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A Game Theory Model for a Differentiated Service-Oriented Internet with Duration-Based Contracts

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Abstract This paper presents a game theory model of a service-oriented Internet in which the network providers compete in usage service rates, quality levels, and duration-based contracts. We formulate the network-based Nash equilibrium conditions as a variational inequality problem, provide qualitative properties of existence and uniqueness, and describe an algorithm, which yields closed-form expressions at each iteration. The numerical examples include sensitivity analysis for price functions at the demand markets as well as variations in the upper bounds on the quality levels for the services.

Keywords Future Internet, service-oriented Internet, duration-based contracts, networks, competition, game theory, variational inequalities

1. Introduction

The Internet, through its evolution, has revolutionized industry, commerce, education, communications, information-gathering and dissemination, as well as entertainment. It provides the backbone for numerous economic transactions and social interactions and has transformed manufacturing, transportation, and finance. There are now approximately 2.5 billion Internet users out of a global population of 7 billion [26].

The Internet is under increasing pressure from consumers with the success in new services creating incentives for further innovation. For example, online video consumption almost doubled in the US from 2012 to 2013 and, according to [10], as of March 2014, Netflix and Google, which owns Youtube, accounted for 47% of all Internet traffic during the evening primetime hours. The demand for video services, however, may result in network congestion that leads to a degradation in the quality of transmission and, hence, the user experience. Internet users, from households to businesses, are increasingly concerned about the quality of Internet services as well as prices. In addition, many existing business models for Internet services involve contracts that require that customers be locked-in for extended periods of time. Such inflexibility is detrimental to users and may also impede innovation in Internet services.

This new business and consumer landscape for the Internet has not gone unnoticed by government research funding agencies. For example, in 2010, the US National Science Foundation (NSF) recognized the success of the Internet, but noted the lack of performance reliability of Internet services. It established the Future Internet Architecture (FIA) project with the goals in funding five of such projects including: *to design and experiment with new network architectures and networking concepts that take into consideration the larger social, economic and legal issues that arise from the interplay between the Internet and society.* (NSF http://www.nets-fia.net/)

Indeed, the evolution of the Internet is now motivating not only the conceptualization of new architectures but also a re-examination of the pricing of Internet services, especially in the context of the Next Generation Internet (NGI). While there has been dramatic success in infrastructure research, resulting in a high bandwidth Internet backbone supporting simple end-to-end connections, there has been less in terms of service-oriented Internet pricing research ([13]). Initially, the Internet was government funded and, thus, free to the user. Later, two pricing models were developed, one, where a flat fee was charged, and, the second, where a basic charge covered a certain time and quantity of data with additional time/data charged incrementally. More than a decade ago, it was realized that such pricing models may not be applicable in the rapidly changing Internet ([12]). For example, in the US, Comcast differentiates its monthly charge for business users based upon the desired download and upload speeds (http://www.comcast.com 8/1/2014). Mediacom Cable not only differentiates among speeds, but adds a limit to the total quantity of data transfer (http://mediacom.com 8/1/2014). In Canada, Rogers also offers similar pricing schemes (http://Rogers.com 8/1/14). For an overview of earlier Internet pricing models see [41].

More specifically, the growing number of Internet users and the emergence of new applications have prompted the introduction of new pricing methods for differentiated network services [2]. The importance of supporting various levels of quality of network service in the Internet, in particular, has recently intensified as a fundamental departure from the traditional best-effort nature of Internet pricing (see, e.g., [16, 40, 39, 17, 38, 1, 43, 31]). Several researchers have focused on two-sided payments in a multi-tier network and evaluated neutral vs. non-neutral networks (see, e.g., [24, 18, 27, 9] with the goal of investigating the effects of charging providers based on supply and demand in the Internet.

However, pricing based on quality and the usage amount, and, as is now typical, contracts of one to two year duration, may result in network congestion since network resource utilization may change over time, unless there are network upgrades. Furthermore, consumers may desire more flexibility and more choices, depending upon their location, and the type of viewing or other experience desired. Hence, we expect that contract duration will become an important feature in the pricing of network services [19] with shorter duration contracts garnering greater interest. A general question then arises: How will the quality and the duration of Internet network contracts affect the pricing?

Our NSF-funded Future Internet Architecture project is known as ChoiceNet (cf. [45, 46]). ChoiceNet, as an example of a new architecture of NGI, aims to enable choices and the associated economic relationships between entities in a future Internet network. ChoiceNet will make it possible for network providers to compete for customers and be rewarded for quality and innovation. An important characteristic of ChoiceNet, and what we expect may also be a feature of other architectures, is the possibility of shorter duration-based contracts. One of the primary goals of such an architecture is to offer consumers more choices, with different contract durations providing more flexibility to consumers. An economic analysis of such an architecture, through a rigorous game theory framework, is essential in order to assess the prices at the demand markets as well as the levels of quality, along with the provider profits.

There exist several early mathematical models in which duration and quality of services are included in the pricing of Internet services (see, e.g., [44, 3, 21, 2, 20]). For example, [14, 21] introduced linear usage for each connection based on the volume and duration (time). [3, 2] proposed pricing Internet services based on the usage charge for the effective bandwidth per unit of time. The amount of effective bandwidth consumed in a contract is a nonlinear function of both users' choices and network parameters. The assumptions in many of these models, however, included monopolistic providers. We believe that it is worth exploring an oligopoly market of Internet network providers which is expected for NGI.

