Supply Chain Network Design of a Sustainable Blood Banking System

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In: *Sustainable Supply Chains: Models, Methods and Public Policy Implications*
T. Boone, V. Jayaraman, and R. Ganeshan, Editors, Springer, London, England, 2012, pp 49-72.

Abstract In this paper, we develop a sustainable network design / redesign model for the complex supply chain of human blood, which is a valuable yet highly perishable product. Specifically, we consider the optimal design (or redesign) of a blood banking system consisting of collection sites, blood centers, testing and processing labs, storage facilities, distribution centers as well as demand points. Our multicriteria system-optimization approach on networks with arc multipliers captures several critical concerns associated with blood banking systems including but not limited to the determination of the optimal capacities and the optimal allocations, the induced supply-side risk, and the induced cost of discarding potentially hazardous blood waste, while the uncertain demand for blood is satisfied as closely as possible.

1 Introduction

Medical waste, also known as *clinical waste*, refers to the waste products that can not be considered as general waste, and that is produced, typically, at health care premises, including hospitals, clinics, and labs. Due to the potentially hazardous nature of medical waste, both the American Dental Association (ADA) and the Centers for Disease Control (CDC) recommend that medical waste be removed in accordance with regulations (Pasupathi et al (2011)).

It is interesting to note that the health care facilities in the United States are second only to the food industry in producing waste, generating more than 6,600 tons per day, and more than 4 billion pounds annually (Fox News (2011)). In addition, according to USA Today (2008), considerable amounts of drugs have been found in 41 million Americans' drinking water due to the improper disposal of unused

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or expired drugs placed in domestic trash or discarded in the waste water. In other countries, up to 4 pounds of waste per hospital bed per day is produced, out of which 0.5 percent might be categorized as risky / potentially hazardous waste (The News International (2011)).

Disposal of medical waste is not only costly to the health care industry, but also may harm the environment. Consequently, poor management of such waste may lead to the contamination of water, the soil, and the atmosphere. While many hospitals choose to have their waste burned so as to avoid polluting the soil through landfills, the incinerators themselves are one of the nation's leading sources of toxic pollutants such as dioxins and mercury (Giusti (2009) and Association of Bay Area Governments (2003)). Thus, minimizing the amount of medical waste throughout the health care supply chains will lead to a cleaner environment, which may, in turn, also reduce illnesses and death.

When it comes to blood supply chains, the scarcity and vitalness of this highly perishable health care product make such supply chains crucial. Hence, the effective design and control of such systems can support the health and well-being of populations and can also positively affect the sustainability of the environment by reducing the associated waste. Indeed, since blood waste is a significant hazard to the environment, a major step in attaining a sustainable blood supply chain is to be able to minimize the outdating of blood products while satisfying the demand.

In this paper, we develop a multicriteria *system-optimization* framework for the supply chain network design of a sustainable blood banking system. The framework allows for the simultaneous determination of optimal link capacities through investments, and the flows on various links, which correspond to such application-based supply chain network activities as: blood collection, the shipment of collected blood, its testing and processing, its storage, its shipment to distribution centers, and, finally, to the points of demand. The system-optimization approach is believed to be mandated for critical supplies (Nagurney, Yu, and Qiang (2011)) in that the demand for such products must be satisfied as closely as possible at minimal total cost. The use of a profit maximization criterion, as in Nagurney (2010a), is not appropriate for an organization such as, for example, the American Red Cross, due to its non-profit status.

In particular, the sustainable supply chain network design model for blood banking developed here is novel for several reasons: 1. it captures the perishability of the product through the use of arc multipliers; 2. it handles the costs associated with the discarding of the medical waste, which could be hazardous, 3. it captures the uncertainty associated with the demand for the product along with the risk associated with procurement of the product, and 4. it allows for total cost minimization and the total risk minimization associated with the design and operation of the blood banking supply chain network.

Our framework is a contribution to the growing literature on sustainable supply chains and to the design of sustainable supply chains, in particular (cf. Nagurney and Nagurney (2010) and the references therein). However, our supply chain network design model for sustainable blood systems focuses not on the minimization of emissions but rather on the minimization of waste. Moreover, it captures the perishability of this product.

Recently, several authors have applied derivations of integer optimization models such as facility location, set covering, allocation, and routing to address the optimization / design of supply chains of blood or other perishable critical products (see Pierskalla (2005), Yang (2006), Sahin, Sural, and Meral (2007), Sivakumar, Ganesh, and Parthiban (2008), Cetin and Sarul (2009), and Ghandforoush and Sen (2010)). Furthermore, inventory management methods (for instance, see Karaesmen, Scheller-Wolf, and Deniz (2011)), Markov models (Boppana and Chalasani (2007)) as well as simulation techniques (Rytila and Spens (2006), and Mustafee, Katsaliaki, and Brailsford (2009)) have also been utilized to handle blood banking systems. Our multicriteria system-optimization approach is quite general and takes into account such critical issues as the determination of optimal capacities and allocations, the induced supply-side risk, uncertain demand, as well as the induced cost of discarding the waste. Furthermore, our mathematical model can be efficiently solved.

The paper is organized as follows. In Section 2, we develop the supply chain network design model for a sustainable blood banking system that allows for the design of such a network from scratch or the redesign of an existing network. Interestingly, the total number of Red Cross testing laboratories in the United States has decreased from 7 to 5 over the past few years, mainly due to the economic situation (Rios (2010)). Although the closure of the testing facilities has reduced the overall costs of the American Red Cross, it has increased the transportation costs corresponding to the blood service divisions that are far from the testing labs. Our model enables the reevaluation of such modifications to a blood supply chain network system. We emphasize that Nagurney, Masoumi, and Yu (2010) proposed an operations management model for blood supply chain networks but did not focus on the more challenging aspect of the design (and redesign) of such supply chains. The notation of our model is based on that model but here we make the crucial extension of including link capacities as decision variables. The formulation and analysis of the model are done through the theory of variational inequalities (see Nagurney (1999)), since this enables the creation of a foundation in which other models, including decentralized ones, can then be constructed.

In Section 3, we propose an algorithmic scheme that yields closed form expressions at each iteration in terms of the product path flows, the link capacities, and the associated Lagrange multipliers. We then apply the algorithm to a spectrum of numerical examples, which illustrate the generality and applicability of our methodological and computational framework. We provide a summary and our conclusions in Section 4.

2 The Sustainable Blood Banking System Supply Chain Network Design Model

In this section, we develop the supply chain network design model for a blood banking system. It is important to mention, at the outset, that our sustainable supply chain network design model is applicable to many perishable products, notably, those associated with health care, as in the case of medicines and vaccines, with minor modifications. For continuity purposes, the notation for our model follows closely that of Nagurney, Masoumi, and Yu (2010).

For definiteness, please refer to Figure 1. Figure 1 depicts a possible network topology of a blood banking system. In this network, the top level (origin) node represents the organization. Every other node in the network denotes a component/facility in the system. A path connecting the origin node to a destination node, corresponding to a demand point, consists of a sequence of directed links which correspond to supply chain network activities that ensure that the blood is collected, processed, and, ultimately, distributed to the demand point. We assume that, in the initial supply chain network topology, as in Figure 1, which serves as a template upon which the optimal supply chain network design is constructed, there exists at least one path joining node 1 with each destination node. This assumption guarantees that the demand at each demand point will be met as closely as possible, given that we will be considering uncertain demand for blood at each demand point. The solution of the model yields the optimal investments associated with the various links as well as the optimal flows, at minimum total cost and risk, as we shall demonstrate and, hence, the optimal sustainable supply chain network design. Our model is sufficiently flexible in that it is capable of handling either the design of the sustainable network from scratch or the redesign of an existing blood banking supply chain network since certain existing link capacities can be either enhanced or reduced.

