Multicriteria Network Equilibrium Modeling with Variable Weights for Decision-Making in the Information Age with Applications to Telecommuting and Teleshopping

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December 2000; revised July 2001
Journal of Economic Dynamics and Control **26**: (2002), pp 1629-1650

Abstract: In this paper, we develop a multicriteria network equilibrium framework for modeling decision-making in the Information Age. We consider distinct classes of decision-makers, each of whom has a set of criteria associated with the decision along with weights which are variable and criterion-dependent. The decisions take place on a network in which links can be either physical, as in the case of transportation, or virtual, as in the case of telecommunications. We derive the equilibrium conditions and establish qualitative properties of the equilibrium pattern. The model enables the prediction of the number of decision-makers that will select particular choices, along with the incurred generalized costs. We then apply the modeling schema to telecommuting versus commuting and to teleshopping versus shopping decision-making.

Acknowledgments: This paper is dedicated to David A. Kendrick in recognition of his work in extending the frontiers of Computational Economics. This research was supported by NSF Grant No.: IIS-002647. This support is gratefully acknowledged. The authors are grateful to Professor Hans M. Amman and to the anonymous referee for helpful comments and suggestions.

1. Introduction

The advent of the Information Age with the increasing availability of new computer and communication technologies, along with the Internet, have transformed the ways in which many individuals work, travel, and conduct their daily activities today. Moreover, the decision-making process itself has been altered through the addition of alternatives which were not, heretofore, possible or even feasible. Indeed, as stated in a recent issue of *The Economist* (2000), "The boundaries for employees are redrawn... as people work from home and shop from work."

Related transformations have occurred in history through such technological innovations as the telegraph and telephone, railroads, electricity, the mass production of the automobile, and/or the introduction of air travel, accompanied by the construction of the underlying infrastructure (cf. Friedlander (1995a, b, 1996)). The Internet, however, may be viewed as being unique in the sense of its speed, very low cost, potential connectivity, and flexibility of usage. Interestingly, all of the above noted technological innovations have been network in nature, with links corresponding, typically, to either transportation links, such as in the case of roads for automobiles and other vehicles, or tracks as in the case of railroads, or to communication links, such as telephone or fiberoptic cables or radio links. Flows on such networks would correspond, respectively, to vehicles, or to messages.

The operation and use of such network systems, however, is done by humans and it is their behavior which affects both the effectiveness and the efficiency of the systems. For example, it is now well-recognized that congestion on urban road networks is a serious problem resulting in $100 billion in lost productivity in the United States alone annually with the figure being approximately $150 billion in Europe (cf. Nagurney (2000a)). Interestingly, congestion is also playing an increasingly prominent role in communication networks and recently discovered paradoxical phenomena therein are closely related to those occurring in transportation networks (cf. Korilis, Lazar, and Orda (1999) and Nagurney and Dong (2000a)).

Furthermore, it is the interaction between transportation and communication networks, and the individuals' use, thereof, which is of particular interest and relevance to the Information Age (see, e.g., Memmott (1963), Jones (1973), Khan (1976), Nilles, et al. (1976), Harkness (1977), Salomon (1986)). Indeed, as noted by Mokhtarian and Salomon (1997), in order to properly address transportation and telecommunication issues today one must ultimately include the transportation network and be able to forecast volumes of flow.

In this paper, we take on the challenge of developing a network equilibrium framework for decision-making in the Information Age. The modeling approach captures choices made possible through transportation and telecommunication mode alternatives. Moreover, it allows for the prediction of the volumes of flow in terms of decision-makers selecting particular choices and the effects of their choices on such possible criteria as time, cost, risk, and/or safety. A network equilibrium framework is natural since not only are now many of the relevant decisions taking place on networks but also the concept of a network – as we demonstrate in this paper – is sufficiently general in an abstract and mathematical setting to also capture many of the salient features comprising decision-making today. Abstract networks in which, for example, nodes need not correspond to physical locations in space and links to physical routes, have been used in a variety of settings in both economics and finance (cf. Nagurney (1999), Nagurney and Siokos (1997), and the references therein.

The first publications in the area of multicriteria decision-making on networks, specifically, transportation networks, were by Schneider (1968) and Quandt (1967). However, they assumed fixed travel times and travel costs. The first flow-dependent such model, which could, hence, handle congestion, was by Dafermos (1981), who assumed two criteria, whereas we consider a finite number. Moreover, our modeling framework considers also elastic demands. For a list of additional references, see Nagurney and Dong (2000b).

This paper is organized as follows. In Section 2, we develop the multiclass, multicriteria network equilibrium models with elastic demand and with fixed demand, respectively. Each class of decision-maker is allowed to have variable weights associated with the criteria which are also permitted to be link-dependent for modeling flexibility purposes. Nagurney and Dong (1999) had earlier introduced variable weights, but in the case of a financial equilibrium model in which each sector of the economy sought to both minimize risk and to maximize net return. We present the governing equilibrium conditions and also give the variational inequality formulations. Variational inequality formulations are necessary since there are no equivalent optimization reformulations of the equilibrium conditions.

In Section 3, we provide some qualitative properties of the equilibrium patterns, notably, existence and uniqueness. In Section 4, we illustrate the usefulness of the multicriteria, multiclass network equilibrim framework by applying it to two distinct areas: telecommuting versus commuting decision-making and teleshopping versus shopping decision-making. In Section 5, we describe how the equilibrium patterns can be computed. For additional background on algorithms and their applications to the solution of problems in economics see the handbook on computational economics edited by Amman, Kendrick, and Rust (1996) and the references therein. For research opportunities, see Kendrick (1993).

2. The Multiclass, Multicriteria Network Equilibrium Models with Variable Weights

In this section, we develop the multicriteria network equilibrium models with variable weights. We present first the elastic demand model and then the fixed demand model. The equilibrium conditions are, subsequently, shown to satisfy finite-dimensional variational inequality problems.

Consider a general network $G = [\mathcal{N}, \mathcal{L}]$, where $\mathcal{N}$ denotes the set of nodes in the network and $\mathcal{L}$ the set of directed links. Let a denote a link of the network connecting a pair of nodes and let p denote a path, assumed to be acyclic, consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. There are n links in the network and n_P paths. Let Ω denote the set of J O/D pairs. The set of paths connecting the O/D pair ω is denoted by P_ω and the entire set of paths in the network by P.

Note that in our framework a link may correspond to an actual physical link of transportation or an abstract or virtual link corresponding to telecommunications. Furthermore, the network representing the problem under study can be as general as necessary and a path may consist also of a set of links corresponding to a combination of physical and virtual choices. A path, hence, abstracts a decision as a sequence of links or possible choices from an origin node, which represents the beginning of the decision, to the destination node, which represents its completion.