Our approach attempts to resolve this void by formulating a competitive oligopoly market of Internet network providers, motivated by ChoiceNet (cf. [45, 46]), although not limited to it, and the economic relationships among them. The entities are able to offer different network services and to create contracts for their users according to the users' desires and needs. The model developed in this paper is straightforward enough to understand for both users and network providers and creates an opportunity to control the total charge for a communication by a modification of the parameters. The users/demand markets select contracts based on three main criteria: the amount of usage contracted for per period of time (the usage rate) during the contract duration, the quality level of service, and the contract duration. Here we consider a reserved usage amount per unit of time. Our earlier work on the network economic game theoretical modeling of Future Internet Architectures focused on introducing quality, with an emphasis on service provision, which is maintained through network transport/provision in [31], and also on capturing the behavior of both content providers and network providers, with the latter competing on price and quality, in [33]. In [32], we modeled the dynamics associated with content and network provider competition in which consumers respond to the prices and the quality of both content provision and network provision. Here, in contrast, our goal is to extend the game theoretical modeling of competitive network providers and services by including not only quality of service but also contract durations as strategic variables, in addition to the reserved usage rates.

This paper is organized as follows. In Section 2, we develop the competitive duration-based contract pricing model for a service-oriented Internet network with differentiated services and derive the variational inequality formulation. We also provide some qualitative properties of the equilibrium pattern. In Section 3, we present the computational scheme, which has nice features for ease of implementation, and compute solutions to a series of numerical examples. We summarize our results and present the conclusions in Section 4.

2. The Competitive Duration-Based Differentiated Service-Oriented Internet Game Theory Model

The focus of our game theory model is on duration-based contracts associated with network provision. We assume m network providers, with a typical provider denoted by i; $i=1,2,\dots,m$ and n users/demand markets, with a typical one denoted by j; $j=1,2,\dots,n$, as shown in Figure 1. A demand market may correspond to an individual, a household, and/or a business. The users reveal their preferences for the network providers' services through the demand price functions, which depend on the service usage rates, the quality of services, and the contract durations. We further detail the model's underlying functions and their generality below.

Let p_{ij} denote the price for transmission of bit units of data (can be individual ones, Megabits, etc.) from network provider i to demand market j, for the selected number of bit units per unit time (the reserved usage rate), at the quality level and the contract duration. Let d_{ij} represent the number of bit units per unit time contracted for between i and j, corresponding to the reserved usage rate, and let q_{ij} denote the contracted quality of service, which ranges between 0 and 100, with 100 denoting perfect quality. T_{ij} represents the duration of the contract between network provider i and demand market j in units of

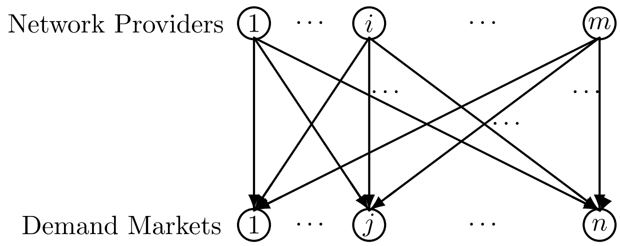

FIGURE 1. The Bipartite Structure of the Competition Among the Network Providers

time. Henceforth, we simply refer to usage rate with the understanding that we mean a reserved usage rate. Indeed, although the consumer may not use up all of his usage rate over the contract duration the network provider still needs to plan as if the user will in order to provide the desired quality of service and to manage the network resources accordingly. In Section 3, we provide specific units in the context of the numerical examples for the prices and decision variables. Here we consider a general framework that can be adapted to any currency, time unit, etc., as needed.

Each network provider i; $i = 1, \dots, m$, is faced, due to technological limitations, with a maximum usage rate to a particular demand market j, $\bar{d}_{ij}$, in terms of the number of megabits per time unit, and may also impose a nonnegative minimum, $\underline{d}_{ij}$, so that

$$\underline{d}_{ij} \le d_{ij} \le \bar{d}_{ij}, \quad \forall i, j. \tag{1}$$

Also, due to technological limitations, network provider i may have a maximum level of quality $\bar{q}_{ij}$ that he can offer, where $\bar{q}_{ij} \le 100$. Hence,

$$0 \le q_{ij} \le \bar{q}_{ij}, \qquad \forall i, j. \tag{2}$$

A single parameter with quality, subject to a bound, as above, was also used in [11].

Finally, the contract durations for a given network provider and demand market pair may also be bounded, with $\bar{T}_{ij}$ denoting the upper bound and $\underline{T}_{ij}$ the nonnegative lower bound, so that

$$\underline{T}_{ij} \le T_{ij} \le \bar{T}_{ij}. \quad \forall i, j. \tag{3}$$

For example, some service providers may decide to have a positive lower bound for the contract duration for ease of management. We group the usage rates for service into the vector $d \in R^{mn}_{+}$, the quality levels into the vector $q \in R^{mn}_{+}$, and the contract durations into the vector $T \in R^{mn}_{+}$.

