Dynamics
of
Global Supply Chain Supernetworks

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Abstract: In this paper, we develop a framework for the modelling, analysis, and computation of solutions to global supply chains. We consider three tiers of decision-makers: manufacturers, who may be located in the same or in different countries; intermediaries, in the form of retailers, who may be either physical or virtual, as in the case of electronic commerce, and consumers at the demand markets who may purchase the product in different currencies in the countries. We model the behavior of the decision-makers, derive the equilibrium conditions, and establish the variational inequality formulation. We then utilize the variational inequality formulation to obtain qualitative properties of the equilibrium product shipment and price pattern. In addition, we propose a dynamic adjustment process for the continuous time adjustment of the product flows and prices and formulate it as a projected dynamical system whose set of solutions coincides with the set of solutions to the variational inequality problem. A discrete-time algorithm is then applied to several numerical supply chain examples. This research extends the recent results surrounding the modelling of supply chains in a network equilibrium setting to the global arena.

Key words: global supply chains, network equilibrium, variational inequalities, dynamical systems

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1. Introduction

The topic of supply chain modelling, analysis, and computation has been the subject of a great deal of research due to the challenging intellectual questions surrounding the efficient and effective production, storage, distribution, and, ultimate sale and consumption of products as well as the practical and financial import of supply chain decision-making. Indeed, numerous books have been written on the subject (cf. [1] – [8]). Moreover, supply chain analysis has been surveyed in several recent articles (cf. Erenguc et al. [9] and Cohen and Huchzermeier [10]).

In addition, the introduction of electronic commerce had opened up new opportunities in terms of both practice and research regarding supply chain modeling and analysis (see Kuglin and Rosenbaum [11]). In particular, electronic commerce (e-commerce) through the Internet has allowed for new connections among suppliers, manufacturers, retailers, and customers both within a particular country as well as beyond national borders. Indeed, the topic of global supply chain analysis (see Kogut and Kulatilaka [12], Vidal and Goetschalckx [13], and Cohen and Huchzermeier [14]) is one where the inclusion of electronic commerce is not only natural but essential.

In this paper, we focus on the development of a global supply chain supernetwork model that captures the interactions among three distinct tiers of decision-makers, notably, manufacturers, retailers, as well as consumers. The idea of *supernetworks*, as utilized in this paper, was introduced by Nagurney and Dong [15] in order to capture the trade-offs associated with transportation versus telecommunication networks in the Information Age. It has recently been applied to construct a single country supply chain network model with electronic commerce by Nagurney et al. [16] and to model financial decision-making with intermediaries, respectively, in a single-country and international context by Nagurney and Ke [17] and Nagurney and Cruz [18]. For applications of supernetworks to shopping versus teleshopping decision-making and commuting versus telecommuting decision-making see [19] and [20].

Here we build upon the work of Nagurney, Dong, and Zhang [21] and Nagurney, et al. [16, 22] and formulate a global supernetwork supply chain model with the following notable features:

1. it handles as many countries, manufacturers, retailers, and demand markets for the product as mandated by the specific application;

2. it predicts, assuming profit-maximizing behavior on the part of the manufacturers and the retailers, not only the equilibrium product shipments between tiers of the decision-makers, but also the equilibrium prices of the product in different currencies at the demand markets in different countries;

3. retailers may be physical or virtual, as in the case of e-commerce;

4. the transaction costs need not be symmetric between tiers of decision-makers, and

5. it allows for the analysis and solution of the equilibrium product flows and prices as well as for the study of the disequilibrium dynamics.

Clearly, the development of such a general modeling framework for global supply chains is timely for several reasons: the increasing dispersion of manufacturing and retail operations around the globe, the introduction and use of new currencies such as the euro, the globalization of the network economy, and the growing role of electronic commerce.

The paper is organized as follows. In Section 2, we develop the model, describe the various decision-makers and their behaviors, and construct the equilibrium conditions, along with the variational inequality formulation. The variables are the equilibrium prices, as well as the equilibrium product shipments between the tiers of decision-makers. In Section 3, we provide a dynamic version of the model. In Section 4, we derive qualitative properties of the equilibrium pattern, under appropriate assumptions, notably, the existence and uniqueness of a solution to the governing variational inequality. We also establish that the dynamic trajectories are well-defined under suitable conditions. In Section 5, we provide the algorithm, which is then applied in Section 6 to several global supply chain supernetwork examples. We conclude the paper with Section 7 in which we summarize our results and suggest directions for future research.

2. The Global Supply Chain Supernetwork Model

In this Section, we develop the global supply chain supernetwork model. The model assumes that the manufacturing firms are involved in the production of a homogeneous product and considers L countries, with I manufacturers in each country, and J retailers, which are not country-specific but, rather, can be either physical or virtual, as in the case of electronic commerce. There are K demand markets for the homogeneous product in each country and H currencies in the global economy. We denote a typical country by l, a typical manufacturer by i, and a typical retailer by j. A typical demand market, on the other hand, is denoted by k and a typical currency by h. We assume, for the sake of generality, that each manufacturer can transact with the retailers in different currencies. Similarly, we assume that the demand for the product in a country can be associated with a particular currency.

The global supply chain supernetwork is now described and depicted graphically in Figure 1. The top tier of nodes consists of the manufacturers in the different countries, with manufacturer i in country l being referred to as manufacturer il and associated with node il. There are, hence, IL top-tiered nodes in the network. The middle tier of nodes consists of the retailers (which need not be country specific) and who act as intermediaries between the manufacturers and the demand markets, with a typical retailer j associated with node j in this (second) tier of nodes in the network. The bottom tier of nodes consists of the demand markets, with a typical demand market k in currency h and country l, being associated with node khl in the bottom tier of nodes. There are, as depicted in Figure 1, J middle (or second) tiered nodes corresponding to the retailers and KHL bottom (or third) tiered nodes in the global supply chain network.

We have identified the nodes in the global supply chain supernetwork and now we turn to the identification of the links joining the nodes in a given tier with those in the next tier. We also associate the product shipments with the appropriate links. We assume that each manufacturer i in country l, involved in the production of a homogeneous product, can transact with a given retailer in any of the H available currencies, as represented by the H links joining each top tier node with each middle tier node j; $j = 1, \ldots, J$. The flow on such a link h joining node il with node j is denoted by q^{il}_{jh} and represents the nonnegative amount of the product produced by manufacturer i in country l and transacted in currency h through

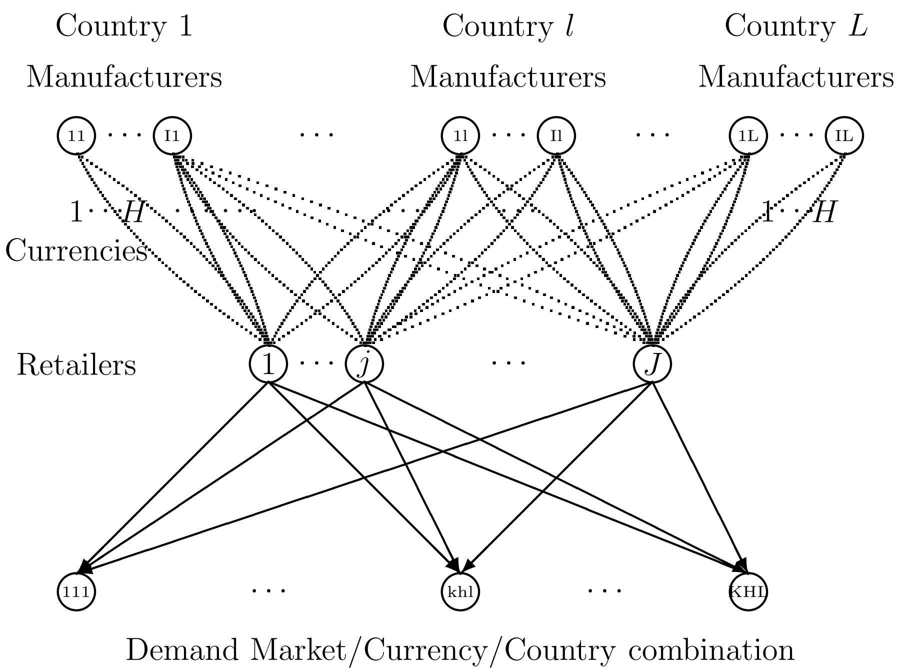

Figure 1: The Structure of the Global Supply Chain Supernetwork

retailer j. We group the product shipments (transactions) associated with manufacturer i in country l into the column vector $q^{il} \in R_+^{JH}$. We then further group all such transactions for all manufacturers in all countries into the column vector $Q^1 \in R_+^{ILJH}$. Note that if a retailer is virtual, then the transaction takes place electronically, although, of course, the product itself may be delivered physically.

