Sustainable Supply Chain and Transportation Networks

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Abstract: In this paper, we show how sustainable supply chains can be transformed into and studied as transportation networks. Specifically, we develop a new supply chain model in which the manufacturers can produce the homogeneous product in different manufacturing plants with associated distinct environmental emissions. We assume that the manufacturers, the retailers with which they transact, as well as the consumers at the demand markets for the product are multicriteria decision-makers with the environmental criteria weighted distinctly by the different decision-makers. We derive the optimality conditions and the equilibrium conditions which are then shown to satisfy a variational inequality problem. We prove that the supply chain model with environmental concerns can be reformulated and solved as an elastic demand transportation network equilibrium problem. Numerical supply chain examples are presented for illustration purposes. This paper, hence, begins the construction of a bridge between sustainable supply chains and transportation networks.

Key Words: supply chains, multicriteria decision-making, environmental concerns, transportation network equilibrium, variational inequalities

1. Introduction

Transportation provides the foundation for the linking of economic activities. Without transportation, inputs to production processes do not arrive, nor can finished goods reach their destinations. In today's globalized economy, inputs to production processes may lie continents away from assembly points and consumption locations, further emphasizing the critical infrastructure of transportation in product supply chains.

At the same time that supply chains have become increasingly globalized, environmental concerns due to global warming and associated security risks regarding energy supplies have drawn the attention of numerous constituencies (cf. Cline (1992), Poterba (1993), and Painuly (2001)). Indeed, companies are increasingly being held accountable not only for their own performance in terms of environmental accountability, but also for that of their suppliers, subcontractors, joint venture partners, distribution outlets and, ultimately, even for the disposal of their products. Consequently, poor environmental performance at any stage of the supply chain may damage the most important asset that a company has, which is its reputation.

In this paper, a significant extension of the supply chain network model of Nagurney and Toyasaki (2003), which introduced environmental concerns into a supply chain network equilibrium framework (see also Nagurney, Dong, and Zhang (2002)), is made through the introduction of alternative manufacturing plants for each manufacturer with distinct associated environmental emissions. In addition, we demonstrate that the new supply chain network equilibrium model can be transformed into a transportation network equilibrium model with elastic demands over an appropriately constructed abstract network or supernetwork. We also illustrate how this theoretical result can be exploited in practice through the computation of numerical examples.

This paper is organized as follows. Section 2 develops the multitiered, multicriteria supply chain network model with distinct manufacturing plants and associated emissions and presents the variational inequality formulation of the governing equilibrium conditions. We also establish that the weights associated with the environmental criteria of the various decision-makers can be interpreted as taxes. Section 3 then recalls the well-known transportation network equilibrium model of Dafermos (1982). Section 4 demonstrates how the new supply chain network model with environmental concerns can be transformed into a transportation network equilibrium model over an appropriately constructed abstract network or supernetwork. This equivalence provides us with a new interpretation of the equilibrium conditions governing sustainable supply chains in terms of path flows. In Section 5 we apply an algorithm developed for the computation of solutions to elastic demand transportation network equilibrium problems to solve numerical supply chain network problems in which there are distinct manufacturing plants available for each manufacturer and emissions associated with production as well as with transportation/transaction and the operation of the retailers are included. The numerical examples illustrate the potential power of this approach for policy analyses.

The contributions in this paper further demonstrate the generality of the concepts of transportation network equilibrium, originally proposed in the seminal book of Beckmann, McGuire, and Winsten (1956) (see also Boyce, Mahmassani, and Nagurney (2005)). Indeed, recently, it has been shown by Nagurney (2006a) that supply chains can be reformulated and solved as transportation network problems. Moreover, the papers by Nagurney and Liu (2005) and Wu et al. (2006) demonstrate, as hypothesized by Beckmann, McGuire, and Winsten (1956), that electric power generation and distribution networks can be reformulated and solved as transportation network equilibrium problems. See also the book by Nagurney (2006b) for a variety of transportation-based supply chain network models and applications and the book by Nagurney (2000) on sustainable transportation networks.