The price of i's service provision to j, p_{ij}, is a function of the reserved usage rates, the quality levels, and the durations of the contracts, as follows

$$p_{ij} = p_{ij}(d, q, T), \qquad \forall i, j. \tag{4}$$

Note that, according to (4), the price of transmission between (i, j) depends not only on the usage per unit time in terms of the number of bit units, the quality of transmission between (i, j), and the contract duration, but also on the values of these variables associated with other network providers and with other demand markets. This functional form also captures that users should be aware of the services offered by the network provider at other demand markets. Indeed, we argue for transparency in ChoiceNet, so that users can make informed decisions.

We assume that the demand price function for each pair (i, j) is monotonically decreasing in its reserved service usage rate and in the duration of the contract between (i, j) but increasing in terms of service quality between the pair.

Each network provider incurs a cost for delivering the service at a specific quality and usage rate and maintaining the quality within the contract duration. We assume that the cost is a convex function ([25], [37]) of the usage rates, the quality levels, and the durations of the contracts. The cost c_{ij} incurred by network provider i for serving j is of the form:

$$c_{ij} = c_{ij}(d, q, T), \qquad \forall i, j. \tag{5}$$

The demand price functions (4) and the cost functions (5) are assumed to be continuous and continuously differentiable. The generality of the expressions in (4) and (5) allows for modeling and application flexibility. Moreover, the cost functions in (5) reveal that the cost on a "link" as depicted in Figure 1 can depend not only on the usage on that link but also on those on the other links. Since there may be competition for network resources, such cost functions can capture competition, albeit at a high level, among the network providers during transmission.

The strategic variables of network provider i are its service usage rates, the quality levels, and the contract durations $\{d_i, q_i$, and $T_i\}$, where $d_i = (d_{i1}, \ldots, d_{in})$, $q_i = (q_{i1}, \ldots, q_{in})$, and $T_i = (T_{i1}, \ldots, T_{in})$.

The utility or profit of network provider i is the difference between his revenue and his total cost and is given by the expression:

$$U_i = \sum_{j=1}^{n} p_{ij} T_{ij} d_{ij} - \sum_{j=1}^{n} c_{ij}, \quad \forall i. \tag{6}$$

In (6), the first term after the equal sign is the total revenue and the second term is the total cost of network provider i.

Let K^i denote the feasible set corresponding to network provider i, where $K^i \equiv \{(d_i, q_i, T_i), \text{such that (1), (2), and (3) hold for } i\}$ and define $K \equiv \prod_{i=1}^{m} K^i$. The network providers compete in a noncooperative manner in the sense of Nash (1950, 1951), each one seeking to maximize his profit. We wish to determine the vectors of the equilibrium service usage rates, quality levels, and contract durations (d^*, q^*, T^*), according to the definition below.

Definition 1: The Differentiated Service-Oriented Internet Network Equilibrium with Contract Durations

A service usage rate, quality, and contract duration pattern $(d^*, q^*, T^*) \in K$ is an equilibrium if, for each network provider i; $i = 1, \ldots, m$:

$$U_i(d_i^*, q_i^*, T_i^*, \hat{d}_i^*, \hat{q}_i^*, \hat{T}_i^*) \geq U_i(d_i, q_i, T_i, \hat{d}_i^*, \hat{q}_i^*, \hat{T}_i^*), \quad \forall (d_i, q_i, T_i) \in K^i, \tag{7}$$

where

$$\hat{d}_i^* = (d_1^*, \ldots, d_{i-1}^*, d_{i+1}^*, \ldots, d_m^*),$$
$$\hat{q}_i^* = (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*),$$

and

$$\hat{T}_i^* = (T_1^*, \ldots, T_{i-1}^*, T_{i+1}^*, \ldots, T_m^*). \tag{8}$$

According to (7), an equilibrium is established if no network provider can unilaterally improve his profit by selecting an alternative vector of reserved service usage rates, quality levels, and durations of contracts, given the decisions of the other network providers.

2.1. Variational Inequality Formulation

Variational inequalities have been used to formulate a spectrum of problems arising in engineering, operations research and the management sciences, transportation science, economics, and finance (cf. [28, 29, 30] and references therein). We now present a variational inequality formulation of the service-oriented Internet network equilibrium.

Theorem 1: Variational Inequality Formulation
Assume that the profit function $U_i(d,q,T)$ is concave with respect to the variables $\{d_{i1},\ldots,d_{in}\}$, $\{q_{i1},\ldots,q_{in}\}$, and $\{T_{i1},\ldots,T_{in}\}$ and is continuous and continuously differentiable for each network provider $i;=1,\ldots,m$. Then, $(d^*,q^*,T^*)\in K$ is an Internet network equilibrium service usage rate, quality, and contract duration pattern according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(d^*,q^*,T^*)}{\partial d_{ij}}\times(d_{ij}-d^*_{ij})-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(d^*,q^*,T^*)}{\partial q_{ij}}\times(q_{ij}-q^*_{ij})$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(d^*,q^*,T^*)}{\partial T_{ij}}\times(T_{ij}-T^*_{ij})\geq 0,\qquad \forall(d,q,T)\in K, \tag{9}$$

