Integrated Multicriteria Network Equilibrium Models
for
Commuting versus Telecommuting

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Abstract: This paper develops integrated multicriteria network equilibrium models which incorporate commuting versus telecommuting choices within a unified framework. The models determine the user-optimized flow patterns based on distinct situations, in which travelers seek to determine their optimal routes of travel given the disutilities associated, respectively, with the origin/destination pairs, the origins, the destinations, or the network system itself. In these "elastic" demand models the members of a class perceive their generalized cost on a route as a weighting of travel time, travel cost, and opportunity cost. The models allow the weights to be not only class-dependent but also link-dependent. The formulation of the governing equilibrium conditions, as well as the qualitative analysis, and the computational procedure are based on finite-dimensional variational inequality theory. The integrated multicriteria network modeling framework is the first to handle commuting versus telecommuting endogenously with predictive equilibrium flows.

1. Introduction

Relationships between transportation and telecommunications have been topics of discussion in the transportation literature for nearly forty years (cf. Memmott (1963), Jones (1973), Khan (1976), Nilles, et al. (1976), Albertson (1977), and Harkness (1977)). The application of telecommuting, in particular, due to its potential to reduce traffic congestion as well as to improve air quality, has been studied from both conceptual (see, e.g., Salomon (1986), Mokhtarian (1990)) as well as empirical (cf. Nilles (1988), Mokhtarian (1991), Mokhtarian, et al. (1995)) perspectives.

Indeed, according to Hu and Young (1992), although commuting's share of total trips (but not miles) is decreasing, commuting, nevertheless, accounts for more trips (26 percent in 1990) and more miles traveled (32 per cent) than any other single purpose. Moreover, as argued by Mokhtarian (1998), it is very likely that a greater proportion of commute trips rather than other types of trips will be amenable to substitution through telecommunications and, consequently, telecommuting most likely has the highest potential for travel reduction of any of the telecommunication applications. Hence, the study of telecommuting and its impacts is a subject meritorious of continued interest and research.

In this paper, we take on the challenge of modeling commuting versus telecommuting within a network equilibrium framework. As stated in Mokhtarian and Salomon (1997), in order to properly address transportation versus telecommunication issues one must ultimately include the transportation network and be able to forecast volumes of flow. Furthermore, "any approach which does not allow that ultimate outcome... will be of limited use in a regional planning context." Towards that end, we develop integrated multicriteria network equilibrium models which explicitly model the choices associated with telecommuting versus commuting. The novelty of the approach lies in that links can represent not only physical links associated with vehicular transportation but also links associated with *virtual* transportation, i.e., telecommuting.

The framework allows the incorporation of individual criteria associated with this process through the construction of generalized costs which include weights associated with the user class's perception of travel time, travel cost, as well as the opportunity cost associated with commuting or telecommuting to work. Hence, individuals who perceive the opportunity cost of telecommuting as being high (since one may not have the opportunity to meet with co-workers and staff) would be characterized by a higher associated opportunity cost weight. On the other hand, a class of "commuter" may be more interested in staying closer to the neighborhood (perhaps to be in proximity to the children's school or to reduce pollution due to vehicular emissions), in which case the opportunity cost weight associated with her transportation link(s) would be higher than that associated with her telecommuting link(s).

We note that multicriteria traffic network models were introduced by Quandt (1967) and Schneider (1968) and explicitly consider that travelers may be faced with several criteria, notably, travel time and travel cost, in selecting their optimal routes of travel. Dial (1979), subsequently, proposed an uncongested model whereas Dafermos (1981) introduced congestion effects and derived an infinite-dimensional variational inequality formulation of her multiclass, multicriteria traffic network equilibrium problem, along with some qualitative properties.

Recently, there has been renewed interest in the formulation, analysis, and computation of multicriteria traffic network equilibrium problems although this approach has not, heretofore, been utilized to examine transportation/telecommunications tradeoffs specifically in the context of telecommuting. Researchers who have considered an infinite-dimensional variational inequality formulation, motivated by Dafermos' (1981) multiclass model, have included Leurent (1993a) who presented an elastic demand formulation but did not allow travel cost to be a function of flow. Leurent (1993b) also proposed a two-criteria model cast as a path-based finite-dimensional variational inequality problem (see, e.g., Leurent (1996)). Dial (1996, 1999), in turn, included a random variable to model the value of time and derived variational inequality formulations for the fixed demand problem. For an overview of multicriteria traffic network equilibrium problems to that date, see Leurent (1998).

The integrated multicriteria network equilibrium models developed here are based on the following contributions:

(1). that of Dafermos (1976), who proposed integrated equilibrium flow models for transportation planning which allowed for the "attractiveness" of origins (residential areas) and destinations (places of work) also to be taken into account in addition to the travel cost associated with route selection, and

(2). those of Nagurney (2000), who developed a multiclass, multicriteria traffic network equilibrium model with fixed travel demands and formulated the governing equilibrium conditions as a finite-dimensional variational inequality problem, and Nagurney and Dong (2000) who then extended the fixed demand model to the case of elastic travel demands and also allowed for distinct weights associated not only with each class of traveler but also with each link.

The modeling framework developed in this paper, in turn, retains the generality of the model developed in Nagurney and Dong (2000), with the following significant new features:

a. It explicitly models telecommuting as a choice on a *transportation* network in which workers who select this option are transported virtually to work.

b. It permits the incorporation of not only the travel cost and the travel time on a link but also the opportunity cost associated with taking that link, with associated weights for the criteria allowed to differ by class and link.

c. It allows for the selection of destinations, or origins, or both destinations and origins, in addition to the optimal routes of travel within the same framework.

The paper is organized as follows. In Section 2, the integrated multicriteria traffic network equilibrium models are developed, and the governing traffic network equilibrium conditions are presented. In addition, we establish how distinct situations or transportation problems can be transformed into a multicriteria traffic assignment-type problem with elastic demands.

In Section 3, we provide the variational inequality formulation of the governing equilibrium conditions and also present some qualitative properties of the solution pattern.

In Section 4, we propose an algorithm for the computation of the equilibrium pattern and provide convergence results. In Section 5, we then apply the algorithm to several numerical examples. We conclude with Section 6 in which we summarize the results.

2. The Integrated Multicriteria Traffic Network Equilibrium Models

In this Section, we develop the integrated multicriteria traffic network equilibrium models with *elastic* travel demands. Specifically, the models correspond to the following situations which reflect distinct circumstances:

Situation 1: Users of the network have predetermined origins, which correspond to their places of residence, and destinations, that is, places of work, and are free to select their travel paths (as well as whether to travel or not). Here "travel" is meant to telecommute or to take a trip. Of course, users may opt to neither telecommute nor take a trip.

Situation 2: Users of the network have predetermined origins and are free to select their destinations as well as their travel paths (as well as whether or not to locate at the origins),

Situation 3: Users of the network have predetermined destinations and are free to choose their origins as well as their travel paths (in addition to whether or not to select a particular destination), and

Situation 4: Users of the network are free to select their origins, their destinations, in addition to their travel paths.

The model permits each class of traveler to perceive the travel cost, the travel time, as well as the opportunity cost on a link in an individual manner. The equilibrium conditions are then shown in Section 3 to satisfy a finite-dimensional variational inequality problem.

2.1 The Notation

We now introduce the notation and then provide the specific problems that correspond to the above four situations.

We consider a general network $G = [\mathcal{N}, \mathcal{L}]$, where $\mathcal{N}$ denotes the set of nodes in the network and $\mathcal{L}$ the set of directed links. Let a denote a link of the network connecting a pair of nodes and let p denote a path, assumed to be acyclic, consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. There are n links in the network and n_P paths. Let W denote the set of J O/D pairs. The set of paths connecting the O/D pair w is denoted by P_w and the entire set of paths in the network by P. Let Y denote the set of origin nodes and Z the set of destination nodes.

Assume that there are k classes of travelers in the network with a typical class denoted by i. Let f_a^i denote the flow of class i on link a and let x_p^i denote the nonnegative flow of class i on path p. The relationship between the link loads by class and the path flows is:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{1}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. Hence, the load of a class of traveler on a link is equal to the sum of the flows of the class on the paths that contain that link.

