Leibovici and Santacreu (2020) investigated the role of international trade of essential supplies in reducing or exacerbating the effects of a pandemic. They observed that a country's imbalance in the trade of essential goods has a significant role in the effects on the country, so that net importers would be worse off than net exporters. By simulating the effects of trade barriers under different scenarios, Grassia et al. (2020) showed that, although a country would benefit from implementing the trade restrictions in isolation, the generalized use of these measures makes most countries' situations worse than theirs in a no-ban scenario. They also estimate that there would be price increases in many countries that impose trade restrictions.

Fiorini, Hoekman, and Yildirim (2020), examining recent examples, argue that, instead of restricting international trade, governments should work with industry actors to find and to improve essential products' supply chain bottlenecks. Nagurney, Besik, and Li (2019) constructed an oligopolistic supply chain network equilibrium with trade policy where firms compete over product quantities and product quality and they are subjected to lower bounds and upper bounds on quality standards. They found that the governments might threaten the consumer welfare of their own nation by imposing trade restrictions. Evenett (2020) argues that, if governments restrict the export of goods, producers will no longer have an incentive to increase the production level to meet the foreign demand. He also points out that manufacturers consider tariffs and non-tariff barriers when deciding whether to sell to foreign buyers.

2.2. Relevant Spatial Price Equilibrium Models

Spatial price equilibrium (SPE) models are very useful in research on the role and impacts of commodity trade instruments, such as tariffs and quotas, in markets where there are multiple supply markets and multiple demand markets. SPE models originated in the works of Samuelson (1952) and Takayama and Judge (1964, 1971). Since then, such models have been expanded and widely applied in practice, with variational inequality theory, in particular, utilized to formulate increasingly more general SPE models (cf. Nagurney (1999, 2006), Daniele (2004), Li, Nagurney, and Yu (2018), Nagurney, Besik, and Dong (2019), and the references therein). Nagurney, Besik, and Dong (2019) constructed a spatial price equilibrium model with multiple countries and supply and demand markets in each country. In that model, a tariff-rate quota, which is a two-tiered tariff, was applied. They provided a unified variational inequality framework for qualitative analysis and algorithmic solution. They also solved and discussed numerical examples inspired by the dairy industry where a tariff-rate quota was imposed by the US on cheese from France. Nagurney, Besik, and Nagurney (2019), inspired by the ongoing trade wars, provided a modeling

and computational framework for competitive global supply chain networks, operating as an oligopoly, where trade instruments are imposed in the form of tariff-rate quotas.

A related research stream that focuses on the movement of people, rather than products, in which regulations have also been incorporated into a network modeling equilibrium framework, is that on human migration (see Nagurney, Daniele, and Nagurney (2020) and Nagurney, Daniele, and Cappello (2021)). In such human migration models, there are utility functions associated with different classes of migrants and origin and destination nodes, rather than supply and demand price functions. The types of regulations typically restrict migration flows, whereas trade measures associated with products often entail pricing measures in the form of tariffs and/or price floors and ceilings. Furthermore, Nagurney, Daniele, and Nagurney (2021) have shown that, with subsidies, a system-optimized solution for human migration can be achieved, even though migrants act independently and unilaterally. It is worthwhile to note that in the Covid-19 pandemic migratory policies continue to be implemented (see Cappello, Daniele, and Nagurney (2021)).

2.3. 2.3 Our Contributions

In this paper, we construct a multiproduct, multicountry spatial price equilibrium model with trade measures that captures the nuances of essential commodity production, trade, and consumption in the Covid-19 pandemic. This paper builds on the work of Nagurney, Besik, and Dong (2019) but with the following significant extensions/modifications:

1. Multiple essential products: Unlike the earlier model, this model can handle multiple products and that is especially important in the Covid-19 pandemic due to the importance of such products as medical supplies, a variety of foods, as well as certain raw materials.

2. Prices and product flows as variables: In the new model, not only are product flows variables but prices at the supply markets and the demand markets in different countries are variables as well. This feature allows us to more seamlessly introduce trade measures in the form of price floors and ceilings, which have been incorporated in this pandemic, and which we elaborate upon in the next contribution below.

3. Multiplicity of trade measures including price floors and ceilings: The model in this paper integrates a plethora of trade measures for multiple products, including tariffs, quotas, as well as price floors and ceilings. We allow for price ceilings on both supply market prices and on demand market prices in the countries. We also allow for positive price floors on supply market prices in the countries with price floors on the demand market prices being set to zero.

4. Special Relevance to the Covid-19 Pandemic: By incorporating multiple distinct trade measures in a unifying framework, the multiproduct, multicountry model is of specific relevance in the pandemic, since governments are using multiple trade measures.

This paper is organized as follows. The multiproduct, multicountry spatial price equilibrium model in price and quantity variables and a multiplicity of trade measures is constructed in Section 3. The governing equilibrium conditions are stated and the variational inequality formulation presented therein. In Section 4, the algorithm and qualitative properties are presented. In Section 5, we apply the modified projection method to compute the equilibrium product flow, the price and the Lagrange multiplier patterns in a series of numerical examples that demonstrate the generality and applicability of our multiproduct,

multicountry SPE model with trade measures. The final section, Section 6, summarizes the results and presents our conclusions.

3. The Multiproduct, Multicountry Spatial Price Equilibrium Model with Trade Measures

We now construct the multiproduct, multicountry spatial price equilibrium model with trade measures for essential products. Each country has multiple possible supply markets as well as multiple possible demand markets. We denote an essential product by h , with $h = 1, \dots, H$. As noted in the Introduction, an essential product can be a medical product, food item, etc. There are n countries in the network economy, with country i ; $i = 1, \dots, n$, having k_i supply markets where the essential products can be produced and l_j ; $j = 1, \dots, n$ demand markets where these product supplies can be delivered to. The total number of supply markets is $n_k = \sum_{i=1}^n k_i$ and the total number of demand markets is $n_l = \sum_{j=1}^n l_j$. Underlying the supply markets and demand markets is a network consisting of a graph $G = [N, L]$, where N is the set of nodes (supply market origin nodes, demand market destination nodes, intermediate nodes for transshipment, etc.) and L is the set of directed links. An illustrative network figure is given in Figure 1. The network topology in Figure 1 can be adapted according to the specific application.

We denote the set of supply markets of country i by O_i and the set of demand markets of country j by D_j . Here, $P_{(j,l)}^{(i,k)}$ denotes the set of paths joining a country supply market origin node (i, k) with a country demand market node (j, l) . $P^{(i,k)}$ denotes the set of paths, such that each path originates at origin node (i, k) and terminates at one of the demand market nodes. $P_{(j,l)}$, in turn, denotes the set of paths, such that each path terminates at the destination node (j, l) and originates at one of the supply market nodes. The set P then denotes the set of all paths joining the country supply market origin nodes with the country demand market destination nodes. There are n_P paths in the network and n_L links.

The basic notation for the model is given in Table 1. All vectors are column vectors. The product link unit transportation cost functions, supply functions, and demand functions are all assumed to be continuous.

We first proceed with the conservation of flow equations. We then identify the relationship between essential product path costs and costs on the links, and provide an amplified discussion of the trade measures, followed by the governing equilibrium conditions. The product unit transportation costs, supply and demand market prices, plus the price floors and ceilings, and the unit tariffs are all in a common currency.

The essential product path flows must be nonnegative, that is:

$$x_p^h \geq 0, \quad \forall p \in P, \forall h. \quad (1)$$

The product link flows, in turn, are related to the product path flows thus:

$$f_a^h = \sum_{p \in P} x_p^h \delta_{ap}, \quad \forall a \in L, \forall h. \quad (2)$$

According to (2), the product flow on a link is equal to the sum of flows of that product on paths that contain that link.

(j,l)