or, equivalently, $(d^*,q^*,T^*)\in K$ is an equilibrium service usage rate, quality, and contract duration pattern if and only if it satisfies the variational inequality:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{l=1}^{n}\frac{\partial c_{il}(d^*,q^*,T^*)}{\partial d_{ij}}-p_{ij}(d^*,q^*,T^*)\times T^*_{ij}-\sum_{l=1}^{n}\frac{\partial p_{il}(d^*,q^*,T^*)}{\partial d_{ij}}\times d^*_{il}\times T^*_{il}\right]\times(d_{ij}-d^*_{ij})$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{l=1}^{n}\frac{\partial c_{il}(d^*,q^*,T^*)}{\partial q_{ij}}-\sum_{l=1}^{n}\frac{\partial p_{il}(d^*,q^*,T^*)}{\partial q_{ij}}\times d^*_{il}\times T^*_{il}\right]\times(q_{ij}-q^*_{ij})$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{l=1}^{n}\frac{\partial c_{il}(d^*,q^*,T^*)}{\partial T_{ij}}-p_{ij}(d^*,q^*,T^*)\times d^*_{ij}-\sum_{l=1}^{n}\frac{\partial p_{il}(d^*,q^*,T^*)}{\partial T_{ij}}\times d^*_{il}\times T^*_{il}\right]$$

$$\times(T_{ij}-T^*_{ij})\geq 0,\qquad \forall(d,q,T)\in K. \tag{10}$$

Proof: (9) follows directly from [15] and [6]. In order to obtain variational inequality (10) from variational inequality (9), we note that:

$$-\frac{\partial U_i}{\partial d_{ij}}=[\sum_{l=1}^{n}\frac{\partial c_{il}}{\partial d_{ij}}-p_{ij}\times T_{ij}-\sum_{l=1}^{n}\frac{\partial p_{il}}{\partial d_{ij}}\times d_{il}\times T_{il}];\quad i=1,\ldots,m;j=1,\ldots,n, \tag{11}$$

$$-\frac{\partial U_i}{\partial q_{ij}}=[\sum_{l=1}^{n}\frac{\partial c_{il}}{\partial q_{ij}}-\sum_{l=1}^{n}\frac{\partial p_{il}}{\partial q_{ij}}\times d_{il}\times T_{il}];\qquad i=1,\ldots,m;j=1,\ldots,n, \tag{12}$$

and

$$-\frac{\partial U_i}{\partial T_{ij}}=[\sum_{l=1}^{n}\frac{\partial c_{il}}{\partial T_{ij}}-p_{ij}\times d_{ij}-\sum_{l=1}^{n}\frac{\partial p_{il}}{\partial T_{ij}}\times d_{il}\times T_{il}];\quad i=1,\ldots,m;j=1,\ldots,n. \tag{13}$$

Multiplying the right-most expression in (11) by $(d_{ij}-d^*_{ij})$ and summing the resultant over all i and all j; multiplying the right-most expression in (12) by $(q_{ij}-q^*_{ij})$ and summing the resultant over all i and j, and, similarly, multiplying the right-most expression in (13) by $(T_{ij}-T^*_{ij})$ and summing the resultant over all i and j yields (10). The conclusion follows. □

We now put variational inequality (10) into standard form (cf. [28]), that is: Determine $X^*\in\mathcal{K}\subset R^N$, such that

$$\langle F(X^*),X-X^*\rangle\geq 0,\qquad \forall X\in\mathcal{K}, \tag{14}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a closed and convex set.

We define the mn-dimensional vectors $X \equiv (d,q,T)$ and $F(X) \equiv (F^1(X), F^2(X), F^3(X))$ with the (i,j)-th component, F^1_{ij}, of $F^1(X)$ given by

$$F^1_{ij}(X) \equiv -\frac{\partial U_i}{\partial d_{ij}}, \tag{15}$$

the (i,j)-th component, F^2_{ij}, of $F^2(X)$ given by

$$F^2_{ij}(X) \equiv -\frac{\partial U_i}{\partial q_{ij}}, \tag{16}$$

and the (i,j)-th component, F^3_{ij}, of $F^3(X)$ given by

$$F^3_{ij}(X) \equiv -\frac{\partial U_i}{\partial T_{ij}}, \tag{17}$$

and with the feasible set $\mathcal{K} \equiv K$. Then, clearly, variational inequality (10) can be put into standard form (14).

The next theorem is immediate from the standard theory of variational inequalities (cf. [22, 28]) since the feasible set $\mathcal{K}$ in our model is compact and the function F that enters variational inequality (14) for our model is, under the imposed assumptions, continuous.

Theorem 2: Existence
A solution X^* to variational inequality (14) is guaranteed to exist.

Theorem 3: Uniqueness
If $F(X)$ is strictly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2, \tag{18}$$

then the solution to variational inequality (14) is unique.

Proof: Follows from the standard theory of variational inequalities.

We know that $F(X)$ is strictly monotone, if $\nabla F(X)$ is positive-definite over the feasible set $\mathcal{K}$.

3. The Algorithm

The feasible set underlying variational inequality (10) consists of box-type constraints, a feature that we will exploit for computational purposes. Specifically, for the computation of the equilibrium pattern, we propose the Euler method, which is induced by the general iterative scheme of [8], and which has been used to compute solutions to numerous network equilibrium problems (see, e.g., [34, 29, 5, 32, 42]).

