

Supply Chain Network Competition
in
Time-Sensitive Markets

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Abstract: We develop a game theory model for supply chain network competition in time-sensitive markets in which consumers respond to the average delivery time associated with the various firms' products. The firms' behavior is captured, along with the supply chain network topologies, with the governing equilibrium concept being that of Nash equilibrium. We derive the variational inequality formulation of the equilibrium conditions and provide illustrative examples. We also identify special cases for distinct applications. An algorithm is proposed, and the framework further illustrated through a case study in which we explore varying sensitivities to the average time delivery with interesting results.

Keywords: supply chains, logistics, networks, time-based competition, game theory, freight services, delay tolerant networks, information asymmetry, variational inequalities

1. Introduction

Timely deliveries of products are essential not only to consumers but to a company's reputation and bottom line. If products are not delivered in a timely manner, as illustrated by the December 2013 holiday season shipping fiasco in the United States, the unfulfilled demands may result in a tremendous loss of good will, anger, frustration, economic losses due to perished and spoiled products, and the potential loss of future business (cf. Ng and Stevens (2014)). A similar example from Sweden was when Christmas trees were home-delivered three days after Christmas due to shipping problems (Ullgren (1999)). Logistics providers commonly face delivery delays. For example, a large freight train operator reports that 5% of its trains are more than 1 hour delayed (GreenCargo (2014)), causing many concerns in the industry about unreliable transport and forcing customers to make costly contingency plans. At the same time, reliability is ranked as one of the most important factors when selecting transport solutions (cf. Laitila and Westin (2000), Maier et al. (2002), Vannieuwenhuysse and Gelder (2003), Danielis et al. (2005), and Floden (2010)).

Hence, in supply chain management, the average time from the placement of an order to a product's delivery has emerged as a key performance indicator (KPI), signifying how effective and efficient a supply chain network is and providing a valuable metric as to the time a product spends in the system. As noted in Ketchen, Jr. et al. (2008), in the 1990s, the average time to fulfill customer orders was measured in weeks, while today, delivery times are being measured in days, or, in some cases, in hours.

In the past decade, in order to increase responsiveness, and to enable comparisons across supply chains, Intel introduced the average delivery time, the Order Fulfillment Lead Time (OFLT), as a KPI (see Hensley et al. (2012)). In military and defense, the Logistics Management Institute (cf. Klapper et al. (1999)) refers to this KPI as the Logistics Response Time (LRT), measured, typically, in days. This supply chain performance measure is one of the four that has been utilized by the U.S. Department of Defense. Moreover, the average time for delivery of critical needs supplies, such as water, food, medicines, and even shelter, in disasters, is also an important performance measure for humanitarian relief chains (cf. Beamon (2008); see also Sheu (2007) and Nagurney, Masoumi, and Yu (2014)). Finally, for pharmaceuticals, more responsive time-efficient supply chains can save lives. As noted in Darabi (2013), the Kenya pharma supply chain improved the average time for sea and air shipments to clear customs from 21.8 days to 5.7 days, and from 2.5 days to 1.8 days, respectively, resulting in more timely deliveries of HIV/AIDS drugs to Kenyans. However, much work still remains globally. The World Bank Logistics Performance Index shows that the median import lead time is more than 3.5 times longer in low performing countries than

in high performing countries (Arvis et al. (2012)).

Because of the recognized competitive advantages associated with speed and timely deliveries (Boyaci and Ray (2003, 2006)), firms are increasingly differentiating their products to include time for delivery and consumers are responding. Online retailers, such as Amazon, have fulfillment/delivery options that often trade-off cost and speed of delivery although it can be noted that customers are not universally willing to pay much extra for faster deliveries (Lukic et al. (2013)), which puts high demands on the supply chain design. Package delivery services, such as the U.S. Post Office, UPS, FEDEX, and DHL, routinely offer multiple delivery options with reduced delivery time coming at increased shipping cost. Similar development has long existed in passenger transport, where, for example, both regular and high speed rail services are offered to the same destinations but at different prices.

Recognition of the criticality of timely deliveries has given rise to increased attention to time-based competition from both practitioners and academics (cf. Stalk (1988), Stalk and Hout (1990), Blackburn et al. (1992), Hum and Sim (1996), So and Song (1998), Blackburn (2012), Nagurney and Yu (2014)). In addition, time-based competition is also relevant to services (So (2000)) including services associated with information and data, such as online content distribution, online commerce, web hosting, etc.

Markets in which consumers are willing to pay a higher price for lower delivery times are referred to as being *time-sensitive*. In the case of perishable products, such as fresh produce, lower average delivery times often correspond to higher product quality (Yu and Nagurney (2013)). The same holds true for certain pharmaceuticals and blood, whose quality may deteriorate with age (see, e.g., Nagurney et al. (2013)). Companies will often make a trade-off between transport time and other characteristics when designing their supply chains. For example, Evans and Harrigan (2005) showed that the U.S. apparel industry adapts to the increasingly rapid fashion changes by moving production where timeliness is important from lower wage locations in Asia to higher wage locations in Mexico and the Caribbean that are closer to the U.S. consumer, thus trading cost for time. Another well-known example is Benneton that uses a postponement strategy by delaying the dying of garments during production to be able to respond faster to trends (Dapiran (1992)). A recent vivid example is the demand for apparel and related products based on the top-grossing Disney animated film *Frozen*, with shipments from manufacturing plants in China increasingly being airlifted because of the demand and willingness of consumers to pay higher prices (see Palmeri (2014)).

Interestingly, for data and information services in isolated, rural, and/or developing parts of the globe, models of delay tolerant networks have been constructed in which both commu-

nication and transportation are used for delivery (see Pentland, Fletcher, and Hasson (2004) and Marentes et al. (2014)). Transportation, in the form of buses, motorcycles, and bicycles, is often used for the mechanical backhaul of the information delivery media. In such delay tolerant networks, where cost and infrastructure play important roles, pricing is also done in accordance with the average time of delivery (see Marentes et al. (2014)). Furthermore, time-based competition is expected to play an important role in future Internet architectures (see, e.g., Rouskas et al. (2013)) in which consumers may be willing to pay higher prices for quicker content deliveries.

Today's supply chains span the globe, which may entail production on one or more continents, transportation over land and sea, storage at multiple locations, and distribution to geographically dispersed demand markets. Large demands at a demand market may be supplied by a firm that has multiple manufacturing plants, multiple distribution centers, and uses multiple modes of transportation. Hence, different pathways in a firm's supply chain network may have been utilized to provide consumers with the product and each of these pathways may have an associated different time. Competition today takes place between supply chains and not individual companies (cf. Zhang (2006) and Christopher (2011)) with the end customer basing his preference on the product delivered and its associated characteristics, such as delivery time. However, given today's complex supply chain network topologies, consumers may not always be aware of the precise manufacturing plant, distribution center, and mode of transport between the two, that were used for a product; nor may they know of the time associated with supply chain network activities such as manufacturing, storage, and distribution. Hence, there may exist information asymmetry. Nevertheless, consumers at demand markets are aware of the average time for the delivery of the ordered products, and respond accordingly through the prices that they are willing to pay. Although there has been work done on information asymmetry in product quality dating to the work of the Nobel laureate Akerlof (1970) (see, also, Spence (1975), Stiglitz (1987), Nagurney and Li (2013), and the references therein) information asymmetry in terms of time has not been explored rigorously in a supply chain network context.

