Ad Valorem Tariffs in Global Supply Chain Networks and Impacts on Labor

Anna Nagurney* and Samirasadat Samadi**
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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Abstract

Global supply chain networks are essential to the production, trade, and consumption of commodities, including agricultural ones. Such networks have been increasingly impacted by trade policies, especially ad valorem tariffs, which are affecting commodity flows, prices, and profits. Furthermore, the labor market plays a critical role in the functioning of supply chain networks. However, the impacts of ad valorem tariffs on labor (and employment) have not yet been quantitatively examined in a competitive global supply chain network framework. This paper constructs an oligopolistic supply chain network equilibrium model that integrates ad valorem tariffs and labor. Firms compete noncooperatively in maximizing their profits by determining their product flows across multiple production sites. Demand markets can be located in different countries, with production and shipment activities subject to labor upper bounds and wages. The governing Nash equilibrium conditions are formulated as a variational inequality. Through Lagrange analysis, an alternative variational inequality is derived with nice features for computations. Illustrative examples are provided along with a global soybean trade case study. Numerical results reveal how such tariffs shift trade flows, reshape labor allocation, and affect demand prices as well as profits, with labor shortages and cost disruptions further negatively compounding the effects.

Key words: supply chains; labor; tariffs; global trade; networks; variational inequalities

* Corresponding author email: nagurney@isenberg.umass.edu

** Second author email: ssamadi@umass.edu

Practitioner Summary

Global supply chain networks form the backbone of international trade and trade flows by linking production sites to demand markets across countries through transportation. Some governments are, increasingly, attempting to alter the product trade flows by instituting policies in the form of tariffs. Tariffs can affect production and transportation decisions and modify demand prices as well as profits. At the same time, labor availability is central to the functioning of supply chain networks and, yet, labor considerations have often been absent from trade policy models. In this paper, we developed a competitive global supply chain network model that explicitly incorporates ad valorem tariffs, which are very common in practice, and labor constraints. Firms in the model have multiple production sites in different countries and ship products to multiple demand markets, with production and shipment activities requiring labor that is subject to site and route specific upper bounds. Wages can vary by location, and labor productivity factors translate labor input into output. Firms compete noncooperatively in product quantities and labor and seek to maximize their own profits. After providing illustrative numerical examples, which are solved analytically, we present a case study of the global soybean trade, focusing on the United States, Brazil, Argentina, and China. The case study numerical examples, whose solutions are computed via an algorithm, reveal the effects of the imposition of ad valorem tariffs, labor restrictions, and climate-related disruptions through changes in production cost functions. The computed equilibrium solutions show that ad valorem tariffs shift production and trade flows toward sources without ad valorem tariffs, alter labor allocation at production sites and along transportation routes, and increase demand prices for consumers. Labor constraints, whether from shortages or immigration policy restrictions, reduce product flows and profitability for the affected firms while creating competitive advantages for less constrained producers. When ad valorem tariffs and increased production cost disruptions occur together, their effects compound, leading to reduced trade flows, higher demand prices, and lower overall profits. The results offer important policy and managerial insights. For policymakers, the results highlight that ad valorem tariffs not only influence prices and trade flows but also interact with labor in shaping competitive outcomes. Ad valorem tariffs can shift market share across countries but may also reduce total output and increase consumer demand prices, particularly when implemented widely. For firms, our model highlights the advantage of diversifying production locations to mitigate the effects of ad valorem tariffs and labor disruptions.

1. Introduction

The global economy is shifting rapidly, with tariffs reemerging as key trade policies, reshaping global supply chains and labor markets. The Organisation for Economic Co-operation and Development (OECD) has downgraded its global economic growth forecast to 2.9% for both 2025 and 2026, down from 3.3% in 2024, explicitly citing escalating trade tensions and newly imposed tariffs (Anand (2025)). The United States exemplifies this trend, as a central player in recent trade policy shifts, having raised its average tariff rate from 2.5% in 2024 to 15.4% in 2025, the highest level since 1938, through broad import duties on nearly all trading partners (Neufeld (2025) and Wiseman (2025)). Consequently, the OECD has revised the US 2025 growth forecast from 2.8% to 1.6%, with a further decline to 1.5% expected in 2026, primarily due to these tariffs and the resultant economic uncertainty (Picchi (2025)). Similarly, China, the world's second-largest economy, is projected to experience a growth drop from 5% to 4.7% in 2025 and 4.3% in 2026 (Wiseman (2025)).

The impact of tariff escalation was already evident in the 2018–2019 US–China trade conflict, which imposed significant ad valorem tariffs, 25% on steel, 10% on aluminum, and up to 25% on $200 billion of Chinese imports, prompting broad retaliatory measures (Office of the United States Trade Representative (2018)). Countries such as China, Mexico, Canada, and the EU, subsequently, imposed counter-tariffs on a broad range of US goods, including on key agricultural products: soybeans, pork, and dairy (Filloon (2018)). As a result, US economic growth decelerated, business investment slowed sharply, and hiring weakened. The agricultural sector faced a surge in bankruptcies, while manufacturing and freight transportation fell to levels unseen since the previous recession (Long (2020)). Recent tariff escalation across sectors signals a renewed wave of trade barriers. In June 2025, the US doubled steel and aluminum tariffs from 25% to 50%, prompting retaliation from Canada, Mexico, and the EU, and driving US aluminum premiums up by a record 54%, while steel prices rose by 7.4% (Swanson and Austen (2025) and Onstad, Jin, and Li (2025)). In terms of seafood, the US imposed a 10% tariff, with China facing 30%. Given that the US imports $16 billion in fish annually but exports just $4.5 billion, consumers are highly exposed to price increases. Brazil and China, major suppliers, are expected to redirect exports to alternative markets (Poidevin (2025)). These examples underscore that tariffs are firmly back as a defining policy of global trade, reshaping markets across sectors and regions.

Tariffs, essentially a tax on foreign goods, raise costs for both consumers and firms (Goreja (2025)). While governments often defend these measures to protect local industries and to preserve domestic jobs (Nagurney and Samadi (2025)), evidence shows that their economic burden falls disproportionately on consumers, through higher prices, and on businesses, through increased input costs and reduced competitiveness. Although the exact magnitude of price changes is uncertain, estimates indicate that 2025 tariffs could push the price level up by 1.5% in the short run, costing households in the United States an average of $2,500 in 2024 dollars (The Budget Lab (2025)). By raising the cost of imported inputs and final goods, tariffs force firms to rethink sourcing, drive up production costs and, ultimately, raise consumer prices (Redding (2023)). This market distortion reduces consumer welfare and leads to resource misallocation (Pomerleau and York (2025)).

Adding to the complexity, today's intricate global supply chain networks, often criss-crossing multiple countries and involving numerous intermediaries, can account for up to 75% of operating costs of some firms (Dutta (2023)). Tariffs intensify pressure on these firms to reassess their entire supply chain, their sourcing, and labor strategies. Such shifts can bring challenges such as securing skilled labor in new regions and managing domestic job displacement (Rahman (2025)). Labor has become a critical, yet frequently overlooked, component of supply chains, with availability constraining throughput from manufacturing to transportation and warehousing. The importance of incorporating labor into models is highlighted by the negative impacts of labor shortages on profits and product availability seen during the COVID-19 pandemic (Nagurney (2023a, b)). Indeed, labor markets in the US are already showing strain from these tariffs, with a projected 0.3 percentage point rise in unemployment and a loss of 376,000 payroll jobs by the end of 2025 (The Budget Lab (2025)). The disruptive effects are already visible internationally: in Lesotho, the mere threat of a 50% US tariff led to widespread factory closures, thousands of layoffs, and severe income loss for textile workers who depend heavily on American demand (Eligon (2025)). Experts anticipate further reductions in hiring and potential layoffs as firms face mounting financial pressure (Lee (2025) and Mohan (2025)).

The impacts of the 2018 US–China trade war were felt immediately across US agriculture. Early in the dispute, agricultural experts noted that the US–China trade war in 2018 exacerbated financial pressures on Wisconsin's farm sector, contributing to price declines and the closure of 638 out of 8,801 dairy farms that year (Ki (2019)). During the same period, retaliatory tariffs reduced US agricultural exports by more than $27 billion, with China driving most of the decline and soybeans alone accounting for about 71 percent of annual losses ($9.4 billion), followed by sorghum ($854 million) and pork ($646 million) (Morgan et al. (2022)). These shocks also weakened labor demand in the US, leading to fewer job postings in farming, fishing, forestry, and other agricultural supply-chain sectors, with input tariffs tending to hit lower-skilled workers harder, while retaliatory export tariffs had a greater impact on higher-skilled labor (Javorcik et al. (2024)). Waugh (2019) further shows that US counties more exposed to Chinese retaliatory tariffs experienced significant employment declines during the trade war in 2018. Looking ahead to recent tariff deals, US agricultural exporters report that retaliatory tariffs have caused major cancellations from China, including the largest cancellation of 12,000 tons of pork orders since 2020, prompting immediate layoffs and raising concerns that reduced cargo volumes could undermine regional employment (LaRocco (2025)). Research suggests that current tariff levels could cost between 955,000 and 3.4 million US jobs once retaliation tariffs are taken into account, implying unemployment increases of 0.6 to 2.0 % (Waugh and Horwich (2025)). In April 2025, China's decision to raise its tariff on US soybeans to 34% resulted in nearly zero purchases, leaving farmers unable to replace this lost market and facing an estimated $5.7 billion decline in soybean exports through October 2025 (Luck et al. (2025)). The ripple effects extend beyond the US since in South Africa farmers warn that the new 30% US tariff will put about 35,000 citrus-sector jobs at risk and could seriously harm the local economies that depend on this industry (Imray (2025)). Officials also caution that the new US tariffs could eliminate as many as 100,000 jobs, with agricultural products, particularly, citrus, table grapes, and wine, being among the hardest hit because these sectors rely heavily on low-skilled labor (Reuters (2025)).

It is important to recognize the variety of tariffs, including unit tariffs, ad valorem tariffs, and tariff rate quotas. Unit tariffs, or specific tariffs, impose a fixed fee per unit of an imported good, regardless of its value, while ad valorem tariffs, are calculated as a percentage of the imported good's value. Tariff-rate quotas blend these approaches by allowing a specified quantity to be imported at a lower tariff, with higher rates applied beyond that volume (O'Reilly (2025)). Ad valorem tariff raise the cost of international transactions and can set off ripple effects throughout supply chain networks. Given that many recent tariffs are ad valorem and influence pricing differently than fixed fees (Simon (2025)) understanding their multifaceted impacts on firms, prices, and, especially, labor, requires a comprehensive network-level perspective. Supply chains are inherently dynamic and interdependent; therefore, analyzing how tariffs spread through these systems to reshape production, trade flows, pricing, and employment calls for advanced analytical tools. However, despite the prominence of ad valorem tariffs in recent trade policy and the growing importance of labor in supply chains (e.g., see the book by Nagurney (2023b)), very few studies integrate both ad valorem tariffs and labor availability within a unified oligopolistic framework. To our knowledge, no prior work has employed a variational inequality-based, game-theoretic model to jointly study ad valorem tariffs and labor constraints in an international oligopoly supply chain network.

In this paper, we address this gap by developing a novel global supply chain network model that, for the first time, simultaneously incorporates ad valorem tariffs and labor constraints within an oligopolistic equilibrium, using the robust framework of variational inequality (VI) theory. This unified approach enables the analysis of firms' joint decisions on labor deployment, product flows, and pricing under strategic interactions and varying tariff rates. Embedding labor dynamics, trade interventions, and strategic competition, within a comprehensive VI framework, provides new theoretical and practical insights into how ad valorem tariffs and the labor market interact to shape equilibrium outcomes and, ultimately, impact the performance and resilience of global supply chain networks. In addition, we demonstrate the relevance and practicality of the methodological framework through both illustrative examples as well as algorithmically solved ones, with the latter ones based on an important commodity in agricultural trade - that of soybeans.

2. Related Literature, Contributions, and Organization of the Paper

We now provide a review of the related literature to provide context for our modeling framework and to highlight how this work contributes to and extends prior research. The structure of the paper follows.

2.1. Related Literature

Tariffs remain powerful trade policy tools, shaping sourcing strategies, cost structures, and competitive dynamics in global supply chains. Among them, ad valorem tariffs, levied as a percentage of the imported good's value, have become especially prevalent, introducing nonlinearities into trade models and yielding distinct strategic and welfare implications. Lockwood and Wong (2000) show that ad valorem tariffs are strategically preferable to specific tariffs, eliciting milder retaliatory responses. Likewise, Niu, Chen, and Wang (2022) find that tariff types significantly affect seller behavior and welfare in cross-border e-commerce, with ad valorem tariffs producing unique outcomes in differentiated markets. In oligopolistic markets, Dixit (1984) and Brander and Spencer (1984) demonstrated that tariffs serve as rent-shifting instruments, transferring profits from foreign to domestic firms, with ad valorem tariffs inducing smaller price distortions while preserving strategic leverage. Parai (1999) extends this by showing that tariffs reduce foreign exports and raise domestic firms' sales and profits in international oligopolies. Francois and Wooton (2010) add another layer by introducing market power and using a general equilibrium model in distribution networks, arguing that imperfect competition in domestic trade and distribution sectors acts as a hidden trade barrier, dampening the benefits of tariff reductions on market access.

Many studies also examine the effects of the 2018–2019 US tariffs, documenting effects on domestic prices, trade flows, welfare losses (Amiti, Redding, and Weinstein (2019), Fajgelbaum and Khandelwal (2022)), import volumes, domestic production shifts, and sourcing (Flaaen, Hortaçsu, and Tintelnot (2020)), as well as export performance, job losses, and reduced market access abroad (Handley, Kamal, and Monarch (2020)). Beyond ad valorem tariffs, the literature also explores other forms, including specific tariffs (Phillips (2024) and Raimondi et al. (2023)) and tariff-rate quotas (Skully (2001), Bishop et al. (2001), Guyomard et al. (2005), Manzo (2007), Hezarkhani, Arisian, and Mansouri (2023), and Maeda, Suzuki, and Kaiser (2001, 2005)). While these studies clarify how tariffs affect prices, welfare, and trade flows, they offer limited insight into how firms adjust their operational decisions, especially labor, in response to tariff shocks. In most tariff-related models, labor is either omitted or treated passively, leaving strategic choices about labor deployment across production sites and transportation routes largely unexplored.

