Tariffs and Quotas in World Trade:
A Unified Variational Inequality Framework

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Abstract:

In this paper, we develop a unified variational inequality framework in the context of spatial price network equilibrium problems that captures world trade policies in the form of tariff rate quotas, which are two-tiered tariffs, and that have been applied in practice to numerous commodities. The spatial price network equilibrium model allows for multiple supply markets and multiple demand markets in different countries as well as multiple transportation routes. The model is qualitatively analyzed and also related to models with trade policies such as ad valorem tariffs and strict quotas. A case study on the dairy industry focusing on the United States and France is presented to illustrate the modeling and computational framework with impacts of stricter quotas, higher over quota tariffs, as well as additional transportation routes assessed. The numerical examples reveal that although domestic producers can gain under tariff rate quotas, consumers may experience higher prices. Given the relevance of tariffs to global trade in the world today and discussions concerning the impacts, the framework constructed in this paper is especially timely.

Key words: networks; tariff rate quotas; spatial price equilibrium; dairy industry; variational inequalities

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1. Introduction

World trade is essential to the global economy through the flow of goods from supply markets in countries to demand markets. Products as diverse as fresh produce and other agricultural and food products such as meat, fish and seafood, cereals, including rice and wheat, and dairy products, to steel and aluminum, and a variety of other commodities, are transported across national boundaries to points of demand. Given the importance of global trade for producers and consumers alike, governments, nevertheless, often turn to trade policies ranging from tariffs and quotas, and their combination, in the form of tariff rate quotas (TRQs), in order to reduce the impact of competitive foreign firms on their demand markets and to protect their less competitive domestic firms. For example, the Uruguay Round in 1996 induced the creation of more than 1,300 new TRQs (cf. Skully (2001)). A TRQ (sometimes also referred to as a tariff quota) is a two-tiered tariff, whereby a lower in-quota tariff is applied to the units of imports until a quota or upper bound is reached and a higher over-quota tariff is applied to all subsequent imports (World Trade Organization (2004)), with, currently, World Trade Organization members having a combined total of 1,425 tariff quotas in their commitments (World Trade Organization (2018)).

Given the relevance of trade policies to world trade, and that tariffs, in particular, are a major issue now in the news (cf. Paquette, Lynch, and Rauhala (2018) and Tankersley (2018)), the development of rigorous mathematical models that can capture trade policies used in practice today and that are computable, providing both equilibrium supply market prices and demand market prices, as well as product flows, is critical. Such models should be sufficiently general to be able to handle multiple supply and demand markets in different countries, trade flows on general networks, and to be able to handle nonlinear cost and price functions that are also asymmetric and flow-dependent.

Spatial price equilibrium (SPE) models, in particular, have attained prominence in the modeling, analysis, and solution of a wide spectrum of commodity trade problems, dating to the seminal contributions of Enke (1951), Samuelson (1952), and Takayama and Judge (1964, 1971). The need to develop extensions, over and above the original SPE models that were reformulated and solved using optimization approaches, especially quadratic programming ones, has also spearheaded advances through the use of methodologies such as complementarity theory as well as variational inequality theory (cf. Asmuth, Eaves, and Peterson (1979), Florian and Los (1982), Dafermos and Nagurney (1984, 1987), Friesz, Harker, and Tobin (1984), Harker (1985), Nagurney and Aronson (1988), Nagurney, Takayama, and Zhang (1995), van den Bergh, Nijkamp, and Rietveld (1996), Nagurney (1999), Daniele (2004), Nagurney (2006), Li, Nagurney, and Yu (2018), and the references therein). In addition, spatial price equilibrium models, due to their practical applications in agricultural and energy and mineral markets, have been constructed to include policies such as ad valorem tariffs (Nagurney, Nicholson, and Bishop (1996a,b)), price policies (Nagurney and Zhao (1993)), and goal targets (Nagurney, Thore, and Pan (1996)) using variational inequality theory, as well as quotas and other tariff schemes using complementarity theory (see, e.g., Rutherford (1995), Nolte (2008), Grant, Hertel, and Rutherford (2009), Johnston and van Kooten (2017), and the references therein).

In this paper, we construct a general spatial price network equilibrium model consisting of countries and multiple supply markets and demand markets in each country under a tariff rate quota regime. Spatial price equilibrium models in which various policy interventions have been incorporated such as tariff rate quotas assume either regions or countries as supply and/or demand markets. Since different supply and demand markets may have, respectively, distinct supply and demand price functions, and trade policies such as tariff rate quotas can impose quotas over multiple countries, we believe that having a greater level of detail is meaningful. Furthermore, rather than assuming only a single path (essentially a link) between a pair of supply and demand market nodes, we allow for multiple paths, each of which is not limited to the same number of links. Different supply markets in a given country may have distinct transport mode options to demand markets in the same or other countries, and, therefore, such options can be represented as paths on the general network, with associated costs. We focus on tariff rate quotas (TRQs) since they have been deemed challenging to formulate and only stylized examples have been reported in an SPE framework (cf. Bishop et al. (2001)). In addition, we construct variants of the general spatial price network equilibrium model with tariff rate quotas, to demonstrate how the latter can be easily adapted to handle unit tariffs, ad valorem tariffs, and/or strict quotas. We also note that, through the use of multiple paths, the evaluation of avenues for transshipment, as a means to avoid tariffs, a topic that has garnered a lot of attention in the popular press recently (cf. Bradshear (2018)), is made possible. Moreover, our framework allows for the investigation of the impacts of TRQs on domestic markets, on both producers and consumers alike, that are imposed on non-domestic markets.

This paper is organized as follows. In Section 2, we first present the general spatial price network equilibrium model with tariff rate quotas, and derive a variational inequality formulation of the governing equilibrium conditions. We then demonstrate in Section 3 how the model can be easily adapted to also handle unit tariffs, ad valorem tariffs, and/or quotas, using a variational inequality framework. In addition, we present several numerical examples for illustrative purposes. In Section 4 we provide qualitative properties and propose a computational scheme in Section 5, which yields closed form expressions at each iteration. A case study in Section 6 on the dairy industry, with a focus on cheese, is then constructed to illustrate the effectiveness of the modeling and computational approach and the type of insights that can be gained. A summary of results, along with conclusions, and suggestions for future research, are provided in Section 7.

2. The Spatial Price Network Equilibrium Model with Tariff Rate Quotas

We consider a spatial price equilibrium problem on a general network consisting of the set of nodes N and the set of links L. In classical spatial price equilibrium problems, the commodity supply prices, trade flows, and demand prices are achieved when the equilibrium conditions stating that: if there is a positive volume of trade of the commodity between a supply market and a demand market, then the commodity price at the demand market is equal to the commodity price at the supply market plus the unit cost of transportation between the two are satisfied (Nagurney (1999)). One crucial feature of spatial price equilibrium problems is the acknowledgment of space, in that the supply and demand markets are spatially separated. In this paper, we extend this idea further by considering multiple countries, each of which can have multiple supply markets and demand markets associated with a homogeneous commodity. We assume that there are m countries with supply markets and n countries with demand markets. A typical country with supply markets is denoted by i and its set of supply markets by $k \in O_i$, whereas a typical country with demand markets is denoted by j with its demand markets denoted by $l \in D_j$. There are a total of k_i supply markets in country i; $i = 1, \ldots, m$, and a total of l_j demand markets in country j; $j = 1, \ldots, n$.

As an illustration, and in order to fix ideas, we present an example of a spatial price network in Figure 1. A top-tiered node in the spatial network, denoted by (i, k), corresponds to supply market k in country i, whereas a bottom-tiered node (j, l) corresponds to demand market l in country j. We consider both domestic and/or international trade of a commodity, meaning that the country and supply market node, i.e., (i, k_i), can be the same or distinct from the country and demand market node, say, (j, l_j).

In addition, we define groups G_g; $g = 1, \ldots, h$, with group G_g consisting of all the countries i and j comprising the group and their supply and demand markets. For typical groups, denoted by G_s and G_r, we have that $G_s \cap G_r = \emptyset, \forall s \neq r, \forall s, r$. Furthermore, for each group G_g there is an associated quota $\bar{Q}_{G_g}$

on the homogeneous product.

Let $P^{(i,k)}_{(j,l)}$ denote the set of paths connecting supply market k of country i with demand market l of country j. A path is denoted by p and the flow of the product on path p by x_p. Also, let $P^{(i,k)}$ denote the set of paths from top-tiered node (i,k) to all the demand markets in all countries (the bottom-tiered nodes); similarly, let $P_{(j,l)}$ denote the set of paths from all the supply markets in all the countries (the top-tiered nodes) to demand market l of country j, that is, the bottom-tiered node (j,l). Paths can correspond to different transport routes as well as mode options. The network topology can take on any form, as mandated by the specific application.

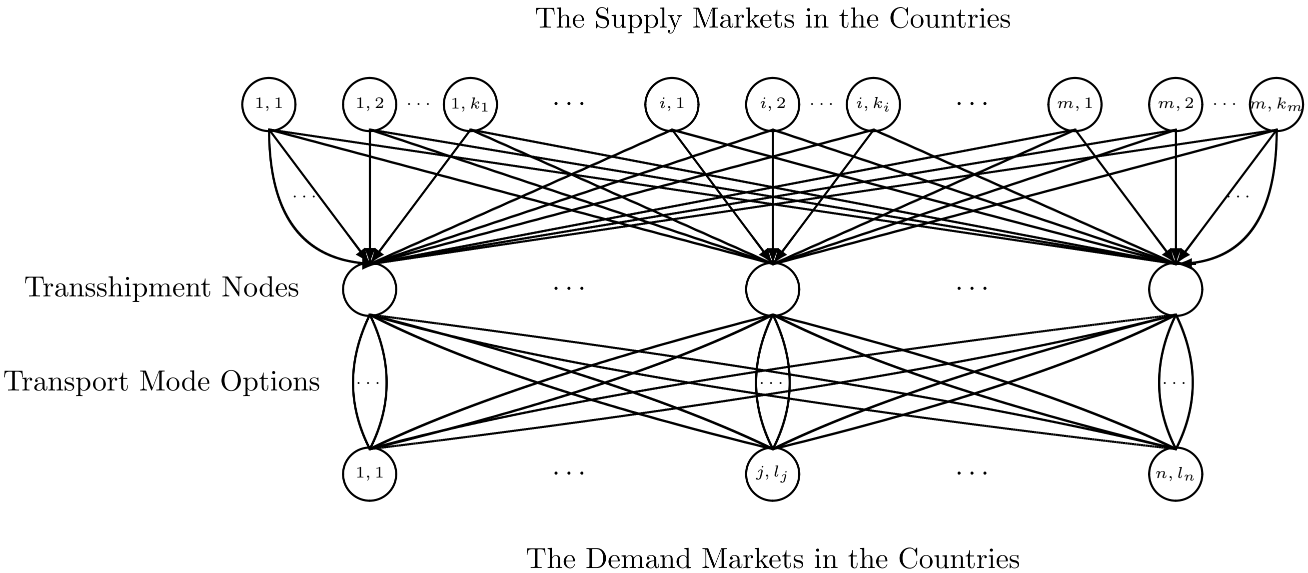

Figure 1: An Example of a Multicountry, Multiple Supply Market and Multiple Demand Market Spatial Price Network

Let f_a denote the flow on link a in the network, $\forall a \in L$. We group all the link flows into the vector $f \in R^{n_L}_+$, where n_L is the number of links in the network. All vectors are column vectors.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P. \tag{1}$$

We group the path flows into the vector $x \in R^{n_P}_+$, where n_P is the number of paths in the network.