In addition, let f_a denote the total flow on link a, where

$$f_a = \sum_{i=1}^{k} f_a^i, \quad \forall a \in \mathcal{L}. \tag{2}$$

Hence, the total load on a link is equal to the sum of the loads of all classes on that link. Group the class link loads into the kn-dimensional column vector $\tilde{f}$ with components: $\{f_a^1, \ldots, f_n^1, \ldots, f_a^k, \ldots, f_n^k\}$ and the total link loads: $\{f_a, \ldots, f_n\}$ into the n-dimensional column vector f. Also, group the class path flows into the kn_P-dimensional column vector $\tilde{x}$ with components: $\{x_{p_1}^1, \ldots, x_{p_{n_P}}^k\}$.

Note that in our framework a link may correspond to an actual physical link of transportation or an abstract or virtual link corresponding to a telecommuting link. Furthermore, the network representing the problem under study can be as general as necessary and a path may also consist of a set of links corresponding to physical and virtual transportation choices such as would occur if a worker were to commute to a work center from which she could then telecommute. In Figure 1, we provide a conceptualization of this idea.

Note that in Figure 1, nodes 1 and 2 represent locations of residences, whereas node 6 denotes the place of work. Work centers from which workers can telecommute are located at nodes 3 and 4 which also serve as intermediate nodes for transportation routes to work. In addition, links: $(1, 6)$, $(3, 6)$, $(4, 6)$, and $(2, 6)$ are telecommunication links depicting virtual transportation to work via telecommuting, whereas all other links are physical links associated with commuting. Hence, the paths $(1, 6)$ and $(2, 6)$ consisting, respectively, of the

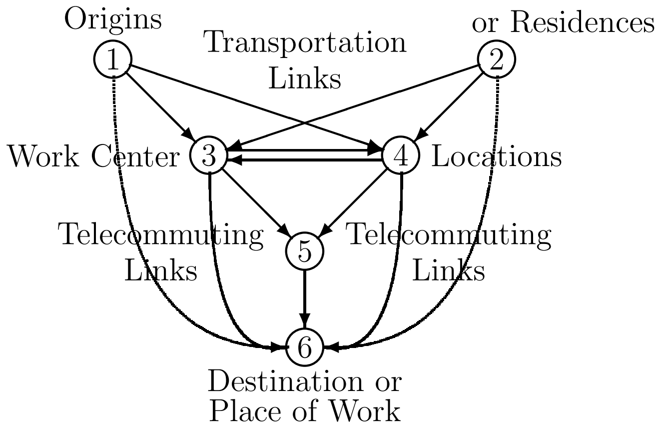

Figure 1: A network conceptualization of commuting versus telecommuting

individual single links represent "going to work" virtually whereas the paths consisting of the links: $(1, 3)$, $(3, 6)$ and $(2, 4)$, $(4, 6)$ represent first commuting to the work centers located at nodes 3 and 4, from which the workers then telecommute. Finally, the remaining paths represent the commuting options for the residents at nodes 1 and 2. The conventional travel paths from node 1 to node 6 are as follows: (1,3), (3,5), (5,6); (1,3), (3,4), (4,5), (5,6); (1,4), (4,5), (5,6), and (1,4), (4,3), (3,5), (5,6). Note that there may be as many classes of users of this network as there are groups who perceive the tradeoffs among travel cost, travel time, and opportunity cost in a similar fashion.

We are now ready to describe the functions associated with the links. We assume, as given, a travel time function t_a associated with each link a in the network, where

$$t_a = t_a(f), \quad \forall a \in \mathcal{L}, \tag{3}$$

and a travel cost function c_a associated with each link a, that is,

$$c_a = c_a(f), \quad \forall a \in \mathcal{L}, \tag{4}$$

with both these functions assumed to be continuous. Note that here we allow for the general situation in which both the travel time and the travel cost can depend on the entire link load pattern, whereas in Dafermos (1981) it was assumed that these functions were separable.

In addition, in order to capture the opportunity costs associated with commuting versus telecommuting tradeoffs, we also introduce an opportunity cost o_a associated with each link in the network, where

$$o_a = o_a(f), \quad \forall a \in \mathcal{L}. \tag{5}$$

For example, we can expect the opportunity cost associated with a telecommunication link to be higher than that of a transportation link since telecommuters do not have the opportunity of socializing and discussing projects face to face with colleagues, among other factors.

We assume that each class of traveler i has his own perception of the trade-offs among travel time, travel cost, and opportunity cost which are represented, respectively, by the nonnegative weights w^i_{1a}, w^i_{2a}, and w^i_{3a}. Here w^i_{1a} denotes the weight associated with class i's travel time on link a, w^i_{2a} denotes the weight associated with class i's travel cost on link a, and w^i_{3a} denotes the weight associated with class i's opportunity cost on link a. The weights w^i_{1a}, w^i_{2a}, and w^i_{3a} are link-dependent and, hence, can incorporate such link-dependent factors as safety, comfort, view, and sociability factors.

We then construct the *generalized* cost of class i associated with link a, and denoted by u^i_a, as:

$$u^i_a = w^i_{1a} t_a + w^i_{2a} c_a + w^i_{3a} o_a, \quad \forall i, \quad \forall a \in \mathcal{L}. \tag{6}$$

In view of (2) – (6), we may write

$$u^i_a = u^i_a(\tilde{f}), \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{7}$$

and group the generalized link costs into the kn-dimensional row vector u with components: $\{u^1_a, \ldots, u^1_n, \ldots, u^k_a, \ldots, u^k_n\}$.

Link-dependent weights were proposed in Nagurney and Dong (2000) and provide a greater level of generality and flexibility in modeling travel decision-making than weights that are identical for the travel time and for the travel cost on all links for a given class (see Nagurney (2000)). Note that in order to model telecommuting versus commuting some links may only have opportunity costs associated with them whereas others may not even include opportunity costs. This can also be achieved by setting the appropriate weights to zero, for example. Note that in the context of Figure 1, a class of traveler i who desires to be closer to home and resides at origin 1 would very likely have a weight $w^i_{3(1,4)}$ which exceeds the weight $w^i_{3(1,6)}$, which reflects that his weight associated with commuting to work exceeds that of telecommuting. On the other hand, a class of traveler who enjoys the socializing associated with her work environment and also resides at node i would have the reverse (relative) ranking of these weights.

Let v^i_p denote the generalized travel cost of class i associated with traveling on path p, where

$$v^i_p = \sum_{a\in\mathcal{L}} u^i_a(\tilde{f})\delta_{ap}, \quad \forall i, \forall p. \tag{8}$$

Hence, the generalized cost, as perceived by a class, associated with traveling on a path is its weighting of the travel times, the travel costs, and the opportunity costs on links which comprise the path.

The total number of trips generated at the origin node y (trip productions) by class i will be denoted by O^i_y. The total number of trips terminating at destination node z (trip attractions) by class i will be denoted by D^i_z. Finally, the travel demand associated with origin/destination (O/D) pair w and class i will be denoted by d^i_w. We let P_y denote the set of paths originating at origin y, P_z the set of paths terminating in destination node z, and recall that P_w is the set of paths connecting the origin/destination pair w.

We group the trip productions into a column vector $O \in R^{kJ}$, the trip attractions into the column vector $D \in R^{kJ}$, and the travel demands into a column vector $d \in R^{kJ}$.

Clearly, the travel demands, trip productions, and trip attractions must satisfy the following conservation of flow equations:

$$d^i_w = \sum_{p\in P_w} x^i_p, \quad \forall i, \forall w, \tag{9}$$

$$O^i_y = \sum_{p\in P_y} x^i_p, \quad \forall i, \forall y, \tag{10}$$

$$D^i_z = \sum_{p\in P_z} x^i_p, \quad \forall i, \forall z. \tag{11}$$

Moreover, if T^i denotes the total number of trips of class i in the network, that is, the total number of trips produced by the class at all origin nodes (and equal to the total number of trips terminating in all destination nodes) we must also have that

$$T^i = \sum_{y \in Y} O^i_y = \sum_{z \in Z} D^i_z = \sum_{p \in P} x^i_p, \quad \forall i. \tag{12}$$

Since we are considering here the elastic case, we need to introduce appropriate disutility functions for the classes. Let λ^i_w denote the travel disutility associated with class i traveling between O/D pair w; λ^i_y – the disutility associated with class i locating at origin node y; λ^i_z – the disutility associated with working at destination node z, and let λ^i denote the disutility associated with residing and working in the network system for class i. We group the disutilities, respectively, into row vectors λ^W, λ^Y, λ^Z, and λ.

