A Supply Chain Network Game Theoretic Framework for Time-Based Competition with Transportation Costs and Product Differentiation

Anna Nagurney
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003
and
School of Business, Economics and Law
University of Gothenburg
Gothenburg, Sweden
and
Min Yu
Pamplin School of Business Administration
University of Portland
Portland, Oregon 97203

In *Optimization in Science and Engineering*
In Honor of the 60th Birthday of Panos M. Pardalos
Th. M. Rassias, C. A. Floudas, and S. Butenko, Editors, Springer, New York, 2014, in press, pp 381-400.

Abstract: In this paper, we developed a supply chain network game theory model with differentiated products and transportation costs in the case of time-based oligopolistic competition. The firms are profit-maximizers and have, as their strategic variables, the product shipments and the guaranteed delivery times to the consumers at the demand markets with the guaranteed delivery times never exceeding the sum of the production time and the transportation time. The demand price functions are functions of the demands for the products at the different demand markets as well as their guaranteed delivery times. The governing Nash equilibrium conditions are formulated as alternative variational inequalities. An algorithm is proposed, which yields closed form expressions, at each iteration, for the product shipments, the guaranteed delivery times, as well as the associated Lagrange multipliers. Supply chain network numerical examples are given to illustrate the modeling and the computational approach.

Key words: supply chains, oligopolies, networks, game theory, Nash equilibrium, product differentiation, transportation costs, time-based competition, build-to-order, made-on-demand, variational inequalities

1. Introduction

Supply chains today span the globe and provide the infrastructure for the production and delivery of goods and services, with more knowledgeable consumers demanding timely deliveries, despite, paradoxically, the great distances that may be involved. Indeed, delivery times are becoming a strategy, as important as productivity, quality, and even innovation (see, e.g., Gunasekaran, Patel, and McGaughey (2004), Christopher (2005), Nagurney (2006), Nagurney and Li (2012), and Yu (2012)). As noted by Ray and Jewkes (2004), practitioners have realized that speed of product delivery is a competitive advantage (Stalk, Jr. and Hout (1990), Blackburn et al. (1992)).

It is now well-recognized (cf. Hum and Sim (1996), Geary and Zonnenberg (2000), and Boyaci and Ray (2003)) that, whether in manufacturing (especially in build-to-order and made-on-demand industries such as certain computers, electronic equipment, specific cars, airplanes, furniture, etc.) or in digitally-based production and delivery (DVDs, online shopping, online content distribution, etc.) speed and consistency of delivery time are two essential components of customer satisfaction, along with price (cf. Handfield and Pannesi (1995), Ballou (1998)). Stalk, Jr., in his seminal *Harvard Business Review* 1988 article, "Time - The next source of competitive advantage," utilized the term *time-based competition*, to single out time as the major factor for sustained competitive advantage. Today, time-based competition has emerged as a paradigm for strategizing about and operationalizing supply chain networks in which efficiency and timeliness matter (see Carter, Melnyk, and Handfield (1995), Vickery et al (1995), Ceglarek et al. (2004), Li et al. (2006), and Nagurney (2006)).

Advances in production and operations management thought and practice, including such revolutionary concepts as time-based competition, have, in turn, provided a rich platform for the accompanying research investigations. The extensive literature review of Hum and Sim (1996) of time-based competition emphasized both its intellectual history as well as the associated mathematical modeling, thereby, constructing a bridge between practice and scholarship on this important topic. They, nevertheless, concluded that much of the time-based focus in modeling was limited to the areas of transportation modeling, leadtime and inventory modeling, and set-up time reduction analysis. Moreover, they argued that the literature emphasized cost minimization but what was needed was the explicit incorporation of time as a significant variable in modeling. The complexity of the production and operations management landscape in the real world could not be adequately captured through an objective function representing simply cost minimization. Gunasekaran and Ngai (2005) further emphasized this shortcoming and the relevance of analyzing the trade-offs between operational costs and delivery time in supply chain management.

Hence, in order to rigorously capture time-based competition within an analytical, computable supply chain framework, one needs to utilize game theory and the appropriate strategic variables with the explicit recognition of time.

Li and Whang (2001) developed an elegant game theory model for time-based competition in which firms choose, as their strategic variables, both prices and production rates and discussed several special cases. Their approach was a generalization of the contributions of Lederer and Li (1997), who, in turn, built on some of the prior research in queuing and delays. However, since the focus in those papers was on operations management, and not on supply chain management, Li and Whang (2001) did not consider the time component associated with the transportation of the products, which is a central issue in increasingly globalized supply chains (cf. Nagurney (2006)). In addition, the underlying functions were assumed to have an explicit structure. Moreover, they assumed that the firms were price-takers. In various industries, as noted above, in which made-to-order and build-to-order strategies are relevant, the underlying industrial organization is that of oligopolies and, imperfect, rather than perfect competition (cf. Tirole (1988) and Vives (1999)). Shang and Liu (2011), in turn, investigated the impacts of promised delivery time and on-time delivery rates under oligopolistic competition. Blackburn (2012) discussed some of the limits of time-based competition quantitatively through the introduction of the marginal value of time derived from a total cost objective function. However, he exclusively focused on inventory costs and did not include transportation costs which are fundamental to global supply chains. Moreover, a single cost-minimizing decision-maker was assumed, whereas in order to appropriately address time-based *competition*, a framework that captures the interactions among decision-makers, that is, firms, in a supply chains, along with the reality of product differentiation, is needed.

In this paper, hence, we develop a game theoretical framework for supply chain network time-based competition, which has the following major, novel features:

(1). firms are assumed to be spatially separated and can compete both on the production side and on the demand side;

(2). firms compete in an oligopolistic manner and, hence, influence the prices;

(3). the time consumption of both production and transportation/shipment supply chain activities is made explicit;

(4). the strategic variables of the firms are quantity variables and guaranteed delivery time variables;

(5). consumers at the demand markets for the substitutable, but differentiated, products respond to both the quantities of the products and to their guaranteed delivery times, as reflected in the prices of the products.

In addition, by capturing the total cost associated with delivery times of each firm, along with their production costs and their transportation costs in their respective objective functions, the marginal cost of time can be quantified in this more general competitive network framework.

The intellectual platform upon which our model is based has several foundational supports. First, it builds upon the existing literature on oligopolistic competition and network equilibria (cf. Dafermos and Nagurney (1987) and Nagurney (1999, 2006)), coupled with the recent modeling advances that incorporate brand/product differentiation and supply chain network competition (see Nagurney and Yu (2011, 2012), Masoumi, Yu, and Nagurney (2012), and Nagurney and Li (2012)). However, unlike Nagurney and Yu (2011), where the goal was to minimize total cost and total time in the supply chain network for a time-sensitive product, which, in that case, was fast fashion apparel, here we focus on the delivery times to the consumers at the demand market. In addition, in contrast to work noted above, in this paper, the consumers reflect their preferences for the different products through both the prices and the guaranteed delivery times, where the guaranteed delivery time here includes the time required for production and for transportation/shipment, with the understanding that different products will be distributed in an appropriate manner (digital products, for example, are distributed via the web).