The link flows are related to the path flows as follows:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

that is, the flow on a link is equal to the sum of flows on paths that contain that link.

Let s_{ik} denote the supply of the product produced at supply market k of country i and let d_{jl} denote the demand for the product in demand market l of country j. We group the supplies into the vector $s \in R^{\sum_{i=1}^m k_i}_+$ and the demands into the vector $d \in R^{\sum_{j=1}^n l_j}_+$. The supplies and demands must satisfy, respectively, the following conservation of flow equations:

$$s_{ik} = \sum_{p \in P^{(i,k)}} x_p, \quad \forall i, k, \tag{3}$$

$$d_{jl} = \sum_{p \in P_{(j,l)}} x_p, \quad \forall j, l. \tag{4}$$

Associated with each supply market k of country i, (i,k), is a supply price function $\pi^{(i,k)}$ and with each demand market l of country j, (j,l), a demand price function $\rho_{(j,l)}$. We assume that, in general, the supply price at a supply market in a country can depend on the vector of supplies and that the demand price at a demand market in a country can depend on the vector of demands so that:

$$\pi^{(i,k)} = \pi^{(i,k)}(s), \quad \forall i, k, \tag{5}$$

$$\rho_{(j,l)} = \rho_{(j,l)}(d), \quad \forall j, l. \tag{6}$$

These functions are assumed to be continuous with the supply price functions being monotonically increasing and the demand functions, on the other hand, monotonically decreasing.

Furthermore, we associate a unit cost c_a with each link a in the network, which includes the unit transportation cost, and, in general, can depend on the vector of link flows f, so that:

$$c_a = c_a(f), \quad \forall a \in L. \tag{7}$$

The link cost functions are also assumed to be continuous and monotonically increasing in order to capture congestion.

The unit cost on a path p, C_p, joining a supply market in a country with a demand market in a country is then given by:

$$C_p = \sum_{a \in L} c_a(f)\delta_{ap}, \quad \forall p \in P. \tag{8}$$

Hence, the cost on a path is equal to the sum of the costs on the links that make up the path.

We now introduce the additional notation for the tariff rate quotas. Recall that a tariff quota is a two-tiered tariff. Let $\tau^u_{G_g}$ denote the tariff, which is fixed and preassigned to group G_g, when the sum of the path flows of the product in the group is less than (under) the quota $\bar{Q}_{G_g}$, for g. Also, let $\tau^o_{G_g}$ denote the fixed and preassigned tariff, which is assigned to group G_g if the sum of the path flows of the product in the group is greater than $\bar{Q}_{G_g}$, for all g. We always have that $\tau^o_{G_g} > \tau^u_{G_g}$. In addition, it is worth mentioning that the under tariffs can be set to zero, depending on the application. Let λ_{G_g}, for all G_g, denote the quota rent equivalent (see, e.g., Skully (2001)), which can have an interpretation of a Lagrange multiplier, and which we expand upon in the network equilibrium conditions below. We group the quota rent equivalents into the vector $\lambda \in R^h$.

The costs, prices, and tariffs are all in a common currency.

Definition 1: Spatial Price Network Equilibrium Conditions Under Tariff Rate Quotas

A product flow pattern $x^* \in R^{n_P}_+$ and quota rent equivalent $\lambda^* \in R^h$ is a spatial price network equilibrium under a tariff rate quota regime if the following conditions hold: For all groups G_g, for all g, and for all pairs of supply and demand markets in the countries: $(i,j),(k,l) \in G_g$, and all paths $p \in P^{(i,k)}_{(j,l)}$:

$$\pi^{(i,k)} + C_p + \tau^u_{G_g} + \lambda^*_{G_g} \begin{cases} = \rho_{(j,l)}, & \textit{if} \quad x^*_p > 0, \\ \geq \rho_{(j,l)}, & \textit{if} \quad x^*_p = 0, \end{cases} \tag{9}$$

and for all g:

$$\lambda^*_{G_g} \begin{cases} = \tau^o_{G_g} - \tau^u_{G_g}, & \textit{if} \quad \sum_{p \in P_{G_g}} x^*_p > \bar{Q}_{G_g}, \\ \leq \tau^o_{G_g} - \tau^u_{G_g}, & \textit{if} \quad \sum_{p \in P_{G_g}} x^*_p = \bar{Q}_{G_g}, \\ = 0, & \textit{if} \quad \sum_{p \in P_{G_g}} x^*_p < \bar{Q}_{G_g}. \end{cases} \tag{10}$$

Equilibrium conditions (9) are an expansion of the Samuelson (1952) and Takayama and Judge (1971) spatial price equilibrium conditions to include, in a novel manner, the incorporation of two-tiered tariffs, depending on the volumes of product flows and quotas in country groups, as stated in (10). See also path formulations of classical spatial price equilibrium problems using path flows by Florian and Los (1982) and Dafermos and Nagurney (1983). Note that, according to the above definition, if the sum of the product path flows is less than the imposed quota for the particular group of countries, then the assigned tariff on those paths is just equal to the "under" quota tariff $\tau^u_{G_g}$ since in this case $\lambda^*_{G_g} = 0$. On the other hand, if the sum of the path flows exceeds the quota (which is allowed in the case of tariff quotas), then the incurred tariff on the associated paths is equal to the "over" quota tariff $\tau^o_{G_g}$. If the sum of the path flows is precisely equal to the quota for the group then the incurred additional payment on the corresponding paths is less than $\tau^u_{G_g}$.

Remark 1

We emphasize that, in the case of a domestic group G_g, the above conditions (9) and (10) are also applicable (and we illustrate this feature in simple numerical examples in Section 3.1), where we set $\tau^u_{G_g} = 0$, and $\tau^o_{G_g} > \tau^u_{G_g}$, with $\bar{Q}_{G_g}$ very large, so that it is never achievable.

In view of (3) and (5), we can redefine the supply price functions as:

$$\hat{\pi}^{(i,k)}(x) \equiv \pi^{(i,k)}(s), \quad \forall i, k, \tag{11}$$

the demand price functions, in view of (4) and (6), as:

$$\hat{\rho}_{(j,l)}(x) \equiv \rho_{(j,l)}(d), \quad \forall j, l, \tag{12}$$

and, in view of (2), (7), and (8), the path and link cost functions:

$$\hat{C}_p(x) \equiv \sum_{a \in L} \hat{c}_a(x)\delta_{ap}, \quad \forall p \in P, \tag{13}$$

where

$$\hat{c}_a(x) \equiv c_a(f), \quad \forall a \in L. \tag{14}$$

The above definitions allow us to construct a variational inequality formulation of the governing spatial price network equilibrium conditions under a tariff rate quota regime in path flows and quota rents in the Theorem below. We define the feasible set $K \equiv \{(x, \lambda) | x \in R^{n_P}_+, \, \lambda \in R^h_+ | 0 \leq \lambda_{G_g} \leq \tau^o_{G_g} - \tau^u_{G_g}, \forall g\}$.

Theorem 1: Variational Inequality Formulation of Spatial Price Network Equilibrium Conditions Under Tariff Rate Quotas

A path flow and quota rent equivalent pattern $(x^*, \lambda^*) \in K$ is a spatial price network equilibrium under a tariff quota regime according to Definition 1 if and only if it satisfies the variational inequality:

$$\sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P^{(i,k)}_{(j,l)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + \lambda^*_{G_g} - \hat{\rho}_{(j,l)}(x^*)\right] \times \left[x_p - x^*_p\right]$$

$$+\sum_{g}\left[\bar{Q}_{G_g}-\sum_{p\in P_{G_g}}x_p^*\right]\times\left[\lambda_{G_g}-\lambda_{G_g}^*\right]\geq 0,\quad \forall(x,\lambda)\in K. \tag{15}$$

Proof: We first establish necessity, that is, we prove that if a path flow and quota rent equivalent pattern $(x^*, \lambda^*) \in K$ satisfies Definition 1 then it also satisfies the variational inequality (15). Note that equilibrium conditions (9), in view of (11) – (14), can be rewritten as: For all groups G_g, for all g, and for all pairs of supply and demand markets: $(i,j),(k,l) \in G_g$, and all paths $p \in P^{(i,k)}_{(j,l)}$:

$$\hat{\pi}^{(i,k)}(x^*)+\hat{C}_p(x^*)+\tau^u_{G_g}+\lambda^*_{G_g}\begin{cases}=\hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x^*_p>0,\\ \geq\hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x^*_p=0.\end{cases} \tag{16}$$

Clearly, for a fixed path $p \in P^{(i,k)}_{(j,l)}$ with $(i,k),(j,l) \in G_g$, and group G_g, (16) implies that

$$\left[\hat{\pi}^{(i,k)}(x^*)+\hat{C}_p(x^*)+\tau^u_{G_g}+\lambda^*_{G_g}-\hat{\rho}_{(j,l)}(x^*)\right]\times\left[x_p-x^*_p\right]\geq 0,\quad \forall x_p\geq 0, \tag{17}$$

since, according to Definition 1, if $x^*_p > 0$, then $\left[\hat{\pi}^{(i,k)}(x^*)+\hat{C}_p(x^*)+\tau^u_{G_g}+\lambda^*_{G_g}-\hat{\rho}_{(j,l)}(x^*)\right]=0$, and (17) holds; on the other hand, if $x^*_p = 0$, then $\left[\hat{\pi}^{(i,k)}(x^*)+\hat{C}_p(x^*)+\tau^u_{G_g}+\lambda^*_{G_g}-\hat{\rho}_{(j,l)}(x^*)\right]\geq 0$, and since, due to the nonnegativity assumption on the path flows: $\left[x_p-x^*_p\right]\geq 0$; hence, the product of these two terms is also ≥ 0, so (17) also holds in this case. Inequality (17) is independent of path p and, therefore, we can sum (17) over all paths, obtaining:

$$\sum_{g}\sum_{(i,j)\in G_g}\sum_{k\in O_i}\sum_{l\in D_j}\sum_{p\in P^{(i,k)}_{(j,l)}}\left[\hat{\pi}^{(i,k)}(x^*)+\hat{C}_p(x^*)+\tau^u_{G_g}+\lambda^*_{G_g}-\hat{\rho}_{(j,l)}(x^*)\right]\times\left[x_p-x^*_p\right]\geq 0,\quad \forall x\in K. \tag{18}$$

Also, for a fixed g, (10) in Definition 1 implies that:

$$\left[\bar{Q}_{G_g}-\sum_{p\in P_{G_g}}x_p^*\right]\times\left[\lambda_{G_g}-\lambda_{G_g}^*\right]\geq 0,\quad 0\leq\lambda_{G_g}\leq\tau^o_{G_g}-\tau^u_{G_g}. \tag{19}$$

Indeed, according to (10), if $\lambda^*_{G_g} = \tau^o_{G_g} - \tau^u_{G_g}$, then the expression before the multiplication sign in (19) is negative, whereas that after the multiplication sign is less than equal to zero and, thus, (19) holds. If $\lambda^*_{G_g} = 0$, then according to (10) the first term in (19) is nonnegative and, due to feasibility, so is the second term; therefore, their product is nonnegative and (19) also holds. Finally, if $\sum_{p\in P_{G_g}} x^*_p = \bar{Q}_g$, then, according to (10), the term to the right-hand-side of the multiplication sign in (19) can be positive or negative, but the multiplication of both terms is still equal to zero and (19) holds.

