Global Supply Chain Networks and Tariff Rate Quotas: Equilibrium Analysis with Application to Agricultural Products

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Abstract:

In this paper, we develop a global supply chain network model in which profit-maximizing firms engage in competition in the production and distribution of products in the presence of quantitative trade policy instruments in the form of tariff rate quotas. Tariff rate quotas are two-tiered tariffs, in which a lower in-quota tariff is applied to the units of imports until a quota or upper bound is attained and then a higher over-quota tariff is applied to all subsequent imports. They are utilized to protect domestic producers in the case of a wide range of products, from agricultural ones to fabrics and even steel, and can be challenging to formulate. We construct the governing set of novel equilibrium conditions associated with the product flows and Lagrange multipliers, which correspond to quota rent equivalents, and derive the variational inequality formulation. Qualitative properties are presented along with an effective algorithm, which is then applied to compute solutions to numerical examples comprising an agricultural product case study on avocados and global trade. This work is the first to model and solve general, competitive supply chain network problems consisting of oligopolistic firms with multiple production sites and demand markets in multiple countries subject to tariff rate quotas.

Key words: supply chains; tariff rate quotas; global trade; networks; variational inequalities; agricultural products

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1. Introduction

Global supply chain networks, driven by profit-maximizing firms wishing to satisfy consumer demand within and across national boundaries, have made possible the production and wide distribution of goods, as varied as food and other agricultural products to textiles and apparel as well as aluminum and steel. The global product flows associated with supply chain networks underpinning world trade have also garnered the attention of government policy makers concerned with the highly competitive environment and possible effects on domestic firms. Examples of policy instruments that have been applied by governments to modify trade patterns, with aspirations of protecting domestic firms have included: tariffs, quotas, and a combination thereof - tariff rate quotas.

For example, a tariff rate quota (TRQ) is a two-tiered tariff, in which a lower in-quota tariff is applied to the units of imports until a quota or upper bound is attained and then a higher over-quota tariff is applied to all subsequent imports (World Trade Organization (2004)). To illustrate the magnitude of TRQs in practice, we note that the Uruguay Round in 1996 induced the creation of more than 1,300 new TRQs (cf. Skully (2001)). Currently, 43 World Trade Organization members have a total of 1,425 tariff quotas in their commitments (World Trade Organization (2018)). TRQs are widely utilized especially in agricultural trade (Manzo (2007)) for products such as: milk and dairy products, bananas, chocolate, sugar, beef, peanuts, eggs, poultry, soybeans, potatoes, among others. In fact, the world's four most important food crops: rice, wheat, corn, and bananas, have all been subject to tariff rate quotas (cf. de Melo (2015) and Gale (2017)).

In the present economic and political climate, tariffs, as well as tariff rate quotas, are garnering prominent attention in the news on world trade with even washing machines in the United States being subject to tariff rate quotas (cf. Office of the United States Trade Representative (2018)). The imposition of tariffs by certain countries, including the United States, is, in turn, leading to retaliation by other countries, such as China, with ramifications across multiple supply chains (cf. Watson (2018)). This new world order is commanding more attention from the research community to construct computable operational mathematical models that enable the assessment of the impacts of trade policy instruments such as tariff rate quotas on consumer prices, trade flows, as well as the profits of producers/firms.

To-date, the inclusion of tariff rate quotas into modeling frameworks that include multiple producers in different countries as well as multiple demand markets has been limited. This may be due, in part, to the observation that tariff rate quotas (TRQs) have been deemed challenging to formulate and only stylized examples have been reported, almost exclusively in a spatial price equilibrium framework (cf. Bishop et al. (2001)). Spatial price equilibrium models are perfectly competitive models, in that therein it is assumed that there are numerous producers, and such models date to Samuelson (1964) and Takayama and Judge (1964, 1971). For more recent treatments and applications of spatial price equilibrium models, utilizing, for example, variational inequality theory, see Nagurney (1999, 2006), Daniele (2004), Li, Nagurney, and Yu (2018), and the references therein). For the inclusion of tariffs rate quotas into spatial price equilibrium models using variational inequality theory, see Nagurney, Besik, and Dong (2019).

In many industrial sectors, however, in which tariff rate quotas are imposed, the more appropriate framework is that of imperfect competition, as in the case of oligopolistic competition (cf. Dafermos and Nagurney (1987), Nagurney and Matsypura (2005), Zhang (2006), Daniele (2010), Qiang et al. (2013), Yu and Nagurney (2013), Li and Nagurney (2017), Nagurney, Yu, and Besik (2017), and Saberi (2018) for some innovative general models of related supply chain network problems), since the number of firms, that is, "players" in the game is not very large. For example, as noted by Guyomard et al. (2005), the world banana market is dominated by a small number of firms and, hence, this raises the importance of addressing imperfect competition in the context of TRQs, which is the focus of this paper.

Shono (2001) relaxed the assumption of perfect competition, and incorporated TRQs, but under the assumption that all countries behave in the same oligopolistic manner and that the computable framework consisted of linear functions. Maeda, Suzuki, and Kaiser (2001, 2005) considered oligopolistic competition and TRQs but assumed that there is a single producer in each country and that it is faced with a separable cost function and that the demand function in each country is also separable, that is, the demand for a product in a country only depends on the price in that country. Furthermore, the transportation costs are assumed to be fixed. In the model in this paper, in contrast, we allow for multiple producers in a country, multiple demand markets in a country, as well as transportation costs that are flow-dependent and the production and demand price functions can depend on vectors of production and consumption volumes, respectively. In addition, we allow for the imposition of TRQs on groups of exporting countries. Moreover, unlike the previous authors, we identify the underlying supply chain network structure of the problem and utilize variational inequality theory for the modeling, qualitative analysis, and computations, rather than complementarity theory.

This paper is organized as follows. In Section 2, we first present the global supply chain network model, consisting of firms that seek to maximize their profits by determining how much of the product to manufacture/produce at the production sites, which can be located in multiple countries, along with the distribution of the product flows to the demand markets, also located in multiple countries, in the presence of tariff rate quotas. We construct the novel system of governing equilibrium conditions, consisting of the Nash equilibrium associated with the firm's profit-maximizing, noncooperative behavior, and the tariff rate quotas, along with the quota rent equivalents, which correspond to Lagrange multipliers, and derive the variational inequality formulation. We highlight how the global supply chain network model can be modified to incorporate related trade policy instruments such as unit tariffs or quotas. We then present several numerical examples for illustrative purposes.

In Section 3 we provide qualitative properties and, in Section 4, we propose a computational scheme, which yields closed form expressions at each iteration. A case study is presented in Section 5 on an agricultural application, focused on avocados, a very popular fruit in the United States, with growing consumer demand even in China. The case study consists of increasingly more general numerical examples, and demonstrates the applicability of the theoretical and computational framework for practice, along with the managerial insights. The results are summarized, along with conclusions and suggestions for future research, in Section 6.

2. The Global Supply Chain Network Model with Tariff Rate Quotas

In the global supply chain network model there are I firms, with a typical firm denoted by i, engaged in the production/manufacture of a homogeneous product and J countries, with a typical country denoted by j. Each firm i; $i = 1, \ldots, I$, has, at its disposal, n^i_j possible locations in country j; $j = 1, \ldots, J$, for the production of its product. Also, in each country j there are n_j demand markets, with the total global number of demand markets denoted by K. The network topology of the supply chain is depicted in Figure 1. In Figure 1 the topmost nodes denote the firms; the middle nodes denote the manufacturing/production site options, and the bottom tier nodes represent the global demand markets. Observe that the topmost nodes are enumerated as: $1, \ldots, I$. The middle tier nodes are delineated as: $(1, 1, 1), \ldots, (I, J, n^I_J)$, with middle tier node (i, j, p) denoting the combination of firm i, country j, and production site p. Finally, the bottom tier nodes in Figure 1 are delineated as: $(1, 1), \ldots, (J, n_J)$ with bottom tier node (j, k) corresponding to demand market k in country j. We also define $\mathcal{J}_i$ as the set of middle tier nodes associated with firm i, that is, nodes: $(i, 1, 1), \ldots, (i, J, n^i_J)$, with $\mathcal{J}$ being the set of all middle tier plant nodes. We further define $\mathcal{K}$ as the set of bottom tier demand market nodes. For simplicity, we refer to a typical middle tier production site (node) by h and a typical bottom tier demand market (node) by l. We enumerate, for simplicity, the former as: $1, \ldots, \sum_{i=1}^{I}\sum_{j=1}^{J} Kn^i_j$, including for the numerical examples.