In particular, at iteration τ of the Euler method, one solves the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{19}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem.

3.1. Explicit Formulae for the Euler Method Applied to the Internet Network Model with Contract Durations

The elegance of this procedure for the computation of solutions to the model with service differentiation and contract durations can be seen in the following explicit formulae. In particular, we have the following closed form expressions for the service usage rates, quality levels, and contract durations $i = 1, \ldots, m; j = 1, \ldots, n$:

$$d^{\tau+1}_{ij} = \max\left\{\underline{d}_{ij}, \min\{\bar{d}_{ij}, d^\tau_{ij} - a_\tau F^1_{ij}(X^\tau)\}\right\}, \tag{20}$$

$$q_{ij}^{\tau+1} = \max\Big\{0\,, \min\{\bar{q}_{ij}\,, q_{ij}^{\tau} - a_{\tau} F_{ij}^{2}(X^{\tau})\}\Big\}, \tag{21}$$

$$T_{ij}^{\tau+1} = \max\Big\{\underline{T}_{ij}\,, \min\{\bar{T}_{ij}, T_{ij}^{\tau} - a_{\tau} F_{ij}^{3}(X^{\tau})\}\Big\}. \tag{22}$$

Note that all the functions to the right of the equal signs in (20) - (22) are evaluated at their respective variables computed at the τ-th iteration. This algorithm can also be interpreted as a discrete-time adjustment process. We now provide the convergence result. The proof is direct from Theorem 5.8 in [34].

Theorem 4: Convergence

In the differentiated service-oriented Internet network game theory model with contract durations, if $F(X) = -\nabla U(d,q,T)$ is strictly monotone at an equilibrium pattern and F is uniformly Lipschitz continuous, then there exists a unique equilibrium service usage rate, quality, and contract duration pattern $(d^*, q^*, T^*) \in K$ and any sequence generated by the Euler method as given by (20) - (22), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (d^*, q^*, T^*).

3.2. Numerical Examples

We implemented the Euler method in Matlab on a VAIO S Series laptop with an Intel Core i7 processor and 12 GB RAM. The algorithm was considered to have converged if, at a given iteration, the absolute value of the difference of each variable differed from its respective value at the preceding iteration by no more than $\epsilon = 10^{-4}$. The sequence $\{a_\tau\}$ was: $(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We initialized the algorithm for all the examples by setting $d_{ij}^0 = \underline{d}_{ij}; q_{ij}^0 = \underline{q}_{ij}; T_{ij}^0 = \underline{T}_{ij}$, $\forall i, j$.

The examples begin with a simple network of two network providers and a single demand market (user), which we then extend to two network providers and two demand markets, and, finally, to two network providers and three demand markets. In the numerical examples, the contract durations, T_{ij}s, are in hours, the reserved service usage rates, d_{ij}s, are in Megabits/second, and, to simplify the presentation, the prices p_{ij} are in cents/Megabit multiplied by 10^{-5}. We use linear demand functions (see [1], [11], and [47]). The data were selected to be consistent with current advertized pricing of ISPs such as COMCAST (http://www.comcast.com).

Example 1

The topology of the first example is given in Figure 2.

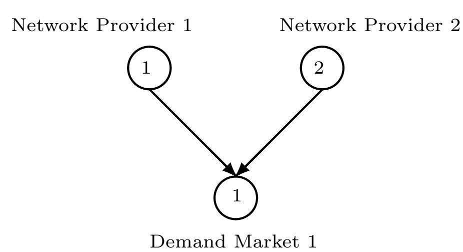

Figure 2. Example 1

The price functions at Demand Market 1 are:

$$p_{11} = 12 - .167\,d_{11} - .0334\,d_{21} + .032\,q_{11} - .0064\,q_{21} - .182\,T_{11} - .0546\,T_{21},$$

$$p_{21} = 12 - .0334\,d_{11} - .167\,d_{21} - .0064\,q_{11} + .032\,q_{21} - .0546\,T_{11} - .182\,T_{21}.$$

These functions reflect that Demand Market 1 is more sensitive to the contract duration than to the service usage rate. The network providers likely use different technologies for their services; therefore, their cost functions are distinct. The cost functions for Network Providers 1 and 2 are, respectively:

$$c_{11} = (.0049\, q_{11}^2 + .001715\, q_{11} + .029\, d_{11})T_{11}, \quad c_{21} = (.0037\, q_{21}^2 + .053\, d_{21}^2)T_{21}.$$

The utility functions of the network providers are:

$$U_1 = p_{11}d_{11}T_{11} - c_{11}, \quad U_2 = p_{21}d_{21}T_{21} - c_{21}.$$

Network Provider 1 can offer services at a higher minimum service usage rate but at a lower minimum contract duration in comparison to Network Provider 2, where:

$$23 \le d_{11} \le 250, \qquad 0 \le q_{11} \le 100, \qquad 8 \le T_{11} \le 40,$$

$$15 \le d_{21} \le 200, \qquad 0 \le q_{21} \le 100, \qquad 11 \le T_{21} \le 40.$$