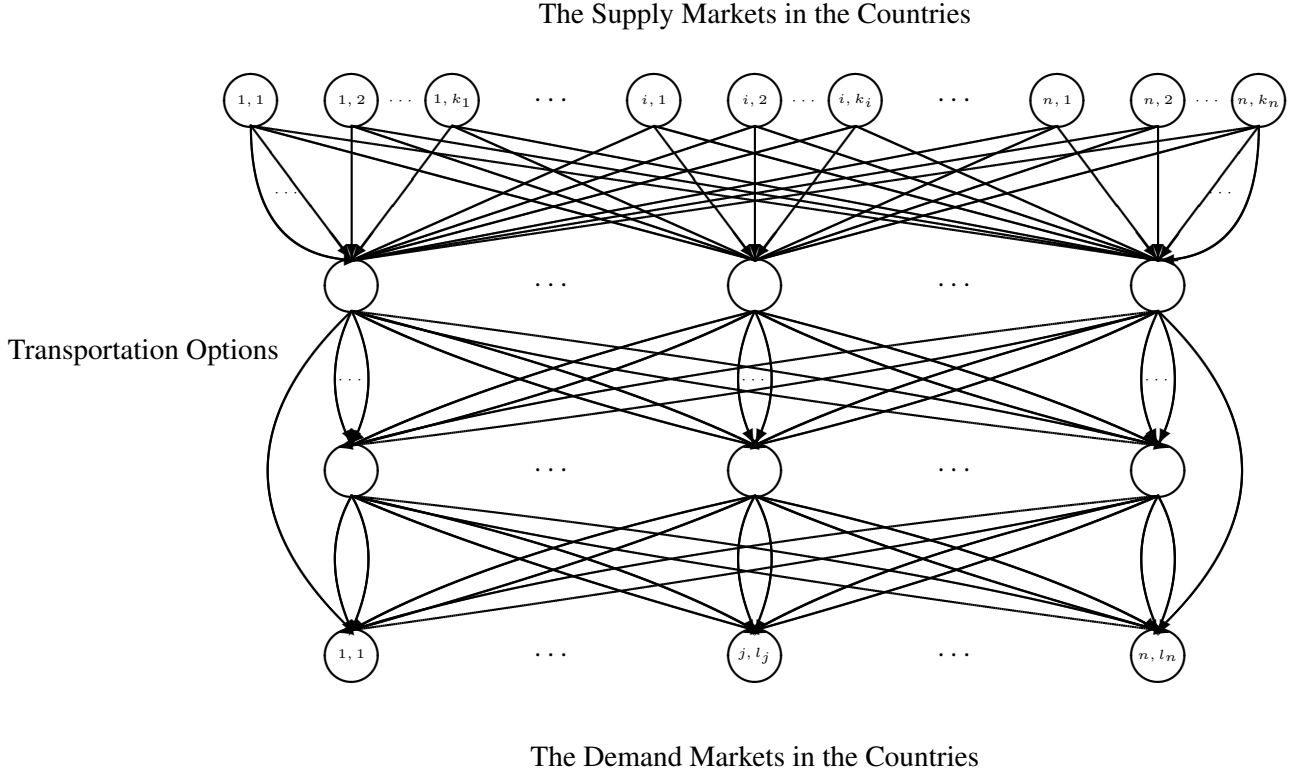


Fig. 1. An Example of a Multicountry, Multiple Supply Market and Multiple Demand Market Spatial Price Network

In addition, we have that the unit transportation cost on a path associated with the transportation of an essential product is equal to the sum of the unit transportation costs on the links that make up the path; in other words:

$$C_p^h(x) = \sum_{a \in L} c_a^h(f) \delta_{ap}, \quad \forall p \in P, \quad (3)$$

where $\delta_{ap} = 1$, if link a is contained in path p , and 0, otherwise.

Our goal is to construct an integrated model in which the impacts of a multiplicity of different trade measures that have been imposed on such essential products as medical supplies and food in the Covid-19 pandemic can be assessed. Specifically, here we consider the following trade measures. First, we allow for a unit tariff τ_{ij}^h imposed by a country j on country i 's essential product h . If there is no such tariff τ_{ij}^h between a pair of countries i and j and product h then we can set the associated value to 0. Also, a country i may wish to support producers of a specific essential product h through a price support in the form of a lower bound on the supply market prices in the form $\underline{\pi}_{(i,k)}^h$, for supply market k in its country. On the other hand, a country j may wish to impose an upper bound on the demand market prices of an

Table 1

Notation for the Multiproduct, Multicountry Spatial Price Equilibrium Model with Trade Measures

Notation	Definition
x_p^h	the flow of essential product h on path p . We group the path flows into the vector $x \in R_+^{Hn_P}$.
f_a^h	the flow of essential product h on link a . We group the product link flows into the vector $f \in R_+^{Hn_L}$.
$\pi_{(i,k)}^h$	the supply price of product h at supply market k in country i . We group all the supply market prices into the vector $\pi \in R^{Hn_k}$.
$\rho_{(j,l)}^h$	the demand price of product h at demand market l in country j . We group all the demand market prices into the vector $\rho \in R^{Hn_l}$.
$c_a^h(f)$	the unit cost of transportation of product h on link a .
$C_p^h(x)$	the unit cost of transportation of product h on path p . We group all the path costs into the vector $C \in R^{Hn_P}$.
$s_{(i,k)}^h(\pi)$	the supply of product h at supply market k of country i . We group all the supply functions into the vector $s(\pi) \in R^{Hn_k}$.
$d_{(j,l)}^h(\rho)$	the demand for product h at demand market l of country j . We group all the demand functions into the vector $d(\rho) \in R^{Hn_l}$.
τ_{ij}^h	the unit tariff imposed by country j on product h from country i .
$\underline{\pi}_{(i,k)}^h$	the supply price lower bound on product h imposed on supply market k of country i by country i .
$\bar{\pi}_{(i,k)}^h$	the supply price upper bound on product h imposed on supply market k of country i by country i .
$\bar{\rho}_{(j,l)}^h$	the demand price upper bound on product h imposed on demand market l of country j by country j .
G_g^h	the group consisting of all the countries i and j comprising the group and the supply markets of i and demand markets of j for which a strict quota of \bar{Q}_g^h on product h is imposed. There is a total of n_G quotas imposed, with n_{G^h} denoting the number of quotas imposed on product h .

essential product h in its country of $\bar{\rho}_{(j,l)}^h$, for its demand market l . We also allow for an upper bound on the supply market price of each product h at each supply market k in each country i of $\bar{\pi}_{(i,k)}^h$. Of course, when such trade measures are not instituted, then we can just set the lower supply market price bound to 0 and the demand market and supply market price ceilings to a high number. In addition, and, as noted in Table 1, a country may wish to impose a quota and it is important to emphasize that this can be either an export quota of an essential product or an import quota of such a product. We handle such trade measures through the use of groups G_g^h , for relevant g s and h s, and associated quotas \bar{Q}_g^h (please refer to Table 1). Associated with each quota there is a corresponding nonnegative Lagrange multiplier λ_g^h . We group these Lagrange multipliers into the vector $\lambda \in R_+^{n_G}$. Also, we define $P_{G_g^h}$ as the set of paths corresponding to group G_g^h .

We define the feasible set

$$K \equiv \{(x, \lambda, \pi, \rho) \in R_+^{H(n_P+n_G+n_k+n_i)}, \text{ and } \underline{\pi}_{(i,k)}^h \leq \pi_{(i,k)}^h \leq \bar{\pi}_{(i,k)}^h, \forall h, i, k, \text{ and } 0 \leq \rho_{(j,l)}^h \leq \bar{\rho}_{(j,l)}^h, \forall h, j, l\}.$$

Definition 1. Multiproduct, Multicountry Spatial Price Equilibrium Conditions Under Trade Measures

A multiproduct, multicountry product flow, quota Lagrange multiplier, supply price, and demand price pattern $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ is an essential product spatial price equilibrium under trade measures of tariffs, quotas, supply price floors and ceilings, and demand price ceilings, if the following conditions hold: For all essential products h and for all groups G_g^h for all g, h , and for all pairs of supply and demand markets in the countries: $(i, j), (k, l) \in G_g^h$, and all paths $p \in P_{(j,l)}^{(i,k)}$, for all i, j, k, l :

$$\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* \begin{cases} = \rho_{(j,l)}^{h*}, & \text{if } x_p^{h*} > 0, \\ \geq \rho_{(j,l)}^{h*}, & \text{if } x_p^{h*} = 0, \end{cases} \quad (4)$$

$$\lambda_{G_g^h}^* \begin{cases} \geq 0, & \text{if } \sum_{p \in P_{G_g^h}} x_p^{h*} = \bar{Q}_{G_g^h}, \\ = 0, & \text{if } \sum_{p \in P_{G_g^h}} x_p^{h*} < \bar{Q}_{G_g^h}; \end{cases} \quad (5)$$

for all products h and all supply markets k in the countries i :

$$s_{(i,k)}^h(\pi^*) \begin{cases} \leq \sum_{p \in P^{(i,k)}} x_p^{h*}, & \text{if } \pi_{(i,k)}^{h*} = \bar{\pi}_{(i,k)}^h, \\ = \sum_{p \in P^{(i,k)}} x_p^{h*}, & \text{if } \underline{\pi}_{(i,k)}^h < \pi_{(i,k)}^{h*} < \bar{\pi}_{(i,k)}^h, \\ \geq \sum_{p \in P^{(i,k)}} x_p^{h*}, & \text{if } \pi_{(i,k)}^{h*} = \underline{\pi}_{(i,k)}^h, \end{cases} \quad (6)$$

plus, for all products h and all demand markets l in the countries j :

$$d_{(j,l)}^h(\rho^*) \begin{cases} \geq \sum_{p \in P_{(j,l)}} x_p^{h*}, & \text{if } \rho_{(j,l)}^{h*} = \bar{\rho}_{(j,l)}^h, \\ = \sum_{p \in P_{(j,l)}} x_p^{h*}, & \text{if } 0 < \rho_{(j,l)}^{h*} < \bar{\rho}_{(j,l)}^h, \\ \leq \sum_{p \in P_{(j,l)}} x_p^{h*}, & \text{if } \rho_{(j,l)}^{h*} = 0. \end{cases} \quad (7)$$

For paths not belonging to any group associated with a quota, we have that (4) holds with the quota Lagrange multiplier removed and (5) also excised.

In the case of the complete prohibition of exports of an essential product h from a country i , as in the case of, for example, certain medical products such as PPEs, ventilators, etc., one could excise the associated paths from the country to other countries for such products. Alternatively, one could define the appropriate group and set the quota for the group equal to 0. On the other hand, and, interestingly, there have also been import bans instituted by several countries due to Covid-19 to reduce the spread of the disease. For example, in that case, if a country j institutes an import ban on a product in its country then all paths from other countries for that product to demand markets in country j would not be used; in effect, they would be eliminated from the network topology for that product. One could also set the quota equal to 0 for the defined group.