We assume that the disutility functions are as follows:

$$\lambda^i_w = \lambda^i_w(d), \quad \forall w, \forall i, \tag{13}$$

$$\lambda^i_y = \lambda^i_y(O), \quad \forall y, \forall i, \tag{14}$$

$$\lambda^i_z = \lambda^i_z(D), \quad \forall z, \forall i, \tag{15}$$

$$\lambda^i = \lambda^i(T), \quad \forall i, \tag{16}$$

where the functions are assumed to be smooth and continuous.

In terms of the above terminology and notation, the above four situations lead to the following transportation problems:

Problem 1: Determine the vector of travel demands d and the path flow pattern x.

Problem 2: Determine the vector of trip productions O and the vector of path flows x along with the origin/destination demand vector d.

Problem 3: Determine the vector of trip attractions D and the path flows x along with the origin/destination demand vector d.

Problem 4: Determine the vector of the total number of trips T along with the trip productions O, the trip attractions D, and the travel demands d.

Note that Dafermos (1976) considered the analogous problems but in the special case where the demands were assumed to be fixed and given (rather than elastic) and there was only a single class of traveler facing a single criterion (rather than multiple classes and multiple criteria).

2.2 The Behavioral Assumption

The behavioral assumption which we utilize here has also been used by Dafermos (1976) in the context of single-criteria, fixed demand integrated traffic network equilibrium models. Note that those models, however, preceded the application of variational inequality theory (cf. Dafermos (1980, 1982)) which is the methodology that we adopt in the subsequent section in order to formulate the governing equilibrium conditions.

Specifically, the behavioral assumption utilized is similar to that underlying traffic assignment models (see also, Beckmann, McGuire, and Winsten (1956)) in that we assume that each class of user in the network selects (subject to constraints) his origin, his destination, and his travel path so as to minimize the generalized cost on the path, given that all other users have made their choices. Note that travel paths in this framework may correspond to telecommuting.

In particular, we have the following traffic network equilibrium conditions for the four problems outlined above:

Traffic Network Equilibrium Conditions

Problem 1: For each class i, for all O/D pairs $w \in W$, and for all paths $p \in P_w$, the flow and demand pattern $(\tilde{x}^*, d^*)$ is said to be in equilibrium if the following conditions hold:

$$v_p^i(\tilde{f}^*) \begin{cases} = \lambda_w^i(d^*), & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_w^i(d^*), & \text{if} \quad x_p^{i*} = 0, \end{cases} \tag{17}$$

In other words, all utilized paths by a class connecting an O/D pair have equal and minimal generalized costs and these costs are equal to the travel disutilities associated with the class and O/D pair.

Problem 2: For each class i, for all origins $y \in Y$, and for all paths $p \in P_y$, the flow and trip production pattern $(\tilde{x}^*, O^*)$ is said to be in equilibrium if the following conditions hold:

$$v_p^i(\tilde{f}^*) \begin{cases} = \lambda_y^i(O^*), & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_y^i(O^*), & \text{if} \quad x_p^{i*} = 0. \end{cases} \tag{18}$$

In this situation, the equilibrium is characterized by all utilized paths for a given class emanating from a given origin node having equal and minimal generalized costs, which, in turn, are equal to the disutility associated with locating at the particular origin node for the class.

Problem 3: For each class i, for all destinations $z \in Z$, and for all paths $p \in P_z$, the flow and trip attraction pattern $(\tilde{x}^*, D^*)$ is said to be in equilibrium if the following conditions hold:

$$v_p^i(\tilde{f}^*) \begin{cases} = \lambda_z^i(D^*), & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_z^i(D^*), & \text{if} \quad x_p^{i*} = 0. \end{cases} \tag{19}$$

Hence, in this case, all the generalized costs on paths for each class terminating in each destination node are equal and minimal. Furthermore, such minimal generalized path costs are also equal to the disutility associated with working at the particular destination node for each class.

Problem 4: For each class i, the flow and total trip pattern $(\tilde{x}^*, T^*)$ is said to be in equilibrium if the following conditions hold:

$$v_p^i(\tilde{f}^*) \begin{cases} = \lambda^i(T^*), & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda^i(T^*), & \text{if} \quad x_p^{i*} = 0. \end{cases} \tag{20}$$

Equilibrium conditions (20) state that for each class, the generalized path costs are equal and minimal and equal to the disutility of that class associated with the network system itself.

We define the feasible sets $\mathcal{K}^i$; $i = 1, \ldots, 4$, underlying the respective problems as $\mathcal{K}^1 \equiv \{(\tilde{f}, d) \,|\, \tilde{x} \geq 0 \, \text{and} \; (1), (2), \, \text{and} \, (9) \, \text{hold}\}$; $\mathcal{K}^2$ is defined similarly except that rather than (9) holding (10) must now hold, whereas $\mathcal{K}^3$ requires that (11) be satisfied instead and $\mathcal{K}^4$ that (12) be satisfied.

Remark

As emphasized by Dial (1996), multiclass, multicriteria traffic network equilibrium models, as is our framework here, subsume multiple modes of transportation as well in that a "path" in this context may correspond to a mode of transportation. Here, since our focus is on telecommuting versus commuting we note that a path, hence, may also correspond to an *alternative* mode of telecommuting.

2.3 Reduction of Problems 2 Through 4 to the Elastic Demand Problem 1

In this Subsection, we show that the integrated network equilibrium models conforming to Situations 2 through 4 can be reduced to the elastic demand model corresponding to Situation 1. Consequently, through the subsequently described transformations, one can then study the distinct situations through the theory applied to the model of Situation 1.

Construction of Expanded Network for Situation/Problem 2

We modify the original network G as follows. Construct a "super" demand or sink node, denoted by ψ, and from each destination node construct then a single link terminating in node ψ. Associate with each link a and class i in this network the generalized cost u_a^i, which is assumed to take on the general form (6). Associate with the artificial links terminating in node ψ a generalized cost equal to zero. Denote by $\hat{p}$, the path which consists of the links in path p plus the artificial link terminating in ψ. In addition, define then the O/D pairs on the expanded network as consisting, respectively, of each of the original origin nodes y and with destination node ψ. Associate with each such O/D pair the disutility function λ_y^i for each class i. Equilibrium condition (17) is then applied to this expanded network. Clearly, in this setting (17) coincides with (18) since the flow pattern x for G gives rise to a flow pattern x' for the expanded network. Moreover, since the generalized costs on the artificial links are zero, we have that the generalized cost on a path for a class in the original network coincides with the generalized cost on the corresponding path for a class on the expanded network.

Hence, the problem of determining an equilibrium pattern for Problem 2 over the network G has been reduced to determining the equilibrium flow pattern for an elastic demand problem (Problem 1) over the expanded network.

Construction of Expanded Networks for Problems 3 and 4

Analogously, the transportation Problems 3 and 4 over a network G can be reduced to an equilibrium problem of the form of Problem 1 over appropriately constructed expanded networks as done for Problem 2 above. Hence, for transportation Problem 3, the expanded network would be constructed from G by adding an artificial super source or origin node ξ, which will be the only origin node of the expanded network for this problem, and then joining every origin node y of G with ξ with an artificial link (ξ, y) with zero generalized cost for each class i. Regarding the transportation Problem 4, in turn, the expanded network would be constructed as follows: Add an artificial super source or origin node ξ, which will be the only origin node of the expanded network, and then join each origin node y of G with an artificial link (y, ξ) with zero generalized cost for each class. In addition, add an artificial super sink or destination node ψ, which will be the only destination node of the expanded network and join each destination node z of G with ψ by an artificial link (z, ψ). These links also have zero generalized costs associated with each class of user.

In Section 5, we provide illustrations of the above constructs through explicit numerical examples for which the equilibrium patterns are then determined.