From each retailer node j; $j = 1, \ldots, J$, we then construct a single link to each node khl with the flow on such a link being denoted by q^j_{khl} and corresponding to the amount of the product transacted between retailer j and demand market k in currency h and country l. The product shipments for all the retailers are grouped into the column vector $Q^2 \in R_+^{JKHL}$.

The notation for the prices is now given. Note that there will be prices associated with each of the tiers of nodes in the global supply chain network. Let ρ^{il}_{1jh} denote the price associated with the product in currency h transacted between manufacturer il and retailer j and group these top tier prices into the column vector $\rho_1 \in R_+^{ILJH}$. Let ρ^j_{2khl}, in turn, denote the price associated with retailer j and demand market k in currency h and country l and group all such prices into the column vector $\rho_2 \in R_+^{JKHL}$. Also, let ρ_{3khl} denote the price of the product at demand market k in currency h and in country l, and group all such prices into the column vector $\rho_3 \in R_+^{KHL}$. Finally, let e_h denote the rate of appreciation of currency h against the basic currency, which can be interpreted as the rate of return earned due to exchange rate fluctuations (see Nagurney and Siokos [23]). These "exchange" rates are grouped into the column vector $e \in R_+^H$.

We now turn to describing the behavior of the various global supply chain network decision-makers represented by the three tiers of nodes in Figure 1. The model is presented, for ease of explosition, for the case of a single homogeneous product. It can also handle multiple products through a replication of the links and added notation. We first focus on the manufacturers. We then turn to the retailers, and, subsequently, to the consumers at the demand markets.

The Behavior of the Manufacturers and their Optimality Conditions

We denote the transaction cost associated with manufacturer il transacting with retailer j for the product in currency h by c^{il}_{jh} and assume that:

$$c^{il}_{jh} = c^{il}_{jh}(q^{il}_{jh}), \quad \forall i, l, j, h, \tag{1}$$

that is, the cost associated with manufacturer i in country l transacting with retailer j for the product in currency h depends on the volume of product flow in the transaction. The transaction cost functions are assumed to be convex and continuously differentiable.

The total transaction costs incurred by manufacturer il are equal to the sum of all of his transaction costs associated with dealing with the distinct retailers in the different currencies. His revenue, in turn, is equal to the sum of the price (rate of return plus the rate of appreciation) that the manufacturer can obtain for the product times the total quantity obtained/purchased of that product. Let now ρ^{il*}_{1jh} denote the actual price charged by the manufacturer il for the product in currency h to retailer j (and that the retailer is willing to pay). Similarly, let e^*_h denote the actual rate of appreciation in currency h. We later discuss how such prices are recovered.

We assume that each manufacturer seeks to maximize his profits, with manufacturer il's profit or utility function denoted by U^{il}. In particular, we assume, as given, a production cost function for manufacturer il and denoted by f^{il}, such that

$$f^{il} = f^{il}(Q^1), \quad \forall i, l. \tag{2}$$

Recall that the vector Q^1 represents all the product flows between the top tier nodes to the middle tier nodes. The function f^{il} is assumed to be strictly convex and continuously differentiable.

We now construct the utility maximization problem facing a manufacturer i in country l. In particular, we can express the optimization problem facing manufacturer il as:

$$\text{Maximize} \quad U^{il} = \sum_{j=1}^{J} \sum_{h=1}^{H} (\rho^{il*}_{1jh} + e^*_h) q^{il}_{jh} - \sum_{j=1}^{J} \sum_{h=1}^{H} c^{il}_{jh}(q^{il}_{jh}) - f^{il}(Q^1), \tag{3}$$

subject to $q_{jh}^{il} \geq 0$, for all j, h. The first term in the utility function (3) denotes the revenue whereas the second term denotes the transaction costs and the last term denotes the production cost.

Here we assume that the manufacturers compete in a noncooperative fashion following Nash [24, 25]. Hence, each manufacturer seeks to determine his optimal strategies, that is production outputs (and shipments), given those of the other manufacturers. The optimality conditions of all manufacturers i; $i = 1, \ldots, I$; in all countries: l; $l = 1, \ldots, L$ simultaneously, under the above assumptions (see also [17], [26] – [29]), can be compactly expressed as: determine $Q^{1*} \in R_+^{IJHL}$ satisfying

$$\sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{h=1}^{H} \left[\frac{\partial f^{il}(Q^{1*})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}} - \rho_{1jh}^{il*} \; - e_h^* \right] \times \left[q_{jh}^{il} - q_{jh}^{il*} \right] \geq 0, \quad \forall Q^1 \in R_+^{IJHL}. \quad (4)$$

Note that (4) is a variational inequality (see [30]) and has a nice economic interpretation. Indeed, if there is a positive amount of the product transacted between a manufacturer in a country and a retailer in a cuurency, then the price plus the appreciation rate is precisely equal to the marginal production and marginal transaction cost associated with the product. If the marginal costs exceed the price plus the appreciation rate, then there will be zero volume of flow of the product between that particular manufacturer and retailer in the currency.

The Behavior of the Retailers and their Optimality Conditions

The retailers (cf. Figure 1), in turn, are involved in transactions both with the manufacturers in the different countries, as well as with the ultimate consumers associated with the demand markets for the product in different countries and currencies and represented by the bottom tier of nodes of the network.

A retailer j is faced with what we term a *handling/conversion* cost, which may include, for example, the cost of handling and storing the product plus the cost associated with transacting in the different currencies. We denote such a cost faced by retailer j by c_j and, in the simplest case, we would have that c_j is a function of $\sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il}$, that is, the handling/conversion cost of a retailer is a function of how much he has obtained of the product from the various manufacturers in the different countries and in what currency the transactions took place in. For the sake of generality, however, we allow the function to depend also on the amounts of the product held and transacted by other retailers and, therefore, we may write:

$$c_j = c_j(Q^1), \quad \forall j. \tag{5}$$

The retailers, which recall can be either physical or virtual, also have associated transaction costs in regards to transacting with the manufacturers, which we assume can be dependent on the type of currency as well as the manufacturer. We denote the transaction cost associated with retailer j transacting with manufacturer il associated with currency h by $\hat{c}^{il}_{jh}$ and we assume that it is of the form

$$\hat{c}^{il}_{jh} = \hat{c}^{il}_{jh}(q^{il}_{jh}), \quad \forall i, l, j, h, \tag{6}$$

that is, such a transaction cost is allowed to depend on the amount of the product transacted in a currency with the particular manufacturer. In addition, we assume that a retailer j also incurs a transaction cost c^j_{khl} associated with transacting with demand market khl, where

$$c^j_{khl} = c^j_{khl}(q^j_{khl}), \quad \forall j, k, h, l. \tag{7}$$

Hence, the transaction costs given in (7) can vary according to the retailer/currency/country combination and are a function of the volume of the product transacted. We assume that the cost functions (5) – (7) are convex and continuously differentiable.

The actual price charged for the product by retailers j is denoted by ρ^{j*}_{2khl}, and is associated with transacting with consumers at demand market k in currency h and country l. Subsequently, we discuss how such prices are arrived at. We assume that the retailers are also profit/utility maximizers.

The utility maximization problem for retailer j, with his utility function expressed by U^j, can, hence, be expressed as:

$$\text{Maximize} \quad U^j = \sum_{k=1}^{k}\sum_{h=1}^{H}\sum_{l=1}^{L}(\rho^{j*}_{2khl} + e^*_h)q^{il}_{jh} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\hat{c}^{il}_{jh}(q^{il}_{jh})$$
$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}c^j_{khl}(q^j_{khl}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}(\rho^{il*}_{1jh} + e^*_h)q^{il}_{jh} \tag{8}$$

subject to: the nonnegativity constraints: $q_{jh}^{il} \geq 0$, $q_{khl}^{j} \geq 0$, for all i, l, h, and

$$\sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} q_{khl}^{j} \leq \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il}. \tag{9}$$

Objective function (8) expresses that the difference between the revenues (given by the first term) minus the handling cost, the two sets of transaction costs, and the payout to the manufacturers (given by the fifth term) should be maximized. The utility function in (8) is concave in its variables under the above posed assumptions.