According to equilibrium conditions (4), there will be a positive flow of an essential product on a path, in equilibrium, if the supply price at the supply market in a country at which the path originates at plus

the unit path cost of transporting the essential product on the path plus the tariff levied plus the quota Lagrange multiplier associated with the group on which the quota is imposed (and the path connects a pair of origin and destination nodes in it) is equal to the price the consumers are willing to pay at the demand market in the destination country. If, on the other hand, the above supply price plus the unit path transportation cost plus tariff plus quota Lagrange multiplier exceeds the price the consumers are willing to pay, then the product flow on that path will be equal to zero. Conditions (5), in turn, reveal that, in equilibrium, if the quota is reached by the path flows associated with the group and product that the quota is imposed on, then the equilibrium Lagrange multiplier for that group and product is positive; otherwise, the Lagrange multiplier is equal to zero.

Equilibrium conditions (6) state that if the equilibrium supply price for an essential product at a supply market in a country is greater than the imposed supply price lower bound, and less than the imposed upper bound, then the supply of the product at the supply market in the country is equal to the product flows out. If the supply price is equal to the imposed supply market lower bound in the country, then there could be excess supply. On the other hand, if the supply price for a product is equal to the imposed supply price upper bound, then the supply at equilibrium could be lower than the shipments out. Equilibrium conditions (7) state that, if the demand price at a demand market in a country for an essential product is equal to the imposed demand market price upper bound, then the demand can exceed the product inflows to that demand market in that country of the medial product. If the equilibrium demand market price is greater than zero and less than the imposed demand price upper bound for the product at the demand market in the country, then the demand for the product at the demand market in the country is equal to the product inflows of that essential product. Finally, if the equilibrium demand price is equal to zero for the product at a demand market in a country, then the demand can be lower.

The above spatial price equilibrium conditions provide a unified framework for a multiplicity of trade measures for essential product supplies in the Covid-19 pandemic. They are extensions of classical spatial price equilibrium conditions due to Samuelson (1952) and Takayama and Judge (1971); see Nagurney (1999, 2006) for many references to spatial price equilibrium models and the more recent work of Nagurney, Besik, and Dong (2019). For example, the above model includes multiple products (of great importance in the case of medical supplies), and concomitant handling of multiproduct tariffs and group quotas, along with the inclusion of supply price floors and ceilings plus demand price ceilings, all on a general network. Furthermore, we identify how to handle complete prohibition of essential product exports or even imports, including those of food products.

The above spatial price equilibrium framework enables decision-makers and policy-makers to evaluate the impacts of different trade measures on product prices and product flows. The impacts of the tightening of trade restrictions can be ascertained, once the model is solved, as well as the impacts of trade liberalizations, such as the reduction in tariffs and the lifting of import or export bans.

We now provide the variational inequality formulation of the above equilibrium conditions in a Theorem.

Theorem 1. *Variational Inequality Formulation of the Multiproduct, Multicountry Essential Product Spatial Equilibrium Conditions Under Trade Measures*

A multiproduct, multicountry product flow, quota Lagrange multiplier, supply price, and demand price pattern $(x^, \lambda^*, \pi^*, \rho^*) \in K$ is an essential product spatial price equilibrium under trade measures of tariffs, quotas, supply price floors and ceilings, and demand price ceilings, according to Definition 1, if*

and only if it satisfies the variational inequality problem:

$$\begin{aligned}
& \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \sum_{(i,j) \in G_g^h} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] \times \left[x_p^h - x_p^{h*} \right] \\
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \sum_{(i,j) \notin G_g^h} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h - \rho_{(j,l)}^{h*} \right] \times \left[x_p^h - x_p^{h*} \right] \\
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \left[\bar{Q}_{G_g^h} - \sum_{p \in P_{G_g^h}} x_p^{h*} \right] \times \left[\lambda_{G_g^h} - \lambda_{G_g^h}^* \right] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[s_{(i,k)}^h(\pi^*) - \sum_{p \in P^{(i,k)}} x_p^{h*} \right] \times \left[\pi_{(i,k)}^h - \pi_{(i,k)}^{h*} \right] \\
& + \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[\sum_{p \in P_{(j,l)}} x_p^{h*} - d_{(j,l)}^h(\rho^*) \right] \times \left[\rho_{(j,l)}^h - \rho_{(j,l)}^{h*} \right] \geq 0, \quad \forall (x, \lambda, \pi, \rho) \in K. \quad (8)
\end{aligned}$$

Proof: See the Appendix.

We now put variational inequality (8) into standard variational inequality form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (9)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in \mathcal{N} -dimensional Euclidean space, F is a given continuous function from \mathcal{K} to $\mathcal{R}^{\mathcal{N}}$, and \mathcal{K} is a given closed, convex set. $\mathcal{N} = H(n_P + n_G + n_k + n_l)$ for our model. We define $X \equiv (x, \lambda, \pi, \rho)$ and $\mathcal{K} \equiv K$. Also, we define $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$, where the components of $F^1(X)$ correspond to the n_P elements with a typical p element as preceding the first multiplication sign in (8); the components of $F^2(X)$ correspond to the n_P elements with a typical such element as immediately preceding the second multiplication sign; the components of $F^3(X)$ correspond to the n_G elements with a typical gh element as preceding the third multiplication sign, and so on.

3.1. Illustrative Examples

In order to fix ideas, we now provide several illustrative examples, the solutions to which can be determined analytically.

Baseline Illustrative Example

We first consider a single country with a single supply market and a single demand market with the network topology depicted in Figure 2.

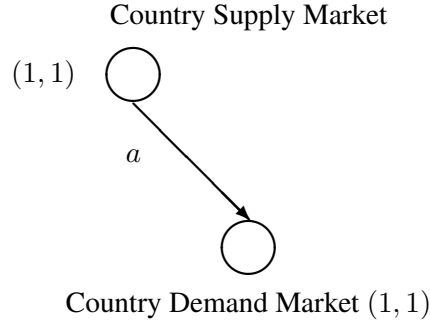


Fig. 2. Network Topology for the Baseline Illustrative Example

For simplicity, there is a single path consisting of a single link joining supply market node $(1, 1)$ with demand market node $(1, 1)$. There is a single essential product and, hence, for simplicity sake, we suppress the superscript $h = 1$ in the functional notation. The supply function is given by:

$$s_{(1,1)}(\pi) = 5\pi_{(1,1)} + 5,$$

and the demand function is given by

$$d_{(1,1)}(\rho) = -\rho_{(1,1)} + 22.$$

The path joining the supply market node with the demand market node is denoted by p_1 and it consists of link a , with a unit cost of $c_a = f_a + 1$ and, therefore, the path cost $C_{p_1} = x_{p_1} + 1$.

Making use of the spatial price equilibrium conditions in Definition 1, and noting that this example has no imposed trade measures, it is clear that the spatial price equilibrium product path flow and supply price and demand price pattern is:

$$x_{p_1}^* = 10, \quad \pi_{(1,1)}^* = 1, \quad \rho_{(1,1)}^* = 12,$$

since $C_{p_1}(x^*) = 11$ and, hence, we have that: $\pi_{(1,1)}^* + C_{p_1}(x^*) = \rho_{(1,1)}^*$. Furthermore, $s_{(1,1)}(\pi^*) = x_{p_1}^* = d_{(1,1)}(\rho^*)$.

Baseline Illustrative Example with Supply Price Floor Added

In the above example we now impose a supply price floor of $\underline{\pi}_{(1,1)} = 2$. A government may do that to support, for example, farmers in terms of the minimum price for their product. Again, referring to the spatial price equilibrium conditions in Definition 1, it is straightforward to determine the spatial equilibrium pattern. Specifically, we now have that:

$$x_{p_1}^* = 9.5, \quad \pi_{(1,1)}^* = 2, \quad \rho_{(1,1)}^* = 12.5.$$

Furthermore, $C_{p_1}(x^*) = 10.5$, and, therefore, $\pi_{(1,1)}^* + C_{p_1}(x^*) = \rho_{(1,1)}^* = 12.5$. Also, $s_{(1,1)}(\pi^*) = 15 > x_{p_1}^* = 9.5$ and $d_{(1,1)}(\rho^*) = 9.5 = x_{p_1}^*$.

Baseline Illustrative Example with Supply Price Floor Plus Demand Price Ceiling Added

Now to the previous example, with the supply price floor, we impose a demand price ceiling of $\bar{\rho}_{(1,1)} = 10$. The new spatial price product flow, supply price and demand price equilibrium pattern is:

$$x_{p_1}^* = 7, \quad \pi_{(1,1)}^* = 2, \quad \rho_{(1,1)}^* = 10,$$

and we have that: $C_{p_1}(x^*) = 8$, so that $\pi_{(1,1)}^* + C_{p_1}(x^*) = \rho_{(1,1)}^*$; $s_1(\pi^*) = 15 > x_{p_1}^* = 7$ and $d_{(1,1)}(\rho^*) = 12 > x_{p_1}^* = 7$.

Baseline Illustrative Example with Supply Price Floor Plus Demand Price Ceiling Plus New Supply Market in Another Country

We now consider the impact of the addition of a new supply market in another country. The network topology is as in Figure 3. The data are as in the immediately preceding example but with the following additions. The supply price function at the supply market in the second country is: $s_{(2,1)}(\pi) = \pi_{(1,2)} + 1$. The cost on link b and, hence, on path $p_2 = (b)$ is $c_b(f) = f_b + 1$ and $C_{p_2}(x) = x_{p_2} + 1$, respectively. Making use of the equilibrium conditions in Definition 1 yields the following equilibrium pattern:

$$x_{p_1}^* = 7, \quad x_{p_2}^* = 5, \quad \pi_{(1,1)}^* = 2, \quad \pi_{(2,1)}^* = 4, \quad \rho_{(1,1)}^* = 10.$$

We also have that: $s_{(1,1)}(\pi^*) = 15 > x_{p_1}^* = 7$; $s_{(2,1)}(\pi^*) = 5 = x_{p_2}^*$ and $d_{(1,1)}(\rho^*) = 12 = x_{p_1}^* + x_{p_2}^*$. Moreover, we have that: $\pi_{(1,1)}^* + C_{p_1}(x^*) = \rho_{(1,1)}^*$ and $\pi_{(2,1)}^* + C_{p_2}(x^*) = \rho_{(1,1)}^*$.