3. Variational Inequality Formulation and Qualitative Properties

In this Section, we present the variational inequality formulation of the equilibrium conditions governing Problem 1 and given by (17) since Problems 2 through 4 can be transformed to this problem. In addition, we present some qualitative properties of the solution to the variational inequality, including some uniqueness results. Subsequently, we investigate properties of the function F that enters the variational inequality formulation of the governing equilibrium conditions for the integrated multicriteria traffic network model.

Specifically, in light of Corollary 1 in Nagurney and Dong (2000), we can write down immediately the variational inequality formulation below.

Theorem 1

The variational inequality formulation of the integrated multicriteria traffic network model with known O/D pair travel disutility functions $\lambda(d)$ satisfying equilibrium condition (17) is given by: Determine $(\tilde{f}, d) \in \mathcal{K}^1$, satisfying

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} u_a^i(\tilde{f}^*) \times (f_a^i - f_a^{i*}) - \sum_{i=1}^{k} \sum_{w \in W} \lambda_w^i(d^*) \times (d_w^i - d_w^{i*}) \geq 0, \qquad \forall (\tilde{f}, d) \in \mathcal{K}^1; \tag{21a}$$

equivalently, in standard variational inequality form (cf. Nagurney (1999)):

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{21b}$$

where $F \equiv (u, \lambda)$, $X \equiv (\tilde{f}, d)$, and $\mathcal{K} \equiv \mathcal{K}^1$.

Observe that if there is only a *single* class of traveler in that all travelers perceive their disutility associated with selecting routes in an identical fashion but the travelers are, nevertheless, multicriteria decision-makers in that they perceive their generalized cost associated with selecting routes as a weighting of the travel times, travel costs, and opportunity costs then the multicriteria model with known O/D pair travel disutility functions collapses to the model of Dafermos (1982) in the case of only a single mode of transportation and in which the "generalized" cost denoted by c_a^1 on a link $a \in \mathcal{L}$ therein coincides with u_a^1.

Moreover, if the generalized cost on a link is a weighted average of only the travel cost and the travel time on a link (with distinct weightings for each class), then the model collapses to the model of Nagurney and Dong (2000) in which also the model with direct demand functions (and not just the inverses) was studied.

We now derive some qualitative properties of the solution to variational inequality (21a).

Note that the feasible set $\mathcal{K}^1$ underlying the variational inequality (21a) is not a compact set as is the case in the fixed demand model (see Nagurney (2000)). Moreover, imposing strong monotonicity conditions on the travel cost, travel time, and opportunity cost functions will not guarantee strong monotonicity of the multiclass link generalized cost functions (7), since all these former functions are assumed to be functions of the total link flows.

Hence, we propose a weaker condition under which the existence of a solution to variational inequality (21a) is guaranteed. Let c, t, and o be given continuous functions with the following properties: There exist positive numbers $\hat{c}, \hat{t}$, and k_2 such that

$$c_a^i(f) \geq \hat{c}, \quad \forall a, \forall i, \quad \forall f \in \mathcal{K}^1, \tag{22}$$

$$t_a^i(f) \geq \hat{t}, \quad \forall a, \forall i, \quad \forall f \in \mathcal{K}^1, \tag{23}$$

$$o_a^i(f) \geq 0, \quad \forall a, \forall i, \quad \forall f \in \mathcal{K}^1, \tag{24}$$

$$d_w^i < k_2, \quad \forall i, \forall w, \qquad \text{with } \lambda_w^i \geq k_1, \tag{25}$$

where $k_1 = \max\limits_{i,a \in \mathcal{L}} w_{1a}^i \hat{c} + w_{2a}^i \hat{t} \geq 0$.

Thus,

$$u_a^i(\tilde{f}) \geq k_1, \quad \forall a, i, \quad \forall f \in \mathcal{K}^1. \tag{26}$$

Conditions (22) – (25) assume only that the uncongested parts of the travel costs and the travel times are not zero and the opportunity costs are nonnegative, with the common belief that congested travel cost and time always exceed the uncongested ones. Condition (25) assumes that travel demands would not be too large (i.e., infinity) if the travel disutility is greater than the maximum weighted uncongested travel cost and travel time.

Referring to Nagurney (1999), we can immediately present the following result.

Theorem 2: Existence

Let c, t, o, and λ be given continuous functions satisfying conditions (22) through (25). Then variational inequality (21a) has at least one solution.

We now turn to examining uniqueness. In particular, we now consider a special case of the above model in which we establish uniqueness not of the vector of class link loads $\tilde{f}^*$ but, rather, the uniqueness of the total link loads f^* and the travel demands d^*.

Specifically, consider a generalized cost function of the form:

$$u_a^i = \psi_a^i t_a + \xi_a^i c_a + (1 - \psi_a^i - \xi_a^i) o_a, \quad \forall a, i, \tag{27}$$

where

$$t_a = g_a(f) + \alpha_a, \quad c_a = g_a(f) + \beta_a, \quad o_a = g_a(f) + \gamma_a, \quad \forall a \in \mathcal{L}. \tag{28}$$

Hence, the generalized cost function u_a^i for each class and link is a weighted average of the travel time, travel cost, and opportunity cost on a link. Moreover, the variable term is identical for the travel time, the travel cost, and opportunity cost on a given link. Here ψ_a^i and ξ_a^i are the link-dependent weights for class i travelers. Assume now that t, c, and o are each strictly monotone in f, that is,

$$\langle (t(f^1) - t(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^1, \quad f^1 \neq f^2, \tag{29}$$

$$\langle (c(f^1) - c(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^1, \quad f^1 \neq f^2, \tag{30}$$

and

$$\langle (o(f^1) - o(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^1, \quad f^1 \neq f^2. \tag{31}$$

Then we have the following:

Theorem 3: Uniqueness of the Total Link Load and Demand Pattern for Problem 1 in a Special Case

The total link load pattern f^* induced by a solution $\tilde{f}^*$ to variational inequality (21a) in the case of generalized cost functions u of the form (27) and (28), is guaranteed to be unique if the *travel time, travel cost, and opportunity cost functions are each strictly monotone increasing in f and the travel disutility function is strictly monotone decreasing in d.*

Proof:

Assume that there are two solutions to variational inequality (21a) given by $(\tilde{f}', d')$ and $(\tilde{f}'', d'')$. Denote the total link load patterns induced by these class patterns through (2) by f' and f'', respectively. Then, since $\tilde{f}', d'$ is assumed to be a solution we must have that

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\left[\psi_a^i(g_a(f')+\alpha_a)+\xi_a^i(g_a(f')+\beta_a)+(1-\psi_a^i-\xi_a^i)(g_a(f')+\gamma_a)\right]\times(f_a^i-f_a^{i'})$$

$$-\sum_{i=1}^{k}\sum_{w\in W}\lambda_w^i(d')\times(d_w^i-d_w^{i'})\geq 0,\quad \forall(f,d)\in\mathcal{K}^1. \tag{32}$$

Similarly, since $(\tilde{f}'', d'')$ is also assumed to be a solution we must have that

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\left[\psi_a^i(g_a(f'')+\alpha_a)+\xi_a^i(g_a(f'')+\beta_a)+(1-\psi_a^i-\xi_a^i)(g_a(f'')+\gamma_a)\right]\times(f_a^i-f_a^{i''})$$

$$-\sum_{i=1}^{k}\sum_{w\in W}\lambda_w^i(d'')\times(d_w^i-d_w^{i''})\geq 0,\quad \forall(f,d)\in\mathcal{K}^1. \tag{33}$$

Let $(f,d)=(f'',d'')$ and substitute into (32). Similarly, let $(f,d)=(f',d')$ and substitute into (33). Adding the two resulting inequalities, after algebraic simplifications, yields

$$\sum_{a\in\mathcal{L}}(g_a(f')-g_a(f''))\times(f_a'-f_a'')-\sum_{i=1}^{k}\sum_{w\in W}(\lambda_w^i(d')-\lambda_w^i(d''))\times(d_w^{i'}-d_w^{i''})\leq 0, \tag{34}$$

which is in contradiction to the assumption that the travel time, travel cost, and opportunity cost functions are strictly monotone increasing and the travel disutility function is strictly monotone decreasing. Hence, we must have that $(f',d')=(f'',d'')$. □

In addition, we now derive the monotonicity property as well as Lipschitz continuity which will be used in establishing convergence of the algorithm in the next Section.