Assume that there are k classes of decision-makers in the network with a typical class denoted by i. Let f_a^i denote the flow of class i on link a and let x_p^i denote the nonnegative flow of class i on path p. The relationship between the link loads by class and the path flows is:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{1}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. Hence, the load of a class of decision-maker on a link is equal to the sum of the flows of the class on the paths that contain that link.

In addition, let f_a denote the total flow on link a, where

$$f_a = \sum_{i=1}^{k} f_a^i, \quad \forall a \in \mathcal{L}. \tag{2}$$

Hence, the total load on a link is equal to the sum of the loads of all classes on that link. Group the class link loads into the kn-dimensional column vector $\tilde{f}$ with components: $\{f_a^1, \ldots, f_n^1, \ldots, f_a^k, \ldots, f_n^k\}$ and the total link loads: $\{f_a, \ldots, f_n\}$ into the n-dimensional column vector f. Group the class path flows into the kn_P-dimensional column vector $\tilde{x}$ with components: $\{x^1_{p_1}, \ldots, x^k_{p_{n_P}}\}$. The demand associated with origin/destination (O/D) pair ω and class i will be denoted by d^i_ω. We group the demands into a column vector $d \in \mathcal{R}^{kJ}$. Clearly, the demands must satisfy the following conservation of flow equations:

$$d^i_\omega = \sum_{p \in P_\omega} x^i_p, \quad \forall i, \forall \omega. \tag{3}$$

We are now ready to describe the functions associated with the links. In particular, we assume that there are H criteria which the decision-makers may utilize in their decision-making with a typical criterion denoted by h. We assume that C_{ha} denotes criterion h associated with link a, where we have that

$$C_{ha} = C_{ha}(f), \quad \forall a \in \mathcal{L}, \tag{4}$$

where C_{ha} is assumed to be a continuous function.

For example, criterion 1 may be time, in which case we would have

$$C_{1a} = C_{1a}(f) = t_a(f), \quad \forall a \in \mathcal{L}, \tag{5}$$

where $t_a(f)$ denotes the time associated with traversing link a. In the case of a transportation link, we would expect the function to be higher than for a telecommunications link. Another relevant criterion may be cost, that is, we may have:

$$C_{2a} = C_{2a}(f) = c_a(f), \quad \forall a \in \mathcal{L}, \tag{6}$$

which might reflect (depending on the link a) an access cost in the case of a telecommunications link, or a transportation or shipment cost in the case of a transportation link. We expect both time and cost to be relevant criteria in decision-making in the Information Age especially since telecommunications is at times a substitute for transportation and it is typically associated with higher speed and lower cost (cf. Mokhtarian (1990)).

In addition, another relevant criterion in evaluating decision-making in the Information Age is opportunity cost since one may expect that this cost would be high in the case of teleshopping, for example, (since one cannot physically experience and evaluate the product) and lower in the case of shopping. Furthermore, in the case of telecommuting, there may be perceived to be a higher associated opportunity cost by some classes of decision-makers who may miss the socialization provided by face-to-face interactions with coworkers and colleagues. Hence, a third possible criterion may be opportunity cost, where

$$C_{3a} = C_{3a}(f) = o_a(f), \quad \forall a \in \mathcal{L}, \tag{7}$$

with $o_a(f)$ denoting the opportunity cost associated with link a. Finally, a decision-maker may wish to associate a safety cost in which case the fourth criterion may be

$$C_{4a} = C_{4a}(f) = s_a(f), \quad \forall a \in \mathcal{L}, \tag{8}$$

where $s_a(f)$ denotes a security or safety cost measure associated with link a. In the case of teleshopping, for example, decision-makers may be concerned with revealing personal or credit information, whereas in the case of transportation, commuters may view certain neighborhood roads as being dangerous.

Assume that each class of decision-maker has a potentially different perception of the tradeoffs among the criteria, which are represented by the nonnegative weights: $w^i_{1a}, \ldots, w^i_{Ha}$. Hence, w^i_{1a} denotes the weight on link a associated with criterion 1 for class i, w^i_{2a} denotes the weight associated with criterion 2 for class i, and so on. Observe that the weights are link-dependent and can incorporate specific link-dependent factors which could include for a particular class factors such as convenience and sociability.

We propose weights associated with the links which are no longer fixed but are allowed to be variable. Moreover, we allow the weights associated with the class and link to be criterion-dependent, that is,

$$w^i_{ha} = w^i_{ha}(C_{ha}), \quad \forall h, a, i. \tag{9}$$

Nagurney and Dong (2000b) were the first to model link-dependent weights but they were assumed fixed and not variable as above. Nagurney, Dong, and Mokhtarian (2000a), in turn, used fixed, link-dependent weights but assumed only three criteria, in particular, travel time, travel cost, and opportunity cost in their integrated multicriteria network equilibrium models for telecommuting versus commuting.

We now state the following definition:

Definition 1: Criterion-Dependent Weight
A class- and link-dependent weight $w^i_{ha} = w^i_{ha}(C_{ha})$ is called a criterion-dependent weight for criterion h and for link a and class i, if it is strictly increasing, convex, smooth, and nonnegative.

Generalized cost functions (in the case of minimization) or value functions (in the case of maximization) have been studied extensively and used for decision problems with multiple criteria by numerous authors, including: Fishburn (1970), Chankong and Haimes (1983), Yu (1985), and Keeney and Raiffa (1993). In particular, assuming that there are q criteria to be minimized, that less is better for each, then $\tilde{C}$ is called a generalized cost function for the q criteria if it is a real-valued function defined on the set of all the possible outcomes such that $\tilde{C}(z_1^1, \cdots, z_q^1) < \tilde{C}(z_1^2, \cdots, z_q^2)$ if and only if $(z_1^1, \cdots, z_q^1)$ is prefered to $(z_1^2, \cdots, z_q^2)$. A constant additive weight generalized cost function is given by:

$$\tilde{C}(z_1, \cdots, z_q) = w_1 z_1 + \cdots + w_q z_q,$$

where the weights w_j's are independent of the z_j's.

Here, we propose a generalized cost function with variable weights and defined as follows. This is the first time that variable weights are proposed for decision-making in a network equilibrium context.