Here we assume that the retailers can also compete in a noncooperative manner with the governing optimality/equilibrium concept being that of Nash. The optimality conditions for all retailers simultaneously, under the above stated assumptions, can, hence, be expressed as the variational inequality problem: determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R_+^{ILJH+JKHL+J}$, such that

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{l=}^{L} \sum_{h=1}^{H} \left[\frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}} + \rho_{1jh}^{il*} + e_h^* + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}} - \gamma_j^* \right] \times \left[q_{jh}^{il} - q_{jh}^{il*} \right]$$

$$+ \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} \left[\frac{\partial c_{khl}^{j}(q_{khl}^{j*})}{\partial q_{khl}^{j}} - \rho_{2khl}^{j*} - e_h^* + \gamma_j^* \right] \times \left[q_{khl}^{j} - q_{khl}^{j*} \right]$$

$$+ \sum_{j=1}^{J} \left[\sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il*} - \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} q_{khl}^{j*} \right] \times \left[\gamma_j - \gamma_j^* \right] \geq 0, \quad \forall (Q^1, Q^2, \gamma) \in R_+^{ILJH+JKHL+J}, \tag{10}$$

where γ_j is the Lagrange multiplier associated with constraint (9) (see [28]), and γ is the J-dimensional column vector of Lagrange multiplers of all the retailers with γ^* denoting the vector of optimal multipliers. Note that γ_j^* serves as the market clearing price for the product at retailer j (as can be seen from the last term in (10)). In particular, its value is positive if the product shipments from the retailer to all the demand markets in the countries and in the various currencies is precisely equal to the product shipments into the retailer from all the manufacturers in all the countries transacted in the different currencies.

The Consumers at the Demand Markets and the Equilibrium Conditions

We now describe the behavior of the consumers located at the demand markets. The consumers take into account in making their consumption decisions not only the price charged for the product by the retailers but also their transaction costs associated with obtaining the product. We let $\hat{c}^j_{khl}$ denote the transaction cost associated with obtaining the product at demand market k in currency h in country l from retailer j and recall that q^j_{khl} is the amount of the product transacted in currency h and flowing between retailer j and consumers at demand market k in country l. We assume that the transaction cost is continuous and of the general form:

$$\hat{c}^j_{khl} = \hat{c}^j_{khl}(Q^2), \quad \forall j, k, h, l. \tag{11}$$

Hence, we allow for the transaction cost (from the perspective of consumers) to depend not only upon the flow of the product from a retailer transacted in a currency to the demand market in the country but also on other product flows in other currencies between retailers and consumers in other demand markets and countries.

Denote the demand for the product at demand market k in currency h in country l by d_{khl} and assume, as given, the continuous demand functions:

$$d_{khl} = d_{khl}(\rho_3), \quad \forall k, h, l, \tag{12}$$

where ρ_3 is the KHL-dimensional column vector containing the demand market prices in all countries and currencies. Thus, according to (12), the demand of consumers for the product in a currency and country depends, in general, not only on the price of the product at that demand market (and currency and country) but also on the prices of the product at the other demand markets (and in other countries and currencies). Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

The consumers take the price charged by the retailer, which was denoted by ρ^{j*}_{2khl} for retailer j, demand market k, currency h, and country l, and the rate of appreciation in the currency, plus the transaction cost, in making their consumption decisions. The equilibrium conditions for the consumers at demand market khl, thus, take the form: for all retailers: $j = 1, \ldots, J$:

$$\rho^{j*}_{2khl} + e^*_h + \hat{c}^j_{khl}(Q^{2*}) \begin{cases} = \rho^*_{3khl}, & \text{if} \quad q^{j*}_{khl} > 0 \\ \geq \rho^*_{3khl}, & \text{if} \quad q^{j*}_{khl} = 0, \end{cases} \tag{13}$$

and

$$d_{khl}(\rho_3^*) \begin{cases} = \sum_{j=1}^{J} q_{khl}^{j*}, & \text{if} \quad \rho_{3khl}^* > 0 \\ \leq \sum_{j=1}^{J} q_{khl}^{j*}, & \text{if} \quad \rho_{3khl}^* = 0. \end{cases} \tag{14}$$

Conditions (13) state that consumers at demand market khl will purchase the product from retailer j, if the price charged by the retailer for the product and the appreciation rate for the currency plus the transaction cost (from the perspective of the consumer) does not exceed the price that the consumers are willing to pay for the product in that currency and country, i.e., ρ_{3khl}^*. Note that, according to (14), if the transaction costs are identically equal to zero, then the price faced by the consumers for the product is the price charged by the retailer for the product and currency in the country plus the rate of appreciation in the currency.

Condition (14), on the other hand, states that, if the price the consumers are willing to pay for the product at a demand market is positive, then the quantity of the product transacted by the retailers with the consumers at the demand market is precisely equal to the demand.

In equilibrium, conditions (13) and (14) will have to hold for all retailers and for all demand markets and these, in turn, can be expressed also as an inequality analogous to those in (4) and (10) and given by: determine $(Q^{2*}, \rho_3^*) \in R_+^{(J+1)KHL}$, such that

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\rho_{2khl}^{j*} + e_h^* + \hat{c}_{khl}^{j}(Q^{2*}) - \rho_{3khl}^*\right] \times \left[q_{khl}^{j} - q_{khl}^{j*}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\sum_{j=1}^{J} q_{khl}^{j*} - d_{khl}(\rho_3^*)\right] \times \left[\rho_{3khl} - \rho_{3khl}^*\right] \geq 0, \quad \forall (Q^2, \rho_3) \in R_+^{(J+1)KHL}. \tag{15}$$

The Equilibrium Conditions for the Global Supply Chain Network Economy

In equilibrium, the product shipments transacted between the manufacturers in the different countries with the retailers must coincide with those that the retailers actually accept from them. In addition, the amounts of the product that are obtained by the consumers in the different countries and currencies must be equal to the amounts that the retailers actually provide. Hence, although there may be competition between decision-makers at the same level of tier of nodes of the supply chain supernetwork there must be, in a sense, cooperation between decision-makers associated with pairs of nodes (through positive flows on the links joining them). Thus, in equilibrium, the prices and product flows must satisfy the sum of the optimality conditions (4) and (10) and the equilibrium conditions (15). We make these relationships rigorous through the subsequent definition and variational inequality derivation below.

Definition 1: Global Supply Chain Equilibrium

The equilibrium state of the supply chain supernetwork is one where the product flows between the tiers of the network coincide and the product flows and prices satisfy the sum of conditions (4), (10), and (15).

The equilibrium state is equivalent to the following:

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the global supply chain network according to Definition 1 are equivalent to the solution of the variational inequality given by: determine $(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in \mathcal{K}$*, satisfying:*

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(Q^{1*})}{\partial q_{jh}^{il}}+\frac{\partial c_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}+\frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}}+\frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}-\gamma_j^*\right]\times\left[q_{jh}^{il}-q_{jh}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\frac{\partial c_{khl}^{j}(q_{khl}^{j*})}{\partial q_{khl}^{j}}+\gamma_j^*+\hat{c}_{khl}^{j}(Q^{2*})-\rho_{3khl}^*\right]\times\left[q_{khl}^{j}-q_{khl}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}q_{jh}^{il*}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{khl}^{j*}\right]\times\left[\gamma_j-\gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\sum_{j=1}^{J}q_{khl}^{j*}-d_{khl}(\rho_3^*)\right]\times\left[\rho_{3khl}-\rho_{3khl}^*\right]\geq 0,\quad \forall (Q^1, Q^2, \gamma, \rho_3)\in\mathcal{K}, \qquad (16)$$

where $\mathcal{K}\equiv\{(Q^1,Q^2,\gamma,\rho_3)|(Q^2,\gamma,\rho_3)\in R_+^{ILJH+JKHL+J+KHL}\}$.

Proof: We first establish that the equilibrium conditions imply variational inequality (16). Indeed, summation of inequalities (4), (10), and (15), after algebraic simplifications, yields variational inequality (16).

We now establish the converse, that is, that a solution to variational inequality (16) satisfies the sum of conditions (4), (10), and (15), and is, hence, an equilibrium.