While much of the trade literature relies on optimization-based or partial equilibrium models, few works employ variational inequality approach to capture the complexity of modern global trade networks. Variational inequality theory is a robust mathematical tool for analyzing equilibrium problems across disciplines, valued for accommodating diverse behavioral assumptions and enabling the study of both perfectly and imperfectly competitive supply chains (Nagurney (1999), and Nagurney and Li (2016)). A classical application appears in spatial price equilibrium models, introduced by Samuelson (1952) and expanded by Takayama and Judge (1964, 1971), which assume perfect competition with many producers (e.g., Nagurney and Aronson (1989), Nagurney and Zhao (1993), Daniele (2004), Nagurney (2006), Li, Nagurney, and Yu (2018), Nagurney and Besik (2022), and Nagurney, Pour, and Samadi (2024)). Beyond perfect competition, variational inequality theory also effectively captures strategic firm behavior in oligopolistic settings, where limited competitors influence market outcomes (Nagurney and Matsypura (2005), Zhang (2006), Qiang et al. (2013), Yu and Nagurney (2013), Li and Nagurney (2017), and Nagurney, Yu, and Besik (2017)).

Early foundational work by Nicholson, Bishop, and Nagurney (1994) and Nagurney, Nicholson, and Bishop (1996a, b) established variational inequality formulations for spatial price equilibrium problems with ad valorem tariffs, demonstrating that the nonlinearities and possible asymmetries introduced by such tariffs make traditional optimization methods inadequate. Subsequent variational inequality based models have integrated unit tariffs along with rerouting to evade tariffs (Nagurney and Samadi (2025)), unit tariffs and tariff-rate quotas within spatial price equilibrium models (Nagurney, Salarpour, and Dong (2022), Nagurney et al. (2023), Nagurney (2022a), and Nagurney, Besik, and Dong (2019)), and in oligopolistic frameworks (Nagurney, Besik, and Nagurney (2019) and Nagurney, Besik, and Li (2019)). Despite the strengths of variational inequality models in trade, most oligopoly studies have overlooked ad valorem tariffs, creating a methodological gap that we seek to address by developing a variational inequality-based framework integrating these tariffs under oligopolistic competition.

Global trade dynamics, especially tariff shocks and supply chain disruptions, have increasingly brought labor market conditions to the forefront. Labor availability, influenced by climate change, demographic shifts, and public health crises, has emerged as a critical determinant of supply chain performance and resilience (Nagurney (2023b), Nagurney and Ermagun (2022), Barnhart (2023), Nagurney (2025)), placing significant pressure on supply chains (Tirschwell, Thomson, and Rouimi (2024)). Nagurney (2021a) was among the first to integrate labor explicitly into a perishable food supply chain network model, using a variational inequality framework, treating labor availability as a constrained input. Building on this, Nagurney (2021b, 2023b) introduced wage-dependent labor in game-theoretic variational inequality models, enabling firms to adjust wages to attract workers under pandemic and climate shocks, and to invest in labor productivity in multiperiod settings. Nagurney (2021c) further extended this to account for multiple labor constraints, on routes, tiers, and the overall network, while examining differentiated products. Extending this concept, Nagurney (2022b) incorporated wage-responsive productivity, showing through Lagrange analysis that firm profitability can improve with strategic wage increases up to a threshold. Related Lagrange-based analysis for network equilibrium problems have been developed in the work of Daniele (2001, 2004, 2006), Barbagallo, Daniele, and Maugeri (2012), Caruso and Daniele (2018), Colajanni et al. (2018), Toyasaki, Daniele, and Wakolbinger (2014), and Nagurney and Daniele (2021).

Further advancing the labor dimension, Nagurney (2022c) developed a supply chain network optimization model that treated labor as a wage-dependent resource and allowed firms to invest in productivity across network routes. Nagurney and Ermagun (2022) introduced generalized efficiency and resilience measures for networks with labor constraints, showing that flexible labor allocation boosts resilience, while productivity disruptions can be more damaging than availability shocks. The focus then turned to sector-specific applications: Nagurney (2023a) modeled a defense-critical supply chain where labor constraints and risk preferences jointly shape equilibrium outcomes, providing performance and resilience metrics for strategic planning. Collectively, these studies highlight the evolving view of labor as both a constraint and a strategic variable in supply chain networks. However, another area of research has focused on optimizing the workforce at an operational level. This includes problems such as multi-shift scheduling and strategic workforce planning (Burns and Narasimhan (1999) and Horn, Elgindy, and Gomez-Iglesias (2016)), skill-based task allocation and flexibility planning (De Bruecker et al. (2015)), joint optimization of worker-task assignment, training, and rotation (Azizi and Liang (2013)), and recruitment strategy under demand and supply uncertainty (Kim et al. (2013)). These works emphasize tactical decisions, which differ in focus and scope from the labor modeling in this paper.

While the above models discussed so far primarily examine tariffs and labor independently, a growing body of work explores how trade policies, particularly tariffs, affect the labor market. Edwards (1988) provided one of the earliest theoretical frameworks linking changes in the terms of trade and import tariffs to labor adjustment. Amity and Davis (2012) and Liu, Xiao, and Qin (2025) extend this by empirically showing how trade shocks, especially input tariffs, drive job reallocation and income divergence. Several recent studies highlight the labor effects of ad valorem tariffs: Xu and Ouyang (2017) attribute rising wage inequality in China to sector-specific ad valorem tariff exposure post-WTO accession. Benguria and Saffie (2020) find that retaliatory tariffs increased unemployment in US agricultural regions, while He, Mau, and Xu (2021) show the US–China trade war reduced Chinese firms' high-skill job postings, altering employment composition. Giovannetti, Marvasi, and Vivoli (2021) report that tariff protection in Egypt lowered real wages and job stability, and Furceri et al. (2018) link tariff hikes to higher unemployment and inequality across countries. Flaaen and Pierce (2019) find that rising input costs and retaliation decreased US manufacturing employment despite limited protective gains. Finally, Ignatenko et al. (2025) simulate the long-run effects of comprehensive US ad valorem tariffs, showing marginal trade balance improvements but global employment contraction and significant welfare losses under retaliation.

While these studies focus on ad valorem tariffs, related work examines how specific tariffs influence labor market outcomes, particularly under oligopoly. Bastos and Kreickemeier (2009) use a two-country general equilibrium model with oligopolistic firms to study the joint effects of specific tariffs and labor unions, showing that wages depend on both trade liberalization and institutional context. Similarly, Ahmed, Marjit, and Chakraborty (2025) develop a general oligopolistic equilibrium model to analyze how changes in specific tariffs affect wages and employment across sectors.

In summary, the literature provides extensive insights into tariffs, labor, and supply chain equilibria, yet no study, to-date, unifies these strands by constructing a variational inequality game-theoretic framework that jointly models ad valorem tariffs and labor dynamics in an oligopolistic supply chain network. This gap is critical given the dominance of ad valorem tariffs in current trade policy and the growing sensitivity of labor markets to trade structures and firm strategies. Our paper fills this void by developing an original game-theoretic model that integrates these elements, offering a more realistic and policy-relevant analysis of global trade tensions and their distributional effects. Furthermore, the methodological framework is applied to the soybean trade, which is very timely, given the present tariff scenario and that this agricultural commodity continues to draw interest from operations researchers (see Cabral and Guimaraes (1994), Geman and Nguyen (2005), and Rettinger amd Minner (2025)).

2.2. Contributions

This paper contributes to the literature in the following key ways:

1. In the model, the firms can operate multiple production sites across countries and supply products to demand markets also located in different countries. They compete in an oligopolistic, noncooperative setting, each maximizing its own profit under endogenous pricing and cost structures.

2. Labor is modeled as a decision variable at production sites and along transportation routes, subject to site- and route-specific bounds. Wages vary by location, capturing heterogeneity across global labor markets.

3. Ad valorem tariffs are imposed on product flows between countries, directly influencing firms' decisions on labor allocation, product flows, and site selection.

4. The governing Nash equilibrium conditions are formulated as a variational inequality (see Nagurney (1999)). We also perform a Lagrange analysis, leading to an alternative variational inequality formulation that facilitates computation for larger-scale networks.

5. The methodological framework is used as the foundation for a case study on the global trade of soybeans, an important agricultural commodity, which has been subject to ad valorem tariffs recently, as well as earlier.

6. By jointly capturing ad valorem tariffs and labor availability within an oligopolistic framework, this work advances theory and offers practical insights into how ad valorem tariffs and labor frictions shape firm decisions on production and workforce deployment, along with the resulting profits, demand prices, product flows, and employment.

2.3. Organization of the Paper

This paper is organized as follows. Section 3 presents the global supply chain network game-theoretic model with multiple competing firms, and formulates the governing equilibrium as a variational inequality. A Lagrange analysis then addresses upper bounds on labor, yielding an alternative variational inequality formulation suitable for large-scale computation. Illustrative examples are provided to highlight our mathematical framework. Section 4 outlines the computational approach, and Section 5 provides numerical examples exploring various labor and tariff scenarios on the global trade of soybeans, an important agricultural commodity. Section 6 concludes the paper with key findings and directions for future research.

3. The Global Supply Chain Network Model with Ad Valorem Tariffs and Labor

The model that we construct is a global supply chain network equilibrium model that incorporates ad valorem tariffs and labor. In the model there are I firms that compete noncooperatively in an oligopolistic manner, with a typical firm denoted by i. We assume that all the costs, prices, wages, and tariffs are in the same currency. Each firm i; $i = 1, \ldots, I$, has n_i possible locations for the production of its product, and the locations can be in different countries. The production sites available for firm i are denoted by j; $j = 1, \ldots, n_i$. Note that a specific firm's production site j differs from the production site j of other firms. There are o demand markets for the products of the firms and these can also be in different countries, with a typical demand market denoted by k. The products are substitutable but differentiated by firm and, therefore, not homogeneous. The supply chain network topology is depicted in Figure 1. In Figure 1, the topmost nodes correspond to the firms; the middle nodes denote the production/manufacturing site options, and the bottom tier nodes correspond to the global demand markets. The top tier nodes are enumerated as: $1, \ldots, I$. The middle tier nodes are delineated as: $(1,1), \ldots, (I, n_I)$, with middle tier node (i,j) corresponding to the combination of firm i and its production site j. The bottom tier nodes in the supply chain network in Figure 1 are enumerated as: $1, \ldots, o$.

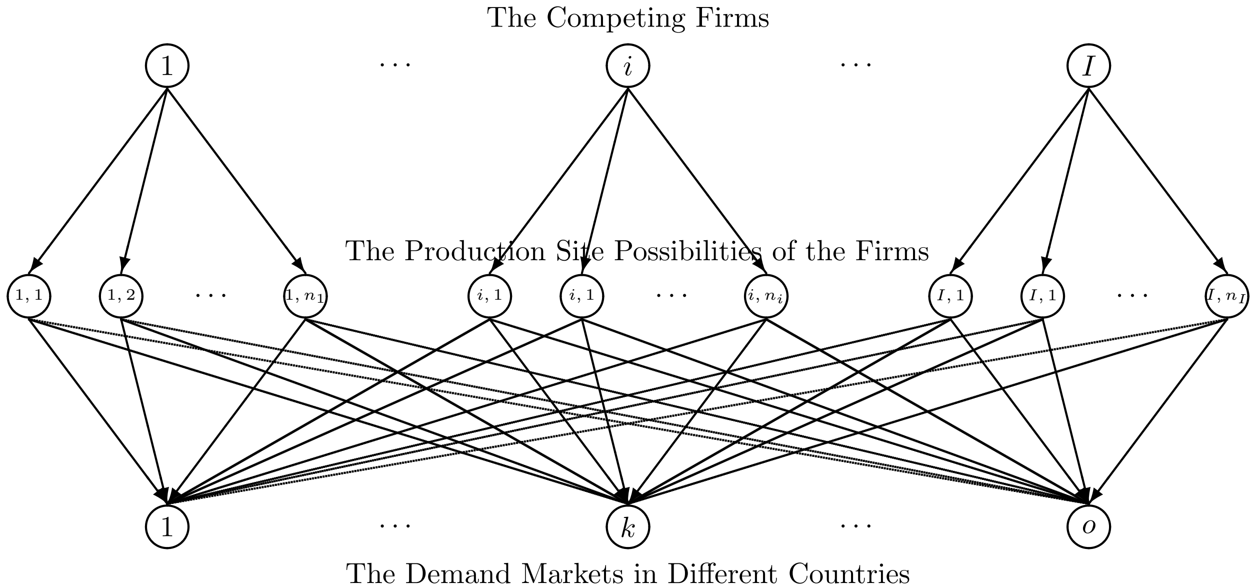

Figure 1: The Global Supply Chain Network with Ad Valorem Tariffs and Labor

The notation for the model appears in Table 1.