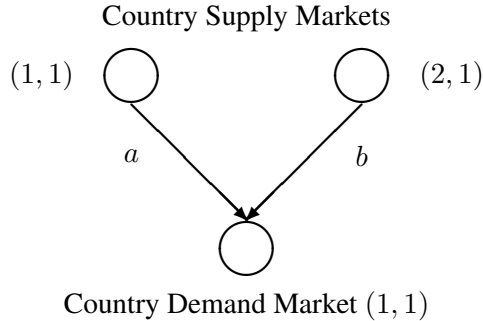


Fig. 3. Network Topology for the Illustrative Example with Two Supply Markets, Each in a Different Country

Baseline Illustrative Example with Supply Price Floor Plus Demand Price Ceiling Plus Supply Market in Another Country Plus Added Tariff

The final illustrative example has the same data as the immediately preceding example but now we consider the impact of an addition of a tariff $\tau_{21} = 2$. The new equilibrium pattern is:

$$x_{p_1}^* = 7, \quad x_{p_2}^* = 4, \quad \pi_{(1,1)}^* = 2, \quad \pi_{(2,1)}^* = 3, \quad \rho_{(1,1)}^* = 10.$$

In addition, we have that: We also have that: $s_{(1,1)}(\pi^*) = 15 > x_{p_1}^* = 7$; $s_{(2,1)}(\pi^*) = 5 = x_{p_2}^*$ and $d_{(1,1)}(\rho^*) = 12 > x_{p_1}^* + x_{p_2}^* = 11$. Moreover, we have that: $\pi_{(1,1)}^* + C_{p_1}(x^*) = \rho_{(1,1)}^*$ and

$\pi_{(2,1)}^* + C_{p_2}(x^*) + \tau_{12} = \rho_{(1,1)}^*$. In this example, as in the preceding ones, the spatial price equilibrium conditions according to Definition 1 hold precisely.

4. The Algorithm and Qualitative Properties

The algorithm that we utilize to solve the variational inequality (9), equivalently (8), governing the multiproduct, multicountry spatial price equilibrium model with trade measures and its variants that is presented in Section 3 is the modified projection method (see Korpelevich (1977) and Nagurney (1999)). This algorithm is guaranteed to converge if the function $F(X)$ that enters the standard variational inequality form is monotone and Lipschitz continuous (see Nagurney (1999)) and a solution exists. The qualitative properties are discussed in the next subsection.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $\tau := 1$ and select η , such that $0 < \eta \leq \frac{1}{L}$, where L is the Lipschitz continuity constant for $F(X)$.

Step 1: Construction and Computation

Compute \hat{X}^τ by solving the variational inequality subproblem:

$$\langle \hat{X}^\tau + (\eta F(X^{\tau-1}) - X^{\tau-1}), X - \hat{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (10)$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + (\eta F(\hat{X}^\tau) - X^{\tau-1}), X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (11)$$

Step 3: Convergence Verification

If $\|X^\tau - X^{\tau-1}\| \leq \epsilon$, with $\epsilon > 0$, a specified tolerance, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

We now provide the explicit formulae for the variables induced by the modified projection method for our model at a given iteration for Step 1 above. Similar explicit formulae can be determined accordingly for Step 2.

Explicit Formulae for Step 1 for the Essential Product Flows on Paths with Quotas

For all h, g , for all $(i, j) \in G_g^h$, and for all (k, l) , for each path $p \in P_{(j,l)}^{(i,k)}$, compute:

$$\hat{x}_p^\tau = \max\{0, x_p^{\tau-1} - \eta(\pi_{(i,k)}^{h(\tau-1)} + C_p^h(x^{\tau-1}) + \tau_{ij}^h + \lambda_{G_g^h}^{\tau-1} - \rho_{(j,l)}^{h(\tau-1)})\}; \quad (12)$$

Explicit Formulae for Step 1 for the Essential Product Flows on Paths without Quotas

For all h, g , for all $(i, j) \notin \cup_g G_g^h$, and for all (k, l) , for each path $p \in P_{(j,l)}^{(i,k)}$, compute:

$$\hat{x}_p^{h\tau} = \max\{0, x_p^{h\tau-1} - \eta(\pi_{(i,k)}^{h(\tau-1)} + C_p^h(x^{\tau-1}) + \tau_{ij}^h - \rho_{(j,l)}^{h(\tau-1)})\}; \quad (13)$$

Explicit Formulae for Step 1 for the Quota Lagrange Multipliers

For all h and for all g , for each group G_g^h , compute:

$$\hat{\lambda}_{G_g^h}^\tau = \max\{0, \lambda_{G_g^h}^{\tau-1} - \eta(\bar{Q}_{G_g^h} - \sum_{p \in P_{G_g^h}} x_p^{h(\tau-1)})\}; \quad (14)$$

Explicit Formulae for Step 1 for the Supply Prices

For all h , for all i , and for all $\forall k$, for each h, i, k , compute:

$$\hat{\pi}_{(i,k)}^{h\tau} = \max\{\underline{\pi}_{(i,k)}^h, \min\{\pi_{(i,k)}^{h(\tau-1)} - \eta(s_{(i,k)}^h(\pi^{\tau-1}) - \sum_{p \in P_{G_g^h}} x_p^{h(\tau-1)}), \bar{\pi}_{(i,k)}^h\}\}; \quad (15)$$

Explicit Formulae for Step 1 for the Demand Price

For all h , for all j , and for all l , for each h, j, l , compute:

$$\hat{\rho}_{(j,l)}^{h\tau} = \max\{0, \min\{\rho_{(j,l)}^{h(\tau-1)} - \eta(\sum_{p \in P_{G_g^h}} x_p^{h(\tau-1)} - d_{(j,l)}^h(\rho^{\tau-1})), \bar{\rho}_{(j,l)}^h\}\}. \quad (16)$$

4.1. Qualitative Properties

In this subsection, we discuss qualitative properties of the function $F(X)$ in (9) for our model required for convergence of the modified projection method. We also provide an existence result.

In the following proposition we show that if the product supply functions, minus the product demand functions, and the product link unit transportation cost functions are monotone in their respective vectors of variables, then the function $F(X)$ in (9) is monotone in (x, λ, π, ρ) . In the following proof we make use of an alternative variational inequality to the one (8), which is in link flows. Let $K^{11} \equiv \{\pi | \pi \in R_+^{H(n_k)}; \underline{\pi}_{(i,k)}^h \leq \pi_{(i,k)}^h \leq \bar{\pi}_{(i,k)}^h, \forall h, i, k\}$; $K^{12} \equiv \{\rho | \rho \in R_+^{H(n_l)}; 0 \leq \rho_{(j,l)}^h \leq \bar{\rho}_{(j,l)}^h, \forall h, j, l\}$; $K^{13} \equiv \{(x, f) | (x, f), \{(2) \text{ holds}\}\}$. Hence, we define the feasible set $K^1 \equiv \{(x, f, \lambda, \pi, \rho) | x \in R_+^{H(n_P)}, \lambda \in R_+^{H(n_G)}, \pi \in R_+^{H(n_k)}, \rho \in R_+^{H(n_l)}; \underline{\pi}_{(i,k)}^h \leq \pi_{(i,k)}^h \leq \bar{\pi}_{(i,k)}^h, \forall h, i, k; 0 \leq \rho_{(j,l)}^h \leq \bar{\rho}_{(j,l)}^h, \forall h, j, l, \text{ and } (2) \text{ holds}\} \equiv K^{11} \times K^{12} \times K^{13}$.

Proposition 1: Monotonicity of $F(X)$ in (9)

We assume that the vector of product supply functions $s(\pi)$, minus the vector of product demand functions $d(\rho)$, and the vector of link product unit transportation cost functions $c(f)$ are all monotone as follows:

$$\langle s(\pi^1) - s(\pi^2), \pi^1 - \pi^2 \rangle \geq 0, \quad \forall \pi^1, \pi^2 \in K^{11}, \quad (17a)$$

$$-\langle d(\rho^1) - d(\rho^2), \rho^1 - \rho^2 \rangle \geq 0, \quad \forall \rho^1, \rho^2 \in K^{12}, \quad (17b)$$

$$\langle c(f^1) - c(f^2), f^1 - f^2 \rangle \geq 0, \quad \forall f^1, f^2 \in K^{13}. \quad (17c)$$

Then, the function that enters variational inequality (8), as in standard form $F(X)$ (9), is monotone, with respect to the product flow vector x , the quota Lagrange multiplier vector λ , the vector of product supply prices π , and the vector of -minus product demand prices ρ , that is, X .