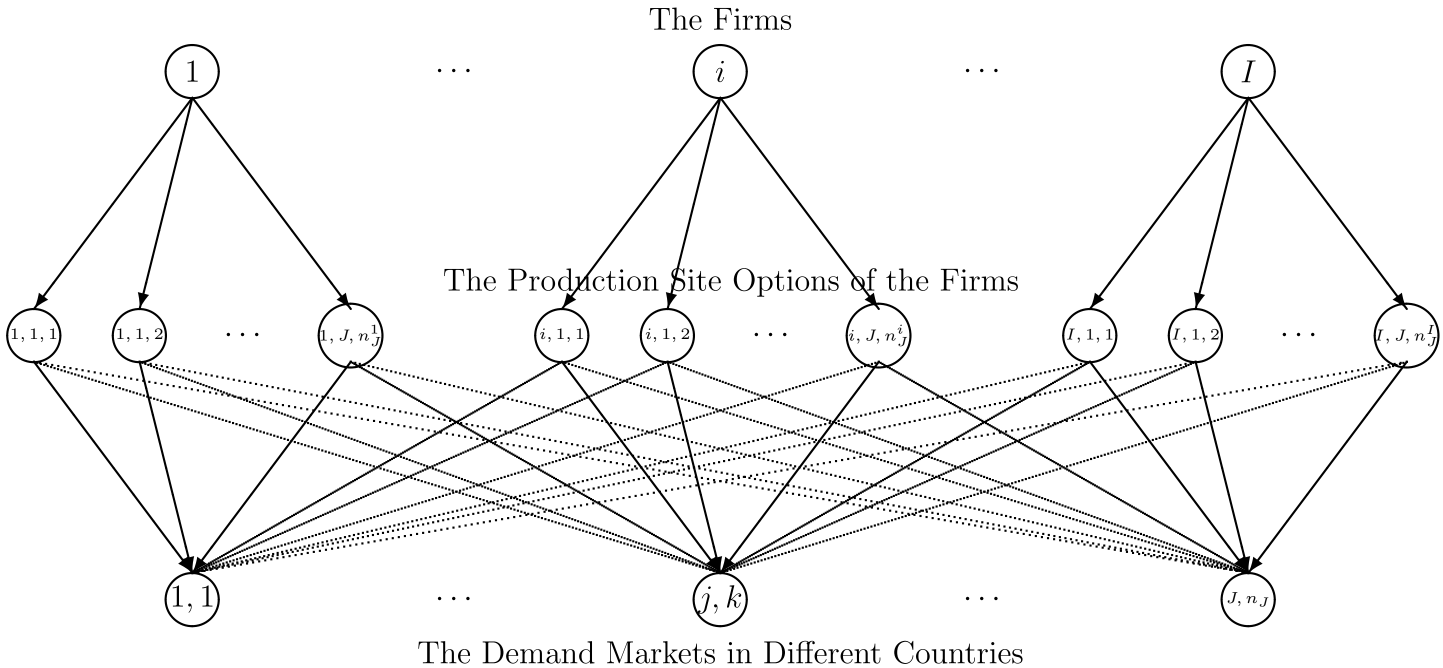

Figure 1: The Global Supply Chain Network with Multiple Firms, Multiple Production Sites in Different Countries, and Multiple Demand Markets

Each firm i; $i = 1, \ldots, I$, has, as its strategic variables, the vector of nonnegative product flows: Q_i where $Q_i = \{Q_{ihl}; h \in \mathcal{J}_i, l \in \mathcal{K}\}$, and Q_{ihl} denotes the volume of the product manufactured/produced by firm i at production site $h \in \mathcal{J}_i$ and then shipped to demand market l for consumption. We group the product flows for each firm i; $i = 1, \ldots, I$ into the vector $Q_i \in R_+^{K \sum_{j=1}^J n_j^i}$. We then group the product flows of all the firms into the vector $Q \in R_+^{\sum_{i=1}^I \sum_{j=1}^J K n_j^i}$. Note that, in the case of an agricultural product, a production site option would correspond to a farm.

In the model, all the costs and prices, as well as tariffs and economic rents associated with the tariff rate quotas, are assessed in a common currency.

Each firm i; $i = 1, \ldots, I$, is faced with a production cost function f_{ih} associated with manufacturing the product at h where we have that:

$$f_{ih} = f_{ih}(Q), \quad \forall h \in \mathcal{J}_i. \tag{1}$$

The production functions are assumed to be convex and continuously differentiable. These production cost functions are associated with the respective links (cf. Figure 1) joining the top nodes with the middle tier nodes.

In addition, each firm i; $i = 1, \ldots, I$, encumbers a transportation cost c_{ihl} associated with transporting the product from production site node h to demand market node l, where

$$c_{ihl} = c_{ihl}(Q), \quad \forall h \in \mathcal{J}_i, \forall l \in \mathcal{K}. \tag{2}$$

The transportation cost functions are also assumed to be convex and continuously differentiable. Observe from (1) and (2) that both the production costs as well as the transportation costs can depend, in general, not only on the particular firm's product flows, but also on those of the other firms. This feature enhances the modeling of competition for resources. The transportation cost functions are associated with the respective links joining the second tier nodes in Figure 1 with the bottom tier nodes.

The demand at each demand node l; $\forall l \in \mathcal{K}$, is denoted by d_l and must satisfy the following conservation of flow equation:

$$\sum_{i=1}^{I} \sum_{h \in \mathcal{J}_i} Q_{ihl} = d_l, \tag{3}$$

that is, the demand for the product at a demand market is equal to the sum of the product flows from all the firms production sites in all the countries to the demand market. We group the demands into the vector $d \in R^K_+$.

The consumers, located at the demand markets, reflect their willingness to pay for the product through the demand price functions ρ_l, $\forall l \in \mathcal{K}$, with these functions being expressed as:

$$\rho_l = \rho_l(d). \qquad (4a).$$

The demand price functions, in turn, are assumed to be continuous and monotone decreasing. Observe that, according to (4a) the price of the product at a demand market in a country can depend on the demands for the product not only at that demand market but also on the demands for the product at other demand markets at the same or other countries.

In view of (3), we can redefine the demand price functions (4a) as follows:

$$\hat{\rho}_l = \hat{\rho}_l(Q) \equiv \rho_l(d), \quad \forall l \in \mathcal{K}. \qquad (4b)$$

We now introduce the notation associated with the tariff rate quotas. We first define the groups G_g; $g = 1, \ldots, n_G$, consisting of the middle tier nodes $\{h\}$ corresponding to the production sites in the countries from which imports are to be restricted under the tariff quota regime and the demand markets $\{l\}$ in the country that is imposing the tariff rate quota. For typical groups, denoted by G_s and G_r, we have that $G_s \cap G_r = \emptyset, \forall s \neq r$, $\forall s, r$. The quota associated with group G_g is denoted by $\bar{Q}_g$; $g = 1, \ldots, n_G$. Note that our framework allows for multiple production sites in multiple countries to be associated with a specific group and the same holds for all the demand markets in a country. Associated with each group G_g is an under-quota tariff $\tau^u_{G_g}$ and an over-quota tariff $\tau^o_{G_g}$, where $\tau^u_{G_g} < \tau^o_{G_g}$.

Let λ_{G_g}, for all G_g, denote the quota rent equivalent (see, e.g., Skully (2001)), which has an interpretation of a Lagrange multiplier, and which we elaborate upon in the global supply chain network equilibrium conditions below. We group the quota rent equivalents into the vector $\lambda \in R^{n_G}$.

We further define $\mathcal{I}^i$ as the set of groups G_g that firm i belongs to.

The Utility Functions of the Global Firms

We now define the utility functions associated with the firms which capture profit maximization. We distinguish the utility functions of firms with production sites in one or more countries belonging to a group, and subject to a tariff rate quota, and those firms without production sites associated with tariff rate quotas. For a firm i in the former category, we define the utility function U_i^G as

$$U_i^G = \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} \hat{\rho}_l(Q) Q_{ihl} - \sum_{h \in \mathcal{J}_i} f_{ih}(Q) - \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} c_{ihl}(Q) - \sum_{G_g \in \mathcal{I}^i} (\tau^u_{G_g} + \lambda^*_{G_g}) \sum_{(h,l) \in G_g} Q_{ihl}, \quad (5a)$$

where $\lambda^*_{G_g}$ is the equilibrium economic rent equivalent for group G_g and it assumes values as in Definition 1 below, and, for a firm i in the latter category, we define its utility function U_i, as

$$U_i = \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} \hat{\rho}_l(Q) Q_{ihl} - \sum_{h \in \mathcal{J}_i} f_{ih}(Q) - \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} c_{ihl}(Q). \quad (5b)$$

We then define $\hat{U}_i \equiv U_i^G$ for all firms i with plants associated with groups and $\hat{U}_i \equiv U_i$ for all firms without plants in countries subject to tariff rate quotas. Observe that firms with objective functions of the form (5b) are not subject to tariff quotas and associated additional costs.