Theorem 4: Monotonicity in a Special Case

Assume that the generalized cost functions u are as in (27) with the travel time, travel cost, and opportunity cost functions differing on a given link only by the fixed cost terms as in (28). Assume also that these functions are monotone increasing in f and that the disutility functions λ are monotone decreasing in d. Then the function that enters the variational inequality problem (21b) governing the multiclass, multicriteria traffic network equilibrium model with disutility functions is monotone.

Proof:

We need to establish that

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{35}$$

where F, X, and $\mathcal{K}$ are as defined following (21b) and with u given by (30) and (31).

We have that

$$\begin{aligned}
&\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle \\
&= \sum_{i=1}^{k} \sum_{a \in \mathcal{L}} \Big[\Big[\psi_a^i (g_a(f^1) + \alpha_a) + \xi_a^i (g_a(f^1) + \beta_a) + (1 - \psi_a^i - \xi_a^i)(g_a(f^1) + \gamma_a) \Big] \\
&- \sum_{i=1}^{k} \sum_{a \in \mathcal{L}} \Big[\psi_a^i (g_a(f^2) + \alpha_a) + \xi_a^i (g_a(f^2) + \beta_a) + (1 - \psi_a^i - \xi_a^i)(g_a^i(f^2) + \gamma_a) \Big] \times \Big[{f_a^i}^1 - {f_a^i}^2 \Big] \\
&- \sum_{i=1}^{k} \sum_{w \in W} \Big[\lambda_w^(d^1) - \lambda_w^i(d^2) \Big] \times \Big[{d_w^i}^1 - {d_w^i}^2 \Big] \\
&= \sum_{i=1}^{k} \sum_{a \in \mathcal{L}} \Big[g_a(f^1) - g_a(f^2) \Big] \times \Big[f_a^1 - f_a^2 \Big] - \sum_{i=1}^{k} \sum_{w \in W} \Big[\lambda_w^i(d^1) - \lambda_w^i(d^2) \Big] \times \Big[{d_w^i}^1 - {d_w^i}^2 \Big].
\end{aligned} \tag{36}$$

But, (36) is greater than or equal to zero, under the assumptions above, and, hence, $F(X)$ is monotone. □

Theorem 5: Lipschitz Continuity

If the generalized cost functions u and the disutility functions λ have bounded first-order derivatives, then the function, $F(X)$, that enters the variational inequality (21b) is Lipschitz continuous, that is, there exists a positive constant L, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{37}$$

Proof:

Denote $F(X) = (F_1(X), \cdots, F_{kL+2kJ}(X))^T$. Since $F_l(X) : R^{kL+2kJ} \mapsto R^1$ is a smooth function, from the Taylor Theorem, we have that, for any $X^1, X^2 \in \mathcal{K}$, there exist $\xi_l \in R^{kL+2kJ}, l = 1, \cdots, kL + 2kW$, such that

$$F_l(X^1) - F_l(X^2) = \nabla F_l(\xi_l)(X^1 - X^2), \qquad l = 1, \cdots, kL + 2kJ. \tag{38}$$

Let

$$\nabla F \equiv \begin{pmatrix} \nabla F_1(\xi_1) & & & & \\ & \ddots & & & \\ & & \nabla F_l(\xi_l) & & \\ & & & \ddots & \\ & & & & \nabla F_{kL+2kJ}(\xi_{kL+2kJ}) \end{pmatrix}. \tag{39}$$

Since the link generalized cost functions and the travel disutility functions have bounded first-order derivatives, there exists an $L \geq 0$, such that

$$\|\nabla F\| \leq L. \tag{40}$$

Therefore, using (39) and the basic properties of the linear norm operator, we have the following:

$$\|F(X^1) - F(X^2)\| = \left\| \begin{pmatrix} \nabla F_1(\xi_1)(X^1 - X^2) \\ \nabla F_1(\xi_1)(X^1 - X^2) \\ \vdots \\ \nabla F_l(\xi_l)(X^1 - X^2) \\ \vdots \\ \nabla F_{kL+2kJ}(\xi_{kL+2kW})(X^1 - X^2) \end{pmatrix} \right\|$$
$$= \|\nabla F(X^1 - X^2)\| \leq \|\nabla F\| \cdot \|X^1 - X^2\|. \tag{41}$$

In view of (40), one can conclude that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \qquad \forall X^1, X^2 \in \mathcal{K}. \tag{42}$$

□

4. The Algorithm

In this Section, an algorithm is presented which can be applied to solve any variational inequality problem in standard form (see (21b)), that is, Determine $X^* \in \mathcal{K}$, satisfying:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

The algorithm is guaranteed to converge provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists). The algorithm is the modified projection method of Korpelevich (1977).

The statement of the modified projection method is as follows, where $\mathcal{T}$ denotes an iteration counter:

Modified Projection Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and let γ be a scalar such that $0 < \gamma < \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. Korpelevich (1977)) (see (37)).

Step 1: Computation

Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (\bar{X}^{\mathcal{T}} + \gamma F(X^{\mathcal{T}-1})^T - X^{\mathcal{T}-1})^T, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{43}$$

Step 2: Adaptation

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (X^{\mathcal{T}} + \gamma F(\bar{X}^{\mathcal{T}})^T - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{44}$$

Step 3: Convergence Verification

If max $|X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} =: \mathcal{T} + 1$, and go to Step 1.

We now discuss the modified projection method more fully. Recall the definition of the projection of X on the closed convex set $\mathcal{K}$, with respect to the Euclidean norm, and denoted by $P_{\mathcal{K}}X$, as

$$y = P_{\mathcal{K}}X = \arg\min_{z\in\mathcal{K}} \|X - z\|. \tag{45}$$

In particular, note that $\bar{X}^{\mathcal{T}}$ generated by the modified projection method as the solution to the variational inequality subproblem (43) is actually the projection of $(X^{\mathcal{T}-1} - \gamma F(X^{\mathcal{T}-1})^T)$ on the closed convex set $\mathcal{K}$. In other words,

$$\bar{X}^{\mathcal{T}} = P_{\mathcal{K}}\left[X^{\mathcal{T}-1} - \gamma F(X^{\mathcal{T}-1})^T\right]. \tag{46}$$

Similarly, $X^{\mathcal{T}}$ generated by the solution to variational inequality subproblem (44) is the projection of $X^{\mathcal{T}-1} - \gamma F(\bar{X}^{\mathcal{T}})^T$ on $\mathcal{K}$, that is,

$$X^{\mathcal{T}} = P_{\mathcal{K}}\left[X^{\mathcal{T}-1} - \gamma F(\bar{X}^{\mathcal{T}})^T\right]. \tag{47}$$

We now give an explicit statement of the modified projection method for the solution of variational inequality problem (21a) for the integrated multicriteria traffic network equilibrium model.

Modified Projection Method for the Solution of Variational Inequality (21a)

Step 0: Initialization

Set $(\tilde{f}^0, d^0) \in \mathcal{K}^1$. Let $\mathcal{T} = 1$ and set γ such that $0 < \gamma < \frac{1}{L}$, where L is the Lipschitz constant for the problem.