It is also important to recognize the literature on time-sensitive products from food to the, already noted, fashion apparel, to even perishable products in healthcare, as well as critical needs products in humanitarian operations; see Nagurney, Yu, Masoumi, and Nagurney (2013) for such a survey. Finally, we note that although the book by Nagurney (2006) contains a spectrum of dynamic supply chain network models the dynamics therein are modeled using projected dynamical systems (cf. Nagurney and Zhang (1996)) without delivery times being explicit strategic variables.

To the best of our knowledge, this is the first paper to synthesize oligopolistic competition, product differentiation, and time-based competition, with guaranteed delivery times as strategic variables, in a computable supply chain network game theoretic model under Nash (1950, 1951) equilibrium.

For the reader, we also highlight the paper by Geunes and Pardalos (2003) and the edited volume of theirs – Geunes and Pardalos (2005), which provide excellent literature overviews of supply chain optimization with the latter also focusing on networks.

This paper is organized as follows. In Section 2 we develop the supply chain network game theoretic model with differentiated products and time-based competition by describing each firm's individual profit-maximizing behavior and the underlying cost functions and demand price functions, with an emphasis on the time element and the network structure. We then define the governing supply chain Nash equilibrium and establish alternative variational inequality formulations. We emphasize that variational inequalities for supply chain network equilibrium problems were first utilized by Nagurney, Dong, and Zhang (2002) and initiated a rich literature. Recent applications have included also supply chain disruptions; see Nagurney and Qiang (2009), Qiang, Nagurney, and Dong (2009), and Wakolbinger and Cruz (2011).

In Section 3, we focus on the variational inequality formulation that has elegant features for computations for which we propose an algorithm that yields, at each iteration, closed form expressions for the product shipments, the delivery times, and the associated Lagrange multipliers with the constraints for the latter. In Section 4, we illustrate the model through a series of numerical examples, which are solved using the algorithm. In Section 5, we conclude the paper with a summary of results and discussion.

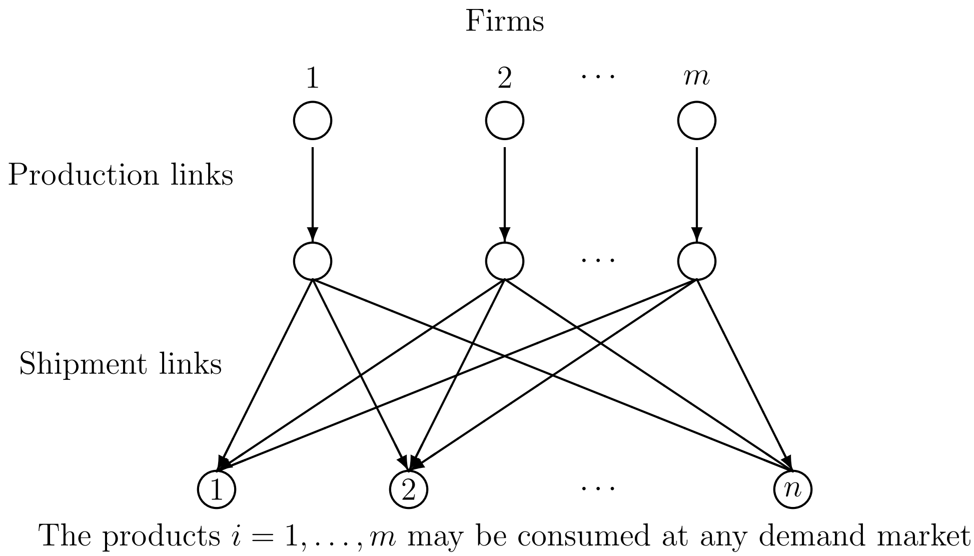

Figure 1: The Network Structure of the Supply Chain Problem

2. The Supply Chain Network Game Theoretic Model with Product Differentiation and Guaranteed Delivery Times

In this Section, we develop a supply chain network model with product differentiation in which the firms have as strategic variables their product shipments to the demand markets and the guaranteed times of the deliveries of the products. The firms compete under the Cournot-Nash equilibrium concept of non-cooperative behavior. The consumers, in turn, signal their preferences for the products through the demand price functions associated with the demand markets, which are, in general, functions of the demands for the products at all the demand markets as well as the guaranteed delivery times of the products, from the manufacturing/production stage to demand market delivery. We assume that there are m firms and n demand markets that can be located in different physical locations. There is a distinct (but substitutable) product produced by each of the m firms and is consumed at the n demand markets. Please refer to Figure 1 for the underlying structure of the supply chain network problem under consideration here. The notation for the model is given in Table 1. The vectors are assumed to be column vectors. The equilibrium solution is denoted by "*".

The model is a strategic model rather than an operational=level model. Hence, we do not consider possible sequencing of jobs for specific demand markets. Such an extension may be considered in future research.

Table 1: Notation for the Game Theoretic Supply Chain Network Model with Product Differentiation and Guaranteed Delivery Times

Notation	**Definition**
Q_{ij}	the nonnegative shipment of firm i's product to demand market j; $i = 1, \ldots, m$; $j = 1, \ldots, n$. We group the $\{Q_{ij}\}$ elements for firm i into the vector $Q_i \in R^n_+$ and all the firms' product shipments into the vector $Q \in R^{mn}_+$.
s_i	the nonnegative output produced by firm i; $i = 1, \ldots, m$. We group the firm production outputs into the vector $s \in R^m_+$.
d_{ij}	the demand for the product produced by firm i at demand market j; $i = 1, \ldots, m$; $j = 1, \ldots, n$. We group the demands into the vector $d \in R^{mn}_+$.
T_{ij}	the guaranteed delivery time of product i, which is produced by firm i, at demand market j; $i = 1, \ldots, m$; $j = 1, \ldots, n$. We group the delivery times of firm i into the vector $T_i \in R^n_+$ and then group all these vectors of all firms into the vector $T \in R^{mn}_+$.
$f_i(s)$	the production cost of firm i; $i = 1, \ldots, m$.
$g_i(T_i)$	the total cost associated with the delivery time of firm i; $i = 1, \ldots, m$.
$p_{ij}(d, T)$	the demand price of the product produced by firm i at demand market j; $i = 1, \ldots, m$; $j = 1, \ldots, n$.
$\hat{c}_{ij}(Q)$	the total transportation cost associated with shipping firm i's product to demand market j; $i = 1, \ldots, m$; $j = 1, \ldots, n$.