In the network in Figure 1, we assume that the organization is considering n_C possible blood collection sites constituting the second tier of the network. Many of these collection sites are mobile or temporary locations while others are permanent sites. In the case of drastic shortages; i.e., natural or man-made disasters, the cognizant organizations are likely to need to import blood products from other regions or even other countries, an aspect that is excluded from this model. The first set of links, connecting the origin node to the second tier, corresponds to the process of blood collection where these collection sites are denoted by: $C_1, C_2, \dots, C_{n_C}$.

The next set of nodes, located in the third tier, consists of the blood centers. There exist, potentially, n_B of these facilities, denoted, respectively, by $B_1, B_2, \dots, B_{n_B}$, to which the whole blood (WB) is shipped after being collected at the collection sites. Thus, the next set of links connecting tiers two and three of the network topology represents the shipment of the collected blood.

The fourth tier of the network is composed of the processing facilities, commonly referred to as the component labs. The number of these potential facilities is given by n_P. These facilities are denoted by $P_1, \dots, P_{n_P}$, respectively, and are typically

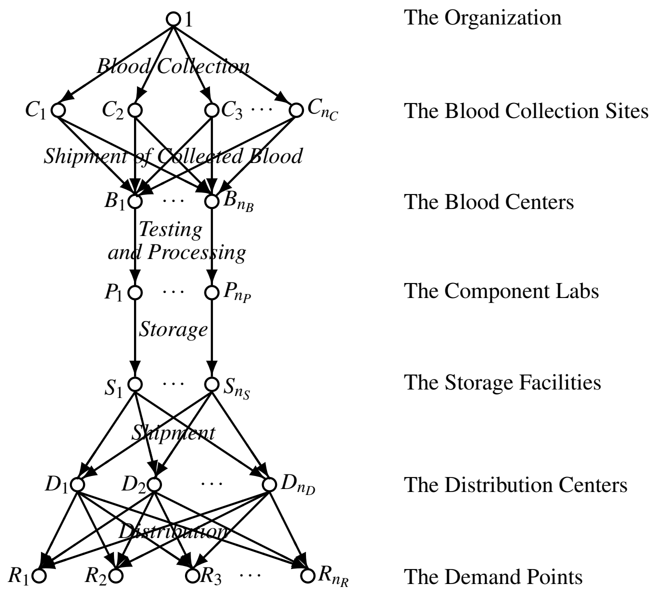

Fig. 1 Initial Supply Chain Network Topology for a Blood Banking System

located within the blood center locations. As discussed in Nagurney, Masoumi, and Yu (2010), at these labs, the collected blood is usually separated into parts, i.e., red blood cells and plasma, since most recipients need only a specific component for transfusions. Every unit of donated whole blood - 450 to 500 milliliters on average - can provide one unit of red blood cells (RBC) and one unit of plasma. In our formulation, what we refer to as the flow of product is, actually, the amount of whole blood (WB) on the first three sets of links. The flow on the links, henceforth, denotes the number of units of red blood cells (RBC) processed at the component labs, which are, ultimately, delivered to the demand points.

The safety of the blood supply is a vital issue for blood service organizations. For example, presently, only 5 testing labs are operating across the United States, and these labs are shared among 36 blood regions. Only a small sample of every donated blood unit is sent to the testing labs, overnight, and these samples are discarded regardless of the results of the tests. Due to the high perishability of many of the blood products, the two processes of testing and separating take place concurrently yet sometimes hundreds of miles away. If the result of a test for a specific unit of donated blood at the testing lab turns out to be positive, the remainder of that unit will be, subsequently, discarded at the corresponding storage facility. In our model, the set of the links connecting the component labs to the storage facilities corresponds to testing and processing, and the costs on these links represent the operational costs of testing and processing combined. The fraction of the flow lost during or, as a result of the testing process, is also included in our model.

The fifth set of nodes denotes the short-term storage facilities. There are, potentially, n_S of such nodes in the network, denoted by $S_1, S_2, \ldots, S_{n_S}$, which are usually located in the same place as the component labs. The links connecting the upper level nodes to the storage facilities denote the procedure of "storage" of the tested and processed blood before it is shipped to be distributed.

The next set of nodes in the network represents the distribution centers, denoted by $D_1, D_2, \ldots, D_{n_D}$, where n_D is the total number of potential such facilities. Distribution centers act as transshipment nodes, and are in charge of facilitating the distribution of blood to the ultimate destinations. The links connecting the storage tier to the distribution centers are of shipment link type.

Finally, the last set of links joining the bottom two tiers of the network are distribution links, and they terminate in n_R demand points. The demands at the demand points $R_1, R_2, \ldots, R_{n_R}$ are denoted by: $d_{R_1}, d_{R_2}, \ldots, d_{R_{n_R}}$, respectively, and the demands are uncertain. Note that, in our design model, the top-tiered node always exists since it represents the organization. Similarly, the bottom-tiered nodes, which correspond to the demand points (such as hospitals and surgical medical centers) also always exist. The solution of our model determines if any of the links should be removed since the optimal solution will yield zero capacities for such links or whether the capacities on links should be increased.

Specific components of the system may physically coincide with some others. Our network model and, hence, the corresponding topology, is process-based rather than location-based, which is compatible with our blood banking problem. Moreover, as mentioned earlier, in general cases of perishable product supply chains, these facilities may be located far apart which can be nicely addressed via our framework.

The possible supply chain network topology, as depicted in Figure 1, is represented by $G = [N, L]$, where N and L denote the sets of nodes and links, respectively. The ultimate solution of the complete model will yield the the optimal capacity modifications on the various links of the network as well as the optimal flows.

The formalism that we utilize is that of multicriteria system-optimization, where the organization wishes to determine at what level the blood collection sites should operate; the same for the blood centers, the component labs, the storage facilities, and the distribution centers. Furthermore, the organization seeks to minimize the total supply-side risk as well as the total costs associated with its blood collection, shipment, processing, storage, and distribution activities, along with the total investment corresponding to the enhancement of link capacities (or their construction from scratch), or the total induced cost of reducing link capacities, as well as the total cost of discarding the waste/perished product over the links. The demands must be satisfied as closely as possible with associated shortage penalties if the demands are not met, in addition to the outdating (surplus) penalties in the case that the organization delivers excess supply to the demand points.

With each link of the network, we associate a unit operational cost function that reflects the cost of operating the particular supply chain activity, that is, the collection of blood at blood drive sites, the shipment of collected blood, the testing and processing, the storage, and the distribution. These links are denoted by a, b, etc. The unit operational cost on link a is denoted by c_a and is a function of flow on that link, f_a. The *total* operational cost on link a is denoted by $\hat{c}_a$, and is constructed as:

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L. \tag{1}$$

The link total cost functions are assumed to be convex and continuously differentiable.

Let w_k denote the pair of origin/destination (O/D) nodes (1, R_k) and let $\mathscr{P}_{w_k}$ denote the set of paths, which represent alternative associated possible supply chain network processes, joining (1, R_k). $\mathscr{P}$ denotes the set of all paths joining node 1 to the destination nodes, and n_p denotes the number of paths.

Let v_k denote the *projected demand* for blood at the demand point $R_k; k = 1, \ldots, n_R$. We assume that the demand at each demand point is uncertain with a known probability distribution. Recall that d_k denotes the actual demand at demand point $R_k; k = 1, \ldots, n_R$, and is a random variable with probability density function given by $\mathscr{F}_k(t)$. Let P_k be the probability distribution function of d_k, that is, $P_k(D_k) = Prob(d_k \leq D_k) = \int_0^{D_k} \mathscr{F}_k(t)d(t)$. Therefore,

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \tag{2}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R, \tag{3}$$

where Δ_k^- and Δ_k^+ represent the shortage and surplus of blood at demand point R_k, respectively.

