Wage-Dependent Labor and Supply Chain Networks in the COVID-19 Pandemic and Beyond

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Abstract:

Labor is a critical resource for the functionality of supply chain networks. Without labor, products cannot be produced, transported, stored, and distributed to consumers at demand markets. Disruptions to labor in the COVID-19 pandemic have further highlighted its importance with many companies now increasing wages in order to attract workers. In this paper, we construct a computable supply chain network framework that includes labor as a resource, along with (hourly) wages that should be paid. We consider both a single firm and multiple firms with the latter engaged in competition with respect to substitutable products. We provide variational inequality formulations for the models without wage ceilings and with wage ceilings, along with a computational procedure. Numerical examples demonstrate the impacts of the addition of electronic commerce; the use of outsourcing, and competition, along with increases, as well as the tightening of wage ceilings on supply chain network links. The general, flexible, holistic framework reveals some unexpected results and also shows the benefits, in terms of profits, of the free market, with no imposed wage upper bounds.

Keywords: labor, wages, supply chains, networks, optimization, game theory

1. Introduction

Labor is essential for the functionality of supply chains from the production of goods to their transport via freight service provision, their storage, and the ultimate distribution to consumers at demand markets. Throughout the COVID-19 pandemic, we have witnessed the impacts of labor disruptions due to workers becoming ill, with many dying. Related issues of labor availability as well as decreases in labor productivity have also affected our societies and economies globally in the pandemic (see, e.g., Bhattarai and Reiley (2020)). Indeed, the news has been replete with worker shortages in various sectors, not only early on in the pandemic, but also 18 months since the declaration of the pandemic, as the vaccination campaigns proceed and economies begin to open up. Examples of labor shortages in the US, for example, have now been noted in manufacturing, and restaurants, as they have started to reopen, as well as in construction (see Morath (2021)). The transportation and logistics sector in the US is also struggling to find workers with surging demand from consumers of various products from household goods to recreation products (DC Veocity Staff (2021)). Some companies are raising wages in order to attract workers, including Walmart, Amazon, and Costco (Casselman and Tankersley (2021)) as are restaurant chains such as McDonald's and Chipotle (Creswell (2021)). According to AP News (2021), wages and benefits grew quickly for workers in the United States in January, February, and March of 2021, the largest gain in more than 13 years, as noted by the US Labor Department.

Labor shortages, exacerbated in the pandemic, and occurring for a plethora of reasons, are also prevalent in the United Kingdom (see Coles (2021)), with the compounding of problems due to Brexit, the European Union (cf. Adascalitei and Weber (2021)), and even parts of Asia, including China (Yifan Xie and Qi (2021)) as well as in Canada and Australia (Wolf (2021)). Indeed, labor shortages are a global phenomenon in the pandemic. Shortages of labor for different supply chain network activities, from production, to freight service provision, and even retail, are dramatically impacting the functionality and efficiency of global supply chains with port congestion resulting from a lack of workers leading to further product delivery delays (cf. Caminiti (2021)).

The healthcare sector, in addition to other sectors in the United States, is contending with labor shortages, with hospitals competing for nurses and hundreds reporting critical staff shortages (Frey (2021)). This is also the situation in parts of the European Union (European Commission (2021)). Clearly, addressing labor issues, within a supply chain context, along with wages of workers, is of societal importance. Indeed, even in the pandemic there has been much discussion as to possible impacts of raising the minimum wage (see Leonhardt (2021)). And, as the pandemic eases in the United States, companies are raising pay in order to attract workers (Casselman (2021)). According to Vacas-Soriano and Aumayr-Pintar (2021), between January 2020 and January 2021, among 21 European Union countries and the United Kingdom with statutory minimum-wage systems, rates have risen in all but the following countries: Belgium, Spain, Greece, and Estonia. However, as noted in Nagurney (2021a), there has been only limited work, to-date, that integrates labor as an important resource in supply chain networks and also addresses competition for labor. Although the economics literature considers labor, also in a competitive setting (see, e.g., Mikesell (1940), Okuguchi (1993), Card and Krueger (1994), de Pinto and Goerke (2020), Matthews (2019), and the references therein), that literature does not capture supply chains in a general, holistic manner as we do here, utilizing the rich conceptual framework of supply chain networks.

In this paper, we construct a supply chain network game theory modeling framework in which labor on supply chain links is an increasing function of the hourly wage. In previous related work (cf. Nagurney (2021a)), the hourly wage of labor was assumed to be fixed. Having labor be elastic, as a function of wages, in a computable mathematical model, enables decision-makers to determine the wages that should be paid. We also construct an extension of the model where each firm imposes an upper bound or ceiling on the wages that it is willing to pay and the wage ceiling can be distinct for different links on a firm's supply chain network. One can then determine the impacts of wage bounds not only on the specific firm's profits, but also on the profits of the other competing firms. This work adds to the literature on the integration of economics and operations research for the inclusion of labor, as a critical resource in the modeling, analysis, and solution of supply chain networks. It is of relevance in the COVID-19 pandemic, and also beyond. Related literature on game theory supply chain network models for differentiated products, as we consider here, using the methodology of variational inequalities, but without the inclusion of labor, has been developed for food applications (Yu and Nagurney (2013) and Besik and Nagurney (2017)) and for pharmaceuticals (Masoumi, Yu, and Nagurney (2012), Nagurney, Li, and Nagurney (2013)). Both of these sectors have been very essential in the COVID-19 pandemic and have also been disrupted (see Chowdhury et al. (2021)). See also Khan et al. (2021), Principato et al. (2020), and Nagurney (2021b) for research on COVID-19 food-related issues, and Nagurney (2021c), Nagurney et al. (2021), Sciacca and Daniele (2021), and Salarpour and Nagurney (2021) for recent research on related medical product shortcomings, including shortages of PPEs.

The paper is organized as follows. In Section 2, we construct the supply chain network model with wage-dependent labor, without wage bounds, and then with wage bounds, and we provide the variational inequality formulations of the governing equilibrium conditions. In Section 3, we propose an algorithm and provide closed form expressions for the computation of the variables for both models, at each iteration, along with conditions for convergence. We then apply the implemented algorithm to solve a series of numerical examples for a single firm and for multiple firms, in order to investigate such scenarios as the introduction of electronic commerce and the outsourcing of production to a lower wage location. We also study the impacts on profits of lowering (and raising) the wages that the firms willing to pay. In Section 4 we summarize our results and present our conclusions.

2. The Supply Chain Network Game Theory Models with Wage-Dependent Labor

We now introduce the supply chain network game theory modeling framework with wage-dependent labor. We first consider the model without wage bounds on links and then discuss the model with such bounds. There are I firms that are engaged in competition as they produce a substitutable product, and provide for its subsequent transportation, storage, and distribution to demand markets. Each firm has its supply chain network as depicted in Figure 1 and the firms sell their products at common demand markets. Observe that, according to Figure 1, the supply chain networks of the individual firms do not have any links in common.