To inequality (16), add the term: $-\rho_{1jh}^{il*}-e_h^*+\rho_{1jh}^{il*}+e_h^*$ to the term in the first set of brackets (preceding the first multiplication sign). Similarly, add the term: $-\rho_{2khl}^{j*}-e_h^*+\rho_{2khl}^{j*}+e_h^*$ to the term in brackets preceding the second multiplication sign in (16). The addition of such terms does not change (16) since the value of these terms is zero and yields:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(Q^{1*})}{\partial q_{jh}^{il}}+\frac{\partial c_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}+\frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}}+\frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}-\gamma_j^*-\rho_{1jh}^{il*}-e_h^*+\rho_{1jh}^{il*}+e_h^*\right]$$

$$\times\left[q_{jh}^{il}-q_{jh}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\frac{\partial c_{khl}^{j}(q_{khl}^{j*})}{\partial q_{khl}^{j}}+\gamma_j^*+\hat{c}_{khl}^{j}(Q^{2*})-\rho_{3khl}^*-\rho_{2khl}^{j*}-e_h^*+\rho_{2khl}^{j*}+e_h^*\right]\times\left[q_{khl}^{j}-q_{khl}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}q_{jh}^{il*}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{khl}^{j*}\right]\times\left[\gamma_j-\gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\sum_{j=1}^{J}q_{khl}^{j*}-d_{khl}(\rho_3^*)\right]\times\left[\rho_{3khl}-\rho_{3khl}^*\right]\geq 0,\quad \forall(Q^1,Q^2,\gamma,\rho_3)\in\mathcal{K},\tag{17}$$

which, in turn, can be rewritten as:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(Q^{1*})}{\partial q_{jh}^{il}}+\frac{\partial c_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}-\rho_{1jh}^{il*}\ -e_h^*\right]\times\left[q_{jh}^{il}-q_{jh}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}}+\rho_{1jh}^{il*}+e_h^*+\frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}-\gamma_j^*\right]\times\left[q_{jh}^{il}-q_{jh}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\frac{\partial c_{khl}^{j}(q_{khl}^{j*})}{\partial q_{khl}^{j}}-\rho_{2khl}^{j*}-e_h^*+\gamma_j^*\right]\times\left[q_{khl}^{j}-q_{khl}^{j*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\rho_{2khl}^{j*}+e_h^*+\hat{c}_{khl}^{j}(Q^{2*})-\rho_{3khl}^*\right]\times\left[q_{khl}^{j}-q_{khl}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}q_{jh}^{il*}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{khl}^{j*}\right]\times\left[\gamma_j-\gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\sum_{j=1}^{J}q_{khl}^{j*}-d_{khl}(\rho_3^*)\right]\times\left[\rho_{3khl}-\rho_{3khl}^*\right]\geq 0. \tag{18}$$

But inequality (18) is equivalent to the sum of conditions (4), (10), and (15), and hence that product and price pattern is an equilibrium according to Definition 1. □

We now put variational inequality (16) into standard form which will be utilized in the subsequent sections. For additional background on variational inequalities and their applications, see the book by Nagurney [30]. In paricular, we have that variational inequality (16) can be expressed as:

$$\langle F(X^*)^T, X-X^*\rangle \geq 0, \quad \forall X\in\mathcal{K}, \tag{19}$$

where $X\equiv(Q^1,Q^2,\gamma,\rho_3)$ and $F(X)\equiv(F_{iljh},F_{jkhl},F_j,F_{khl})_{i=1,\ldots,I;l=1,\ldots,L;j=1,\ldots,J;h=1,\ldots,H}$, and the specific components of F are given by the functional terms preceding the multiplication signs in (16), respectively. The term $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space.

We now describe how to recover the prices associated with the first two tiers of nodes in the supply chain network. Clearly, the components of the vector ρ_3^* are obtained directly from the solution of variational inequality (16) as will be demonstrated explicitly through several numerical examples in Section 6. In order to recover the second tier prices associated with the retailers and the appreciation rates one can (after solving variational inequality (16) for the particular numerical problem) *either* (cf. (13)) set $\rho_{2khl}^{j*}+e_h^*=\rho_{3khl}^*-\hat{c}_{khl}(Q^{2*})$, for any j,k,h,l such that $q_{khl}^{j*}>0$, *or* (cf. (10)) for any $q_{khl}^{j*}>0$, set $\rho_{2khl}^{j*}+e_h^*=\frac{\partial c_{khl}^j(q_{khl}^{j*})}{\partial q_{khl}^j}-\gamma_j^*$.

Similarly, from (10) we can infer that the top tier prices comprising the vector ρ_1^* can be recovered (once the variational inequality (16) is solved with particular data) thus: for any i,l,j,h, such that $q_{jh}^{il*}>0$, set $\rho_{1jh}^{il*}+e_h^*{=}\gamma_j^*-\frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}}-\frac{\partial\hat{c}_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}}$.

3. The Dynamic Adjustment Process

In this Section, we propose a dynamic adjustment process which describes the disequilibrium dynamics as the various global supply chain decision-makers adjust their product shipments between the tiers and the prices associated with the different tiers adjust as well. Importantly, the set of stationary points of the projected dynamical system which formulates the dynamic adjustment process will coincide with the set of solutions to the variational inequality problem (16). We begin by describing the dynamics underlying the prices of the product at the various demand markets and currencies in the various countries.

Demand Market Price Dynamics

We assume that the rate of change of the price ρ_{3khl}, denoted by $\dot{\rho}_{3khl}$ is equal to the difference between the demand for the product at the demand market and currency and country and the amount of the product actually available at that particular market. Hence, if the demand for the product at the demand market (and currency and country) at an instant in time exceeds the amount available from the various retailers, then the price will increase; if the amount available exceeds the demand at the price, then the price will decrease. Moreover, it is guaranteed that the prices do not become negative. Thus, the dynamics of the price ρ_{3khl} for each k, h, l can be expressed as:

$$\dot{\rho}_{3khl} = \begin{cases} d_{khl}(\rho_3) - \sum_{j=1}^{J} q^j_{khl}, & \text{if} \quad \rho_{3khl} > 0 \\ \max\{0, d_{khl}(\rho_3) - \sum_{j=1}^{J} q^j_{khl}\} & \text{if} \quad \rho_{3khl} = 0. \end{cases} \tag{20}$$

The Dynamics of the Product Shipments between the Retailers and the Demand Markets

The rate of change of the product shipment q^j_{khl}, in turn, and denoted by $\dot{q}^j_{khl}$, is assumed to equal to the difference between the price the consumers are willing to pay for the product at the demand market and currency and country minus the price charged and the various transaction costs. Here we also guarantee that the product shipments do not become negative. Hence, we may write: that for every j, k, h, l

$$\dot{q}^j_{khl} = \begin{cases} \rho_{3khl} - \frac{\partial c^j_{khl}(q^j_{khl})}{\partial q^j_{khl}} - \hat{c}^j_{khl}(Q^2) - \gamma_j, & \text{if} \quad q^j_{khl} > 0 \\ \max\{0, \rho_{3khl} - \frac{\partial c^j_{khl}(q^j_{khl})}{\partial q^j_{khl}} - \hat{c}^j_{khl}(Q^2) - \gamma_j\} & \text{if} \quad q^j_{khl} = 0. \end{cases} \tag{21}$$

Hence, according, to (21), if the price that the consumers are willing to pay for the product in the currency and demand market and country exceeds the price that the retailer charges and the various transaction costs, then the volume of flow of the product to that demand market will increase; otherwise, it will decrease (or remain unchanged).