The following conservation of flow equations must hold:

$$s_i = \sum_{j=1}^{n} Q_{ij}, \quad i = 1, \dots, m, \tag{1}$$

$$d_{ij} = Q_{ij}, \quad i = 1, \dots, m; \; j = 1, \dots, n, \tag{2}$$

$$Q_{ij} \geq 0, \quad i = 1, \dots, m; \; j = 1, \dots, n. \tag{3}$$

Consequently, the quantity of the product produced by each firm is equal to the sum of the amounts shipped to all the demand markets; the quantity of a firm's product consumed at a demand market is equal to the amount shipped from the firm to that demand market, and the product shipments must be nonnegative.

As noted in the Introduction, the firms are also competing with time, that is, the guaranteed delivery times are strategic variables. Since each product must be manufactured and then delivered, as depicted in Figure 1, we need to account for the time consumption associated with these supply chain network activities. Hence, associated with each firm and demand market pair, we also have the following constraint:

$$t_i s_i + h_i + t_{ij} Q_{ij} + h_{ij} \leq T_{ij}, \quad i = 1, \dots, m; \; j = 1, \dots, n, \tag{4a}$$

where t_i, h_i, t_{ij}, and h_{ij} are all positive parameters. The first two terms in (4a) reflect the actual time consumption associated with producing product i and the second two terms reflect the actual time consumption associated with delivering product i to demand market j. Constraint (4a), thus, guarantees that the product of each firm i will be produced and shipped to demand market j within the guaranteed delivery time T_{ij} determined by firm i.

Note that, according to (4a), the supply chain network activities of production / manufacturing and transportation are functions, respectively, of how much is produced and of how much is transported. Indeed, it may take longer to produce a larger quantity of product and also (since the product may need to be loaded/unloaded) to deliver a larger volume of product to a demand point. The fixed terms h_i and h_{ij} denote the physical lower bounds of the time needed to produce and to transport product i to demand market j, respectively. Even in the case of digital products there will be a lower bound, albeit, small, in size. In light of (1), (4a) also ensures that the guaranteed delivery time strategic variables will be nonnegative. Furthermore, the total transportation cost functions $\hat{c}_{ij}$; $i = 1, \dots, m$; $j = 1, \dots, n$ since they, for the sake of generality, are functions of the product shipment pattern, capture possible congestion or competition for shipment resources (see also Nagurney (2006) and the references therein). Of course, a special case of (4a) and (4b) is when some (or all) of the parameters t_i; $i=1,\ldots,m$ and t_{ij}; $i=1,\ldots,m$; $j=1,\ldots,n$ are identically equal to zero. The transportation costs that we consider, as a special case, capture the possibility of fixed transportation costs between firm and demand market pairs.

In view of (1), we may rewrite ($4a$) in product shipment variables only, that is,

$$t_i \sum_{j=1}^{n} Q_{ij} + h_i + t_{ij}Q_{ij} + h_{ij} \leq T_{ij}, \quad i=1,\ldots,m;\ j=1,\ldots,n. \tag{4b}$$

In our numerical examples, we illustrate different realizations of constraint (4b) in which we show that sometimes there may be a slack associated with (4b) in the equilibrium solution and sometimes not.

A firm's production cost may depend not only on its production output but also on that of the other firms. This is reasonable since firms which produce substitutable products may compete for the resources needed to produce their products. Also, in lieu of the time consumption (cf. ($4a,b$)) associated with producing a product the production costs $f_i(s)$; $i=1,\ldots,m$, also capture the cost associated with the timely production of different levels of output. Due to the conservation of flow equation (1), we can define the production cost functions $\hat{f}_i$; $i=1,\ldots,m$, in quantity shipments only, that is

$$\hat{f}_i = \hat{f}_i(Q) \equiv f_i(s), \quad i=1,\ldots,m. \tag{5}$$

The production cost functions (5) are assumed to be convex and continuously differentiable.

It is important to emphasize that faster guaranteed delivery may be more costly, since it may require additional capacity and may be dependent on the operational efficiency (cf. So (2000), Boyaci and Ray (2003), Ray and Jewkes (2004), Cachon and Zhang (2006), Nagurney and Yu (2011), and Yu (2012)). For example, shipping costs of Amazon.com were doubled when the guaranteed delivery time was decreased from one week to two days (So and Song (1998)). This is captured in our functions g_i; $i=1\ldots,m$, which are also assumed to be convex and continuously differentiable.

In view of (2), we may define demand price functions, $\hat{p}_{ij}$, for all (i,j), in terms of the product shipments, that is:

$$\hat{p}_{ij} = \hat{p}_{ij}(Q,T) \equiv p_{ij}(d,T), \quad i=1,\ldots,m;\ j=1,\ldots,n. \tag{6}$$

We note that including both product quantities as well as guaranteed delivery time into demand functions has a tradition in economics as well as in operations research and marketing (cf. Hill and Khosla (1992), Lederer and Li (1997), So and Song (1998), Boyaci and Ray

(2003, 2006), Ray and Jewkes (2004), Shang and Liu (2011), and the references therein). The demand price functions (6) and the total transportation cost functions $\hat{c}_{ij}$; $i = 1, \ldots, m$ and $j = 1, \ldots, n$, are assumed to be continuous and continuously differentiable.

Representing both the production cost (5) and the demand price functions (6) as functions of the product shipments, along with the time delivery constraints (4b) and the total cost function associated with the guaranteed delivery times, yields an elegant formulation of the supply chain network game with strategic variables being the product shipments and the delivery times, as we shall establish in Theorem 1.

The strategic variables of firm i are its product shipments $\{Q_i\}$ where $Q_i = (Q_{i1}, \ldots, Q_{in})$ and its guaranteed delivery times $\{T_i\}$, note that $T_i = (T_{i1}, \ldots, T_{in})$.

The profit or utility U_i of firm i; $i = 1, \ldots, m$, is, hence, given by the expression

$$U_i = \sum_{j=1}^{n} \hat{p}_{ij} Q_{ij} - \hat{f}_i - g_i - \sum_{j=1}^{n} \hat{c}_{ij}, \tag{7}$$

which is the difference between its total revenue and its total costs.

In view of (1)-(7), one may write the profit as a function solely of the shipment pattern and delivery times, that is,

$$U = U(Q, T), \tag{8}$$

where U is the m-dimensional vector with components: $\{U_1, \ldots, U_m\}$.

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, T_i) | Q_i \geq 0,$ and (4b) is satisfied for $i\}$ and $K \equiv \prod_{i=1}^{m} K^i$.

In the oligopolistic market mechanism, the m firms supply their products in a non-cooperative fashion, each one trying to maximize its own profit. We seek to determine an equilibrium product shipment and delivery time pattern (Q^*, T^*), according to the definition below (see also Cournot (1838) and Nash (1950, 1951)).