Proof. Using (2) and (3) we have the following:

$$\begin{aligned}
& \sum_{h=1}^H \sum_{i=1}^n \sum_{j=1}^n \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} C_p^h(x^*) \times [x_p^h - x_p^{h*}] = \sum_{h=1}^H \sum_{p \in P} C_p^h(x^*) \times [x_p^h - x_p^{h*}] \\
& = \sum_{h=1}^H \sum_{p \in P} \left[\sum_{a \in L} c_a^h(f^*) \delta_{ap} \right] \times [x_p^h - x_p^{h*}] = \sum_{h=1}^H \sum_{a \in L} c_a^h(f^*) \times \left[\sum_{p \in P} \delta_{ap} x_p^h - \sum_{p \in P} \delta_{ap} x_p^{h*} \right] \\
& = \sum_{h=1}^H \sum_{a \in L} c_a^h(f^*) \times [f_a^h - f_a^{h*}]. \tag{18}
\end{aligned}$$

Now, using (18), we can rewrite variational inequality (8) as follows:

$$\begin{aligned}
& \sum_{h=1}^H \sum_{i=1}^n \sum_{j=1}^n \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + \tau_{ij}^h - \rho_{(j,l)}^{h*} \right] \times [x_p^h - x_p^{h*}] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{j=1}^n \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[C_p^h(x^*) \right] \times [x_p^h - x_p^{h*}] \\
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \sum_{(i,j) \in G_g^h} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\lambda_{G_g^h}^* \right] \times [x_p^h - x_p^{h*}] \\
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \left[\bar{Q}_{G_g^h} - \sum_{p \in P_{G_g^h}} x_p^{h*} \right] \times [\lambda_{G_g^h} - \lambda_{G_g^h}^*] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[s_{(i,k)}^h(\pi^*) - \sum_{p \in P_{(i,k)}} x_p^{h*} \right] \times [\pi_{(i,k)}^h - \pi_{(i,k)}^{h*}] \\
& + \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[\sum_{p \in P_{(j,l)}} x_p^{h*} - d_{(j,l)}^h(\rho^*) \right] \times [\rho_{(j,l)}^h - \rho_{(j,l)}^{h*}] \\
& = \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} \right] \times [x_p^h - x_p^{h*}] + \sum_{h=1}^H \sum_{i=1}^n \sum_{j=1}^n \sum_{l=1}^{l_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\tau_{ij}^h - \rho_{(j,l)}^{h*} \right] \times [x_p^h - x_p^{h*}] \\
& + \sum_{h=1}^H \sum_{a \in L} c_a^h(f^*) \times [f_a^h - f_a^{h*}] + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \sum_{G_g^h} \sum_{p \in P_{G_g^h}} \left[\lambda_{G_g^h}^* \right] \times [x_p^h - x_p^{h*}]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \left[\bar{Q}_{G^h} - \sum_{p \in P_{G^h}} x_p^{h*} \right] \times \left[\lambda_{G^h} - \lambda_{G^h}^* \right] + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[s_{(i,k)}^h(\pi^*) \right] \times \left[\pi_{(i,k)}^h - \pi_{(i,k)}^{h*} \right] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[- \sum_{p \in P_{(i,k)}} x_p^{h*} \right] \times \left[\pi_{(i,k)}^h - \pi_{(i,k)}^{h*} \right] + \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \sum_{p \in P_{(j,l)}} x_p^{h*} \times \left[\rho_{(j,l)}^h - \rho_{(j,l)}^{h*} \right] \\
& + \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[-d_{(j,l)}^h(\rho^*) \right] \times \left[\rho_{(j,l)}^h - \rho_{(j,l)}^{h*} \right] \geq 0, \quad \forall (x, f, \pi, \lambda, \rho) \in K^1. \tag{19}
\end{aligned}$$

We can now establish that $F(X)$ is monotone. For any $X^1 = (x^1, \lambda^1, \pi^1, \rho^1) \in K, X^2 = (x^2, \lambda^2, \pi^2, \rho^2) \in K,$

$$\begin{aligned}
\langle F(X^1) - F(X^2), X^1 - X^2 \rangle & = \langle F(x^1, \lambda^1, \pi^1, \rho^1) - F(x^2, \lambda^2, \pi^2, \rho^2), \begin{bmatrix} x^1 - x^2 \\ \lambda^1 - \lambda^2 \\ \pi^1 - \pi^2 \\ \rho^1 - \rho^2 \end{bmatrix} \rangle \\
& = \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[\pi_{(i,k)}^{h1} - \pi_{(i,k)}^{h2} \right] \times \left[\sum_{p \in P_{(i,k)}^{(i,k)}} x_p^{h1} - \sum_{p \in P_{(i,k)}^{(i,k)}} x_p^{h2} \right] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{j=1}^n \sum_{l=1}^{l_j} \left[-\rho_{(j,l)}^{h1} + \rho_{(j,l)}^{h2} \right] \times \left[\sum_{p \in P_{(j,l)}^{(i,k)}} x_p^{h1} - \sum_{p \in P_{(j,l)}^{(i,k)}} x_p^{h2} \right] \\
& + \sum_{h=1}^H \sum_{a \in L} \left[c_a^h(f^1) - c_a^h(f^2) \right] \times \left[f_a^{h1} - f_a^{h2} \right] \\
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \left[\lambda_{G^h}^1 - \lambda_{G^h}^2 \right] \times \left[\sum_{p \in P_{G^h}} x_p^{h1} - \sum_{p \in P_{G^h}} x_p^{h2} \right] \\
& + \sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \left[- \sum_{p \in P_{G^h}} x_p^{h1} + \sum_{p \in P_{G^h}} x_p^{h2} \right] \times \left[\lambda_{G^h}^1 - \lambda_{G^h}^2 \right] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[s_{(i,k)}^h(\pi^1) - s_{(i,k)}^h(\pi^2) \right] \times \left[\pi_{(i,k)}^{h1} - \pi_{(i,k)}^{h2} \right] \\
& + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[- \sum_{p \in P_{(i,k)}} x_p^{h1} + \sum_{p \in P_{(i,k)}} x_p^{h2} \right] \times \left[\pi_{(i,k)}^{h1} - \pi_{(i,k)}^{h2} \right]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[\sum_{p \in P_{(j,l)}} x_p^{h1} - \sum_{p \in P_{(j,l)}} x_p^{h2} \right] \times \left[\rho_{(j,l)}^{h1} - \rho_{(j,l)}^{h2} \right] \\
& + \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[-d_{(j,l)}^h(\rho^1) + d_{(j,l)}^h(\rho^2) \right] \times \left[\rho_{(j,l)}^{h1} - \rho_{(j,l)}^{h2} \right] \\
= & \sum_{h=1}^H \sum_{a \in L} \left[c_a^h(f^1) - c_a^h(f^2) \right] \times \left[f_a^{h1} - f_a^{h2} \right] + \sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[s_{(i,k)}^h(\pi^1) - s_{(i,k)}^h(\pi^2) \right] \times \left[\pi_{(i,k)}^{h1} - \pi_{(i,k)}^{h2} \right] \\
& - \sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[d_{(j,l)}^h(\rho^1) - d_{(j,l)}^h(\rho^2) \right] \times \left[\rho_{(j,l)}^{h1} - \rho_{(j,l)}^{h2} \right]. \tag{20}
\end{aligned}$$

With the assumptions (17a, b, c) on the product supply functions, minus the product demand functions, and the link product unit transportation cost functions, we can conclude that expression (20) is greater than or equal to zero. Therefore, $F(X)$ is monotone. \square

Definition 2. Lipschitz Continuity

A function $F(X)$ is Lipschitz continuous if the following condition holds:

$$\|F(X^1) - F(X^2)\| \leq L \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{21}$$

where $L > 0$ is known as the Lipschitz constant.

Remark 1. $F(X)$ is Lipschitz continuous for our model provided that the link product unit transportation cost functions, the product supply functions, and the product demand functions have bounded first order derivatives (see also Dong, Zhang, and Nagurney (2004) and Li and Nagurney (2015)).

Only monotonicity and Lipschitz continuity of $F(X)$ are required for the convergence of the modified projection method, provided a solution exists.

Remark 2. Observe that equilibrium conditions (4) and (5) may be re-expressed without the use of the Lagrange multiplier vector λ^* , leading to a variational inequality in path flows, supply prices, and demand prices and over a feasible set that includes the quota constraints. Since the link unit cost functions and the supply and the demand functions are all assumed to be continuous, and since the supply prices and the demand prices are bounded and the product flows are as well under the assumption that there are quotas imposed on all product flows (and these may actually be very large), existence of a solution is guaranteed under the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)).

5. Numerical Examples

In this section, we present algorithmically computed solutions to spatial price numerical examples in order to show the types of insights that can be obtained, when trade measures are imposed. Furthermore,

the examples yield not only qualitative information, but also quantitative results. The spatial price network topology is as in Figure 4. Specifically, we consider a network with two countries, with a single supply market in each country and with two demand markets in each country. The product being produced, shipped, and demanded is that of N95 masks. The prices, the price floors, and the price ceilings are all in a common currency. The modified projection method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the implementation of the algorithm and the solution of these numerical examples. The algorithm was initialized as follows. We divided a demand of 100 equally among the paths to each demand market and we set the initial demand market prices at 1 and the initial supply market prices at 0 (except for Examples 4 and 5 where the supply price at the supply market of Country 1 is initialized to the supply price floor of 20). The algorithm was deemed to have converged if the absolute value of the successive variable iterates differed by no more than 10^{-4} .