The expected values of the shortage (Δ_k^-) and the surplus (Δ_k^+) are given by:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathscr{F}_k(t)d(t), \qquad k = 1, \ldots, n_R, \tag{4}$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - t)\mathscr{F}_k(t)d(t), \qquad k = 1, \ldots, n_R. \tag{5}$$

As in Nagurney, Masoumi, and Yu (2010), we associate a relatively large penalty of λ_k^- with the shortage of a unit of blood at demand point R_k, where λ_k^- corresponds to the social cost of a death or a severe injury of a patient, due to a blood shortage. Since blood is highly perishable and will be outdated if not used within a certain period after being delivered, the outdating penalty of λ_k^+ is assigned to the unit of a possible supply surplus. This surplus penalty is charged to the organization because human blood is scarce, and the cognizant organization seeks to minimize the amount of outdated blood at demand points, which actually dominates the amount of blood waste during the other activities of blood banking, for example, within the American Red Cross network (Rios (2010)). Hence, in our framework, λ_k^+, in the case of blood (as for other perishable products), includes the cost of short-term inventory holding (cold storage), and the discarding cost of the outdated product. Analogous examples of penalty costs, due to excessive supplies, as well as to shortages, can be found in the literature (see, e.g., Dong, Zhang, and Nagurney (2004), and Nagurney, Yu, and Qiang (2011)). These penalties can be assessed by the authority who is contracting with the organization to deliver the blood.

Thus, the expected total penalty at demand point $k; k = 1, \ldots, n_R$, is:

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+). \tag{6}$$

Also, as in Nagurney, Masoumi, and Yu (2010), we associate with every link a in the network, a multiplier α_a, which corresponds to the percentage of loss over that link. This multiplier lies in the range (0,1], for the network activities, where α_a means that $\alpha_a \times 100\%$ of the initial flow on link a reaches the successor node of that link, with $\alpha_a = 1$, hence, reflecting that there is no waste/loss on link a. For example, the average percentage of loss due to the testing process was reported to be 1.7% (Sullivan et al. (2007)); consequently, the corresponding multiplier, α_a, would be equal to $1 - 0.017 = 0.983$.

As mentioned earlier, f_a denotes the (initial) flow on link a. Let f'_a denote the final flow on that link; i.e., the flow that reaches the successor node of the link. Therefore,

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{7}$$

The waste/loss on link a, denoted by w_a, is equal to:

$$w_a = f_a - f'_a = (1 - \alpha_a) f_a, \qquad \forall a \in L. \tag{8}$$

The organization is also responsible for disposing this waste which is potentially hazardous. Contractors are typically utilized to remove and dispose of the waste. The corresponding discarding cost, y_a, is a function of the waste, w_a, which is charged to the organization:

$$y_a(w_a) = y_a(f_a - f'_a) = y_a\big((1 - \alpha_a) f_a\big), \qquad \forall a \in L. \tag{9}$$

Since α_a is constant, and known apriori, a new total discarding cost function, $\hat{z}_a$, can be defined accordingly, which is a function of the flow, f_a, and is assumed to be convex and continuously differentiable:

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L. \tag{10}$$

Let x_p represent the (initial) flow of blood (or a general perishable product) on path p joining the origin node with a destination node. The path flows must be nonnegative, that is,

$$x_p \geq 0, \qquad \forall p \in \mathscr{P}, \tag{11}$$

since the product will be collected, shipped, etc., in nonnegative quantities.

Let μ_p denote the multiplier corresponding to the loss on path p, which is defined as the product of all link multipliers on links comprising that path, that is,

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in \mathscr{P}. \tag{12}$$

The projected demand at demand point R_k, v_k, is the sum of all the final flows on paths joining $(1, R_k)$:

$$v_k \equiv \sum_{p \in \mathscr{P}_{w_k}} x_p \mu_p, \qquad k = 1, \ldots, n_R. \tag{13}$$

Indeed, although the amount of blood that originates on a path p is x_p, the amount (due to perishability) that actually arrives at the destination of this path is $x_p \mu_p$.

As discussed in Nagurney, Masoumi, and Yu (2010), the multiplier, α_{ap}, is the product of the multipliers of the links on path p that precede link a in that path. This multiplier can be expressed as:

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a' < a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \tag{14}$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p. Recall that δ_{ap} is defined as equal to 1 if link a is contained in path p; otherwise, it is equal to zero, and Ø denotes the null set. In other words, α_{ap} is equal to the product of all link multipliers preceding link a in path p. If link a is not contained in path p, then α_{ap} is set to zero. If a belongs to the first set of links, the blood collection links, this multiplier is equal to 1. The relationship between the link flow, f_a, and the path flows is as follows:

$$f_a = \sum_{p \in \mathscr{P}} x_p \, \alpha_{ap}, \qquad \forall a \in L. \tag{15}$$

The blood supply chain organization not only wishes to determine which facilities should operate and at what level, but also is interested in possibly redesigning the existing capacities with the demand being satisfied as closely as possible, and the total cost and risk being minimized. Let $\bar{u}_a$ denote the nonnegative existing capacity on link $a, \forall a \in L$. The organization can enhance/reduce the capacity of link a by $u_a, \forall a \in L$. The total investment cost of adding capacity u_a on link a, or contrarily, the induced cost of lowering the capacity by u_a, is denoted by $\hat{\pi}_a$, and is a function of the change in capacity:

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \qquad \forall a \in L. \tag{16}$$

The total capacity investment cost functions can be interpreted as the cost of purchasing/renting additional equipments, hiring extra staff personnel, and expanding the transportation fleet. On the other hand, the total cost corresponding to capacity reduction typically includes the relocation of equipment, the reallocation of personnel as well as the storing of surplus equipment. These functions are also assumed to be convex and continuously differentiable. We group the link capacity changes into the vector u. Similarly, the path flows, the link flows, and the projected demands are grouped into the respective vectors x, f, and v.

The total cost minimization objective faced by the organization includes the total cost of operating the various links, the total discarding cost of waste/loss over the links, the total cost of capacity modification, and the expected total blood supply shortage cost as well as the total discarding cost of outdated blood at the demand points. This optimization problem can be expressed as:

$$\text{Minimize} \quad \sum_{a\in L}\hat{c}_a(f_a)+\sum_{a\in L}\hat{z}_a(f_a)+\sum_{a\in L}\hat{\pi}_a(u_a)+\sum_{k=1}^{n_R}\left(\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+)\right) \tag{17}$$

subject to: constraints (11), (13), and (15), and

$$f_a \leq \bar{u}_a + u_a, \qquad \forall a \in L, \tag{18}$$

$$-\bar{u}_a \leq u_a, \qquad \forall a \in L. \tag{19}$$

Constraint (18) guarantees that the flow on a link cannot exceed the new capacity on that link. Furthermore, the change in link capacities can take on positive/negative values corresponding to the enhancement/reduction of the capacities. Constraint (19) guarantees that the flow on a link will not be negative by imposing a lower bound for this link capacity change (see Nagurney (2010b)).

Observe that if $\bar{u}_a = 0,\ \forall a \in L$, then the redesign model converts to a "design from scratch" model in that there will be no capacities on the link apriori. Both models of redesign and design are consistent with the presented network topology in Figure 1.

As mentioned earlier, the minimization of total costs is not the only objective of suppliers of perishable goods. A major challenge for a blood service organization is to capture the risk associated with different activities in the blood supply chain network. Unlike the demand, which can be projected according to the historical data, albeit with some uncertainty involved, the amount of donated blood at the collection sites has been observed to be highly stochastic. For example, although blood donors may make appointments in advance, donors may miss their appointments due to traffic delays, bad weather, etc.

As in Nagurney, Masoumi, and Yu (2010), we introduce a total risk function $\hat{r}_a$ corresponding to link a for every blood collection link. This function is assumed to be convex and continuously differentiable, and a function of the flow, that is, the amount of collected blood, on its corresponding link. The organization attempts to minimize the total risk over all links connecting the first two tiers of the network, denoted by $L_1 \subset L$. The remainder of the links in the network, i.e., the shipment of collected blood, the processing, the storage, shipment, and the distribution links, comprise the set L_1^C. The subset L_1 and its complement L_1^C partition the entire set of links L, that is, $L_1 \cup L_1^C = L$.

Thus, the risk minimization objective function for the organization can be expressed as:

$$\text{Minimize} \quad \sum_{a \in L_1} \hat{r}_a(f_a), \tag{20}$$

where $\hat{r}_a = \hat{r}_a(f_a)$ is the total risk function on link a.