In this paper, we focus on supply chain network oligopolistic competition in which firms produce substitutable, but differentiated, products and, hence, consumers respond to each firm's product as a separate brand. The model that we develop captures the time associated with the various supply chain network activities of manufacturing (production), transportation, storage, and ultimate distribution to the demand markets. Each firm can possess multiple manufacturing plants, may have access to multiple distribution centers, serves multiple demand markets and can avail itself of multiple transport options. When it comes to the

latter some may be owned in-house and some may be purchased/contracted for as in the case of third party logistics providers. The setting depends on the specific application. Hence, depending on the specific application, the network topology can be adapted accordingly, as we also demonstrate. The link time consumption functions can depend on the volume of link flow since it may take longer to produce greater quantities and to ship them (due, for example, to loading and unloading, etc.); and the same holds for ultimate distribution. Consumers reflect their preferences for the brands through the demand price functions, which, in general, can depend on the entire demand pattern and on the average times associated with delivery at the demand markets of all the products. These can also be adapted for the application of concern. In contrast to the competitive supply chain network model of Nagurney and Yu (2014), which also made use of link time consumption functions, this new model is not limited to a single path from a firm to a demand market. Moreover, here we do not consider time for delivery as a strategic variable in our game theoretic framework but, rather, as a product characteristic that consumers react to, with the understanding that the average time is the characteristic (as average quality is in the framework of quality competition under information asymmetry according to Akerlof (1970)).

We consider the operations of the competing firm's supply chains through their various supply chain network activities, which can include production, storage/distribution, as well as transportation and shipment. We highlight our earlier work on supply chain network design, which, nonetheless, did not consider time issues (cf. Nagurney and Nagurney (2010), Nagurney (2010)). It is also worth noting the work of Lin and Chen (2008), who, in contrast, consider the network design problem for time-definite freight delivery of common carriers, a third party logistics service provider in supply chains, and propose a generalized hub-and-spoke network.

The paper is organized as follows. In Section 2, we develop the game theory model for supply chain network competition in time-sensitive markets in which consumers react to the average time of delivery associated with the various firms' products which represent brands. The behavior of the firms is captured, along with the supply chain network topologies, and the governing equilibrium concept is that of Nash (1950, 1951) equilibrium. We derive the variational inequality formulation of the equilibrium conditions and present an existence result. Examples of industries in which our framework is relevant include food and tobacco industries in the U.S. as well as vaccine production. For example, Bhuyan and Lopez (1997) demonstrated empirically that 37 out of 40 food and tobacco manufacturing industries in the U.S. exhibited substantial oligopoly power. These included such industries as the cereal preparation industry which ranked at the top of those studied with the highest degree of

oligopoly power, with flour and grain, soft drinks, distilled liquor, and pickled sauces also having high degrees of oligopoly power. The dried fruit and vegetable industries were found to have the lowest degree of oligopoly power in the same study with such perishable and time-sensitive food items as poultry, butter, and cheese. Winfree et al. (2004) also identified oligopoly power in the pear industry, which would fit our framework since fresh fruit is perishable. Also, as noted by the World Health Organization (2014), since there are very few manufacturers of vaccines that meet international quality standards, many of the individual vaccine markets are in the form of oligopolies. Vaccines are clearly another time-sensitive product (see also Nagurney et al. (2013)) that fits into our framework. Furthermore, we note that, as noted by Cortez (2012), the fashion and luxury goods market structure is an oligopoly by nature, with fast fashion (cf. Nagurney and Yu (2012)) representing a time-sensitive product fitting the supply chain network model developed in this paper.

In Section 3, we provide several illustrative examples. We also identify how this framework may be used in distinct applications, such as in delay tolerant networks and web hosting.

In Section 4, we propose a computational procedure and further illustrate our framework through a case study in which we explore varying sensitivities to the average time of delivery of the products at the demand markets. We find that, as the sensitivity to the average delivery time at a demand market increases, the firms respond accordingly but at the expense of decreased profits. Interestingly, we find that the average time for delivery at the other demand market also decreases, although not as significantly. The case study demonstrates the usefulness of the theoretical and computational framework. In Section 5, we summarize our results and present our conclusions.

2. The Supply Chain Network Model for Time-Sensitive Markets

In our framework, the various supply chain activities of each competing firm, which consist of production, transport/shipment, storage, and distribution of a substitutable but, differentiated, product are represented as links in a supply chain network. The profit-maximizing firms compete in a noncooperative manner in an oligopolistic fashion to provide their products to the demand markets. The supply chain network economy is represented by all the firms' supply chain networks, as depicted in Figure 1. Each of the supply chain activities is time-consuming and Figure 1 also depicts the progression in time of the various supply chain activities as a product moves on a path of the network from an origin node, represented by a firm, to a destination node, represented by a demand market. The firms have perfect information available as to the product flows on the paths as well as the associated times. However, the consumers, located at the demand markets, are only aware of the *average time*

for the delivery of the products at the demand markets, and respond to the average time of receipt of the products through the demand price functions. This information asymmetry is reasonable since consumers may not have the information (or resources) to track the products as they proceed through the supply chain network from the origin(s) to the destination(s). Moreover, they may not even be interested in such specifics.

L^i denotes the set of directed links representing the supply chain network economic activities associated with firm i ; $i = 1, \dots, I$ and $G = [N, L]$ denotes the graph consisting of the set of nodes N and the set of links L in Figure 1, where $L \equiv \cup_{i=1, \dots, I} L^i$.

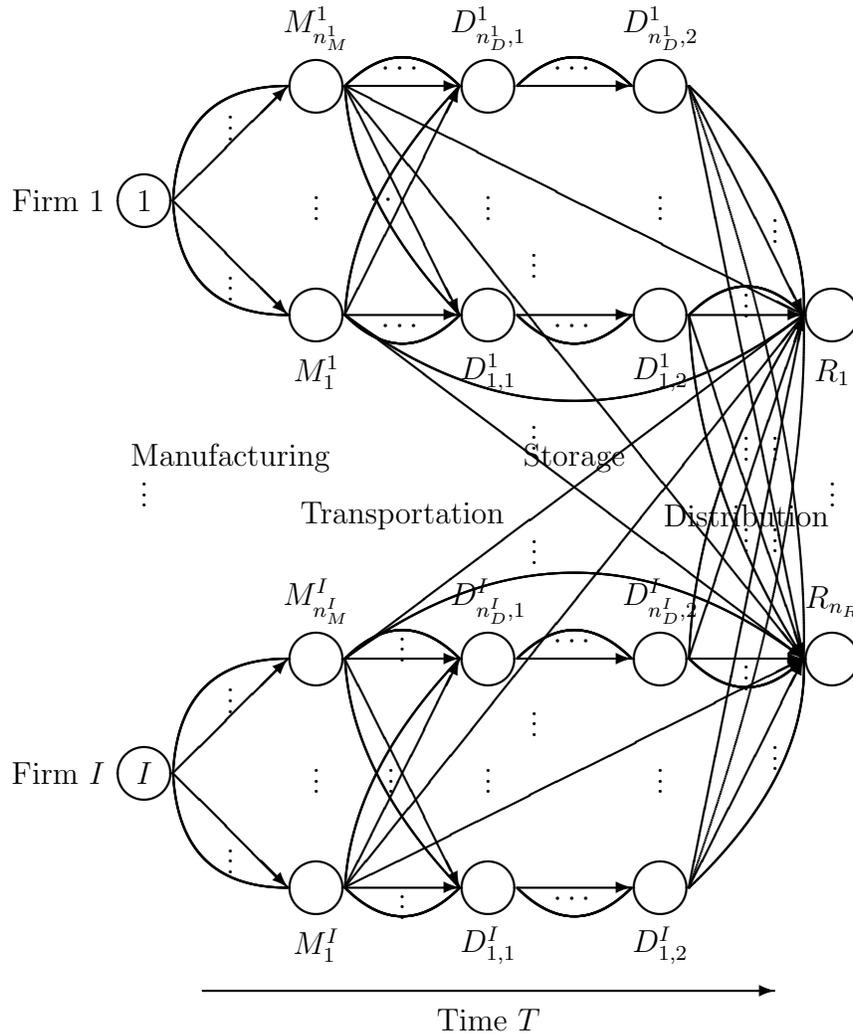


Figure 1: The Supply Chain Network Topology with Progression in Time

The left-most nodes: $i = 1, \dots, I$ in Figure 1, corresponding to the respective firms, are connected, via multiple links, to their manufacturing plant nodes, which are denoted, respectively, by: $M_1^i, \dots, M_{n_M}^i$, and these links represent the manufacturing links. The