The inequality (19) holds for any g, and, therefore, we can conclude that summation over all g yields:

$$\sum_{g}\left[\bar{Q}_{G_g}-\sum_{p\in P_{G_g}}x_p^*\right]\times\left[\lambda_{G_g}-\lambda_{G_g}^*\right]\geq 0,\quad \forall\lambda\in K. \tag{20}$$

Summation of (18) and (20) gives us variational inequality (15) and necessity is established.

We now prove sufficiency, that is, a solution $(x^*, \lambda^*) \in K$ of variational inequality (15) also satisfies the supply chain network equilibrium conditions (9) and (10) of Definition 1.

Assume that a pattern $(x^*, \lambda^*) \in K$ satisfies variational inequality (15). Let $\lambda_{G_g} = \lambda^*_{G_g}$ for all g and let $x_q = x^*_q$ for all paths q except for path $p \in P^{(i,k)}_{(j,l)}$. Substitution of these into (15) then yields:

$$\left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + \lambda^*_{G_g} - \hat{\rho}_{(j,l)}(x^*)\right] \times \left[x_p - x^*_p\right] \geq 0, \quad \forall x_p \geq 0. \tag{21}$$

But (21) implies that equilibrium conditions (9) must hold for path p and, hence, for any path.

Similarly, setting $x_p = x^*_p$, $\forall p \in P$, and $\lambda_{G_g} = \lambda^*_{G_g}$ for all $g \neq r$, and substituting the resultants into variational inequality (15) gives us:

$$\left[\bar{Q}_{G_r} - \sum_{p \in P_{G_r}} x^*_p\right] \times \left[\lambda_{G_r} - \lambda^*_{G_r}\right] \geq 0, \quad \forall \lambda_{Gr} \text{ such that } 0 \leq \lambda_{G_r} \leq \tau^o_{G_r} - \tau^u_{G_r}. \tag{22}$$

And (22) implies that equilibrium conditions (10) must hold for any r. The proof is complete. □

We now put variational inequality (15) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{23}$$

where X and $F(X)$ are $\mathcal{N}$-dimensional vectors, $\mathcal{K}$ is a closed, convex set, and F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$.

Indeed, we can define $X \equiv (x, \lambda)$ and $F(X) \equiv (F_1(X), F_2(X))$, where $F_1(X)$ consists of n_P elements: $\hat{\pi}^{(i,k)}(x) + \hat{C}_p(x) + \tau^u_{G_g} + \lambda_{G_g} - \hat{\rho}_{(j,l)}(x)$, for all paths $p \in P$, and $F_2(X)$ consists of h elements, with the g-th element given by: $\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x_p$. Also, here $\mathcal{N} = n_P + h$ and $\mathcal{K} \equiv K$.

3. Variants of the Spatial Price Network Equilibrium Model

In this Section, we present several variants of the model constructed in Section 2. This is being done in order to demonstrate the flexibility of the variational inequality formalism for policy modeling in the context of world trade.

First, an extension of the classical spatial price network equilibrium model is given, and from this model policies in the form of ad valorem tariffs and then quotas incorporated.

Without loss of generality, we let the set of paths P correspond to all the paths joining the country supply market nodes with the country demand market nodes with the number of paths being n_P. In the absence of a tariff rate quota regime, we then have the following definition, with reference to (16):

Definition 2: (Classical) Spatial Price Network Equilibrium Conditions

A product flow pattern $x^* \in R^{n_P}_+$ is a spatial price network equilibrium if the following conditions hold: For all pairs of supply and demand markets $(i,k), (j,l)$ and all paths $p \in P^{(i,k)}_{(j,l)}$:

$$\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) \begin{cases} = \hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x^*_p > 0, \\ \geq \hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x^*_p = 0. \end{cases} \tag{24}$$

The following variational inequality is immediate using similar arguments as in the proof of Theorem 1.

Corollary 1: Variational Inequality Formulation of a Spatial Price Network Equilibrium

A product flow pattern $x^* \in R^{n_P}_+$ is a spatial price network equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$\sum_{(i,k)} \sum_{(j,l)} \sum_{p \in P^{(i,k)}_{(j,l)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x^*_p \right] \geq 0, \quad \forall x^* \in R^{n_P}_+. \tag{25}$$

Remark 2

There are alternative variational inequalities to (25). The first alternative would utilize the conservation of flow equations and vectors s and d and retain the path flows; the second alternative formulation would then further utilize the link/path conservation of flow equations and have the supplies, demands, and link flows be the variables. For an example of a model with the latter, but not including explicit countries and regions, see Nagurney (1999).

Also, unit tariffs can be easily incorporated into the variational inequality (25) by simply adding the tariff on the appropriate link(s) of the appropriate paths.

We now consider ad valorem tariffs, with $1 + \tau_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, denoting the ad valorem tariff associated with product flows from country i to j as in the modification of Definition 2 to yield Definition 3 below. Typically, the τ_{ij}s are in the range $0 < \tau_{ij} \leq 1$.

Definition 3: Spatial Price Network Equilibrium Conditions with Ad Valorem Tariffs

A product flow pattern $x^* \in R^{n_P}_+$ is a spatial price network equilibrium under an ad valorem tariff regime if the following conditions hold: For all pairs of supply and demand markets $(i,k), (j,l)$ and all paths $p \in P^{(i,k)}_{(j,l)}$:

$$(\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*))(1 + \tau_{ij}) \begin{cases} = \hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x^*_p > 0, \\ \geq \hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x^*_p = 0. \end{cases} \tag{26}$$

The following corollary generalizes results of Nagurney, Nicholson, and Bishop (1996a,b) to multiple countries and multiple paths connecting each pair of supply and demand markets as well as non-fixed transportation costs.

Corollary 2: Variational Inequality Formulation of a Spatial Price Network Equilibrium with Ad Valorem Tariffs

A product flow pattern $x^* \in R^{n_P}_+$ is a spatial price network equilibrium under an ad valorem tariff regime according to Definition 3 if and only if it satisfies the variational inequality:

$$\sum_{(i,k)} \sum_{(j,l)} \sum_{p \in P^{(i,k)}_{(j,l)}} \left[(\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*))(1 + \tau_{ij}) - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x^*_p \right] \geq 0, \quad \forall x^* \in R^{n_P}_+. \tag{27}$$

We now consider strict quotas. Note that, in contrast, under a tariff rate quota regime, the tariffs are two-tiered, and the quota can be exceeded, but then there is a higher associated tariff. In practice, these can be set very high.

Specifically, we let $\hat{\hat{Q}}_g$ denote the strict quota for g; $g = 1, \ldots, \hat{h}$ and we let $\lambda_{\hat{G}_g}$ denote the tariff/tax, which is, in effect, a Lagrange multiplier, and we refer to it as such, if the quota is reached for group $\hat{G}_g$. Then we have the following spatial price network equilibrium conditions:

Definition 4: Spatial Price Network Equilibrium Conditions Under Strict Quotas

A product flow pattern $x^* \in R_+^{n_P}$ and Lagrange multiplier pattern $\hat{\lambda}^* \in R_+^{\hat{h}}$ is a spatial price network equilibrium under strict quotas if the following conditions hold: For all pairs of supply and demand markets $(i,k), (j,l)$ and all paths $p \in P_{(j,l)}^{(i,k)}$:

$$\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \hat{\lambda}^*_{\hat{G}_g} \begin{cases} = \hat{\rho}_{(j,l)}(x^*), & \textit{if} \quad x_p^* > 0, \\ \geq \hat{\rho}_{(j,l)}(x^*), & \textit{if} \quad x_p^* = 0, \end{cases} \tag{28}$$

and for all g:

$$\hat{\lambda}^*_{\hat{G}_g} \begin{cases} \geq 0, & \textit{if} \quad \sum_{p \in P_{\hat{G}_g}} x_p^* = \hat{\hat{Q}}_{\hat{G}_g}, \\ = 0, & \textit{if} \quad \sum_{p \in P_{\hat{G}_g}} x_p^* < \hat{\hat{Q}}_{\hat{G}_g}. \end{cases} \tag{29}$$

The following corollary follows using similar arguments as in the proof of Theorem 1.

Corollary 3: Variational Inequality Formulation of a Spatial Price Network Equilibrium with Strict Quotas

A product flow pattern $x^* \in R_+^{n_P}$ and Lagrange multiplier pattern $\hat{\lambda}^* \in R_+^{\hat{h}}$ is a spatial price network equilibrium under strict quotas according to Definition 4 if and only if it satisfies the variational inequality:

$$\sum_{(i,k)} \sum_{(j,l)} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \hat{\lambda}^*_{\hat{G}_g} - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x_p^* \right]$$

$$+ \sum_g \left[\hat{\hat{Q}}_{\hat{G}_g} - \sum_{p \in P_{\hat{G}_g}} x_p^* \right] \times \left[\hat{\lambda}_{G_g} - \hat{\lambda}^*_{\hat{G}_g} \right] \geq 0, \quad \forall x^* \in R_+^{n_P}, \forall \hat{\lambda}^* \in R_+^{\hat{h}}. \tag{30}$$

3.1 Illustrative Examples

For definiteness, and in order to illustrate the model, with the focus on the TRQ one, we now provide several illustrative examples. We consider two countries with a single supply market in each country of a commodity, denoted by $(1,1)$ and $(2,1)$, respectively. There is a single demand market in country 1, $(1,1)$, which is spatially separated from its domestic supply market $(1,1)$. The network topology for the examples is as in Figure 2.