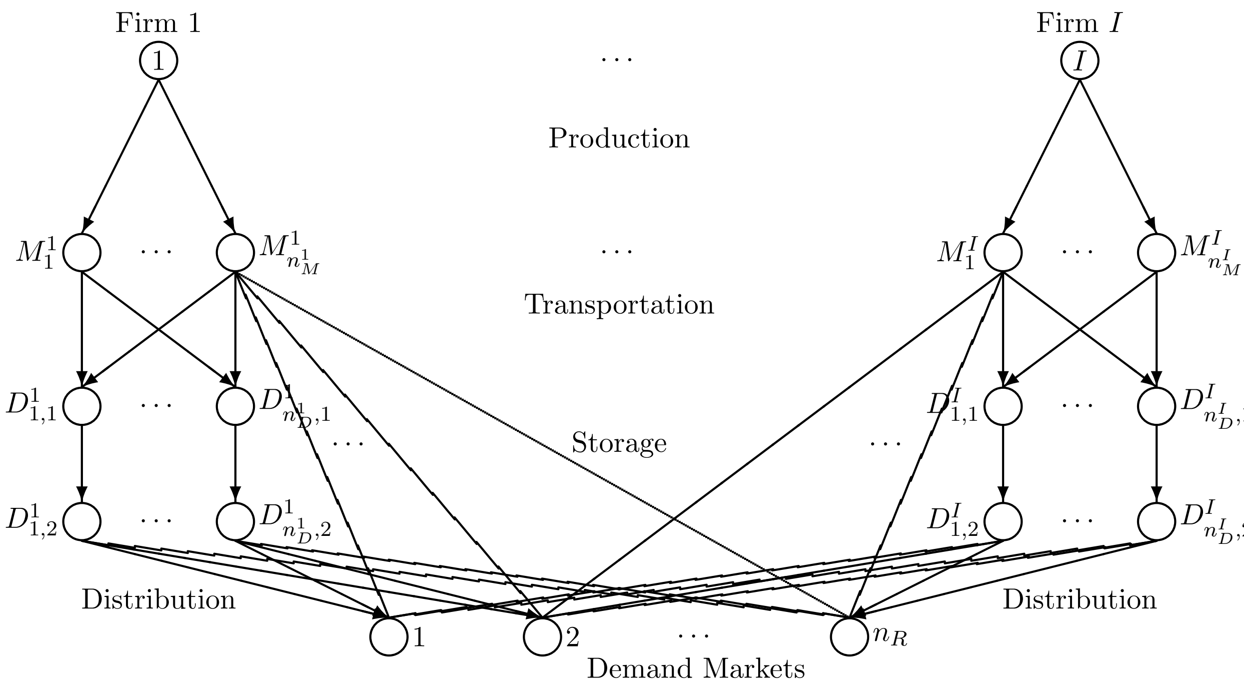

Figure 1: The Supply Chain Network Topology of the Game Theory Models with Wage-Dependent Labor

Each firm i; $i = 1, \ldots, I$, has available n_M^i production facilities, n_D^i distribution centers, and can sell its product at n_R demand markets. L^i denotes the links making up the supply chain network of firm i; $i = 1, \ldots, I$, with n_{L^i} elements. The links of L^i include firm i's links to its production nodes; the links from production nodes to the distribution centers, the storage links, and the links from the distribution centers to the demand markets. L is the complete set set of links in the supply chain network economy, where $L = \cup_{i=1}^I L^i$, consisting of n_L elements. We let $G = [N, L]$ represent the graph made up of the set of nodes N and the set of links L as in Figure 1. Note that Figure 1 also includes direct links from production sites to demand markets since electronic commerce may be available and has clearly been of significance in the COVID-19 pandemic. The supply chain network topology in Figure 1 can be adapted to the specific application under investigation with links and nodes added and/or removed accordingly. Each firm competes noncooperatively with the other firms, and seeks to determine its optimal product quantities on its supply chain network pathways that maximize its profits, along with the labor volumes, which are a function of the hourly wages that each of the firm is willing to pay for its supply chain network economic activities of production, transportation, storage, and distribution. We elaborate on the wage-dependence of labor, as well as on labor productivity further below in this section.

Table 1 contains the basic notation for the model. All vectors are column vectors.

Table 1: Notation for the Supply Chain Game Theory Model with Wage-Dependent Labor

Notation	Definition
P^i_k	the set of paths in firm i's supply chain network terminating in demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.
P^i	the set of all n_{P^i} paths of firm i; $i = 1, \ldots, I$.
P	the set of all n_P paths in the supply chain network economy.
$x_p; p \in P^i_k$	the nonnegative flow on path p originating at firm node i and terminating at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. Firm i's product path flows are grouped into the vector $x^i \in R^{n_{P^i}}_+$. The vector x^i is the vector of strategic variables of firm i. All the firms' product path flows are grouped into the vector $x \in R^{n_P}_+$.
f_a	the nonnegative flow of the product on link a, $\forall a \in L$. All the link flows are grouped into the vector $f \in R^{n_L}_+$.
l_a	the labor on link a (usually denoted in person hours).
α_a	positive factor relating input of labor to output of product flow on link a, $\forall a \in L$.
γ_a	positive factor relating input of wage to labor on link a, $\forall a \in L$.
d_{ik}	the demand for the product of firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. We group the $\{d_{ik}\}$ elements for firm i into the vector $d^i \in R^{n_R}_+$ and all the demands into the vector $d \in R^{I \times n_R}_+$.
$\hat{c}_a(f)$	the total operational cost associated with link a, $\forall a \in L$.
π_a	wage for a unit of labor on link a per hour the cognizant firm is willing to pay, at its links $a \in L^i$ for $i = 1, \ldots, I$.
$\bar{\pi}_a$	upper bound of wage on link a that the cognizant firm is willing to pay, at its links $a \in L^i$ for $i = 1, \ldots, I$.
$\rho_{ik}(d)$	the demand price function for the product of firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.

We first present the conservation of flow equations and then the equations relating link flows to labor and also the equations relating labor on the links to the wages associated with the respective links.

The path flows must be nonnegative; that is, for each firm i:

$$x_p \geq 0, \quad \forall p \in P^i, \quad \forall i. \tag{1}$$

Also, the demand for each firm's product at each demand market must be equal to the sum of the product flows from the firm to that demand market. Hence, for each firm i: $i = 1, \ldots, I$:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \ldots, n_R. \tag{2}$$

The link flows of each firm i; $i = 1, \ldots, I$, depend on the path flows as follows:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L^i, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. In other words, the flow of a firm's product on a link is equal to the sum of that product's flows on paths that use that link.

Having introduced the conservation of flow equations, we now turn to describing the important relationships between labor and the product link flows and also wages and labor. Specifically, as we have done in our previous supply chain network research with labor and game theory (see Nagurney (2021a)), we assume a linear production function as in economics, where

$$f_a = \alpha_a l_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I, \tag{4}$$

so that the greater the value of α_a, the more productive the labor on the link.

Furthermore, since here we are introducing wage-dependent labor in order to ascertain what wages should be paid by the firms, we assume that

$$l_a = \gamma_a \pi_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{5}$$

According to (5), there is greater labor availability at higher wages. This is reflected also in data and policy-making (see Domm (2021)).

We now introduce the utility functions of each of the firms. Then, we utilize some of the above expressions to express each utility functions exclusively in terms of its product path flow variables, which are its strategic variables.

The utility function of firm i, U^i; $i = 1, \ldots, I$, is the profit, given by the difference between its revenue and its total costs:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a. \tag{6}$$

The first term after the equal sign in (6) is the firm's revenue. The second term after the equal sign captures all the total link operational costs of the firm, whereas the last term in (6) is the firm's expenses for labor in its supply chain network.

The utility functions U_i; $i = 1, \ldots, I$, are assumed to be concave, with the demand price functions being monotone decreasing and continuously differentiable and with the total operational link cost functions being convex and also continuously differentiable.