The optimization problem faced by each firm i; $i = 1, \ldots, I$, in the noncooperative game is, hence,

$$\text{Maximize} \quad \hat{U}_i(Q, \lambda^*) \quad (6)$$

subject to:

$$Q_{ihl} \geq 0, \quad \forall h \in \mathcal{J}_i, \forall l \in \mathcal{K}. \quad (7)$$

Definition 1: Global Supply Chain Network Equilibrium Under Tariff Rate Quotas

A product flow pattern Q^* and quota rent equivalent λ^* is a global supply chain network equilibrium under tariff rate quotas if, for each firm i; $i = 1, \ldots, I$, the following conditions hold:

$$\hat{U}_i(Q_i^*, Q_{-i}^*, \lambda^*) \geq \hat{U}_i(Q_i, Q_{-i}^*, \lambda^*), \quad \forall Q_i \in K_i, \tag{8}$$

where $Q_{-i}^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*)$, and $K_i \equiv \{Q_i | Q_i \in R_+^{\sum_{j=1}^J K n_j^i}\}$

and for all groups G_g:

$$\lambda_{G_g}^* \begin{cases} = \tau_{G_g}^o - \tau_{G_g}^u, & \text{if} \quad \sum_{i=1}^I \sum_{(h,l)\in G_g} Q_{ihl}^* > \bar{Q}_{G_g}, \\ \leq \tau_{G_g}^o - \tau_{G_g}^u, & \text{if} \quad \sum_{i=1}^I \sum_{(h,l)\in G_g} Q_{ihl}^* = \bar{Q}_{G_g}, \\ = 0, & \text{if} \quad \sum_{i=1}^I \sum_{(h,l)\in G_g} Q_{ihl}^* < \bar{Q}_{G_g}. \end{cases} \tag{9}$$

Objective function (5a), along with the system (9), captures the two-tiered structure of tariff rate quotas in that, if the quota for a group G_g is not attained, then only an under-quota tariff is encumbered by the firms with plants and demand markets in the group; if the quota is exceeded, then the over-quota tariff is encumbered, and, if the imports are exactly equal to the quota, then the additional cost to the firms in the group is less than the over-quota tariff.

We define the feasible set $\bar{K} \equiv \prod_{i=1}^I K_i$ and the feasible set $\mathcal{H} \equiv \{(Q, \lambda) | Q \in \bar{K}, \lambda \in R_+^{n_G} | 0 \leq \lambda_{G_g} \leq \tau_{G_g}^o - \tau_{G_g}^u, \forall g\}$.

Under the above imposed assumptions, the firms' utility functions are concave and continuously differentiable. Hence, we have the following theorem.

Theorem 1: Variational Inequality Formulation of the Global Supply Chain Network Equilibrium Under Tariff Rate Quotas

A product flow and quota rent equivalent pattern $(Q^*, \lambda^*) \in \mathcal{H}$ is a global supply chain network equilibrium under tariff rate quotas according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^I \sum_{h\in\mathcal{J}_i} \sum_{l\in\mathcal{K}} \frac{\partial \hat{U}_i(Q^*, \lambda^*)}{\partial Q_{ihl}} \times (Q_{ihl} - Q_{ihl}^*)$$

$$+\sum_g \left[\bar{Q}_{G_g} - \sum_{i=1}^I \sum_{(h,l)\in G_g} Q_{ihl}^*\right] \times \left[\lambda_{G_g} - \lambda_{G_g}^*\right] \geq 0, \quad \forall (Q, \lambda) \in \mathcal{H}. \tag{10}$$

Proof: Under the assumptions that the utility functions are concave and continuously differentiable, we know that, following Gabay and Moulin (1980); see also Nagurney (1999), Nash equilibria (1950, 1951) the above form (8) can be formulated as the solution to the variational inequality: determine $Q^* \in \bar{K}$, such that:

$$-\sum_{i=1}^{I}\sum_{h\in\mathcal{J}_i}\sum_{l\in\mathcal{K}}\frac{\partial \hat{U}_i(Q^*,\lambda^*)}{\partial Q_{ihl}}\times(Q_{ihl}-Q^*_{ihl})\geq 0,\quad \forall Q\in\bar{K}. \tag{11}$$

System (9), in turn, implies that for $\lambda^*_{G_g}$, such that $0\leq\lambda^*_{G_g}\leq\tau^o_{G_g}-\tau^u_{G_g}$:

$$\left[\bar{Q}_{G_g}-\sum_{i=1}^{I}\sum_{(h,l)\in G_g}Q^*_{ihl}\right]\times\left[\lambda_{G_g}-\lambda^*_{G_g}\right]\geq 0,\quad \forall\lambda_{G_g}\,\text{such that}\,0\leq\lambda_{G_g}\leq\tau^o_{G_g}-\tau^u_{G_g}. \tag{12a}$$

Indeed, according to (9), if $\lambda^*_{G_g}=\tau^o_{G_g}-\tau^u_{G_g}$, then the expression before the multiplication sign in (12a) is negative, whereas that after the multiplication sign is less than equal to zero and, thus, (12a) holds. If $\lambda^*_{G_g}=0$, then according to (9) the first term in (12a) is nonnegative and, due to feasibility, so is the second term; therefore, their product is nonnegative and (12a) also holds. Finally, if $\sum_{i=1}^{I}\sum_{(h,l)\in G_g}Q^*_{ihl}=\bar{Q}_{G_g}$, then, according to (9), the term to the right-hand-side of the multiplication sign in (12a) can be positive or negative, but the multiplication of both terms is still equal to zero and (12a) holds.

The inequality (12a) holds for any g, and, therefore, we can conclude that summation over all g yields (12b):

$$\sum_g\left[\bar{Q}_{G_g}-\sum_{i=1}^{I}\sum_{(h,l)\in G_g}Q^*_{ihl}\right]\times\left[\lambda_{G_g}-\lambda^*_{G_g}\right]\geq 0,\quad \forall\lambda_{G_g}\in R^{n_G}_+\,\text{such that}\,0\leq\lambda_{G_g}\leq\tau^o_{G_g}-\tau^u_{G_g},\forall g. \tag{12b}$$

Summation of (11) and (12b) gives us variational inequality (10) and necessity is established.

We now prove sufficiency, that is, a solution $(Q^*,\lambda^*)\in\mathcal{H}$ of variational inequality (10) also satisfies the global supply chain network equilibrium conditions (8) and (9) of Definition 1. The equivalence to (8) has already been proven above.

Assume that a pattern $(Q^*,\lambda^*)\in\mathcal{H}$ satisfies variational inequality (10). Setting $Q_{ihl}=Q^*_{ihl}$, $\forall i,h,l$, and $\lambda_{G_g}=\lambda^*_{G_g}$ for all $g\neq r$, substitution of the resultants into variational inequality (10), yields:

$$\left[\bar{Q}_{G_r}-\sum_{i=1}^{I}\sum_{(h,l)\in G_r}Q^*_{ihl}\right]\times\left[\lambda_{G_r}-\lambda^*_{G_r}\right]\geq 0,\quad \forall\lambda_{G_r}\text{ such that }0\leq\lambda_{G_r}\leq\tau^o_{G_r}-\tau^u_{G_r} \tag{13}$$

and (13) implies that equilibrium conditions (9) must hold for any r. The proof is complete. $\square$

We now put variational inequality (10) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{L} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{L}, \tag{14}$$

where X and $F(X)$ are $\mathcal{N}$-dimensional vectors, $\mathcal{L}$ is a closed, convex set, and F is a given continuous function from $\mathcal{L}$ to $R^{\mathcal{N}}$.