We now present the conservation of flow equations. The production output at firm i's site j, s^i_j, is equal to the total shipments of firm i's product to all the demand markets; that is:

$$s^i_j = \sum_{k=1}^{o} Q^i_{jk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{1}$$

whereas the demand for firm i's product at demand market k, d^i_k, must be satisfied by the firm's product shipments from all the firm's production sites to each demand market; hence:

$$d^i_k = \sum_{j=1}^{n_i} Q^i_{jk}, \quad i = 1, \ldots, I; k = 1, \ldots, o. \tag{2}$$

Table 1: Notation for the Global Supply Chain Network Model with Tariffs and Labor

Notation	Definition
s^i_j	the nonnegative production output (supply) of firm i at its production site j; $j = 1, \ldots, n_i$. We group the production outputs for each i; $i = 1, \ldots, I$, into the vector $s^i \in R^{n_i}_+$. We then further group all such vectors into the vector $s \in R_+^{\sum_{i=1}^I n_i}$.
Q^i_{jk}	the nonnegative amount of firm i's product produced at its site j and shipped to demand market k. The $\{Q^i_{jk}\}$ elements for all j and k are grouped into the vector $Q^i \in R^{n_i o}_+$. We then further group the Q^i; $i = 1, \ldots, I$, into the vector $Q \in R_+^{\sum_{i=1}^I n_i o}$.
d^i_k	the demand for firm i's product at demand market k. We group the demands for firm i's product for each $i = 1, \ldots, I$, into the vector $d^i \in R^o_+$ and then group the demands for all i into the vector $d \in R^{Io}_+$.
l^i_j	the labor (in hours) available at firm i's production site j; $i = 1, \ldots, I$; $j = 1, \ldots, n_i$.
l^i_{jk}	the labor (in hours) available for shipping firm i's product for $i = 1, \ldots, I$ from its production site j; $j = 1, \ldots, n_i$ to demand market k; $k = 1, \ldots, o$.
$f^i_j(s)$	the production cost at firm i's site j.
$\hat{c}^i_{jk}(Q)$	the total transportation cost associated with shipping firm i's product, produced at site j, to demand market k.
$\rho^i_k(d)$	the demand price function for firm i's product at demand market k.
w^i_j	hourly wage at firm i's production site j.
w^i_{jk}	hourly wage for shipping firm i's product from its production site j to demand market k.
β^i_j	positive factor relating inputs of labor at firm i's production site j to the amount of product produced there.
β^i_{jk}	positive factor relating inputs of labor to shipment volume of firm i's product from its production site j to demand market k.
$\bar{l}^i_j$	upper bound on labor hours of availability at production site j of firm i.
$\bar{l}^i_{jk}$	upper bound on labor hours of availability for shipment of firm i's product produced at its site j to demand market k.
τ^i_{jk}	the ad valorem tariff rate between the country that production site j of firm i is in and the country of demand market k

Furthermore, all the product shipments must be nonnegative:

$$Q^i_{jk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, o. \tag{3}$$

Using an approach as in Nagurney (2023a, b), relating labor to production outputs, as in economics, we have that the relationships, under the assumption of linearity between product outputs and labor, are at the production sites, and in transportation, respectively, as follows:

$$s^i_j = \beta^i_j l^i_j, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{4}$$

and

$$Q^i_{jk} = \beta^i_{jk} l^i_{jk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, o. \tag{5}$$

Additional information on linear and other production functions can be found in Samuelson and Marks (2012).

Also, the labor capacities cannot be exceeded, and the labor hours are nonnegative, so that

$$0 \leq l^i_j \leq \bar{l}^i_j, \quad i = 1, \ldots, I; j = 1, \ldots, n_i \tag{6}$$

and

$$0 \leq l^i_{jk} \leq \bar{l}^i_{jk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, o. \tag{7}$$

Each firm i; $i = 1, \ldots, I$, seeks to maximize its utility, U^i, consisting of its net revenue, subject to its constraints. Therefore, the optimization problem faced by each firm i; $i = 1, \ldots, I$, is given by:

$$\text{Maximize}\, U^i = \sum_{j=1}^{n_i} \sum_{k=1}^{o} \frac{\rho^i_k(d) Q^i_{jk}}{(1 + \tau^i_{jk})} - \sum_{j=1}^{n_i} f^i_j(s) - \sum_{j=1}^{n_i} \sum_{k=1}^{o} \hat{c}^i_{jk}(Q) - \sum_{j=1}^{n_i} w^i_j l^i_j - \sum_{j=1}^{n_i} \sum_{k=1}^{o} w^i_{jk} l^i_{jk} \tag{8}$$

subject to: (1) – (7) for i.

The first term in (8) represents the revenue under the ad valorem tariff rate. See Okuguchi and Yamazaki (1994) for a discussion in the case of a duopoly but for a much simpler model without transportation and multiple demand markets and with only a single production site for each firm and no labor. The second term corresponds to all the site production costs whereas the third term in (8) captures the total transportation costs. The last two terms in (8) are the total wage payouts for production and for transportation, respectively.

We now provide some redefinitions of the production cost and demand price functions in order to allow the strategic variables of the firms to be their product shipments.

Because of expression (1), one can redefine the production cost functions (cf. Table 1) in terms of product shipments, thus:

$$\hat{f}^i_j = \hat{f}^i_j(Q) \equiv f^i_j(s), \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{9}$$

On the demand side, because of (2), one can redefine the demand price functions in terms of product shipments as:

$$\hat{\rho}^i_k = \hat{\rho}^i_k(Q) \equiv \rho^i_k(d), \quad i = 1, \ldots, I; k = 1, \ldots, o. \tag{10}$$

The production cost and the transportation cost functions are assumed to be convex and continuously differentiable and the demand price functions to be monotonically decreasing in demands, and continuously differentiable. We will also replace the labor variables in the firms' objective functions with their product shipment equivalents. Towards that end, in view of (4) and (1), we have that:

$$\frac{\sum_{k=1}^{o} Q^i_{jk}}{\beta^i_j} = l^i_j, \quad i = 1, \dots, I; j = 1, \dots, n_i. \tag{11}$$

Also, in view of (5), we have that

$$\frac{Q^i_{jk}}{\beta^i_{jk}} = l^i_{jk}, \quad i = 1, \dots, I; j = 1, \dots, n_i; k = 1, \dots, o. \tag{12}$$

We can now re-express each firm's optimization problem (as stated above) in its production shipment variables only thus:

$$\text{Maximize} \quad \hat{U}^i(Q)$$

$$= \sum_{j=1}^{n_i}\sum_{k=1}^{o} \frac{\hat{\rho}^i_k(Q) Q^i_{jk}}{(1+\tau^i_{jk})} - \sum_{j=1}^{n_i} \hat{f}^i_j(Q) - \sum_{j=1}^{n_i}\sum_{k=1}^{o} \hat{c}^i_{jk}(Q) - \sum_{j=1}^{n_i} w^i_j \frac{\sum_{k=1}^{o} Q^i_{jk}}{\beta^i_j} - \sum_{j=1}^{n_i}\sum_{k=1}^{o} w^i_{jk} \frac{Q^i_{jk}}{\beta^i_{jk}} \tag{13}$$

subject to:

$$\frac{\sum_{k=1}^{o} Q^i_{jk}}{\beta^i_j} \le \bar{l}^i_j, \quad j = 1, \dots, n_i, \tag{14}$$

$$\frac{Q^i_{jk}}{\beta^i_{jk}} \le \bar{l}^i_{jk}, \quad j = 1, \dots, n_i; k = 1, \dots, o, \tag{15}$$

$$Q^i_{jk} \ge 0, \quad j = 1, \dots, n_i; k = 1, \dots, o. \tag{16}$$

The utility functions of all the firms are assumed to be concave and continuously differentiable.

Remark

In the global supply chain network equilibrium model with ad valorem tariffs and labor we assume that the firms compete noncooperatively. The functions in the firm utility expressions (13) are quite general in that competition can take place at the demand markets, through the demand price functions, and also at the production sites, through the production cost functions, plus through the transportation cost functions. Indeed, note that the demand price function of a firm i can depend not only on its own vector of strategies, in the form of its commodity shipments Q^i, but also on those of the other firms since we have that $\hat{\rho}^i_k(Q)$ is a function of the vector of commodity shipments Q. Similarly, the production cost of firm i, at each of its production sites j, $\hat{f}^i_j(Q)$, can depend not only on the firm's vector of commodity shipments but also on those of the other firms. In addition, competition can also occur in transportation through the cost function $\hat{c}^i_{jk}(Q)$ of firm i associated with transportation of its product from each of its production sites j; $j = 1, \dots, n_i$, to each of its demand markets k; $k = 1, \dots, o$.

One can also adapt the objective function (8) for unit tariffs: t^i_{jk}; $i = 1, \dots, I$; $j = 1, \dots, n_i$, and $k = 1, \dots, o$ by removing the denominator $(1 + \tau^i_{jk})$ in the first expression in (8) after the first summation signs and adding the term: $\sum_{j=1}^{n_i}\sum_{k=1}^{o} t^i_{jk} Q^i_{jk}$.

For tariff rate quotas and supply chain networks in an oligopolistic framework, but without labor, see Nagurney, Besik, and Nagurney (2019). That model can be extended with the inclusion of labor, using an approach similar to the one in this paper.

We define the feasible sets:

$$K^i \equiv \{Q^i | Q^i \,\text{satisfies}\, (14) - (16)\}, \quad i = 1, \ldots, I \tag{17a}$$

and

$$K \equiv \prod_{i=1}^{I} K^i. \tag{17b}$$

We now state the governing Nash Equilibrium conditions.

Definition 1: Global Supply Chain Network Nash Equilibrium Under Ad Valorem Tariffs

A product shipment pattern $Q^* \in K$ is a global supply chain network Nash Equilibrium under ad valorem tariffs if, for each firm i; $i = 1, \ldots, I$, the following equilibrium condition holds:

$$\hat{U}^i(Q^{i*}, Q^{-i*}) \geq \hat{U}^i(Q^i, Q^{-i*}), \quad \forall Q^i \in K^i, \tag{18}$$

where $Q^{-i*} \equiv (Q^{1*}, \ldots, Q^{i-1*}, Q^{i+1*}, \ldots, Q^{I*})$.

According to (18), a Nash Equilibrium is established if no firm, unilaterally, with its selected strategies, can improve upon its utility, given the strategies of the other firms.

The variational inequality formulation of the above Nash Equilibrium conditions is given in Theorem 1 below.

Theorem 1: Variational Inequality Formulation of the Global Supply Chain Network Nash Equilibrium Under Ad Valorem Tariffs

A product shipment pattern $Q^* \in Ki$ is a global supply chain network Nash Equilibrium under ad valorem tariffs according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o} \frac{\partial \hat{U}^i(Q^*)}{\partial Q^i_{jk}} \times (Q^i_{jk} - Q^{i*}_{jk}) \geq 0, \quad \forall Q \in K. \tag{19}$$

Proof: Under the assumptions that the utility functions, as in the form (18), are concave and continuously differentiable, we know that, according to Gabay and Moulin (1980); see also Nagurney (1999), Nash (1950, 1951), the associated Nash Equilibrium pattern can be formulated as the solution to the variational inequality: determine $Q^* \in K$, satisfying (19). □

A solution to variational inequality (19) is guaranteed to exist since, under our assumptions, the marginal utilities are all continuous and the feasible set K is compact due to the constraints

(14) and (15) on all firms $i = 1, \ldots, I$. This result follows from the classical theory of variational inequalities; see Kinderlehrer and Stampacchia (1980).

Note that the labor values associated with production and with transportation can be recovered using equations (11) and (12) once the variational inequality problem (19) is solved.

We now construct an alternative variational inequality to the one in (19) that includes Lagrange multipliers. The alternative variational inequality we will use for the solution of larger-scale numerical examples.

Define $V(Q)$ as

$$V(Q) \equiv -\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\frac{\partial \hat{U}^i(Q^*)}{\partial Q^i_{jk}} \times (Q^i_{jk} - Q^{i*}_{jk}) \tag{20}$$

and note that the VI (19) can be rewritten as

$$\text{Minimize}_K V(Q) = V(Q^*) = 0. \tag{21}$$

In order to formulate the Lagrange function, we reformulate the constraints with the associated Lagrange multiplier next to the corresponding constraint:

$$e^i_j = \frac{\sum_{k=1}^{o} Q^i_{jk}}{\beta^i_j} - \bar{l}^i_j \leq 0, \quad \lambda^i_j, \forall i, \forall j, \tag{22}$$

$$\phi^i_{jk} = \frac{Q^i_{jk}}{\beta^i_{jk}} - \bar{l}^i_{jk} \leq 0, \quad \mu^i_{jk}, \forall i, \forall j, \forall k, \tag{23}$$

$$g^i_{jk} = -Q^i_{jk}, \quad \epsilon^i_{jk}, \forall i, \forall j, \forall k, \tag{24}$$

and

$$\Gamma(Q) = (e^i_j, \phi^i_{jk}, g^i_{jk}), \forall i, \forall j, \forall k. \tag{25}$$

The Lagrange function is now constructed with e being the vector of all e^i_js; ϕ being the vector of all ϕ^i_{jk}s, and g being the vector of all g^i_{jk}s, and denoted by $L(Q, \lambda, \mu, \epsilon)$:

$$L(Q,\lambda,\mu,\epsilon) \equiv -\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\frac{\partial \hat{U}^i(Q^*)}{\partial Q^i_{jk}} \times (Q^i_{jk} - Q^{i*}_{jk})$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i} e^i_j\lambda^i_j + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\phi^i_{jk}\mu^i_{jk} + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o} g^i_{jk}\epsilon^i_{jk},$$

$$\forall Q \in R_+^{\sum_{i=1}^{I} n_i o}, \quad \forall \lambda \in R_+^{\sum_{i=1}^{I} n_i}, \mu \in R_+^{\sum_{i=1}^{I} n_i o}, \forall \epsilon \in R_+^{\sum_{i=1}^{I} n_i o}. \tag{26}$$

The feasible set K is convex and the Slater condition is satisfied. Indeed, we know that $\Gamma(Q)$ is convex and $\exists \bar{Q} \in R_+^{\sum_{i=1}^{I} n_i o}$ such that $\Gamma(\bar{Q}) < 0$, since we can always identify a small enough shipment pattern. Therefore, if Q^* is a minimal solution to (21), there exist Lagrange multipliers $\lambda^* \in R_+^{\sum_{i=1}^I n_i}$, $\mu^* \in R_+^{\sum_{i=1}^I n_i o}$, and $\epsilon^* \in R_+^{\sum_{i=1}^I n_i o}$ such that the vector $(Q^*, \lambda^*, \mu^*, \epsilon^*)$ is a saddle point of the Lagrange function (26):

$$L(Q^*, \lambda, \mu, \epsilon) \le L(Q^*, \lambda^*, \mu^*, \epsilon^*) \le L(Q, \lambda^*, \mu^*, \epsilon^*) \tag{27}$$

and

$$e_j^{i*} \lambda_j^{i*} = 0, \quad \forall i, \forall j, \tag{28}$$

$$\phi_{jk}^{i*} \mu_{jk}^{i*} = 0, \quad \forall i, \forall j, \forall k, \tag{29}$$

$$g_{jk}^{i*} \epsilon_{jk}^{i*} = 0, \quad \forall i, \forall j, \forall k. \tag{30}$$

From the right-hand side of (27), it follows that $Q^* \in R_+^{\sum_{i=1}^I n_i o}$ is a minimal point of the function $L(Q, \lambda^*, \mu^*, \epsilon^*)$ in the whole space $R_+^{\sum_{i=1}^I n_i o}$ and, therefore:

$$\frac{\partial L(Q^*, \lambda^*, \mu^*, \epsilon^*)}{\partial Q_{jk}^i} = -\frac{\partial \hat{U}^i(Q^*)}{\partial Q_{jk}^i} + \frac{\lambda_j^{i*}}{\beta_j^i} + \frac{\mu_{jk}^{i*}}{\beta_{jk}^i} - \epsilon_{jk}^{i*} = 0, \quad \forall i, \forall j, \forall k, \tag{31}$$

together with conditions (28) - (30).