The Dynamics of the Prices at the Retailers

The prices at the retailers, whether they are physical or virtual, must reflect supply and demand conditions as well. In particular, we let $\dot{\gamma}_j$ denote the rate if change in the market clearing price associated with retailer j and we propose the following dynamic adjustment for every retailer j:

$$\dot{\gamma}_j = \begin{cases} \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{khl}^{j} - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q_{jh}^{il}, & \text{if} \quad \gamma_j > 0 \\ \max\{0, \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{khl}^{j} - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q_{jh}^{il}\}, & \text{if} \quad \gamma_j = 0. \end{cases} \tag{22}$$

The Dynamics of the Product Shipments between Manufacturers and Retailers

The dynamics of the product shipments transacted in the different currencies are now described. Note that in order for a shipment to take place in practice there must be agreement between the pair of decision-makers. Towards that end, we let $\dot{q}_{jh}^{il}$ denote the rate of change of the product shipment from manufacturer il to retailer j transacted via currency h and have that for every i, l, j, h:

$$\dot{q}_{jh}^{il} = \begin{cases} \gamma_j - \frac{\partial f^{il}(Q^1)}{\partial q_{jh}^{il}} - \frac{\partial c_{jh}^{il}(q_{jh}^{il})}{\partial q_{jh}^{il}} - \frac{\partial c_j(Q^1)}{\partial q_{jh}^{il}} - \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il})}{\partial q_{jh}^{il}}, & \text{if} \quad q_{jh}^{il} > 0 \\ \max\{0, \gamma_j - \frac{\partial f^{il}(Q^1)}{\partial q_{jh}^{il}} - \frac{\partial c_{jh}^{il}(q_{jh}^{il})}{\partial q_{jh}^{il}} - \frac{\partial c_j(Q^1)}{\partial q_{jh}^{il}} - \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il})}{\partial q_{jh}^{il}}\}, & \text{if} \quad q_{jh}^{il} = 0. \end{cases} \tag{23}$$

The Projected Dynamical System

Consider now a dynamical system in which the demand market prices evolve according to (20); the product shipments between retailers and demand markets evolve according to (21); the prices at the retailers evolve according to (22), and the product shipments between manufacturers and retailers evolve according to (23). Let X and $F(X)$ be as defined following (18) and recall that the feasible set $\mathcal{K}$ is simply the nonnegative orthant. Then the dynamic model described by (20)- (23) can be rewritten as a projected dynamical system (see [31],

[32]) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{24}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = (Q^{10}, Q^{20}, \gamma^0, \rho_3^0)$ is the initial point corresponding to the initial product flow and price pattern. Note that since the feasible set $\mathcal{K}$ is simply the nonnegative orthant the projection operation takes on a very simple form as revealed through (20) – (23).

The trajectory of (24) describes the dynamic evolution of and the dynamic interactions among the product flows and prices. The dynamical system (24) is non-classical since it has a discontinuous right-hand side due to the projection operation. Such dynamical systems were introduced by Dupuis and Nagurney [31] and have been used to study a plethora of dynamic models in economic, finance, and transportation (see Nagurney and Zhang [32]). In addition, the projected dynamical systems methodology has been used to-date to formulate dynamical supply chain network models, with and without electronic commerce, respectively, by Nagurney et al. [16, 22].

Here we apply the methodology, for the first time, to global supply chain modelling and amalysis. The following result is immediate from Dupuis and Nagurney [31].

Theorem 2: Set of Stationary Points Coincides with Set of Equilibrium Points

The set of stationary points of the projected dynamical system (24) coincides with the set of solutions of the variational inequality problem (19) and, thus, with the set of equilibrium points as defined in Definition 1.

With Theorem 2, we see the the dynamical system proposed in this section, provides the disequilibrium dynamics prior to the steady or equilibrium state of the global supply chain network. Hence, once, a stationary point of the projected dynamical system is reached that point (consisting of product shipments and prices) also satisfies variational inequality (19) (equivalently, (16)) and is, therefore, a global supply chain equilibrium.

4. Qualitative Properties

In this Section, we provide some qualitative properties of the solution to variational inequality (16). In particular, we derive existence and uniqueness results. We also investigate properties of the function F (cf. (19)) that enters the variational inequality of interest here. Finally, we establish that the trajectories of the projected dynamical system (24) are well-defined under reasonable assumptions.

Since the feasible set is not compact we cannot derive existence simply from the assumption of continuity of the functions. Nevertheless, we can impose a rather weak condition to guarantee existence of a solution pattern. Let

$$\mathcal{K}_b = \{(Q^1, Q^2, \gamma, \rho_3) | 0 \leq Q^1 \leq b_1;\ 0 \leq Q^2 \leq b_2; 0 \leq \gamma \leq b_3;\ 0 \leq \rho_3 \leq b_4\}, \tag{25}$$

where $b = (b_1, b_2, b_3, b_4) \geq 0$ and $Q^1 \leq b_1; Q^2 \leq b_2; \gamma \leq b_3; \rho_3 \leq b_4$ means that $q^{il}_{jh} \leq b_1$; $q^j_{khl} \leq b_2$; $\gamma_j \leq b_3$; and $\rho_{3khl} \leq b_4$ for all i, l, j, k, h. Then $\mathcal{K}_b$ is a bounded closed convex subset of $R^{ILJH+JKHL+J+KHL}$. Thus, the following variational inequality

$$\langle F(X^b)^T, X - X^b \rangle \geq 0, \quad \forall X^b \in \mathcal{K}_b, \tag{26}$$

admits at least one solution $X^b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and F is continuous. Following Kinderlehrer and Stampacchia [33] (see also Theorem 1.5 in Nagurney [30]), we then have:

Theorem 3

Variational inequality (16) admits a solution if and only if there exists a $b > 0$, such that variational inequality (19) admits a solution in $\mathcal{K}_b$ with

$$Q^{1b} < b_1, \quad Q^{2b} < b_2, \quad \gamma^b < b_3, \quad \rho^b_3 < b_4. \tag{27}$$

Theorem 4: Existence

Suppose that there exist positive constants M, N, R, with $R > 0$, such that:

$$\frac{\partial f^{il}(Q^1)}{\partial q^{il}_{jh}} + \frac{\partial c^{il}_{jh}(q^{il}_{jh})}{\partial q^{il}_{jh}} + \frac{\partial c_j(Q^1)}{\partial q^{il}_{jh}} + \frac{\partial \hat{c}^{il}_{jh}(q^{il}_{jh})}{\partial q^{il}_{jh}} \geq M, \quad \forall Q^1 \text{ with } q^{il}_{jh} \geq N, \forall i, l, j, h, \tag{28}$$

$$\frac{\partial c^j_{khl}(q^j_{khl})}{\partial q^j_{khl}} + \hat{c}^j_{khl}(Q^2) \geq M, \quad \forall Q^2 \ with \ q^j_{khl} \geq N, \quad \forall j,k,h,l, \tag{29}$$

$$d_k(\rho_3) \leq N, \quad \forall \rho_3 \ with \ \rho_{3khl} > R, \quad \forall k,h,l. \tag{30}$$

Then variational inequality (16); equivalently, variational inequality (19), admits at least one solution.

Proof: Follows using analogous arguments as the proof of existence for Proposition 1 in Nagurney and Zhao [34] (see also the existence proof in [17]). □

Assumptions (28) and (29) are reasonable from an economics perspective, since when the product flow between a manufacturer and a retailer or a retailer and demand market is large, we can expect the corresponding sum of the associated marginal costs of production, transaction, and handling to exceed a positive lower bound. Moreover, in the case where the demand price of the product in a currency and country as perceived by consumers at a demand market is high (cf. (30)), we can expect that the demand for the product at the demand market to not exceed a positive bound.

We now establish additional qualitative properties both of the function F that enters the variational inequality problem (cf. (16) and (19)), as well as uniqueness of the equilibrium pattern. Since the proofs of Theorems 5 and 6 below are similar to the analogous proofs in Nagurney and Ke [17] they are omitted here. Additional background on the properties establish below can be found in the books by Nagurney and Siokos [23] and Nagurney [30].

Theorem 5: Monotonicity

Suppose that the production cost functions f^{il}; $i = 1,\ldots,I$ are strictly convex and that the c^{il}_{jh}, c_j, $\hat{c}^{il}_{jh}$, and c^j_{khl} functions are convex; the $\hat{c}^j_{khl}$ functions are monotone increasing, and the d_k functions are monotone decreasing functions, for all i,l,j,h,k. Then the vector function F that enters the variational inequality (16) is monotone, that is,

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{31}$$

Monotonicity plays a role in the qualitative analysis of variational inequality problems similar to that played by convexity in the context of optimization problems.

Theorem 6: Strict Monotonicity

Assume all the conditions of Theorem 5. In addition, suppose that one of the families of convex functions $c^{il}_{jh}; i = 1, ..., I; l = 1, \dots, L; j = 1, ..., J; h = 1, \dots, H$, c_j; $j = 1, ..., J$; $\hat{c}^{il}_{jh}$; $i = 1, \dots, I; l = 1, \dots, L; j = 1, \dots, J; h = 1, \dots, H$, *and* c^j_{khl}; $j = 1, \dots, J; k = 1, \dots, K; h = 1, \dots, H$, *and* $l = 1, \dots, L$, *is a family of strictly convex functions. Suppose also that* $\hat{c}^j_{khl}$; $j = 1, ..., J$; $k = 1, ..., K$; $h = 1, \dots, H$; $l = 1, \dots, L$, *and* $-d_k$; $k = 1, ..., K$, *are strictly monotone. Then, the vector function* F *that enters the variational inequality (16) is strictly monotone, with respect to* (Q^1, Q^2, ρ_3), *that is, for any two* X', X'' *with* $(Q^{1'}, Q^{2'}, \rho_3') \neq (Q^{1''}, Q^{2''}, \rho_3'')$

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0. \tag{32}$$

Theorem 7: Uniqueness

Assuming the conditions of Theorem 6, there must be a unique product flow pattern (Q^{1*}, Q^{2*}), *and a unique demand price price vector* ρ_3^* *satisfying the equilibrium conditions of the global supply chain network. In other words, if the variational inequality (16) admits a solution, then that is the only solution in* (Q^1, Q^2, ρ_3).