multiple links reflect different possible manufacturing technologies at the plants and have associated with them distinct times associated with production as well as distinct total costs, as we elaborate fully below. The links from the manufacturing plant nodes, in turn, are connected to the distribution center nodes of each firm i ; $i = 1, \dots, I$, which are denoted by $D_{1,1}^i, \dots, D_{n_D,1}^i$. These links represent the transportation/shipment links between the manufacturing plants and the distribution centers where the product is stored. We allow for different possible modes of transportation, as represented by distinct links, since, in the case of time-sensitive products, it is important to quantitatively evaluate the impact of different options in terms of both time and cost. Different modes of transportation may include: rail, air, truck, ship, as feasible, or intermodal transport. The links joining nodes $D_{1,1}^i, \dots, D_{n_D,1}^i$ with nodes $D_{1,2}^i, \dots, D_{n_D,2}^i$ for $i = 1, \dots, I$ correspond to the storage links. The multiple storage links represent the available storage options. Also, with the storage links there are associated different times and costs since there may be different options for loading and unloading the product shipments and inventorying, etc. Finally, there are multiple possible shipment/distribution links, as mandated by the specific application, joining the nodes $D_{1,2}^i, \dots, D_{n_D,2}^i$ for $i = 1, \dots, I$ with the demand market nodes: R_1, \dots, R_{n_R} . Here, we also allow for multiple modes of transportation, as depicted by the links in Figure 1. Hence, a firm can allow for multiple delivery options, as reflected by multiple links, to a demand market, if that is appropriate for the application. We refer to the right-most nodes in Figure 1 as demand markets, noting that they may correspond to retailers, as needed by the specific application.

In addition, we allow for the possibility that a firm may wish to have the product transported directly from a manufacturing plant to a demand market, and avail itself of one or more transportation shipment modes. We can expect that the less time-consuming transport modes, i.e., the faster ones (such as air), will be more expensive, and that the slower ones (truck, for example) will be less expensive. These may also represent specific choices such as express service, next day delivery, and so on. Our general supply chain network topology, given in Figure 1, can be adapted to include options such as ship transport, depending on the needs of the decision-maker, and the specific locations of the manufacturing plants, the distribution centers, as well as the demand markets.

The notation for the model is given in Table 1.

Since the time aspect is the critical feature in our supply chain network competition model, we first present and discuss the time consumption functions. Each link a , $a \in L$, has

Table 1: Notation for the Supply Chain Network Model in Time-Sensitive Markets with Information Asymmetry

Notation	Definition
$x_p; p \in P_k^i$	the nonnegative flow of firm i 's product to demand market k ; $i = 1, \dots, I$; $k = 1, \dots, n_R$. We group the $\{x_p\}$ elements for firm i into the vector $X_i \in R_+^{n_{P^i}}$ and all the firms' product flows into the vector $x \in R_+^{n_P}$.
f_a	the nonnegative flow of the product on link a . We group the link flows into the vector $f \in R_+^{n_L}$.
d_{ik}	the demand for the product of firm i at demand market k ; $i = 1, \dots, I$; $k = 1, \dots, n_R$. We group the demands into the vector $d \in R_+^{I \times n_R}$.
$\hat{c}_a(f)$	the total cost associated with link a .
$t_a(f_a)$	the unit product time consumption function for the activity associated with link a .
T_p	the unit product time to complete the activities associated with path p .
T_{ik}^{ave}	the average time to complete all activities associated with firm i ; $i = 1, \dots, I$ and demand market k ; $k = 1, \dots, n_R$; we group the T_{ik}^{ave} into the vector $T^{ave} \in R_+^{I \times n_R}$.
$\rho_{ik}(d, T^{ave})$	the demand price of the product of firm i at demand market k ; $i = 1, \dots, I$; $k = 1, \dots, n_R$.

associated with it the time function $t_a(f_a)$, which is of the specific form:

$$t_a = t_a(f_a) = g_a f_a + h_a, \quad \forall a \in L, \quad (1)$$

where $h_a > 0$ and $g_a \geq 0$. Observe that, through the representation in (1), we capture that different supply chain network activities may be sensitive to the volume of product handled on the link, in terms of time requirement per unit of product. Indeed, it is reasonable, for example, to expect that the per unit time to load a truck will take longer if more of the product has to be loaded. The same holds for production since a capacity may be reached. The time consumption functions apply to all the supply chain network links and associated economic activities. Of course, if a link is characterized by a fixed time per unit of product then we can simply set $g_a = 0$ for such a link. Similar time consumption functions were utilized in Nagurney and Yu (2014) but only in supply chain network topologies with two links per path. Nagurney, Masoumi, and Yu (2014), in turn, considered such activity link time functions, but in the case of an optimization supply chain network model for disaster relief, in which there is no competition. Moreover, in the former study, the firms had guaranteed delivery times as strategic variables and since there was only a single path between each firm and demand market pair there was no average time concept nor information asymmetry.

The unit time on a path $p \in P_k^i$ for a product for $i = 1, \dots, I$; $k = 1, \dots, n_R$, is then given by the sum of the link consumption times on links that comprise the path, that is:

$$T_p = \sum_{a \in L} t_a \delta_{ap}, \quad \forall p \in P_k^i, \forall i, \forall k, \quad (2)$$

where $\delta_{ap} = 1$, if link a is contained in path p , and 0, otherwise.

As mentioned earlier, the consumers at the demand markets reflect their preferences through the demand price functions, which depend on the average time. Specifically, the average time T_{ik}^{ave} associated with firm i 's product at demand market k is computed by:

$$T_{ik}^{ave} = \frac{\sum_{p \in P_k^i} T_p x_p}{\sum_{p \in P_k^i} x_p}, \quad \forall i, \forall k. \quad (3)$$

For convenience, we refer to the average time as the average delivery time, with the understanding that this time reflects the completion of all the associated activities as represented by the links that comprise the paths.

We now present the conservation of flow equations and then describe the link total cost functions and the demand price functions that are used to construct the individual firms' objective functions and optimization problems.

Specifically, the following conservation of flow equations must hold:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad \forall i, \forall k, \quad (4)$$

since the demand produced by each firm must be satisfied at each demand market. If d_{ik} is equal to zero for any i, k then we simply remove that demand market k from consideration for firm i and (3) is still well-defined.

Moreover, the path flows must be nonnegative, that is:

$$x_p \geq 0, \quad \forall p \in P. \quad (5)$$

Furthermore, the expression that relates the link flows to the path flows is given by:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L. \quad (6)$$

In view of (3) and (4), and the demand price functions defined in Table 1, we may reexpress the demand price functions $\rho_{ik}(d, T^{ave})$, as follows:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) \equiv \rho_{ik}(d, T^{ave}), \quad \forall i, \forall k. \quad (7)$$

We assume that the demand price functions are continuous, continuously differentiable, and monotone decreasing in both the product demand at the specific demand market and the average time. Hence, consumers, located at the demand markets, are willing to pay a higher price for a lower average time of delivery at the demand markets. Similarly, the demand for a product is higher if the price is lower. Of course, a special case of the demand price functions is that of $\rho_{ik} = \rho_{ik}(d, T_{ik}^{ave})$, $\forall i, k$, which implies that consumers at each demand market are only concerned with the average delivery time of each product at their demand market.

The total cost on a link, be it a manufacturing/production link, a shipment/distribution link, or a storage link is assumed, in general, to be a function of the product flows on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \quad (8)$$

The above total cost expressions capture competition among the firms for resources used in the manufacture, transportation, storage, and distribution of their products. We assume that the total cost on each link is convex, continuous, and continuously differentiable.