Indeed, we can define $X \equiv (Q, \lambda)$ and $F(X) \equiv (F_1(X), F_2(X))$, where $F_1(X)$ consists of $\sum_{i=1}^{I} \sum_{j=1}^{J} Kn_j^i$ elements: $-\frac{\partial \hat{U}_i(Q,\lambda)}{\partial Q_{ihl}}$ for all i, h, l, and $F_2(X)$ consists of n_G elements, with the g-th element given by: $\left[\bar{Q}_{G_g} - \sum_{i=1}^{I} \sum_{(h,l) \in G_g} Q_{ihl}\right]$. Also, here $\mathcal{N} = \sum_{i=1}^{I} \sum_{j=1}^{J} Kn_j^i + n_G$ and $\mathcal{L} \equiv \mathcal{H}$.

In the absence of tariff rate quotas, all the firms are faced with utility functions of the form (5b) and the following corollary is immediate.

Corollary 1: Variational Inequality Formulation for Global Supply Chain Network Without Tariff Rate Quotas

In the absence of tariff rate quotas, the equilibrium of the resulting global supply chain network model collapses to the solution of the variational inequality: determine $Q^* \in \bar{K}$, satisfying:

$$-\sum_{i=1}^{I} \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} \frac{\partial U_i(Q^*)}{\partial Q_{ihl}} \times (Q_{ihl} - Q_{ihl}^*) \geq 0, \quad \forall Q \in \bar{K}. \tag{15}$$

Remark 1: Unit Tariffs

In this paper, the focus is on the complex policy regime of tariff rate quotas in an oligopolistic setting. We note that the above framework can be adapted to handle the simpler trade policy of unit tariffs, with firms then facing utility functions of the form (5b) with an appended term: $-\sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} \tau_{hl} Q_{ihl}$, where τ_{hl} denotes the unit tariff assessed on a product flow from h to l, with $\tau_{hl} = 0$, if h, l corresponds to a production site and demand market pair in countries not under a tariff, and with identical country pair nodes assessed the same positive tariff. Variational inequality (15) would then apply with the utility functions U_i; $i = 1, \ldots, I$, modified as above.

Remark 2: Strict Quotas

On the other hand, if there is a strict quota regime, then for those firms i that are affected, the utility function U_i^G in (5a) is modified to U_i^Q as:

$$U_i^Q = \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} \hat{\rho}_l(Q) Q_{ihl} - \sum_{h \in \mathcal{J}_i} f_{ih}(Q) - \sum_{h \in \mathcal{J}_i} \sum_{l \in \mathcal{K}} c_{ihl}(Q) - \sum_{G_g \in \mathcal{I}^i} \lambda^*_{G_g} \sum_{(h,l) \in G_g} Q_{ihl}, \tag{16}$$

where the groups G_g, $\forall g$, now correspond to those node pairs under strict quotas.

The Nash Equilibrium conditions (8) are still relevant but the system (9) is replaced with the system below: for all groups G_g:

$$\bar{Q}_{G_g} - \sum_{i=1}^{I} \sum_{(h,l) \in G_g} Q^*_{ihl} \begin{cases} = 0, & \text{if} \quad \lambda^*_{G_g} > 0, \\ \geq 0, & \text{if} \quad \lambda_{G_g} = 0. \end{cases} \tag{17}$$

Letting $\hat{U}_i \equiv U_i^Q$, for all i under strict quotas, the governing variational inequality remains as in (10) but with a new feasible set: $\mathcal{H}^1 \equiv \{(Q, \lambda) | Q \in K, \lambda \in R_+^{n_G}, \forall g\}$. The interpretation of the $\lambda^*_{G_g}$, which were economic rents in the tariff rate quota global supply chain network model, clearly correspond to Lagrange multipliers associated with the constraints on the product imports in the form of strict quotas.

2.1 Illustrative Examples

We now, for illustrative purposes, present the following examples, which can be easily solved. The first numerical example is a supply chain network example without tariff rate quotas, for which the variational inequality (15) applies. For the second example, we impose a tariff rate quota and associated tariffs on the first example to demonstrate the changes in trade flows, demand market prices, as well as firm profits. We then raise the over-quota tariff in the third example. The equilibrium solutions to the second and third examples satisfy variational inequality (10). The examples have the underlying supply chain network topology depicted in Figure 2. Specifically, there are two firms. The first firm has the option of producing in a domestic plant, denoted by node $(1, 1, 1)$ whereas the second firm has the option of producing in a nondomestic plant, denoted by node $(2, 2, 1)$. There is a single domestic demand market denoted by node $(1, 1)$.

Note that, as described earlier, according to Figure 2, we enumerate the middle tier nodes (the production sites) as 1 and 2 with 1 corresponding to the first production site node $(1, 1, 1)$ and 2 corresponding to the second production site node $(2, 2, 1)$. Hence, we are interested in determining the equilibrium product flows Q^*_{111} and Q^*_{221}.

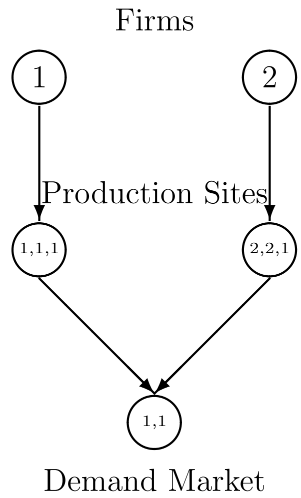

Figure 2: The Global Supply Chain Network Topology for Illustrative Examples

The production cost functions for Firms 1 and 2, respectively, are:

$$f_{11}(Q) = 2Q_{111}^2 + Q_{111} + 20, \quad f_{22}(Q) = Q_{221}^2 + Q_{221} + 10.$$

The transportation cost functions for the first firm and the second firm, are, respectively:

$$c_{111}(Q) = .1Q_{111}^2 + .5Q_{111}, \quad c_{221}(Q) = .5Q_{221}^2 + Q_{221}.$$

The demand price function is:

$$\rho_1(d) = -d_1 + 1000,$$

or, equivalently:

$$\hat{\rho}_1(Q) = -(Q_{111} + Q_{221}) + 1000.$$

The utility functions of the two firms, in the absence of tariff rate quotas, are:

$$\hat{U}_1 = U_1 = \hat{\rho}_1(Q)Q_{111} - f_{11}(Q) - c_{111}(Q), \quad \hat{U}_2 = U_2 = \hat{\rho}_1(Q)Q_{221} - f_{22}(Q) - c_{221}(Q).$$

The governing equilibrium conditions can, hence, be formulated as the variational inequality problem (15). Furthermore, since it is reasonable that both product trade flows will be positive, substitution of the above data into the marginal utility expressions, yields:

$$-\frac{\partial U_1(Q^*)}{\partial Q_{111}} = 6.2Q_{111}^* + Q_{221}^* - 998.50 = 0$$

and also

$$-\frac{\partial U_2(Q^*)}{\partial Q_{221}} = Q_{111}^* + 5Q_{221}^* - 998.00 = 0,$$

which correspond to the following system of equations:

$$6.2Q_{111}^* + Q_{221}^* = 998.50$$

$$Q_{111}^* + 5Q_{221}^* = 998.00,$$

with solution: $Q_{111}^* = 133.15$ and $Q_{221}^* = 172.97$. The equilibrium demand, hence, is 306.12 and the incurred equilibrium price at the demand market, ρ_1, is: 693.88. The profits of the firms are: $U_1 = 54,939.66$ and $U_2 = 74,786.55$.

Observe from the above equilibrium solution that the second firm, with a nondomestic plant, has a volume of exports of the product to the demand market that is approximately 30% higher than that of Firm 1, the domestic plant.

We now impose a tariff rate quota and the associated under-quota and over-quota tariffs as follows. There is a single group G_1 consisting of the nondomestic plant and the demand market with: $\bar{G}_1 = 100$ and $\tau_{G_1}^u = 50$ and $\tau_{G_1}^o = 200$. The governing variational inequality is now the one in (10).

Firm 1 retains its utility function $\hat{U}_1 = U_1$ of the form (5b), whereas Firm 2's utility function $\hat{U}_2$ is now U_2^G, of the form (5a). Hence, we know, assuming positive trade flows, which is reasonable, that

$$-\frac{\partial U_1(Q^*)}{\partial Q_{111}} = 6.2Q_{111}^* + Q_{221}^* - 998.50 = 0,$$

whereas

$$-\frac{\partial U_2(Q^*, \lambda^*)}{\partial Q_{221}} = Q_{111}^* + 5Q_{221}^* + \lambda_{G_1}^* + \tau_{G_1}^u - 998.00 = 0.$$

We also know that the equilibrium system (9) must hold.