Path p_1 consists of the link joining top-tiered node $(1,1)$ with bottom-tiered node $(1,1)$ and path p_2 consists of the link joining top-tiered node $(2,1)$ with bottom-tiered node $(1,1)$. There are two groups, with group G_1 consisting of the domestic production and consumption sites: top-tiered node $(1,1)$ and

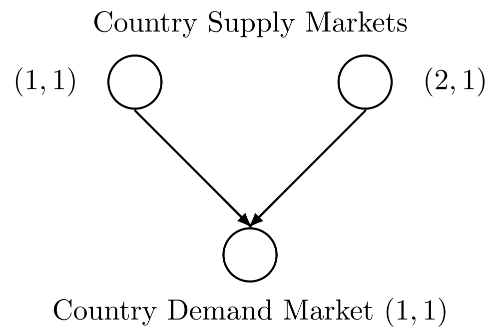

Figure 2: Network Topology for the Illustrative Example

bottom-tiered node $(1,1)$, and group G_2 consisting of top-tiered node $(2,1)$, which is not domestic, and bottom-tiered node $(1,1)$.

Illustrative Example 1

We are interested in evaluating the impact of a tariff rate quota (TRQ), but we first present a classical SPE example and then build the TRQ one from it.

The supply price functions are:

$$\hat{\pi}^{(1,1)}(x) = 5x_{p_1} + 5, \quad \hat{\pi}^{(2,1)}(x) = x_{p_2} + 2,$$

the unit path cost functions are:

$$\hat{C}_{p_1}(x) = x_{p_1} + 2, \quad \hat{C}_{p_2}(x) = x_{p_2} + 3,$$

and the demand price function is:

$$\hat{\rho}_{(1,1)}(x) = -(x_{p_1} + x_{p_2}) + 18.$$

Observe that the non-domestic supply market has a lower supply market price function but a higher unit transportation cost function than the domestic supply and demand markets.

In the classical model, without any imposed TRQs, the spatial price equilibrium conditions, according to (24), are, with the realization that $x^*_{p_1} > 0$ and $x^*_{p_2} > 0$:

$$\hat{\pi}^{(1,1)}(x^*) + \hat{C}_{p_1}(x^*) = \hat{\rho}_{(1,1)}(x^*),$$

$$\hat{\pi}^{(2,1)}(x^*) + \hat{C}_{p_2}(x^*) = \hat{\rho}_{(1,1)}(x^*),$$

and, hence, numerically, we have that, at equilibrium:

$$5x^*_{p_1} + 5 + x^*_{p_1} + 2 = -(x^*_{p_1} + x^*_{p_2}) + 18,$$

$$x^*_{p_2} + 2 + x^*_{p_2} + 3 = -(x^*_{p_1} + x^*_{p_2}) + 18,$$

which, after algebraic simplification, results in the following system of equations:

$$7x^*_{p_1} + x^*_{p_2} = 11,$$

$$x^*_{p_1} + 3x^*_{p_2} = 13,$$

with solution: $x^*_{p_1} = 1$ and $x^*_{p_2} = 4$, with $\hat{\pi}^{(1,1)} = 10$, $\hat{C}_{p_1} = 3$, and $\hat{\rho}_{(1,1)} = 13$, and $\hat{\pi}^{(2,1)} = 6$, $\hat{C}_{p_2} = 7$, so, indeed, the classical spatial price equilibrium conditions are satisfied (cf. (24)).

We now impose the following tariff rate quota, which allows us, once the problem is solved, to investigate the impacts on trade flows and incurred prices.

For the domestic pair, there is no under tariff, and the over tariff, which will never be applied, since we impose a very high quota for this paper, is as follows:

$$\tau^u_{G_1} = 0, \quad \tau^o_{G_1} = 1, \quad \bar{Q}_{G_1} = 100,$$

and for the non-domestic, domestic pair of nodes, we have the following tariffs and associated quota:

$$\tau^u_{G_2} = 2, \quad \tau^o_{G_2} = 4, \quad \bar{Q}_{G_2} = 3.$$

Returning to the governing equilibrium conditions for the SPE model with TRQs, given by (9) and (10), and knowing that $\lambda^*_{G_1} = 0$, we know that now the following equations must be satisfied:

$$5x^*_{p_1} + 5 + x^*_{p_1} + 2 = -(x^*_{p_1} + x^*_{p_2}) + 18,$$

$$x^*_{p_2} + 2 + x^*_{p_2} + 3 + \tau^u_{G_2} + \lambda^*_{G_2} = -(x^*_{p_1} + x^*_{p_2}) + 18.$$

Moreover, we know that, if $x^*_{p_2} = 3$, then $\lambda^*_{G_2} = \tau^o_{G_2} - \tau^u_{G_2} \leq 2$.

Substituting $x^*_{p_2} = 3$ into the above system of equations, we obtain:

$$7x^*_{p_1} + 3 = 11,$$

$$x^*_{p_1} + 9 + \lambda^*_{G_2} = 11.$$

Solving now the above simplified system, we obtain:

$$x^*_{p_1} = 1\frac{1}{7}, \quad \lambda^*_{G_2} = \frac{6}{7},$$

since we have that $x^*_{p_2} = 3$.

Observe that, by imposing the TRQ, as above, the domestic supply market now produces more than in the classical SPE numerical example above, whereas the non-domestic supply market exports less. The consumers, nevertheless, now pay a higher unit price of $\hat{\rho}_{(1,1)} = 13\frac{6}{7}$ as opposed to 13. Also, we now have: $\hat{\pi}^{(1,1)} = 10\frac{5}{7}$, $\hat{C}_{p_1} = 3\frac{1}{7}$, and $\hat{\pi}^{(2,1)} = 5$, $\hat{C}_{p_2} = 6$.

Illustrative Example 2

We construct another TRQ example, beginning with a classical SPE counterpart to which we, as in Example 1, then impose tariffs rate quotas. The network topology is as in Figure 2. The same markets are domestic as in Example 1.

The supply price functions are now:

$$\hat{\pi}^{(1,1)}(x) = 3x_{p_1} + 1, \quad \hat{\pi}^{(2,1)}(x) = x_{p_2} + 1,$$

the unit path cost functions are:

$$\hat{C}_{p_1}(x) = 2x_{p_1} + 1, \quad \hat{C}_{p_2}(x) = x_{p_2} + 1,$$

and the demand price function is:

$$\hat{\rho}_{(1,1)}(x) = -2(x_{p_1} + x_{p_2}) + 26.$$

Proceeding as in Example 1, and realizing that $x^*_{p_1} > 0$ and $x^*_{p_2} > 0$, the classical SPE conditions (24) yield a system of equations, which, when solved algebraically, yield the following solution: $x^*_{p_1} = 2$ and $x^*_{p_2} = 5$, with the following incurred supply prices, unit path transportation costs, and demand price: $\hat{\pi}^{(1,1)} = 7$, $\hat{C}_{p_1} = 5$, and $\hat{\rho}_{(1,1)} = 12$, and $\hat{\pi}^{(2,1)} = 6$, $\hat{C}_{p_2} = 6$. Clearly, the classical spatial price equilibrium conditions are satisfied (cf. (24)).

We now impose the following tariff rate quota. We set: $\tau^u_{G_2} = 3$, $\tau^o_{G_2} = 6$, and $\bar{Q}_{G_2} = 2$.

Utilizing TRQ equilibrium conditions (9) and (10), the solution is: $x^*_{p_1} = 2\frac{1}{2}$, $x^*_{p_2} = 3\frac{1}{4} > 2$, and $\lambda^*_{G_2} = 3$. Observe that, in this example, the imports from the second supply market, which is not domestic, exceeds the imposed quota and, therefore, the higher tariff is in effect. The incurred prices and costs are: $\hat{\pi}^{(1,1)} = 8\frac{1}{2}$, $\hat{C}_{p_1} = 6$, and $\hat{\rho}_{(1,1)} = 14\frac{1}{2}$, and $\hat{\pi}^{(2,1)} = 4\frac{1}{4}$, $\hat{C}_{p_2} = 4\frac{1}{4}$. The supply price at the domestic supply market has increased, whereas that of the non-domestic one has decreased. More of the domestic product is consumed than previously without the TRQ and less of the non-domestic one. Again, the consumers experience a higher demand market price.

4. Qualitative Properties

In this Section we provide qualitative properties of the function F (cf. (23)) corresponding to the SPE model with TRQs, which are needed for convergence of the algorithmic scheme in Section 5. In addition, we provide existence results for a solution X^* of the variational inequality (23).

In the following proposition we establish that if the supply price functions, the link cost functions, and minus the demand price functions are monotone in their respective vectors of variables, then the function F as in (23) is monotone in (x, λ). Conditions that guarantee monotonicity of these functions is that their respective Jacobian matrices are positive semidefinite over the feasible set. In the proof of Proposition 1 we also derive an alternative variational inequality to (15), which includes link flows. We, hence, define the feasible set $K^1 \equiv \{(s, f, d, x, \lambda) | x \in R^{n_P}_+, \lambda \in R^h_+, 0 \leq \lambda_{G_g} \leq \tau^o_{G_g} - \tau^u_{G_g}, \forall g, \text{and}\,(2),(3),(4)\,\text{hold}\}$.

Proposition 1: Monotonicity of $F(X)$ in (23)

Assume that the vector of supply price functions, $\pi(s)$, the vector of link transportation cost functions, $c(f)$, and the vector of minus the demand price functions $-\rho(d)$, are all monotone as follows:

$$\langle \pi(s^1) - \pi(s^2), s^1 - s^2 \rangle \geq 0, \quad \forall (s^1, s^2) \in K^1, \tag{31a}$$

$$\langle c(f^1) - c(f^2), f^1 - f^2 \rangle \geq 0, \quad \forall (f^1, f^2) \in K^1, \tag{31b}$$

$$-\langle \rho(d^1) - \rho(d^2), d^1 - d^2 \rangle \geq 0, \quad \forall (d^1, d^2) \in K^1. \tag{31c}$$

Then the function that enters the VI (15), as in standard form $F(X)$ in (23), is monotone, with respect to the path flows x and the Lagrange multipliers λ, $X = (x, \lambda)$.