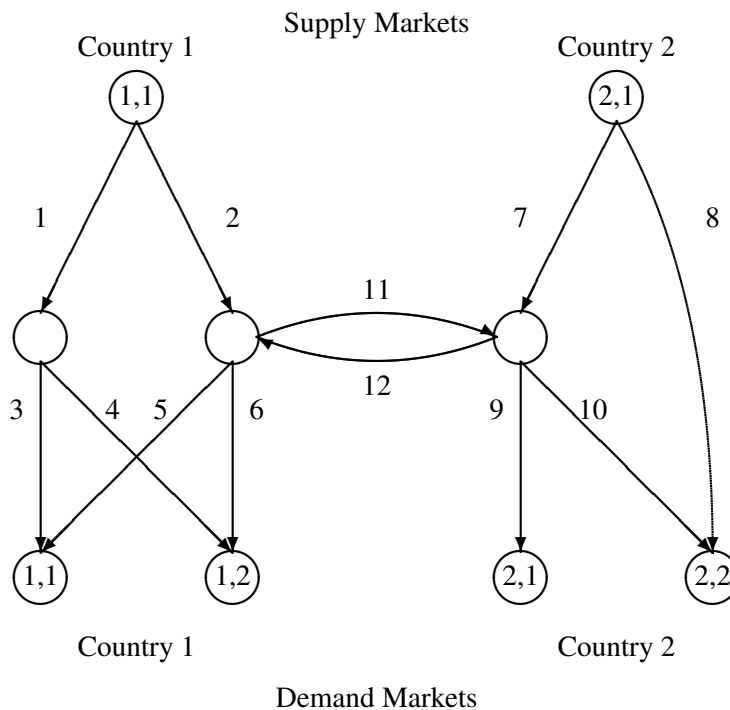


Fig. 4. Network for the Spatial Price Equilibrium Examples Solved Algorithmically

These numerical examples are stylized but, nevertheless, are grounded in realistic data. Country 1, for example, is inspired by China and Country 2 by the United States. The unit of flow is a kilogram of N95 masks with a kilogram corresponding to 100-150 masks.

The cost of air, sea, and land transportation depends on various factors, but according to Freightos (2020) and WTO (2020c), air cargo shipping cost ranges from \$2 to \$4 per kilogram. In our network, we assume links 1, 2, 7, 8, 11, and 12 are air freight links. Links 11 and 12 are international long distance

air shipping links between the two countries, so they have the highest unit shipping costs. Other links also have different rates depending on the distance between the origin and the destination.

The computed equilibrium path flows for Examples 1 through 5 are given in Table 2 and the computed equilibrium link flows in Table 3. We report the computed equilibrium prices, and the supplies and demands at the markets according to their functional formulae in the numerical examples, as well as the associated flows out of the supply markets and associated flows into the demand markets. Please refer to equilibrium conditions (5), (6), and (7).

Example 1 - No Trade Measures

In order to be able to ascertain the impact of a specific trade measure, or combination thereof, we first consider a numerical example in a pure form, that is, one in which no trade measures are imposed.

The data for this example are as follows. The paths are:

$$p_1 = (1, 3), \quad p_2 = (2, 5), \quad p_3 = (7, 12, 5), \quad p_4 = (1, 4), \quad p_5 = (2, 6), \quad p_6 = (7, 12, 6), \\ p_7 = (2, 11, 9), \quad p_8 = (7, 9), \quad p_9 = (2, 11, 10), \quad p_{10} = (7, 10), \quad p_{11} = (8).$$

The supply functions at the supply markets are:

$$s_{(1,1)}^1(\pi) = 25\pi_{(1,1)}^1 + 3000, \quad s_{(2,1)}^1(\pi) = 22\pi_{(2,1)}^1 + 1000.$$

The demand functions at the demand markets are:

$$d_{(1,1)}^1(\rho) = -0.5\rho_{(1,1)}^1 + 1800, \quad d_{(1,2)}^1(\rho) = -0.5\rho_{(1,2)}^1 + 1500, \\ d_{(2,1)}^1(\rho) = -1.5\rho_{(2,1)}^1 + 1000, \quad d_{(2,2)}^1(\rho) = -1.0\rho_{(2,2)}^1 + 2500.$$

The link unit transportation cost functions are:

$$c_1^1(f) = .01f_1^1 + 3, \quad c_2^1(f) = .02f_2^1 + 2, \quad c_3^1(f) = .03f_3^1 + 2, \quad c_4^1(f) = .06f_4^1 + 1, \quad c_5^1(f) = .5f_5^1 + 1, \\ c_6^1(f) = .02f_6^1 + 2, \quad c_7^1(f) = .4f_7^1 + 4, \quad c_8^1(f) = .1f_8^1 + 3, \quad c_9^1(f) = .5f_9^1 + 1, \quad c_{10}^1(f) = .05f_{10}^1 + 1, \\ c_{11}^1(f) = .2f_{11}^1 + .1f_{12}^1 + 4, \quad c_{12}^1(f) = .3f_{12}^1 + .20f_{11}^1 + 5.$$

The computed equilibrium path flows and equilibrium link flows are reported in Tables 2 and 3, respectively. We now report the computed equilibrium prices. Observe that Country 2 exports no N95 masks to Country 1, since the equilibrium link flow on link 12 is equal to 0.00.

The computed equilibrium supply market prices are:

$$\pi_{(1,1)}^{1*} = 32.55, \quad \pi_{(2,1)}^{1*} = 48.57,$$

and the computed equilibrium demand market prices are:

$$\rho_{(1,1)}^{1*} = 109.16, \quad \rho_{(1,2)}^{1*} = 88.50, \quad \rho_{(2,1)}^{1*} = 397.14, \quad \rho_{(2,2)}^{1*} = 223.08.$$

The spatial price equilibrium conditions (5), (6), and (7) hold with excellent accuracy. Observe that the supply market price in Country 1 is lower than that at the supply market in Country 2; the same for the demand market prices in Country 1 as opposed to those in Country 2. Recall that the prices, which are

in a common currency, are for a kilogram of N95 masks, which corresponds to 100-150 masks. Hence, the above prices are quite reasonable; see also Nagurney et al. (2020). Also, it is worth noting that all the supply market prices and all the demand markets prices are positive and none are at a value of 0. In fact, the flow of N95 masks out of supply market 1 is equal to 3813.82, and the flow out of supply market in Country 2 is equal to 2068.57; furthermore, the flows of N95 masks into the demand markets are, respectively: 1745.42, 1455.75, 404.30, and 2276.92. These values are precisely equal to their respective values according to the supply function and demand function formulae above (note equations (6) and (7) of the equilibrium conditions).

Example 2 - Data as in Example 1 but with a Tariff Imposed by Country 2 on Imports of N95 Masks from Country 1

We now evaluate the impact of the imposition of a unit tariff by Country 2 on the imports of N95 masks from Country 1. This, of course, allows us to also compare the removal of such a tariff, which would correspond to Example 1. The data are as in Example 1 but with the following unit tariff: $\tau_{12}^1 = 2$. Please refer to the computed equilibrium path flows and the equilibrium link flows, respectively, in Tables 2 and 3.

The computed equilibrium supply market prices are now:

$$\pi_{(1,1)}^{1*} = 32.35, \quad \pi_{(2,1)}^{1*} = 48.76,$$

and the equilibrium demand market prices are:

$$\rho_{(1,1)}^{1*} = 108.95, \quad \rho_{(1,2)}^{1*} = 88.24, \quad \rho_{(2,1)}^{1*} = 397.52, \quad \rho_{(2,2)}^{1*} = 223.57.$$

Clearly, under the imposed tariff, consumers at the two demand markets in Country 2 experience a higher price for the N95 masks than they did in Example 1, whereas consumers at demand markets in Country 1 experience reduced prices, as compared to the values in Example 1.

The supply of N95 masks at the supply market in Country 1 according to the functional formula is: 3808.80 and the supply at the supply market in Country 2 is: 2072.76. The demands at the demand markets for N95 masks are now, according to their functional formulae, respectively: 1745.24, 1455.89, 403.73, and 2276.43. Both the supply flows and the demand flows correspond precisely to the values of the respective function evaluated at the equilibrium prices (see equilibrium conditions (6) and (7)).

We then proceeded to conduct sensitivity analysis to determine at what value of the tariff there would be zero flows of the N95 masks from Country 1 to Country 2. We observed that when the tariff τ_{12}^1 was 234 or higher the flow on link 11 was zero (and if it were 233 or lower there would be a positive volume of flow on link 11. Furthermore, under a unit tariff of 234, the computed equilibrium supply market prices were

$$\pi_{(1,1)}^{1*} = 9.15, \quad \pi_{(2,1)}^{1*} = 70.79,$$

and the computed equilibrium demand market prices were:

$$\rho_{(1,1)}^{1*} = 84.56, \quad \rho_{(1,2)}^{1*} = 57.89, \quad \rho_{(2,1)}^{1*} = 441.50, \quad \rho_{(2,2)}^{1*} = 280.34.$$

One can see that consumers in Country 2 suffer, in that, the higher the tariff that is levied by their country's government, the higher the demand market prices for the N95 masks.

The supply at supply market 1 in Country 1, as represented by the flow, at the unit tariff of 234, is equal to 3808.80 and that at the supply market in Country 2 equal to 2072.76; the demands at the demand markets, as represented by the flow, are, respectively: 1745.52, 1455.89, 403.73, and 2276.43. Again, as expected, since the equilibrium prices are not equal to 0, which would correspond, to price floors at that value, the above flow values coincide precisely with the corresponding functions evaluated at the computed equilibrium prices.