We postulate that $\lambda_{G_1}^* = \tau_{G_1}^o - \tau_{G_1}^u = 150$, in which case $Q_{221}^* > \bar{Q}_{G_1} = 100$. The above two equations then simplify to:

$$6.2Q_{111}^* + Q_{221}^* = 998.50$$

$$Q_{111}^* + 5Q_{221}^* = 798.00,$$

whose solution yields the equilibrium trade flow pattern: $Q_{111}^* = 139.82$ and $Q_{221}^* = 131.64$, resulting in demand of: 271.45 and a demand market price: $\rho_1 = 728.54$. Observe that Firm 2 exports above the quota and, hence, encumbers the over-quota tariff. Also, note that the demand price increases, which has a negative effect on consumers. Firm 1 with the domestic production site benefits in terms of a higher product flow to the demand market, and an increased utility (profit) of: $U_1 = 60,580.50$ while the imports from Firm 2's nondomestic production site are greatly reduced and its utility $U_2^G = 43,310.09$. Hence, Firm 1 gains, and Firm 2 loses under the tariff rate quota regime, with consumers also losing in that the price of the product increases from 693.88 to 728.54 at the demand market. The domestic government, which imposed the tariff rate quota, garners 26,328.00 in total tariff proceeds.

We now raise the over-quota tariff $\tau^o_{G_1}$ from 200 to 400 and keep the remainder of the data as in the immediately preceding example. The new equilibrium solution is: $Q^*_{111} = 144.92$ and $Q^*_{221} = 100.00$, which is the quota, with $\lambda^*_{G_1} = 303.08$. It is easy to verify that, indeed, equilibrium conditions (8) and (9) are satisfied. The equilibrium demand is now: 244.92 with a demand price $\rho_1 = 755.08$. The consumers are now faced with an even higher price for the product at the demand market.

3. Qualitative Properties

In this Section we first provide existence results for the solution of variational inequality (10) in standard form (14). We then discuss qualitative properties of the function $F(X)$ in (14) required for convergence of the algorithm presented in Section 4 that is applied to compute solutions to numerical examples comprising the case study in Section 5.

Since the feasible set $\mathcal{H}$ is not compact, existence of a solution X^* is not guaranteed from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)) even though $F(X)$ is continuous under our imposed assumptions. Nevertheless, we have existence under not unreasonable assumptions as in the following theorem.

Theorem 2: Existence of a Solution X^* to (14)

Existence of a solution X^* to the variational inequality governing the global supply chain network model with tariff rate quotas given by (14); equivalently, (10), is guaranteed.

Proof: Clearly, the quota rent equivalents comprising the vector λ are bounded because of the definition of the feasible set $\mathcal{H}$. The demands at the demand markets in the various countries, in turn, are bounded since the demand prices are monotonically decreasing and negative prices are not relevant. Furthermore, although demands may be large they will not be infinite because the demand is bounded at each demand market in each country. Consequently, the product trade flows are also bounded and, therefore, existence of a solution X^* is guaranteed. $\square$

In the following proposition we provide a condition for $F(X)$ in (14) to be monotone.

Proposition 1: Monotonicity of $F(X)$ in (14)

$F(X)$ in (14) is monotone, that is,

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{L}, \tag{18}$$

if and only if $\hat{F}(X)$ is monotone, where the (i,h,l)-component of $\hat{F}(X)$, $\forall i,h,l$, consists of

$$\left[\sum_{j \in \mathcal{J}_i} \frac{\partial f_{ij}(Q)}{\partial Q_{ihl}} + \sum_{j \in \mathcal{J}_i} \sum_{k \in \mathcal{K}} \frac{\partial c_{ijk}(Q)}{\partial Q_{ihl}} - \hat{\rho}_l(Q) - \sum_{j \in \mathcal{J}_i} \sum_{k \in \mathcal{K}} \frac{\partial \hat{\rho}_k(Q)}{\partial Q_{ihl}} Q_{ijk} \right]. \tag{19}$$

Proof: In constructing $\langle F(X^1) - F(X^2), X^1 - X^2 \rangle$ with $F(X)$ as defined following (14), algebraic simplification yields:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle$$

$$= \sum_{i=1}^{I} \sum_{h \in \mathcal{J}_i} \sum_{l} \left[\sum_{j \in \mathcal{J}_i} \frac{\partial f_{ij}(Q^1)}{\partial Q_{ihl}} + \sum_{j \in \mathcal{J}_i} \sum_{k \in \mathcal{K}} \frac{\partial c_{ijk}(Q^1)}{\partial Q_{ihl}} - \hat{\rho}_l(Q^1) - \sum_{j \in \mathcal{J}_i} \sum_{k \in \mathcal{K}} \frac{\partial \hat{\rho}_k(Q^1)}{\partial Q_{ihl}} Q^1_{ijk} \right]$$

$$- \sum_{i=1}^{I} \sum_{h \in \mathcal{J}_i} \sum_{l} \left[\sum_{j \in \mathcal{J}_i} \frac{\partial f_{ij}(Q^2)}{\partial Q_{ihl}} + \sum_{j \in \mathcal{J}_i} \sum_{k \in \mathcal{K}} \frac{\partial c_{ijk}(Q^2)}{\partial Q_{ihl}} - \hat{\rho}_l(Q^2) - \sum_{j \in \mathcal{J}_i} \sum_{k \in \mathcal{K}} \frac{\partial \hat{\rho}_k(Q^2)}{\partial Q_{ihl}} Q^2_{ijk} \right] \times \left[Q^1_{ihl} - Q^2_{ihl} \right] \tag{20}$$

and the conclusion follows. □

Definition 3: Lipschitz Continuity

A function $F(X)$ is Lipschitz continuous on $\mathcal{L}$ if the following condition holds:

$$\|F(X') - F(X'')\| \leq L \|X' - X''\|, \quad \forall X', X'' \in \mathcal{L}, \tag{21}$$

where $L > 0$ is known as the Lipschitz constant.

For convergence of the algorithmic scheme described in the next Section, which we utilize for computational purposes in our case study, only monotonicity and Lipschitz continuity of $F(X)$ are required, provided that a solution exists.

4. The Algorithm

In this Section, we present the algorithm to solve the variational inequality (14), equivalently, (10), governing the global supply chain network model, with oligopolistic competition, and with tariff rate quotas and its variants described in Section 2. The algorithm that we utilize for the computation of the product trade flow and economic rent equivalent equilibrium pattern is the modified projection method (see Korpelevich (1977)). The steps of the algorithm are:

Step 0: Initialization

Initialize with $X^0 \in \mathcal{L}$. Set $t := 1$ and select β, such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant (cf. (21)) for function F in the variational inequality problem.

Step 1: Construction and Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{L}. \tag{22}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{L}. \tag{23}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, for $\epsilon > 0$, a specified tolerance, then, stop; otherwise, set $t := t + 1$ and go to Step 1.

The feasible set of variational inequality (10) is very simple; in particular, it is of box-type. Hence, both Steps 1 and 2 of the modified projection method above (cf. (22) and (23)) result in closed form expressions for the product trade flows as well as the economic rent equivalents at each iteration, as given below.

Closed Form Expressions for the Product Trade Flows and Economic Rent Equivalents at an Iteration

We now present the closed form expressions for the solution of the variational inequality subproblems (22). Similar ones for (23) can be analogously constructed. The closed form expressions for the product trade flows at iteration t are:

For each Q_{ihl} with (h,l) associated with a group G_g, $\forall g$, compute:

$$\bar{Q}^t_{ihl} = \max\{0, \beta(-\sum_{j\in\mathcal{J}_i}\frac{\partial f_{ij}(Q^{t-1})}{\partial Q_{ihl}} - \sum_{j\in\mathcal{J}_i}\sum_{k\in\mathcal{K}}\frac{\partial c_{ijk}(Q^{t-1})}{\partial Q_{ihl}} + \hat{\rho}_l(Q^{t-1})$$

$$+\sum_{j\in\mathcal{J}_i}\sum_{k\in\mathcal{K}}\frac{\partial \hat{\rho}_k(Q^{t-1})}{\partial Q_{ihl}}Q^{t-1}_{ijk} - \tau^u_{G_g} - \lambda^{t-1}_{G_g}) + Q^{t-1}_{ihl}\}, \tag{24a}$$

and for each Q_{ihl} with (h,l) not associated with a tariff rate quota group, compute:

$$\bar{Q}^t_{ihl} = \max\{0, \beta(-\sum_{j\in\mathcal{J}_i}\frac{\partial f_{ij}(Q^{t-1})}{\partial Q_{ihl}} - \sum_{j\in\mathcal{J}_i}\sum_{k\in\mathcal{K}}\frac{\partial c_{ijk}(Q^{t-1})}{\partial Q_{ihl}} + \hat{\rho}_l(Q^{t-1})$$

$$+\sum_{j\in\mathcal{J}_i}\sum_{k\in\mathcal{K}}\frac{\partial \hat{\rho}_k(Q^{t-1})}{\partial Q_{ihl}}Q^{t-1}_{ijk}) + Q^{t-1}_{ihl}\}. \tag{24b}$$