The profit of a firm is the difference between its revenue and its total costs (see, e.g., Yu and Nagurney (2013) and the references therein), where the total costs are the total operational costs in its supply chain network. Here we introduce the novel demand price functions that capture the average times of delivery applicable to time-sensitive markets. Hence, the profit function of firm i , denoted by U_i , is given by:

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d, T^{ave}) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f). \quad (9)$$

From (9) we can see that each firm is aware that consumers respond, through the demand price functions, to the average time of the delivery of the products at the demand markets but the firms themselves know the times (along with the shipment volumes) of all the paths in their respective supply chain networks.

Since we are dealing with Cournot-Nash oligopolistic competition, the decision variables of the firms are their product path flows on their respective supply chain network, with the path flows also corresponding to their strategies in game theory parlance. Let X_i denote the vector of path flows associated with firm i ; $i = 1, \dots, I$, where $X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}$, $P^i \equiv \cup_{k=1, \dots, n_R} P_k^i$, and n_{P^i} denotes the number of paths from firm i to the demand markets. Thus, X is the vector of all the firms' strategies, that is, $X \equiv \{\{X_i\} | i = 1, \dots, I\}$.

Through the use of the conservation of flow equations (4), (5), and (6) and the functions (7) and (8), we define $\hat{U}_i(X) \equiv U_i$ for all firms i ; $i = 1, \dots, I$, with the I -dimensional vector

\hat{U} consisting of the vector of profits of all the firms:

$$\hat{U} = \hat{U}(X). \quad (10)$$

In the Cournot-Nash oligopolistic market framework, each firm selects its product path flows (quantities) in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below.

Definition 1: Supply Chain Network Cournot-Nash Equilibrium

A path flow pattern $X^* \in K = \prod_{i=1}^I K_i$ constitutes a supply chain network Cournot-Nash equilibrium if for each firm $i; i = 1, \dots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \quad (11)$$

where $\hat{X}_i^* \equiv (X_1^*, \dots, X_{i-1}^*, X_{i+1}^*, \dots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{Pi}}\}$.

Hence, an equilibrium is established if no firm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other firms.

Next, we derive the variational inequality formulations of the Cournot-Nash equilibrium for the supply chain network with information asymmetry in time-sensitive markets satisfying Definition 1, in terms of path flows (see Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), Nagurney (2006), and Nagurney et al. (2013)).

Theorem 1

Assume that, for each firm $i; i = 1, \dots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i , and is continuously differentiable. Then $X^* \in K$ is a supply chain network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^I \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \quad (12)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i . Variational inequality (12), in turn, for our model, is equivalent to the variational inequality in path flows: determine the vector of equilibrium path flows $x^* \in K^1$ such that:

$$\sum_{i=1}^I \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} x_q^* \right] \times [x_p - x_p^*] \geq 0, \quad \forall x \in K^1, \quad (13)$$

where $K^1 \equiv \{x|x \in R_+^{n_P}\}$, and for each path p ; $p \in P_k^i$; $i = 1, \dots, I$; $k = 1, \dots, n_R$,

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}. \quad (14)$$

Proof: See the Appendix.

Variational inequalities (13) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (15)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n -dimensional Euclidean space. Let $X \equiv x$ and

$$F(X) \equiv \left[\begin{array}{l} \frac{\partial \hat{C}_p(x)}{\partial x_p} - \hat{\rho}_{ik}(x) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} x_q; \\ p \in P_k^i; i = 1, \dots, I; k = 1, \dots, n_R \end{array} \right], \quad (16)$$

and $\mathcal{K} \equiv K^1$, then (13) can be re-expressed as (15).

Since the feasible set K^1 is not compact, we cannot obtain the existence of a solution simply based on the assumption of the continuity of F . However, the demand d_{ik} for each firm i 's product; $i = 1, \dots, I$ at every demand market R_k ; $k = 1, \dots, n_R$, may be assumed to be bounded, since the population requiring these products is finite (although it may be large). Consequently, in light of (3) and (4), we have that:

$$\mathcal{K}_b \equiv \{x|0 \leq x \leq b\}, \quad (17)$$

where $b > 0$ and $x \leq b$ means that $x_p \leq b$ for all $p \in P_k^i$; $i = 1, \dots, I$, and $k = 1, \dots, n_R$. Then \mathcal{K}_b is a bounded, closed, and convex subset of K^1 . Thus, the following variational inequality

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X \in \mathcal{K}_b, \quad (18)$$

admits at least one solution $X^b \in \mathcal{K}_b$, since \mathcal{K}_b is compact and F is continuous. Therefore, following Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), we have the following theorem:

Theorem 2: Existence

There exists at least one solution to variational inequality (13), since there exists a $b > 0$, such that variational inequality (18) admits a solution in \mathcal{K}_b with

$$x^b \leq b. \tag{19}$$

3. An Illustrative Example and Variants

We now present an illustrative example, along with several variants.

Consider the supply chain network topology given in Figure 2 in which there are two firms, each of which potentially supplies a single demand market, represented as node R_1 , and has, at its disposal, a single manufacturing plant and a single distribution center. The links are labeled in the figure.

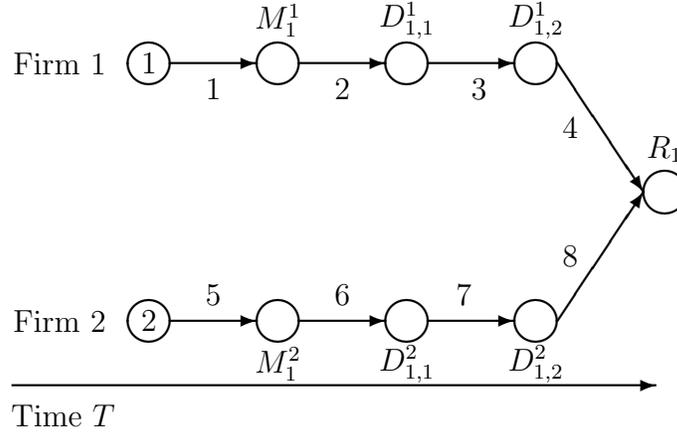


Figure 2: Supply Chain Network Topology for an Illustrative Example and Two Variants

Specifically, Firm 1 is based in the U.S., and its manufacturing plant M_1^1 is also located in the U.S., as is its distribution center. Firm 2 is based in Asia, where its manufacturing plant M_1^2 is located; however, its distribution center is in the U.S. The demand market R_1 is located in the U.S. Note that, in Figure 2, links 1 and 5 denote manufacturing links; links 2,4,6, and 8 denote transportation links, with links 3 and 7 corresponding to storage at the distribution centers.

Because there is only a single path $p_1 = (1, 2, 3, 4)$ connecting Firm 1 with demand market R_1 and a single path $p_2 = (5, 6, 7, 8)$ connecting Firm 2 with R_1 the average time expressions (cf. (3)) become $T_{11}^{ave} = T_{p_1}$ and $T_{21}^{ave} = T_{p_2}$. The time durations are in terms of days.

The demand price functions (cf. Table 1) are as below.

$$\rho_{11}(d, T^{ave}) = -2d_{11} - d_{21} - 3T_{11}^{ave} + 3T_{21}^{ave} + 100, \quad (20)$$

$$\rho_{21}(d, T^{ave}) = -3d_{21} - d_{11} - 2T_{21}^{ave} + 2T_{11}^{ave} + 100. \quad (21)$$

These demand price functions, can be reexpressed in path flows, as in (7), and, here, because of the topology in Figure 2, become:

$$\hat{\rho}_{11}(x) = -2x_{p_1} - x_{p_2} - 3T_{p_1} + 3T_{p_2} + 100,$$

$$\hat{\rho}_{21}(x) = -3x_{p_2} - x_{p_1} - 2T_{p_2} + 2T_{p_1} + 100,$$

where, in view of the link time unit functions given in Table 1, $T_{p_1} = 3.5x_{p_1} + 14$ and $T_{p_2} = 3.1x_{p_2} + 14$.