Proof: Using (2), (3), (4), (7), and (8), and recalling that $G_s \cap G_r = \emptyset, \forall s \neq r, \forall s, r$, we have the following equations:

$$s_{ik} = \sum_{p \in P_{(i,k)}} x_p = \sum_{j=1}^{n} \sum_{l=1}^{l_j} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p = \sum_{g} \sum_{(i,j) \in G_g} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p, \tag{32}$$

$$d_{jl} = \sum_{p \in P_{(j,l)}} x_p = \sum_{i=1}^{m} \sum_{k=1}^{k_i} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p = \sum_{g} \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p, \tag{33}$$

and

$$\sum_{g} \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \hat{C}_p(x^*) \times \left[x_p - x_p^*\right] = \sum_{p \in P} \hat{C}_p(x) \times \left[x_p - x_p^*\right]$$

$$= \sum_{p \in P} \left[\sum_{a \in L} c_a(f) \delta_{ap}\right] \times \left[x_p - x_p^*\right] = \sum_{a \in L} c_a(f) \times \left[\sum_{p \in P} \delta_{ap} x_p - \sum_{p \in P} \delta_{ap} x_p^*\right] = \sum_{a \in L} c_a(f) \times \left[f_a - f_a^*\right]. \tag{34}$$

Using now (32), (33), and (34), and also (11) and (12), we can rewrite VI (15) as follows:

$$\sum_{g} \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \hat{\pi}^{(i,k)}(x^*) \times \left[x_p - x_p^*\right]$$

$$+ \sum_{g} \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \hat{C}_p(x^*) \times \left[x_p - x_p^*\right]$$

$$+ \sum_{g} \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\tau_{G_g}^u + \lambda_{G_g}^*\right] \times \left[x_p - x_p^*\right]$$

$$- \sum_{g} \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \hat{\rho}_{(j,l)}(x^*) \times \left[x_p - x_p^*\right]$$

$$+ \sum_{g} \left[\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x_p^*\right] \times \left[\lambda_{G_g} - \lambda_{G_g}^*\right]$$

$$= \sum_{i,k} \pi^{(i,k)}(s^*) \times \left[s_{ik} - s_{ik}^*\right] + \sum_{a \in L} c_a(f^*) \times \left[f_a - f_a^*\right]$$

$$+ \sum_{g} \sum_{p \in G_g} \left[\tau_{G_g}^u + \lambda_{G_g}^*\right] \times \left[x_p - x_p^*\right] - \sum_{j,l} \rho_{(j,l)}(d^*) \times \left[d_{jl} - d_{jl}^*\right]$$

$$+ \sum_{g} \left[\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x_p^*\right] \times \left[\lambda_{G_g} - \lambda_{G_g}^*\right] \geq 0, \quad \forall (s, f, d, x, \lambda) \in K^1. \tag{35}$$

Now we can prove that $F(X)$ is monotone. For any $X^1 = (x^1, \lambda^1) \in K, X^2 = (x^2, \lambda^2) \in K$,

$$\langle F(X)^1 - F(X^2), X^1 - X^2 \rangle = \langle (F(x^1, \lambda^1) - F(x^2, \lambda^2)), \begin{bmatrix} x^1 - x^2 \\ \lambda^1 - \lambda^2 \end{bmatrix} \rangle$$

$$
= \sum_{i,k} (\left[\pi^{(i,k)}(s^1) - \pi^{(i,k)}(s^2) \right] \times \left[s^1_{ik} - s^2_{ik} \right] + \sum_{a \in L} \left[c_a(f^1) - c_a(f^2) \right] \times \left[f^1_a - f^2_a \right]
$$

$$
+ \sum_g \left[\lambda^1_{G_g} - \lambda^2_{G_g} \right] \times \left[\sum_{p \in P_{G_g}} x^1_p - \sum_{p \in P_{G_g}} x^2_p \right] - \sum_{j,l} \left[\rho_{(j,l)}(d^1) - \rho_{(j,l)}(d^2) \right] \times \left[d^1_{jl} - d^2_{jl} \right]
$$

$$
+ \sum_g \left[- \sum_{p \in P_{G_g}} x^1_p + \sum_{p \in P_{G_g}} x^2_p \right] \times \left[\lambda^1_{G_g} - \lambda^2_{G_g} \right]
$$

$$
= \sum_{i,k} \left[\pi^{(i,k)}(s^1) - \pi^{(i,k)}(s^2) \right] \times \left[s^1_{ik} - s^2_{ik} \right] + \sum_{a \in L} \left[c_a(f^1) - c_a(f^2) \right] \times \left[f^1_a - f^2_a \right]
$$

$$
- \sum_{j,l} \left[\rho_{(j,l)}(d^1) - \rho_{(j,l)}(d^2) \right] \times \left[d^1_{jl} - d^2_{jl} \right]. \tag{36}
$$

With the assumptions (31a, b, c) on the functions in the statement of Proposition 1, we can conclude that expression (36) is greater or equal to zero. Thus, $F(X)$ is monotone. □

In addition, we can expect that $F(X)$ for our model is Lipschitz continuous under assumptions that the underlying supply price, demand price, and link cost functions are continuous and have bounded second order derivatives (cf. Nagurney (1999)). The definition of Lipschitz continuity is given below for easy reference.

Definition 5: Lipschitz Continuity

A function $F(X)$ is Lipschitz continuous if the following condition holds:

$$
\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \tag{37}
$$

where $L > 0$ is known as the Lipschitz constant.

For convergence of the algorithmic scheme described in the next Section, which we utilize for computational purposes in our case study, only monotonicity and Lipschitz continuity of $F(X)$ are required, provided that a solution exists.

We now turn to establishing the existence of a solution X^* to VI (15); equivalently, (23). Note that the feasible set $\mathcal{K}$ is not compact and, hence, existence of a solution would not immediately follow from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)) although the supply price, demand price, and link cost functions are assumed to be continuous.

Theorem 5: Existence of a Solution

Existence of a solution X^* to the VI for our model given by (15); equivalently, (23), is guaranteed.

Proof: We know that the quota rent equivalents comprising the vector λ are bounded because of the definition of the feasible set $\mathcal{K}$. The demands, in turn, are bounded since the demand prices are monotonically decreasing and negative prices are not relevant. Furthermore, although demands may be large they will not be infinite since the population and, hence, demand, is bounded at each demand market. Therefore, the product path flows are also bounded and, consequently, existence of a solution X^* is guaranteed.

5. The Algorithm

In this Section, we present the algorithm to solve the variational inequality (23), equivalently (15), governing the spatial equilibrium model with tariff rate quotas and its variants that is presented in Section 3. The algorithm that we propose for the computation of the equilibrium pattern is the modified projection method of Korpelevich (1977).

The steps of the algorithm for the spatial price problem with tariff rate quotas are as follows:

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $t := 1$ and select φ, such that $0 < \varphi \leq \frac{1}{L}$, where L is the Lipschitz constant for function F in the variational inequality problem.

Step 1: Construction and Computation

Compute $\bar{X}^{t-1}$ by solving the variational inequality subproblem:

$$\langle \bar{X}^{t-1} + \varphi F(X^{t-1}) - X^{t-1}, X - \bar{X}^{t-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{38}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \varphi F(\bar{X}^{t-1}) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{39}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, for $\epsilon > 0$, a specified tolerance, then, stop; otherwise, set $t := t + 1$ and go to Step 1.

We now further discuss the modified projection method. The projection map of X, $P_{\mathcal{K}}$, on the feasible set $\mathcal{K}$ is given by:

$$P_{\mathcal{K}}(X) = \operatorname*{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{40}$$

where $\| \cdot \| = \langle X, X \rangle$. The solution to the variational inequality subproblem (38), $\bar{X}^t$, generated by the modified projection method is the projection of $\bar{X}^{t-1} - \varphi F(X^{t-1})$ on the closed convex set $\mathcal{K}$. Therefore, we have:

$$\bar{X}^t = P_{\mathcal{K}} \left[X^t - \varphi F(X^{t-1}) \right]. \tag{41}$$

The solution to the variational subproblem (39), in turn, corresponds to the the following:

$$X^t = P_{\mathcal{K}} \left[X^{t-1} - \varphi F(\bar{X}^{t-1}) \right]. \tag{42}$$

The feasible set is box-type; therefore, the projections (41) and (42) decompose across the coordinates of the feasible set.

Next, we state the expanded form of $F(X)$ for our specific model:

Step 0: Initialization

Set $(x^0, \lambda^0) \in K$. Let $t = 1$ and set φ so that $0 < \varphi \leq \frac{1}{L}$.

Step 1: Construction and Computation

Compute $(\bar{x}^t, \bar{\lambda}^t) \in K$ by solving the variational inequality subproblem:

$$\sum_g \sum_{(i,j)\in G_g} \sum_{k\in O_i} \sum_{l\in D_j} \sum_{p\in P^{(i,k)}_{(j,l)}} [\, \bar{x}^t_p + \varphi(\hat{\pi}^{(i,k)}(x^{t-1}) + \hat{C}_p(x^{t-1}) + \tau^u_{G_g} + \lambda^{t-1}_{G_g} - \hat{\rho}_{(j,l)}(x^{t-1})) - x^{t-1}_p] \; \times [\, x_p - \bar{x}^t_p]$$

$$+\sum_g \left[\bar{\lambda}^t_{G_g} + \varphi(\bar{Q}_{G_g} - \sum_{p\in P_{G_g}} x^{t-1}_p) - \lambda^{t-1}_{G_g} \right] \times \left[\lambda_{G_g} - \bar{\lambda}^t_{G_g} \right] \geq 0, \quad \forall (x,\lambda) \in K. \tag{43}$$

Step 2: Adaptation

Compute $(x^t, \lambda^t) \in K$ by solving the variational inequality subproblem:

$$\sum_g \sum_{(i,j)\in G_g} \sum_{k\in O_i} \sum_{l\in D_j} \sum_{p\in P^{(i,k)}_{(j,l)}} [\, x^t_p + \varphi(\hat{\pi}^{(i,k)}(\bar{x}^t) + \hat{C}_p(\bar{x}^t) + \tau^u_{G_g} + \bar{\lambda}^t_{G_g} - \hat{\rho}_{(j,l)}(\bar{x}^t)) - x^{t-1}_p] \; \times [\, x_p - x^t_p]$$

$$+\sum_g \left[\lambda^t_{G_g} + \varphi(\bar{Q}_{G_g} - \sum_{p\in P_{G_g}} \bar{x}^t_p) - \lambda^{t-1}_{G_g} \right] \times \left[\lambda_{G_g} - \lambda^t_{G_g} \right] \geq 0, \quad \forall (x,\lambda) \in K. \tag{44}$$

Step 3: Convergence Verification

If $|x^t_p - {x_p}^{t-1}| \leq \epsilon$, for all $p \in P^{(i,k)}_{(j,l)}$, $\forall i,k,j,l$, and $|\lambda^t_{G_g} - {\lambda_{G_g}}^{t-1}| \leq \epsilon$, for all $g = 1,\ldots,h$, with $\epsilon > 0$, a specified tolerance, then, stop; otherwise, set $t := t+1$ and go to Step 1.

Closed Form Expressions

We present the closed form expressions for the solution of problems (43). Analogous ones for (44) can be determined accordingly.

The closed form expressions for the path flows at iteration t are as follows.

For each path $p \in P^{(i,k)}_{(j,l)}$, $\forall i,k,j,l$, compute:

$$\bar{x}^t_p = \max\{0, \varphi(\hat{\rho}_{(j,l)}(x^{t-1}_p) - \hat{\pi}^{(i,k)}(x^{t-1}) - \hat{C}_p(x^{t-1}) - \tau^u_{G_g} - \lambda^{t-1}_{G_g}) + x^{t-1}_p\}, \tag{45}$$

and the following is the closed form expression for all the quota rent equivalents for group G_g; $g = 1,\ldots,h$:

$$\bar{\lambda}^t_{G_g} = \max\{0, \min\{\varphi(\sum_{p\in P_{G_g}} x^{t-1}_p - \bar{Q}_{G_g}) + \lambda^{t-1}_{G_g}, \tau^o_{G_g} - \tau^u_{G_g}\}\}. \tag{46}$$

We now provide the convergence result for the modified projection method.