Proof: Under the strict monotonicity result of Theorem 6, uniqueness follows from the standard variational inequality theory (cf. [33]) □

Theorem 8: Lipschitz Continuity

The function that enters the variational inequality problem (16) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \textit{ where } L > 0, \tag{33}$$

under the following conditions:

(i). f^{il}, c^{il}_{jh}, c_j, $\hat{c}^{il}_{jh}$, c^j_{khl} *have bounded second-order derivatives, for all* i, l, j, h, k*;*

(ii). $\hat{c}^j_{khl}$, *and* d_k *have bounded first-order derivatives for all* j, k, h, l*.*

Proof: The result is direct by applying a mid-value theorem from calculus to the vector function F that enters the variational inequality problem (16). □

Theorem 9: Existence and Uniqueness

Assume the conditions of Theorem 8. Then, for any $X_0 \in \mathcal{K}$*, there exists a unique solution* $X_0(t)$ *to the initial value problem (24).*

Note that Theorem 9, unlike Theorems 4 and 7, is concerned with the existence of a unique trajectory. Theorems 4 and 7, on the other hand, are concerned with the existence and uniqueness of an equilibrium pattern. Hence, according to Theorem 9, the dusequilibrium dynamics of the global supply chain network are well-defined. Also, for completeness, we now provide a stability result (see Zhang and Nagurney [35]). First we recall the following:

Definition 2: Stability of the System

The system defined by (24) is stable if, for every X_0 *and every equilibrium point* X^**, the Euclidean distance* $\|X^* - X_0(t)\|$ *is a monotone nonincreasing function of time* t*.*

We now provide a stability result.

Theorem 10: Stability of the Global Supply Chain Network

Assume the conditions of Theorem 5. Then the dynamical system (24) underlying the global supply chain is stable.

Proof: Under the assumptions of Theorem 5, $F(X)$ is monotone and, hence, the conclusion follows directly from Theorem 4.1 of Zhang and Nagurney [35]. □

In the next Section, we propose a discrete-time algorithm, the Euler method, which will track the dynamic trajectory until a stationary state; equivalently, an equilibrium point is attained satisfying Definition 1.

5. The Algorithm

In this Section, we consider the computation of solutions to variational inequality (16); equivalently, the stationary points of (24). The algorithm that we propose is the Euler-type method, which is induced by the general iterative scheme of Dupuis and Nagurney [31]. The realization of the Euler method for this model (for further details see also Nagurney and Dong [15]) is as follows, where $\mathcal{T}$ denotes an iteration counter:

The Euler Method

Step 0: Initialization

Set $(Q^{10}, Q^{20}, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T} = 1$ and set the sequence $\{\alpha_\mathcal{T}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_\mathcal{T}$, $\alpha_\mathcal{T} > 0$, $\alpha_\mathcal{T} \to 0$, as $\mathcal{T} \to \infty$.

Step 1: Computation

Compute $(Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, \gamma^\mathcal{T}, \rho_3^\mathcal{T}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[q_{jh}^{il\mathcal{T}} + \alpha_\mathcal{T}\left(\frac{\partial f^{il}(Q^{1\mathcal{T}-1})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il\mathcal{T}-1})}{\partial q_{jh}^{il}} + \frac{\partial c_j(Q^{1\mathcal{T}-1})}{\partial q_{jh}^{il}} + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il\mathcal{T}-1})}{\partial q_{jh}^{il}}\right.\right.$$
$$\left.\left.-\gamma_j^{\mathcal{T}-1}\right) - q_{jh}^{il\mathcal{T}-1}\right] \times \left[q_{jh}^{il} - q_{jh}^{il\mathcal{T}}\right]$$
$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[q_{khl}^{j\mathcal{T}} + \alpha_\mathcal{T}\left(\frac{\partial c_{khl}^{j}(q_{khl}^{j\mathcal{T}-1})}{\partial q_{khl}^{j}} + \gamma_j^{\mathcal{T}-1} + \hat{c}_{khl}^{j}(Q^{2\mathcal{T}-1}) - \rho_{3khl}^{\mathcal{T}-1}\right) - q_{khl}^{j\mathcal{T}-1}\right] \times \left[q_{khl}^{j} - q_{khl}^{j\mathcal{T}}\right]$$
$$+\sum_{j=1}^{J}\left[\gamma_j^\mathcal{T} + \alpha_\mathcal{T}\left(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q_{jh}^{il\mathcal{T}-1} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{khl}^{j\mathcal{T}-1}\right) - \gamma_j^{\mathcal{T}-1}\right] \times \left[\gamma_j - \gamma_j^\mathcal{T}\right]$$
$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\rho_{3khl}^\mathcal{T} + \alpha_\mathcal{T}\left(\sum_{j=1}^{J} q_{khl}^{j\mathcal{T}-1} - d_{khl}(\rho_3^{\mathcal{T}-1})\right) - \rho_{3khl}^{\mathcal{T}-1}\right] \times \left[\rho_{3khl} - \rho_{3khl}^\mathcal{T}\right] \geq 0,$$
$$\forall (Q^1, Q^2, \gamma, \rho_3) \in \mathcal{K}. \tag{34}$$

Step 2: Convergence Verification

If $|q_{jh}^{il\mathcal{T}} - q_{jh}^{il\mathcal{T}-1}| \leq \epsilon$, $|q_{khl}^{j\mathcal{T}} - q_{khl}^{j\mathcal{T}-1}| \leq \epsilon$, $|\gamma_j^\mathcal{T} - \gamma_j^{\mathcal{T}-1}| \leq \epsilon$, $|\rho_{3khl}^\mathcal{T} - \rho_{3khl}^{\mathcal{T}-1}| \leq \epsilon$, for all $i = 1, \cdots, I$; $l = 1, \ldots, L$; $j = 1, \cdots, J$; $h = 1, \ldots, H$; $k = 1, \cdots, K$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Variational inequality subproblem (34) can be solved explicitly and in closed form. For completeness, and also to illustrate the simplicity of the proposed computational procedure in the context of the global supply chain network model, we provide the explicit formulae for the computation of the $Q^{1\mathcal{T}}$, the $Q^{2\mathcal{T}}$, the $\gamma^{\mathcal{T}}$, and the $\rho_3^{\mathcal{T}}$ (cf. (34)) below.

Computation of the Products Flows

In particular, compute, at iteration $\mathcal{T}$, the $q_{jh}^{il\mathcal{T}}$ according to:

$$q_{jh}^{il\mathcal{T}} = \max\{0, q_{jh}^{il\mathcal{T}-1} - \alpha_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il\mathcal{T}-1})}{\partial q_{jh}^{il}} + \frac{\partial c_j(Q^{1\mathcal{T}-1})}{\partial q_{jh}^{il}} + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il\mathcal{T}-1})}{\partial q_{jh}^{il}} - \gamma_j^{\mathcal{T}-1})$$

$$\forall i, l, j, h, \tag{35}$$

and the $q_{khl}^{j\mathcal{T}}$s, according to:

$$q_{khl}^{j\mathcal{T}} = \max\{0, q_{khl}^{j\mathcal{T}-1} - \alpha(\frac{\partial c_{khl}^{j}(q_{khl}^{j\mathcal{T}-1})}{\partial q_{khl}^{j}} + \gamma_j^{\mathcal{T}-1} + \hat{c}_{khl}^{j}(Q^{2\mathcal{T}-1}) - \rho_{3khl}^{\mathcal{T}-1})\}, \forall j, k, h, l. \tag{36}$$

Computation of the Prices

At iteration $\mathcal{T}$, compute the $\gamma_j^{\mathcal{T}}$s according to:

$$\gamma_j^{\mathcal{T}} = \max\{0, \gamma_j^{\mathcal{T}-1} - \alpha_{\mathcal{T}}(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q_{jh}^{il\mathcal{T}-1} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{khl}^{j\mathcal{T}-1})\}, \quad \forall j, \tag{37}$$

whereas the $\rho_{3khl}^{\mathcal{T}}$s are computed explicitly and in closed form according to:

$$\rho_{3khl}^{\mathcal{T}} = \max\{0, \rho_{3khl}^{\mathcal{T}-1} - \alpha_{\mathcal{T}}(\sum_{j=1}^{J} q_{khl}^{j\mathcal{T}-1} - d_{khl}(\rho_3^{\mathcal{T}-1}))\}, \quad \forall k, h, l. \tag{38}$$

Hence, at a given iteration, all the product shipments and the prices can be solved explicitly and in closed form using the above simple formulae. Note that these computations can be done simultaneously, that is, in parallel. The algorithm also can be interpreted as a discrete-time adjustment process in which the product shipments between tiers are adjusted as well as the prices at the tiers until the equilibrium state is reached. Convergence conditions for the algorithm can be found in [31] and [32].