Step 1: Computation

Compute $(\bar{\tilde{f}}^{\mathcal{T}}, \bar{d}^{\mathcal{T}}) \in \mathcal{K}^1$ by solving the variational inequality subproblem:

$$\sum_i \sum_{a\in\mathcal{L}} (\bar{f}_a^{i\,\mathcal{T}} + \gamma(u_a^i(\tilde{f}^{\mathcal{T}-1})) - f_a^{i\,\mathcal{T}-1}) \times (f_a^i - \bar{f}_a^{i\,\mathcal{T}}) + \sum_i \sum_{w\in W} (\bar{d}_w^{i\,\mathcal{T}} - \gamma\lambda_w^i(d^{\mathcal{T}-1}) - d_w^{i\,\mathcal{T}-1}) \times (d_w^i - \bar{d}_w^{i\,\mathcal{T}})$$

$$\geq 0, \quad \forall (\tilde{f}, d) \in \mathcal{K}^1. \tag{48}$$

Step 2: Adaptation

Compute $(\tilde{f}^{\mathcal{T}}, d^{\mathcal{T}}) \in \mathcal{K}^1$ by solving the variational inequality subproblem:

$$\sum_i \sum_{a \in \mathcal{L}} ({f_a^i}^{\mathcal{T}} + \gamma(u_a^i(\bar{\tilde{f}}^{\mathcal{T}}) - {f_a^i}^{\mathcal{T}-1}) \times (f_a^i - {f_a^i}^{\mathcal{T}}) + \sum_i \sum_{w \in W} ({d_w^i}^{\mathcal{T}} - \gamma\lambda_w^i(\bar{d}^{\mathcal{T}}) - {d_w^i}^{\mathcal{T}-1}) \times (d_w^i - {d_w^i}^{\mathcal{T}})$$
$$\geq 0, \quad \forall (\tilde{f}, d) \in \mathcal{K}^1. \tag{49}$$

Step 3: Convergence Verification

If $|{f_a^i}^{\mathcal{T}} - {f_a^i}^{\mathcal{T}-1}| \leq \epsilon$, $|{d_w^i}^{\mathcal{T}} - {d_w^i}^{\mathcal{T}-1}| \leq \epsilon$, for all $i = 1, \cdots, k$; $w \in W$, and all $a \in \mathcal{L}$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

We now state the convergence result for the modified projection method for this model. Note that the algorithm may converge even for functions of more general form provided that they are monotone and Lipschitz continuous. Theorem 6 identifies specific functions for which monotonicity can be readily established.

Theorem 6: Convergence

Assume that u takes the form of (27) and (28) and is monotone increasing, and also satisfies conditions (22) through (25). Assume that the disutility functions λ are known and are monotone decreasing. Also assume that u and λ have bounded first-order derivatives. Then the modified projection method described above converges to the solution of the variational inequality (21a) (and (21b)).

Proof:

According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (21b), provided that the function that enters the variational inequality, F is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 2. Lipschitz continuity, in turn, follows from Theorem 5 under the assumption that the generalized cost functions have bounded first-order derivatives.

The conclusion follows. $\square$

5. Numerical Examples

In this Section, we present two numerical examples which illustrate Situations 1 and 4 described in Section 2 and for which we provide solutions to the associated Problems 1 and 4. Specifically, we utilize the modified projection method for the solution of variational inequality (21a) discussed in the preceding section in order to compute the multiclass, multicriteria network equilibrium pattern. For each of the examples, we consider the same network topology, the same number of classes, and the same travel time, travel cost, and opportunity costs on the links, with the same associated weights. The examples differ in the situations expressed and in the disutility functions. Moreover, we construct the expanded network for Situation/Problem 4 which allows us to solve Problem 4 as a multicriteria traffic assignment problem corresponding to Situation/Problem 1.

The traffic network examples consist of two classes of users. The convergence criterion was that the absolute value of the path flows at two successive iterations was less than or equal to ϵ with ϵ set to 10^{-3}. The γ parameter used in the modified projection method (see Section 4) was set to .1. The demands were initialized to 100 for each O/D pair and the demand was equally distributed among all the paths connecting the O/D pair to construct the initial path flow pattern.

For the solution of the variational inequality subproblems (48) and (49), we utilized the Euler method (see Nagurney and Zhang (1996)).

We report the CPU time (exclusive of input and output) and the number of iterations required for convergence of the modified projection method for the numerical examples.

Example 1 – Situation/Problem 1

The first numerical example had the topology depicted in Figure 2. It corresponds to Problem/Situation 1 in which the O/D pairs are known and the associated O/D pair travel disutility functions are given. Links 1 through 13 are transportation links whereas links 14 and 15 are telecommunication links. The network consisted of ten nodes, fifteen links, and two O/D pairs where $w_1 = (1, 8)$ and $w_2 = (2, 10)$ with travel travel disutility functions by class given by: $\lambda^1_{w_1} = -d^1_{w_1} + 1200$, $\lambda^1_{w_2} = -d^1_{w_2} + 1200$, $\lambda^2_{w_1} = -2d^2_{w_1} + 1200$, and $\lambda^2_{w_2} = -d^2_{w_2} + 1100$. The paths connecting the O/D pairs were: for O/D pair w_1: $p_1 =$

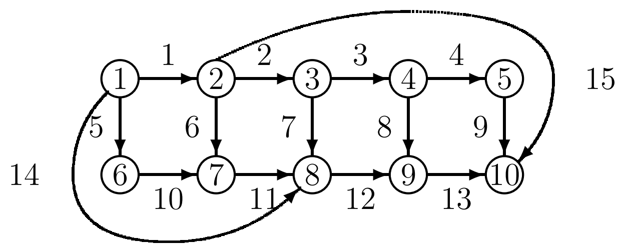

Figure 2: Network Topology for Example 1 – Situation/Problem 1

$(1,2,7)$, $p_2 = (1,6,11)$, $p_3 = (5,10,11)$, $p_4 = (14)$, and for O/D pair w_2: $p_5 = (2,3,4,9)$, $p_6 = (2,3,8,13)$, $p_7 = (2,7,12,13)$, $p_8 = (6,11,12,13)$, and $p_9 = (15)$. The weights were constructed as follows: For class 1, the weights were: $w^1_{1,1} = .25$, $w^1_{2,1} = .25$, $w^1_{3,1} = 1.$, $w^1_{1,2} = .25$, $w_{2,2} = .25$, $w^1_{3,2} = 1.$, $w^1_{1,3} = .4$, $w^1_{2,3} = .4$, $w^1_{3,3} = 1.$, $w^1_{1,4} = .5$, $w^1_{2,4} = .5$, $w^1_{3,4} = 2.$, $w^1_{1,5} = .4$, $w^1_{2,5} = .5$, $w^1_{3,5} = 1.$, $w^1_{1,6} = .5$, $w^1_{2,6} = .3$, $w^1_{3,6} = 2.$, $w^1_{1,7} = .2$, $w^1_{2,7} = .4$, $w^1_{3,7} = 1.$, $w^1_{1,8} = .3$, $w^1_{2,8} = .5$, $w^1_{3,8} = 1.$, $w^1_{1,9} = .6$, $w^1_{2,9} = .2$, $w^1_{3,9} = 2.$, $w^1_{1,10} = .3$, $w^1_{2,10} = .4$, $w^1_{3,10} = 1.$, $w^1_{1,11} = .2$, $w^1_{2,11} = .7$, $w^1_{3,11} = 1.$, $w^1_{1,12} = .3$, $w^1_{2,12} = .4$, $w^1_{3,12} = 1.$, $w^1_{1,13} = .2$, $w^1_{2,13} = .3$, $w^1_{3,13} = 2.$, $w^1_{1,14} = .5$, $w^1_{2,14} = .2$, $w^1_{3,14} = .1$, $w^1_{1,15} = .5$, $w^1_{2,15} = .3$, $w^1_{3,15} = .1$,

For class 2: $w^2_{1,1} = .5$, $w^2_{2,1} = .5$, $w^2_{3,1} = .5$, $w^2_{1,2} = .5$, $w^2_{2,2} = .4$, $w^2_{3,2} = .4$, $w^2_{1,3} = .4$, $w^2_{2,3} = .3$, $w^2_{3,3} = .7$, $w^2_{1,4} = .3$, $w^2_{2,4} = .2$, $w^2_{3,4} = .6$, $w^2_{1,5} = .5$, $w^2_{2,5} = .4$, $w^2_{3,5} = .5$, $w^2_{1,6} = .7$, $w^2_{2,6} = .6$, $w^2_{3,6} = .7$, $w^2_{1,7} = .4$, $w^2_{2,7} = .3$, $w^2_{3,7} = .8$, $w^2_{1,8} = .3$, $w^2_{2,8} = .2$, $w^2_{3,8} = .6$, $w^2_{1,9} = .2$, $w^2_{2,9} = .3$, $w^2_{3,9} = .9$, $w^2_{1,10} = .1$, $w^2_{2,10} = .4$, $w^2_{3,10} = .8$, $w^2_{1,11} = .4$, $w^2_{2,11} = .5$, $w^2_{3,11} = .9$, $w^2_{1,12} = .5$, $w^2_{2,12} = .5$, $w^2_{3,12} = .7$, $w^2_{1,13} = .4$, $w^2_{2,13} = .6$, $w^2_{3,13} = .9$, $w^2_{1,14} = .3$, $w^2_{2,14} = .4$, $w^2_{3,14} = 1.$, $w^2_{1,15} = .2$, $w^2_{2,15} = .3$, $w^2_{3,15} = .2$,

The generalized link cost functions were constructed according to (6).