Theorem 6: Convergence

Assume that the function $F(X)$ that enters the variational inequality (23) (or (15)) satisfies the conditions in Proposition 1 and following, that is, monotonicity and Lipschitz continuity. Then, the modified projection method described above converges to a solution of the variational inequality (23), or, equivalently, (15).

Proof: As stated in Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem (23) (or (15)), provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. $\square$

6. A Case Study on the Dairy Industry

In this Section, we focus on a case study based on the dairy industry in the United States. In 2002, the dairy industry in the United States generated 20 billion dollars in sales value (Hadjigeorgalis (2005)). In recent years, the dairy industry in the United States has experienced a shift towards larger operations with more than 500 cows, in which the dairy farmers can hold more inventory and increase their production of milk. The large size operations accounted for nearly 60% of all milk produced in 2009, a substantial increase from 39% in 2001 (USDA (2018)). The dairy production in the US occurs primarily in the states of: California, Wisconsin, New York, Pennsylvania, Idaho, Minnesota, New Mexico, Michigan, Texas, and Washington (NASS (2004), USDA (2018)). Since 2002, the US is a net importer of dairy products, especially of cheese, according to Hadjigeorgalis (2005). The international trade in the dairy industry is governed by the 1994 Uruguay Round of Multilateral Trade Negotiations (cf. United States. Office of the U.S. Trade Representative (1994)). Furthermore, in the United States tariff-rate quotas are imposed on most of the dairy products. The US is an exporter of some dairy products such as dry nonfat milk products, with the major export market for the United States of nonfat dry milk being Mexico.

For our case study, we focus on the dairy industry, specifically, on cheese in the United States. Cheese is usually contained in a preserved form in which the main protein and milk fat are not exposed to rapid deterioration from microorganisms. According to Hadjigeorgalis (2005), the European Union dominates the cheese market in the United States by having an import value of 69%, where the closest export country is New Zealand with only 10%. Therefore, it is clear that the European Union and the United States engage in a crucial amount of agricultural trade and it is worthwhile to analyze this relationship in this case study. Especially, in 2018, with the ongoing trade wars (see Tankersley (2018)) and disputes between many countries all over the world, we believe that a case study on the dairy industry and the evaluation of the impact of tariff rate quotas are worth investigating (cf. Paquette, Lynch, and Rauhala (2018)). Furthermore, according to Chatellier (2017), the Netherlands, Germany, and France are the top exporters of dairy products in the European Union, and cheese is counted as the top dairy product exported from France. Therefore, for this case study, we focus on France as a producing country from the European Union.

We implemented the modified projection method in MATLAB on an OS X 10.12.6 system. The code in Matlab is executed on a Macbook Pro laptop with a 2.8 GHz Intel Core i5 processor and 8GB 1600 MHz DDR3 memory. The parameter φ is set to 0.3 with the convergence tolerance being 10^{-6}, that is, the modified projection method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. The algorithm was initialized with each path flow set equal to 1, and with each quota rent equivalent (the Lagrange multiplier) for the variational inequality (15) set equal to 0. In the following examples we modify the spatial price network and also the associated tariff values to generate insights.

6.1 Baseline Example

The first example serves as a baseline example. The computations are made utilizing the closed form expressions provided for the modified projection method in Section 5. The spatial price network structure for the Baseline Example is given in Figure 3. For this example, the producing countries are the United States and France, which belongs to the European Union, with each having two supply markets.

Furthermore, The United States has supply markets: Southwest and Midwest, represented in Figure 3 by the top-tiered nodes $(1,1)$ and $(1,2)$, respectively, whereas France has supply markets: South and North, represented, respectively, by the top-tiered nodes $(2,1)$ and $(2,2)$ in Figure 3.

The transshipment node is assumed to be located in the East region of the United States, with the cheese from the South and the North regions of France arriving in the East region of the United States, and then being transported to the demand markets. Furthermore, we assume that cheese produced at a dairy farm in the Southwest and the Midwest of the United States is sent to the East region of the United States to be distributed to the demand markets. The United States has three demand markets, depicted by the bottom-tiered nodes in Figure 3: $(1,2)$, $(1,3)$, and $(1,4)$, corresponding, without loss of generality to: the Midwest, the Northeast, and the Southeast of the United States, respectively.

There are two groups, G_1 and G_2. The first group, G_1, contains the domestic supply markets and demand markets in the United States, and includes the nodes $(1,1)$, $(1,2)$ (both supply and demand ones), $(1,3)$, and $(1,4)$. The group G_2 represents the supply markets in France, specifically, the South and the North, and the demand markets in the US, represented by the bottom-tiered nodes in Figure 3: $(1,2)$, $(1,3)$, and $(1,4)$. The examples are constructed from the data collected from the website of the European Commission (2018). We also retrieved information from the United Nations (M49) area tool (cf. United Nations (1999)), in which they define countries and regions for regional and preferential trade agreements.

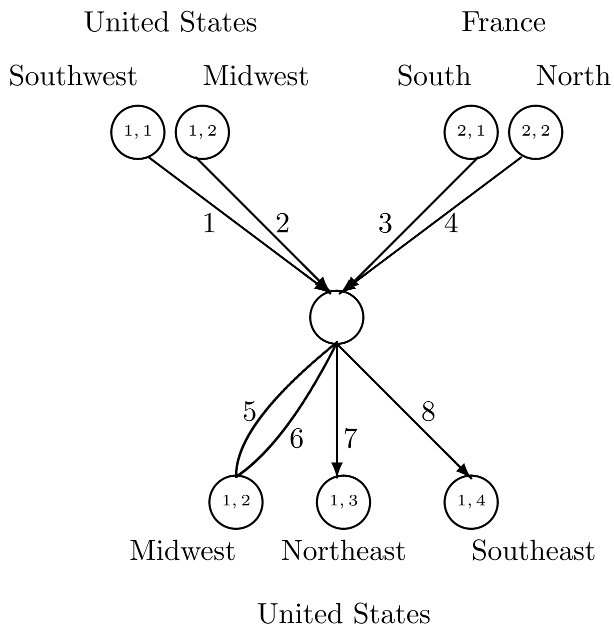

Figure 3: Spatial Network Structure of the Baseline Example

In the spatial price network in Figure 3, there are eight links in the set $L = (1,2,3,4,5,6,7,8)$. Link 1 depicts the transportation of cheese from the Southwest of the United States to the transshipment point, located in the United States, such as Philadelphia, by rail transport. Link 2 represents the railway transport from the Midwest of the United States to the transshipment point. Links 3 and 4 represent, respectively, sea transportation and air transportation to the transshipment point from the South and North of France. We assume that link 5 represents the air transportation mode, whereas links 6, 7, and 8 are for the truck transportation. Having air transportation from the transshipment node to the Midwest region of the United States is acceptable, since cheese is perishable. Furthermore, there are sixteen paths in the spatial price network, and they are as follows: path $p_1 = (1,5)$, $p_2 = (1,7)$, $p_3 = (1,8)$, $p_4 = (2,5)$,

$p_5 = (2,7)$, $p_6 = (2,8)$, $p_7 = (3,5)$, $p_8 = (3,7)$, $p_9 = (3,8)$, $p_{10} = (4,5)$, $p_{11} = (4,7)$, $p_{12} = (4,8)$, $p_{13} = (1,6)$, $p_{14} = (2,6), p_{15} = (3,6)$, and $p_{16} = (4,6)$.

The data for the supply price functions, link cost functions, and the demand price functions are now given. The supply price functions are:

$$\pi^{(1,1)}(s) = .03s_{11} + .02s_{12} + .01s_{21} + .01s_{22} + 3,$$

$$\pi^{(1,2)}(s) = .03s_{12} + .02s_{11} + .01s_{21} + .01s_{22} + 4,$$

$$\pi^{(2,1)}(s) = .02s_{21} + .01s_{11} + .01s_{12} + .01s_{22} + 1,$$

$$\pi^{(2,2)}(s) = .02s_{22} + .01s_{11} + .01s_{12} + .01s_{21} + 2.$$

We assume that the fixed supply price term for the cheese produced in the United States at nodes $(1,1)$ and $(1,2)$ is higher than those supply price function terms for the cheese produced in France, at nodes $(2,1)$ and $(2,2)$. The reasoning behind this assumption is that the European Union has a major role in milk production, responsible for 20.1% of the world production, followed by the United States with 12% (Chatellier (2017)). Since cheese is produced from milk, it is reasonable to assume for this case study that France, as a part of the European Union, has an advantage over production of cheese against its competitor, the United States. Hence, the fixed supply price term for the cheese produced in the South and North of France is lower at nodes (2,1) and (2,2).

The link cost functions, in which the link flows are converted to path flows for the algorithm according to (13) and (14), are:

$$c_1(f) = .001f_1^2 + .01f_1,\ c_2(f) = .001f_2^2 + .005f_2,\ c_3(f) = .001_3^2 + .06f_3,$$

$$c_4(f) = .002f_4^2 + 0.1f_4,\ c_5(f) = .002f_5^2 + 0.1f_5,\ c_6(f) = .001_6^2 + .06f_6,$$

$$c_7(f) = .001_7^2 + .06f_7,\ c_8(f) = .001_8^2 + .06f_8.$$

The cost functions are constructed, according to the transportation modes. Links that represent truck, railway, or sea transportation are cheaper than air transportation, represented by links 4 and 5.

Furthermore, the demand price functions are as follows:

$$\rho_{(1,2)}(d) = -0.01d_{12} - 0.0075d_{13} - 0.005d_{14} + 9,$$

$$\rho_{(1,3)}(d) = -0.01d_{13} - 0.0075d_{12} - 0.005d_{14} + 10,$$

$$\rho_{(1,4)}(d) = -0.0075d_{14} - 0.005d_{12} - 0.0025d_{13} + 11.$$

For the Baseline Example, we assume similar demand price functions for cheese to be sold in the Midwest, the Southeast, and the Northeast of the United States, with the cheese demand price function fixed term in the Midwest being lower than those in the Southeast and the Northeast. Furthermore, the cheese price fixed terms in the Northeast are higher than in the Southeast, resulting from a different consumer profile. Additionally, the price for cheese in the Southeast region is less sensitive to the demand than in the other demand market regions, due to the consumer profile and the larger population. Under these assumptions, we constructed the above demand price functions.