The closed form expression for the quota rent equivalent for group G_g; $g = 1, \ldots, n_G$, is:

$$\bar{\lambda}^t_{G_g} = \max\{0, \min\{\beta(\sum_{i=1}^{I}\sum_{(h,l)\in G_g} Q^{t-1}_{ihl} - \bar{Q}_{G_g}) + \lambda^{t-1}_{G_g}, \tau^o_{G_g} - \tau^u_{G_g}\}\}. \tag{25}$$

We now provide the convergence result for the modified projection method for the global supply chain network model with tariff rate quotas.

Theorem 3: Convergence

Assume that the function $F(X)$ that enters the variational inequality (14) (or (10)) satisfies the conditions in Proposition 1 and following, that is, monotonicity and Lipschitz continuity. Then, the modified projection method outlined above converges to a solution of the variational inequality (14), or, equivalently, (10).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (14), if the function F that enters the variational inequality is monotone and Lipschitz continuous and a solution exists. $\square$

5. Case Study on Avocados

In this Section, we present a case study consisting of multiple numerical examples focused on avocados, an agricultural product that is formally a fruit. Mexico, according to Larmer (2018), produces more avocados than any other country in the world, about a third of the global total, principally, in the state of Michoacan. In 2017, Mexico exported more than 1.7 billion pounds of Haas avocados to the US with about 90% of the avocados imported from

Mexico to the United States coming from Michoacan, with the Mexican state of Jalisco the second-largest avocado-producing state in Mexico, accounting for about 6 percent of total Mexican production. The volume of avocado imports into the United States has surpassed even the volume by weight of bananas imported into the United States, surprisingly, since bananas are considered the world's most popular fruit. Because of the current popularity of avocados, the fruit has been dubbed "green gold" in Michoacan (cf. Cruz (2017)), with consumers in the United States clamoring for avocados since they are also a principal ingredient in guacamole as well as part of avocado toast. In the United States, California is the principal avocado growing region, and, according to gro-intelligence.com (2015), as much as 95% of Californian avocados are of the especially tasty Hass variety, which is also Mexico's leading variety.

US domestic avocado consumption has risen to approximately 6.5 pounds per person annually, as compared to only 1.4 in 1990. The United States is among the world's top ten avocado producers, producing between 160,000 and 270,000 tons of avocados a year (U.S. International Trade Commission (2016)). In terms of other major demand markets, Mexico was the largest supplier of avocados to China until 2017 (Larmer (2018); see also Bradsher (2017)). For further, detailed recent background on US trade agreements, including a case study of avocados, see the United States International Trade Commission (2016).

The political climate in the United States has promulgated investigations into the North American Free Trade Agreement (NAFTA) and reconsideration and reevaluation of it (cf. Donnan and Wasson (2018)). The United States' recent imposition of a variety of tariffs, in turn, has resulted in retaliatory tariffs by multiple countries, notably, by Mexico and China, and on agricultural products produced in the US, in particular (cf. Filloon (2018)). Our case study, hence, focuses on avocados, which has been called "the fruit of global trade" (Larmer (2018)). Although the case study is stylized, managerial insights can be gained and these are also relevant to other products, specifically, agricultural ones.

The modified projection method, as outlined in Section 4, was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computations. The algorithm was initialized with the variables all set equal to zero. the convergence tolerance $\epsilon = 10^{-5}$. The contraction parameter β was set to .3, except were noted.

Example 1: Baseline Example Without Tariff Rate Quotas

In the first example considered in our case study, the global supply chain network for avocados (cf. Figure 3) consists of two firms. Firm 1 is located in the United States and Firm 2 is located in Mexico. Firm 1 has two US production sites available, both located in California, and denoted by nodes $(1,1,1)$ and $(1,1,2)$. Node $(1,1,1)$ corresponds to San Diego county where approximately 60% of the avocados cultivated in the United States are grown, and node $(1,1,2)$ corresponds to the San Luis Obispo area in California, another area known for avocado cultivation. Firm 2 also has two production sites available, but located in Mexico, in Michoacan and Jalisco, respectively, and denoted by nodes $(2,2,1)$ and $(2,2,2)$. There is a single demand market in this example, located in the United States, and denoted by node $(1,1)$.

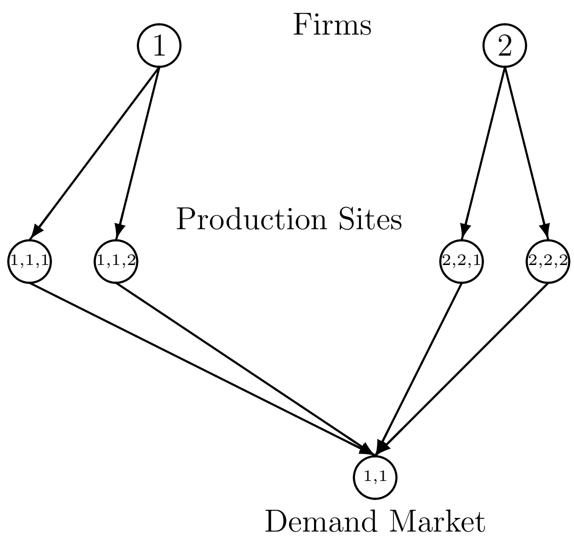

Figure 3: Global Supply Chain Network Topology for Case Study Examples 1 and 2

Example 1 of our case study serves as the baseline. Hence, in the example, there are no tariff rate quotas and the governing variational inequality is the one in (15). Observe that in order to solve this variational inequality we can still apply the modified projection method with Steps 2 and 3 (cf. (22) and (23)) yielding variational inequality subproblems in product trade flows only with the explicit formulae for (22) consisting of (24b); and, analogously, for (23).

We consider the time horizon of a week and the quantities of avocados are reported in millions of pounds. The currency is US dollars.

Recall that we enumerate the middle tier nodes as: $1,2,3,4$, corresponding, respectively, to nodes: $(1,1,1)$, $(1,1,2)$, $(2,2,1)$, and $(2,2,2)$ in Figure 3. Hence, we are interested in determining the equilibrium avocado product flows: Q^*_{111}, Q^*_{121}, Q^*_{231}, and Q^*_{241}.

The cost data are as follows. The production cost functions faced by Firm 1 at its two production sites are, respectively:

$$f_{11}(Q) = .005Q_{111}^2 + .8Q_{111}, \quad f_{12}(Q) = .01Q_{121}^2 + 1.1Q_{121},$$

whereas the transportation cost functions associated with Firm 1 transporting the avocados to the demand market are:

$$c_{111}(Q) = .1Q_{111}^2 + .5Q_{111}, \quad c_{121}(Q) = .1Q_{121}^2 + .4Q_{121}.$$

The production cost functions faced by Firm 2, with the production sites at the two locations in Mexico, are, respectively:

$$f_{23}(Q) = .0005Q_{231}^2 + .15Q_{231}, \quad f_{24}(Q) = .0005Q_{241}^2 + .5Q_{241},$$

and its transportation costs to the demand market are:

$$c_{231}(Q) = .04Q_{231}^2 + .5Q_{231}, \quad c_{241}(Q) = .045Q_{241}^2 + .5Q_{241}.$$

The demand price function is:

$$\rho_1(d) = -.01d_1 + 3.$$

The modified projection method yielded the equilibrium avocado product flow pattern:

$$Q_{111}^* = 5.63, \quad Q_{121}^* = 4.52, \quad Q_{231}^* = 20.75, \quad Q_{241}^* = 15.24,$$

with an incurred demand price per pound of avocados, $\rho_1 = 2.54$ at the demand market. Since consumers in the United States consume about 80% of their avocados from Mexico and about 20% from the United States, the above results are very reasonable and also correspond well to the weekly consumption of avocados by US consumers. The equilibrium demand market price is also reasonable.