The sustainable supply chain network design problem for a blood banking system can be expressed as a multicriteria decision-making problem. The organization seeks to determine the optimal levels of blood processed on each supply chain network link coupled with the optimal levels of capacity escalation/reduction in its blood banking supply chain network activities subject to the minimization of the total cost (operational and discarding) as well as the minimization of the total supply risk. The weight associated with the total cost objective, (17), serves as the numeraire, and is set equal to 1. On the other hand, corresponding to the total supply risk objective, (20), a weight of θ is assigned by the decision-maker. Thus, the multicriteria optimization problem is:

$$\begin{aligned} \text{Minimize} \quad & \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{a \in L} \hat{\pi}_a(u_a) \\ & + \sum_{k=1}^{n_R} \left(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) \right) + \theta \sum_{a \in L_1} \hat{r}_a(f_a) \end{aligned} \tag{21}$$

subject to: constraints (11), (13), (15), (18), and (19).

The above optimization problem is in terms of link flows. It can also be expressed in terms of path flows:

$$\begin{aligned} \text{Minimize} \quad & \sum_{p \in \mathscr{P}} \left(\hat{C}_p(x) + \hat{Z}_p(x) \right) + \sum_{a \in L} \hat{\pi}_a(u_a) \\ & + \sum_{k=1}^{n_R} \left(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) \right) + \theta \sum_{p \in \mathscr{P}} \hat{R}_p(x) \end{aligned} \tag{22}$$

subject to: constraints (11), (13), (15), (18), and (19), where the total operational cost function, $\hat{C}_p(x)$, the total discarding cost function, $\hat{Z}_p(x)$, and the total risk function, $\hat{R}_p(x)$, corresponding to path p are, respectively, derived from $C_p(x)$, $Z_p(x)$, and $R_p(x)$ as follows:

$$\begin{aligned} \hat{C}_p(x) &= x_p \times C_p(x), && \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R, \\ \hat{Z}_p(x) &= x_p \times Z_p(x), && \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R, \\ \hat{R}_p(x) &= x_p \times R_p(x), && \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R, \end{aligned} \tag{23}$$

with the unit cost functions on path p, i.e., $C_p(x), Z_p(x)$, and $R_p(x)$, in turn, defined as below:

$$\begin{aligned} C_p(x) &\equiv \sum_{a \in L} c_a(f_a) \alpha_{ap}, && \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R, \\ Z_p(x) &\equiv \sum_{a \in L} z_a(f_a) \alpha_{ap}, && \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R, \end{aligned}$$

$$R_p(x) \equiv \sum_{a \in L_1} r_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R. \tag{24}$$

Next, we present some preliminaries that help us to express the total shortage as well as the total discarding costs of outdated blood at the demand points solely in terms of path flow variables. Observe that, for each O/D pair w_k:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = \mu_p \left[P_k \left(\sum_{p \in \mathscr{P}_{w_k}} x_p \mu_p \right) - 1 \right], \qquad \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R. \tag{25}$$

Similarly, for the surplus, we have:

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = \mu_p P_k \left(\sum_{p \in \mathscr{P}_{w_k}} x_p \mu_p \right), \qquad \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R. \tag{26}$$

Proof: See Nagurney, Masoumi, Yu (2010) for the proofs of (25) and (26). □

We associate the Lagrange multiplier γ_a with constraint (18) for link a, and we denote the optimal Lagrange multiplier by $\gamma_a^*, \forall a \in L$. The Lagrange multipliers may be interpreted as shadow prices. We group these Lagrange multipliers, respectively, into the vectors γ and γ^*.

Let K denote the feasible set such that:

$$K \equiv \{(x, u, \gamma) | x \in R_+^{n_p}, \text{ (19) holds, and } \gamma \in R_+^{n_L}\}. \tag{27}$$

Before stating the variational inequality formulation of the problem, we recall a lemma that formalizes the construction of the partial derivatives of the total operational cost, the total discarding cost, and the total risk functions with respect to a path flow.

Lemma 1. *The partial derivatives of the total operational cost, the total discarding cost, and the total risk of a path with respect to a path flow are, respectively, given by:*

$$\frac{\partial (\sum_{q \in \mathscr{P}} \hat{C}_q(x))}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R,$$
$$\frac{\partial (\sum_{q \in \mathscr{P}} \hat{Z}_q(x))}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R,$$
$$\frac{\partial (\sum_{q \in \mathscr{P}} \hat{R}_q(x))}{\partial x_p} \equiv \sum_{a \in L_1} \frac{\partial \hat{r}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R. \tag{28}$$

Proof: See Nagurney, Masoumi, and Yu (2010) for the proof. □

We now derive the variational inequality formulation of the problem in terms of path flows and link flows.

Theorem 1. *The optimization problem (22), subject to its constraints, is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal capacity adjustments, and the vector of optimal Lagrange multipliers $(x^*, u^*, \gamma^*) \in K$, such that:*

$$\sum_{k=1}^{n_R} \sum_{p \in \mathscr{P}_{w_k}} \left[\frac{\partial(\sum_{q\in\mathscr{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q\in\mathscr{P}} \hat{Z}_q(x^*))}{\partial x_p} + \lambda_k^+ \mu_p P_k \left(\sum_{p\in\mathscr{P}_{w_k}} x_p^* \mu_p \right) \right.$$

$$\left. -\lambda_k^- \mu_p \left(1 - P_k \left(\sum_{p\in\mathscr{P}_{w_k}} x_p^* \mu_p \right) \right) + \sum_{a\in L} \gamma_a^* \delta_{ap} + \theta \; \frac{\partial(\sum_{q\in\mathscr{P}} \hat{R}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*]$$

$$+ \sum_{a\in L} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \gamma_a^* \right] \times [u_a - u_a^*] + \sum_{a\in L} \left[\bar{u}_a + u_a^* - \sum_{p\in\mathscr{P}} x_p^* \alpha_{ap} \right] \times [\gamma_a - \gamma_a^*] \geq 0,$$

$$\forall (x,u,\gamma) \in K. \tag{29}$$

The variational inequality (29), in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and the link capacity adjustments, and the vector of optimal Lagrange multipliers $(f^*, v^*, u^*, \gamma^*) \in K^1$, such that:

$$\sum_{a\in L} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \gamma_a^* + \theta \; \frac{\partial \hat{r}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{a\in L} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \gamma_a^* \right] \times [u_a - u_a^*] + \sum_{k=1}^{n_R} \left[\lambda_k^+ P_k(v_k^*) - \lambda_k^- (1 - P_k(v_k^*)) \right] \times [v_k - v_k^*]$$

$$+ \sum_{a\in L} [\bar{u}_a + u_a^* - f_a^*] \times [\gamma_a - \gamma_a^*] \geq 0, \qquad \forall (f,v,u,\gamma) \in K^1, \tag{30}$$

where K^1 denotes the feasible set as defined below:

$$K^1 \equiv \{(f,v,u,\gamma) | \exists x \geq 0, \; (11),(13),(15), \text{ and } (19) \text{ hold, and } \gamma \geq 0\}. \tag{31}$$

Proof. First, we prove the result for path flows (cf. (29)).