In view of (2), we can define demand price functions $\tilde{\rho}_{ik}(x) \equiv \rho_{ik}(d)$, $\forall i$, $\forall k$, and, in view of (3), we can define the total operational link cost functions $\tilde{c}_a(x) \equiv \hat{c}_a(f)$, $\forall a \in L$. Furthermore, using (5), (4), and then (3), we observe that

$$\pi_a l_a = \frac{l_a^2}{\gamma_a} = \frac{(\frac{f_a}{\alpha_a})^2}{\gamma_a} = \frac{\left(\sum_{p \in P} x_p \delta_{ap}\right)^2}{\alpha_a^2 \gamma_a}, \quad \forall a \in L. \tag{7}$$

Therefore, the utility functions U^i; $i = 1, \ldots, I$, in (6) can be re-expressed as:

$$U^i = \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \tilde{\rho}_{ik}(x) x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{\left(\sum_{p \in P} x_p \delta_{ap}\right)^2}{\alpha_a^2 \gamma_a}. \tag{8}$$

2.1 Equilibrium Conditions and Variational Inequality Formulations

We now state the Nash (1950,1951) equilibrium conditions governing this noncooperative game theory model and then present the variational inequality formulation.

The feasible set K_i for firm i is defined as: $K_i \equiv \{x^i | x^i \in R_+^{n_{P^i}}$, for $i = 1, \ldots, I$. Also, $K \equiv \prod_{i=1}^I K_i$.

Definition 1: Supply Chain Network Nash Equilibrium for Model with Wage-Dependent Labor with No Wage Bounds

A path flow pattern $x^* \in K$ is a supply chain network Nash Equilibrium if for each firm i; $i = 1, \ldots, I$:

$$U^i(x^{i*}, \hat{x}^{i*}) \geq U^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \tag{9}$$

where $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{I*})$.

From (9), we know that a Nash equilibrium is established when no firm, acting unilaterally, can improve upon its utility, which represents its profits.

Applying the classical theory of Nash equilibria and variational inequalities, under our imposed assumptions on the underlying functions (cf. Gabay and Moulin (1980) and Nagurney (1999)) it follows that the solution to the above Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem: determine $x^* \in K$, such that

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\frac{\partial U^i(x^*)}{\partial x_p}\times(x_p-x_p^*)\geq 0,\quad \forall x\in K. \tag{10}$$

Variational Inequality Formulation for Model Without Wage Bounds

We now expand variational inequality (10), which yields: determine $x^* \in K$, such that

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}2\frac{(\sum_{p\in P}x_p^*\delta_{ap})}{\alpha_a^2\gamma_a}\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial\tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*]\geq 0,\ \forall x\in K, \tag{11}$$

where

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p}\equiv\sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap},\quad \forall p\in P^i,\forall i,\quad \text{and}\quad \frac{\partial\tilde{\rho}_{il}(x)}{\partial x_p}\equiv\frac{\partial\rho_{il}(d)}{\partial d_{ik}},\forall p\in P_k^i,\forall i,\forall k. \tag{12}$$

Once the equilibrium is computed - as we discuss in the next section - we can determine the labor values on the links using (4) and also the wages on the links using (5).

Variational Inequality Formulation for Model With Wage Bounds

We now extend the above model to introduce upper bounds on wages that the firms are willing to pay per hour their workers. We allow for distinct upper limits on different links. Specifically, the model remains as above except for the addition of the following constraints:

$$\pi_a\leq\bar{\pi}_a,\quad \forall a\in L. \tag{13}$$

Using the previous expressions (4) and (5), (13) becomes

$$\sum_{p\in P}x_p\delta_{ap}\leq\alpha_a\gamma_a\bar{\pi}_a,\quad \forall a\in L. \tag{14}$$

We can define the feasible set $K_1^i\equiv\{x^i\geq 0,\ \text{and (14) holds for all}\ a\in L^i\}$, and $K_1\equiv\prod_{i=1}^{I}K_1^i$. With the inclusion of the wage link upper bounds the statement of the Nash equilibrium according to Definition 1 is still appropriate but over the feasible set K_1. The variational inequality (11) also holds but with the new feasible set K_1. It is worth noting that since there are bounds on the wages, the link flows are, hence, bounded, as are the path flows and, therefore, the feasible set K_1 is compact. Since all the functions in (11) are continuous, under our imposed assumptions, we know then from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)) that a solution exists.

For computational purposes, we now propose the following variational inequality in the case of wage link bounds. We associate the Lagrange multiplier λ_a, with each link constraint as in (14). We group the Lagrange multipliers into the vector $\lambda \in R_+^{n_L}$. We define the feasible set $K_2 \equiv \{x|x \geq 0, \text{ and } \lambda \geq 0\}$ and, referring to Nagurney (2021a), it follows that the alternative variational inequality to the one in (11), to include the bounds on wages, is: determine $(x^*, \lambda^*) \in K_2$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}2\frac{(\sum_{p\in P}x_p^*\delta_{ap})}{\alpha_a^2\gamma_a}\delta_{ap}+\sum_{a\in L}\lambda_a^*\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial\tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*]$$

$$+\sum_{a\in L}\left[\bar{\pi}_a\alpha_a\gamma_a-\sum_{p\in P}x_p^*\delta_{ap}\right]\times[\lambda_a-\lambda_a^*]\geq 0,\ \forall (x,\lambda)\in K_2, \tag{15}$$

What is especially notable is that the variational inequality (11) as well as the one in (15) is characterized by a very simple feasible set, which is the nonnegative orthant, but of different dimensions. This feature, as we will see in the next section, allows for an iterative scheme, which yields closed form expressions for the variables at each iteration and, hence, is easy to implement.

Remark

It is importance to emphasize that the above game theory models, as a special case, include the single firm counterpart. These are also contributions to the literature since a firm can "independently" then investigate what wages it should charge and the potential impacts of having higher (or tighter) bounds on wages that it is willing to pay workers, whether when it comes to production at its various production sites, or to transportation and distribution, as well as storage. In the numerical examples in the next section, we first present a series of single firm examples, and then we provide results for multifirm examples.

3. The Algorithm and Numerical Examples

We use the algorithm of Korpelevich (1977) for the computation of solutions to our numerical examples. We first outline the algorithm and the form that it takes for our model(s). We then provide solutions to single firm examples in Section 3.2 and to multifirm examples in Section 3.3

3.1 The Algorithm

The algorithm is guaranteed to converge if the function $F(X)$ that enters the variational inequality problem: determine $X^* \in \mathcal{K}$, where

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\mathcal{K} \subset R^{\mathcal{N}}$, F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space, is monotone and Lipschitz continuous, provided that a solution exists.

We now, for easy reference, recall the definitions of monotonicity and Lipschitz continuity of $F(X)$. The function $F(X)$ is said to be monotone, if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{17}$$

and it is Lipschitz continuous, if there exists a constant $\mathcal{L} > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \mathcal{L}\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{18}$$

We now put variational inequality (15) into standard form (16). We define $X \equiv (x, \lambda)$ consisting of $n_P + n_L$ elements and $F(X)$, also consisting of $n_P + n_L$ elements, with $F(X) \equiv (F^1(X), F^2(X))$, with the p-th element of $F^1(X)$ being: $\frac{\partial \tilde{C}_p(x)}{\partial x_p} + \sum_{a \in L^i} 2 \frac{(\sum_{p \in P} x_p \delta_{ap})}{\alpha_a^2 \gamma_a} \delta_{ap} + \sum_{a \in L} \lambda_a \delta_{ap} - \tilde{\rho}_{ik}(x) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} x_q$ and the j-th element of $F^2(X)$ being: $\bar{\pi}_j \alpha_j \gamma_j - \sum_{p \in P} x_p \delta_{jp}$. Also, we set $\mathcal{L} \equiv K_2$ and $\mathcal{N} \equiv n_P + n_L$. Then, clearly, variational inequality (15) can be put into standard form (16).