We now state the following theorem.

Theorem 2: Alternative Variational Inequality Formulation

Conditions (28) through (30) and (31) correspond to a variational inequality equivalent to VI (19) given by: determine $(Q^*, \lambda^*, \mu^*, \epsilon^*) \in R_+^{\sum_{i=1}^I n_i o} + R_+^{\sum_{i=1}^I n_i} + R_+^{2\sum_{i=1}^I n_i o}$ *such that:*

$$\left[-\sum_{i=1}^I \sum_{j=1}^{n_i} \sum_{k=1}^o \frac{\partial \hat{U}^i(Q^*)}{\partial Q_{jk}^i} + \frac{\lambda_j^{i*}}{\beta_j^i} + \frac{\mu_{jk}^{i*}}{\beta_{jk}^i} - \epsilon_{jk}^{i*} \right] \times \left[Q_{jk}^i - Q_{jk}^{i*} \right]$$

$$+ \sum_{i=1}^I \sum_{j=1}^{n_i} \left[\bar{l}_j^i - \frac{\sum_{k=1}^o Q_{jk}^{i*}}{\beta_j^i} \right] \times \left[\lambda_j^i - \lambda_j^{i*} \right] + \sum_{i=1}^I \sum_{j=1}^{n_i} \sum_{k=1}^o \left[\bar{l}_{jk}^i - \frac{Q_{jk}^{i*}}{\beta_{jk}^i} \right] \times \left[\mu_{jk}^i - \mu_{jk}^{i*} \right]$$

$$+ \sum_{i=1}^I \sum_{j=1}^{n_i} \sum_{k=1}^o Q_{jk}^{i*} \times \left[\epsilon_{jk}^i - \epsilon_{jk}^{i*} \right] \ge 0, \quad \forall (Q, \lambda, \mu, \epsilon) \in R_+^{\sum_{i=1}^I n_i o + \sum_{i=1}^I n_i + 2\sum_{i=1}^I n_i o}, \tag{32}$$

or simplified as: determine $(Q^*, \lambda^*, \mu^*) \in R_+^{2\sum_{i=1}^I n_i o} + R_+^{\sum_{i=1}^I n_i}$ *such that:*

$$\left[-\sum_{i=1}^I \sum_{j=1}^{n_i} \sum_{k=1}^o \frac{\partial \hat{U}^i(Q^*)}{\partial Q_{jk}^i} + \frac{\lambda_j^{i*}}{\beta_j^i} + \frac{\mu_{jk}^{i*}}{\beta_{jk}^i} \right] \times \left[Q_{jk}^i - Q_{jk}^{i*} \right] + \sum_{i=1}^I \sum_{j=1}^{n_i} \left[\bar{l}_j^i - \frac{\sum_{k=1}^o Q_{jk}^{i*}}{\beta_j^i} \right] \times \left[\lambda_j^i - \lambda_j^{i*} \right]$$

$$+ \sum_{i=1}^I \sum_{j=1}^{n_i} \sum_{k=1}^o \left[\bar{l}_{jk}^i - \frac{Q_{jk}^{i*}}{\beta_{jk}^i} \right] \times \left[\mu_{jk}^i - \mu_{jk}^{i*} \right] \ge 0, \quad \forall (Q, \lambda, \mu) \in R_+^{2\sum_{i=1}^I n_i o + \sum_{i=1}^I n_i}. \tag{33}$$

Proof: First, note that a vector Q^* satisfying (31) and (28) - (30) also satisfies VI (32). Now we establish that it also satisfies VI (19).

If one multiplies (31) for a fixed i, j, and k, by $(Q^i_{jk} - Q^{i*}_{jk})$ and sums the resultant over all i, j, k, one gets:

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\frac{\partial \hat{U}^i(Q^*)}{\partial Q^i_{jk}} \times (Q^i_{jk} - Q^{i*}_{jk}) = \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\left[-\frac{\lambda^{i*}_j}{\beta^i_j} - \frac{\mu^{i*}_{jk}}{\beta^i_{jk}} + \epsilon^{i*}_{jk}\right] \times \left[Q^i_{jk} - Q^{i*}_{jk}\right]. \tag{34}$$

Expanding the right-hand side of (34) gives us:

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\left[-\frac{\lambda^{i*}_j}{\beta^i_j} - \frac{\mu^{i*}_{jk}}{\beta^i_{jk}} + \epsilon^{i*}_{jk}\right] \times Q^i_{jk} - \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\left[-\frac{\lambda^{i*}_j}{\beta^i_j} - \frac{\mu^{i*}_{jk}}{\beta^i_{jk}} + \epsilon^{i*}_{jk}\right] \times Q^{i*}_{jk}, \tag{35}$$

which, after applying (14) and (15), gives us:

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[\bar{l}^i_j - \frac{\sum_{k=1}^{o} Q^i_{jk}}{\beta^i_j}\right]\lambda^{i*}_j + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\left[\bar{l}^i_{jk} - \frac{Q^i_{jk}}{\beta^i_{jk}}\right]\mu^{i*}_{jk} + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\epsilon^{i*}_{jk}Q^i_{jk} - \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\epsilon^{i*}_{jk}Q^{i*}_{jk}. \tag{36}$$

We know that the first two terms in (36) are nonnegative because of constraints (14) and (15), respectively, and since the associated Lagrange multipliers λ^* and μ^* are also nonnegative. The third term is also nonnegative because the Lagrange multipliers ϵ^* and the commodity shipments are all nonnegative. Finally, the last term in (36) is zero because of (16).

VI (33) then follows from (32) since the nonnegativity of the commodity shipments is guaranteed by the feasible set in (33). The conclusion follows. □

Expanding the VI (33) using (13) gives us:

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\left[\sum_{h=1}^{n_i}\frac{\partial \hat{f}^i_h(Q^*)}{\partial Q^i_{jk}} + \sum_{h=1}^{n_i}\sum_{l=1}^{o}\frac{\partial \hat{c}^i_{hl}(Q^*)}{\partial Q^i_{jk}} + \frac{w^i_j}{\beta^i_j} + \frac{w^i_{jk}}{\beta^i_{jk}} - \frac{1}{1+\tau^i_{jk}}\left(\hat{\rho}^i_k(Q^*) + \sum_{h=1}^{n_i}\sum_{l=1}^{o}\frac{\partial \hat{\rho}^i_l(Q^*)}{\partial Q^i_{jk}}Q^{i*}_{hl}\right)\right.$$
$$\left. + \frac{\lambda^{i*}_j}{\beta^i_j} + \frac{\mu^{i*}_{jk}}{\beta^i_{jk}}\right] \times \left[Q^i_{jk} - Q^{i*}_{jk}\right] + \sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[\bar{l}^i_j - \frac{\sum_{k=1}^{o} Q^{i*}_{jk}}{\beta^i_j}\right] \times \left[\lambda^i_j - \lambda^{i*}_j\right]$$
$$+ \sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{o}\left[\bar{l}^i_{jk} - \frac{Q^{i*}_{jk}}{\beta^i_{jk}}\right] \times \left[\mu^i_{jk} - \mu^{i*}_{jk}\right] \geq 0, \quad \forall (Q, \lambda, \mu) \in R_+^{2\sum_{i=1}^{I} n_i o + \sum_{i=1}^{I} n_i}. \tag{37}$$

In variational inequality (37), the indices h and l are summation indices used to capture the interdependencies among all production sites and demand markets in the cost and demand price functions. Specifically, h ranges over all production sites of firm i (i.e., $h = 1, \ldots, n_i$) and l ranges over all demand markets (i.e., $l = 1, \ldots, o$). These summation indices differ from j and k, which represent the specific production site and demand market associated with the decision variable Q^i_{jk}

in the variational inequality. The partial derivatives $\frac{\partial \hat{f}_h^i(Q^*)}{\partial Q_{jk}^i}$, $\frac{\partial \hat{c}_{hl}^i(Q^*)}{\partial Q_{jk}^i}$, and $\frac{\partial \hat{\rho}_l^i(Q^*)}{\partial Q_{jk}^i}$ capture how changes in shipment Q_{jk}^i affect the cost at each production site h and on each transportation route (h, l), as well as the demand price at each demand market l, thereby reflecting the competitive interactions in the oligopolistic supply chain model.

We now put variational inequality (37) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{38}$$

where X and $F(X)$ are N-dimensional vectors, $\mathcal{K}$ is a closed, convex set, and F is a given continuous function from $\mathcal{K}$ to R^N.

We set $\mathcal{K} \equiv R_+^{2\sum_{i=1}^I n_i o + \sum_{i=1}^I n_i}$, $X \equiv (Q, \lambda, \mu)$, and $N \equiv 2\sum_{i=1}^I n_i o + \sum_{i=1}^I n_i$. Also, we define the vector $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, where

$$F_1(X) = \left[\sum_{h=1}^{n_i} \frac{\partial \hat{f}_h^i(Q)}{\partial Q_{jk}^i} + \sum_{h=1}^{n_i} \sum_{l=1}^{o} \frac{\partial \hat{c}_{hl}^i(Q)}{\partial Q_{jk}^i} + \frac{w_j^i}{\beta_j^i} + \frac{w_{jk}^i}{\beta_{jk}^i} - \frac{1}{1+\tau_{jk}^i} \left(\hat{\rho}_k^i(Q) + \sum_{h=1}^{n_i} \sum_{l=1}^{o} \frac{\partial \hat{\rho}_l^i(Q)}{\partial Q_{jk}^i} Q_{hl}^i \right) \right.$$

$$\left. + \frac{\lambda_j^i}{\beta_j^i} + \frac{\mu_{jk}^i}{\beta_{jk}^i}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, o \right], \tag{39}$$

$$F_2(X) = \left[\bar{l}_j^i - \frac{\sum_{k=1}^o Q_{jk}^i}{\beta_j^i}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i \right], \tag{40}$$

$$F_3(X) = \left[\bar{l}_{jk}^i - \frac{Q_{jk}^i}{\beta_{jk}^i}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, o \right]. \tag{41}$$

Then, clearly VI (37) can be rewritten as variational inequality (38), with the above definitions.

3.1. Illustrative Examples

In this subsection, several stylized illustrative examples are provided to demonstrate how the model behaves under different ad valorem tariff conditions. The examples are simple and easily solvable to highlight the mechanisms of the model. We assume that there are two firms in Example 1, Firm 1 and Firm 2, competing to sell their products at a single demand market, Demand Market 1. Each firm has a single available production site located in a different country. The network topology for Examples 1 through 3 is depicted in Figure 2.

Example 1: Baseline Example

The hourly wages, upper bounds on the labor hours, and labor productivity factors are:

$$w_1^1 = 5, \quad w_1^2 = 8, \quad w_{11}^1 = 10, \quad w_{11}^2 = 12, \quad \bar{l}_1^1 = 200, \quad \bar{l}_1^2 = 200,$$

$$\bar{l}_{11}^1 = 150, \quad \bar{l}_{11}^2 = 150, \quad \beta_1^1 = 2, \quad \beta_1^2 = 2, \quad \beta_{11}^1 = 2, \quad \beta_{11}^2 = 2.$$

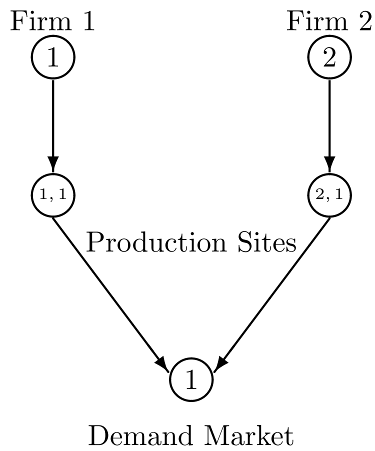

Figure 2: Global Supply Chain Network for Examples 1, 2, and 3

The production cost functions for Firm 1 and Firm 2 at their productions sites are:

$$\hat{f}^1_1(Q) = {Q^1_{11}}^2 + 0.5Q^1_{11} + 12, \quad f^2_1(Q) = 1.5{Q^2_{11}}^2 + 0.3Q^2_{11} + 15.$$

The total transportation cost functions for the first firm and the second firm, are, respectively:

$$\hat{c}^1_{11}(Q) = 0.8{Q^1_{11}}^2 + 8, \quad \hat{c}^2_{11}(Q) = 1.2{Q^2_{11}}^2 + 5.$$

The demand price functions for the products of Firm 1 and Firm 2 at the Demand Market 1 are:

$$\hat{\rho}^1_1(Q) = -Q^1_{11} - 0.4Q^2_{11} + 180, \quad \hat{\rho}^2_1(Q) = -0.5Q^1_{11} - 1.2Q^2_{11} + 160.$$

In Example 1, which serves as a baseline example, no ad valorem tariffs are imposed on the firms; therefore:

$$\tau^1_{11} = \tau^2_{11} = 0.$$

The simple structure of the supply chain network enables us to directly formulate the conservation of flow equations (1) and (2) as:

$$s^1_1 = Q^1_{11}, \quad s^2_1 = Q^2_{11}, \quad d^1_1 = Q^1_{11}, \quad d^2_1 = Q^2_{11}.$$

We make the assumption that $Q^{1*}_{11} > 0$ and $Q^{2*}_{11} > 0$ and that none of the bounds on labor are tight so all the Lagrange multipliers would be equal to 0.00 at the equilibrium. From VI (37), we then have the following expressions:

$$\frac{\partial \hat{f}^1_1(Q^*)}{\partial Q^1_{11}} + \frac{\partial \hat{c}^1_{11}(Q^*)}{\partial Q^1_{11}} + \frac{w^1_1}{\beta^1_1} + \frac{w^1_{11}}{\beta^1_{11}} - \left(\hat{\rho}^1_1(Q^*) + \frac{\partial \hat{\rho}^1_1(Q^*)}{\partial Q^1_{11}} Q^{1*}_{11}\right) = 0, \tag{42}$$

$$\frac{\partial \hat{f}^2_1(Q^*)}{\partial Q^2_{11}} + \frac{\partial \hat{c}^2_{11}(Q^*)}{\partial Q^2_{11}} + \frac{w^2_1}{\beta^2_1} + \frac{w^2_{11}}{\beta^2_{11}} - \left(\hat{\rho}^2_1(Q^*) + \frac{\partial \hat{\rho}^2_1(Q^*)}{\partial Q^2_{11}} Q^{2*}_{11}\right) = 0. \tag{43}$$

Inserting the corresponding functions and parameters into equations (42) and (43), we obtain the following system of equations:

$$5.6Q_{11}^{1*} + 0.4Q_{11}^{2*} = 172.00,$$

$$0.5Q_{11}^{1*} + 7.8Q_{11}^{2*} = 149.70.$$

with solution: $Q_{11}^{1*} = 29.47$ and $Q_{11}^{2*} = 17.30$. The equilibrium values of the product shipments, the labor hours, the demand prices, and the profits are reported in Table 2, and all Lagrange multipliers are, indeed equal to 0.00.