Definition 1: A Supply Chain Network Equilibrium with Product Differentiation and Delivery Times

A product shipment and delivery time pattern $(Q^*, T^*) \in K$ is said to constitute a network equilibrium if for each firm $i; i = 1, \ldots, m$,

$$U_i(Q_i^*, T_i^*, \hat{Q}_i^*, \hat{T}_i^*) \geq U_i(Q_i, T_i, \hat{Q}_i^*, \hat{T}_i^*), \quad \forall (Q_i, T_i) \in K^i, \tag{9}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*); \quad \textit{and} \quad \hat{T}_i^* \equiv (T_1^*, \ldots, T_{i-1}^*, T_{i+1}^*, \ldots, T_m^*). \tag{10}$$

According to (9), an equilibrium is established if no firm can unilaterally improve its profits by selecting an alternative vector of product shipments and delivery times of its product, given the decisions of the other firms.

Variational Inequality Formulations

We now derive alternative variational inequality formulations of the above supply chain network equilibrium with product differentiation in the following theorem.

Theorem 1

Assume that for each firm i the profit function $U_i(Q,T)$ is concave with respect to the variables $\{Q_{i1},\ldots,Q_{in}\}$, and $\{T_{i1},\ldots,T_{in}\}$, and is continuous and continuously differentiable. Then $(Q^*,T^*)\in K$ is a supply chain network equilibrium according to Definition 1 if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(Q^*,T^*)}{\partial Q_{ij}}\times(Q_{ij}-Q^*_{ij})-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(Q^*,T^*)}{\partial T_{ij}}\times(T_{ij}-T^*_{ij})\geq 0,\quad \forall (Q,T)\in K, \tag{11}$$

or, equivalently, $(Q^*,T^*,\gamma^*)\in K^1$ is an equilibrium product shipment and guaranteed delivery time pattern if and only if it satisfies the variational inequality

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial \hat{f}_i(Q^*)}{\partial Q_{ij}}+\sum_{l=1}^{n}\frac{\partial \hat{c}_{il}(Q^*)}{\partial Q_{ij}}-\sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q^*,T^*)}{\partial Q_{ij}}\times Q^*_{il}-\hat{p}_{ij}(Q^*,T^*)+\sum_{l=1}^{n}\gamma^*_{il}t_i+\gamma^*_{ij}t_{ij}\right]$$
$$\times(Q_{ij}-Q^*_{ij})+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial g_i(T^*_i)}{\partial T_{ij}}-\sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q^*,T^*)}{\partial T_{ij}}\times Q^*_{il}-\gamma^*_{ij}\right]\times(T_{ij}-T^*_{ij})$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[T^*_{ij}-t_i\sum_{l=1}^{n}Q^*_{il}-t_{ij}Q^*_{ij}-h_i-h_{ij}\right]\times\left[\gamma_{ij}-\gamma^*_{ij}\right]\geq 0,\quad \forall (Q,T,\gamma)\in K^1, \tag{12}$$

where $K^1\equiv\{(Q,T,\gamma)|\, Q\geq 0,\, T\geq 0,\, \gamma\geq 0\}$ with γ being the mn-dimensional vector with component (i,j) consisting of the element γ_{ij} corresponding to the Lagrange multiplier associated with the (i,j)-th constraint (4b).

Proof: (11) follows directly from Gabay and Moulin (1980) and Dafermos and Nagurney (1987).

In order to obtain variational inequality (12), we note that, for a given firm i, under the imposed assumptions, (11) holds if and only if (see, e.g., Bertsekas and Tsitsiklis (1989)) the following holds:

$$\sum_{j=1}^{n}\left[\frac{\partial \hat{f}_i(Q_i^*)}{\partial Q_{ij}}+\sum_{l=1}^{n}\frac{\partial \hat{c}_{il}(Q^*)}{\partial Q_{ij}}-\sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q^*,T^*)}{\partial Q_{ij}}\times Q_{il}^*-\hat{p}_{ij}(Q^*,T^*)+\sum_{l=1}^{n}\gamma_{il}^*t_i+\gamma_{ij}^*t_{ij}\right]$$

$$\times(Q_{ij}-Q_{ij}^*)+\sum_{j=1}^{n}\left[\frac{\partial g_i(T_i^*)}{\partial T_{ij}}-\sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q^*,T^*)}{\partial T_{ij}}\times Q_{il}^*-\gamma_{ij}^*\right]\times(T_{ij}-T_{ij}^*)$$

$$+\sum_{j=1}^{n}\left[T_{ij}^*-t_i\sum_{l=1}^{n}Q_{il}^*-t_{ij}Q_{ij}^*-h_i-h_{ij}\right]\times\left[\gamma_{ij}-\gamma_{ij}^*\right]\geq 0,\quad \forall(Q_i,T_i,\gamma_i)\in K_i^1, \tag{13}$$

where $K_i^1 \equiv \{(Q_i,T_i,\gamma_i)|(Q_i,T_i,\gamma_i)\in R_+^{3n}\}$, with $\{\gamma_i\}=(\gamma_{i1},\ldots,\gamma_{in})$.

But (13) holds for each firm i; $i=1,\ldots,m$, and, hence, the summation of (13) yields variational inequality (12). The conclusion follows.□

We now put variational inequality (12) into standard form (cf. Nagurney (1999)): determine $X^*\in\mathcal{K}\subset R^N$, such that

$$\langle F(X^*)^T, X-X^*\rangle \geq 0,\quad \forall X\in\mathcal{K}, \tag{14}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a closed and convex set.

We define the $3mn$-dimensional vector $X\equiv(Q,T,\gamma)$ and the $3mn$-dimensional row vector $F(X)=(F^1(X),F^2(X),F^3(X))$ with the (i,j)-th component, F_{ij}^1, of $F^1(X)$ given by

$$F_{ij}^1(X)\equiv\frac{\partial \hat{f}_i(Q)}{\partial Q_{ij}}+\sum_{l=1}^{n}\frac{\partial \hat{c}_{il}(Q)}{\partial Q_{ij}}-\sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q,T)}{\partial Q_{ij}}\times Q_{il}-\hat{p}_{ij}(Q,T)+\sum_{l=1}^{n}\gamma_{il}t_i+\gamma_{ij}t_{ij}, \tag{15}$$

the (i,j)-th component, F_{ij}^2, of $F^2(X)$ given by

$$F_{ij}^2(X)\equiv\frac{\partial g_i(T_i)}{\partial T_{ij}}-\sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q,T)}{\partial T_{ij}}\times Q_{il}-\gamma_{ij}, \tag{16}$$

and the (i,j)-th component, F_{ij}^3, of $F^3(X)$ given by

$$F_{ij}^3(X)=T_{ij}-t_i\sum_{l=1}^{n}Q_{il}-t_{ij}Q_{ij}-h_i-h_{ij}, \tag{17}$$

and with the feasible set $\mathcal{K}\equiv K$. Then, clearly, variational inequality (12) can be put into standard form (14).