In the next Section, we apply this algorithm to solve several global supply chain network examples.

6. Numerical Examples

In this Section, we apply the Euler method described in the preceding section to several numerical examples. The algorithm was implemented in FORTRAN and the computer system used was a DEC Alpha system located at the University of Massachusetts at Amherst.

The convergence criterion used was that the absolute value of the product flows and prices between two successive iterations differed by no more than 10^{-4}. For the examples, the sequence $\{\alpha_\mathcal{T}\}$ was set to $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$ in the algorithm. We initialized the Euler method as follows: all the initial product flows and prices were set to one.

We solved two sets of numerical examples for a total of 5 examples. Detailed descriptions are given below.

Example 1

The first set of numerical examples consisted of one country, two manufacturers, two currencies, two retailers, and two demand markets for the product. Hence, $L = 1$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$, for this and the subsequent two numerical examples. The global supply chain network for the first three examples is depicted in Figure 2.

The data for the first example were constructed for easy interpretation purposes. The transaction cost functions faced by the manufacturers associated with transacting with the retailers (cf. (1)) were given by:

$$c^{il}_{jh}(q^{il}_{jh}) = .5(q^{il}_{jh})^2 + 3.5q^{il}_{jh}, \quad \text{for } i = 1, 2; l = 1; j = 1, 2; h = 1, 2.$$

The production cost functions were

$$f^{il}(Q^1) = (\sum_{j=1}^{2} \sum_{h=1}^{2} q^{il}_{jh})^2, \quad \text{for } i = 1, 2; l = 1.$$

The handling costs of the retailers (see (5)) were given by:

$$c_j(Q^1) = .5(\sum_{i=1}^{2} \sum_{h=1}^{2} q^{i1}_{jh})^2, \quad \text{for } j = 1, 2.$$

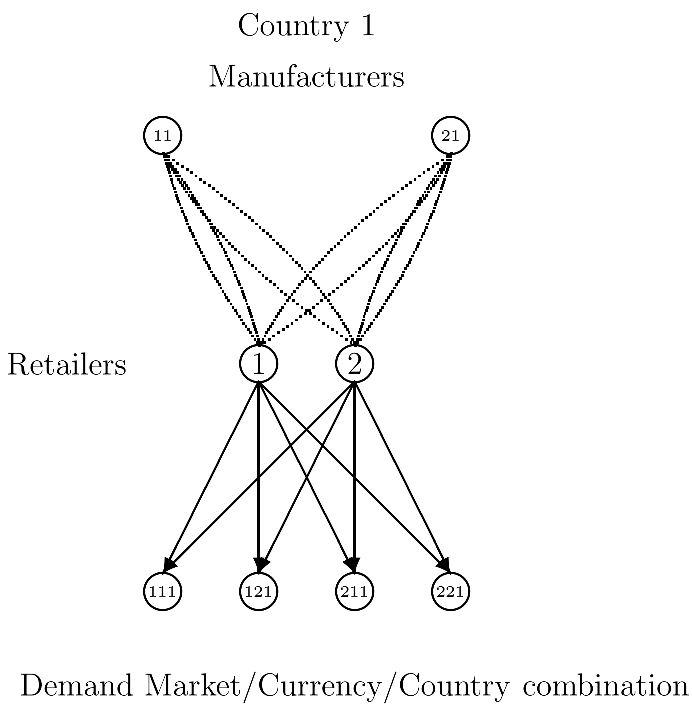

Figure 2: Global Supply Chain Network for the First Set of Numerical Examples

The transaction costs of the retailers associated with transacting with the source manufacturers were (cf. (6)) given by:

$$\hat{c}^{il}_{jh}(q^{il}_{jh}) = 1.5{q^{il}_{jh}}^2 + 3q^{il}_{jh}, \quad \text{for } i = 1,2; l = 1; j = 1,2; h = 1,2.$$

The demand functions at the demand markets (cf. (12)) were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000.$$

and the transaction costs between the retailers and the consumers at the demand markets (see (11)) were given by:

$$\hat{c}^{j}_{khl}(Q^2) = q^{j}_{khl} + 5, \quad \text{for } j = 1,2; k = 1,2; h = 1,2; l = 1.$$

We assumed for this and the subsequent examples that the transaction costs as perceived by the retailers and associated with transacting with the demand markets were all zero, that is, $c^j_{khl}(q^j_{khl}) = 0$, for all j, k, h, l.

The Euler method converged in 196 iterations and yielded the following equilibrium product shipment pattern:

$$q^{il*}_{jh} = 15.605, \quad \forall i, l, j, h, \quad q^{j*}_{khl} = 15.605, \quad \forall j, k, h, l.$$

The vector γ^* had components: $\gamma^*_1 = \gamma^*_2 = 256.190$, and the computed demand prices at the demand markets were: $\rho^*_{3111} = \rho^*_{3121} = \rho^*_{3211} = \rho^*_{3221} = 276.797$.

We also, for completeness, recover the equilibrium prices associated with the source agents according to the discussion following (19). In particular, $\rho^{il*}_{1jh} + e^*_h = 143.95$, for all i, l, j, h.

It is easy to verify that the optimality/equilibrium conditions were satisfied with good accuracy.

Example 2

In the second example, we kept the data as in Example 1 except for the following changes: we increased the demand associated with the product at the first demand market in the first currency so that the new demand functions were now given by:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1010,$$

with the demand functions for the second demand market (and second currency) remaining unchanged.

The Euler method converged in 198 iterations and yielded the following equilibrium product shipment pattern:

$$q_{jh}^{il*} = 15.643, \quad \forall i, l, j, h,$$

$$q_{111}^{1*} = 18.100,\ q_{121}^{1*} = 14.100,\ q_{211}^{1*} = 15.191,\ q_{221}^{1*} = 15.191,$$

$$q_{111}^{2*} = 18.100,\ q_{121}^{2*} = 14.100,\ q_{211}^{2*} = 15.191,\ q_{221}^{2*} = 15.191.$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 256.840$, and the computed demand prices at the demand markets were: $\rho_{3111}^* = 279.942, \rho_{3121}^* = 275.943, \rho_{3211}^* = 277.033, \rho_{3221}^* = 277.033.$

Clearly, since the demand for the product in the first demand market in the country and currency increased, the product shipments between the two retailers and that demand market increased (as did the equilibrium price at that demand market).

We also computed (as discussed in Example 1) the new equilibrium prices associated with the top tier of nodes in the supply chain network which were now given by $\rho_{1jh}^{il*} + e_h^* = 144.316$.

Example 3

In the third and final example in this set, we kept the data as in Example 2 except for the following changes: we increased the production cost functions of each manufacturer by a factor of two.

The Euler method now required 219 iterations for convergence and yielded the following new equilibrium product shipment pattern:

$$q_{jh}^{il*} = 10.751, \quad \forall i, l, j, h,$$

$$q_{111}^{1*} = 13.206,\ q_{121}^{1*} = 9.206,\ q_{211}^{1*} = 10.297,\ q_{221}^{1*} = 10.297,$$

$$q_{111}^{2*} = 13.206,\ q_{121}^{2*} = 9.206,\ q_{211}^{2*} = 10.297,\ q_{221}^{2*} = 10.297.$$

The vector γ^* now had components $\gamma_1^* = \gamma_2^* = 264.534$. The demand market prices, in turn, were now given by: $\rho_{3111}^* = 282.738$, $\rho_{3121}^* = 278.739$, $\rho_{3211}^* = 279.830$, $\rho_{3221}^* = 279.830$. The prices at the top tier of nodes were now: $\rho_{1jh}^{il*} + e_h^* = 186.274$. Hence, when the production costs associated with manufacturing the product increased, the quantities produced and shipped by both the manufacturers in the country decreased. The prices of the product increased at all the demand markets since the product was scarcer.