Next, we define the necessary parameters for the tariff rate quota regime. Recall that the two-tiered tariff requires an under quota tariff, $\tau^u_{G_g}$, and an over quota tariff, $\tau^o_{G_g}$, for each group. Since we include the domestic production in the United States for Group G_1, $\tau^u_{G_1} = 0$ and $\tau^o_{G_1} = 0$. According to the World Trade Organization (2018), the United States imposes the in quota fresh cheese tariff as 1.128 dollars per kg of cheese and the over quota tariff as 2.126 dollars per kg of cheese for the European Union, including the cheese exported from France. We assumed that $\tau^u_{G_2} = 1$ dollar per kilogram and $\tau^o_{G_2} = 2$ dollars per kilogram for Group G_2. We analyze a short time period such as a week. The quotas for the groups are assigned as $\bar{Q}_{G_1} = 10000$, which is a large number, since Group G_1 includes the domestic production and consumption, and the quota for Group G_2 is assumed to be $\bar{Q}_{G_2} = 100$ kilograms, imposed on France by the United States for this group. The data is collected from the report by USDA (2018).

The computed equilibrium cheese product path flows in kilograms for the Baseline Example and the equilibrium path costs are reported in Table 1.

Table 1: Equilibrium Path Flows and Incurred Path Costs for the Baseline Example

Path	Path Links	x^*_p	$\hat{C}_p(x^*)$
p_1	(1,5)	5.30	3.25
p_2	(1,7)	9.50	4.26
p_3	(1,8)	11.92	5.52
p_4	(2,5)	2.40	2.36
p_5	(2,7)	6.60	3.38
p_6	(2,8)	9.02	4.64
p_7	(3,5)	5.06	5.82
p_8	(3,7)	9.26	6.83
p_9	(3,8)	11.67	8.10
p_{10}	(4,5)	0.99	4.98
p_{11}	(4,7)	5.19	6.00
p_{12}	(4,8)	7.60	7.26
p_{13}	(1,6)	7.25	3.25
p_{14}	(2,6)	4.34	2.36
p_{15}	(3,6)	7.00	5.82
p_{16}	(4,6)	2.93	4.98

Furthermore, the equilibrium link flows and the incurred link costs for the Baseline Example are given in Table 2.

The computed equilibrium quota rent equivalents are: $\lambda^*_{G_1} = 0$ and $\lambda^*_{G_2} = 0$.

Notice that, for this example, we assumed a large quota for France and, therefore, the quota rent equivalent is zero. This is modified in the next example.

Also, the computed equilibrium cheese supply and demand, in kilograms, are:

$$s^*_{11} = 33.99,\ s^*_{12} = 22.37,\ s^*_{21} = 33.00,\ s^*_{22} = 16.72,$$

$$d^*_{12} = 35.30, \quad d^*_{13} = 30.56, \quad d^*_{14} = 40.23.$$

The total amount of cheese shipment from the Southwest and Midwest regions of the United States is larger than the cheese shipments from the South and North of France.

Table 2: Equilibrium Link Flows and Incurred Link Costs for the Baseline Example

Link a	f_a^*	$c_a(f^*)$
1	33.99	1.49
2	22.37	0.61
3	33.00	3.06
4	16.72	2.23
5	13.76	1.75
6	21.53	1.75
7	30.56	2.76
8	40.23	4.03

Furthermore, the incurred supply prices per kilogram of cheese in dollars are:

$$\pi^{(1,1)} = 4.96, \ \pi^{(1,2)} = 5.84, \ \pi^{(2,1)} = 2.39, \ \pi^{(2,2)} = 3.22.$$

Observe that the supply prices of cheese are higher in the Southwest and the Midwest regions of the United States than in the South and North regions of France. This should have given an advantage to France to export their cheese in a larger amount to the United States; however, since the transportation costs are larger for France, and their cheese is also subject to an under tariff of $\tau^u_{G_2} = 1$ (cf. (9) and (10)) large export amounts are not achieved.

Next, we report the incurred demand prices, per kilogram of cheese, at the equilibrium, in dollars:

$$\rho_{(1,2)} = 8.21, \quad \rho_{(1,3)} = 9.22, \quad \rho_{(1,4)} = 10.49.$$

The demand price for cheese in the Southeast of the US is higher than it is in the Northeast. This is reasonable due to the demographic differences in these demand regions.

The equilibrium conditions (9) and (10) are satisfied with excellent accuracy.

6.2 Change in Quotas Example

This example is constructed from the Baseline Example and has the spatial price network topology in Figure 3 as well as the same data, except for the following. In this example, we decrease the quota of Group G_2. In 2018, the turmoil in world trade generated an environment in which the import countries started to modify their tariffs and tariff rate quotas to maintain their political agendas (cf. Paquette, Lynch, and Rauhala (2018)). In a broader sense, import countries can impose stricter quotas and higher under quota and over quota tariffs on their trade partners to support their domestic production. We, hence, modify the tariffs and quotas to investigate the associated impacts on the economy.

For the Change in Quotas Example, we consider the situation in which the United States imposes a stricter quota on cheese that is exported from France's supply regions. Therefore, the quota on Group G_2, $\bar{Q}_{G_2}$, is tightened to 35 kilograms. The supply price functions, link cost functions, and the demand price functions remain as in the Baseline Example.

The computed equilibrium cheese product path flows, in kilograms, are reported in Table 3.

Table 3: Equilibrium Path Flows and Incurred Path Costs for the Change in Quotas Example

Path	Path Links	x_p^*	$\hat{C}_p(x^*)$
p_1	(1,5)	5.88	3.26
p_2	(1,7)	10.04	4.27
p_3	(1,8)	12.48	5.53
p_4	(2,5)	3.21	2.37
p_5	(2,7)	7.36	3.38
p_6	(2,8)	9.80	4.63
p_7	(3,5)	3.65	5.97
p_8	(3,7)	7.80	6.98
p_9	(3,8)	10.24	8.23
p_{10}	(4,5)	0.00	5.12
p_{11}	(4,7)	4.03	6.13
p_{12}	(4,8)	6.47	7.38
p_{13}	(1,6)	7.73	3.06
p_{14}	(2,6)	5.05	2.37
p_{15}	(3,6)	5.49	5.97
p_{16}	(4,6)	1.72	5.12

When the quota for Group G_2 is decreased, the path flows are adjusted accordingly. Notice that the flow on path p_{10} is equal to 0.00.

The equilibrium link flows and associated incurred link costs for this example are given in Table 4.

Table 4: Equilibrium Link Flows and Incurred Link Costs for the Change in Quotas Example

Link a	f_a^*	$c_a(f^*)$
1	36.14	1.66
2	25.45	0.77
3	27.21	2.37
4	12.23	1.52
5	12.76	1.60
6	20.01	1.60
7	29.25	2.61
8	39.01	3.86

The computed equilibrium quota rent equivalents are: $\lambda_{G_1}^* = 0$ and $\lambda_{G_2}^* = 1$.

Furthermore, the computed equilibrium supplies and demands, in kilograms, are:

$$s_{11}^* = 36.14, \; s_{12}^* = 25.45, \; s_{21}^* = 27.21, \; s_{22}^* = 12.23,$$

$$d_{12}^* = 32.78, \quad d_{13}^* = 29.25, \quad d_{14}^* = 39.01.$$

The equilibrium quota rent equivalent for Group G_2 is positive and at its maximum value, $\tau_{G_2}^o - \tau_{G_2}^u = 2 - 1 = 1$. The reason for having a positive quota rent equivalent for Group G_2 is that the summation of path flows originating from the South and the North of France; equivalently, the sum of the supplies from France, which is equal to 39.45, is over the imposed quota of 35.

It is clear that the equilibrium supply of cheese from the South and the North of France decreases from the respective values in the Baseline Example, due to the stricter quota on Group G_2 as well as the tariffs. The equilibrium domestic supply in the United States, represented by s_{11}^* and s_{12}^*, increases. Additionally, the equilibrium demand values decrease.

The incurred supply prices per kilogram of cheese in dollars at the equilibrium are:

$$\pi^{(1,1)} = 4.98,\ \pi^{(1,2)} = 5.88,\ \pi^{(2,1)} = 2.28,\ \pi^{(2,2)} = 3.13.$$

The supply prices of cheese in the Southwest and the Midwest of the United States increase from their values in the Baseline Example. On the other hand, the supply prices for the South and North of France decrease to try to recover any competitive advantage.

The incurred equilibrium demand prices, per kilogram of cheese, in dollars, are:

$$\rho_{(1,2)} = 8.25, \quad \rho_{(1,3)} = 9.26, \quad \rho_{(1,4)} = 10.51.$$

The demand prices increase in all three demand markets in the United States from their previous values, meaning that the cheese supply in the United States from the Southwest and the Midwest is not sufficient to keep the demand prices as equal or lower than they were in the Baseline Example. This shows that having a trade quota could result in a demand price increase, affecting negatively the consumers in the economy. This was also observed in the illustrative examples in Section 3.1.

We now proceed to conduct sensitivity analysis for this example. In particular, we investigate the impact of an increase in the over quota tariff $\tau^o_{G_2}$ on the production outputs of both France and the United States.

The algorithm is re-run for three different over quota tariffs, $\tau^o_{G_2} = 3$ dollars, $\tau^o_{G_2} = 5$ dollars, and $\tau^o_{G_2} = 8$ dollars for Group G_2. The equilibrium link flows for the production links $(1, 2, 3, 4)$, under different over quota tariffs are reported in Table 5.

Table 5: Equilibrium Supplies under Different Over Quota Tariffs on France for Example 6.2

Over Quota Tariffs	s_{11}^*	s_{12}^*	s_{21}^*	s_{22}^*
$\tau^o_{G_2} = 2$	36.14	25.45	27.21	12.23
$\tau^o_{G_2} = 3$	38.43	28.57	21.04	7.34
$\tau^o_{G_2} = 5$	42.89	34.38	6.18	0.00
$\tau^o_{G_2} = 8$	44.10	35.90	0.00	0.00

Observe from Table 5 that, when the over quota tariff increases, the supply of cheese from France decreases, represented by s_{21}^* and s_{22}^*. However, the supply of cheese produced in the United States in the Southwest and Midwest regions experiences a steady increase, demonstrating vividly the positive impact on producers in the US. Furthermore, notice that the equilibrium supplies decrease, as the over quota tariffs increase for the supply of cheese from North and the South regions of France. When the over quota tariff is high for Group G_2, the supply of cheese from the North and the South regions of France stay under their quota and the algorithm converges to lower supply amounts for Group G_2.

Table 6: Demand Prices under Different Over Quota Tariffs on France for Example 6.2

Over Quota Tariffs	$\rho_{(1,2)}$	$\rho_{(1,3)}$	$\rho_{(1,4)}$
$\tau^o_{G_2} = 2$	8.25	9.26	10.51
$\tau^o_{G_2} = 3$	8.30	9.30	10.54
$\tau^o_{G_2} = 5$	8.40	9.39	10.60
$\tau^o_{G_2} = 8$	8.42	9.42	10.62

Additionally, we report the demand prices under different over quota tariffs in Table 6. Observe that the demand price increases for the cases in which the over quota tariffs are larger, meaning that the consumers are getting affected negatively from the increase in tariffs.