The travel time functions and the travel cost functions were given by:

$$t_1(f) = .00005f_1^4+4f_1+2f_3+2,\ t_2(f) = .00003f_2+2f_2+f_5+1,\ t_3(f) = .00005f_3^4+f_3+.5f_2+3,$$

$$t_4(f) = .00003f_4^4 + 7f_4 + 3f_1 + 1, \quad t_5(f) = 5f_5 + 2, \quad t_6(f) = .00007f_6^4 + 3f_6 + f_9 + 4,$$

$$t_7(f) = 4f_7 + 6, \quad t_8(f) = .00001f_8^4 + 4f_8 + 2f_{10} + 1, \quad t_9(f) = 2f_9 + 8,$$

$$t_{10}(f) = .00003f_{10}^4 + 4f_{10} + f_{12} + 7, \quad t_{11}(f) = .00004f_{11}^4 + 6f_{11} + 2f_{13} + 2,$$

$$t_{12}(f) = .00002f_{12}f^4 + 4f_{12} + 2f_5 + 1, \quad t_{13}(f) = .00003f_{13}^4 + 7f_{13} + 4f_{10} + 8,$$

$$t_{14}(f) = f_{14} + 2, \quad t_{15}(f) = f_{15} + 1,$$

and

$$c_1(f) = .00005f_1^4+5f_1+1, \quad c_2(f) = .00003f_2^4+4f_2+2f_3+2, \quad c_3(f) = .00005f_3^4+3f_3+f_1+1,$$

$$c_4(f) = .00003f_4^4 + 6f_4 + 2f_6 + 4, \quad c_5(f) = 4f_5 + 8, \quad c_6(f) = .00007f_6^4 + 7f_6 + 2f_2 + 6,$$

$$c_7(f) = 8f_7 + 7, \quad c_8(f) = .00001f_8^4 + 7f_8 + 3f_5 + 6, \quad c_9(f) = 8f_9 + 5,$$

$$c_{10}(f) = .00003f_{10}^4 + 6f_{10} + 2f_8 + 3, \quad c_{11}(f) = .00004f_{11} + 4f_{11} + 3f_{10} + 4,$$

$$c_{12}(f) = .00002f_{12} + 6f_{12} + 2f_9 + 5,$$

$$c_{13}(f) = .00003f_1^4 + 9f_{13} + 3f_8 + 3,$$

$$c_{14}(f) = .1f_{14} + 1, \quad c_{15}(f) = .2f_{15} + 1.$$

The opportunity cost functions on the links were as follows:

$$o_1(f) = 2f_1 + 4, \quad o_2(f) = 3f_2 + 2, \quad o_3(f) = f_3 + 4,$$

$$o_4(f) = f_4 + 2, \quad o_5(f) = 2f_5 + 1, \quad o_6(f) = f_6 + 2,$$

$$o_7(f) = f_7 + 3, \quad o_8(f) = 2f_8 + 1, \quad o_9(f) = 3f_9 + 2,$$

$$o_{10}(f) = f_{10} + 1, \quad o_{11}(f) = 4f_{11} + 3, \quad o_{12}(f) = 3f_{12} + 2,$$

$$o_{13}(f) = f_{13} + 1, \quad o_{14}(f) = 6f_{14} + 1, \quad o_{15}(f) = 7f_{15} + 4.$$

Observe that the opportunity costs associated with links 14 and 15 were high since these are telecommunication links and users by choosing these links forego the opportunities associated with working and associatin with colleagues from a face to face perspective. Note, however, that the weights for class 1 associated with the opportunity costs on the telecommunication links are low (relative to those of class 2). This has the interpretation that class 1 does not weigh such opportunity costs highly and may, for example, prefer to be working from the home for a variety, including familial reasons. Also, note that class 1 weighs the travel time on the telecommunication links more highly than class 2 does.

The modified projection method converged in 2.42 CPU seconds and required 20 iterations for convergence. It yielded the following equilibrium multiclass link load pattern:

$$f_1^{1*} = 0.0000, \quad f_2^{1*} = 0.0000, \quad f_3^{1*} = 0.0000, \quad f_4^{1*} = 0.0000, \quad f_5^{1*} = 0.0000,$$

$$f_6^{1*} = 0.0000, \quad f_7^{1*} = 0.0000 \quad f_8^{1*} = 0.0000, \quad f_9^{1*} = 0.0000, \quad f_{10}^{1\,*} = 0.0000,$$

$$f_{11}^{1\,*} = 0.0000, \quad f_{12}^{1\,*} = 0.0000, \quad f_{13}^{1\,*} = 0.0000,$$

$$f_{14}^{1\,*} = 545.6185, \quad f_{15}^{1\,*} = 515.6567,$$

$$f_1^{2*} = 30.1559, \quad f_2^{2*} = 60.6264, \quad f_3^{2*} = 33.8925, \quad f_4^{2*} = 25.0791, \quad f_5^{2*} = 41.3280,$$

$$f_6^{2*} = 6.1290, \quad f_7^{2*} = 26.7339, \quad f_8^{2*} = 8.8134, \quad f_9^{2*} = 25.0791, \quad f_{10}^{2\,*} = 41.3280,$$

$$f_{11}^{2\,*} = 47.4570, \quad f_{12}^{2\,*} = 2.7070, \quad f_{13}^{2\,*} = 11.5204,$$

$$f_{14}^{2\,*} = 0.0000, \quad f_{15}^{2\,*} = 0.0000.$$

with total link loads given by:

$$f_1^* = 30.1559, \quad f_2^* = 60.6264, \quad f_3^* = 33.8925, \quad f_4^* = 25.0791, \quad f_5^* = 41.3280,$$

$$f_6^* = 6.1290, \quad f_7^* = 26.7339, \quad f_8^* = 8.8134, \quad f_9^* = 25.0791, \quad f_{10}^* = 41.3280,$$

$$f_{11}^* = 47.4570, \quad f_{12}^* = 2.7070, \quad f_{13}^* = 11.5204,$$

$$f_{14}^* = 545.6185, \quad f_{15}^* = 515.6567,$$

and induced by the equilibrium multiclass path flow pattern:

$$x_{p_1}^{1\ *} = 0.0000, \quad x_{p_2}^{1\ *} = 0.0000, \quad x_{p_3}^{1\ *} = 0.0000, \quad x_{p_4}{}^{*} = 545.6185,$$

$$x_{p_5}^{1\ *} = 0.0000, \quad x_{p_6}^{1\ *} = 0.0000, \quad x_{p_7}^{1\ *} = 0.0000, \quad x_{p_8}^{1\ *} = 0.0000,$$

$$x_{p_9}^{1\ *} = 515.6568.$$

$$x_{p_1}^{2\ *} = 24.8494, \quad x_{p_2}^{2\ *} = 5.3065, \quad x_{p_3}^{2\ *} = 41.3280,$$

$$x_{p_4}^{2\ *} = 0.000, \quad x_{p_5}^{2\ *} = 25.0791, \quad x_{p_6}^{2\ *} = 8.8134, \quad x_{p_7}^{2\ *} = 1.8845,$$

$$x_{p_8}^{2\ *} = .8225, \quad x_{p_9}^{2\ *} = 1063.4005.$$

The generalized path costs were: for Class 1, O/D pair w_1:

$$v_{p_1}^1 = 832.6793, \quad v_{p_2}^1 = 937.7325, \quad v_{p_3}^1 = 1177.2548, \quad v_{p_4}^1 = 611.3099,$$

for Class 1, O/D pair w_2:

$$v_{p_5}^1 = 1190.1959, \quad v_{p_6}^1 = 943.0282, \quad v_{p_7}^1 = 829.0654, \quad v_{p_8}^1 = 934.1187,$$

$$v_{p_9}^1 = 649.9652.$$

for Class 2, O/D pair w_1:

$$v_{p_1}^2 = 1056.8972, \quad v_{p_2}^2 = 1057.0726, \quad v_{p_3}^2 = 1057.0931,$$

$$v_{p_4}^2 = 3455.0918,$$

and for Class 2, O/D pair w_2:

$$v_{p_5}^2 = 1063.2885, \quad v_{p_6}^2 = 1063.2863, \quad v_{p_7}^2 = 2063.2919, \quad v_{p_8}^2 = 1063.4670,$$

$$v_{p_9}^2 = 7350.8740.$$

The combination of the modified projection method embedded with the Euler method for the solution of the traffic network subproblems of Steps 1 and 2 yielded accurate solutions in a timely manner. Indeed, the equilibrium conditions were satisfied with good accuracy.