Definition 2: Generalized Cost Function with Variable Weights
We construct the generalized cost of class i associated with link a and denoted by C_a^i as:

$$C_a^i = \sum_{h=1}^{H} w_{ha}^i (C_{ha}) C_{ha}, \quad \forall i, \quad \forall a \in \mathcal{L}. \tag{10}$$

For example, the variable weights as described above have the interpretation of *penalizing* those criteria which are higher, or, in a sense, dominating. In lieu of (2) – (10), we may write

$$C_a^i = C_a^i(\tilde{f}), \quad \forall i, \quad \forall a \in \mathcal{L}, \tag{11}$$

and group the generalized link costs into the kn_L-dimensional column vector C with components: $\{C_a^1, \ldots, C_{n_L}^1, \ldots, C_a^k, \ldots, C_{n_L}^k\}$.

Let C_p^i denote the generalized cost of class i associated with path p in the network where

$$C_p^i = \sum_{a \in \mathcal{L}} C_a^i(\tilde{f}) \delta_{ap}, \quad \forall i, \quad \forall p. \tag{12}$$

Hence, the generalized cost associated with a class and a path is that class's weighted combination of the various criteria on the links that comprise the path.

Note from the structure of the criteria on the links as expressed by (4) and the generalized cost structure assumed for the different classes on the links according to (9) and (10), that we are explicitly assuming that the relevant criteria are functions of the total flows on the links, where recall that the total flows (see (2)) correspond to the total number of decision-makers of all classes that selects a particular link. This is not unreasonable since we can expect that the greater the number of decision-makers that select a particular link (which comprises a part of a path), the greater the congestion on that link and, hence, we can expect the time of traversing the link as well as the cost to increase.

In the case of the elastic demand model, we assume, as given, the inverse demand functions λ_ω^i for all classes i and all O/D pairs ω, where:

$$\lambda_\omega^i = \lambda_\omega^i(d), \quad \forall i, \forall \omega, \tag{13}$$

where these functions are assumed to be smooth and continuous. We group the inverse demand functions into a column vector $\lambda \in \Re^{kJ}$.

The Behavioral Assumption

We assume that the decision-making involved in the problem is repetitive in nature such as, for example, in the case of commuting versus telecommuting, or shopping versus teleshopping. The behavioral assumption that we propose, hence, is that decision-makers select their paths so that their generalized costs are minimized. (We do not, at this time, propose a tatonnement or adjustment process, although this can be done in our context through the use of the theory of projected dynamical systems, following Nagurney and Zhang (1996)). Specifically, the behavioral assumption utilized is similar to that underlying traffic network assignment models (see, e.g., Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969), and Dafermos (1982)) in that we assume that each class of decision-maker in the network selects a path so as to minimize the generalized cost on the path, given that all other decision-makers have made their choices. Such an idea was also used in the context of multicriteria traffic networks by Dafermos (1981), Leurent (1993a), Dial (1996), Marcotte (1998), and Nagurney (2000b), among others.

In particular, we have the following network equilibrium conditions for the problem outlined above:

Multicriteria Network Equilibrium Conditions for the Elastic Demand Case

For each class i, for all O/D pairs $\omega \in \Omega$, and for all paths $p \in P_\omega$, the flow pattern $\tilde{x}^*$ is said to be *in equilibrium* if the following conditions hold:

$$C_p^i(\tilde{f}^*) \begin{cases} = \lambda_\omega^i(d^*), & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_\omega^i(d^*), & \text{if} \quad x_p^{i*} = 0. \end{cases} \tag{14}$$

In other words, all utilized paths by a class connecting an O/D pair have equal and minimal generalized costs and the generalized cost on a used path by a class is equal to the inverse demand/travel disutility for that class and the O/D pair that the path connects.

In the case of the fixed demand model, in which the demands in (3) are now assumed known and fixed, the multicriteria network equilibrium conditions now take the form:

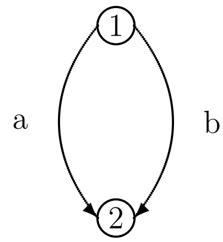

Figure 1: Network example for variable weights

Multicriteria Network Equilibrium Conditions for the Fixed Demand Case
For each class i, for all O/D pairs $\omega \in \Omega$, and for all paths $p \in P_\omega$, the flow pattern $\tilde{x}^*$ is said to be *in equilibrium* if the following conditions hold:

$$C_p^i(\tilde{f}^*) \begin{cases} = \lambda_\omega^i, & \text{if} \quad x_p^{i*} > 0 \\ \geq \lambda_\omega^i, & \text{if} \quad x_p^{i^*} = 0, \end{cases} \tag{15}$$

where now the λ_ω^i denotes simply an indicator representing the minimal incurred generalized path cost for class i and O/D pair ω. Equilibrium conditions (15) state that all used paths by a class connecting an O/D pair have equal and minimal generalized costs. Unused paths may have higher generalized costs.

An Example

In order to further emphasize the need for variable weights within this behavioral decision-making framework, we present a small illustrative example. For a depiction of the network structure of the problem, see Figure 1.

Assume that there is a single class of decision-maker faced with three criteria which are: time, cost, and opportunity cost. There are two possible options: a or b. Assume that the class of decision-maker weighs each of the criteria equally and assigns a fixed weight of $\frac{1}{3}$ to each criteria. Assume also that the induced criteria are as in Table 1.

According to the behavioral principle that proposes the selection of the path with minimal generalized cost and, assuming the equal fixed weights of $\frac{1}{3}$ as above, the decision-maker would select option a. However, it is reasonable that a rational decision-maker would prefer option b, since the opportunity cost associated with alternative a is so high relative to the other two criteria. A proper generalized cost function should, in this case, impose a higher weight for the opportunity cost as a penalty.

It is clear that, in practice, more attention will generally be paid to a particular criterion when it is higher. Hence, we have introduced generalized cost functions with variable weights

Table 1: Data for small example

Criteria	a	b
Time	10	45
Cost	5	45
Opportunity Cost	105	45
Weighted Average	40	45

which we believe may better model the weights associated with criteria than constant weights and produce more appropriate generalized cost functions. Of course, fixed weights are special cases of our more general weighting structure.

We now present the variational inequality formulations of the equilibrium conditions governing the elastic demand and the fixed demand problems, respectively, given by (14) and (15). In Section 3, we then present some qualitative properties of the solutions to the variational inequality problems.

Specifically, in light of Corollary 1 in Nagurney and Dong (2000b), we can write down immediately the variational inequality formulation below.