The steps of the modified projection method are given below, with τ denoting an iteration counter:

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\mathcal{L}}$, where $\mathcal{L}$ is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{19}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{20}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The elegance of this procedure for the computation of solutions to both the supply chain network game theory models with wage-dependent labor can be seen in the following explicit formulae.

Explicit Formulae at Iteration τ for the Product Path Flows in Step 1

In particular, we have the following closed form expressions for the path flows in Step 1 for the solution of variational inequality (15):

$$\bar{x}_p^\tau = \max\{0, x_p^{\tau-1} + \beta(\tilde{\rho}_{ik}(x^{\tau-1}) + \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^{\tau-1})}{\partial x_p} \sum_{q \in P_l^i} x_q^{\tau-1} - \frac{\partial \tilde{C}_p(x^{\tau-1})}{\partial x_p} - \sum_{a \in L} \lambda_a^{\tau-1} \delta_{ap} - \sum_{a \in L^i} 2 \frac{(\sum_{p \in P} x_p^{\tau-1} \delta_{ap})}{\alpha_a^2 \gamma_a} \delta_{ap})\},$$

$$\forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{21}$$

Explicit Formulae at Iteration τ for the Lagrange Multipliers in Step 1

Similarly, we have the following closed form expressions for the Lagrange multipliers in Step 1 at an iteration τ:

$$\lambda_a^\tau = \max\{0, \lambda_a^{\tau-1} + \beta(\sum_{p \in P} x_p^{\tau-1} \delta_{ap} - \bar{\pi}_a \alpha_a \gamma_a)\}, \quad \forall a \in L. \tag{22}$$

The analogues of expressions (21) and (22) for Step 2 follow easily.

As for the solution of variational inequality (11), for the model without wage upper bounds or ceilings, (22) is no longer needed, whereas (21) still holds for Step 1 of the modified projection method but with the term: $-\sum_{a \in L} \lambda_a^{\tau-1} \delta_{ap}$ removed.

The modified projection method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computations. The algorithm was initialized with a demand of 40 for each firm-demand market pair, with the demand then equally divided among the associated path flows. In the case of the examples with the wage ceilings, the Lagrange multipliers were all initialized to zero. The convergence tolerance was 10^{-7} in that the absolute difference between two successive variable iterates differed by no more than this amount. The β parameter was set to .01 for all the examples.

The numerical examples are stylized but reflect reasonable wages and correspond to a product that is fairly expensive. The insihts gaineed, nevertheless, have broader ramifications.

3.2 Numerical Results for Single Firm Examples

The supply chain network topology for Examples 1 and 2 is as given in Figure 2. The firm has two production sites at its disposal, a single distribution center for storage, and serves two demand markets. Example 1 is without wage ceilings, whereas Example 2 is constructed from Example 1 but includes wage ceilings.

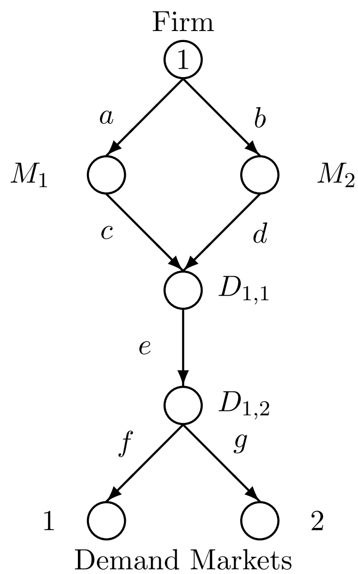

Figure 2: Supply Chain Network Topology for Examples 1 and 2

Example 1 – Single Firm, No Wage Ceilings

The total operational link cost functions are:

$$\hat{c}_a(f) = 2f_a^2, \quad \hat{c}_b(f) = 2f_b^2, \quad \hat{c}_c(f) = .5f_c^2, \quad \hat{c}_d = .5f_d^2,$$

$$\hat{c}_e(f) = f_e^2 + 2f_e, \quad \hat{c}_f(f) = .5f_f^2, \quad \hat{c}_g(f) = .5f_g^2.$$

The demand price functions are:

$$\rho_{11}(d) = -5d_{11} + 800, \quad \rho_{12}(d) = -5d_{12} + 850.$$

The alpha link parameters are:

$$\alpha_a = 55, \quad \alpha_b = 50, \quad \alpha_c = 35, \quad \alpha_d = 35, \quad \alpha_e = 60, \quad \alpha_f = 38, \quad \alpha_g = 36,$$

and the gamma link parameters are:

$$\gamma_a = .1, \quad \gamma_b = .1, \quad \gamma_c = .09, \quad \gamma_d = .07, \quad \gamma_e = .08 \quad \gamma_f = .06, \quad \gamma_g = .08.$$

The paths are defined as: path $p_1 = (a, c, e, f)$, path $p_2 = (b, d, e, f)$, path $p_3 = (a, c, e, g)$, and path $p_4 = (b, d, e, g)$.

The modified projection method yields the following equilibrium path flow pattern:

$$x^*_{p_1} = 19.39, \quad x^*_{p_2} = 19.36, \quad x^*_{p_3} = 21.66, \quad x^*_{p_4} = 21.63.$$

The equilibrium link flows, labor values, and hourly wages are reported in Table 2.

The demand price at the first demand market is 606.27 and at the second demand market the price is: 633.52, with the respective equilibrium demands of: 38.75 and 43.30.

The firm earns a profit of: 33,816.98.

Example 2 – Single Firm, Wage Ceilings

Example 2 has the identical data to the data in Example 1, except that now we impose wage ceilings of 15 on all the links. Hence, the firm is not willing to pay more than the minimum wage that has been much discussed in the news in the US lately (cf. Cooper, Mokhiber, and Zipperer (2021)).

The equilibrium path flow pattern is now:

$$x^*_{p_1} = 16.87, \quad x^*_{p_2} = 16.85, \quad x^*_{p_3} = 19.15, \quad x^*_{p_4} = 19.13.$$

The demand price at the first demand market is now: 631.37 and at the second demand market the price is: 658.63, with the respective equilibrium demands of: 33.73 and 38.27. Hence, the demand prices now rise at both demand markets, hurting the consumers.

The equilibrium link flows, labor values, and hourly wages are reported in Table 2. Observe that the equilibrium wage on link e is at the upper bound of 15. The Lagrange multiplier associated with the wage ceiling on link e is, hence, positive, and it is equal to 100.73. All other equilibrium Lagrange multipliers are equal to 0.00.

It can be seen, from Table 2, that the hourly wages now decrease on all the supply chain network economic links as compared to their respective values in Example 1.

The firm earns a profit of: 33,311.27. Hence, the firm suffers a decrease in profits by imposing an upper bound on the wages that it is willing to pay. This result speaks to the advantage of having wages freely equilibrate without wage ceilings.

Notation	Equilibrium Value	
	Example 1	**Example 2**
f_a^*	41.05	36.02
f_b^*	40.99	35.98
f_c^*	41.05	36.02
f_d^*	40.99	35.98
f_e^*	82.04	72.00
f_f^*	38.75	33.73
f_g^*	43.30	38.27
l_a^*	.75	.66
l_b^*	.82	.72
l_c^*	1.17	1.03
l_d^*	1.17	1.03
l_e^*	1.37	1.20
l_f^*	1.02	.89
l_g^*	1.20	1.06
π_a^*	7.46	6.55
π_b^*	8.20	7.20
π_c^*	13.03	11.44
π_d^*	16.73	14.68
π_e^*	17.09	15.00
π_f^*	16.99	14.79
π_g^*	15.03	13.29

Table 2: Equilibrium Link Flows, Labor Values, and Hourly Wages for Examples 1 and 2

Example 3 – Single Firm, Electronic Commerce, No Wage Ceilings

In Examples 3 and 4 the supply chain network topology of the firm is as given in Figure 3. In these examples we are interested in investigating the impact of the introduction of electronic commerce as represented by links h and i in Figure 3. The data for Example 3 is identical to the data in Example 1 but with the following additions.