We now present two examples in order to illustrate some of the above concepts and results.

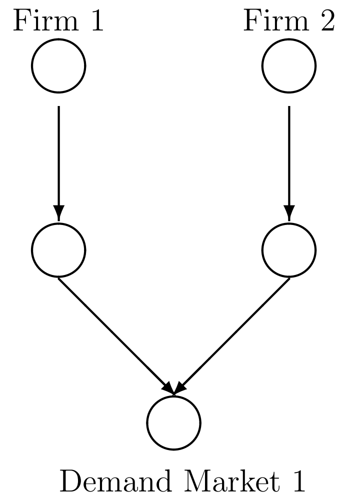

Figure 2: The Network Structure for the Illustrative Examples

Illustrative Examples

Consider a supply chain network oligopoly problem consisting of two firms and one demand market, as depicted in Figure 2.

Example 1

We assume that these two firms are located in the same area. Both of them adopt similar technologies for the production and delivery of their highly substitutable products. Therefore, the production and transportation cost functions of Firms 1 and 2 are identical. Meanwhile, consumers at the demand market are indifferent between the products of Firms 1 and 2. The production cost functions are:

$$f_1(s) = 2s_1^2 + 3s_1, \quad f_2(s) = 2s_2^2 + 3s_2,$$

so that (cf. (5)):

$$\hat{f}_1(Q) = 2Q_{11}^2 + 3Q_{11}. \quad \hat{f}_2(Q) = 2Q_{21}^2 + 3Q_{21}.$$

The total transportation cost functions are:

$$\hat{c}_{11}(Q_{11}) = Q_{11}^2 + Q_{11}, \quad \hat{c}_{21}(Q_{21}) = Q_{21}^2 + Q_{21},$$

the total cost functions associated with delivery times are:

$$g_1(T_1) = T_{11}^2 - 30T_{11} + 400, \quad g_2(T_2) = T_{21}^2 - 40T_{21} + 450,$$

and the demand price functions are assumed to be:

$$p_{11}(d,T) = 300 - 2d_{11} - 0.5d_{21} - T_{11} + 0.2T_{21}, \quad p_{21}(d,T) = 300 - 2d_{21} - 0.5d_{11} - T_{21} + .2T_{11}$$

so that (cf. 6)):

$$\hat{p}_{11}(Q,T) = 300 - 2Q_{11} - 0.5Q_{21} - T_{11} + 0.2T_{21}, \quad \hat{p}_{21}(Q,T) = 300 - 2Q_{21} - 0.5Q_{11} - T_{21} + .2T_{11}.$$

The above nonlinear cost functions, although hypothetical, were constructed to capture the potential resource competition and congestion in the production and delivery activities. Moreover, the total cost associated with delivery times decreases if the delivery time is increased in a certain range. However, the slower delivery may also be costly since resources could be used elsewhere.

The parameters associated with the production time consumption are:

$$t_1 = 0, \quad h_1 = 1, \quad t_2 = 0, \quad h_2 = 1,$$

and the parameters associated with the transportation time consumption are:

$$t_{11} = 0, \quad h_{11} = 1, \quad t_{21} = 0, \quad h_{21} = 1,$$

which means that the actual production times and the actual transportation times of these two firms are fixed.

Hence, for Firm 1, the following guaranteed delivery time constraint must be satisfied:

$$1 + 1 \leq T_{11},$$

and for Firm 2, the corresponding guaranteed delivery time constraint is:

$$1 + 1 \leq T_{21}.$$

The equilibrium product shipment and guaranteed delivery time pattern is:

$$Q^*_{11} = 28.14, \quad Q^*_{21} = 27.61, \quad T^*_{11} = 2.00, \quad T^*_{21} = 6.19,$$

and the corresponding Lagrange multipliers are:

$$\gamma^*_{11} = 2.14, \quad \gamma^*_{21} = 0.00.$$

Furthermore, the equilibrium prices associated with these two products are:

$$p_{11} = 229.15, \quad p_{21} = 224.91,$$

and the profits of the two firms are:

$$U_1 = 3,616.20, \quad U_2 = 3,571.90.$$

In this example, Firm 2's guaranteed delivery time, which is 6.19, is longer than the actual delivery time, which is 2, mainly because the total cost associated with delivery time would increase notably if Firm 2 were to reduce its guaranteed delivery time.

Example 2

Example 2 has the same data as Example 1 except that now the actual production times and the actual transportation times of Firms 1 and 2 depend on how much is produced and how much is shipped, respectively, that is,

$$t_1 = 0.2, \quad t_2 = 0.3, \quad t_{11} = 0.1, \quad t_{21} = 0.2.$$

The new equilibrium product shipment and guaranteed delivery time pattern is:

$$Q_{11}^* = 27.06, \quad Q_{21}^* = 26.13, \quad T_{11}^* = 10.12, \quad T_{21}^* = 15.07,$$

and the corresponding Lagrange multipliers are:

$$\gamma_{11}^* = 17.30, \quad \gamma_{21}^* = 16.26.$$

The equilibrium prices associated with these two products are:

$$p_{11} = 225.70, \quad p_{21} = 221.17,$$

and the profits of the two firms are:

$$U_1 = 3,603.89, \quad U_2 = 3,551.89.$$

This example shows that Firm 1 attracts more consumers with a notably shorter guaranteed delivery time, although the price of its product is higher than that of Firm 2's product. Due to its competitive advantage in delivery time performance, Firm 1 achieves a relatively higher profit.

Example 3

Example 3 has the same data as Example 2 except that now Firm 2 has reduced its production cost by improving its operational efficiency. The production cost function of Firm 2 is now given by:

$$f_2(s) = s_2^2 + 2s_2,$$

so that (cf. (5)):

$$\hat{f}_2(Q_{21}) = Q_{21}^2 + 2Q_{21}.$$

The equilibrium product shipment and guaranteed delivery time pattern is:

$$Q_{11}^* = 26.86, \quad Q_{21}^* = 31.75, \quad T_{11}^* = 10.06, \quad T_{21}^* = 17.87,$$

and the corresponding Lagrange multipliers are:

$$\gamma_{11}^* = 16.97, \quad \gamma_{21}^* = 27.49.$$

The equilibrium prices associated with these two products are:

$$p_{11} = 223.93, \quad p_{21} = 207.22,$$

and the profits of the two firms are:

$$U_1 = 3,543.33, \quad U_2 = 4,413.00.$$

As a result of its lower production cost, Firm 2 is able to provide consumers with its product at an appealing price. Hence, the demand for Firm 2's product increases remarkably, even with a longer guaranteed delivery time, while there is a slight decrease in the demand for Firm 1's product. Therefore, in this example, Firm 2's profit improves significantly.