Example 3 - Data as in Example 2 but with Demand Price Bounds on Demand Markets in Country 2

In Example 3, the data remain as in Example 2, except that now we impose demand price ceilings on the N95 masks at the demand markets in Country 2. The government in Country 2 is concerned about the prices of the N95 at its demand markets and sets the following demand market price ceilings:

$$\bar{\rho}_{(2,1)}^1 = 200.00, \quad \bar{\rho}_{(2,2)}^1 = 100.00.$$

The computed equilibrium flow patterns for Example 3 are reported in Tables 2 and 3. In addition, for completeness, and for comparison with the preceding example, we now report the computed equilibrium supply and demand market prices:

$$\pi_{(1,1)}^{1*} = 16.61, \quad \pi_{(2,1)}^{1*} = 5.37,$$

and the computed equilibrium demand market prices:

$$\rho_{(1,1)}^{1*} = 92.40, \quad \rho_{(1,2)}^{1*} = 67.65, \quad \rho_{(2,1)}^{1*} = 200.00, \quad \rho_{(2,2)}^{1*} = 100.00.$$

The supply market price decreases at the supply market in Country 1, as compared to the supply market price in Example 2, and also decreases at the supply market in Country 2. The demand market prices decrease at the demand markets in Country 1, and also at the demand markets in Country 2, where they attain values at the imposed price ceilings.

The supply at supply market 1 in Country 1, as represented by the flow, is now equal to 3415.23, and that at the supply market in Country 2 is equal to 1118.04. The demands at the demand markets are, respectively: 1753.80, 1466.18, 217.90, and 1095.37. The flow values for the supply markets coincide with the respective functional values evaluated at the computed equilibrium prices. The flow values into the demand markets in Country 1 coincide with the corresponding demand functions evaluated at the computed equilibrium prices. However, since the equilibrium demand prices at both demand markets in Country 2 are at the imposed price ceiling (see also equilibrium condition (7)), the demand functions evaluated at the computed equilibrium prices, which are for the two demand markets in Country 2 equal to 700.00 and 2400, respectively, exceed the flow into the respective demand market.

As compared to Example 2, the flows of the N95 masks to demand markets in Country 2 decrease precipitously, and are at about 50%, which is detrimental to the health of those who require them as well as to the containment of contagion. Clearly, price ceilings at demand markets may reduce the purchase price at the demand market, but at the expense of volume of essential product. Interestingly, consumers at demand markets in Country 1 gain in terms of an increased volume of flow into their demand markets of the N95 masks, but at higher prices, as compared to the demand market prices in Example 2.

Example 4 - Data as in Example 3 but with a Supply Price Floor at the Supply Market of Country 2

Country 1 is concerned that the supply price of the N95 masks at its supply market is low, and, in order to protect producers, it has instituted a price floor of: $\pi_{(1,1)}^1 = 20$. The rest of the data for Example 4 is as in Example 3.

The computed equilibrium path flows and equilibrium link flows are reported in Tables 2 and 3, respectively.

The computed equilibrium supply market prices are now:

$$\pi_{(1,1)}^{1*} = 20.00, \quad \pi_{(2,1)}^{1*} = 5.40,$$

and the equilibrium demand market prices are:

$$\rho_{(1,1)}^{1*} = 95.69, \quad \rho_{(1,2)}^{1*} = 70.82, \quad \rho_{(2,1)}^{1*} = 200.00, \quad \rho_{(2,2)}^{1*} = 100.00.$$

Also, the supply at supply market 1 in Country 1, according to the supply function formula, is now equal to 3500.00, whereas the flow out is equal to 3398.90. The supply at the supply market in Country 2, according to the supply function formula, is equal to 1118.90, and this is exactly the value of the sum of flows out of this supply market. The demands at the demand markets according to the demand functions are, respectively: 1752.16, 1464.60, 700.00, and 2400.00, with the first two corresponding, respectively to the flow in to each demand market. The flow at Country 2's demand markets, in turn, are, respectively: 216.83 and 1084.22.

One can see that, at the supply market in Country 1, the supply price is at the supply price floor of 20 and that the supply market price at the supply market in Country 2 has risen, albeit slightly. The prices that the consumers pay at the two demand markets in Country 1 have also risen (as compared to their values in Example 3). Hence, although producers in Country 1 enjoy a higher price, consumers in Country 1 pay more for the N95 masks. As for the demand markets in Country 2, the demand market prices remain at the imposed price ceilings (as was the case in Example 3). Interestingly, although the supply price floor is enacted in Country 1 at its supply market, consumers at both demand markets in Country 2 experience a lower volume of N95 masks at their demand markets.

Example 5 - Data as in Example 4 but with an Export Quota of 100 Imposed by Country 1 on Country 2

In Example 5, the government of Country 1 is getting concerned about rising cases of Covid-19 and institutes an export quota for the N95 masks of 100. Please refer to Tables 2 and 3 for the computed equilibrium path flows and equilibrium link flows in Tables 2 and 3, respectively.

First, observe that, in Example 5, the quota of 100 is met, since the sum of the path flows on paths p_7 and p_9 is equal to 100; equivalently, one can see that, given the network topology for this set of problems, the equilibrium link flow on link 11 is 100.00. The associated computed Lagrange multiplier is $\lambda^{1*} = 21.11$, when we suppress the group notation.

The computed equilibrium supply market prices are now:

$$\pi_{(1,1)}^{1*} = 20.00, \quad \pi_{(2,1)}^{1*} = 5.65,$$

and the equilibrium demand market prices are:

$$\rho_{(1,1)}^{1*} = 95.47, \quad \rho_{(1,2)}^{1*} = 69.80, \quad \rho_{(2,1)}^{1*} = 200.00, \quad \rho_{(2,2)}^{1*} = 100.00.$$

Also, the flow of N95 masks out of supply market 1 in Country 1, according to the supply function, is now equal to 3500.00, whereas the flow out is equal to 3317.37.

The supply at the supply market in Country 2, according to the supply function formula, is equal to 1124.27, and this is exactly the value of the sum of flows out of this supply market.

The demands at the demand markets according to the demand functions are, respectively: 1752.27, 1465.20, 700.00, and 2400.00, and correspond, respectively, for the first two demand markets to the flow to each demand market. In the case of the demand markets in Country 2, the flow into its demand markets is: 210.07 and 1014.20, respectively. The supply flows of the N95 masks in both countries now decrease, which is detrimental to the health of those who require them as well as to the containment of contagion. The demand markets in Country 2 receive fewer of the N95 masks than in Example 4. The prices of N95 masks in the demand markets in Country 2 are bounded by the price ceilings but the price in the supply market of Country 2 has increased.

Path	Equilibrium Path Flows				
	Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5
p_1	1664.01	1663.97	1661.00	1659.09	1657.04
p_2	81.41	81.55	92.80	93.06	95.23
p_3	0.00	0.00	0.00	0.00	0.00
p_4	504.46	503.58	434.68	431.94	417.77
p_5	951.29	952.30	1031.50	1032.65	1047.33
p_6	0.00	0.00	0.00	0.00	0.00
p_7	266.93	256.04	130.97	125.58	91.30
p_8	137.36	138.69	86.93	91.24	118.77
p_9	345.71	342.35	64.27	56.56	8.70
p_{10}	216.17	216.04	114.77	111.71	92.02
p_{11}	1715.03	1718.04	916.33	915.94	913.48

Table 2
Equilibrium Path Flows for Examples 1-5

These numerical examples illustrate the impacts of trade measures not only on the country imposing the trade measure(s) but also on other countries. Interestingly, it may so happen that a trade measure imposed by a country, hoping to help its citizens, may actually adversely affect its consumers, but may positively affect those in another country.

Link	Equilibrium Link Flows				
	Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5
1	2168.47	2167.56	2095.68	2091.03	2074.81
2	1645.34	1641.24	1319.55	1307.86	1242.56
3	1664.01	1663.97	1661.00	1659.09	1657.04
4	504.46	503.58	434.68	431.94	417.77
5	81.41	81.55	92.80	93.06	95.23
6	951.29	952.30	1031.50	1032.65	1047.33
7	353.54	354.72	201.71	202.95	210.79
8	1715.03	1718.04	916.33	915.94	913.48
9	404.30	403.73	217.90	216.83	210.07
10	561.88	558.39	179.04	168.28	100.72
11	612.64	607.39	195.24	182.15	100.00
12	0.00	0.00	0.00	0.00	0.00

Table 3
Equilibrium Link Flows for Examples 1-5

6. Summary and Conclusions

The Covid-19 pandemic has disrupted lives and economies around the globe and has resulted in both a healthcare and an economic disaster. The novel coronavirus that causes Covid-19 is very contagious and knowledge about how it is spread, and how to mitigate the spread is emerging. Many governments of different countries, seeking to protect their citizens, along with essential workers, has instituted a variety of trade measures on essential products in the pandemic. Examples of such trade measures include: tariffs, quotas, as well as price supports in the form of price floors and ceilings.

In this paper, we provide a unified variational inequality framework for quantifying the impacts of a plethora of trade measures on the flows of essential products from supply markets to demand markets within a country and across countries. We introduce a multiproduct, multicountry spatial price equilibrium model, which has both product flows, as well as supply market prices and demand market prices as variables. It can also handle multiple paths, consisting of multiple numbers of links between supply markets and demand markets. We state the governing equilibrium conditions, in the presence of the trade measures, and derive a variational inequality formulation, which is then used to obtain qualitative results as well as solutions to illustrative examples and numerical examples, with the latter solved via an implemented algorithm.

The modeling and algorithmic framework enables policy makers and decision-makers to quantify the impacts of different trade measures, both individually or jointly, and to ascertain who may benefit and who may lose. For example, our computed solutions to numerical examples reveal that unexpected results

may occur. We find, for example, that, with a price ceiling at the demand markets imposed by a country, consumers benefit in terms of a reduction of prices at the demand markets in the country, but experience a loss in terms of volumes of flow of the essential products. Interestingly, under this scenario, a competing country may experience an increase in the flow of the essential product to its demand markets. Hence, a country may think that it is benefiting its own consumers, but actually helps those in another country and not its own. Also, imposing a price floor at a supply market in a country may help producers in securing a higher price for the essential product, consumers in the country bear a higher price for the essential product. Clearly, this is also not a positive outcome in a pandemic.