Theorem 1: Variational Inequality Formulation of the Elastic Demand Model
The variational inequality formulation of the multicriteria network model with elastic demand satisfying equilibrium conditions (14) is given by: Determine $(\tilde{f}^*, d^*) \in \mathcal{K}^1$*, satisfying*

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} C_a^i(\tilde{f}^*) \times (f_a^i - f_a^{i*}) - \sum_{i=1}^{k} \sum_{\omega \in \Omega} \lambda_\omega^i(d^*) \times (d_\omega^i - d_\omega^{i*}) \geq 0, \quad \forall (\tilde{f}, d) \in \mathcal{K}^1, \qquad (16a)$$

where $\mathcal{K}^1 \equiv \{(\tilde{f}, d) | \tilde{x} \geq 0,$ *and (1), (2), and (3) hold*$\}$*; equivalently, in standard variational inequality form (cf. Nagurney (1999)):*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \qquad (16b)$$

where $F \equiv (C, \lambda)$*,* $X \equiv (\tilde{f}, d)$*, and* $\mathcal{K} \equiv \mathcal{K}^1$*.*

Hence, a flow and demand pattern satisfies equilibrium conditions (14) if and only if it also satisfies the variational inequality problem (16a) or (16b).

Also, we have immediately due to the proof of Theorem 1 in Nagurney (2000a) the following result:

Theorem 2: Variational Inequality Formulation of the Fixed Demand Model
The variational inequality formulation of the fixed demand multiciteria network equilibrium

model satisfying equilibrium conditions (15) is given by: Determine $\tilde{f}^* \in \mathcal{K}^2$, *satisfying*

$$\sum_{i=1}^{k} \sum_{a \in \mathcal{L}} C_a^i(\tilde{f}^*) \times (f_a^i - f_a^{i*}) \geq 0, \qquad \forall \tilde{f} \in \mathcal{K}^2, \tag{17a}$$

where $\mathcal{K}^2 \equiv \{\tilde{f} | \exists \tilde{x} \geq 0,$ *and satisfying (1), (2), and (3), with d known*$\}$; *equivalently, in standard variational inequality form:*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17b}$$

where $F \equiv C$, $X \equiv \tilde{f}$, *and* $\mathcal{K} \equiv \mathcal{K}^2$.

Therefore, a flow pattern satisfies equilibrium conditions (15) if and only if it satisfies variational inequality (17a) or (17b).

Both (16) and (17) are finite-dimensional variational inequality problems. Finite-dimensional variational inequality formulations were also obtained by Nagurney (2000b) for her bicriteria fixed demand traffic network equilibrium model in which the weights were fixed and only class-dependent. Nagurney and Dong (2000b), in turn, formulated an elastic demand traffic network problem with two criteria and weights which were fixed but class- and link-dependent as a finite-dimensional variational inequality problem. The first use of a finite-dimensional variational inequality formulation of a multicriteria network equilibrium problem is due to Leurent (1993b), who, however, only allowed one of the two criteria to be flow-dependent. Moreover, although his model was an elastic demand model, the demand functions were separable and not class-dependent as are ours.

We note that a plethora of equilibrium problems including spatial price equilibrium problems, oligopolistic market equilibrium problems, as well as general financial equilibrium problems have been formulated and solved as finite-dimensional variational inequality problems (cf. Nagurney (1999)). Such an approach allows one to formulate and study equilibrium problems, the equilibrium conditions of which cannot be reformulated as the solutions to optimization problems. This allows one to study problems in which the underlying functions do not have symmetric Jacobians as was assumed in classical spatial price equilibrium problems (cd. Samuelson (1952) and Takayama and Judge (1971)) and traffic network equilibrium problems (cf. Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969)).

3. Qualitative Properties

We now derive some qualitative properties of the solutions to variational inequalities (16) and (17), in particular, existence and uniqueness results. We first present the existence results and then the uniqueness results.

We first consider the variational inequality (17) governing the fixed demand model. Noting that the feasible set $\mathcal{K}^2$ is compact and that the function C is assumed to be continuous, we have, immediately, from the standard theory of variational inequalities (see the books by Kinderlehrer and Stampacchia (1980) and Nagurney (1999)), the following existence result.

Theorem 3: Existence of an Equilibrium in the Case of the Fixed Demand Model *Let C_{ha} be given continuous functions, for all links $a \in \mathcal{L}$ and for all criteria h. Also, let the weights w^i_{ha} be as defined in Definitions 1–2 for all a, h, and i. Then, variational inequality (17a) (equivalently, (17b)) has at least one solution.*

In the case of the elastic demand model, however, the feasible set $\mathcal{K}^1$ is no longer compact as $\mathcal{K}^2$ is in the case of the fixed demand model. Nevertheless, we can impose the following conditions, under which the existence of a solution to variational inequality (16a) will be guaranteed. The conditions are generalizations of those used by Dafermos (1986) and Nagurney and Dong (2000b) to establish existence in their elastic demand models.

In particular, let C_{ha} be given continuous criterion functions for all h and a with the following properties: There exist positive numbers $\hat{C}$ and k_2, such that

$$C_{ha}(f) \geq \hat{C}, \quad \forall a, h, \quad \forall f \in \mathcal{K}^1, \tag{18}$$

$$\lambda^i_\omega(d) < k_1, \quad \forall i, \forall \omega, \quad \text{with} \quad d^i_\omega \geq k_2, \tag{19}$$

where $k_1 = \min_{h,i,a} w^i_{ha}(\hat{C})\hat{C} \geq 0$. Thus, we have that

$$C^i_a(\tilde{f}) \geq k_1, \quad \forall a, \forall i, \quad \forall f \in \mathcal{K}^1. \tag{20}$$

Condition (18) assumes only that the fixed parts of the criterion functions are not zero. Condition (19) assumes that the inverse demands would not be too large.

Referring now to Nagurney (1999) and to Dafermos (1986) (see also Nagurney and Dong (2000b)), and the references therein, we can immediately present the following result:

Theorem 4: Existence of an Equilibrium for the Model with Elastic Demand *Let C_{ha}, for all links a and for all criteria h, and λ be given continuous functions satisfying conditions (18) through (20). Then, variational inequality (16a) (equivalently (16b)) has at least one solution.*

We now turn to examining uniqueness for both models. Although we cannot expect uniqueness of an equilibrium multiclass link load pattern to hold, in general, we, nevertheless, show that, in a special case of the above model(s), we can establish uniqueness not of the vector of class link loads $\tilde{f}^*$ but, rather, of the total link loads f^*.

Specifically, consider linear weight functions for w^i_{ha}, for all a, h, i as follows: $w^i_{ha} = \psi^i_{ha} C_{ha}$, which yields the generalized cost function for each class i and each link a of the form:

$$C^i_a = \sum_{h=1}^{H} \psi^i_{ha} C^2_{ha}, \quad \forall a, \forall i, \tag{21}$$

and assume that each criterion function is of the special form: where

$$C_{ha} = g_a(f) + \alpha_{ha}, \quad \forall a \in \mathcal{L}, \forall h. \tag{22}$$

Note that (22) implies that criteria differ from one another by their fixed terms.