There are now two additional paths defined as:

$$p_5 = (a, h), \quad p_6 = (b, i).$$

The data on the electronic commerce links are:

$$\hat{c}_h(f) = f_h^2, \quad \hat{c}_i(f) = f_i^2, \quad \alpha_h = 40, \quad \alpha_i = 45, \quad \gamma_h = .1, \quad \gamma = .1$$

Also, with the advent of electronic commerce we modified the demand price functions to:

$$\rho_{11}(d) = -d_{11} + 800, \quad \rho_{12}(d) = -d_{12} + 850.$$

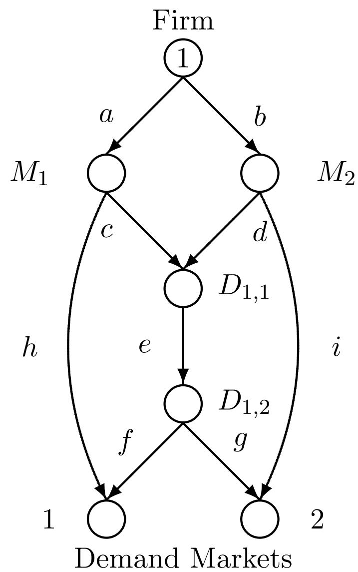

Figure 3: Supply Chain Network Topology for Examples 3 and 4

The modified projection method yields the following equilibrium path flow pattern:

$$x^*_{p_1} = 11.20, \quad x^*_{p_2} = 9.22, \quad x^*_{p_3} = 17.88, \quad x^*_{p_4} = 15.90, \quad x^*_{p_5} = 80.14, \quad x^*_{p_6} = 85.03.$$

It is clear that the greatest product volumes are on the pathways associated with the electronic commerce links.

The demand price at the first demand market is now: 699.43 and at the second demand market the price is: 731.20, with the respective equilibrium demands of: 100.57 and 118.80.

The profit earned by the firm is: 90,663.52.

Example 4 – Single Firm, Electronic Commerce, Wage Ceilings

Example 4 has the identical data to that in Example 3, but now wage ceilings of 15 are imposed on all the links.

The new equilibrium path flow pattern is:

$$x^*_{p_1} = 8.48, \quad x^*_{p_2} = 3.46, \quad x^*_{p_3} = 15.93, \quad x^*_{p_4} = 10.90, \quad x^*_{p_5} = 58.09, \quad x^*_{p_6} = 60.64.$$

The profit of the firm now drops to: 83,385.75. The pathways associated with the electronic commerce links are, again, the ones that have the highest volumes of product flowing on them. In Example 4, both links a and b have equilibrium wages at the imposed wage ceilings of 15. The associated equilibrium Lagrange multipliers are, respectively: 212.48 and 252.59. All other Lagrange multipliers are equal to 0.00.

The demand price at the first demand market is now: 729.97 and at the second demand market the price is: 762.53, with the respective equilibrium demands of: 70.03 and 87.47. The demand prices now rise (as they did in Example 2) signaling that the consumers are, in a sense, worse-off, under the imposed wage ceilings. The profit of the firm is also now lower than in Example 3, wherrin there are no imposed wage ceilings.

The equilibrium link flows, labor values, and hourly wages are reported in Table 3 for both Examples 3 and 4. The wage for each of the supply chain network links is lower in Example 4 than in Example 3. The workers also suffer in that their hourly wages decrease.

Notation	**Equilibrium Value**	
	Example 3	**Example 4**
f_a^*	109.22	82.50
f_b^*	110.15	75.00
f_c^*	29.08	24.41
f_d^*	25.12	14.36
f_e^*	54.20	38.78
f_f^*	20.43	11.95
f_g^*	33.77	26.83
f_h^*	80.14	58.09
f_i^*	85.03	60.64
l_a^*	1.99	1.50
l_b^*	2.20	1.50
l_c^*	.83	.70
l_d^*	.72	.41
l_e^*	.90	.65
l_f^*	.54	.31
l_g^*	.94	.75
l_h^*	2.00	1.45
l_i^*	1.89	1.35
π_a^*	19.86	15.00
π_b^*	22.03	15.00
π_c^*	9.23	7.75
π_d^*	10.25	5.86
π_e^*	11.29	8.08
π_f^*	8.96	5.24
π_g^*	11.73	9.32
π_h^*	20.03	14.52
π_i^*	18.90	13.47

Table 3: Equilibrium Link Flows, Labor Values, and Hourly Wages for Examples 3 and 4

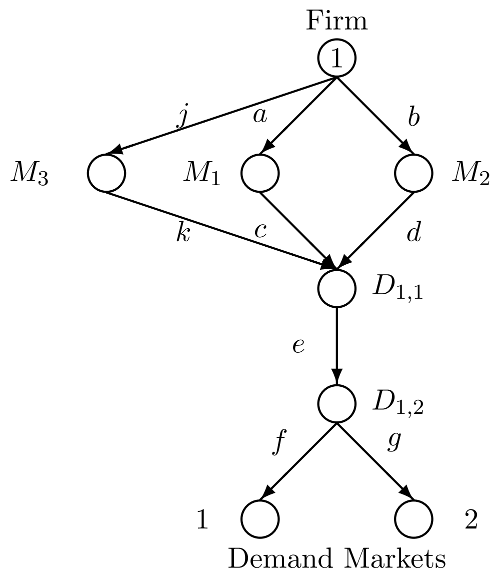

Figure 4: Supply Chain Network Topology for Examples 5, 6, and 7

Example 5 – Single Firm, Outsourcing, No Wage Ceilings

Examples 5 through 7 have the supply chain network topology depicted in Figure 4. In these examples, we are interested in evaluating the impacts when the firm makes use of another production site that it outsources the production to. The production site has lower manufacturing costs but is located a bit further from the firm's distribution center.

The data for Example 5 are identical to the data in Example 1 except for the new data to represent links j and k as in Figure 4. Specifically, the total operational costs on these links are:

$$\hat{c}_j(f) = f_j^2, \quad \hat{c}_k(f) = f_k;$$

the alpha and gamma parameters on these two new links are:

$$\alpha_j = 45, \quad \alpha_k = 35, \quad \gamma_j = .07, \quad \gamma_k = .06.$$

The new paths are: path $p_7 = (j, k, e, f)$ and path $p_8 = (j, k, e, g)$.

The equilibrium path flow pattern is:

$$x^*_{p_1} = 13.24, \quad x^*_{p_2} = 13.22, \quad x^*_{p_3} = 14.76, \quad x^*_{p_4} = 14.74, \quad x^*_{p_7} = 16.65, \quad x^*_{p_8} = 18.16.$$

The profit of the firm is now: 37,406.73. The firm enjoys a sizeable increase in profit, as compared to that it earns in Example 1.

The demand price at the first demand market is: 584.47, and at the second demand market: 611.71, with a demand at the first market of 43.11 and a demand of 47.66 at the second demand market. The demand prices drop at both demand markets, as compared to their values in Example 1, a plus for consumers. The equilibrium link flows, labor values, and hourly wages are given in Table 4. The wages decrease at the two original production sites but increase at the distribution center, which is now handling an increase in product volume.