Firm 1 achieves a utility (profit) of 6.09 in millions of dollars whereas Firm 2 enjoys a utility (profit) of 34.63 in millions of dollars.

Example 2: Tariff Rate Quotas on Avocados from Mexico

Example 2 is constructed from Example 1 and has the same data except that we now incorporate tariff rate quotas. In particular, the United States, interested in protecting domestic avocado production, assigns the tariff rate quota on group G_1, which consists of the Mexican production sites that ship to the United States, and the demand market, of

$\bar{Q}_1 = 30$, and with under-quota and over-quota tariffs of: $\tau^u_{G_1} = .25$ and $\tau^o_{G_1} = .50$. The governing variational inequality for this problem is, hence, (10). We applied the modified projection method as described in Section 4 and it yielded the following equilibrium avocado product trade flow and economic rent equivalent pattern:

$$Q^*_{111} = 5.88, \quad Q^*_{121} = 4.76, \quad Q^*_{231} = 17.60, \quad Q^*_{241} = 12.40,$$

$$\lambda^*_{G_1} = .09.$$

The demand market price per pound of avocados is now: $\rho_1 = 2.59$. The consumers are faced with a higher price. Observe that the imports from Mexico to the United States are precisely equal to the imposed quota.

The utility (profit) of Firm 1 is now: 6.69 in millions of dollars and that of Firm 2: 24.18 in millions of dollars. The imposition of the TRQs increases the profit of the US firm by about 10% and decreases the profit of the Mexican firm by about 33%.

The US government acquires tariff payments of 10.24 in millions of dollars.

The equilibrium avocado trade flows for Example 1 and Example 2 are displayed in the bar graph in Figure 4.

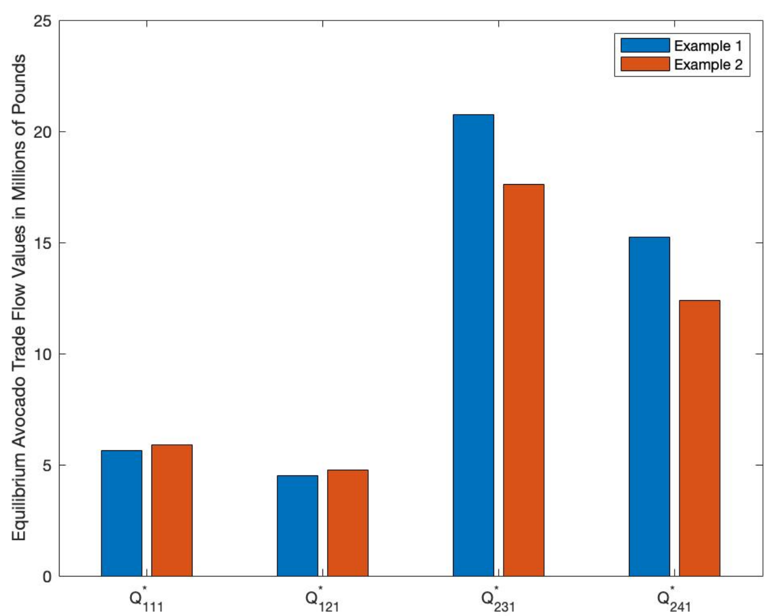

Figure 4: Equilibrium Avocado Trade Flows in Example 1 and in Example 2

The blue and red bars indicate the equilibrium avocado product flows in Example 1 and in Example 2, respectively, in millions of pounds. It is graphically clear that the equilibrium avocado trade flows from Mexico exhibit a decline in Example 2 from their respective values in Example 1, whereas those from the United States exhibit an increase. The TRQ imposed by the US on avocados from Mexico clearly benefits the US firm.

Example 3: Addition of a New Production Site in the United States

Example 3 has the same data as Example 2 except that now Firm 1 has added a production site in Florida, denoted by node $(1, 1, 3)$, as in the supply chain network depicted in Figure 5. This node is enumerated as production site 5. The same tariff rate quota and under-quota and over-quota tariffs apply as in Example 2.

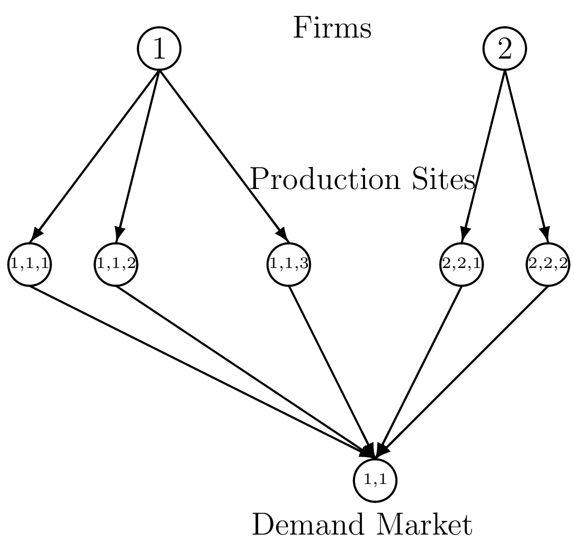

Figure 5: Global Supply Chain Network Topology for Case Study Example 3

The production cost at the Florida site is:

$$f_{15}(Q) = .0025Q_{151}^2 + .7Q_{151}$$

and the transportation cost to the demand market is:

$$c_{151}(Q) = .1Q_{151}^2 + .2Q_{151}.$$

The new computed equilibrium avocado product flow pattern is now

$$Q_{111}^* = 5.57, \quad Q_{121}^* = 4.45, \quad Q_{151}^* = 7.56, \quad Q_{231}^* = 17.60, \quad Q_{241}^* = 12.40.$$

The volume of imports from Mexico remain at the quota $\bar{G}_1 = 30$ million pounds and the equilibrium $\lambda_{G_1}^* = .02$.

The utility (profit) of Firm 1 is now 12.36 in millions of dollars and 24.18 for Firm 2 in millions of dollars. The demand market price at the equilibrium has now dropped to: 2.52. The almost doubling of profits for Firm 1 in this example signals that it should expand the number of its production sites. Consumers also benefit since the demand market price decreases. The US government now acquires tariff payments of 8.10 in millions of dollars.

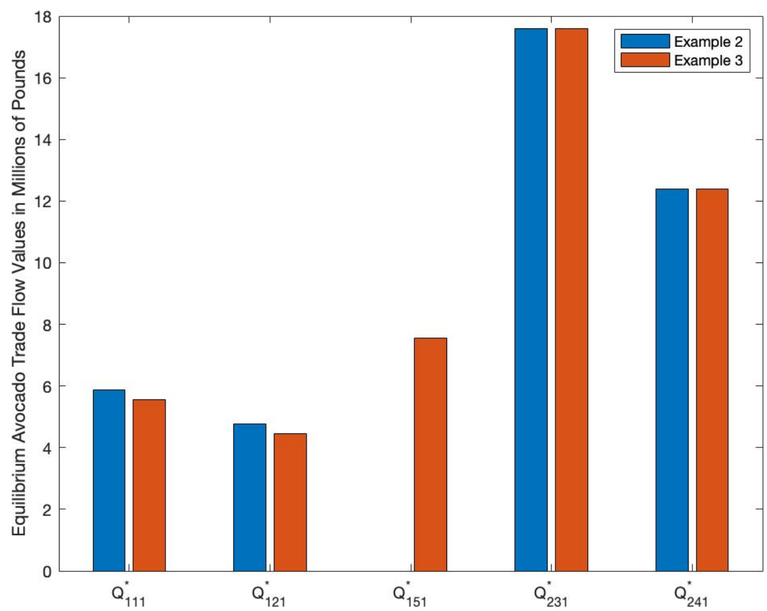

Figure 6: Equilibrium Avocado Trade Flows in Example 2 and in Example 3

The equilibrium avocado trade flow pattern in Example 2 and in Example 3 are displayed in Figure 6. There is a decline in equilibrium product flows, Q^*_{111} and Q^*_{121}, generated by the addition of the new production site, but a sizeable flow from from the new site.

Example 4: Addition of a New Demand Market in China

In Example 4, we explore the impact of the addition of a new demand market, denoted by node $(2,1)$ in Figure 7, and representing China. The remainder of the data is as in Example 3. There has been growing interest among consumers in China for avocados (cf. Daniels (2018)). China does not assign a tariff rate quota to the imports of avocados in this example. In this example, we have the contraction parameter $\beta = .1$.