6.3 Increase in the Number of Paths Example

In this example we explore the impact of additional paths. In particular, we use the same data as in the preceding example but now we introduce two new paths, paths p_{17} and p_{18}, with path p_{17} connecting the Midwest supply node with the Midwest demand node and with path p_{18} connecting the South supply market of France with the demand market of the Southeast of the US. The spatial price network topology is now as given in Figure 4. Path p_{17} consists of the new link 9 and path p_{18} is comprised of the new link 10. These paths represent direct routes without transshipment from supply markets to demand markets. The link cost functions on the new links are: $c_9(f) = .001f_9^2 + .001f_9$ and $c_{10}(f) = .001f_{10}^2 + .01f_{10}$.

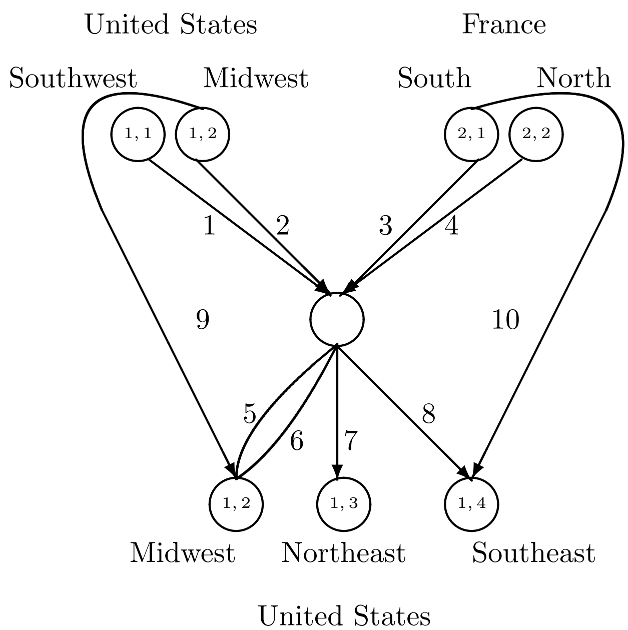

Figure 4: Spatial Price Network for the Increase in the Number of Paths Example

The computed equilibrium path flows and the incurred path costs are reported in Table 7 and the computed equilibrium link flows and the incurred link costs in Table 8.

The computed equilibrium quota rent equivalents are now: $\lambda^*_{G_1} = 0$ and $\lambda^*_{G_2} = 1$, whereas the computed

Table 7: Equilibrium Path Flows and the Incurred Path Costs for the Increase in the Number of Paths Example

Path	Path Links	x_p^*	$\hat{C}_p(x^*)$
p_1	(1,5)	4.87	2.09
p_2	(1,7)	8.90	3.15
p_3	(1,8)	11.95	4.58
p_4	(2,5)	0.01	0.97
p_5	(2,7)	4.04	2.04
p_6	(2,8)	7.09	3.46
p_7	(3,5)	1.81	4.31
p_8	(3,7)	0.01	5.38
p_9	(3,8)	8.89	6.80
p_{10}	(4,5)	0.04	4.05
p_{11}	(4,7)	3.46	5.11
p_{12}	(4,8)	6.51	6.54
p_{13}	(1,6)	6.04	2.09
p_{14}	(2,6)	1.18	0.97
p_{15}	(3,6)	2.98	4.31
p_{16}	(4,6)	0.60	4.05
p_{17}	(9)	30.78	0.97
p_{18}	(10)	64.51	6.80

Table 8: Equilibrium Link Flows and the Incurred Link Costs for the Increase in the Number of Paths Example

Link a	f_a^*	$c_a(f^*)$
1	31.77	1.32
2	12.33	0.21
3	19.53	1.55
4	10.62	1.28
5	6.74	0.76
6	10.80	0.76
7	22.25	1.83
8	34.45	3.25
9	30.78	0.97
10	64.51	4.80

equilibrium supplies and demands, in kilograms, are:

$$s_{11}^* = 31.77,\ s_{12}^* = 43.12,\ s_{21}^* = 84.05,\ s_{22}^* = 10.62,$$

$$d_{12}^* = 48.33, \quad d_{13}^* = 22.25, \quad d_{14}^* = 98.97.$$

Observe that the equilibrium supply from the Midwest of the United States and the equilibrium supply from the South of France have increased substantially from their values in Example 6.2. The reason is that the cost of the additional paths p_{17} and p_{18} is lower than that of the other paths in the network. Hence, even though the cheese supplied from the South of France is subject to an over-quota tariff, it can still export its cheese to the demand market of the Southwest in the United States. Furthermore, notice that the total equilibrium supply from France, which is 89.19, is strictly over the quota, that is, 35; therefore, the equilibrium quota rent for Group 2, $\lambda^*_{G_2}$, is positive.

The incurred supply prices per kilogram of cheese in dollars at the equilibrium are now:

$$\pi^{(1,1)} = 5.76, \ \pi^{(1,2)} = 6.87, \ \pi^{(2,1)} = 3.53, \ \pi^{(2,2)} = 3.80,$$

and the incurred equilibrium demand prices, per kilogram of cheese, in dollars, are:

$$\rho_{(1,2)} = 7.85, \quad \rho_{(1,3)} = 8.92, \quad \rho_{(1,4)} = 10.34.$$

The addition of new paths in this example results in an increase in the equilibrium supply prices at all the supply markets in the countries as compared to their values in Example 6.2. Conversely, the equilibrium demand prices at all the demand markets in the United States decrease from their values in Example 6.2. Hence, both producers and consumers gain when there are alternative competitive transportation routes.

We now present a similar sensitivity analysis for this example, as was done for Example 6.2, in which we compute the solutions to variants of Example 6.3 for different over quota tariffs: $\tau^o_{G_2} = 3$ dollars, $\tau^o_{G_2} = 5$ dollars, and $\tau^o_{G_2} = 8$ dollars for Group G_2. In Table 8, we report the results of the sensitivity analysis.

Table 9: Equilibrium Supplies under Different Over Quota Tariffs on France for Example 6.3

Over Quota Tariffs	s^*_{11}	s^*_{12}	s^*_{21}	s^*_{22}
$\tau^o_{G_2} = 2$	31.77	43.12	84.05	10.62
$\tau^o_{G_2} = 3$	34.14	48.15	71.95	5.54
$\tau^o_{G_2} = 5$	38.21	56.27	46.01	0.00
$\tau^o_{G_2} = 8$	40.08	62.02	11.34	0.00

Observe from Table 9 that the equilibrium supply from the countries and their supply regions change drastically under different over quota tariffs. It is interesting to observe, for example, that when the imposed over quota tariff, $\tau^o_{G_2} = 5$ dollars on France, the equilibrium supply, denoted by s^*_{22}, from the North of France drops to 0. The cheese supplied from the South of France is still positive under different over quota tariff values; however, it shows a steady decline with the increase in over quota tariffs. The reason for this is that the cost of the path p_{18} is lower. Notice that, for the case $\tau^o_{G_2} = 8$ dollars, the total equilibrium supply for France is 38.50, which is closer to their quota, meaning that, even with the lower path cost, the over quota tariff becomes too expensive for the South of France to export its cheese to the United States. Similar to the sensitivity analysis in Example 6.2, the equilibrium domestic supply (domestic production) in the United States, in the Southwest and the Midwest, represented by s^*_{11} and s^*_{12}, respectively, demonstrates an increase under the increase in over quota tariffs on France. This is also an interesting result, since countries usually impose tariff rate quotas on certain export products to increase their domestic production. However, notice also that, when the over quota tariff changes from $\tau^o_{G_2} = 5$ to $\tau^o_{G_2} = 8$, the change in the domestic production is very small. This means that the countries imposing tariff rate quotas on products should be careful as to how high of an over quota tariff they impose. Having a computationally tractable model such as the one presented here allows for the evaluation of the impacts of changes to quotas, under and over quota tariffs, changes in the spatial price network topology, as well as the underlying functions.

We report the demand prices in Table 10, under different over quota tariffs. Notice that the demand price increases, as expected, when the over quota tariffs increase.

Table 10: Demand Prices under Different Over Quota Tariffs on France for Example 6.3

Over Quota Tariffs	$\rho_{(1,2)}$	$\rho_{(1,3)}$	$\rho_{(1,4)}$
$\tau^o_{G_2} = 2$	7.85	8.92	10.34
$\tau^o_{G_2} = 3$	7.91	8.97	10.37
$\tau^o_{G_2} = 5$	8.02	9.09	10.44
$\tau^o_{G_2} = 8$	8.13	9.20	10.48

7. Summary, Conclusions, and Suggestions for Future Research

With numerous commodities from fresh produce to metals such as aluminum and steel, utilized in many product supply chains, criss-crossing the globe from points of production to locations of demand, world trade is essential for producers and consumers alike. Policy makers, as well as governments, in turn, are increasingly utilizing policy trade instruments in an attempt to protect domestic producers from competition. In fact, in the past year or so, the discussions of various trade policies, including tariffs, as well as their potential impacts, have garnered wide global news attention and coverage.

In this paper, we took up the challenge of providing a unified variational inequality framework for the modeling, analysis, and computation of solutions to a general spatial network equilibrium problem with multiple countries and supply and demand markets in each country, as well as multiple routes joining the supply markets with the demand markets, in the presence of two-tiered tariffs in the form of tariff rate quotas. A tariff rate quota policy consists of an under quota tariff (applied when the imposed quota is not exceeded) and an over quota tariff (applied when the quota is exceeded). We presented the governing equilibrium conditions, derived the variational inequality formulation, and also provided variational inequality formulations of the analogues of the spatial price network equilibrium models with ad valorem tariffs and with strict quotas, respectively. Qualitative properties were also obtained and an effective computational algorithm, along with conditions for convergence, outlined.

Illustrative examples were given as well as a case study consisting of larger numerical examples for which the equilibrium path flows, and link flows, as well as the equilibrium quota rents, and the supply and demand market prices, and path costs, reported. The case study focused on the dairy industry and a tariff rate quota imposed on the US on cheese from France. Sensitivity analysis demonstrated the impacts on production outputs (both US domestic and imports from France) of a tightening of the imposed quota as well as increases in the over tariff rate. The case study included adding routes between supply and demand markets in different countries. The results show that tariff rate quotas may protect domestic producers, but at the expense of the consumers. On the other hand, adding competitive alternative transportation routes may help both domestic producers and exporters as well as consumers.

Possibilities for future research include constructing a general spatial price network equilibrium model with differentiated products as well as ascertaining the introduction of tariff rate quota regimes in the case of supply chain network oligopolistic competition. We expect the the work herein can serve as a foundation for the above models which would capture alternative industrial organizations of distinct trade product sectors.

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