Example 6 – Single Firm, Outsourcing, Wage Ceilings on All Links, Tighter Ones on Outsourcing Links

Example 6 has the identical data to that in Example 5 except that now we are interested in the impacts of wage ceilings. The price ceilings on the original links remain as in Example 2, with a value of 15 each, but the outsourcing links have lower wage bounds (since the authorities there can pay lower wages) so we have that the wage bounds are equal to 10 on both link j and link k.

The equilibrium path flow pattern is now:

$$x^*_{p_1} = 12.00, \quad x^*_{p_2} = 11.98, \quad x^*_{p_3} = 13.52, \quad {p_4}^* = 13.50, \quad x^*_{p_7} = 9.74, \quad x^*_{p_8} = 11.26.$$

The path flow is lower on each path as compared to the corresponding path flow in Example 5.

The profit of the firm is: 35,667.08. The profit is lower than in Example 5 and we are seeing, consistently, that lower wages result in a lower profit for the firm.

The demand price at the first demand market is now: 631.37, and at the second demand market: 658.63, higher in both cases than the demand market prices in Example 5. The demand at the first market is now 33.73, whereas the demand at the second demand market is now 38.27. The equilibrium link flows, labor values, and hourly wages are given in Table 4. Wages now decrease on all the links. The Lagrange multiplier on link e is 153.52 and that on link k is 43.35 since the wages associated with these two supply chain network links are at the imposed wage ceilings associated with these links. All other link Lagrange multipliers are equal to 0.00, at the equilibrium.

Example 7 – Single Firm, Outsourcing, Tighter Wage Ceilings on All Links

Example 7 has the same data as Example 6 except now the firm has all of the wage ceilings set to 10 (rather than 15). Therefore, the wage ceilings on all links in the supply chain network in Figure 4 are equal to 10.

The computed equilibrium path flow pattern is now:

$$x^*_{p_1} = 6.65, \quad x^*_{p_2} = 6.64, \quad x^*_{p_3} = 8.16, \quad x^*_{p_4} = 8.15, \quad x^*_{p_7} = 8.45, \quad x^*_{p_8} = 9.96.$$

Under lower wage ceilings, the path flow is lower on each path as compared to the corresponding path flow in Example 6.

Notation	Equilibrium Value		
	Example 5	**Example 6**	**Example 7**
f_a^*	28.00	25.52	14.81
f_b^*	27.96	25.48	14.79
f_c^*	28.00	25.52	14.81
f_d^*	27.96	25.48	14.79
f_e^*	90.77	72.00	48.00
f_f^*	43.11	33.73	21.73
f_g^*	47.66	38.27	26.27
f_j^*	34.81	21.00	18.41
f_k^*	34.81	21.00	18.41
l_a^*	.51	.46	.17
l_b^*	.56	.51	.30
l_c^*	.80	.73	.42
l_d^*	.80	.73	.42
l_e^*	1.51	1.2	.80
l_f^*	1.14	.89	.57
l_g^*	1.32	1.06	.73
l_j^*	.77	.47	.41
l_k^*	.99	.60	.53
π_a^*	5.09	4.64	2.69
π_b^*	5.59	5.10	2.96
π_c^*	8.89	8.10	4.70
π_d^*	11.41	10.40	6.04
π_e^*	18.91	15.00	10.00
π_f^*	18.91	14.79	9.53
π_g^*	16.55	13.29	9.12
π_j^*	11.05	6.67	5.84
π_k^*	16.58	10.00	8.77

Table 4: Equilibrium Link Flows, Labor Values, and Hourly Wages for Examples 5, 6, and 7

The profit of the firm now 29,115.85. The profit is lower than in Example 6, further evidence that lower wage ceilings, which result in lower wages, yield a lower profit for the firm.

The demand price at the first demand market is now: 691.36, and at the second demand market: 718.73, higher in both cases than the demand market prices in Example 6. The demand at the first market has now dropped to 21.73, whereas the demand at the second demand market has now decreased to 26.27. The computed equilibrium link flows, labor values, and hourly wages are given in Table 4. The Lagrange multiplier on link e is 387.75, with all other Lagrange multipliers equal to 0.00.

3.2 Numerical Results for Multifirm Examples

The multifirm numerical examples, Examples 8 through 11, have the supply chain network topology depicted in Figure 5. Observe that the supply chain network for Firm 1 is identical to its network in Examples 1 and 2 and the data are also as therein except that the demand price functions now have a term to capture competition with the other firm. Firm 2 also has at its disposal two production sites, a single distribution center, and sells its product at the same demand markets as does Firm 1.

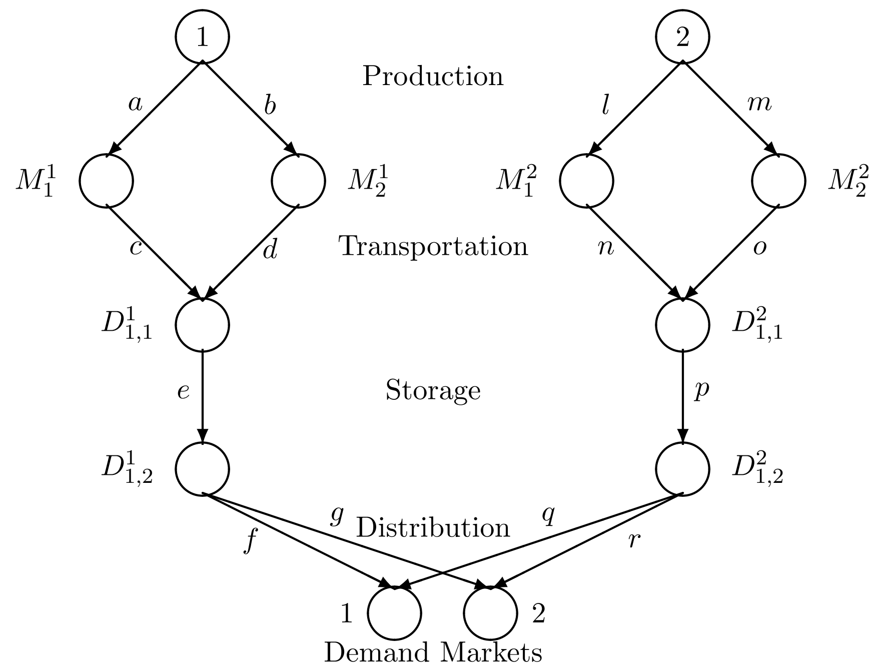

Figure 5: The Supply Chain Network Topology for the Numerical Examples 8 Through 11

Example 8 – Two Firm Example, No Wage Ceilings

The data for Firm 1 are identical to the data in Example 1 but with the demand price functions now including cross-terms to capture competition with the other firm, that is:

$$\rho_{11}(d) = -5d_{11} - 2d_{21} + 800, \quad \rho_{12}(d) = -5d_{12} - d_{22} + 850.$$

The data for the supply chain network of Firm 2 are: The total operational link cost functions are:

$$\hat{c}_l(f) = 1.5f_l^2, \quad \hat{c}_m(f) = 1.5f_m^2 + f_m, \quad \hat{c}_n(f) = f_n^2 + 2f_n, \quad \hat{c}_o = f_o^2,$$

$$\hat{c}_p(f) = .5f_p^2, \quad \hat{c}_q(f) = .5f_q^2 + f_q, \quad \hat{c}_r(f) = f_r^2 + 2f_r.$$

The demand price functions are:

$$\rho_{21}(d) = -3d_{21} - d_{11} + 700, \quad \rho_{22}(d) = -5d_{22} - .5d_{12} + 750.$$

The alpha link parameters are:

$$\alpha_l = 40, \quad \alpha_m = 45, \quad \alpha_n = 30, \quad \alpha_o = 30, \quad \alpha_p = 55, \quad \alpha_q = 39, \quad \alpha_r = 35,$$

and the gamma link parameters are:

$$\gamma_l = .1, \quad \gamma_m = .1, \quad \gamma_n = .09, \quad \gamma_o = .09, \quad \gamma_p = .09 \quad \gamma_q = .07, \quad \gamma_r = .1.$$

Observe that Firm 2 is better at attracting workers with its gamma link parameters the same or higher than those of Firm 1 for similar supply chain network activities.