The remainder of the input data for this example are reported in Table 2.

Table 2: Total Link Operational Cost and Link Unit Time Functions, and Equilibrium Link Flow Solution for Illustrative Example

Link a	\hat{c}_a	$t_a(f_a)$	f_a^*
1	$5f_1^2 + 10f_1$	$.5f_1 + 10$	3.31
2	$2f_2$	$f_2 + 1$	3.31
3	$f_3^2 + f_3$	$f_3 + 1$	3.31
4	$3f_4$	$f_4 + 2$	3.31
5	$f_5^2 + 5f_5$	$.1f_5 + 4$	4.64
6	$8f_6$	$f_6 + 7$	4.64
7	$f_7^2 + f_7$	$f_7 + 1$	4.64
8	$2f_8$	$f_8 + 2$	4.64

Because of the simplicity of the supply chain network topology, the equilibrium path flows, which induce the equilibrium link flows reported in Table 2, can be computed by solving a 2×2 system of equations (under the assumption that both paths are used and, hence, have positive flows), yielding for path p_1 , $x_{p_1}^* = 3.31$, and for path p_2 , $x_{p_2}^* = 4.64$.

The values of the incurred demand market prices at the equilibrium are: $\rho_{11} = 97.10$, $\rho_{21} = 77.20$ with the average time values being: $T_{11}^{ave} = T_{p_1} = 25.59$ and $T_{21}^{ave} = T_{p_2} = 28.37$. Firm 1 earns a profit of 202.74 whereas Firm 2 earns a profit of 240.79. Hence, Firm 2 compensates for its greater distance from the demand market by lower manufacturing times as well as lower manufacturing costs.

Variant 1

Consider the following variant. Firm 1 has enhanced its manufacturing process, which has resulted in greater efficiency and time reduction with the consequence that its unit time function on link 1 has been reduced to: $t_1(f_1) = .5f_1 + 5$. All other data remain as in Table 2 and the demand price functions remain unchanged.

The new equilibrium path flow pattern is: $x_{p_1}^* = 3.64$ and $x_{p_2}^* = 4.28$ with the equilibrium link flow pattern being $f_1^* = f_2^* = f_3^* = f_4^* = 3.64$ and $f_5^* = f_6^* = f_7^* = f_8^* = 4.28$. The values of the average times are: $T_{11}^{ave} = T_{p_1} = 21.72$ and $T_{21}^{ave} = T_{p_2} = 27.62$. Hence, the demand for Firm 1's product has increased, since it has reduced its (average) time for the delivery of its product and that for Firm 2's product has decreased, due to the competition. The values of the incurred demand market prices at the equilibrium are now: $\rho_{11} = 105.06$, $\rho_{21} = 72.47$. Firm 1 earns a profit of 244.47 whereas Firm 2 earns a profit of 204.91. Hence, Firm 1's profit has increased and now surpasses that of Firm 2's.

Variant 2

In the second variant we kept the data as in Variant 1 but now Firm 1 has reduced its operational cost associated with the manufacturing link 1 so that $\hat{c}_a(f) = 2.5f_1^2 + 10f_1$, with the remainder of the data unchanged. The new equilibrium path flow pattern is: $x_{p_1}^* = 4.25$ and $x_{p_2}^* = 4.44$ with the equilibrium link flow pattern being $f_1^* = f_2^* = f_3^* = f_4^* = 4.25$ and $f_5^* = f_6^* = f_7^* = f_8^* = 4.44$. The values of the average times are: $T_{11}^{ave} = T_{p_1} = 23.86$ and $T_{21}^{ave} = T_{p_2} = 27.77$. Hence, the demand for Firm 1's product has again increased, even though its delivery time has increased (slightly). In fact, Firm 1 now has almost the same size market share as that of Firm 2. The values of the incurred demand market prices at the equilibrium are now: $\rho_{11} = 98.79$ and $\rho_{21} = 74.62$. Consumers are increasingly attracted to Firm 1's product. Firm 1 earns a profit of 288.40 whereas Firm 2 earns a profit of 220.87. Firm 1's profit has further increased, whereas that of Firm 2 has also increased, but not as significantly.

Variant 3

In the third variant of the Illustrative Example, we assumed that the Asian firm, Firm 2, is very concerned by the increasing success of Firm 1 and has acquired a second manufacturing plant, M_2^2 , as depicted in Figure 3, which is located in the U.S. Firm 2 still, however, utilizes its distribution center in the U.S. and serves the same demand market as before. Also, the demand price functions remain as in (20) and (21). However, notice that the average time expression T_{21}^{ave} does not take on the form T_{p_2} as in the preceding examples, since now there

is another path $p_3 = (9, 10, 7, 8)$ available for Firm 2.

The data, for convenience of the reader, are reported in Table 3. The input data are as for Variant 2 but with new data added for the two new links 9 and 10 in Figure 3. The data in Table 3 reflect that the Asian firm's manufacturing plant is now closer to its distribution center (and its demand market) in the U.S. The equilibrium path flow pattern, is now: for Firm 1: $x_{p_1}^* = 4.01$ and for Firm 2: $x_{p_2}^* = 2.78$ and $x_{p_3}^* = 2.23$ with the induced equilibrium link flow pattern as reported in Table 3. The values of the incurred demand market prices at the equilibrium are now: $\rho_{11} = 94.18$ and $\rho_{21} = 76.13$. Firm 1 now earns a profit of 257.21 and Firm 2 a profit of 255.97. The values of the average times are: $T_{11}^{ave} = T_{p_1} = 23.03$ and $T_{21}^{ave} = \frac{T_{p_2}x_{p_2} + T_{p_3}x_{p_3}}{x_{p_2} + x_{p_3}} = 25.44$, where $T_{p_2} = 27.09$ and $T_{p_3} = 23.38$.

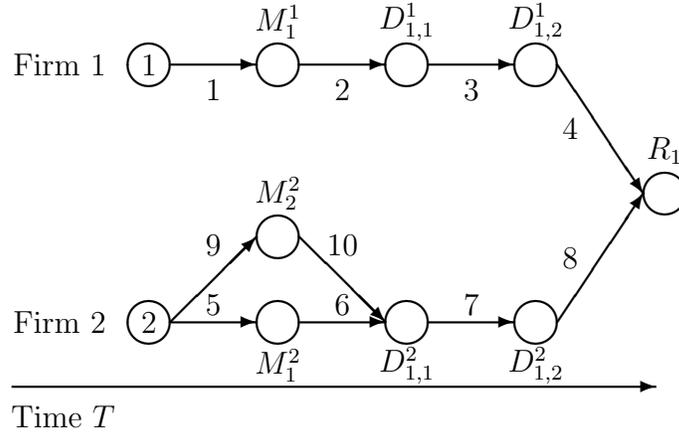


Figure 3: Supply Chain Network Topology for Variants 3 and 4

Table 3: Total Link Operational Cost and Link Unit Time Functions for Variant 3, and Equilibrium Link Flow Solution

Link a	\hat{c}_a	$t_a(f_a)$	f_a^*
1	$2.5f_1^2 + 10f_1$	$.5f_1 + 5$	4.01
2	$2f_2$	$f_2 + 1$	4.01
3	$f_3^2 + f_3$	$f_3 + 1$	4.01
4	$3f_4$	$f_4 + 2$	4.01
5	$f_5^2 + 5f_5$	$.1f_5 + 4$	2.78
6	$8f_6$	$f_6 + 7$	2.78
7	$f_7^2 + f_7$	$f_7 + 1$	5.01
8	$2f_8$	$f_8 + 2$	5.01
9	$3f_9^2 + 10f_9$	$.5f_9 + 5$	2.23
10	$2f_{10}$	$f_{10} + 2$	2.23

Firm 2, by adding a new manufacturing plant that is closer to its distribution center and demand market has reduced its average time and now enjoys higher profit. Firm 1, in turn, because of the increased competition from Firm 2, now suffers a decrease in profits.