3. The Algorithm

The feasible set underlying variational inequality (12) is the nonnegative orthant, a feature that we will exploit for computational purposes. Specifically, we will apply the Euler-type method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), where, at iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one must solve the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{18}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (12).

As demonstrated in Dupuis and Nagurney (1993) and in Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces this algorithmic scheme, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other supply chain and network oligopoly models can be found in Nagurney and Zhang (1996), Nagurney, Dupuis, and Zhang (1994), Nagurney (2010), Nagurney and Yu (2012), and in Nagurney, Yu, and Qiang (2011).

Explicit Formulae for the Euler Method Applied to the Supply Chain Network Model

The elegance of this procedure for the computation of solutions to our model with product differentiation and time deliveries can be seen in the following explicit formulae. In particular, we have the following closed form expression for all the product shipments $i = 1, \ldots, m; j = 1, \ldots, n$:

$$Q_{ij}^{\tau+1} = \max\{0, Q_{ij}^{\tau} + a_\tau(-F_{ij}^1(X^\tau))\}, \tag{19}$$

and the following closed form expression for all the guaranteed delivery time values $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$T_{ij}^{\tau+1} = \max\{0, T_{ij}^{\tau} + a_\tau(-F_{ij}^2(X^\tau))\} \tag{20}$$

with the Lagrange multipliers being computed for all $i = 1, \ldots, m$; $j = 1, \ldots, n$ according to:

$$\gamma_{ij}^{\tau+1} = \max\{0, \gamma_{ij}^{\tau} + a_\tau(-F_{ij}^3(X^\tau))\}\}; \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{21}$$

In the next Section, we apply the Euler method to compute solutions to additional numerical supply chain network problems.

4. Numerical Examples

We implemented the Euler method, as described in Section 3, using Matlab. The convergence criterion was $\epsilon = 10^{-6}$; that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each product shipment, each guaranteed delivery time value, and each Lagrange multiplier differed from its respective value at the preceding iteration by no more than ϵ. We set the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \cdots)$.

In this Section, we considered a supply chain network consisting of three firms and two demand markets, which are geographically separated (as depicted in Figure 3). Consumers at Demand Market 2 are more sensitive with respect to guaranteed delivery times than consumers at Demand Market 1.

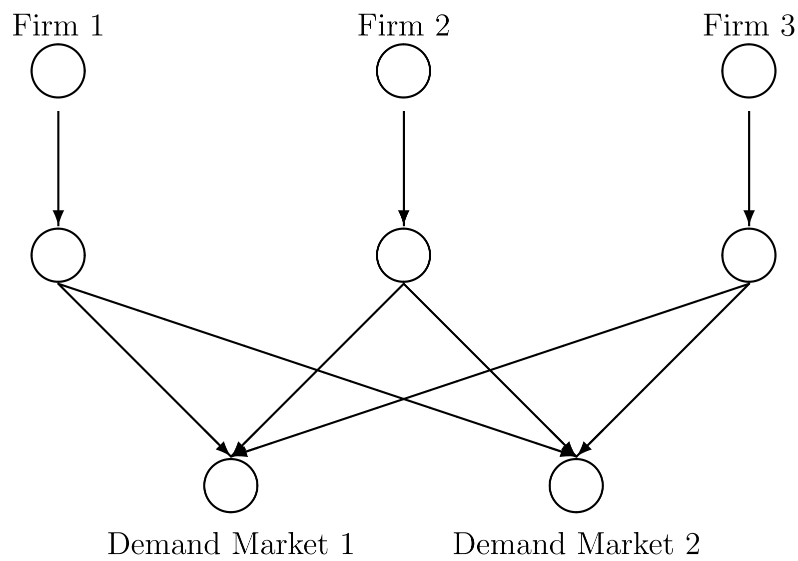

Figure 3: The Network Structure for the Numerical Examples

Example 4

The cost functions, demand price functions, and parameters associated with time consumption are as follows:

Firm 1:

$$f_1(s) = s_1^2 + 0.5s_1s_2 + 0.5s_1s_3, \quad g_1(T_1) = T_{11}^2 + T_{12}^2 - 30T_{11} - 40T_{12} + 650,$$

$$\hat{c}_{11}(Q_{11}) = Q_{11}^2 + 0.5Q_{11}, \quad \hat{c}_{12}(Q_{12}) = Q_{12}^2 + Q_{12},$$

$$p_{11}(d,T) = 400 - 2d_{11} - d_{21} - 0.8d_{31} - 1.2T_{11} + 0.3T_{21} + 0.2T_{31},$$

$$p_{12}(d,T) = 400 - 1.5d_{12} - 0.5d_{22} - 0.8d_{32} - 2T_{12} + 0.2T_{22} + 0.3T_{32},$$

$$t_1 = 0.8, \quad h_1 = 1.5, \quad t_{11} = 0.4, \quad h_{11} = 1.5, \quad t_{12} = 0.5, \quad h_{12} = 1.5;$$

Firm 2:

$$f_2(s) = 1.5s_2^2 + 0.8s_1s_2 + 0.8s_2s_3, \quad g_2(T_2) = T_{21}^2 + T_{22}^2 - 30T_{21} - 30T_{22} + 480,$$

$$\hat{c}_{21}(Q_{21}) = Q_{21}^2 + Q_{21}, \quad \hat{c}_{22}(Q_{22}) = Q_{22}^2 + Q_{22},$$

$$p_{21}(d,T) = 400 - 2d_{21} - d_{11} - d_{31} - 1.2T_{21} + 0.2T_{11} + 0.2T_{31},$$

$$p_{22}(d,T) = 400 - 1.5d_{22} - 0.5d_{12} - 0.5d_{32} - 2T_{22} + 0.3T_{12} + 0.3T_{32},$$

$$t_2 = 0.6, \quad h_2 = 1.5, \quad t_{21} = 0.4, \quad h_{21} = 1.3, \quad t_{22} = 0.4, \quad h_{22} = 1.3;$$

Firm 3:

$$f_3(s) = 2s_3^2 + 0.8s_1s_3 + 0.8s_2s_3, \quad g_3(T_3) = 0.8T_{31}^2 + 0.8T_{32}^2 - 25T_{31} - 20T_{32} + 400,$$

$$\hat{c}_{31}(Q_{31}) = 1.5Q_{31}^2 + Q_{31}, \quad \hat{c}_{32}(Q_{32}) = Q_{32}^2 + 1.5Q_{32},$$

$$p_{31}(d,T) = 400 - 2d_{31} - 0.8d_{11} - d_{21} - 1.2T_{31} + 0.2T_{11} + 0.3T_{21},$$

$$p_{32}(d,T) = 400 - 1.5d_{32} - 0.8d_{12} - 0.5d_{22} - 2T_{32} + 0.3T_{12} + 0.2T_{22},$$

$$t_3 = 0.3, \quad h_3 = 1, \quad t_{31} = 0.2, \quad h_{31} = 1, \quad t_{32} = 0.1, \quad h_{32} = 1.$$

We utilized (5) and (6) to construct the production cost functions and the demand price functions, respectively, in shipment variables, for all examples in this Section.