Assume now that g is strictly monotone, that is,

$$\langle (g(f^1) - g(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^1, \quad f^1 \neq f^2, \tag{23a}$$

and that g has a lower bound, that is,

$$g_a(f) \geq \hat{G} > 0, \quad \forall a, f, \tag{23b}$$

where $\hat{G}$ is a constant. Then, it is easy to prove that g^2 is strictly monotone, that is,

$$\langle (g^2(f^1) - g^2(f^2))^T, f^1 - f^2 \rangle > 0, \quad \forall f^1, f^2 \in \mathcal{K}^1, \quad f^1 \neq f^2. \tag{23c}$$

Also, assume that the inverse demand function λ is strictly monotone decreasing, that is,

$$-\langle (\lambda(d^1) - \lambda(d^2)), d^1 - d^2 \rangle > 0, \quad \forall d^1, d^2 \in \mathcal{K}^1, \quad d^1 \neq d^2. \tag{23d}$$

Thus, we have the following:

Theorem 5: Uniqueness of the Equilibrium Total Link Load Pattern for the Elastic Demand Model in a Special Case

The total link load pattern f^* induced by a solution $\tilde{f}^*$ to variational inequality (16a) in the case of generalized link cost functions C of the form (21) and (22), and the demand pattern d^* are guaranteed to be unique if g statisfies conditions (23a) and (23b), λ satisfies condition (23d), and $\sum_{h=1}^{H} \alpha_{ha} \psi^i_{ha} = \rho$, where ρ is a constant greater than or equal to zero, and $\sum_{h=1}^{H} \psi^i_{ha} = 1, \forall i, a$.

Proof: Assume that there are two solutions to variational inequality (16a) given by $(\tilde{f}', d')$ and $(\tilde{f}'', d'')$. Denote the total link load patterns induced by these class patterns through

(2) by f' and f'', respectively. Then, since $(\tilde{f}', d')$ is assumed to be a solution, we must have that

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\left[\psi_{1a}^{i}(g_a(f')+\alpha_{1a})^2+\psi_{2a}^{i}(g_a(f')+\alpha_{2a})^2+\ldots+(1-\sum_{h=1}^{H-1}\psi_{ha}^{i})(g_a(f')+\alpha_{Ha})^2\right]\times(f_a^i-f_a^{i'})$$

$$-\sum_{i=1}^{k}\sum_{\omega\in\Omega}\lambda_{\omega}^{i}(d')\times(d_{\omega}^{i}-d_{\omega}^{i'})\geq 0,\quad \forall(\tilde{f},d)\in\mathcal{K}^1. \tag{24}$$

Similarly, since $\tilde{f}''$ is also assumed to be a solution we must have that

$$\sum_{i=1}^{k}\sum_{a\in\mathcal{L}}\left[\psi_{1a}^{i}(g_a(f'')+\alpha_{1a})^2+\psi_{2a}^{i}(g_a(f'')+\alpha_{2a})^2+\ldots+(1-\sum_{h=1}^{H-1}\psi_{ha}^{i})(g_a(f'')+\alpha_{Ha})^2\right]\times(f_a^i-f_a^{i''})$$

$$-\sum_{i=1}^{k}\sum_{\omega\in\Omega}\lambda_{\omega}^{i}(d'')\times(d_{\omega}^{i}-d_{\omega}^{i''})\geq 0,\quad \forall(\tilde{f},d)\in\mathcal{K}^1. \tag{25}$$

Let $(\tilde{f}, d) = (\tilde{f}'', d'')$ and substitute into (24). Similarly, let $(\tilde{f}, d) = (\tilde{f}', d')$ and substitute into (25). Adding the two resulting inequalities, after algebraic simplifications, yields

$$\sum_{a\in\mathcal{L}}(g_a^2(f')-g_a^2(f''))\times(f_a'-f_a'')+\sum_{a\in\mathcal{L}}2\rho(g_a(f')-g_a(f''))\times(f_a'-f_a'')$$

$$-\sum_{i=1}^{k}\sum_{\omega\in\Omega}(\lambda_{\omega}^{i}(d')-\lambda_{\omega}^{i}(d''))\times(d_{\omega}^{i'}-d_{\omega}^{i''})\leq 0, \tag{26}$$

which is in contradiction to the assumption that g^2 and g are strictly monotone increasing and that λ is strictly monotone decreasing. Thus, we must have that $(f', d') = (f'', d'')$. $\square$

Note that, given the proof of Theorem 5, we can immediately obtain the analogous uniqueness result for the fixed demand model governed by variational inequality (16a). Indeed, we now state the following corollary:

Corollary 1: Uniqueness of the Equilibrium Total Link Load Pattern for the Fixed Demand Model in a Special Case

The total link load pattern f^* induced by a solution $\tilde{f}^*$ to variational inequality (17a) in the case of generalized link cost functions C of the form (21) and (22) is guaranteed to be unique if g statisfies conditions (23a) and (23b) and $\sum_{h=1}^{H}\alpha_{ha}\psi_{ha}^{i}=\rho$, where ρ is a constant greater than or equal to zero, and $\sum_{h=1}^{H}\psi_{ha}^{i}=1, \forall i, a$.

4. Applications

In this section, we describe two applications of the multiclass, multicriteria network equilibrium framework with variable weights introduced in Section 2. In Section 4.1, we apply the fixed demand multicriteria network equilibrium model to telecommuting versus commuting, whereas in Section 4.2, we apply the elastic demand one to teleshopping versus shopping.

4.1 Modeling Telecommuting versus Commuting Decision-Making

In this subsection, we apply the fixed demand model to telecommuting versus commuting decision-making. According to Hu and Young (1999, Tables 8 and 9), person-trips and person-miles of commuting increased between 1990 and 1995, both in absolute terms and as a share of all personal travel. Constituting 18% of all person-trips and 22% of all person-miles in 1995, commuting is the single most common trip purpose. Furthermore, as argued by Mokhtarian (1998) (see also Mokhtarian (1991)), it is very likely that a greater proportion of commute trips rather than other types of trips will be amenable to substitution through telecommunications. Consequently, telecommuting most likely has the highest potential for travel reduction of any of the telecommunication applications. Therefore, the study if telecommuting and its impacts is a subject worthy of continued interest and research. Furthermore, recent legislation that allows federal employees to select telecommuting as an option (see US (2000)), underscores the practical importance of this topic. The decision-makers in the context of this application are travelers, who seek to determine their *optimal* routes of travel from their origins, which are residences, to their destinations, which are their places of work.