The convexity of $\hat{C}_p$, $\hat{Z}_p$, and $\hat{R}_p$ for all paths p holds since $\hat{c}_a$, $\hat{z}_a$, and $\hat{r}_a$ were assumed to be convex for all links a. The convexity of $\hat{\pi}_a$ was also assumed to hold. We just need to verify that $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$ is also convex. We have:

$$\frac{\partial^2}{\partial {x_p}^2}\left[\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+)\right]=\lambda_k^-\ \frac{\partial^2 E(\Delta_k^-)}{\partial {x_p}^2}+\lambda_k^+\ \frac{\partial^2 E(\Delta_k^+)}{\partial {x_p}^2},$$
$$\forall p \in \mathscr{P}_{w_k}; k=1,\ldots,n_R. \qquad (32)$$

Substituting the first order derivatives from (25) and (26) into (32) yields:

$$\frac{\partial^2}{\partial {x_p}^2}\left[\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+)\right]=$$

$$=\lambda_k^-\ \frac{\partial}{\partial x_p}\mu_p\left[P_k\left(\sum_{p\in\mathscr{P}_{w_k}} x_p\mu_p\right)-1\right]\ +\ \lambda_k^+\ \frac{\partial}{\partial x_p}\mu_p P_k\left(\sum_{p\in\mathscr{P}_{w_k}} x_p\mu_p\right)$$
$$=(\lambda_k^-+\lambda_k^+)(\mu_p)^2\mathscr{F}_k\left(\sum_{p\in\mathscr{P}_{w_k}} x_p\mu_p\right)>0, \qquad \forall p\in\mathscr{P}_{w_k}; k=1,\ldots,n_R. \quad (33)$$

The above inequality holds provided that $(\lambda_k^-+\lambda_k^+)$, i.e., the sum of shortage and surplus penalties, is assumed to be positive. Hence, $\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+)$, and, as a consequence, the multicriteria objective function in (22) is also convex.

Since the objective function (22) is convex and the feasible set K is closed and convex, the variational inequality (29) follows from the standard theory of variational inequalities (see Nagurney (1999)).

As for the proof of the variational inequality (30), now that (29) is established, we can apply the equivalence between partial derivatives of total costs on paths and partial derivatives of total costs on links from Lemma 1. Also, from (13) and (15), we can rewrite the formulation in terms of link flows and projected demands rather than path flows. Thus, the second part of Theorem 1, that is, the variational inequality in link flows (30), also holds. □

Note that variational inequality (29) can be put into standard form (see Nagurney (1999)) as follows: determine $X^* \in \mathscr{K}$ such that:

$$\langle F(X^*)^T, X-X^*\rangle \geq 0, \qquad \forall X\in\mathscr{K}, \qquad (34)$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product in n-dimensional Euclidean space.

If we define the feasible set $\mathscr{K}\equiv K$, and the column vector $X\equiv(x,u,\gamma)$, and $F(X)\equiv(F_1(X),F_2(X),F_3(X))$, where:

$$F_1(X)=\left[\frac{\partial(\sum_{q\in\mathscr{P}}\hat{C}_q(x))}{\partial x_p}+\frac{\partial(\sum_{q\in\mathscr{P}}\hat{Z}_q(x))}{\partial x_p}+\lambda_k^+\mu_p P_k\left(\sum_{p\in\mathscr{P}_{w_k}} x_p\mu_p\right)\right.$$

$$-\lambda_k^- \mu_p \left(1 - P_k\left(\sum_{p\in\mathscr{P}_{w_k}} x_p\mu_p\right)\right) + \sum_{a\in L}\gamma_a\delta_{ap} + \theta\,\frac{\partial(\sum_{q\in\mathscr{P}}\hat{R}_q(x))}{\partial x_p};$$

$$p \in \mathscr{P}_{w_k};\ k = 1,\ldots,n_R\Big],$$

$$F_2(X) = \left[\frac{\partial\hat{\pi}_a(u_a)}{u_a} - \gamma_a; \quad a \in L\right],$$

and

$$F_3(X) = \left[\bar{u}_a + u_a - \sum_{p\in\mathscr{P}} x_p\alpha_{ap}; \quad a \in L\right], \tag{35}$$

then variational inequality (29) can be re-expressed in standard form (34).

We will utilize variational inequality (29) in path flows for our computations since our proposed computational procedure will yield closed form expressions at each iteration. Once we have solved problem (29), by using (15), which relates the links flows to the path flows, we can obtain the solution f^* which, along with u^*, minimizes the total cost as well as the total supply risk (cf. (21)) associated with the design of the supply chain network of a blood banking system.

We now present the algorithm for the solution of the sustainable blood banking supply chain network design followed by several numerical examples.

3 The Algorithm and the Numerical Examples

In this Section, we first recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of the sustainable blood bank supply chain design problem governed by variational inequality (29) (see also (34)) induces subproblems that can be solved explicitly and in closed form.

Specifically, at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)), one computes:

$$X^{\tau+1} = P_{\mathscr{K}}(X^\tau - a_\tau F(X^\tau)), \tag{36}$$

where $P_{\mathscr{K}}$ is the projection on the feasible set $\mathscr{K}$ and F is the function that enters the variational inequality problem (34).

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein. Applications of this Euler method to the solution of oligopolistic supply chain network design problems can be found in Nagurney (2010a).

3.1 Explicit Formulas for the Euler Method Applied to the Sustainable Blood Supply Chain Network Design Variational Inequality (29)

The elegance of this procedure for the computation of solutions to the sustainable blood supply chain network design problem modeled in Section 2 can be seen in the following explicit formulas. In particular, (36) for the blood supply chain network design problem governed by variational inequality problem (29) yields the following closed form expressions for the blood product path flows, the capacity adjustments corresponding to various links, and the Lagrangian multipliers, respectively:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^- \mu_p(1 - P_k(\sum_{p \in \mathscr{P}_{w_k}} x_p^\tau \mu_p)) - \lambda_k^+ \mu_p P_k(\sum_{p \in \mathscr{P}_{w_k}} x_p^\tau \mu_p)$$

$$- \frac{\partial(\sum_{q \in \mathscr{P}} \hat{C}_q(x^\tau))}{\partial x_p} - \frac{\partial(\sum_{q \in \mathscr{P}} \hat{Z}_q(x^\tau))}{\partial x_p} - \sum_{a \in L} \gamma_a^\tau \delta_{ap} - \theta \frac{\partial(\sum_{q \in \mathscr{P}} \hat{R}_q(x^\tau))}{\partial x_p})\},$$

$$\forall p \in \mathscr{P}_{w_k}; k = 1, \ldots, n_R; \quad (37)$$

$$u_a^{\tau+1} = \max\{-\bar{u}_a, u_a^\tau + a_\tau(\gamma_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L; \quad (38)$$

$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^\tau + a_\tau(\sum_{p \in \mathscr{P}} x_p^\tau \alpha_{ap} - \bar{u}_a - u_a^\tau)\}, \quad \forall a \in L. \quad (39)$$

We applied the Euler method to compute solutions to numerical blood supply chain network problems. The initial prospective network topology used in our numerical examples consisted of two blood collection sites, two blood centers, two component labs, two storage facilities, two distribution centers, and three demand points, as depicted in Figure 2. The Euler method (cf. (37), (38), and (39)) for the solution of variational inequality (29) was implemented in Matlab. A Microsoft Windows System at the University of Massachusetts Amherst was used for all the computations. We set the sequence $\{a_\tau\} = .1(1, \frac{1}{2}, \frac{1}{2}, \cdots)$, and the convergence tolerance was $\varepsilon = 10^{-6}$. The algorithm was initialized by setting the projected demand at each demand point and all other variables equal to zero.

Example 1
In this example, we assumed that the existing capacities of all links in the network were zero; hence, the goal was to design a sustainable blood supply chain network from scratch.