The equilibrium product shipment and guaranteed delivery time pattern, the Lagrange multipliers, and the prices are reported in Tables 1 and 2.

Note that, in Example 4, Firm 1 has a slight advantage over its competitors in Demand Market 1, despite the longer guaranteed delivery time, perhaps as a consequence of the lower price. Firm 3 captures the majority of the market share at Demand Market 2, due to consumers' preference for timely delivery. However, Firm 2 attains the lowest profit, as compared to its rivals, since Firm 2 is neither cost-effective enough nor sufficiently time-efficient.

Example 5

Example 5 has the identical data to that in Example 4, except that consumers at Demand Market 2 are becoming even more time-sensitive. The new demand price functions are now given by:

$$p_{12}(d,T) = 400 - 1.5d_{12} - 0.5d_{22} - 0.8d_{32} - 3T_{12} + 0.2T_{22} + 0.3T_{32},$$

$$p_{22}(d,T) = 400 - 1.5d_{22} - 0.5d_{12} - 0.5d_{32} - 3T_{22} + 0.3T_{12} + 0.3T_{32},$$

$$p_{32}(d,T) = 400 - 1.5d_{32} - 0.8d_{12} - 0.5d_{22} - 3T_{32} + 0.3T_{12} + 0.2T_{22}.$$

We also provided the solutions to Example 5 in Tables 2 and 3.

In Example 5, Firm 1 and Firm 3 still dominate Demand Markets 1 and 2, respectively. Consumers' increasing time sensitivity at Demand Market 2 has forced all these three firms to shorten their guaranteed delivery times. The decrease in Firm 3's profit is negligible, while the profits of Firms 1 and 2 shrink notably. The results in Examples 4 and 5 suggest that delivery times, as a strategy, are particularly influential in time-based competition.

Table 2: The Equilibrium Product Shipment and Guaranteed Delivery Time Patterns, the Lagrange Multipliers, and the Prices for Examples 4 and 5

		Example 4				Example 5			
Firm	Demand Market	Q^*	T^*	γ^*	p	Q^*	T^*	γ^*	p
1	1	18.05	36.44	64.54	302.76	19.30	35.34	63.84	299.96
	2	14.73	36.59	62.65	288.15	11.48	33.36	61.15	268.47
2	1	15.96	29.10	47.36	308.78	17.01	28.32	47.04	305.76
	2	17.23	29.61	63.67	311.74	14.19	27.19	66.94	295.34
3	1	17.14	17.63	23.78	330.18	17.47	17.24	23.55	327.49
	2	23.55	16.56	53.60	328.05	21.69	15.91	70.54	318.91

Table 3: The Profits of Firms 1, 2, and 3 in Examples 4 and 5

	Firm 1	Firm 2	Firm 3
Example 4	6,097.14	5,669.63	6,782.11
Example 5	5,697.97	5,3072.64	6,560.58

5. Summary and Conclusions

In this paper, we developed a rigorous modeling and computational framework for time-based competition in supply chain networks using game theory and variational inequality theory.

Specifically, the firms are assumed to compete in an oligopolistic manner using as strategic variables not only their product shipments to the various demand markets, under brand differentiation, but also their guaranteed delivery times. Here the guaranteed delivery times provide upper bounds on the sum of the production time and the transportation time between the firm and demand market pairs. All firms are assumed to be profit-maximizers and subject to production and transportation costs. The consumers, in turn, reflect their preferences for the firms' brands or products through the demand price functions which are functions of not only the demands for the firms' products at the different demand markets but also their guaranteed delivery times.

Numerical supply chain network examples were presented to illustrate the generality of the proposed model with a complete reporting of the input data and the computed equilibrium product shipments and guaranteed delivery times, along with the Lagrange multipliers associated with the delivery time constraints.

The modeling and analytical framework can be used as the foundation for the investigation of supply chain networks in the case of build to order and made on demand products. It can also be extended in several directions through the inclusion of multiple options of transportation and multiple technologies for production. One may also incorporate additional tiers of suppliers. Nevertheless, we have laid out the foundations for time-based competition in supply chain networks with this study that enables numerous explorations both theoretical and empirical with a focus on particular industrial sectors.

Acknowledgments

The first author's research was supported, in part, by the School of Business, Economics and Law at the University of Gothenburg through its Visiting Professor Program.

This research was also supported, in part, by the National Science Foundation (NSF) grant CISE #1111276, for the NeTS: Large: Collaborative Research: Network Innovation Through Choice project awarded to the University of Massachusetts Amherst.

The above support is gratefully acknowledged.

The authors dedicate this paper to Professor Panos M. Pardalos whose scholarship, leadership, and friendship have made our community richer in numerous ways.

References

Ballou, R.H., 1998. *Business logistics management*. Upper Saddle River, NJ: Prentice Hall.

Bertsekas, D.P. and Tsitsiklis, J.N., 1989. *Parallel and distributed computation: Numerical methods*. Englewood Cliffs, NJ: Prentice Hall.

Blackburn, J.D., 2012. Valuing time in supply chains: Establishing limits of time-based competition. *Journal of Operations Management*, 30, 396-405.

Blackburn, J.D., Elrod, T., Lindsley, W.B. and Zahorik, A.J., 1992. The strategic value of response time and product variety. In *Manufacturing strategy: Process and content*, Voss, C.A. (Editor), London, England: Chapman and Hall, pp 261-281.

Boyaci, T. and Ray, S., 2003. Product differentiation and capacity cost interaction in time and price sensitive markets. *Manufacturing Service Operations Management*, 5 (1), 18-36.

Boyaci, T. and Ray, S., 2006. The impact of capacity costs on product differentiation in delivery time, delivery reliability, and price. *Production and Operations Management*, 15

(2), 179-197.

Cachon, G.P. and Zhang, F., 2006. Procuring fast delivery: Sole sourcing with information asymmetry. *Management Science*, 52 (6), 881-896.

Carter, P.L., Melnyk, S.A. and Handfield, R.B., 1995. Identifying the basic process strategies for time-based competition. *Production and Inventory Management Journal*, 1st Quarter, 65-70.

Ceglarek, D., Huang, W., Zhou, S., Ding, Y., Kuma, R. and Zhou, Y., 2004. Time-based competition in multistage manufacturing: Stream-of-variation analysis (SOVA) methodology – Review. *International Journal of Flexible Manufacturing Systems*, 16, 11-44.

Christopher, M., 2005. *Logistics and supply chain management: Creating value-adding networks*, third edition. London, UK: Prentice-Hall/Financial Times.