Applying the Euler algorithm, the equilibrium solution and the incurred prices at equilibrium are, after 2,957 iterations:

$$d_{11}^* = 28.28,\; d_{21}^* = 20.97,$$

$$T_{11}^* = 17.83,\; T_{21}^* = 17.39,$$

$$q_{11}^* = 92.17,\; q_{21}^* = 90.63,$$

$$p_{11} = 4.75,\; p_{21} = 5.73.$$

The Jacobian matrix of $F(X) = -\nabla U(d,q,T)$, denoted by $J(d_{11}, d_{21}, T_{11}, T_{21}, q_{11}, q_{21})$, for this problem evaluated at $X^* = (d_{11}^*, d_{21}^*, T_{11}^*, T_{21}^*, q_{11}^*, q_{21}^*)$ is:

$$J = \begin{bmatrix} 5.96 & 0.59 & 3.25 & 0.97 & -0.575 & 0.115 \\ 0.58 & 7.655 & 0.95 & 3.16 & 0.11 & -0.56 \\ 3.25 & 0.94 & 10.29 & 1.54 & 0 & 0.18 \\ 0.70 & 3.16 & 1.15 & 7.63 & 0.13 & 0 \\ -0.57 & 0 & 0 & 0 & 0.17 & 0 \\ 0 & -0.56 & 0 & 0 & 0 & 0.13 \end{bmatrix}.$$

The eigenvalues of $\frac{1}{2}(J + J^T)$ are: 0.08, 0.11, 4.28, 4.49, 9.45, and 13.43, which are all positive. Therefore, both the uniqueness of the equilibrium solution and the conditions for convergence of the algorithm are guaranteed.

Hence, the contract period for Network Provider 1 at Demand Market 1 is 17.83 hours and that for Network Provider 2 is 17.39 hours. The revenue in cents for Network Provider 1 for the contract is $p_{11}d_{11}T_{11} \times 10^{-5} \times 3600$ seconds/hour = 86.26 cents. Network Provider 1 faces a cost of $c_{11} \times 10^{-5} \times 3600$ seconds/hour=27.37 cents for this contract and earns a profit of 58.91 cents. Note that this is the profit for a single user for the specific contract. The quality provided by Network Provider 1 of its service is higher than that provided by Network Provider 2. Network Provider 2, on the other hand, earns 75.15 cents in revenue, has 33.61 cents in cost, which results in a profit of 41.54 cents.

In this example, if the contract duration was 1 month, the revenue of a network provider per user would be approximately $35, which is consistent with today's Internet pricing from service providers such as COMCAST (http://www.comcast.com).

Example 2 and Sensitivity Analysis in Price

This example has the identical data to that of Example 1 except that Demand Market 2 is added as in Figure 3.

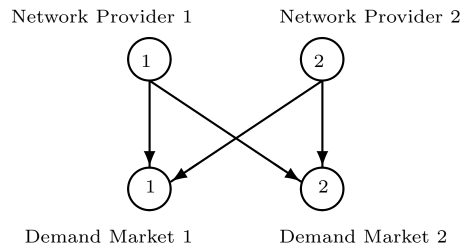

FIGURE 3. Example 2

The price functions for Demand Market 1 are as in Example 1. Demand Market 2 is less sensitive to the contract duration, the quality, and the service usage rate than Demand Market 1. The price functions for Demand Market 2 are:

$$p_{12} = 6 - .063\,d_{12} - .0126\,d_{22} + .026\,q_{12} - .0052\,q_{22} - .117\,T_{12} - .0351\,T_{22},$$

$$p_{22} = 6 - .0126\,d_{12} - .063\,d_{22} - .0052\,q_{12} + .026\,q_{22} - .0351\,T_{12} - .117\,T_{22}.$$

The cost functions for the network providers are:

$$c_{1j} = (.0049\,q_{1j}^2 + .001715\,q_{1j} + .029\,d_{1j})T_{1j},\ j = 1,2; \quad c_{2j} = (.0037\,q_{2j}^2 + .053\,d_{2j}^2)T_{2j},\ j = 1,2.$$

The bounds on the variables are:

$$23 \le d_{1j} \le 250, \qquad 0 \le q_{1j} \le 100, \qquad 8 \le T_{1j} \le 40, \qquad j = 1,2,$$

$$15 \le d_{2j} \le 200, \qquad 0 \le q_{2j} \le 100, \qquad 11 \le T_{2j} \le 40, \qquad j = 1,2.$$

The utilities of Network Providers 1 and 2 are, respectively:

$$U_1 = p_{11}d_{11}T_{11} + p_{12}d_{12}T_{12} - (c_{11} + c_{12}), \quad U_2 = p_{21}d_{21}T_{21} + p_{22}d_{22}T_{22} - (c_{21} + c_{22}).$$

The Jacobian of $F(X)$ is also positive-definite for this example.

The computed equilibrium, after 6,244 iterations, is:

$$d_{11}^* = 28.28,\ d_{12}^* = 45.39,\ d_{21}^* = 20.98,\ d_{22}^* = 20.71,$$

$$T_{11}^* = 17.83,\ T_{12}^* = 15.18,\ T_{21}^* = 17.39,\ T_{22}^* = 12.47,$$

$$q_{11}^* = 92.16,\ q_{12}^* = 100.00,\ q_{21}^* = 90.72,\ q_{22}^* = 72.64.$$

At equilibrium, the prices of network services are:

$$p_{11} = 4.75,\ p_{12} = 2.89,\ p_{21} = 5.73,\ p_{22} = 3.50.$$

Following the methodology used for Example 1, it follows that the revenue of Network Provider 1 is now 157.87 cents and his cost is 54.93 cents. Therefore, Network Provider 1 earns 102.94 cents for providing the services to the two demand markets. Network Provider 2's profit is now 55.12 cents at a revenue of 107.75 cents and a cost of 52.63 cents.