In this baseline example without ad valorem tariffs, Firm 1 produces and ships substantially more of its product to Demand Market 1 than Firm 2 does of its product, with outputs of 29.47 and 17.30 units, respectively. This higher production by Firm 1 is supported by greater labor hours at both the production site and along the transportation route, each at 14.73, compared to 8.65 for Firm 2. Equilibrium demand prices are \$143.60 for Firm 1's product and \$124.49 for Firm 2's. The combination of higher shipment and demand price results in Firm 1 achieving a profit of \$2,413.12, well above Firm 2's profit of \$1,147.62. This example, hence, serves as a baseline for assessing the impacts of ad valorem tariffs in the subsequent two examples.

Table 2: Equilibrium Solutions, Demand Prices, and Profits for Examples 1, 2, and 3

Variable	**Example 1**	**Example 2**	**Example 3**
Q_{11}^{1*}	29.47	24.40	20.92
Q_{11}^{2*}	17.30	17.62	12.06
l_{11}^{1*}	14.73	12.20	10.46
l_{11}^{2*}	8.65	8.81	6.03
l_{1}^{1*}	14.73	12.20	10.46
l_{1}^{2*}	8.65	8.81	6.03
$\hat{\rho}_1^1$	143.60	148.55	154.25
$\hat{\rho}_1^2$	124.49	126.65	135.06
$\hat{U}^1$	2,413.12	1,503.32	1,041.70
$\hat{U}^2$	1,147.62	1,191.84	481.10

Example 2: Ad Valorem Tariff Imposed Only on Firm 1's Product Produced at its Production Site

In Example 2, the data are the same as in Example 1 except that now it is assumed that a 30% ad valorem tariff is imposed at Demand Market 1 on Firm 1's product produced at its production site:

$$\tau_{11}^1 = 0.3, \quad \tau_{11}^2 = 0.$$

We make similar assumptions as to those made for Example 1 and solve VI (37). By incorporating the ad valorem tariff rate $\tau_{11}^1 = 0.3$ into equation (42), we obtain:

$$\frac{\partial \hat{f}_1^1(Q^*)}{\partial Q_{11}^1} + \frac{\partial \hat{c}_{11}^1(Q^*)}{\partial Q_{11}^1} + \frac{w_1^1}{\beta_1^1} + \frac{w_{11}^1}{\beta_{11}^1} - \frac{1}{1+\tau_{11}^1}\left(\hat{\rho}_1^1(Q^*) + \frac{\partial \hat{\rho}_1^1(Q^*)}{\partial Q_{11}^1} Q_{11}^{1*}\right) = 0. \tag{44}$$

Equation (43) remains the same as in Example 1, since no ad valorem tariffs are imposed on Firm 2. By substituting the functions and parameters into (43) and (44), we obtain the following system of equations:

$$5.13Q_{11}^{1*} + 0.30Q_{11}^{2*} = 130.46,$$

$$0.5Q_{11}^{1*} + 7.8Q_{11}^{2*} = 149.70.$$

The equilibrium outputs, labor hours, the demand prices, and the profits are then computed accordingly.

The imposition of a 30% ad valorem tariff on Firm 1's product in Example 2 leads to a clear shift in market dynamics. Firm 1's output falls to 24.40 units, with labor hours at its production site and on the transport route dropping by 17.17%, from 14.73 to 12.20, and its profit declining sharply to \$1,503.32. Meanwhile, Firm 2 slightly increases its output to 17.62 units and raises labor hours at both the production site and route by 1.9% to 8.81, increasing its profit to \$1,191.84 Demand prices for both firms rise compared to the baseline Example 1. These results underscore how the ad valorem tariff weakens Firm 1's position while allowing Firm 2 to expand its market share. All the equilibrium Lagrange multipliers are equal to zero as in Example 1.

Example 3: Equal Ad Valorem Tariffs on Both Competing Firms

Example 3 has the same data as in Example 1 except that both firms face identical trade policy conditions, with a 60% ad valorem tariff imposed on their products from their production sites to Demand Market 1:

$$\tau_{11}^1 = \tau_{11}^2 = 0.6.$$

Under the same assumptions as made in Examples 1 and 2, and by incorporating ad valorem tariffs into VI (37), we obtain the following expressions:

$$\frac{\partial \hat{f}_1^1(Q^*)}{\partial Q_{11}^1} + \frac{\partial \hat{c}_{11}^1(Q^*)}{\partial Q_{11}^1} + \frac{w_1^1}{\beta_1^1} + \frac{w_{11}^1}{\beta_{11}^1} - \frac{1}{1+\tau_{11}^1}\left(\hat{\rho}_1^1(Q^*) + \frac{\partial \hat{\rho}_1^1(Q^*)}{\partial Q_{11}^1}Q_{11}^{1*}\right) = 0, \tag{45}$$

$$\frac{\partial \hat{f}_1^2(Q^*)}{\partial Q_{11}^2} + \frac{\partial \hat{c}_{11}^2(Q^*)}{\partial Q_{11}^2} + \frac{w_1^2}{\beta_1^2} + \frac{w_{11}^2}{\beta_{11}^2} - \frac{1}{1+\tau_{11}^2}\left(\hat{\rho}_1^2(Q^*) + \frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{11}^2}Q_{11}^{2*}\right) = 0, \tag{46}$$

which results with incorporation of the data in the following system of equations:

$$4.85Q_{11}^{1*} + 0.25Q_{11}^{2*} = 104.50,$$

$$0.31Q_{11}^{1*} + 6.90Q_{11}^{2*} = 89.70.$$

Table 2 reports the equilibrium values for the product shipments, the labor hours, the demand prices, and profits. When both firms face 60% ad valorem tariffs in Example 3, production drops sharply to 20.92 for Firm 1 and to 12.06 for Firm 2. Both firms experience a sharp reduction in labor hours at the production sites and along the routes: Firm 1's labor hours drop by 29.0%

and Firm 2's by 30.2% at the sites and along the respective routes as compared to results for Example 1. The resulting decreases in product supply raise the demand prices higher. Firm 1's profit declines to \$1,041.70, while Firm 2 experiences a more dramatic drop in profits to \$481.10. These results underscore how ad valorem tariffs can constrain market shares, reduce profits, and reshape competitive dynamics when imposed more broadly.

Example 4: No Ad Valorem Tariffs with Each Firm Operating Two Production Sites

Example 4 has the same data as that in Example 1, except that now Firm 1 and Firm 2 each have two available production sites located in different countries that produce and ship products to Demand Market 1, as shown in Figure 3. We set new parameters for the added production sites thus:

$$w^1_2 = 6, \quad w^2_2 = 7, \quad w^1_{21} = 11, \quad w^2_{21} = 13, \quad \bar{l}^1_2 = 180, \quad \bar{l}^2_2 = 180,$$
$$\bar{l}^1_{21} = 140, \quad \bar{l}^2_{21} = 140, \quad \beta^1_2 = 2, \quad \beta^2_2 = 2, \quad \beta^1_{21} = 2, \quad \beta^2_{21} = 2.$$

The production cost functions for Firm 1 and Firm 2 at their new productions sites are:

$$\hat{f}^1_2(Q) = {Q^1_{21}}^2 + 0.3Q^1_{21} + 10, \quad \hat{f}^2_2(Q) = 1.2{Q^2_{21}}^2 + 0.3Q^2_{21} + 15.$$

The total transportation cost functions for the shipments from added Firm 1 and Firm 2's production sites to the Demand Market 1 are:

$$\hat{c}^1_{21}(Q) = 0.9{Q^1_{21}}^2 + 8, \quad \hat{c}^2_{21}(Q) = 1.1{Q^2_{21}}^2 + 6.$$

The demand price functions for the products of Firm 1 and Firm 2 at the Demand Market 1 are:

$$\hat{\rho}^1_1(Q) = -(Q^1_{11} + Q^1_{21}) - 0.4(Q^2_{11} + Q^2_{21}) + 180, \quad \hat{\rho}^2_1(Q) = -0.5(Q^1_{11} + Q^1_{21}) - 1.2(Q^2_{11} + Q^2_{21}) + 160.$$

In Example 4, similar to Example 1, no ad valorem tariffs are imposed on the firms:

$$\tau^1_{11} = \tau^1_{21} = \tau^2_{11} = \tau^2_{21} = 0.$$

According to the supply chain network topology of this example, the conservation of flow equations are:

$$s^1_1 = Q^1_{11}, \quad s^1_2 = Q^1_{21}, \quad s^2_1 = Q^2_{11}, \quad s^2_2 = Q^2_{21}, \quad d^1_1 = Q^1_{11} + Q^1_{21}, \quad d^2_1 = Q^2_{11} + Q^2_{21}.$$

Using VI (37) and, assuming that all product shipments are positive at equilibrium and that all Lagrange multipliers, in turn, are equal to 0.00, we obtain the following system:

$$\frac{\partial \hat{f}^1_1(Q^*)}{\partial Q^1_{11}} + \frac{\partial \hat{c}^1_{11}(Q^*)}{\partial Q^1_{11}} + \frac{w^1_1}{\beta^1_1} + \frac{w^1_{11}}{\beta^1_{11}} - \left(\hat{\rho}^1_1(Q^*) + \frac{\partial \hat{\rho}^1_1(Q^*)}{\partial Q^1_{11}} Q^{1*}_{11} + \frac{\partial \hat{\rho}^1_1(Q^*)}{\partial Q^1_{21}} Q^{1*}_{21}\right) = 0, \tag{47}$$

$$\frac{\partial \hat{f}^1_2(Q^*)}{\partial Q^1_{21}} + \frac{\partial \hat{c}^1_{21}(Q^*)}{\partial Q^1_{21}} + \frac{w^1_2}{\beta^1_2} + \frac{w^1_{21}}{\beta^1_{21}} - \left(\hat{\rho}^1_1(Q^*) + \frac{\partial \hat{\rho}^1_1(Q^*)}{\partial Q^1_{21}} Q^{1*}_{21} + \frac{\partial \hat{\rho}^1_1(Q^*)}{\partial Q^1_{11}} Q^{1*}_{11}\right) = 0, \tag{48}$$

$$\frac{\partial \hat{f}^2_1(Q^*)}{\partial Q^2_{11}} + \frac{\partial \hat{c}^2_{11}(Q^*)}{\partial Q^2_{11}} + \frac{w^2_1}{\beta^2_1} + \frac{w^2_{11}}{\beta^2_{11}} - \left(\hat{\rho}^2_1(Q^*) + \frac{\partial \hat{\rho}^2_1(Q^*)}{\partial Q^2_{11}} Q^{2*}_{11} + \frac{\partial \hat{\rho}^2_1(Q^*)}{\partial Q^2_{21}} Q^{2*}_{21}\right) = 0, \tag{49}$$

$$\frac{\partial \hat{f}_2^2(Q^*)}{\partial Q_{21}^2}+\frac{\partial \hat{c}_{21}^2(Q^*)}{\partial Q_{21}^2}+\frac{w_2^2}{\beta_2^2}+\frac{w_{21}^2}{\beta_{21}^2}-\left(\hat{\rho}_1^2(Q^*)+\frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{21}^2}Q_{21}^{2*}+\frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{11}^2}Q_{11}^{2*}\right)=0. \tag{50}$$

Inserting the corresponding functions and parameters into the above equations (47) – (50), we obtain the following system of equations:

$$5.6Q_{11}^{1*}+2Q_{21}^{1*}+0.4Q_{11}^{2*}+0.4Q_{21}^{2*}=172.00,$$

$$2Q_{11}^{1*}+5.8Q_{21}^{1*}+0.4Q_{11}^{2*}+0.4Q_{21}^{2*}=171.20,$$

$$0.5Q_{11}^{1*}+0.5Q_{21}^{1*}+7.8Q_{11}^{2*}++2.4Q_{21}^{2*}=149.70,$$

$$0.5Q_{11}^{1*}+0.5Q_{21}^{1*}+2.4Q_{11}^{2*}+7Q_{21}^{2*}=149.70,$$

with solution: $Q_{11}^{1*}=21.59, Q_{21}^{1*}=20.25, Q_{11}^{2*}=12.12$ and $Q_{21}^{1*}=14.23$. The equilibrium solutions for labor hours, demand prices, and profits are reported in Table 3. And, again, as assumed, all the equilibrium Lagrange multipliers are equal to 0.00.

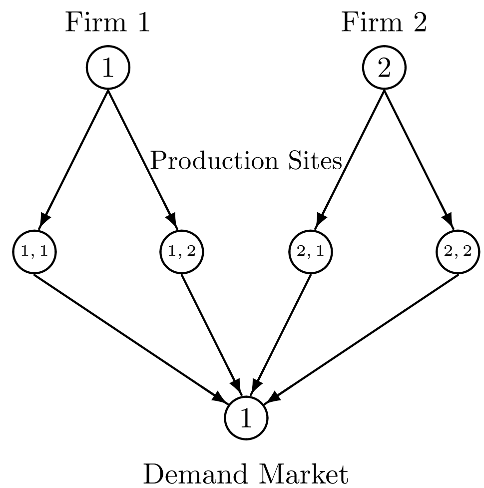

Figure 3: Global Supply Chain Network for Examples 4, 5, and 6

In Example 4, both firms operate two production sites in different countries without any ad valorem tariffs. Firm 1 produces 21.59 and 20.25 units at its sites, supported by labor hours of 10.79 and 10.12, respectively. Firm 2's sites produce 12.12 and 14.23 units, with corresponding labor hours of 6.06 and 7.11. Labor hours along the transportation routes from Firm 1 and Firm 2's production sites to Demand Market 1 mirror the same values, respectively. The higher overall production and shipments, as compared to Example 1, keep demand prices even lower in this example, at \$127.62 for Firm 1's product and \$107.46 for Firm 2's. Profits are highest in Example 4, compared to the previous examples, with Firm 1 earning \$3,332.73 and Firm 2: \$1,656.93. This example highlights how an expanded supply chain network without ad valorem tariffs allows firms to distribute production efficiently, sustaining lower prices for consumers, while maximizing profits.