Note that, in our framework, a link may correspond to an actual physical link of transportation or an abstract or virtual link corresponding to a telecommuting link. Furthermore, the network representing the problem under study can be as general as necessary and a path may also consist of a set of links corresponding to physical and virtual transportation choices such as would occur if a worker were to commute to a work center from which she could then telecommute. In Figure 2, we provide a conceptualization of this idea.

Observe that, in Figure 2, nodes 1 and 2 represent locations of residences, whereas node 6 denotes the place of work. Work centers from which workers can telecommute are located at nodes 3 and 4 which also serve as intermediate nodes for transportation routes to work. The links: $(1,6)$, $(3,6)$, $(4,6)$, and $(2,6)$ are telecommunication links depicting virtual transportation to work via telecommuting, whereas all other links are physical links associated with commuting. Hence, the paths $(1,6)$ and $(2,6)$ consisting, respectively, of the individual single links represent "going to work" virtually whereas the paths consisting of the links: $(1,3)$, $(3,6)$ and $(2,4)$, $(4,6)$ represent first commuting to the work centers located at nodes 3 and 4, from which the workers then telecommute. Finally, the remaining paths represent

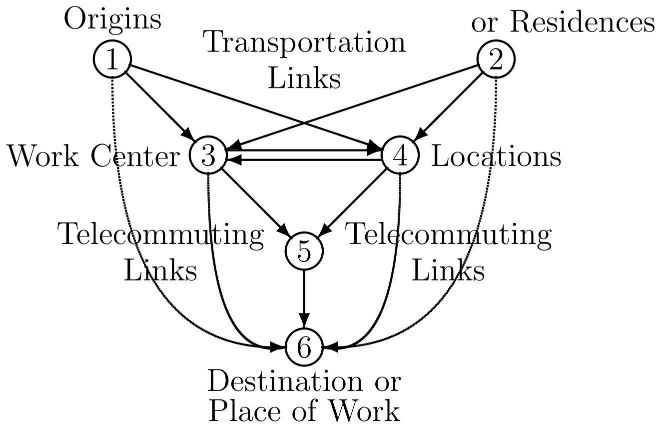

Figure 2: A network conceptualization of commuting versus telecommuting

the commuting options for the residents at nodes 1 and 2. The conventional travel routes from node 1 to node 6 are as follows: (1,3), (3,5), (5,6); (1,3), (3,4), (4,5), (5,6); (1,4), (4,5), (5,6), and (1,4), (4,3), (3,5), (5,6). Note that there may be as many classes of users of this network as there are groups who perceive the tradeoffs among the criteria in a similar fashion.

Of course, the network depicted in Figure 2 is illustrative, and the actual network can be much more complex with numerous paths depicting the physical transportation choices from one's residence to one's work location. Similarly, one can further complexify the telecommunication link/path options. Also, we emphasize, that a *path* within this framework is sufficiently general to also capture a choice of mode, which, in the case of transportation could correspond to busses, trains, or subways (i.e., public transit) and, of course, to the use of cars (i.e., private vehicles) (see also, e.g., Dial (1996)). Similarly, the concept of path can be used to represent a distinct telecommunications option.

In the model, since the decision-makers are travelers, the path flows and link flows by class would correspond, respectively, to the number of travelers of the class selecting a particular path and link. Hence, the conservation of flow equations (1) and (2) would apply and since we have assumed a fixed demand model (of course, one could also consider an elastic demand version, which would have location choice implications), the expression (3) must also be satisfied, with the travel demand d^i_ω associated with class i traveling between origin/destination pair ω assumed known and given.

The Criteria

We now turn to a discussion of the criteria, which we expect to be reasonable in the context of decision-making in this particular application. Recall that the first multicriteria traffic network models, due to Schneider (1968) and Quandt (1967), considered two criteria and these were travel time and travel cost. Of course, telecommuting was not truly an option in those days. Dafermos (1981), Leurent (1993a, b), Marcotte (1998), as well as Nagurney (2000b) also considered those two criteria. Nagurney, Dong, and Mokhtarian (2000a), in turn, focused on the development of an integrated multicriteria network equilibrium model, which was the first to consider telecommuting versus commuting tradeoffs. They considered three criteria: travel time, travel cost, and an opportunity cost to trade-off the oppportunity cost associated with not being physically able to interact with colleagues. Here, we now propose variable weights and a fourth criterion, that of safety. We note, however, that the network equilibrium model with fixed demands in Section 2 can actually handle any number of criteria, provided that the number is finite.

Hence, we propose the four criteria, given by (5) through (8), and representing, respectively, travel time, travel cost, the opportunity cost, and safety cost. We consider variable weights for each class, link, and criterion given by (9) and a generalized link cost for each class given by (10). Thus, the generalized cost on a path as perceived by a class of traveler is given by (12). Variable weights are quite reasonable for this application, since, for example, if the opportunity cost associated with telecommuting gets too high, certain classes of decision-makers may opt to commute, instead.

The behavioral assumption is that travelers of a particular class are assumed to choose the paths associated with their origin/destination pair so that the generalized cost on that path is minimal. An equilibrium is assumed reached, hence, when the multicriteria network equilibrium conditions (15) are satisfied. Hence, only those paths connecting an O/D pair are utilized such that the generalized costs on the paths, as perceived by a class, are equal and minimal. The governing variational inequality for this problem is given by (17a); equivalently, by (17b), and with the existence result following from Theorem 2 and the uniqueness result from Corollary 1.

4.2 Modeling Teleshopping versus Shopping Decision-Making

In this subsection, we propose a multicriteria network equilibrium model for teleshopping versus shopping. The model generalizes the model proposed in Nagurney, Dong, and Mokhtarian (2000b) to the case of elastic demands and variable weights. Furthermore, we allow destinations, which will now correspond to locations where the commodity is received, to not necessarily correspond to the same origin at which the shopping experience was initiated. Moreover, we allow the number of origins to be distinct from the number of destinations.

Although there is now a growing body of transportation literature on telecommuting (cf.

Mokhtarian (1998)), the topic of teleshopping, which is a newer concept, has received less attention to date. In particular, shopping refers to a set of activities in which consumers seek and obtain information about products and/or services, conduct a transaction transferring ownership or right to use, and spatially relocatethe product or service to the new owner (Mokhtarian and Salomon (1997)). Teleshopping, in turn, refers to a case in which one or more of those activities is conducted through the use of telecommunication technologies. Today, much attention is focused on the Internet as the technology of interest, and Internet-based shopping is, indeed, increasing. In this setting, teleshopping represents the consumer's role in "Business-to-Consumer" (B2C) electronic commerce. Although the model we propose in this subsection is in the context of Internet-based shopping, the model can apply more broadly.