The new paths associated with Firm 2's supply chain network are:

$$p_9 = (l, n, p, q), \quad p_{10} = (m, o, p, q), \quad p_{11} = (l, n, q, r), \quad p_{12} = (m, o, p, r).$$

Note that in Example 8 there are now wage ceilings. in the subsequent numerical examples, there are wage ceilings on Firm 1's network only in Example 9; then on Firm 2's network only in Example 10, and, finally, there are wage ceilings on all the supply chain network links in Example 11.

The modified projection method yields the following equilibrium path flow pattern:

$$x^*_{p_1} = 17.65, \quad x^*_{p_2} = 17.63, \quad x^*_{p_3} = 19.61, \quad x^*_{p_4} = 19.58.$$

$$x^*_{p_9} = 18.07, \quad x^*_{p_{10}} = 18.18, \quad x^*_{p_{11}} = 39.66, \quad x^*_{p_{12}} = 39.78.$$

The equilibrium link flows, labor values, and hourly wages are reported in Table 5 for this, as well as the subsequent numerical examples.

The demand price of Firm 1's product at the first demand market is 551.14 and at the second demand market the price is: 574.14, with the respective equilibrium demands of: 35.28 and 39.19. The demand price of Firm 2's product at the first demand market is 555.99 and at the second demand market the price is: 650.96, with the respective equilibrium demands of: 36.24 and 79.44.

Firm 1 earns a profit of: 27,857.52 and Firm 2 earns a profit of: 40,874.88. Clearly, Firm 2 is more competitive. Firm 1 now, with competition, has much lower profit than it did in Example 1. Firm 2 is able to attract more labor for its supply chain network activities than Firm 1 and, on the majority of its supply chain network links, pays a higher wage than Firm 1 does (for the analogous activities).

Notation	**Equilibrium Value**			
	Example 8	**Example 9**	**Example 10**	**Example 11**
f_a^*	37.26	36.02	38.65	36.02
f_b^*	37.21	35.98	38.60	35.98
f_c^*	37.26	36.02	38.65	36.02
f_d^*	37.21	35.98	38.60	35.98
f_e^*	74.47	72.00	77.26	72.00
f_f^*	35.28	34.03	36.76	34.13
f_g^*	39.19	37.97	40.50	37.87
f_l^*	57.73	57.80	37.02	37.02
f_m^*	57.96	58.03	37.23	37.23
f_n^*	57.73	57.80	37.02	37.02
f_o^*	57.96	58.03	37.23	37.23
f_p^*	115.69	115.83	74.25	74.25
f_q^*	36.24	36.35	21.75	21.75
f_r^*	79.44	79.47	52.50	52.50
l_a^*	.68	.66	.70	.66
l_b^*	.74	.72	.77	.72
l_c^*	1.06	1.03	1.10	1.03
l_d^*	1.06	1.03	1.10	1.03
l_e^*	1.24	1.20	1.29	1.20
l_f^*	.93	.90	.97	.90
l_g^*	1.09	1.05	1.12	1.05
l_l^*	1.44	1.46	.93	.93
l_m^*	1.29	1.29	.83	.83
l_n^*	1.92	1.92	1.23	1.23
l_o^*	1.93	1.92	1.24	1.24
l_p^*	2.10	2.11	1.35	1.35
l_q^*	.93	.93	.58	.58
l_r^*	2.27	2.27	1.50	1.50
π_a^*	6.77	6.55	7.03	6.55
π_b^*	7.44	7.20	7.72	7.20
π_c^*	11.83	11.44	12.27	11.44
π_d^*	15.19	14.68	15.76	14.68
π_e^*	15.51	15.00	16.10	15.00
π_f^*	15.47	14.93	16.12	14.97
π_g^*	13.61	13.18	14.06	14.15
π_l^*	14.43	14.45	9.25	9.25
π_m^*	12.88	12.90	8.27	8.27
π_n^*	21.38	21.41	13.71	13.71
π_o^*	21.47	21.49	13.79	13.79
π_p^*	23.37	23.40	15.00	15.00
π_q^*	13.28	13.32	7.97	7.97
π_r^*	22.70	22.71	15.00	15.00

Table 5: Equilibrium Link Flows, Labor Values, and Hourly Wages for Examples 8 Through 11

Example 9 – Two Firm Example, Wage Ceilings Only on Firm 1's Supply Chain Network

Example 9 has the same data as Example 8 but now we impose wage ceilings of 15 on all the supply chain network links of Firm 1 only.

The modified projection method converges to the following equilibrium path flow pattern:

$$x^*_{p_1} = 17.03, \quad x^*_{p_2} = 17.00, \quad x^*_{p_3} = 19.00, \quad x^*_{p_4} = 18.97.$$

$$x^*_{p_9} = 18.12, \quad x^*_{p_{10}} = 18.23, \quad x^*_{p_{11}} = 39.68, \quad x^*_{p_{12}} = 39.79.$$

The demand price of Firm 1's product at the first demand market is now: 557.13 and at the second demand market the price is: 580.69, with the respective equilibrium demands of: 34.03 and 37.97. The demand price of Firm 2's product at the first demand market is 556.91 and at the second demand market the price is: 651.94, with the respective equilibrium demands of: 36.35 and 79.47. The demand market prices of Firm 1's product rise whereas those of Firm 2's product only a very small amount.

Firm 1 earns a profit of: 27,818.46 and Firm 2 earns a profit of: 40,968.69. This result is quite interesting. Firm 1 has lower profits under wage ceilings, and, in fact, "helps" Firm 2 to achieve higher profits. This example demonstrates the importance of having a holistic supply chain network framework since changes associated with a firm can actually have impacts on other firms and, sometimes, in not expected ways. The Lagrange multiplier associated with link e is: 24.64, since the wage on the link is at its upper bound. All other Lagrange multipliers are equal to 0.00.

Additional results are reports in Table 5.

Firm 2, again, pays its workers more than Firm 1 (except on link q) on similar links and attracts more labor for those activities than does Firm 1.

Example 10 – Two Firm Example, Wage Ceilings Only on Firm 2's Supply Chain Network

Example 10 has the same data as Example 8 but now we impose wage ceilings of 15 on all the supply chain network links of Firm 2 only.

The modified projection method converges to the following equilibrium path flow pattern:

$$x^*_{p_1} = 18.39, \quad x^*_{p_2} = 18.37, \quad x^*_{p_3} = 20.26, \quad x^*_{p_4} = 20.23.$$

$$x^*_{p_9} = 10.82, \quad x^*_{p_{10}} = 10.93, \quad x^*_{p_{11}} = 26.20, \quad x^*_{p_{12}} = 26.30.$$

Firm 1 has increases of product flow on all of its paths, as compared to its flows in Example 9; in contrast, Firm 2 now experiences decreases in all its product path flows.

The demand price of Firm 1's product at the first demand market is now: 572.69 and at the second demand market the price is: 595.02, with the respective equilibrium demands of: 36.76 and 40.50. The demand price of Firm 2's product at the first demand market is 597.99 and at the second demand market the price is: 677.25, with the respective equilibrium demands of: 21.75 and 52.50. The demand market prices of Firm 1's product rise whereas those of Firm 2's product only a very small amount.