Table 3: Equilibrium Solutions, Demand Prices, and Profits for Examples 4, 5, and 6

Variable	**Example 4**	**Example 5**	**Example 6**
Q_{11}^{1*}	21.59	10.00	10.00
Q_{21}^{1*}	20.25	10.00	10.00
Q_{11}^{2*}	12.12	13.15	9.00
Q_{21}^{2*}	14.23	15.44	9.00
l_{11}^{1*}	10.79	5.00	5.00
l_{21}^{1*}	10.12	5.00	5.00
l_{11}^{2*}	6.06	6.57	4.50
l_{21}^{2*}	7.11	7.72	4.50
l_1^{1*}	10.79	10.00	10.00
l_2^{1*}	10.12	10.00	10.00
l_1^{2*}	6.06	6.57	6.00
l_2^{2*}	7.11	7.72	6.00
$\hat{\rho}_1^1$	127.62	148.56	152.80
$\hat{\rho}_1^2$	107.46	115.69	128.40
$\hat{U}^1$	3,332.73	1,557.16	1,562.98
$\hat{U}^2$	1,656.93	1,960.02	851.28

Example 5: Ad Valorem Tariffs and Labor Disruption at Firm 1

In Example 5 the data are the same as in Example 4 except that Firm 1 faces 30% and 50% ad valorem tariffs on shipments from its two production sites, respectively, to the Demand Market 1, while Firm 2 remains with no ad valorem tariffs:

$$\tau_{11}^1 = 0.3, \quad \tau_{21}^1 = 0.5, \quad \tau_{11}^2 = 0, \quad \tau_{21}^2 = 0.$$

In addition to these ad valorem tariff rates, we assume a labor disruption that effectively reduces the upper bounds on labor hours available at Firm 1's production sites and also causes a loss of productivity. This example captures how, tariffs, coupled with labor challenges can affect production, product prices, and profits. We set the labor parameters, thus:

$$\bar{l}_1^1 = \bar{l}_2^1 = 10,$$

$$\beta_1^1 = \beta_2^1 = 1.$$

The remainder of the data is as in Example 4.

Under the same assumptions as made in Example 4, but now assuming, which is reasonable, positive equilibrium Lagrange multipliers associated with the labor hour constraints at Firm 1's production sites, and by incorporating the ad valorem tariff rates and parameters into VI (37), we obtain the following expressions:

$$\frac{\partial \hat{f}_1^1(Q^*)}{\partial Q_{11}^1} + \frac{\partial \hat{c}_{11}^1(Q^*)}{\partial Q_{11}^1} + \frac{w_1^1}{\beta_1^1} + \frac{w_{11}^1}{\beta_{11}^1} - \frac{1}{1+\tau_{11}^1}\left(\hat{\rho}_1^1(Q^*) + \frac{\partial \hat{\rho}_1^1(Q^*)}{\partial Q_{11}^1} Q_{11}^{1*} + \frac{\partial \hat{\rho}_1^1(Q^*)}{\partial Q_{21}^1} Q_{21}^{1*}\right) + \frac{\lambda_1^{1*}}{\beta_1^1} = 0, \quad (51)$$

$$\frac{\partial \hat{f}_2^1(Q^*)}{\partial Q_{21}^1}+\frac{\partial \hat{c}_{21}^1(Q^*)}{\partial Q_{21}^1}+\frac{w_2^1}{\beta_2^1}+\frac{w_{21}^1}{\beta_{21}^1}-\frac{1}{1+\tau_{21}^1}\left(\hat{\rho}_1^1(Q^*)+\frac{\partial \hat{\rho}_1^1(Q^*)}{\partial Q_{21}^1}Q_{21}^{1*}+\frac{\partial \hat{\rho}_1^1(Q^*)}{\partial Q_{11}^1}Q_{11}^{1*}\right)+\frac{\lambda_2^{1*}}{\beta_2^1}=0, \quad (52)$$

$$\bar{l}_1^1-\frac{Q_{11}^{1*}}{\beta_1^1}=0, \quad (53)$$

$$\bar{l}_2^1-\frac{Q_{21}^{1*}}{\beta_2^1}=0. \quad (54)$$

Equations (49) and (50) remain the same as in Example 4, since no ad valorem tariffs are imposed on Firm 2. By substituting the functions and parameters into equations (49) -– (54), and using the known values of $Q_{11}^{1*}=10.00$ and $Q_{21}^{1*}=10.00$ from (53) and (54), the system reduces to the following:

$$0.30Q_{11}^{2*}+0.30Q_{21}^{2*}+\lambda_1^{1*}=61.36,$$

$$0.26Q_{11}^{2*}+0.26Q_{21}^{2*}+\lambda_2^{1*}=43.60,$$

$$7.8Q_{11}^{2*}+2.4Q_{21}^{2*}=139.7,$$

$$2.4Q_{11}^{2*}+7Q_{21}^{2*}=139.7.$$

The resulting equilibrium solution for the product shipments, the labor hours, the demand prices, and the profits are computed and presented in Table 3.

In Example 5, the combination of ad valorem tariffs on Firm 1's products, 30% at site 1 and 50% at site 2, and a simultaneous labor disruption creates a compounded shock to its operations. Firm 1's production at both sites are at exactly 10 units due to tight labor hour upper bounds, a sharp drop from the 21.59 and 20.25 units it produced in the Example 4. Labor hours at these production sites are fully utilized, with the associated Lagrange multipliers positive such that $\lambda_1^{1*}=52.77$ and $\lambda_2^{1*}=36.16$. Labor hours along the transportation routes from Firm 1's sites also drop to 5, more than a 50% reduction, reflecting reduced shipping activity due to lower product shipments.

In contrast, Firm 2 increases its production to 13.15 and 15.44 units, respectively, taking advantage of the reduced market presence of Firm 1. This increase is supported by higher labor hours both at the production sites and along the transportation routes, enabling Firm 2 to expand its market share.

On the demand side, the demand prices rise to \$148.56 for Firm 1's and \$115.69 for Firm 2's products, both notably higher than in the Example 4. The profit implications are substantial. Firm 1's profit falls from \$3,332.73 in Example 4 to \$1,557.16, reflecting both lower output and constrained labor hours. Meanwhile, Firm 2, unaffected by tariffs or labor shortages, benefits from the shift in market dynamics, increasing its profit to \$1,960.02. All the remaining equilibrium Lagrange multipliers are equal to 0.00.

This example highlights how the interplay between ad valorem tariffs and labor constraints can dramatically reshape competition and profits across global supply chains.

Example 6: Ad Valorem Tariffs and Labor Disruption on Both Competing Firms

Example 6 has the same data as that in Example 5, except that now Firm 2 also faces ad valorem tariffs at 40% and 70% on its two production sites, respectively:

$$\tau_{11}^1 = 0.3, \quad \tau_{21}^1 = 0.5, \quad \tau_{11}^2 = 0.4, \quad \tau_{21}^2 = 0.7.$$

In addition, there is a labor disruption at both Firm 2's production sites, which reduces available labor hours and labor productivity to:

$$\bar{l}_1^2 = \bar{l}_2^2 = 6.00,$$

$$\beta_1^2 = \beta_2^2 = 1.50.$$

Under the same assumptions as in Example 5, but now with both firms having positive Lagrange multipliers on the labor hour constraints at their production sites, we incorporate these tariff rates and parameters into VI (37) to obtain the following expressions:

$$\frac{\partial \hat{f}_1^2(Q^*)}{\partial Q_{11}^2} + \frac{\partial \hat{c}_{11}^2(Q^*)}{\partial Q_{11}^2} + \frac{w_1^2}{\beta_1^2} + \frac{w_{11}^2}{\beta_{11}^2} - \frac{1}{1+\tau_{11}^2}\left(\hat{\rho}_1^2(Q^*) + \frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{11}^2} Q_{11}^{2*} + \frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{21}^2} Q_{21}^{2*}\right) + \frac{\lambda_1^{2*}}{\beta_1^2} = 0, \quad (55)$$

$$\frac{\partial \hat{f}_2^2(Q^*)}{\partial Q_{21}^2} + \frac{\partial \hat{c}_{21}^2(Q^*)}{\partial Q_{21}^2} + \frac{w_2^2}{\beta_2^2} + \frac{w_{21}^2}{\beta_{21}^2} - \frac{1}{1+\tau_{21}^2}\left(\hat{\rho}_1^2(Q^*) + \frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{21}^2} Q_{21}^{2*} + \frac{\partial \hat{\rho}_1^2(Q^*)}{\partial Q_{11}^2} Q_{11}^{2*}\right) + \frac{\lambda_2^{2*}}{\beta_2^2} = 0, \quad (56)$$

$$\bar{l}_1^2 - \frac{Q_{11}^{2*}}{\beta_1^2} = 0, \quad (57)$$

$$\bar{l}_2^2 - \frac{Q_{21}^{2*}}{\beta_2^2} = 0. \quad (58)$$

Equations (51)–(54) remain unchanged from Example 5. By substituting the functions and parameters into equations (51) -– (58), and incorporating the equilibrium values of product shipments determined by the binding constraints (53), (54), (57), and (58) as $Q_{11}^{1*} = 10.00, Q_{21}^{1*} = 10.00, Q_{11}^{2*} = 9.00$, and $Q_{21}^{2*} = 9.00$, we obtain the following positive Lagrange multipliers:

$$\lambda_1^{1*} = 55.96, \quad \lambda_2^{1*} = 38.92, \quad \lambda_1^{2*} = 16.27, \quad \lambda_2^{2*} = 10.07.$$

The equilibrium solutions for labor hours, demand prices, and profits are reported in Table 3.

In Example 6, both firms face ad valorem tariffs and binding available labor hours at their production sites. As a direct result, production at all four sites for both firms hits the labor-imposed upper bounds: Firm 1 produces 10 units at each of its sites, while Firm 2 is constrained to 9 units at both of its sites, reflecting also the new lower productivity. All the other equilibrium Lagrange multipliers are equal to zero.

The impact extends throughout the supply chain network. Labor hours along the transportation routes from Firm 2's production sites to Demand Market 1 drop sharply from 6.57 and 7.72 in

Example 5 to just 4.50 and 4.50 in Example 6, mirroring the lower product shipments. Meanwhile, labor hours on the routes from Firm 1's sites to Demand Market 1 remain unchanged from Example 5. As a result of these reduced shipments, demand prices rise for both firms' products, both noticeably higher than in the previous examples.

In terms of profitability, Firm 1's profit slightly improves over that in Example 5, reaching \$1,562.98, due to higher prices offsetting its limited production. Meanwhile, Firm 2's profit falls sharply to \$851.28, illustrating how being disrupted simultaneously by new ad valorem tariffs and labor disruption places it at a competitive disadvantage. This example shows that, as ad valorem tariffs raise costs and affect product shipments, firms adjust operations, but, when labor also becomes scarce or less productive, both competitive firms are forced to operate at reduced scales. Example 6 further underscores, that when both ad valorem tariffs and labor pressures converge, the impacts can extend beyond individual firms, leading to a reshaping of the market equilibrium.

4. The Computational Procedure

In this section, we present the algorithm used to compute the equilibrium solutions for the series of numerical examples in Section 5. We employ the modified projection method of Korpelevich (1977), which guarantees convergence provided that the function $F(X)$ in variational inequality (38), equivalently (37), is monotone and Lipschitz continuous. These conditions are expressed as follows:

For monotonicity:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{59}$$

And for Lipschitz continuity with constant $\eta > 0$:

$$||F(X^1) - F(X^2)|| \leq \eta ||X^1 - X^2||, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{60}$$

The steps of the algorithm are now presented, followed by the closed-form expressions for Step 1, which compute the product flows and the associated Lagrange multipliers at iteration t:

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $t = 1$ and let β be a scalar such that $o < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{61}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{62}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

Explicit Formulae at Iteration t for the Product Flows in Step 1

The closed-form expressions for the product flows in (61) for variational inequality (37) are:

$$\bar{Q}^{it}_{jk} = \max\left\{0,\ Q^{it-1}_{jk} + \beta\left(\frac{1}{1+\tau^i_{jk}}(\hat{\rho}^i_k(Q^{t-1}) + \sum_{h=1}^{n_i}\sum_{l=1}^{o}\frac{\partial \hat{\rho}^i_l(Q^{t-1})}{\partial Q^i_{jk}} Q^{it-1}_{hl})\right.\right.$$
$$\left.\left. - \sum_{h=1}^{n_i}\frac{\partial \hat{f}^i_h(Q^{t-1})}{\partial Q^i_{jk}} - \sum_{h=1}^{n_i}\sum_{l=1}^{o}\frac{\partial \hat{c}^i_{hl}(Q^{t-1})}{\partial Q^i_{jk}} - \frac{w^i_j}{\beta^i_j} - \frac{w^i_{jk}}{\beta^i_{jk}} - \frac{\lambda^{it-1}_j}{\beta^i_j} - \frac{\mu^{it-1}_{jk}}{\beta^i_{jk}}\right)\right\}, \quad \forall i, j, k. \tag{63}$$

Explicit Formulae at Iteration t for the Production Labor Hours Lagrange Multipliers in Step 1

The closed-form expressions for the Lagrange multipliers associated with production sites in (61) for variational inequality (37) are:

$$\bar{\lambda}^{it}_j = \max\left\{0,\ \lambda^{it-1}_j + \beta(\frac{\sum_{k=1}^{o} Q^{it-1}_{jk}}{\beta^i_j} - \bar{l}^i_j)\right\}, \quad \forall i, j. \tag{64}$$

Explicit Formulae at Iteration t for the Shipment Labor Hours Lagrange Multipliers in Step 1

The closed-form expressions for the Lagrange multipliers associated with shipment labor hours in (61) for variational inequality (37) are:

$$\bar{\mu}^{it}_{jk} = \max\left\{0,\ \mu^{it-1}_{jk} + \beta(\frac{Q^{it-1}_{jk}}{\beta^i_{jk}} - \bar{l}^i_{jk})\right\}, \quad \forall i, j, k. \tag{65}$$

The explicit formulae for the variables in (62) in Step 2 easily follow.