It is interesting to see the separation by classes in the equilibrium solution. Note that all members of class 1, whether residing at node 1 or node 2, were telecommuters, whereas all members of class 2 chose to commute to work. This outcome is realistic, given the weight assignments of the two classes on the opportunity costs associated with the links (as well as the weight assignments associated with the travel times).

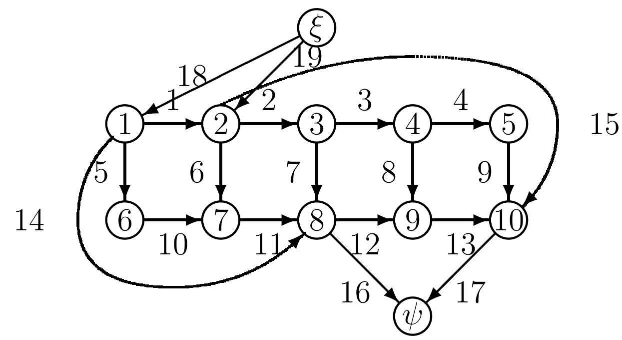

Figure 3: Expanded Network Topology for Example 2 – Situation/Problem 4

Example 2 – Situation/Problem 4

In the final example, we kept the origin nodes as nodes 1 and 2, and the destination nodes as nodes 8 and 10. This example is an illustration of Situation/Problem 4 in which all users of the network must determine their places of residence, their places of work, as well as their routes of travel between the origins and destinations. Hence, as discussed in Section 2 for this situation, we constructed a single supersource node ξ and a single supersink node ψ with abstract links 18 and 19 connecting the supersource node to nodes 1 and 2 and abstract links 16 and 17 connecting nodes 8 and 10 to the supersink node, as depicted in Figure 3. The generalized costs on the abstract links were set to zero as also discussed in Section 2. We retained the link cost structure of the preceding example for the two classes.

The disutility functions were $\lambda^1(T) = -T^1 + 1200$, $\lambda^2(T) = -2T^2 + 1200$, which corresponded to the disutility functions for each class for the *single* abstract O/D pair (ψ, ξ).

The modified projection method converged in 6.45 CPU seconds and required 123 iterations for convergence. Here, for simplicity, we summarize the results by reporting the computed equilibrium link loads on the original links (exclusive of the abstract links: 16 through 19).

The algorithm yielded the following equilibrium multiclass link load pattern:

$$f_1^{1*} = 0.0000, \quad f_2^{1*} = 0.0000, \quad f_3^{1*} = 0.0000, \quad f_4^{1*} = 0.0000, \quad f_5^{1*} = 0.0000,$$

$$f_6^{1*} = 0.0000, \quad f_7^{1*} = 0.0000 \quad f_8^{1*} = 0.0000, \quad f_9^{1*} = 0.0000, \quad f_{10}^{1\ *} = 0.0000,$$

$$f_{11}^{1\ *} = 0.0000, \quad f_{12}^{1\ *} = 0.0000, \quad f_{13}^{1\ *} = 0.0000,$$

$$f_{14}^{1\ *} = 398.3354, \quad f_{15}^{1\ *} = 354.1955,$$

$$f_1^{2*} = 0.0000, \quad f_2^{2*} = 64.5888, \quad f_3^{2*} = 19.8592, \quad f_4^{2*} = 15.0886, \quad f_5^{2*} = 28.7413,$$

$$f_6^{2*} = 22.0290, \quad f_7^{2*} = 44.7296, \quad f_8^{2*} = 4.7706, \quad f_9^{2*} = 15.0886, \quad f_{10}^{2\ *} = 28.7413,$$

$$f_{11}^{2\ *} = 50.7703, \quad f_{12}^{2\ *} = 0.0000, \quad f_{13}^{2\ *} = 4.7706,$$

$$f_{14}^{2\ *} = 0.0000, \quad f_{15}^{2\ *} = 0.0000.$$

with total link loads given by:

$$f_1^* = 0.0000, \quad f_2^* = 64.5888, \quad f_3^* = 19.8592, \quad f_4^* = 15.0886, \quad f_5^* = 28.7413,$$

$$f_6^* = 22.0290, \quad f_7^* = 44.7296, \quad f_8^* = 4.7706, \quad f_9^* = 15.0886, \quad f_{10}^* = 28.7413,$$

$$f_{11}^* = 50.7703, \quad f_{12}^* = 0.0000, \quad f_{13}^* = 4.7706,$$

$$f_{14}^* = 398.3354, \quad f_{15}^* = 354.1955.$$

We now also summarize the computed equilibrium path flow patterns: All path flows for each class were zero except for the following:

The flow on the path for class 1 consisting of the links: $(18, 14, 16)$, was 398.3354 with a generalized path cost equal to 447.4352. This corresponds to members of class 1 choosing to locate at node 1 and to telecommute to work at node 8. In addition, the flow on the path for class 1 consisting of the links: $(19, 15, 17)$ was 354.1954 with a generalized path travel cost of 447.4865. This corresponds to members of class 1 locating at node 2 and telecommuting to work to destination node 10. Note that, as in Example 1, no members of class 1 chose to commute to work; rather, they all chose to telecommute.

In terms of class 2, the modified projection method computed the following nontrivial path flows: The path consisting of links: $(18, 5, 10, 11, 16)$ had a path flow of 28.7413 with generalized path cost of 969.2819. The path consisting of links: $(19, 2, 3, 4, 9, 17)$ had a path flow of 15.0886 with a generalized cost of 969.2819; the path consisting of the links: $(19, 2, 3, 8, 13, 17)$ had a path flow for class 2 of 4.7706 with a generalized cost of 969.2818; the path consisting of links: $(19, 2, 7, 16)$ had a path flow for class 2 of 44.7296 with a generalized path cost of 929.2819, and, finally, the path consisting of the links: $(19, 6, 11, 16)$ had a flow for class 2 of 22.0290 with a generalized cost of 929.2819. The generalized costs of paths with zero flow for each class exceeded the generalized costs on used paths for the corresponding class. Hence, the equilibrium conditions corresponding to Problem 4 given by (20) were met with good accuracy.

6. Summary and Conclusions

In this paper, we have developed integrated multicriteria traffic network equilibrium models which determine the equilibrium path and link loads corresponding to commuting and telecommuting choices. The models are elastic demand models and depict four distinct transportation situations or problems in which users of the network must determine their optimal routes of travel given their O/D pairs and associated travel disutilities, or, instead, their origins or destinations or both their origins and destinations, given their appropriate disutilities. Here we allowed for three criteria associated with the links and these were: the travel time, the travel cost, as well as the opportunity cost associated with telecommuting/commuting. We allowed the weights associated with the criteria to be not only class-dependent but also link-dependent.

We established the equilibrium conditions for all for situations/problems and then demonstrated how the three latter problems could be transformed to the first problem via the construction of the appropriate expanded network. We presented the variational inequality formulation of the governing equilibrium conditions, provided qualitative properties, and proposed an algorithm along with convergence results. Finally, we demonstrated both the model and the algorithm through numerical examples.

This work represents the first theoretical framework for the investigation of equilibrium patterns associated with commuting/telecommuting.

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