We note that outside the work of Nagurney, Dong, and Mokhtarian (2000b), there has been essentially no study of the transportation impacts of teleshopping beyond speculation (e.g., Gould (1998), Mokhtarian and Salomon (1997)).

We assume that consumers are engaged in the purchase of a product which they do so in a repetitive fashion, say, on a weekly basis. The product may consist of a single good, such as a book, or a bundle of goods, such as food. We also assume that there are locations, both virtual and physical, where the consumers can obtain information about the product. The virtual locations are accessed through telecommunications via the Internet whereas the physical locations represent more classical shopping venues such as stores and require physical travel to reach.

The consumers may order/purchase the product, once they have selected the appropriate location, be it virtual or physical, with the former requiring shipment to the consumers' locations and the latter requiring, after the physical purchase, transportation of the consumer with the product to its final destination (which we expect, typically, to be his residence or, perhaps, place of work).

We refer the reader to the network conceptualization of the problem given in Figure 3 and further identify the above concepts with the corresponding network component. The idea of such a shopping network was proposed in Nagurney, Dong, and Mokhtarian (2000b). Here, however, we propose several generalizations.

Observe that the network depicted in Figure 3 consists of four levels of nodes with the first (top) level and the last (bottom) level corresponding to the locations (destinations) of the consumers involved in the purchase of the product. There are a total of $m+2N+M$ nodes in the network with the number of consumer locations (origins) given by m and the number of information locations given by N where N also corresponds to the number of shopping sites.

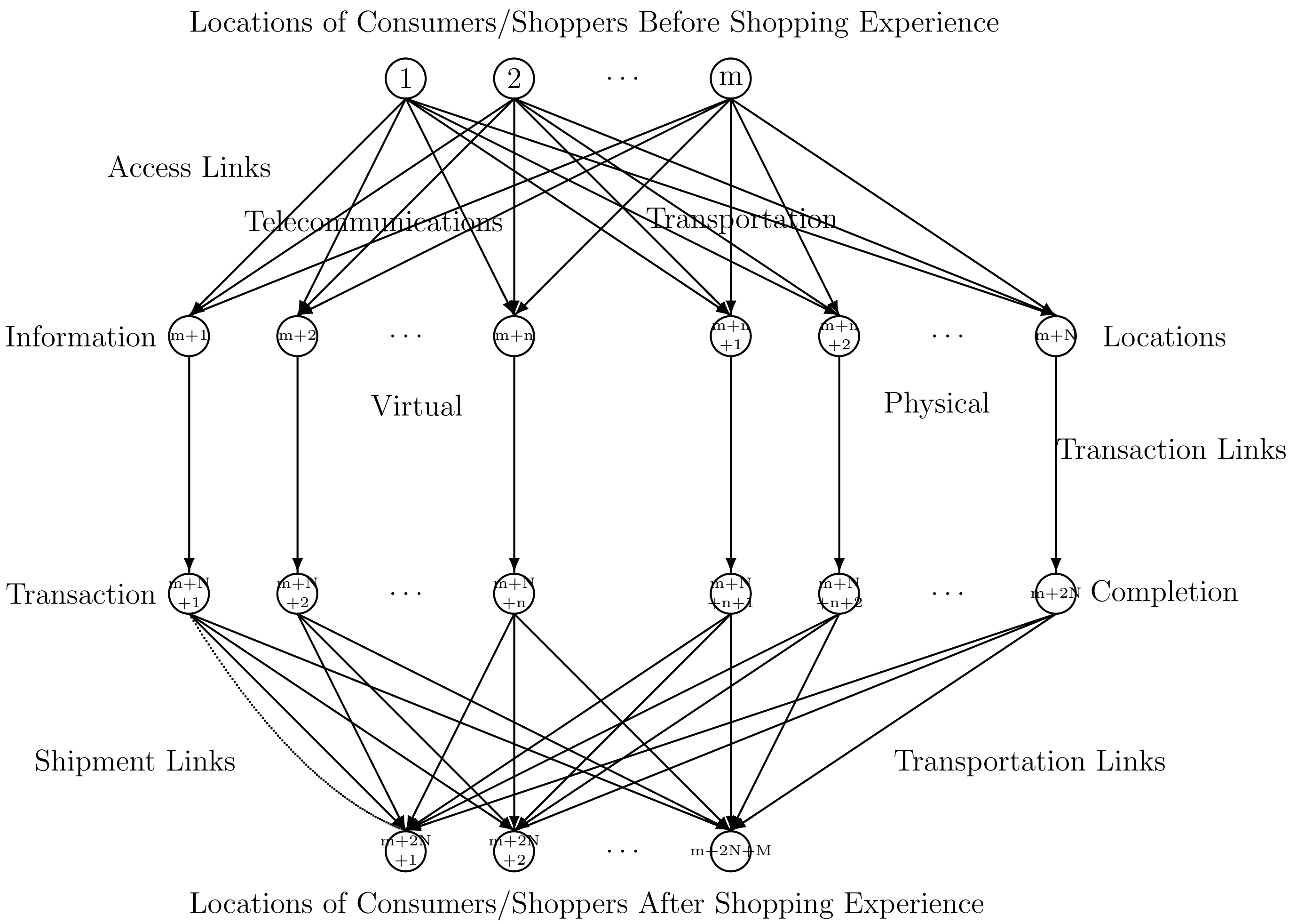

Figure 3: A Network Framework for Teleshopping versus Shopping

The number of consumer locations associated with the destinations is given by M. We denote the consumer location nodes (before the shopping experience) at the top level of nodes by: $1, \ldots, m$, with a typical such node denoted by j. We emphasize that each location may have many consumers. The second level of nodes, in turn, corresponds to the information locations (and where the transactions also take place), with nodes: $m+1, \ldots, m+n$ representing the virtual or Internet-based locations and nodes: $m+n+1, \ldots, m+N$ denoting the physical locations of information corresponding to stores, for example. Such a typical node is denoted by κ. The third level of nodes corresponds to the completion of the transaction with nodes: $m+N+1, \ldots, m+N+n$ corresponding to Internet sites where the product could have been purchased (and where we have assumed information has also been made available in the previous level of nodes) and nodes: $m+N+n+1, \ldots, m+2N$ corresponding to the completion of the transaction at the physical stores. A typical such node is denoted by l. The bottom level of the nodes are enumerated as: $m+2N+1, \ldots m+2N+M$ and denote the locations of the consumers following the completion of the shopping experience. Note that we have, for flexibility purposes, let the number of nodes in the top level be distinct from the number at the bottom level. In Nagurney, Dong, and Mokhtarian (2000b), it was assumed that these two numbers were the same and that consumers who ordered from a particular location had the commodity either shipped to that same location or purchased the product and then physicalled went to that location.