5. Numerical Examples with Algorithmically Computed Solutions

In this section, we examine the global soybean supply chain, an essential agricultural commodity, and present numerical examples illustrating its trade structure, flows, and labor implications under ad valorem tariffs. The solutions to these examples are algorithmically computed. Soybeans have become the leading agricultural commodity in global trade, serving as a key input for livestock feed, biofuels, and plant-based foods, and underpinning significant portions of the world economy (Langthaler (2025), Fraanje and Garnett (2020), and Rgultig (2025a)). The market's value was estimated at USD 169.65 billion in 2024 and is projected to reach USD 255.39 billion by 2033 (Research and Market (2025)). Since the 1950s, soybean production has risen fifteen-fold and shifted from Asia to the US, Brazil, and Argentina, which now produce 80% of global output (Banqu (2023)). In 2023, Brazil, the US, and Argentina were the largest exporters, shipping 101.9, 48.7, and 1.8 million tons, respectively, while China, the EU, and Argentina led imports with 106, 15.3, and 10.3 million tons (Food and Agriculture Organization (2024)).

China's centrality in the soybean trade cannot be overstated. As the top global importer, accounting for roughly 60% of total soybean imports and serving as the primary customer for both the US and Brazil, China plays a pivotal role in shaping global trade flows and pricing (Voora et al. (2024) and Colussi et al. (2024)). Over the past decade, Brazil has become increasingly dependent on Chinese demand: by 2023, 73% of its soybean exports went to China, whereas 51% of the US exports of soybeans went to China (Colussi et al. (2024)). Brazil overtook the US as the world's top soybean exporter in 2013 and has since widened that gap (Caraway (2025)).

Given the high concentration of trade among the three dominant exporters, the US, Brazil, and Argentina, and their mutual influence on global supply and pricing, the soybean export market is best modeled as an oligopoly. Empirical studies of China's soybean imports show extreme market concentration, classifying the market as a "highest oligopoly" (Yan et al. (2023)). Additionally, the Soybean Market (2025) report emphasizes that the global soybean sector is shaped by a handful of powerful agribusinesses, such as Cargill, ADM, and Bunge, which dominate the market through integrated production, wide processing networks, and advanced supply chain systems. These firms control a significant share of global soybeans exports, especially from North and South America, where more than 80% of the world's soybeans are produced. Together, these structural features support the use of an oligopolistic framework to capture trade and labor dynamics in this sector.

The US–China trade conflict, beginning in 2018, marked a sharp turning point for the soybean trade. After the US imposed 25% ad valorem tariffs on Chinese goods, China retaliated with equivalent tariffs on key US exports, including soybeans (Adjemian, Smith and He (2021)). This led to a 75% drop in US soybean exports to China, with values falling from USD 12 billion in 2017 to just USD 3 billion in 2018 (Lacoume (2025)). As US soybeans lost competitiveness, China shifted demand to Brazil, whose exports to China rose by 48% over the same period (FreightAmigo (2025)). The restructuring of global soybean supply chains in favor of Brazil has persisted, altering market power and price-setting dynamics. Renewed tariffs in 2025 signal another escalation: the US reimposed ad valorem tariffs on Chinese goods in early spring, prompting China to retaliate with duties up to 125% on US imports, including soybeans, raising the risk of another major supply chain shock see uSMART (2025)). As of 11 August 2025, an executive order extended the deadline for high US ad valorem tariffs on Chinese goods by 90 days, until mid-November 2025. Without this extension, US and China's ad valorem tariffs would have returned to the levels of 145% and 125%, which were later reduced in May to 30% and 10% (Breuninger and Javers (2025)).

This shift in trade flows not only changed global market shares but also directly impacted labor across the soybean supply chain. The global soybean sector provides employment for many people around the world. In 2022, an estimated 223,000 full-time jobs in the US were tied to soybean production (Feed and Grain (2023)), while nearly 240,000 Brazilian farms were cultivating soybeans as of 2017 (Bicudo Da Silva et al. (2020)). Given that labor demand closely follows export volumes, ad valorem tariffs create ripple effects on employment throughout the sector.

The numerical examples in this section were solved using the modified projection method, described in Section 4 and implemented in MATLAB on a Mac system. A convergence tolerance of 10^{-3} was applied, meaning that the algorithm was considered to have converged when the absolute difference between successive values of product flows and Lagrange multipliers was less than or equal to that threshold. The step-size parameter β in the algorithm was set to 0.15.

5.1 Example 7 - Baseline Example

The first example in this case study, Example 7, serves as the baseline scenario. The supply chain network topology is depicted in Figure 4. It features two firms, both based in the US. Firm 1 and Firm 2 represent stylized versions of Cargill and Archer Daniels Midland Company (ADM), two leading US soybean exporters with extensive domestic and international production networks (Rgultig (2025b) and US Import Data (2024)). Firm 1 operates three production sites, located across the US, Brazil, and Argentina, while Firm 2 manages two production sites in the US and Brazil, as labeled in Figure 4. The single demand market, Demand Market 1, represents China, reflecting its role as the world's largest soybean importer. The currency is US dollars.

The parameter values used in this example are motivated by recent labor market data. For production activities, the wage at US sites is set to \$17 per hour, based on the 2023 average US farm wage of \$17.55 per hour (Economic Research Service (2025)). In Brazil, the average farmworker earns 13.44 BRL per hour, equivalent to approximately \$3 per hour, while in Argentina the average farmworker wage is 2,882.87 ARS per hour, also about \$3 per hour (Salary Expert (2025)). For transportation activities, the hourly wage in the United States is set to \$22, reflecting the 2025 average wage for transportation workers of \$21.83 per hour (ZipRecruiter (2025)). In Brazil, transportation workers earn an average of 27.09 BRL per hour, equivalent to about \$5 per hour, and in Argentina the average transportation wage is 4,850.44 ARS per hour, equivalent to about \$4 per hour (Salary Expert (2025)).

Labor productivity is captured by setting: $\beta^i_j = 0.7, \forall i, j$, and $\beta^i_{jk} = 0.7, \forall i, j, k$, linking production and shipment volumes to labor hours. These values are informed by agricultural data showing that soybean and corn production require approximately 2.1 labor hours per acre, with an average yield of 1.27 metric tons per acre (equivalent to 1.39 tons per acre)(Langemeier (2022) and Good in Every Grain (2021)).

The corresponding upper bounds on labor hours are specified as 3×10^7 for all the transportation routes, 3×10^7 for the production sites located at Brazil, and 2×10^7 for the production sites at the US and Argentina. In Example 7, the ad valorem tariff rates on shipments from Firm 1's and Firm 2's production sites to the Demand Market 1 are equal to zero. The prices and costs are per ton and the units for the soybean shipments are tons.

The functional forms used in the numerical examples follow established conventions in supply chain network modeling and economic theory. The quadratic production and transportation cost functions capture the economic principle of increasing marginal costs as a higher volume of product increases per-unit costs of production and transportation (Li and Nagurney (2017), Nagurney, Besik, and Nagurney (2019) and Nagurney, Besik, and Yu (2018)). Additionally, the quadratic forms ensure that the marginal cost terms in the variational inequality formulation depend on the product shipment variable, which is used to obtain well-defined equilibrium solutions where optimal solutions balance marginal revenues and marginal costs. Similar quadratic functions for production and transportation costs have been employed in Nagurney, Besik, and Li (2019) in their oligopolistic soybean supply chain analysis. The demand price functions take the standard form of a monotone decreasing linear function with cross-price effects between competing firms' products. It is important to note that our model accommodates various functional forms beyond the quadratic specifications used in our examples, provided they satisfy the necessary conditions of continuity and convexity for the cost functions as well as the demand price functions being continuous and monotonically decreasing

The production cost functions of Firm 1 and Firm 2 at their production sites are:

$$\hat{f}_1^1(Q) = 10^{-5} {Q_{11}^1}^2 + 90, \quad \hat{f}_2^1(Q) = 4.3 \times 10^{-6} {Q_{21}^1}^2 + 80,$$

$$\hat{f}_3^1(Q) = 7.3 \times 10^{-5} {Q_{31}^1}^2 + 100, \quad \hat{f}_1^2(Q) = 1.8 \times 10^{-5} {Q_{11}^2}^2 + 90,$$

$$\hat{f}_2^2(Q) = 9 \times 10^{-6} {Q_{21}^2}^2 + 80.$$

The transportation cost functions for the shipments from Firm 1 and Firm 2's production sites to Demand Market 1 are:

$$\hat{c}_{11}^1(Q) = 2 \times 10^{-5} {Q_{11}^1}^2 + 40, \quad \hat{c}_{21}^1(Q) = 5 \times 10^{-6} {Q_{21}^1}^2 + 30,$$

$$\hat{c}_{31}^1(Q) = 6 \times 10^{-5} {Q_{31}^1}^2 + 45, \quad \hat{c}_{11}^2(Q) = 10^{-5} {Q_{11}^2}^2 + 30,$$

$$\hat{c}_{21}^2(Q) = 4 \times 10^{-6} {Q_{21}^2}^2 + 15.$$

The demand price functions for the products of Firm 1 and Firm 2 at Demand Market 1 in dollars are:

$$\rho_1^1(d) = -1.8 \times 10^{-5} d_1^1 - 10^{-5} d_1^2 + 950, \quad \rho_1^2(d) = -10^{-5} d_1^1 - 2.5 \times 10^{-5} d_1^2 + 1050.$$

Using the modified projection method, the equilibrium product flows, labor hours at the production sites and transportation routes, demand prices, demands, and profits are calculated and reported in Table 4.

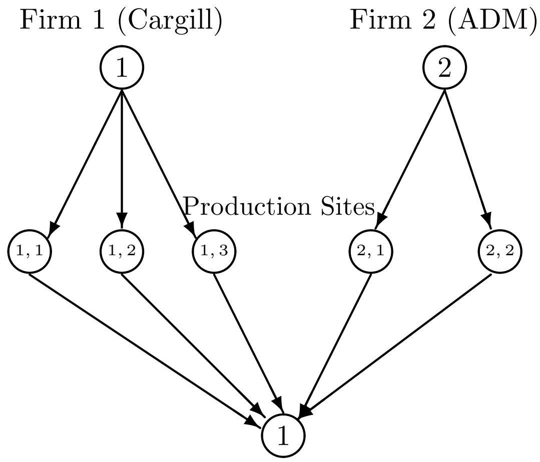

Figure 4: The Soybean Supply Chain Network Topology for Examples 7, 8, 9, and 10

Table 4: Equilibrium Solutions, Demand Prices, and Profits for Examples 7, 8, 9, and 10

	Example 7	**Example 8**	**Example 9**	**Example 10**
Q_{11}^{1*}	3,065,167.00	2,382,688.56	1,400,000.00	2,850,274.69
Q_{21}^{1*}	12,268,105.34	12,736,900.30	13,433,546.57	11,332,297.71
Q_{31}^{1*}	863,239.02	896,019.96	944,735.71	1,027,857.05
Q_{11}^{2*}	3,496,410.27	2,781,649.05	1,400,000.00	3,331,122.34
Q_{21}^{2*}	9,234,026.64	9,728,068.38	10,668,295.98	8,570,370.78
l_{11}^{1*}	4,378,810.01	3,403,840.80	2,000,000.00	4,071,820.99
l_{21}^{1*}	17,525,864.77	18,195,571.86	19,190,780.81	16,188,996.73
l_{31}^{1*}	1,233,198.61	1,280,028.51	1,349,622.44	1,468,367.22
l_{11}^{2*}	4,994,871.82	3,973,784.35	2,000,000.00	4,758,746.20
l_{21}^{2*}	13,191,466.63	13,897,240.55	15,240,422.83	12,243,386.84
l_1^{1*}	4,378,810.01	3,403,840.80	2,000,000.00	4,071,820.99
l_2^{1*}	17,525,864.77	18,195,571.86	19,190,780.81	16,188,996.73
l_3^{1*}	1,233,198.61	1,280,028.51	1,349,622.44	1,468,367.22
l_1^{2*}	4,994,871.82	3,973,784.35	2,000,000.00	4,758,746.20
l_2^{2*}	13,191,466.63	13,897,240.55	15,240,422.83	12,243,386.84
d_1^{1*}	16,196,511.37	16,015,608.82	15,778,282.28	15,210,429.45
d_1^{2*}	12,730,436.91	12,509,717.43	12,068,295.98	11,901,493.13
$\hat{\rho}_1^1$	531.16	536.62	545.31	557.20
$\hat{\rho}_1^2$	569.77	577.10	590.51	600.36
$\hat{U}^1$	6,502,623,987.76	6,265,508,717.39	6,507,263,407.57	5,933,698,880.38
$\hat{U}^2$	5,502,368,550.04	5,185,244,846.39	5,392,080,170.32	4,902,279,307.20

In Example 7, for both firms, exports from Brazil exceed those from the US, mirroring real-world trade patterns in which Brazil has surpassed the United States as China's top soybean supplier (Colussi et al. (2024)). The export quantities generated in the model closely align with actual trade data. As noted in industry reports, Cargill accounts for over 15% of US soybean exports (Soybean Market (2025)). Given that the US exported 22.13 million tons of soybeans to China in 2024 (CGTN News (2025)), the model's result for Cargill's export from its US site approximates this market share quite well. Similarly, the results for Cargill's shipments from Brazil and Argentina are consistent with 2024 exports, 74 million tons for Brazil (Soto (2025)) and 4.1 million tons for Argentina (NAFB News Service (2025)), assuming that Cargill maintains a comparable market presence in these countries.