In order to understand the impact of changes in price functions, we denoted the constant term in the price functions of Demand Market 2 as p_0 and allowed p_0 to vary from 6 (its initial value) in both p_{12} and p_{22} to 18 in increments of 2. The results are reported in Figure 4. We see that not only the prices that Demand Market 2 is charged, but, also, the service usage rates and the durations of the contracts for this demand market increase, which leads to higher profits for not only Network Provider 2 but, also, interestingly, for Network Provider 1 (cf. Figure 5).

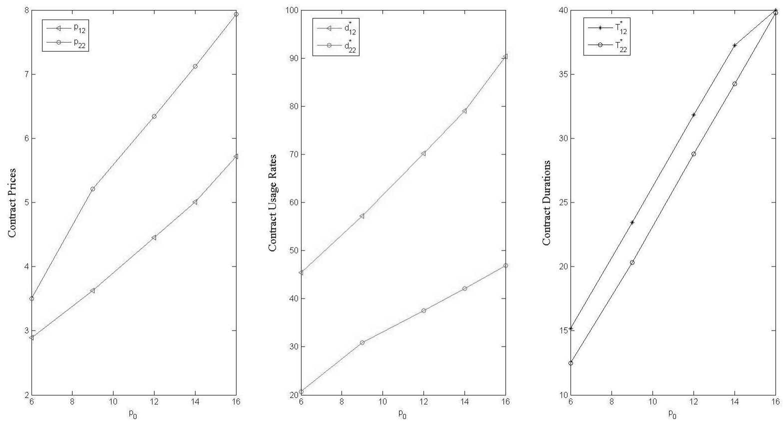

FIGURE 4. Effect of Increasing p_0 on Demand Market 2's Contracts

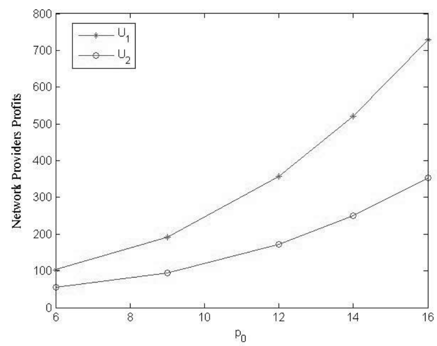

FIGURE 5. Effect of Increasing p_0 on Network Providers' Profits

Example 3 and Sensitivity Analysis in Quality Upper Bounds

Example 3 extends Example 2 to include a third demand market as shown in Figure 6.

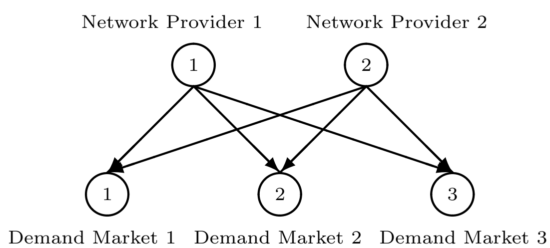

FIGURE 6. Example 3

The price and demand functions for Demand Markets 1 and 2 are as in Example 2. The price functions for Demand Market 3 are:

$$p_{13} = 9 - .115\, d_{13} - .023\, d_{23} + .028\, q_{13} - .0056\, q_{23} - .211\, T_{13} - .0633\, T_{23},$$

$$p_{23} = 9 - .023\, d_{13} - .115\, d_{23} - .0056\, q_{13} + .028\, q_{23} - .0633\, T_{13} - .211\, T_{23}.$$

The cost functions for Demand Market 3 are:

$$c_{1j} = (.0049\, q_{1j}^2 + .001715\, q_{1j} + .029\, d_{1j})T_{1j},\ j = 3, \quad c_{2j} = (.0037\, q_{2j}^2 + .053\, d_{2j}^2)T_{2j},\ j = 3,$$

with those for Demand Markets 1 and 2 as in Example 2.

The bounds on the variables are:

$$23 \le d_{1j} \le 250, \qquad 0 \le q_{1j} \le 100, \qquad 8 \le T_{1j} \le 40, \qquad j = 1, 2, 3,$$

$$15 \le d_{2j} \le 200, \qquad 0 \le q_{2j} \le 100, \qquad 11 \le T_{2j} \le 40, \qquad j = 1, 2, 3.$$

The utility functions of Network Providers 1 and 2 are:

$$U_1 = p_{11}d_{11}T_{11} + p_{12}d_{12}T_{12} + p_{13}d_{13}T_{13} - (c_{11} + c_{12} + c_{13}),$$

$$U_2 = p_{21}d_{21}T_{21} + p_{22}d_{22}T_{22} + p_{23}d_{23}T_{23} - (c_{21} + c_{22} + c_{23}).$$

The Jacobian of $F(X)$ for this example is also positive-definite.