We assumed that R_1 was a small surgical center while R_2 and R_3 were large hospitals with higher demand for red blood cells. The demands at these demand

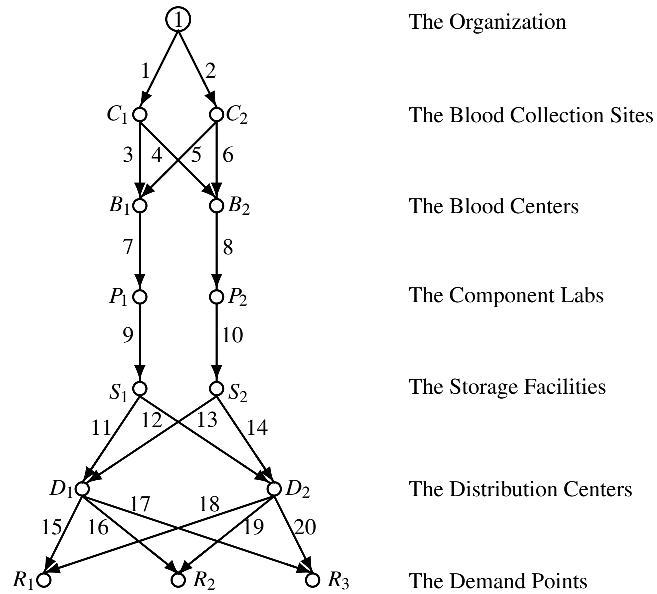

Fig. 2 The Supply Chain Network Topology for the Numerical Examples

points followed the uniform probability distribution on the intervals [5,10], [40,50], and [25,40], respectively. Hence,

$$P_1\Big(\sum_{p\in\mathscr{P}_{w_1}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_1}} \mu_p x_p - 5}{5}, \qquad P_2\Big(\sum_{p\in\mathscr{P}_{w_2}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_2}} \mu_p x_p - 40}{10},$$

$$P_3\Big(\sum_{p\in\mathscr{P}_{w_3}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_3}} \mu_p x_p - 25}{15},$$

where $w_1 = (1,R_1), w_2 = (1,R_2),$ and $w_3 = (1,R_3)$.

The shortage and outdating penalties for each of the three demand points - defined by the organization, such as the American Red Cross Regional Division Management - were:

$$\lambda_1^- = 2800, \quad \lambda_1^+ = 50,$$
$$\lambda_2^- = 3000, \quad \lambda_2^+ = 60,$$
$$\lambda_3^- = 3100, \quad \lambda_3^+ = 50.$$

The total risk functions corresponding to the blood collection links were:

$$\hat{r}_1(f_1) = 2f_1^2, \text{ and } \hat{r}_2(f_2) = 1.5f_2^2,$$

and the weight associated with the risk criterion, θ, was 0.7.

The total cost functions corresponding to the capacity adjustment are as reported in Table 1. In addition, the multipliers corresponding to the links, the total cost functions, and the total discarding cost functions are also reported there. Most of these numbers have been selected based on the average historical data for the American Red Cross Northeast Division Blood Services (Rios (2010)).

Table 1 also provides the computed optimal solutions.

Table 1 Total Cost, Total Discarding Cost, and Total Investment Cost Functions, and Solution for Numerical Example 1

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2+15f_1$	$.8f_1^2$	$.8u_1^2+u_1$	47.18	47.18	76.49
2	.99	$9f_2^2+11f_2$	$.7f_2^2$	$.6u_2^2+u_2$	39.78	39.78	48.73
3	1.00	$.7f_3^2+f_3$	$.6f_3^2$	$u_3^2+2u_3$	25.93	25.93	53.86
4	.99	$1.2f_4^2+f_4$	$.8f_4^2$	$2u_4^2+u_4$	19.38	19.38	78.51
5	1.00	$f_5^2+3f_5$	$.6f_5^2$	$u_5^2+u_5$	18.25	18.25	37.50
6	1.00	$.8f_6^2+2f_6$	$.8f_6^2$	$1.5u_6^2+3u_6$	20.74	20.74	65.22
7	.92	$2.5f_7^2+2f_7$	$.5f_7^2$	$7u_7^2+12u_7$	43.92	43.92	626.73
8	.96	$3f_8^2+5f_8$	$.8f_8^2$	$6u_8^2+20u_8$	36.73	36.73	460.69
9	.98	$.8f_9^2+6f_9$	$.4f_9^2$	$3u_9^2+2u_9$	38.79	38.79	234.74
10	1.00	$.5f_{10}^2+3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2+2u_{10}$	34.56	34.56	375.18
11	1.00	$.3f_{11}^2+f_{11}$	$.3f_{11}^2$	$u_{11}^2+u_{11}$	25.90	25.90	52.80
12	1.00	$.5f_{12}^2+2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2+u_{12}$	12.11	12.11	37.34
13	1.00	$.4f_{13}^2+2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2+1.5u_{13}$	17.62	17.62	64.92
14	1.00	$.6f_{14}^2+f_{14}$	$.4f_{14}^2$	$u_{14}^2+2u_{14}$	16.94	16.94	35.88
15	1.00	$.4f_{15}^2+f_{15}$	$.7f_{15}^2$	$.5u_{15}^2+1.1u_{15}$	5.06	5.06	6.16
16	1.00	$.8f_{16}^2+2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2+3u_{16}$	24.54	24.54	37.36
17	.98	$.5f_{17}^2+3f_{17}$	$.5f_{17}^2$	$2u_{17}^2+u_{17}$	13.92	13.92	56.66
18	1.00	$.7f_{18}^2+f_{18}$	$.7f_{18}^2$	$u_{18}^2+u_{18}$	0.00	0.00	1.00
19	1.00	$.6f_{19}^2+4f_{19}$	$.4f_{19}^2$	$u_{19}^2+2u_{19}$	15.93	15.93	33.86
20	.98	$1.1f_{20}^2+5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2+u_{20}$	12.54	12.54	21.06

As seen in Table 1, the optimal capacity on link 18 was zero (and, as expected, so was its flow), which means that D_1 was the only distribution center to serve the demand point R_1. The values of the total investment cost and the cost objective criterion, (17), were $42,375.96$ and $135,486.43$, respectively.

The computed amounts of projected demand for each of the three demand points were:

$$v_1^* = 5.06, \quad v_2^* = 40.48, \text{ and } v_3^* = 25.93.$$

It is interesting to note that for all the demand points, the values of the projected demand were closer to the lower bounds of their uniform probability distributions due to the relatively high cost of setting up a new blood supply chain network from scratch.

Next, we examine the effect of increasing the shortage penalties - while retaining the other costs - with the purpose of reducing the risk of having shortages at our demand points.

Example 2

Example 2 had the exact same data as Example 1 with the exception of the penalties per unit shortage. The new penalties corresponding to the demand points 1,2, and 3 were as follows:

$$\lambda_1^- = 28000, \qquad \lambda_2^- = 30000, \qquad \lambda_3^- = 31000.$$

Table 2 shows the optimal solution for Example 2; that is, when the shortage penalties are ten times larger than those of Example 1.

Table 2 Total Cost, Total Discarding Cost, and Total Investment Cost Functions, and Solution for Numerical Example 2

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2+15f_1$	$.8f_1^2$	$.8u_1^2+u_1$	63.53	63.53	102.65
2	.99	$9f_2^2+11f_2$	$.7f_2^2$	$.6u_2^2+u_2$	53.53	53.53	65.23
3	1.00	$.7f_3^2+f_3$	$.6f_3^2$	$u_3^2+2u_3$	34.93	34.93	71.85
4	.99	$1.2f_4^2+f_4$	$.8f_4^2$	$2u_4^2+u_4$	26.08	26.08	105.34
5	1.00	$f_5^2+3f_5$	$.6f_5^2$	$u_5^2+u_5$	24.50	24.50	50.00
6	1.00	$.8f_6^2+2f_6$	$.8f_6^2$	$1.5u_6^2+3u_6$	27.96	27.96	86.89
7	.92	$2.5f_7^2+2f_7$	$.5f_7^2$	$7u_7^2+12u_7$	59.08	59.08	839.28
8	.96	$3f_8^2+5f_8$	$.8f_8^2$	$6u_8^2+20u_8$	49.48	49.48	613.92
9	.98	$.8f_9^2+6f_9$	$.4f_9^2$	$3u_9^2+2u_9$	52.18	52.18	315.05
10	1.00	$.5f_{10}^2+3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2+2u_{10}$	46.55	46.55	504.85
11	1.00	$.3f_{11}^2+f_{11}$	$.3f_{11}^2$	$u_{11}^2+u_{11}$	35.01	35.01	71.03
12	1.00	$.5f_{12}^2+2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2+u_{12}$	16.12	16.12	49.36
13	1.00	$.4f_{13}^2+2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2+1.5u_{13}$	23.93	23.93	87.64
14	1.00	$.6f_{14}^2+f_{14}$	$.4f_{14}^2$	$u_{14}^2+2u_{14}$	22.63	22.63	47.25
15	1.00	$.4f_{15}^2+f_{15}$	$.7f_{15}^2$	$.5u_{15}^2+1.1u_{15}$	9.33	9.33	10.43
16	1.00	$.8f_{16}^2+2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2+3u_{16}$	29.73	29.73	44.62
17	.98	$.5f_{17}^2+3f_{17}$	$.5f_{17}^2$	$2u_{17}^2+u_{17}$	19.89	19.89	80.55
18	1.00	$.7f_{18}^2+f_{18}$	$.7f_{18}^2$	$u_{18}^2+u_{18}$	0.00	0.00	1.00
19	1.00	$.6f_{19}^2+4f_{19}$	$.4f_{19}^2$	$u_{19}^2+2u_{19}$	18.99	18.99	39.97
20	.98	$1.1f_{20}^2+5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2+u_{20}$	18.98	18.98	31.37