Variant 4

Variant 4 had the same data as Variant 3, with the input data as reported in Table 3, except that now Firm 2 engages in additional marketing to inform U.S. consumers that it has invested in a plant in the U.S., with the result that the demand price function for its product has now been changed from the one in (26) to:

$$\rho_{21}(d, T^{ave}) = -3d_{21} - d_{11} - 2T_{21}^{ave} + 2T_{11}^{ave} + 200. \quad (22)$$

Hence, the consumers at the demand market are now willing to pay a higher price for Firm 2's product.

The equilibrium path flow pattern, is now: for Firm 1: $x_{p_1}^* = 5.17$ and for Firm 2: $x_{p_2}^* = 6.24$ and $x_{p_3}^* = 4.08$ with the induced equilibrium link flow pattern given by: $f_1^* = f_2^* = f_3^* = f_4^* = 5.17$, $f_5^* = f_6^* = 6.24$, $f_7^* = f_8^* = 10.32$, and $f_9^* = f_{10}^* = 4.08$. The values of the incurred demand market prices at the equilibrium are now: $\rho_{11} = 116.89$ and $\rho_{21} = 138.83$. Firm 1 now earns a profit of 428.28 and Firm 2 a profit of 1076.26. The values of the average times are now: $T_{11}^{ave} = T_{p_1} = 27.11$ and $T_{21}^{ave} = 39.63$ with $T_{p_2} = 41.50$ and $T_{p_3} = 36.76$. The demand for Firm 2's product has more than doubled as compared to the demand in Variant 3, whereas its profit has more than quadrupled. Interestingly, the profit for Firm 1 has also increased, by about 60%, demonstrating the impact of one firm's decision on the other, which may not be immediately apparent or expected. Hence, the impacts of competition are revealed through the equilibrium solutions.

3.1 Additional Applications - Delay Tolerant Networks and Web Hosting

Below, we describe additional applications, which can be addressed using the above general competitive network model.

As noted in the Introduction, the supply chain network model developed above can also be used in certain data and information delivery applications, including those in isolated regions (whether rural and/or underdeveloped) in which there may be inadequate telecommunication infrastructure due to costs. In order to capture competition among providers of such services, we can envision a topology as depicted in Figure 4, which is a special case of the one in Figure 1, to reflect that each firm has costs associated with the operation of

its telecommunication infrastructure with the transport being provided by either a real-time connection represented by a link joining its “manufacturing” node with each demand market or by a physical mechanical backhaul option denoted by the alternative such link. Consumers in such markets, as noted by Marentes et al. (2014), respond to the average time associated with the delivery of the data and information, which can use a combination of such delivery services (real-time and physical). Firms are profit-maximizers.

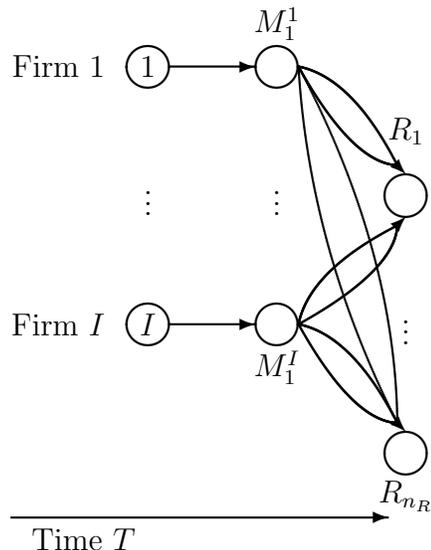


Figure 4: Topology for Competition in Delay Tolerant Networks

Also, if one were to add additional links joining the “manufacturing” nodes with the demand markets in the network in Figure 4, a possible separate application would be that of web-hosting. In such an application, consumers would select the web-host(s) that provide the appropriate time-average responsiveness (and price) for the desired information, whether that of videos, news, and the like. The links terminating in the demand markets would correspond to the aggregations of telecommunication options. Of course, one could also fully represent the underlying network topology and incorporate the time consumption functions, as well, depending on the level of accuracy needed.

4. A Numerical Case Study

In this section, we present a numerical case study that focuses on the sensitivity of consumers at the demand markets to the average delivery times.

The algorithm that we utilize for the computations is the Euler method, introduced by Dupuis and Nagurney (1993), which has been applied to a plethora of supply chain network problems (see, e.g., Nagurney (2006), Nagurney et al. (2013), Nagurney, Yu, and Floden

(2013), and the references therein). For the convenience of the reader, we now recall the Euler method and demonstrate its realization in the context of our new model, which yields explicit, closed form expressions in the path flows, at each iteration. Specifically, at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \quad (23)$$

where $P_{\mathcal{K}}$ is the projection on the feasible set \mathcal{K} and F is the function that enters the variational inequality problem (15) where $F(X)$ for our model is as in (16).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \rightarrow 0$, as $\tau \rightarrow \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of network oligopolies can be found in Nagurney and Zhang (1996), Nagurney (2010), Nagurney and Yu (2012), and Masoumi, Yu, and Nagurney (2012).

Explicit Formulae for the Euler Method Applied to the Supply Chain Network for Time-Sensitive Markets Variational Inequality (15) with $F(X)$ as in (16)

The elegance of this procedure for the computation of solutions to the supply chain network problem for time-sensitive markets can be seen in the following explicit formulae. In particular, we have the following closed form expressions for the product path flows:

$$x_p^{\tau+1} = \max\left\{0, x_p^{\tau} + a_{\tau}(\hat{\rho}_{ik}(x^{\tau}) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^{\tau})}{\partial x_p} \sum_{q \in P_l^i} x_q^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p})\right\},$$

$$\forall p \in P_k^i; i = 1, \dots, I; k = 1, \dots, n_R, \quad (24)$$

where $\frac{\partial \hat{\rho}_{il}}{\partial x_p}$ is as given in (A.5), with x replaced by x^{τ} .

This case study is an extension of Variant 3 in Section 2.1. The U.S. firm, Firm 1, has one manufacturing plant and one distribution center, both of which are located in the U.S. The Asian firm, Firm 2, has two manufacturing plants, as depicted in Figure 5. One manufacturing plant, M_1^2 , is located in Asia; while the other one, M_2^2 , is located in the U.S., as is its distribution center. In contrast to Variant 3, in this case study, each firm servers two geographically separated demand markets in the U.S. Therefore, there are two new distribution links 11 and 12 connecting the respective distribution centers and the demand market R_2 . There are two paths for Firm 1 and four paths for Firm 2. We label the paths as follows (please refer to Figure 5): for Firm 1:

$$p_1 = (1, 2, 3, 4), \quad p_2 = (1, 2, 3, 11)$$

and for Firm 2:

$$\begin{aligned}
 p_3 &= (5, 6, 7, 8), & p_4 &= (9, 10, 7, 8), \\
 p_5 &= (5, 6, 7, 12), & p_6 &= (9, 10, 7, 12),
 \end{aligned}$$

We implemented the Euler method (cf. (24)) for the solution of variational inequality (13), using Matlab. We set the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots)$, and the convergence tolerance was 10^{-6} . In other words, the absolute value of the difference between each path flow in two consecutive iterations was less than or equal to this tolerance. We initialized the algorithm by setting the path flows equal to 5.