The new equilibrium solution, computer after 8,681 iterations, is:

$$d_{11}^* = 31.48,\ d_{12}^* = 45.39,\ d_{13}^* = 30.16,\ d_{21}^* = 23.55,\ d_{22}^* = 20.71,\ d_{23}^* = 19.87,$$

$$T_{11}^* = 20.31,\ T_{12}^* = 15.18,\ T_{13}^* = 13.49,\ T_{21}^* = 19.84,\ T_{22}^* = 12.47,\ T_{23}^* = 13.00,$$

$$q_{11}^* = 100.00,\ q_{12}^* = 100.00,\ q_{13}^* = 76.77,\ q_{21}^* = 100.00,\ q_{22}^* = 72.64,\ q_{23}^* = 67.11.$$

The equilibrium prices are:

$$p_{11} = 5.29,\ p_{12} = 2.89,\ p_{13} = 3.77,\ p_{21} = 6.43,\ p_{22} = 3.50,\ p_{23} = 4.57.$$

Following the methodology used for Examples 1 and 2, we determine that Network Provider 1 earns a profit of 169.81 cents and Network Provider 2 a profit of 99.21 cents. The total cost of Network Provider 1 is now 78.71 cents and that of Network Provider 2: 83.71 cents.

In order to investigate the effects of the maximum quality level of the network providers on their profits, we conducted a numerical sensitivity analysis. Quality disruptions may occur for various reasons, including natural, man-made, or technological. We used Example 3 as a baseline but varied the quality upper bounds from 10 through 100 in increments of 10 with both providers having the same quality upper bound. The profits/utilities of the providers are displayed in Figure 7.

The results show that the profit of Network Provider 1 increases as the maximum quality level increases, while the profit of Network Provider 2 is less sensitive to changes in the maximum quality level. Also, Figure 7 reveals that, when both providers have similar maximum levels of quality, Network Provider 1's utility/profit is highest when the maximum quality level for both providers is at 100, while the highest utility/profit for Network Provider 2 is obtained when the maximum quality level for both is at 60.

Additional sensitivity analysis results are given in Figure 8 to investigate the impact on profits of distinct quality level upper bounds for the providers. The results in Figure 8 reveal that each network provider is better off at a higher maximum quality level while the maximum quality level of the other provider is fixed. Also, each provider benefits by increasing his maximum quality level bound while the maximum quality level of the other provider is lower.

For each provider, the lowest utility/profit occurs when that provider has the lowest level of maximum quality (10) whereas the other provider has the highest maximum quality level (100).

These examples illustrate the importance of computations in gaining insights that may not be obvious otherwise because of the number of decision-makers, demand markets, and the complexity of interactions among them.

Note that if $F(X)$ is monotone, a property that would be satisfied for utility functions as in Theorem 1, in which case $\nabla F(X)$ is positive-semidefinite, then an algorithm such as the extragradient method of [23] could be utilized, with $F(X)$ also being Lipschitz continuous.

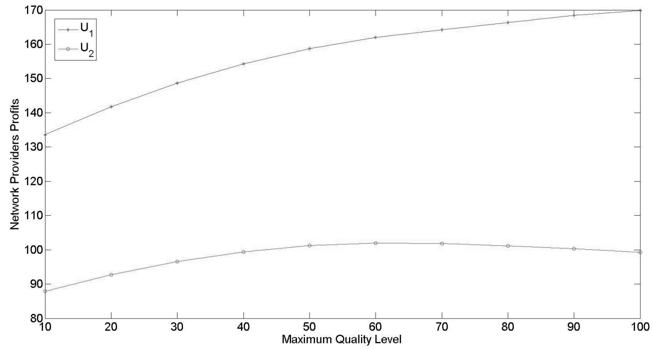

FIGURE 7. Effect of Increasing Maximum Quality Level on Network Providers' Profits - Same for Both Providers

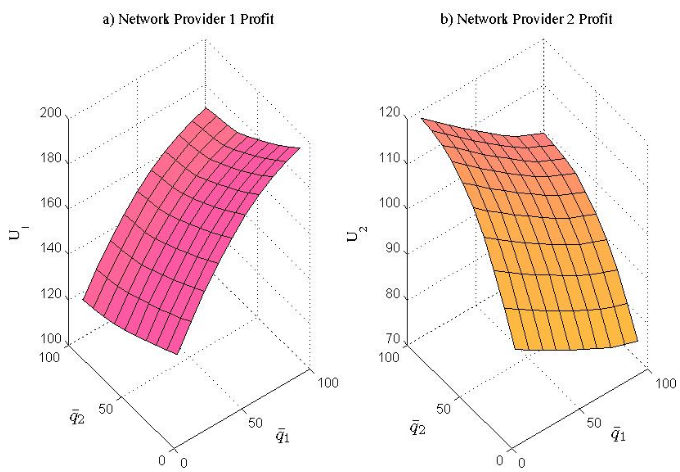

FIGURE 8. Effect of Increasing Maximum Quality Level on Network Providers' Profits - Different for Each Provider

4. Summary

In this paper, we developed a game theory model for a differentiated service-oriented Internet with duration-based contracts and quality competition. The theoretical formalism was established using variational inequalities, which provides us with tools for both qualitative analysis and computational schemes. Numerical examples, supplemented with sensitivity analysis, demonstrated the efficacy of both the model and algorithmic framework.

Acknowledgments

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