A comparison of the optimal capacities in Examples 1 and 2 confirms that raising the shortage penalties, while keeping all operational and investment costs constant, increased the level of activities in all the network links, except for link 18 which stayed inactive. Due to the increased capacities, the new projected demand values were:

$$v_1^* = 9.33, \quad v_2^* = 48.71, \text{ and } v_3^* = 38.09.$$

As seen above, unlike Example 1, here the projected demand values were closer to the upper bounds of their uniform probability distributions. As a result, the values of the total investment cost and the cost objective criterion, were $75,814.03$ and $177,327.31$, respectively, which were significantly higher than Example 1.

Example 3

In this example, we assumed positive capacities for all the activities of the supply chain network. Thus, the problem became one of redesigning an existing blood supply chain network as opposed to designing one from scratch.

The existing capacity for each link, $\bar{u}_a$, was chosen close to the corresponding optimal solution for capacity, u_a^*, in Example 1, as reported in Table 3. All other parameters were the same as in Example 1.

Table 3 Total Cost, Total Discarding Cost, and Total Investment Cost Functions, Initial Capacities, and Solution for Numerical Example 3

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2+15f_1$	$.8f_1^2$	$.8u_1^2+u_1$	48.00	54.14	6.14	10.83
2	.99	$9f_2^2+11f_2$	$.7f_2^2$	$.6u_2^2+u_2$	40.00	43.85	3.85	5.62
3	1.00	$.7f_3^2+f_3$	$.6f_3^2$	$u_3^2+2u_3$	26.00	29.64	3.64	9.29
4	.99	$1.2f_4^2+f_4$	$.8f_4^2$	$2u_4^2+u_4$	20.00	22.35	2.35	10.39
5	1.00	$f_5^2+3f_5$	$.6f_5^2$	$u_5^2+u_5$	19.00	20.10	1.10	3.20
6	1.00	$.8f_6^2+2f_6$	$.8f_6^2$	$1.5u_6^2+3u_6$	21.00	22.88	1.88	8.63
7	.92	$2.5f_7^2+2f_7$	$.5f_7^2$	$7u_7^2+12u_7$	44.00	49.45	5.45	88.41
8	.96	$3f_8^2+5f_8$	$.8f_8^2$	$6u_8^2+20u_8$	37.00	41.40	4.40	72.88
9	.98	$.8f_9^2+6f_9$	$.4f_9^2$	$3u_9^2+2u_9$	39.00	43.67	4.67	30.04
10	1.00	$.5f_{10}^2+3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2+2u_{10}$	35.00	38.95	3.95	44.70
11	1.00	$.3f_{11}^2+f_{11}$	$.3f_{11}^2$	$u_{11}^2+u_{11}$	26.00	29.23	3.23	7.45
12	1.00	$.5f_{12}^2+2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2+u_{12}$	13.00	13.57	0.57	2.72
13	1.00	$.4f_{13}^2+2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2+1.5u_{13}$	18.00	22.05	4.05	16.07
14	1.00	$.6f_{14}^2+f_{14}$	$.4f_{14}^2$	$u_{14}^2+2u_{14}$	17.00	16.90	-0.10	1.81
15	1.00	$.4f_{15}^2+f_{15}$	$.7f_{15}^2$	$.5u_{15}^2+1.1u_{15}$	6.00	6.62	0.62	1.72
16	1.00	$.8f_{16}^2+2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2+3u_{16}$	25.00	25.73	0.73	4.03
17	.98	$.5f_{17}^2+3f_{17}$	$.5f_{17}^2$	$2u_{17}^2+u_{17}$	14.00	18.92	4.92	20.69
18	1.00	$.7f_{18}^2+f_{18}$	$.7f_{18}^2$	$u_{18}^2+u_{18}$	0.00	0.00	0.00	1.00
19	1.00	$.6f_{19}^2+4f_{19}$	$.4f_{19}^2$	$u_{19}^2+2u_{19}$	16.00	17.77	1.77	5.53
20	.98	$1.1f_{20}^2+5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2+u_{20}$	13.00	12.10	-0.62	0.00

As expected, in Example 3, because of the positive link capacities a priori, the computed values of capacity adjustment, u_a^*, were relatively small. Therefore, the optimal Lagrangian multipliers, γ_a^*, which denote the shadow prices of constraints (18), $\forall a \in L$, were considerably smaller than their counterparts in Example 1. Furthermore, the respective values of the capacity investment cost and the cost criterion were 856.36 and $85,738.13$.

It is also important to note that, for links 14 and 20, the optimal amounts of capacity adjustment were negative, meaning that the existing capacities were slightly higher than the optimal levels given the probability distribution demands.

The Euler method in Example 3 computed the following projected demand values:

$$v_1^* = 6.62, \quad v_2^* = 43.50, \text{ and } v_3^* = 30.40.$$

Example 4

Example 4 was another case of redesigning the blood supply chain network, this time with increased demands. The existing capacities, the shortage penalties, and the cost functions were the same as in Example 3.

The new demands at the three hospitals followed a uniform probability distribution on the intervals [10,17], [50,70], and [30,60], respectively. Thus, the cumulative distribution functions corresponding to the above demands were:

$$P_1\Big(\sum_{p\in\mathscr{P}_{w_1}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_1}} \mu_p x_p - 10}{7}, \qquad P_2\Big(\sum_{p\in\mathscr{P}_{w_2}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_2}} \mu_p x_p - 50}{20},$$

$$P_3\Big(\sum_{p\in\mathscr{P}_{w_3}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_3}} \mu_p x_p - 30}{30}.$$

Table 4 reports the corresponding cost functions as well as the computed optimal solution for Example 4.

As seen in Table 4, a 50% increase in demand resulted in significant positive capacity changes as well as positive flows on all 20 links in the network, including link 18, which was not constructed/used under our initial demand scenarios. The values of the total investment function and the cost criterion were $5,949.18$ and $166,445.73$, respectively, and the projected demand values were now:

$$v_1^* = 10.65, \quad v_2^* = 52.64, \text{ and } v_3^* = 34.39.$$

Example 5

Example 5 was similar to Example 4, but now the demand suffered a decrease from the original demand scenario rather than the increase that we studied in Example 4. The new demand at demand points 1, 2, and 3 followed a uniform probability distribution on the intervals [4,7], [30, 40], and [15,30], respectively, with the following functions:

$$P_1\Big(\sum_{p\in\mathscr{P}_{w_1}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_1}} \mu_p x_p - 4}{3}, \qquad P_2\Big(\sum_{p\in\mathscr{P}_{w_2}} \mu_p x_p\Big) = \frac{\sum_{p\in\mathscr{P}_{w_2}} \mu_p x_p - 30}{10},$$

Table 4 Total Cost, Total Discarding Cost, and Total Investment Cost Functions, Initial Capacities, and Solution for Numerical Example 4