We now discuss the links connecting the nodes in the network in Figure 3. There are four sets of links in the network. A typical link (j, κ) connecting a top level node (consumers' location) j to an information node κ at the second level corresponds to an *access* link for information. The links terminating in nodes: $m+1, \ldots, m+n$ of the second level correspond to telecommunication access links and the links terminating in nodes: $m+n+1, \ldots, m+N$ correspond to (aggregated) transportation links.

As can be seen from Figure 3, from each second tier node κ there emanates a link to a node l, which corresponds to a completion of a transaction node. The first mn such links correspond to virtual orders, whereas the subsequent links denote physical orders/purchases. Finally, we have links emanating from the transaction nodes to the consumers' (final) destination nodes, with the links emanating from transaction nodes: $m+N+1, \ldots, m+N+n$ denoting shipment links (since the product, once ordered, must be shipped to the consumer), and the links emanating from transaction nodes: $m+N+n+1, \ldots, m+2N+M$ representing physical transportation links to the consumers' destinations. Note that, in the case of the latter links, the consumers (after purchasing the product) transport it with themselves, whereas in the former case, the product is shipped to the consumers. Observe that in our network framework we explicitly allow for alternative modes of shipping the product which is represented by an additional link (or links) connecting a virtual transaction node with the consumers' location.

We emphasize that the above network construction captures the *electronic dissemination* of goods (such as books or music, for example) in that an alternative shipment link in the bottom tier of links may correspond to the virtual or electronic shipment of the product.

Having fixed the above ideas we are now ready to present the notation which will allow us to clarify the costs, demands, and flows on the network. In addition, we introduce the behavior of the shoppers, who are assumed to be multicriteria decision-makers. Recall that, as mentioned in the Introduction, the shoppers can now shop from work and have their purchase delivered either to their work or to their home location.

An origin/destination pair in this network corresponds to a pair of nodes from the top tier in Figure 3 to the bottom tier. In the shopping network framework, a path consists of a sequence of choices made by a consumer. For example, the path consisting of the links: $(1, m+1), (m+1, m+N+1), (m+N+1, m+2N+1)$ would correspond to consumers located at location 1 accessing virtual location $m+1$ through telecommunications, placing an order at the site for the product, and having it shipped to them. The path consisting of the links: $(m, m+N)$, $(m+N, m+2N)$, and $(m+2N, m+2N+M)$, on the other hand, could reflect that consumers at location m (which could be a work location or home) drove to the store at location $m+N$, obtained the information there concerning the product, completed the transaction, and then drove to node M. Note that a path represents a sequence of possible options for the consumers. The flows, in turn, reflect *how many* consumers of a particular class actually select the particular paths and links, with a zero flow on a path corresponding to the situation that no consumer elects to choose that particular sequence of links.

The conservation of flow equations associated with the different classes of shoppers are given by (1) (2), and (3).

The Criteria

The criteria that we believe are relevant to decision-making in this application are: time, cost, opportunity cost, and safety or security risk, that is, (5) through (8), where, in contrast to the telecommuting application time need not be restricted simply to *travel* time and, depending on the associated link may include transaction time. In addition, the cost is not exclusively a travel cost but depends on the associated link and can include the transaction cost as well as the product price, or shipment cost. Moreover, the opportunity cost now arises when shoppers on the Internet cannot have the physical experience of trying the good or the actual sociableness of the shopping experience itself. Finally, the safety or security risk cost now can reflect not only the danger of certain physical transportation links but also the potential of credit card fraud, etc.

We assume variable weights for each class, link, and criterion given by (9) and a generalized link cost for each class given by (10). The generalized path cost for a class of consumer is given by (12).

Also, we assume that we are given inverse demand functions which reflect the "price" that the consumers of each class and O/D pair are willing to pay for the shopping experience as a functions of demand. Hence, we assume inverse demand functions of the form (13).

The behavioral assumption is that consumers of a particular class are assumed to choose the paths associated with an O/D pair so that their generalized path costs are minimal. An equilibrium, hence, in the elastic demand model must satisfy conditions (14), which also require that if there is positive demand for a class and O/D pair, then the minimum generalized path cost is equal to the inverse demand for that class and O/D pair. The governing variational inequality is given by (16a); equivalently, (16b). The existence result appears in Theorem 4, and the uniqueness result in Theorem 5.

5. A Computational Procedure

In this section, an algorithm is presented which can be applied to solve any variational inequality problem in standard form, that is, Determine $X^* \in \mathcal{K}$, satisfying:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

The algorithm is the modified projection method of Korpelevich (1977) and is guaranteed to converge provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists). Existence for the models has been established in Theorems 3 and 4. Lipschitz continuity of F (cf. Nagurney (1999)) for the elastic demand model follows under the assumptions that the generalized link cost functions C and the inverse demand functions λ have bounded first-order derivatives. For the fixed demand model, only the former condition need to be satisfied. Monotonicity, on the other hand, can be obtained using similar arguments as preceding the uniqueness Theorem 5 and Corollary 1, but assuming monotonicity, rather than strict monotonicity of the functions.

The statement of the modified projection method is as follows, where $\mathcal{T}$ denotes an iteration counter:

Modified Projection Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and let γ be a scalar such that $0 < \gamma < \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. Korpelevich (1977)).

Step 1: Computation
Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (\bar{X}^{\mathcal{T}} + \gamma F(X^{\mathcal{T}-1})^T - X^{\mathcal{T}-1})^T, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{27}$$

Step 2: Adaptation
Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (X^{\mathcal{T}} + \gamma F(\bar{X}^{\mathcal{T}})^T - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{28}$$

Step 3: Convergence Verification
If max $|X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} =: \mathcal{T} + 1$, and go to Step 1.

Numerical examples, for the sake of brevity, will be presented elsewhere.

In this paper, we have presented a conceptual framework, which is based on theoretical rigor, for modeling decision-making in the Information Age. The modeling schema is a multicriteria network equilibrium one in which there can be a finite number of criteria used in decision-making and each class of decision-maker can assign weights to the criteria which are link- and class-dependent and variable. We presented both an elastic demand and a fixed demand model, gave the variational inequality formulations of the governing equilibrium conditions, and established existence and uniqueness results. We then applied the framework to telecommuting versus commuting and to teleshopping versus shopping. Finally, we proposed an algorithm and discussed convergence for the computation of solutions to such problems.

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