Cournot, A.A., 1838. *Researches into the mathematical principles of the theory of wealth*, English translation. London, England: MacMillan.

Dafermos, S. and Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. *Regional Science and Urban Economics*, 17, 245-254.

Dupuis, P. and Nagurney, A., 1993. Dynamical systems and variational inequalities. *Annals of Operations Research*, 44, 9-42.

Gabay, D. and Moulin, H., 1980. On the uniqueness and stability of Nash equilibria in noncooperative games. In *Applied stochastic control of econometrics and management science*, Bensoussan, A., Kleindorfer, P. and Tapiero, C.S. (Editors), Amsterdam, The Netherlands: North-Holland, pp 271-294.

Geary, S. and Zonnenberg, J.P., 2000. What it means to be best in class. *Supply Chain Management Review*, 4, 42-49.

Geunes, J. and Pardalos, P.M., 2003. Network optimization in supply chain management and financial engineering: An annotated bibliography. *Networks*, 42(2), 66-84.

Geunes, J. and Pardalos, P.M., Eds., 2005. *Supply chain optimization*, Springer.

Gunasekaran, A. and Ngai, E.W.T., 2005. Build-to-order supply chain management: A literature review and framework for development. *Journal of Operations Management*, 23, 423-451.

Gunasekaran, A., Patel, C. and McGaughey, R.E., 2004. A framework for supply chain performance measurement. *International Journal of Production Economics*, 87, 333-347.

Handfield, R.B. and Pannesi, R.T., 1995. Antecedents of lead-time competitiveness in make-to-order manufacturing firms. *International Journal of Production Research*, 33 (2), 511-537.

Hill, A.V. and Khosla, I.S., 1992. Models for optimal lead time reduction. *Production and Operations Management*, 1 (2), 185-197.

Hum, S.-H. and Sim, H-H., 1996. Time-based competition: Literature review and implications for modelling. *International Journal of Operations and Production Management*, 16, 75-90.

Lederer, P.J. and Li, L., 1997. Pricing, production, scheduling and delivery-time competition. *Operations Research*, 45 (3), 407-420.

Li, S., Ragu-Nathan, B., Ragu-Nathan, T.S. and Subba Rao, S., 2006. The impact of supply chain management practices on competitive advantage and organizational performance. *Omega*, 34 (2), 107-124.

Li, L. and Whang, S., 2001. Game theory models in operations management and information systems. In *Game theory and business applications*, Chatterjee, K. and Samuelson, W.F. (Editors), Dordrecht, The Netherlands: Kluwer Academic Publishers, pp 95-131.

Masoumi, A.H., Yu, M. and Nagurney, A., 2012. A supply chain generalized network oligopoly model for pharmaceuticals under brand differentiation and perishability. *Transportation Research E*, 48, 762-780.

Nagurney, A., 1999. *Network economics: A variational inequality approach*, second and revised edition. Boston, MA: Kluwer Academic Publishers.

Nagurney, A., 2006. *Supply chain network economics: Dynamics of prices, flows, and profits.* Cheltenham, England: Edward Elgar Publishing.

Nagurney, A., 2010. Supply chain network design under profit maximization and oligopolistic competition. *Transportation Research E*, 46, 281-294.

Nagurney, A., Dong, J., Zhang, D., 2002. A supply chain network equilibrium model. *Transportation Research E* 38(5), 281-303.

Nagurney, A., Dupuis, P. and Zhang, D., 1994. A dynamical systems approach for network oligopolies and variational inequalities. *Annals of Regional Science*, 28, 263-283.

Nagurney, A. and Li, D., 2012. A dynamic spatial oligopoly model with transportation costs, product differentiation, and quality competition. Isenberg School of Management, University of Massachusetts, Amherst, Massachusetts.

Nagurney, A., Yu, M., Masoumi, A.H. and Nagurney, L.S., 2013. *Networks against time: Supply chain analytics for perishable products.* Springer Science + Business Media, New York.

Nagurney, A. and Qiang, Q., 2009. *Fragile networks: Identifying vulnerabilities and synergies in an uncertain world.* John Wiley & Sons, Hoboken, New Jersey.

Nagurney, A. and Yu, M., 2011. Fashion supply chain management through cost and time minimization from a network perspective. In *Fashion supply chain management: Industry and business analysis*, Choi, T.M. (Editor), Hershey, PA: IGI Global, pp 1-20.

Nagurney, A. and Yu, M., 2012. Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. *International Journal of Production Economics*, 135, 532-540.

Nagurney, A., Yu, M. and Qiang, Q., 2011. Supply chain network design for critical needs with outsourcing. *Papers in Regional Science*, 90, 123-142.

Nagurney, A. and Zhang, D., 1996. *Projected dynamical systems and variational inequalities with applications.* Boston, MA: Kluwer Academic Publishers.

Nash, J.F., 1950. Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences USA*, 36, 48-49.

Nash, J.F., 1951. Noncooperative games. *Annals of Mathematics*, 54, 286-298.

Qiang, Q., Nagurney, A. and Dong, J., 2009. Modeling of supply chain risk under disruptions with performance measurement and robustness analysis. In: T. Wu and J. Blackhurst, eds. *Managing supply chain risk and vulnerability: tools and methods for supply chain decision makers.* Berlin: Springer, 91111.

Ray, S. and Jewkes, E.M., 2004. Customer lead time management when both demand and price are lead time sensitive. *European Journal of Operational Research*, 153, 769-781.

Shang, W. and Liu, L., 2011. Promised delivery time and capacity games in time-based competition. *Management Science*, 57 (3), 599-610.

So, K.C., 2000. Price and time competition for service delivery. *Manufacturing and Service Operations Management*, 2, 392-409.

So, K.C. and Song, J.-S., 1998. Price, delivery time guarantees and capacity selection. *European Journal of Operational Research*, 111, 28-49.

Stalk, Jr., G., 1988. Time – The next source of competitive advantage. *Harvard Business Review*, 66 (4), 41-51.

Stalk, Jr., G. and Hout, T.M., 1990. *Competing against time: How time-based competition is reshaping global markets*. New York: Free Press.

Tirole, J., 1988. *The theory of industrial organization*. Cambridge, MA: MIT Press.

Vickery, S.K., Droge, C.L.M., Yeomans, J.M. and Markland, R.E., 1995. Time-based competition in the furniture industry. *Production and Inventory Management Journal*, 4th quarter, 14-21.

Vives, X., 1999. *Oligopoly pricing: Old ideas and new tools*. Cambridge, MA: MIT Press.

Wakolbinger, T. and Cruz, J., 2011. Supply chain disruption risk management through strategic information acquisition and sharing and risk sharing contracts. *International Journal of Production Research*, 49(13), 4063-4084.

Yu, M., 2012. *Analysis, design, and management of supply chain networks with applications to time-sensitive products*, Ph.D. Thesis, University of Massachusetts, Amherst, MA.