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2+15f_1$	$.8f_1^2$	$.8u_1^2+u_1$	48.00	65.45	17.45	28.92
2	.99	$9f_2^2+11f_2$	$.7f_2^2$	$.6u_2^2+u_2$	40.00	53.36	13.36	17.03
3	1.00	$.7f_3^2+f_3$	$.6f_3^2$	$u_3^2+2u_3$	26.00	35.87	9.87	21.74
4	.99	$1.2f_4^2+f_4$	$.8f_4^2$	$2u_4^2+u_4$	20.00	26.98	6.98	28.91
5	1.00	$f_5^2+3f_5$	$.6f_5^2$	$u_5^2+u_5$	19.00	24.43	5.43	11.86
6	1.00	$.8f_6^2+2f_6$	$.8f_6^2$	$1.5u_6^2+3u_6$	21.00	27.87	6.87	23.60
7	.92	$2.5f_7^2+2f_7$	$.5f_7^2$	$7u_7^2+12u_7$	44.00	59.94	15.94	234.92
8	.96	$3f_8^2+5f_8$	$.8f_8^2$	$6u_8^2+20u_8$	37.00	50.21	13.21	178.39
9	.98	$.8f_9^2+6f_9$	$.4f_9^2$	$3u_9^2+2u_9$	39.00	52.94	13.94	85.77
10	1.00	$.5f_{10}^2+3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2+2u_{10}$	35.00	47.24	12.24	134.64
11	1.00	$.3f_{11}^2+f_{11}$	$.3f_{11}^2$	$u_{11}^2+u_{11}$	26.00	35.68	9.68	20.35
12	1.00	$.5f_{12}^2+2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2+u_{12}$	13.00	16.20	3.20	10.61
13	1.00	$.4f_{13}^2+2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2+1.5u_{13}$	18.00	26.54	8.54	32.23
14	1.00	$.6f_{14}^2+f_{14}$	$.4f_{14}^2$	$u_{14}^2+2u_{14}$	17.00	20.70	3.70	9.40
15	1.00	$.4f_{15}^2+f_{15}$	$.7f_{15}^2$	$.5u_{15}^2+1.1u_{15}$	6.00	10.30	4.30	5.40
16	1.00	$.8f_{16}^2+2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2+3u_{16}$	25.00	30.96	5.96	11.34
17	.98	$.5f_{17}^2+3f_{17}$	$.5f_{17}^2$	$2u_{17}^2+u_{17}$	14.00	20.95	6.95	28.81
18	1.00	$.7f_{18}^2+f_{18}$	$.7f_{18}^2$	$u_{18}^2+u_{18}$	0.00	0.35	0.35	1.69
19	1.00	$.6f_{19}^2+4f_{19}$	$.4f_{19}^2$	$u_{19}^2+2u_{19}$	16.00	21.68	5.68	13.36
20	.98	$1.1f_{20}^2+5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2+u_{20}$	13.00	14.14	1.14	2.83

$$P_3\Big(\sum_{p\in\mathscr{P}_{w_3}}\mu_p x_p\Big)=\frac{\sum_{p\in\mathscr{P}_{w_3}}\mu_p x_p-15}{15}.$$

Table 5 displays the optimal solution to this example.

As expected, most of the computed capacity changes were negative as a result of the diminished demand for blood at our demand points. Accordingly, the projected demand values were as follows:

$$v_1^*=5.52,\quad v_2^*=35.25,\ \text{ and }\ v_3^*=23.02.$$

The value of the total cost criterion for this Example was $51,221.32$.

Table 5 Total Cost, Total Discarding Cost, and Total Investment Cost Functions, Initial Capacities, and Solution for Numerical Example 5

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	γ_a^*
1	.97	$6f_1^2+15f_1$	$.8f_1^2$	$.8u_1^2+u_1$	48.00	43.02	-0.62	0.00
2	.99	$9f_2^2+11f_2$	$.7f_2^2$	$.6u_2^2+u_2$	40.00	34.54	-0.83	0.00
3	1.00	$.7f_3^2+f_3$	$.6f_3^2$	$u_3^2+2u_3$	26.00	23.77	-1.00	0.00
4	.99	$1.2f_4^2+f_4$	$.8f_4^2$	$2u_4^2+u_4$	20.00	17.54	-0.25	0.00
5	1.00	$f_5^2+3f_5$	$.6f_5^2$	$u_5^2+u_5$	19.00	15.45	-0.50	0.00
6	1.00	$.8f_6^2+2f_6$	$.8f_6^2$	$1.5u_6^2+3u_6$	21.00	18.40	-1.00	0.00
7	.92	$2.5f_7^2+2f_7$	$.5f_7^2$	$7u_7^2+12u_7$	44.00	38.99	-0.86	0.00
8	.96	$3f_8^2+5f_8$	$.8f_8^2$	$6u_8^2+20u_8$	37.00	32.91	-1.67	0.00
9	.98	$.8f_9^2+6f_9$	$.4f_9^2$	$3u_9^2+2u_9$	39.00	34.43	-0.33	0.00
10	1.00	$.5f_{10}^2+3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2+2u_{10}$	35.00	30.96	-0.19	0.00
11	1.00	$.3f_{11}^2+f_{11}$	$.3f_{11}^2$	$u_{11}^2+u_{11}$	26.00	23.49	-0.50	0.00
12	1.00	$.5f_{12}^2+2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2+u_{12}$	13.00	10.25	-0.33	0.00
13	1.00	$.4f_{13}^2+2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2+1.5u_{13}$	18.00	18.85	0.85	4.57
14	1.00	$.6f_{14}^2+f_{14}$	$.4f_{14}^2$	$u_{14}^2+2u_{14}$	17.00	12.11	-1.00	0.00
15	1.00	$.4f_{15}^2+f_{15}$	$.7f_{15}^2$	$.5u_{15}^2+1.1u_{15}$	6.00	5.52	-0.48	0.63
16	1.00	$.8f_{16}^2+2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2+3u_{16}$	25.00	20.68	-2.14	0.00
17	.98	$.5f_{17}^2+3f_{17}$	$.5f_{17}^2$	$2u_{17}^2+u_{17}$	14.00	16.15	2.15	9.59
18	1.00	$.7f_{18}^2+f_{18}$	$.7f_{18}^2$	$u_{18}^2+u_{18}$	0.00	0.00	0.00	1.00
19	1.00	$.6f_{19}^2+4f_{19}$	$.4f_{19}^2$	$u_{19}^2+2u_{19}$	16.00	14.58	-1.00	0.00
20	.98	$1.1f_{20}^2+5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2+u_{20}$	13.00	7.34	-0.62	0.00

4 Summary and Conclusions

In this paper, we developed a sustainable supply chain network design model for a highly perishable health care product – that of human blood. The process incorporated the determination of the optimal capacities of the various activities of a blood banking system, consisting of such activities as the procurement of, the testing and processing of, and the distribution of this product. The model has several novel features:

1. it captures the perishability of this life-saving product through the use of arc multipliers;
2. it contains discarding costs associated with waste/disposal;
3. it determines the optimal enhancement/reduction of link capacities as well as the determination of the capacities from scratch;
4. it can capture the cost-related effects of shutting down specific modules of the supply chain due to an economic crisis;
5. it handles uncertainty associated with demand points;
6. it assesses costs associated with shortages/surpluses at the demand points, and
7. it quantifies the supply-side risk associated with procurement.

We illustrated the model through several numerical examples, which vividly demonstrate the flexibility and generality of our sustainable supply chain network design model for blood banking systems. For the sake of generality, and the establishment of the foundations that will enable further extensions and applications, we used a variational inequality approach for both model formulation and solution.

The framework developed here can be applied, with appropriate adaptation, to other perishable products, also in the health care arena, such as medicines and vaccines, as well as to agricultural products, including food.

Acknowledgements This research was supported, in part, by the John F. Smith Memorial Foundation. This support is gratefully acknowledged.

The authors acknowledge Mr. Len Walker, the Director of Business Development for the American Red Cross Blood Services in the greater Boston area, who shared valuable information on the subject. They also thank Dr. Jorge Rios, the Medical Director for the American Red Cross Northeast Division Blood Services, who graciously agreed to be interviewed twice, and who provided enlightening thoughts on the model.

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