Firm 1 earns a profit of: 29,975.13 and Firm 2 earns a profit of: 35,586.85. Firm 1 now benefits profit-wise from Firm 2 imposing wage ceilings, whereas Firm 2 suffers a decrease in profits as compared to Example 9 and Example 8. Firm 2 now, on three links pays its workers lower wages than does Firm 1 for the corresponding links.

The Lagrange multiplier associated with link p is: 246.32 and that associated with link r is: 7.32. All other Lagrange multipliers are equal to 0.00. Please refer to Table 5 for additional results on the equilibrium link flows, labor values, and hourly wages.

Example 11 – Two Firm Example, Wage Ceilings on All Supply Chain Network Links

Example 11 has the same data as Example 8 but now we impose wage ceilings of 15 on all the supply chain network links of both Firm 1 and Firm 2.

The modified projection method now computes the following equilibrium path flow pattern:

$$x^*_{p_1} = 17.08, \quad x^*_{p_2} = 17.06, \quad x^*_{p_3} = 18.94, \quad x^*_{p_4} = 18.92.$$

$$x^*_{p_9} = 10.82, \quad x^*_{p_{10}} = 10.93, \quad x^*_{p_{11}} = 26.20, \quad x^*_{p_{12}} = 26.30.$$

The demand price of Firm 1's product at the first demand market is now: 585.83 and at the second demand market the price is: 608.17, with the respective equilibrium demands of: 34.13 and 37.87. The demand price of Firm 2's product at the first demand market is 600.61 and at the second demand market the price is: 678.57, with the respective equilibrium demands of: 21.75 and 52.50.

The wages on links: e, p, and r are at the respective imposed wage ceilings with equilibrium Lagrange multipliers, respectively, of: 52.73, 248.95, and 6.01. Also, in this example, Firm 2 now pays lower wages than Firm 1 on two (both only two) links for analogous activities.

Firm 1 earns a profit of 29,836.55, whereas Firm 2 earns a profit of 35,713.00. Interestingly, with both firms imposing wage ceilings of the same value on all their supply chain network links, Firm 1

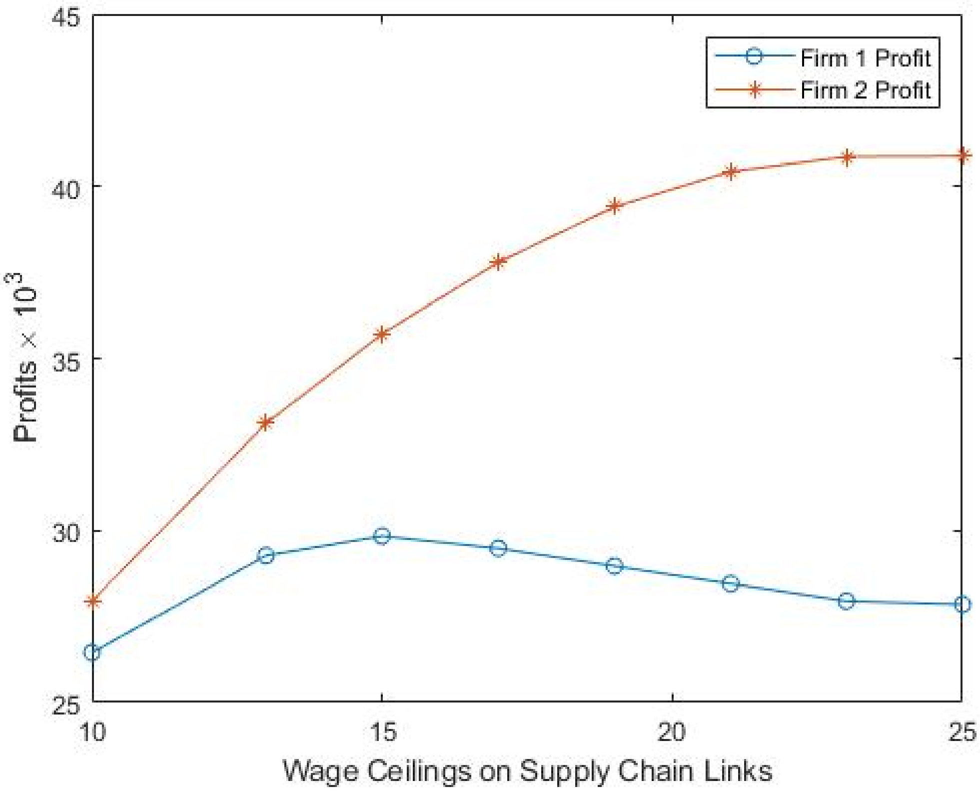

Figure 6: Sensitivity Analysis for Different Wage Ceilings and Effects in the Firms' Profits

earns a greater profit than it did in Example 8, with no wage ceilings but Firm 2, in contrast, earns a lower profit than it did in Example 8. It has two of its links with wages at the ceiling, whereas Firm 1 has only one of its links at the wage upper bound.

Please refer to Table 5 for additional results for this example.

We emphasize that the above numerical results demonstrate the importance of computing solutions to supply chain network problems using a full network perspective so that decisions made on one part of the supply chain and these could include those made by a specific firm, are also analyzed with respect to impacts on other firms. The above results are stylized but, nevertheless, reveal the type of information that can be obtained and assessed from product path and link flows to wages that should be paid, and the labor involved.

In Figure 6, we display the results of sensitivity analysis for Example 11. In particular, we report the profit of Firm 1 and of Firm 2, when the wage ceilings are at 10, for all links; at 13 for all links, and then at 15, 17, and 19, 21, 23, and 25 for all links.

As can be seen from Figure 6, the profit of Firm 2 continues to increase until it reaches a plateau, which is to be expected. The profit of Firm 1, in contrast, increases, and then decreases and then plateaus. The sensitivity analysis further demonstrates the importance of a holistic approach to supply chain network modeling, analysis, and computations.

4. Summary and Conclusions

The COVID-19 pandemic has vividly revealed the importance of labor to the functionality of supply chain networks and the associated products. As the world moves forward and, hopefully, one day before too long, past the pandemic, it is clear that labor and labor shortages are permeating the news. Many companies have started to raise wages in order to attract workers.

In this paper, we construct a supply chain network framework consisting of one or more firms engaged in the production, transportation, storage, and distribution of a product that is substitutable. The framework assumes linear production functions as well as labor availability that is an increasing function of the wages paid. The firms are profit-maximizers, compete, and seek to determine their optimal product flows on the supply chain network pathways to the demand markets, along with the wages that should be paid on the various supply chain network links, and the associated labor values. We consider a model without wage ceilings (upper bounds) and then a model with wage ceilings. The work is highly relevant since there is much discussion now as to what workers should be paid in the United States as well as in many other countries.

We present numerical examples for a single firm, followed by examples of multiple firms, which are solved using the proposed algorithm that has nice features at each iteration for implementation. The supply chain network models with wage-dependent labor, without and with wage ceilings, are all uniformly formulated and solved as variational inequality problems.

The numerical results clearly reveal the importance of a holistic approach to supply chain network modeling since decisions made by a specific firm can have unexpected impacts on other competing firms in the supply chain network economy. Furthermore, our results strongly suggest that having wages and labor equilibrate without any price ceilings is beneficial for an individual firm and also for firms engaged in competition.

This research adds to the application of variational inequalities to supply chain network problems of relevance in the COVID-19 pandemic and beyond and also advances the literature integrating concepts from operations research and economics.

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