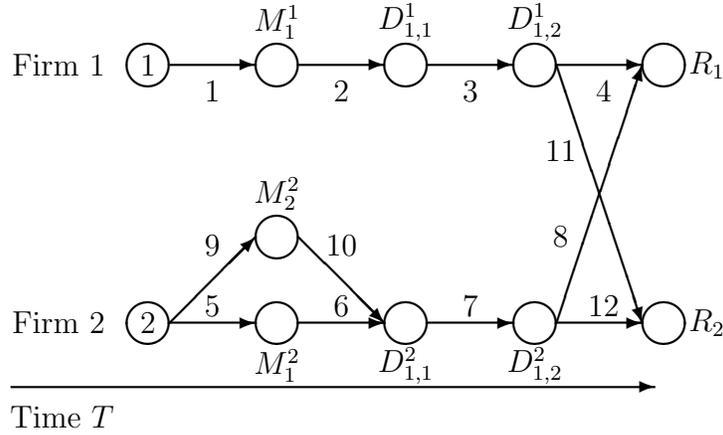


Figure 5: Supply Chain Network Topology for the Case Study

Case Study Example 1

The consumers at demand market R_1 are more sensitive to the average delivery times than those at demand market R_2 are. The demand price functions are:

$$\begin{aligned}
 \rho_{11}(d, T^{ave}) &= -2d_{11} - d_{21} - 3T_{11}^{ave} + 3T_{21}^{ave} + 100, \\
 \rho_{12}(d, T^{ave}) &= -3d_{12} - d_{22} - 2T_{12}^{ave} + 2T_{22}^{ave} + 100, \\
 \rho_{21}(d, T^{ave}) &= -2d_{21} - d_{11} - 3T_{21}^{ave} + 3T_{11}^{ave} + 100, \\
 \rho_{22}(d, T^{ave}) &= -3d_{22} - d_{12} - 2T_{22}^{ave} + 2T_{12}^{ave} + 100.
 \end{aligned}$$

The link operational cost and link unit time functions are provided in Table 4, along with the computed equilibrium link flow solution.

Table 4: Total Link Operational Cost and Link Unit Time Functions, and Equilibrium Link Flow Solution for Case Study Example 1

Link a	\hat{c}_a	$t_a(f_a)$	f_a^*
1	$2.5f_1^2 + 10f_1$	$.5f_1 + 5$	5.06
2	$2f_2$	$f_2 + 1$	5.06
3	$f_3^2 + f_3$	$f_3 + 1$	5.06
4	$3f_4$	$f_4 + 2$	2.33
5	$f_5^2 + 5f_5$	$.1f_5 + 4$	3.83
6	$8f_6$	$f_6 + 7$	3.83
7	$f_7^2 + f_7$	$f_7 + 1$	6.65
8	$2f_8$	$f_8 + 2$	3.26
9	$3f_9^2 + 10f_9$	$.5f_9 + 5$	2.81
10	$2f_{10}$	$f_{10} + 2$	2.81
11	$3f_{11}$	$f_{11} + 2$	2.73
12	$2f_{12}$	$f_{12} + 2$	3.39

The equilibrium demands at the demand markets are:

$$\text{Firm 1: } d_{11}^* = 2.33, \quad d_{12}^* = 2.73,$$

$$\text{Firm 2: } d_{21}^* = 3.26, \quad d_{22}^* = 3.39.$$

The average delivery times are:

$$\text{Firm 1: } T_{11}^{ave} = 23.98, \quad T_{12}^{ave} = 24.38,$$

$$\text{Firm 2: } T_{21}^{ave} = 24.67, \quad T_{22}^{ave} = 28.25.$$

The incurred demand prices are:

$$\text{Firm 1: } \rho_{11} = 94.17, \quad \rho_{12} = 96.17,$$

$$\text{Firm 2: } \rho_{21} = 89.06, \quad \rho_{22} = 79.36.$$

The profits of Firms 1 and 2 are:

$$U_1 = 311.33, \quad U_2 = 372.94.$$

Since Firm 2 is capable of providing competitive delivery service at a significantly lower price, Firm 2 dominates both demand markets, leading to a higher profit.

Case Study Example 2

This example has the identical data to that of Case Study Example 1 except that the consumers at demand market R_1 are now more sensitive with respect to the average delivery times. The demand price functions associated with demand market R_1 are now:

$$\begin{aligned}\rho_{11}(d, T^{ave}) &= -2d_{11} - d_{21} - 4T_{11}^{ave} + 4T_{21}^{ave} + 100, \\ \rho_{21}(d, T^{ave}) &= -2d_{21} - d_{11} - 4T_{21}^{ave} + 4T_{11}^{ave} + 100.\end{aligned}$$

Case Study Example 3

This example has the same data as Case Study Example 1 but now the consumers at the demand market R_1 are even more sensitive with respect to the average delivery times, with the demand price functions associated with demand market R_1 given by:

$$\begin{aligned}\rho_{11}(d, T^{ave}) &= -2d_{11} - d_{21} - 5T_{11}^{ave} + 5T_{21}^{ave} + 100, \\ \rho_{21}(d, T^{ave}) &= -2d_{21} - d_{11} - 5T_{21}^{ave} + 5T_{11}^{ave} + 100.\end{aligned}$$

The computed equilibrium demands, average delivery times, prices, and profits for Case Study Examples 1-3 are reported in Table 5. In Table 6, the computed equilibrium path flows and the unit time on each path are provided.

A comparison of the results in Case Study Examples 1-3 demonstrates that the average delivery times from both Firms 1 and 2 to the demand market R_1 decline significantly, due to the consumers' increased sensitivity to the average delivery times; and both firms charge higher prices for the timely delivery. It is very interesting to observe that the average delivery times to demand market R_2 also decrease, but slightly. For Firm 2, there are two paths for each demand market. In order to fulfill the high requirement for timely delivery, Firm 2 mainly relies on its manufacturing plant in the U.S., M_2^2 , to satisfy the demand at demand market R_1 ; while its Asian manufacturing plant M_1^2 covers all the demand at demand market R_2 . The results show that the firms are trading cost for time when reacting to the changed customer preferences. This is well in line with real world observed behavior, such as for the perishable products and fashion items discussed in the Introduction.

Moreover, from Table 6, we see that Firm 2's path p_6 is never used at equilibrium in our three case study examples, and, hence, always has equilibrium flow equal to 0. Also, Firm 2's equilibrium path flow on its path p_3 decreases in Case Study Examples 2 and 3, and reaches the value of 0 in the latter. The fastest delivery time is achieved by Firm 2 on its

Table 5: Computed Equilibrium Demands, Average Delivery Times, Demand Prices, and Profits for Case Study Examples 1-3

		Example 1	Example 2	Example 3
Demands for Firm 1's Product	d_{11}^*	2.33	1.93	1.66
	d_{12}^*	2.73	2.79	2.82
Demands for Firm 2's Product	d_{21}^*	3.26	2.65	2.37
	d_{22}^*	3.39	3.53	3.48
Firm 1's Average Delivery Times	T_{11}^{ave}	23.98	22.72	21.86
	T_{12}^{ave}	24.38	23.58	23.02
Firm 2's Average Delivery Times	T_{21}^{ave}	24.67	22.79	21.77
	T_{22}^{ave}	28.25	27.61	27.15
Demand Prices of Firm 1's Product	ρ_{11}	94.17	93.80	93.85
	ρ_{12}	96.17	96.17	96.32
Demand Prices of Firm 2's Product	ρ_{21}	89.06	92.48	94.06
	ρ_{22}	79.36	78.56	78.49
Profit of Firm 1		311.33	295.64	285.51
Profit of Firm 2		372.94	354.39	341.51

path p_4 (21.77) in Case Study Example 3 and the slowest by Firm 2 on its path p_5 (28.25) in Case Study Example 1.

5. Summary and Conclusions

In this paper, we develop a supply chain network model consisting of multiple firms who compete with one another in both quantities of their product and in the average time of their delivery to the demand markets.

The contributions of the paper to the literature are as follows. First, the new model is relevant to time-sensitive product markets, in which consumers are willing to pay a higher price for lower average times of delivery. Products of relevance include certain food products, vaccines and other healthcare products, specific fashion products, as well as products that are in demand because of feature events. Each firm may have at its disposal one or more manufacturing plants, and avail itself of distribution centers and different modes of transport for shipping to multiple demand markets. The products of the firms are differentiated by brands. This work is inspired, in part, by recent failures of timely deliveries of products in the real-world.