The demand price function at the demand market in China is:

$$\rho_2 = -.01d_2 + 7$$

since Chinese consumers are willing to pay a high price for avocados.

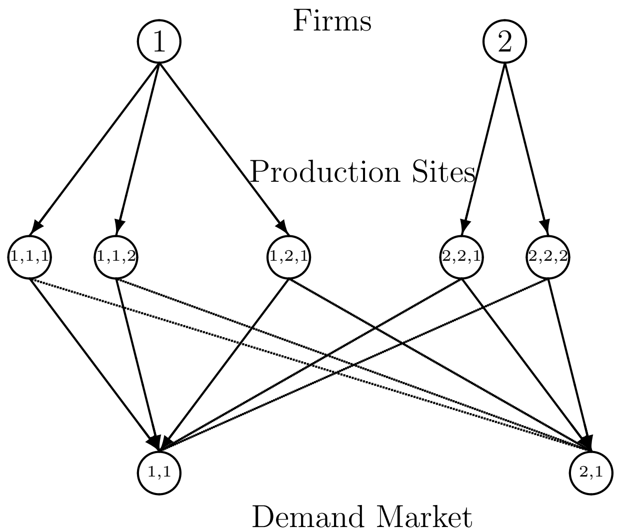

Figure 7: Global Supply Chain Network Topology for Case Study Examples 4 and 5

The transportation cost functions from the production sites in the United States to China are:

$$c_{112} = .15Q_{112}^2 + Q_{112}, \quad c_{122} = .15Q_{122}^2 + Q_{122}, \quad c_{152} = 2Q_{152}^2 + 1.1Q_{152}$$

and the transportation cost functions from the production sites in Mexico to China are:

$$c_{232} = .05Q_{232}^2 + 1.5Q_{232}, \quad c_{242} = .1Q_{242}^2 + Q_{242}.$$

The new computed equilibrium avocado product flow pattern is now

$$Q_{111}^* = 5.03, \quad Q_{112}^* = 13.33, \quad Q_{121}^* = 3.48, \quad Q_{122}^* = 11.96, \quad Q_{151}^* = 7.60, \quad Q_{152}^* = 1.07,$$

$$Q_{231}^* = 17.51, \quad Q_{232}^* = 40.09, \quad Q_{241}^* = 12.49, \quad Q_{242}^* = 21.82.$$

The incurred demand market price at the equilibrium is $\rho_1 = 2.54$ in the United States and, in China, the demand market price is: $\rho_2 = 6.12$ at the equilibrium. The latter is very reasonable given price data from China (cf. freshplaza.com (2017)).

The utility (profit) of Firm 1 is now 68.35 and that of Firm 2 is: 174.97.

The imports from Mexico to the United States are at the quota with $\lambda_{G_1}^* = .01$. With the opening of a major new market for avocados, the utilities (profits) of both firms increase significantly, with those of Firm 1 more than five-fold, and those of Firm 2 about seven-fold. The US government income from tariff payments is now: 7.8 in millions of dollars.

Example 5: Tariff Rate Quota Imposed by China on Imports from the United States

In Example 5 for our case study, we investigate the impact of China imposing a retaliatory tariff rate quota on imports of avocados from the United States. The data remain as in Example 4 except that now we have G_2 consisting of the production sites corresponding to the United States and the demand market in China. The added data are: $\bar{Q}_{G_2} = 15$ and $\tau^u_{G_2} = 1$ and $\tau^o_{G_2} = 2$. The contraction parameter $\beta = .1$, as in Example 4.

The modified projection method yielded the following equilibrium avocado product flow pattern:

$$Q^*_{111} = 5.25, \quad Q^*_{112} = 7.80, \quad Q^*_{121} = 3.92, \quad Q^*_{122} = 6.58, \quad Q^*_{151} = 7.58, \quad Q^*_{152} = .63,$$

$$Q^*_{231} = 17.50, \quad Q^*_{232} = 40.99, \quad Q^*_{241} = 12.48, \quad Q^*_{242} = 22.30.$$

The demand prices are now: $\rho_1 = 2.53$ for a pound of avocados in the United States and $\rho_2 = 6.22$ for a pound of avocados in China. The equilibrium economic rents are: $\lambda^*_{G_1} = 0.00$ and $\lambda^*_{G_2} = .87$. The imports from the United States to China are at the imposed quota of 15 and, hence, the positive equilibrium economic rent. The imports to the United States from Mexico are just under the imposed quota of 30 and equal to 29.97.

Firm 1 now has a reduced utility (profit) of 30.60 in millions of dollars, whereas Firm 2 has a utility (profit) of 181.67 in millions of dollars. Under the tariff quota regime imposed by China on the United States, Firm 1 experiences a drop in profits of over 50% as compared to Example 4, whereas Firm 2 enjoys a small increase in profits. The US government gathers 7.49 million in tariff payments, whereas the Chinese government gains 28.05 million dollars in tariff payments. The Chinese government clearly benefits from the imposition of the tariff rate quota against the United States; however, consumers in China must pay a higher price.

We illustrate the equilibrium avocado trade flow pattern in Example 4 and Example 5 in Figure 8. The United States is the loser under this retaliatory TRQ imposed by China against the US.

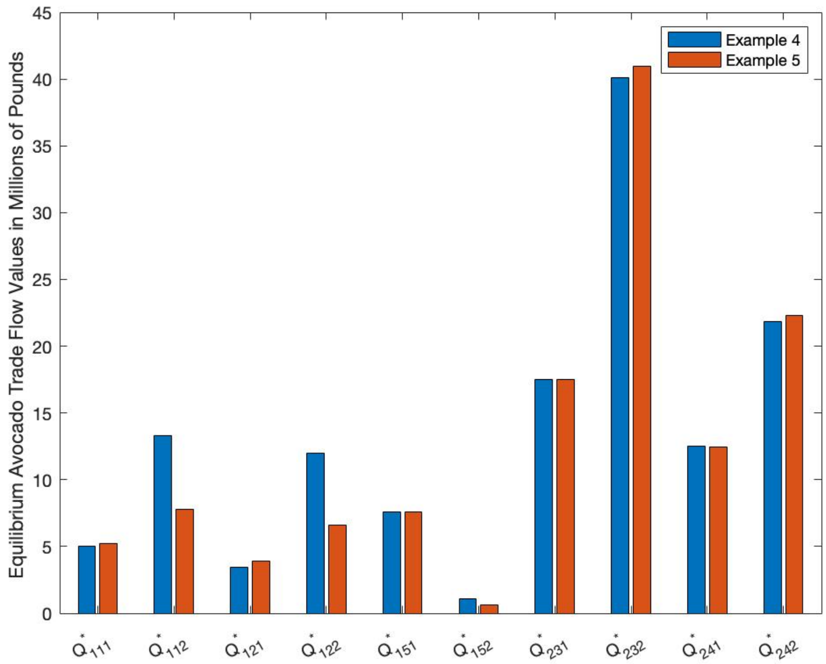

Figure 8: Equilibrium Avocado Trade Flows in Example 4 and in Example 5

6. Summary and Conclusions

In this paper, we constructed a modeling and computational framework for competitive global supply chain networks in the presence of trade policies in the form of tariff rate quotas. Our work is inspired by the timeliness of the topic, as evidenced by various recent tariffs being imposed by multiple countries. To-date, there has been limited modeling work integrating oligopolistic firms, competing globally, in the presence of such trade policies, which have been challenging to model. We also demonstrated how the model can be adapted to handle tariffs or strict quotas. The theoretical framework utilized for the formulation, analysis, and computation of the equilibrium product flow and economic rent equivalent patterns is the theory of variational inequalities.

This paper provides illustrative numerical examples and also a case study focused on the agricultural sector, in particular, the global market for avocados, a fruit that has been experiencing a rise in popularity and consumer demand. The numerical examples that comprise the case study quantify impacts of tariff rate quotas on consumer prices, on product flows, as well as on the firms' profits. We also investigate the impacts of new production sites, new demand markets, and retaliatory tariffs. The results demonstrate that TRQs can be effective in reducing product flows from countries on which they are imposed but at the expense of the consumers in terms of prices. Governments that acquire the tariff payments, however, gain financially.

Future research may include multiple product supply chain networks and tariff rate quotas, as well as multiperiod modeling, and the exploration of the feasibility of incorporating quality into agricultural products that are perishable along with minimum quality standards and a variety of trade policies, as noted in this paper.

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