Example 4

In the second set of numerical examples, the global supply chain network was as given in Figure 3. These two examples consisted of two countries with two manufacturers in each country; two currencies, two retailers, and two demand markets. Hence, $L = 2$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$.

The data for the first example in this set was constructed for easy interpretation purposes and to create a baseline from which the simulations could be conducted. In fact, we essentially "replicated" the data for the first country as it appeared in Example 1 in order to construct the data for the second country.

The transaction cost functions faced by the manufacturers associated with transacting with the retailers were given by:

$$c_{jh}^{il}(q_{jh}^{il}) = .5(q_{jh}^{il})^2 + 3.5q_{jh}^{il}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2.$$

The production cost functions were the same for all the manufacturers and were given as described in Example 1.

The handling costs of the retailers (since the number of retailers in this set is still equal to two) remained as in Example 1, that is, they were given by:

$$c_j(Q^1) = .5(\sum_{i=1}^{2}\sum_{h=1}^{2} q_{jh}^{i1})^2, \quad \text{for } j = 1,2.$$

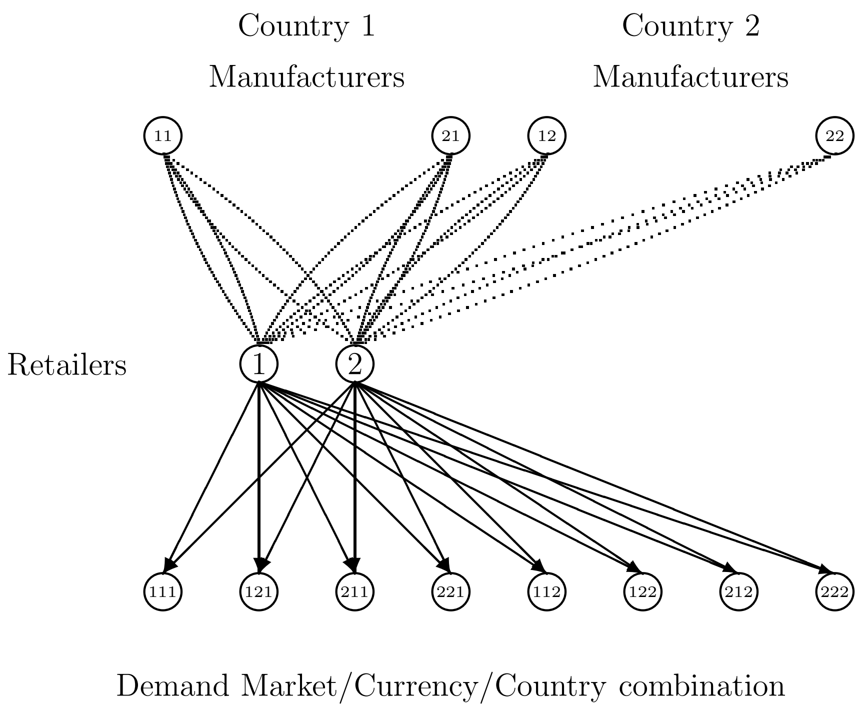

Figure 3: Global Supply Chain Network for the Second Set of Numerical Examples

The transaction costs of the retailers associated with transacting with the manufacturers in the two countries were given by:

$$\hat{c}^{il}_{jh}(q^{il}_{jh}) = 1.5{q^{il}_{jh}}^2 + 3q^{il}_{jh}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2.$$

The demand functions at the demand markets were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000,$$

$$d_{112}(\rho_3) = -2\rho_{3112} - 1.5\rho_{3122} + 1000, \quad d_{122}(\rho_3) = -2\rho_{3122} - 1.5\rho_{3112} + 1000,$$

$$d_{212}(\rho_3) = -2\rho_{3212} - 1.5\rho_{3222} + 1000, \quad d_{222}(\rho_3) = -2\rho_{3222} - 1.5\rho_{3212} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}^{j}_{khl}(q^{j}_{khl}) = q^{j}_{khl} + 5, \quad \text{for } j = 1,2; k = 1,2; l = 1,2.$$

The Euler method converged in 318 iterations and yielded the following equilibrium product shipment pattern:

$$q^{il*}_{jh} = 12.712, \quad \forall i, l, j, h, \quad q^{j*}_{khl} = 12.712, \quad \forall j, k, h, l.$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 260.739$, and the computed demand prices at the demand markets were: $\rho^*_{3111} = \rho^*_{3121} = \rho^*_{3211} = \rho^*_{3221}$ $\rho^*_{3112} = \rho^*_{3122} = \rho^*_{3212} = \rho^*_{3222} =$ 278.450. The top-tiered prices $\rho^{il*}_{jh} + e^*_h = 117.908$, for all i, l, j, h.

Example 5

Example 5 was constructed from the preceding example as follows. We kept the data as in Example 4 except that we changed the demand functions associated with the second country so that:

$$d_{112}(\rho_3) = -2\rho_{3112} - 1.5\rho_{3122} + 1010, \quad d_{122}(\rho_3) = -2\rho_{3122} - 1.5\rho_{3112} + 1020,$$

$$d_{212}(\rho_3) = -2\rho_{3212} - 1.5\rho_{3222} + 1030, \quad d_{222}(\rho_3) = -2\rho_{3222} - 1.5\rho_{3212} + 1040.$$

The Euler method again converged in 318 iterations and yielded the following equilibrium product shipment pattern:

$$q_{jh}^{il*} = 12.877, \quad \forall i, l, j, h,$$

$$q_{111}^{1*} = 10.605,\ q_{121}^{1*} = 10.605,\ q_{211}^{1*} = 10.605,\ q_{221}^{1*} = 10.605,$$

$$q_{112}^{1*} = 11.332\, q_{122}^{1*} = 15.332,\ q_{212}^{1*} = 14.968,\ q_{222}^{1*} = 18.968,$$

$$q_{111}^{2*} = 10.605\, q_{121}^{2*} = 10.605,\ q_{211}^{2*} = 10.605,\ q_{221}^{2*} = 10.605,$$

$$q_{112}^{2*} = 11.323,_{122}^{2*} = 15.332,\ q_{212}^{2*} = 14.968,\ q_{222}^{2*} = 18.968.$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 264.051$, and the computed demand prices at the demand markets were: $\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = 279.654$ $\rho_{3112}^* = 280.381, \rho_{3122}^* = 284.381, \rho_{3212}^* = 284.017, \rho_{3222}^* = 284.017$.

The $\rho_{1jh}^{il*} + e_h^*$ were identically equal to 119.198 for all i, l, j, h.

These examples (although stylized) have been presented to show both the model and the computational procedure. Obviously, different input data and dimensions of the problems solved will affect the equilibrium product shipment and price patterns. One now has a powerful tool with which to explore the effects of perturbations to the data as well as the effects of changes in the number of manufacturers, retailers countries, currencies, and/or demand markets.

6. Summary and Conclusions

In this paper, we have developed a framework for the modelling, analysis, and computation of solutions to global supply chain supernetworks, consisting of multiple tiers of decision-makers, who compete within a tier but cooperate between tiers. We modeled the behavior of the decision-makers, consisting of manufacturers in different countries, retailers, as well as consumers at different demand markets and countries who can purchase the product in different currencies. We then derived the optimality conditions as well as the governing equilibrium conditions which were then formulated as a variational inequality problem.

The framework allows for the handling of as many countries, as many manufacturers in each country, as many currencies in which the products can be obtained, and as many retailers, as mandated by the specific application. Furthermore, the generality of the transaction costs and the supernetwork framework allows for the capture of electronic commerce in that the retailers need not be country specific and can transact either virtually or physically with both the manufacturers and the consumers. The variational inequality problem was then utilized to obtain qualitative properties of the equilibrium product shipment and price pattern.

In addition, we proposed a dynamic adjustment process by which the product shipments between tiers of the supernetwork can adjust as well as the prices associated with the tiers and showed that it can be formulated as a projected dynamical system. We also established, under reasonable conditions, stability of the global supply chain network system and then applied a discrete-time algorithm to compute the solutions to several numerical examples.

This framework generalizes the recent work of Nagurney et al. [16, 22] in supply chain network modelling and analysis to the global dimension.

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