We describe the underlying network topology, identify the behavior of the firms, who are assumed to be profit-maximizing, and construct the time consumption functions associated

Table 6: Computed Equilibrium Path Flows and Path Unit Times for Case Study Examples 1-3

				Example 1		Example 2		Example 3	
Firm	Manufacturing Plant	Demand Market	Path	x_p^*	T_p	x_p^*	T_p	x_p^*	T_p
Firm 1	M_1^1 (in U.S.)	R_1	p_1	2.33	23.98	1.93	22.72	1.66	21.86
	M_1^1 (in U.S.)	R_2	p_2	2.73	24.38	2.79	23.58	2.82	23.02
Firm 2	M_1^2 (in Asia)	R_1	p_3	0.44	28.12	0.01	26.72	0.00	26.04
	M_2^2 (in U.S.)	R_1	p_4	2.81	24.13	2.64	22.78	2.37	21.77
	M_1^2 (in Asia)	R_2	p_5	3.39	28.25	3.53	27.61	3.48	27.15
	M_2^2 (in U.S.)	R_2	p_6	0.00	24.26	0.00	23.67	0.00	22.88

with the various supply chain network activities of manufacturing, transport, storage, and distribution. In addition, we identify how our general model can also be used to investigate delay tolerant networks and web-hosting. It also has relevance to time-based competition in future Internet architectures for content provision.

We utilize variational inequality theory for the formulation of the governing equilibrium conditions under Cournot-Nash, and also provide qualitative properties of the equilibrium state. Our proposed computational procedure has nice features for implementation. This is the first time that such a general model for time-sensitive markets with such a general network topology has been constructed and formulated.

We provide an illustrative example with variants as well as a numerical case study in which we report the equilibrium solution, the average times, the individual path times, and the profits achieved by the firms. The numerical results reveal that the firms are trading cost for time when reacting to the changed customer preferences. This supports real world observed behavior for time-sensitive products ranging from fast fashion to perishable products such as food and vaccines.

The framework also contributes to the literature in terms of managerial insights. For example, the results imply that, in the case of consumers' increasing time-sensitivity, it might be profitable to bring some manufacturing back to the domestic facilities from the off-shore plants. Our results further support such strategic decision-making in supply chain management as appropriate location selection for both manufacturing plants and distribution centers. In addition, the results indicate that, when firms attempt to shorten the average delivery time to consumers at one time-sensitive demand market, the average delivery time to the other demand markets may also decrease. This demonstrates synergistic/opportunistic

effects that may not have been immediately available without a rigorous computational framework as we have proposed. This is why it is imperative to capture holistically the supply chain network activities of firms as well as their interactions using a game theoretic framework in the context of time-sensitive markets.

Finally, the model contributes to the literature in information asymmetry since the firms are aware of the product path flows through their supply chains, along with the associated times, but consumers, located at the demand markets, are only aware of the average time for their delivery.

Future work may include further disaggregating the supply chain networks of the firms in order to further detail the production and transportation choices, in the form of routes, for example. Clearly, supply chain network design for time-sensitive products would also be of interest.

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Appendix

Proof of Theorem 1: Variational inequality (12) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). In view of (4) and (7), we may rewrite the profit functions as follows: for each firm i , $\forall i$,

$$\hat{U}_i(X) \equiv U_i = \sum_{l=1}^{n_R} \hat{\rho}_{il}(x) \sum_{q \in P_l^i} x_q - \sum_{b \in L^i} \hat{c}_b(f). \quad (\text{A.1})$$

Observe now that

$$\nabla_{X^i} \hat{U}_i(X) = \left[\frac{\partial \hat{U}_i}{\partial x_p}; p \in P_k^i; k = 1, \dots, n_R \right], \quad (\text{A.2})$$

where for each path p ; $p \in P_k^i$,

$$\begin{aligned} \frac{\partial \hat{U}_i}{\partial x_p} &= \frac{\partial \left[\sum_{l=1}^{n_R} \hat{\rho}_{il}(x) \sum_{q \in P_l^i} x_q - \sum_{b \in L^i} \hat{c}_b(f) \right]}{\partial x_p} = \sum_{l=1}^{n_R} \frac{\partial \left[\hat{\rho}_{il}(x) \sum_{q \in P_l^i} x_q \right]}{\partial x_p} - \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial x_p} \\ &= \hat{\rho}_{ik}(x) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} x_q - \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \frac{\partial f_a}{\partial x_p} \\ &= \hat{\rho}_{ik}(x) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P_l^i} x_q - \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}. \end{aligned} \quad (\text{A.3})$$

In view of (1), (2) and (6), the average time associated with firm i 's product at demand market k can be rewritten as a function of path flows:

$$T_{ik}^{ave} = \frac{\sum_{q \in P_k^i} \sum_{a \in L^i} [g_a (\sum_{o \in P^i} x_o \delta_{ao}) + h_a] \delta_{aq} x_q}{\sum_{q \in P_k^i} x_q}, \quad \forall i, \forall k. \quad (\text{A.4})$$

Also, for completeness, according to the chain rule, the partial derivative $\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p}$ can be expanded as follows: for each path p ; $p \in P_k^i$,

$$\begin{aligned}
\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} &= \frac{\partial \rho_{il}(d, T^{ave})}{\partial x_p} = \sum_{j=1}^{n_R} \frac{\partial \rho_{il}(d, T^{ave})}{\partial d_{ij}} \frac{\partial d_{ij}}{\partial x_p} + \sum_{j=1}^{n_R} \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ij}^{ave}} \frac{\partial T_{ij}^{ave}}{\partial x_p} \\
&= \frac{\partial \rho_{il}(d, T^{ave})}{\partial d_{ik}} \frac{\partial d_{ik}}{\partial x_p} + \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ik}^{ave}} \frac{\partial T_{ik}^{ave}}{\partial x_p} + \sum_{j \neq k} \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ij}^{ave}} \frac{\partial T_{ij}^{ave}}{\partial x_p} \\
&= \frac{\partial \rho_{il}(d, T^{ave})}{\partial d_{ik}} \\
&\quad + \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ik}^{ave}} \frac{\left[\sum_{a \in L^i} (g_a \sum_{o \in P^i} x_o \delta_{ao} + h_a) \delta_{ap} + \sum_{q \in P_k^i} \sum_{a \in L^i} g_a x_q \delta_{aq} \delta_{ap} \right] \left[\sum_{q \in P_k^i} x_q \right] - \sum_{q \in P_k^i} \sum_{a \in L^i} (g_a \sum_{o \in P^i} x_o \delta_{ao} + h_a) \delta_{aq} x_q}{\left[\sum_{q \in P_k^i} x_q \right]^2} \\
&\quad + \sum_{j \neq k} \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ij}^{ave}} \frac{\sum_{q \in P_j^i} \sum_{a \in L^i} g_a x_q \delta_{aq} \delta_{ap}}{\sum_{q \in P_j^i} x_q} \\
&= \frac{\partial \rho_{il}(d, T^{ave})}{\partial d_{ik}} \\
&\quad + \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ik}^{ave}} \frac{\left[\sum_{a \in L^i} (g_a \sum_{o \in P^i} x_o \delta_{ao} + h_a) \delta_{ap} \right] \left[\sum_{q \in P_k^i} x_q \right] - \sum_{q \in P_k^i} \sum_{a \in L^i} (g_a \sum_{o \in P^i} x_o \delta_{ao} + h_a) \delta_{aq} x_q}{\left[\sum_{q \in P_k^i} x_q \right]^2} \\
&\quad + \sum_{j=1}^{n_R} \frac{\partial \rho_{il}(d, T^{ave})}{\partial T_{ij}^{ave}} \frac{\sum_{q \in P_j^i} \sum_{a \in L^i} g_a x_q \delta_{aq} \delta_{ap}}{\sum_{q \in P_j^i} x_q}. \tag{A.5}
\end{aligned}$$

By making use of the definition of $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ in (14), variational inequality (13) is immediate. \square