In constructing a general, computable spatial price equilibrium model in both quantity and price variables and with a spectrum of trade instruments, we have enriched the portfolio of spatial price equilibrium models which incorporate relevant policies in the form of trade measures, and which are being actively imposed now by governments around the world in the pandemic.

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Appendix A

A.1. Appendix

Proof. Proof of Theorem 1:

We first establish necessity, that is, we prove that if the vector $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ satisfies the spatial price equilibrium conditions according to Definition 1, then it also satisfies variational inequality (8). Note that, for a fixed path $p \in P_{(j,l)}^{(i,k)}$ with $(i, j), (k, l) \in G_g^h$, essential product $h \in H$, and group G_g^h , (4) implies that:

$$\left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] \times [x_p - x_p^*] \geq 0, \quad \forall x_p \geq 0. \quad (A1)$$

Indeed, according to (4), if $x_p^* > 0$, then $\left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] = 0$, so (A1) holds. Also, if $x_p^* = 0$, then $\left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] \geq 0$, and, since, due to the nonnegativity assumption on the path flows, $[x_p - x_p^*] \geq 0$, the product of these two terms is also nonnegative and (A1) holds. Inequality (A1) is independent of path p , essential product h , and group G_g^h ; therefore, summation over all essential products, groups, and paths, yields:

$$\sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \sum_{(i,j) \in G_g^h} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] \times [x_p - x_p^*] \geq 0 \quad \forall x \in K. \quad (A2)$$

Using similar arguments, we can conclude that

$$\sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \sum_{(i,j) \notin G_g^h} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\pi_{(i,k)}^{h*} + C_p^h(x^*) + \tau_{ij}^h - \rho_{(j,l)}^{h*} \right] \times [x_p - x_p^*] \geq 0, \quad \forall x \in K. \quad (A3)$$

Now, (5) implies that, for fixed g and h , if $\lambda_{G_g^h}^*$ satisfies (5), then:

$$\left[\bar{Q}_{G_g^h} - \sum_{p \in P_{G_g^h}} x_p^{h*} \right] \times [\lambda_{G_g^h} - \lambda_{G_g^h}^*] \geq 0, \quad \forall \lambda_{G_g^h} \geq 0. \quad (A4)$$

Summation of (A4) over all h and g yields:

$$\sum_{h=1}^H \sum_{g=1}^{n_{G^h}} \left[\bar{Q}_{G_g^h} - \sum_{p \in P_{G_g^h}} x_p^{h*} \right] \times [\lambda_{G_g^h} - \lambda_{G_g^h}^*] \geq 0, \quad \forall \lambda \in K. \quad (A5)$$

Also, (6) implies that, if $\pi_{(i,k)}^{h*}$ satisfies (6), then:

$$\left[s_{(i,k)}^h(\pi^*) - \sum_{p \in P^{(i,k)}} x_p^{h*} \right] \times [\pi_{(i,k)}^h - \pi_{(i,k)}^{h*}] \geq 0, \quad \forall \underline{\pi}_{(i,k)}^h \leq \pi_{(i,k)}^h \leq \bar{\pi}_{(i,k)}^h. \quad (A6)$$

But inequality (A6) holds for any h, i , and k , so summation over all h, i , and k yields:

$$\sum_{h=1}^H \sum_{i=1}^n \sum_{k=1}^{k_i} \left[s_{(i,k)}^h(\pi^*) - \sum_{p \in P^{(i,k)}} x_p^{h*} \right] \times \left[\pi_{(i,k)}^h - \pi_{(i,k)}^{h*} \right] \geq 0, \quad \forall \pi \in K. \quad (A7)$$

Analogously, (7) implies that if $\rho_{(j,l)}^{h*}$ satisfies (7), then:

$$\left[\sum_{p \in P_{(j,l)}} x_p^{h*} - d_{(j,l)}^h(\rho^*) \right] \times \left[\rho_{(j,l)}^h - \rho_{(j,l)}^{h*} \right] \geq 0, \quad 0 \leq \rho_{(j,l)}^h \leq \bar{\rho}_{(j,l)}^h. \quad (A8)$$

Since inequality (A8) holds for any h, j , and l , we can conclude that summation over all h, j , and l yields:

$$\sum_{h=1}^H \sum_{j=1}^n \sum_{l=1}^{l_j} \left[\sum_{p \in P_{(j,l)}} x_p^{h*} - d_{(j,l)}^h(\rho^*) \right] \times \left[\rho_{(j,l)}^h - \rho_{(j,l)}^{h*} \right] \geq 0, \quad \forall \rho \in K. \quad (A9)$$

Now, summing up (A2), (A3), (A5), (A7), and (A9) gives us variational inequality (8) and necessity has been established.

We now establish sufficiency, that is, we show that, if $(x^*, \lambda^*, \pi^*, \rho^*) \in K$ satisfies variational inequality (8) then it also satisfies the spatial price equilibrium conditions (4) - (7).

Let $\pi_{(i,k)}^h = \pi_{(i,k)}^{h*}, \forall (h, i, k), \rho_{(j,l)}^h = \rho_{(j,l)}^{h*}, \forall (h, j, l), \lambda_{G_g^h} = \lambda_{G_g^h}^*, \forall (h, g)$, and $x_p = x_p^*, \forall p \neq q$. Substitution of these values into (8), yields:

$$\left[\pi_{(i,k)}^{h*} + C_q^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] \times [x_q - x_q^*] \geq 0, \quad \forall x_q \geq 0. \quad (A10)$$

Inequality (A10) implies that, if $x_q^* = 0$, then $\left[\pi_{(i,k)}^{h*} + C_q^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] \geq 0$, and if $x_q^* > 0$, then $\left[\pi_{(i,k)}^{h*} + C_q^h(x^*) + \tau_{ij}^h + \lambda_{G_g^h}^* - \rho_{(j,l)}^{h*} \right] = 0$. Since this condition holds for any q in a group, we conclude that (A10) implies that equilibrium conditions (4) must hold for any path in a group.

Similarly, let $\pi_{(i,k)}^h = \pi_{(i,k)}^{h*}, \forall (h, i, k), \rho_{(j,l)}^h = \rho_{(j,l)}^{h*}, \forall (h, j, l), x_p = x_p^*, \forall p \neq q$, and $(i, j), (k, l) \notin \cup_g G_g^h$. Substituting these values into (8), we obtain:

$$\left[\pi_{(i,k)}^{h*} + C_q^h(x^*) + \tau_{ij}^h - \rho_{(j,l)}^{h*} \right] \times [x_q - x_q^*] \geq 0, \quad \forall x_q \geq 0. \quad (A11)$$

And (A11) implies that equilibrium conditions (4) must hold for any path q not in a group.

By setting now $\pi_{(i,k)}^h = \pi_{(i,k)}^{h*}, \forall (h, i, k), \rho_{(j,l)}^h = \rho_{(j,l)}^{h*}, \forall (h, j, l), x_p = x_p^*, \forall p$, and $\lambda_{G_s^h} = \lambda_{G_s^h}^*, \forall (h, g) \neq (s, t)$ in variational inequality (8), we get:

$$\left[\bar{Q}_{G_s^t} - \sum_{p \in P_{G_s^t}} x_p^{t*} \right] \times \left[\lambda_{G_s^t} - \lambda_{G_s^t}^* \right] \geq 0, \quad \forall \lambda_{G_s^t} \geq 0. \quad (A12)$$

(A12), in turn, implies that equilibrium conditions (5) must hold for any s and t .

Analogously, we now let $\rho_{(j,l)}^h = \rho_{(j,l)}^{h*}, \forall (h, j, l)$, $x_p = x_p^*, \forall p$, $\lambda_{G_g^h} = \lambda_{G_g^h}^*, \forall (h, g)$, and $\pi_{(i,k)}^h = \pi_{(i,k)}^{h*}, \forall (h, i, k) \neq (r, u, w)$. Substitution into (8) yields:

$$\left[s_{ru}^w(\pi^*) - \sum_{p \in P(r,u)} x_p^{w*} \right] \times [\bar{\pi}_{ru}^w - \pi_{ru}^{w*}] \geq 0, \quad \forall \underline{\pi}_{ru}^w \leq \pi_{ru}^w \leq \bar{\pi}_{ru}^w. \quad (A13)$$

(A13) implies that equilibrium conditions (6) must hold for any r , u , and w .

Finally, we set $x_p = x_p^*, \forall p$, $\lambda_{G_g^h} = \lambda_{G_g^h}^*, \forall (h, g)$, $\pi_{(i,k)}^h = \pi_{(i,k)}^{h*}, \forall (h, i, k)$, and $\rho_{(j,l)}^h = \rho_{(j,l)}^{h*}, \forall (h, j, l) \neq (e, o, v)$. Substitution of these values into variational inequality (8) yields:

$$\left[\sum_{p \in P(e,o)} x_p^{v*} - d_{eo}^v(\rho^*) \right] \times [\rho_{eo}^v - \rho_{eo}^{v*}] \geq 0, \quad 0 \leq \rho_{eo}^v \leq \bar{\rho}_{eo}^v. \quad (A14)$$

And (A14) implies that equilibrium conditions (7) must hold for any e , o , and v .

□