ADM also generates soybean flows that are in a plausible range. While exact firm-level export data are unavailable, ADM is recognized as one of the top three companies in the global soybean sector alongside Cargill and Bunge (Soybean Market (2025) and US Import Data (2024)). Its simulated exports from the US and Brazil align with its important, though somewhat smaller, role in global soybean trade than Cargill. These computed export volumes align with recent empirical data. In 2024, Brazil accounted for 71% of China's soybean imports, while the US share declined to 21% (Hanrahan (2025)). In this example, exports from Brazil (production sites located in Brazil) represent 74.33% of total shipments to China, and US-based exports account for 22.68%.

In terms of labor allocation, Cargill employs substantially more labor at its production site at Brazil than at its site the US and in Argentina, consistent with its higher export volume and broader operational scale. Assuming 2,080 annual working hours per full-time employee (Food and Agriculture Organization (2024)), the modeled labor inputs correspond to approximately 2,105 workers at Cargill's US site, 8,425 workers at its Brazilian site, and 592 workers at its Argentine site. For ADM, the estimates translate to about 2,401 workers at its US site and 6,342 workers at its Brazilian site. According to Intellectual Market Insights (2025), ADM employs approximately 40,000 people worldwide, while Cargill is significantly larger, with a global workforce of around 155,000 employees. Both firms operate across the full soybean supply chain, from production to export, and maintain strong presences across the globe. Given this scale and infrastructure, the modeled workforce levels fall well within a plausible range for firms with extensive international operations. Labor hours available for each transportation route follow the same pattern.

At equilibrium, the model predicts that ADM's soybeans have a slightly higher price in China, \$569.77 per ton, than Cargill's, \$531.16 per ton, both of which are broadly consistent with the observed 2025 average Chinese import price of 463 dollars per ton (IndexBox (2025)). Despite this modest pricing advantage, Cargill earns substantially higher profits, driven by its scale of production and greater output levels. Notably, all Lagrange multipliers associated with labor constraints are zero in this example, indicating that no labor upper bounds are binding in the baseline example.

5.2 Example 8 - China's Retaliatory Ad Valorem Tariff on US Soybean Exports

In Example 8, the data are the same as in Example 7, except that we now examine the impact of a newly imposed 25% ad valorem tariff by China on US soybean imports, announced in April 2025 in direct response to US tariff measures earlier that month (uSMART (2025)). The supply chain network structure remains as in Example 7, but we incorporate a tariff on shipments from US production sites of both firms as follows:

$$\tau^1_{11} = \tau^2_{21} = 0.25.$$

The computed equilibrium solution, demand prices, and profits are reported in Table 4.

The equilibrium results reveal a striking shift in trade flows: exports from all US sites drop, and both Cargill and ADM reallocate their production to Brazil and Argentina, where no tariffs apply, consistent with reports that high Chinese tariffs on US soybeans have redirected product flows toward South American producers before, particularly Brazil and Argentina (Draper and Nicas (2025)).For Firm 1, exports from the US site drop by 22.26%, while shipments from the US site for Firm 2 decline by 20.44%. This result aligns with economic theory, which predicts that ad valorem tariffs reduce the competitiveness of affected exporters and encourage buyers to switch to lower-cost sources.

A similar pattern was observed in April 2025, when Chinese orders dropped by more than 97% in a single week, from 72,800 tons to just 1,800 tons. At the time, Chinese officials emphasized that US grain imports, including soybeans, could be easily substituted due to abundant global supply (Herzlich and Fickenscher (2025)). The 125% tariff imposed by China is expected to significantly raise the cost of US soybeans, rendering them uncompetitive and shifting demand to Brazilian and Argentine suppliers (Glauber, Gianatiempo, and Piñeiro (2025) and Cao and Thukral (2025)). The ad valorem tariff slightly raises equilibrium demand prices in China for both firms, to \$536.62 and \$577.10 per ton, respectively, as it reduces total quantities sold in the China for both firms.

The effects of the tariff extend to labor allocation as well. Labor hours at US production sites and their related transportation routes decrease as exports fall. As production shifts to Brazil and Argentina, labor hours increase accordingly: by 3.82% at Cargill's Brazilian site, by 3.79% at its Argentine site, and by 5.35% at ADM's Brazilian site. Transportation labor usage from these countries to China rises by the same proportion. Despite the reallocation, profits decline, by 3.64% for Firm 1 and by 5.76% for Firm 2. This impact aligns with recent real-world developments: following China's imposition of ad valorem tariffs on US soybeans, abundant supply and robust output from Brazil and other South American exporters helped to stabilize global prices and maintained trade flows (ADV and Prakash (2025)), thereby preventing significant profit losses. Ultimately, the ad valorem tariff reduces labor hours and soybean flows, along with the overall demand in China, while increasing demand prices for consumers. All Lagrange multipliers remain zero in this example, indicating that no labor upper bounds are binding at the equilibrium.

5.3 Example 9 - Immigration Policy and its Impact on Available Labor at the US Production Sites

Example 9 has the same data as that in Example 7, except that we now examine the impact of the current farm labor shortage in the US due to increased immigration enforcement. The US agricultural sector is heavily dependent on foreign labor, with approximately 86% of its workforce being foreign-born, and 45% comprising undocumented workers (Rosenbloom (2022)). In 2025, intensified immigration enforcement and the failure to renew working authorizations led to an estimated 70% of the agricultural workforce in key regions, especially California, stopping work (The Guardian (2025)). Labor shortages and enforcement actions targeting undocumented workers are resulting in crop losses and disruptions in harvest (Farmonaut (2025)).

To simulate the labor shortage, we reduce the available labor hours at both firms' US production sites to $\bar{l}_1^1 = \bar{l}_1^2 = 2 \times 10^6$ hours. The computed equilibrium solutions and the demand prices and profits are displayed in Table 4.

Under these conditions, the reduced available labor hours at US production sites significantly disrupt output for both firms. Relative to Example 7, Firm 1's shipments from the US to Demand Market 1 decline by 54.32%, while Firm 2's drop by 59.95%, both constrained by the binding labor upper bounds. Accordingly, the Lagrange multipliers at these sites are positive, with $\lambda_1^{1*} = 85.10$ and $\lambda_1^{2*} = 108.28$, and labor hours are reduced to the imposed upper bound of 2,000,000 hours. To compensate, both firms reallocate production to Brazil and Argentina: ADM increases its Brazilian exports by 15.53%, while Cargill's shipments from Brazil and Argentina rise by 9.49% and 9.44%, respectively. This production shift leads to corresponding increases in labor hours at these sites and along the associated transportation routes to the Demand Market 1.

These adjustments help to stabilize market conditions. Compared to Example 7, demand for Firm 1 and Firm 2's soybeans at the Demand Market 1 decreases slightly by 2.58% and 5.20%, respectively. As a result of reduced demand, demand prices increase marginally to \$545.31 and \$590.51 per ton. Despite the labor shortage in the US, profits decrease by only 2.00% for Firm 2 and increase by 0.07% for Firm 1, due to reallocation of production to unconstrained South American production sites. This example isolates the effect of labor shortage shocks, apart from ad valorem tariffs. The results demonstrate how even partial reductions in US farm labor without ad valorem tariffs can substantially reshape trade flows and labor allocation decisions, pushing production, in the case of soybeans, toward unconstrained sites in South America.

5.4 Example 10 - Combined Impact of Ad Valorem Tariffs Imposed on US and Brazilian Drought-Induced Production Cost Increases

In Example 10, we jointly examine the impact of climate-related disruptions on soybean production costs in Brazil and the imposition of a 25% ad valorem tariff on soybean shipments from the US to China. Brazil has faced some of its most severe drought conditions recently, with over one-third of the country affected. The adverse weather, marked by persistent dryness and high temperatures, has disrupted planting schedules and imposed significant stress on yields, particularly in southern states such as Mato Grosso and Rio Grande do Sul. These two states alone account for a large portion of Brazil's soybean output (Odiase (2024) and Samora (2025)). In March 2025, Mato Grosso state authorities reported increased cost estimates for the 2025-26 soybean production, citing higher prices for seeds and fertilizers (Giannetti (2025)). Adverse weather conditions, particularly drought, increase production costs through multiple mechanisms. First, drought stress reduces crop yields, requiring farmers to plant additional acreage or higher-density crops to meet production targets, thereby increasing seed costs per unit of output. As a result of yield reductions, the per-unit fixed costs increase, as the same equipment, land, and expenses are spread over fewer tons of output. Additionally, crops are also more vulnerable to pests and disease under drought, leading to higher spending on crop protection. To represent these pressures, we increase both the quadratic and fixed parameters in the production cost functions at the two Brazilian production sites. The other data remains identical to Example 7, except for the US ad valorem tariff parameters:

$$\tau_{11}^{1} = \tau_{21}^{2} = 0.25.$$

New production cost functions for Brazilian sites are:

$$\hat{f}_2^1(Q) = 7 \times 10^{-6} {Q_{11}^{1}}^{2} + 150, \quad \hat{f}_2^2(Q) = 1.3 \times 10^{-5} {Q_{11}^{1}}^{2} + 150.$$

Table 4 reports the computed equilibrium solution, the prices, and the profits for Example 10.

The simultaneous ad valorem tariff on US shipments and production cost increases in Brazil generate a more complex reallocation of trade flows than the individual disruption in Examples 7 and 8. For Firm 1, US exports are 19.62% higher than in Example 8 but remain 7.01% below the baseline in Example 7. Firm 2 follows a similar pattern, with US exports 19.75% higher than in Example 8 yet still 4.72% lower than in Example 7. This reflects that, despite the ad valorem tariff, firms continue to utilize US capacity to offset the stronger production cost disadvantages in Brazil, leading to higher US flows than in the tariff-only Example 8, but still below the baseline example. This adjustment matches expectations that Brazil's drought could delay planting and growing, thereby extending the US soybean export window in February 2025 (Ever Ag (2024)).

Exports for Firm 1 from Brazil drop by 7.62% below Example 7, due to the higher production costs. Similarly, for Firm 2, Brazilian exports are 7.18% below Example 7. These reductions closely mirror the impact of Brazil's 2022 drought, during which soybean output declined by 9.20% (Colussi, Schnitkey, and Zulauf (2022)). This trend also aligns with recent yield downgrades in Brazil, where the soybean yield forecast for Rio Grande do Sul was reduced from 57 to 52 bags per hectare (8.70%) by January 2025, with further downward revisions anticipated (Freitas (2025)).Shipments from Argentine rise steadily across all examples, reaching a 19.06% increase over Example 7.

This reallocation is also evident in the labor hours, which follow a similar pattern to the shipment changes. US production sites, despite the ad valorem tariffs, retain more labor than in Example 8 but less than in Example 7. Brazilian site labor hours decline to their lowest levels across all examples due to the increased production costs. In contrast, Argentina, benefiting from higher production and shipments, increases its labor hours at the production site. Labor hours along the transportation routes display the same pattern.

Compared to all previous scenarios, Example 10 yields the highest demand prices and the lowest demand quantities and profits. Firm 1's demand price reaches \$557.20 and Firm 2's: \$600.36, while demand quantities for both firms fall to their lowest levels across all examples. Profits are also at their lowest: Firm 1's profit decreases by 5.29% from Example 8 and by 8.74% from Example 7, while Firm 2's profit drops by 5.45% from Example 8 and by 10.90% from Example 7. All Lagrange multipliers remain zero in this example.

6. Summary and Conclusions

Global supply chain networks form the backbone of international trade by linking production sites to demand markets across countries via transportation. Some governments, increasingly, are attempting to alter such trade flows by instituting trade policies such as tariffs, which affect production and transportation decisions, and modify demand prices as well as profits. At the same time, labor availability is central to the functioning of supply chain networks and, yet, labor considerations have often been absent from trade policy models. The investigation of the impacts of ad valorem tariffs also on labor and, hence, employment, calls for rigorous methodological frameworks that can capture their joint effects within competitive global supply chains.

In this paper, we developed an oligopolistic global supply chain network model that explicitly incorporates ad valorem tariffs and labor constraints. Firms in the model have multiple production sites in different countries and ship products to multiple demand markets, with production and shipment activities requiring labor that is subject to site and route specific upper bounds. Wages can vary by location, and labor productivity factors translate labor input into output. Firms compete noncooperatively in product quantities and labor and seek to maximize their own profits. The governing Nash equilibrium conditions are formulated as a variational inequality problem. Then, through Lagrange analysis, an alternative variational inequality is derived, with nice features for algorithmic solution.

After providing illustrative numerical examples, which are solved analytically, we present a case study of the global soybean trade, focusing on the United States, Brazil, Argentina, and China. Numerical examples, computed via the modified projection method, which results in closed form expressions at each iteration for the product flows and the Lagrange multipliers, reveal the effects of the imposition of ad valorem tariffs, labor restrictions, and climate-related disruptions through changes in production cost functions. The computed equilibrium solutions show that ad valorem tariffs shift production and trade flows toward sources without ad valorem tariffs, alter labor allocation at production sites and along transportation routes, and increase demand prices for consumers. Labor constraints, whether from shortages or immigration policy restrictions, reduce product flows and profitability for the affected firms while creating competitive advantages for less constrained producers. When ad valorem tariffs and increased production cost disruptions occur together, their effects compound, leading to reduced trade flows, higher demand prices, and lower overall profits.

The results offer important policy and managerial insights. For policymakers, the results highlight that ad valorem tariffs not only influence prices and trade flows but also interact with labor in shaping competitive outcomes. Ad valorem tariffs can shift market share across countries but may also reduce total output and increase consumer demand prices, particularly when implemented widely. For firms, our model highlights the advantage of diversifying production locations to mitigate the effects of ad valorem tariffs and labor disruptions. An interesting direction for future research is to extend the model to incorporate specific minimum wages, which would impose lower bounds on wages at production sites and along transportation routes and directly influence labor allocation decisions. Another possible extension is to incorporate product quality as a decision variable alongside product flows and labor, with demand prices depending on both, and quality subject to minimum standards. In addition, the framework could be generalized to multitiered supply chain networks with intermediate suppliers, assembly processes, and storage. Another extension would be to integrate workforce optimization more explicitly with trade policies, capturing how decisions on workforce allocation, flexibility, and capacity adjustments interact with tariffs and affect overall supply chain performance. Finally, another promising direction is to examine how alternative production cost function specifications affect equilibrium behavior in oligopolistic supply chain networks with labor constraints and trade policies, such as tariffs. We leave such work for future research.

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No potential conflict of interest was reported by the authors.

Data availability statement

All data are included in the paper.

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