2. The Supply Chain Model with Alternative Manufacturing Plants and Environmental Concerns

In this Section we develop the supply chain network model that includes manufacturing plants as well as multicriteria decision-making associated with environmental concerns. We consider I manufacturers, each of which generally owns and operates M manufacturing plants. Each manufacturing plant is associated with a different primary production process and energy consumption combination with associated environmental emissions. There are also J retailers, T transportation/transaction modes between each retailer and demand market, with a total of K demand markets, as depicted in Figure 1. The majority of the needed notation is given in Table 1. An equilibrium solution is denoted by "*". All vectors are

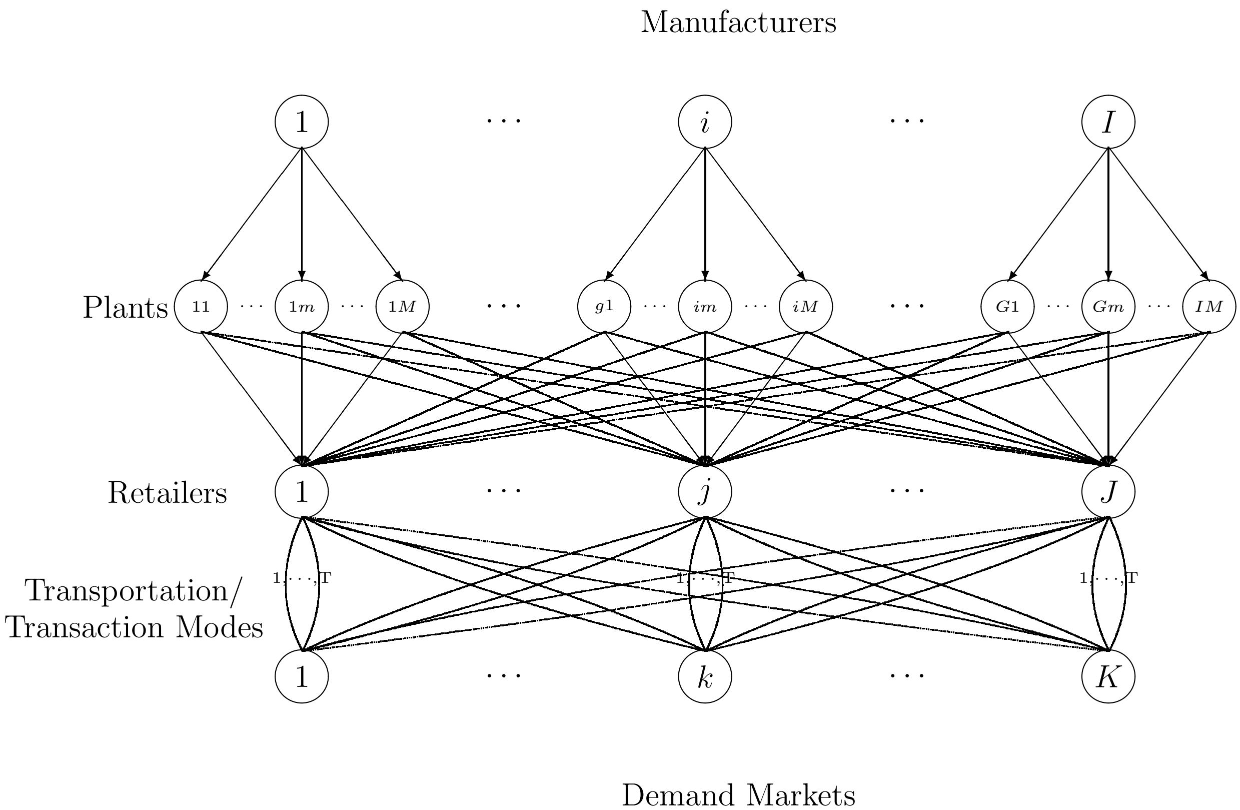

Figure 1: The Supply Chain Network with Manufacturing Plants

assumed to be column vectors, except where noted otherwise.

The top-tiered nodes in the supply chain network in Figure 1, enumerated by $1, \ldots, i \ldots, I$, represent the I manufacturers, who are the decision-makers who own and operate the manufacturing plants denoted by the second tier of nodes in the network. The manufacturers produce a homogeneous product using the different plants and sell the product to the retailers in the third tier.

Table 1: Notation for the Supply Chain Model

Notation	**Definition**
q_{im}	quantity of product produced by manufacturer i using plant m, where $i = 1, \ldots, I; m = 1, \ldots, M$
q_m	I-dimensional vector of the product generated by manufacturers using plant m with components: $q_{1m}, \ldots, q_{Im}$
q	IM-dimensional vector of all the production outputs generated by the manufacturers at the plants
Q^1	IMJ-dimensional vector of flows between the plants of the manufacturers and the retailers with component imj denoted by q_{imj}
Q^2	JTK-dimensional vector of product flows between retailers and demand markets with component jtk denoted by q^t_{jk} and denoting the flow between retailer j and demand market k via transportation/transaction mode t
d	K-dimensional vector of market demands with component k denoted by d_k
$f_{im}(q_m)$	production cost function of manufacturer i using plant m with marginal production cost with respect to q_{im} denoted by $\frac{\partial f_{im}}{\partial q_{im}}$
$c_{imj}(q_{imj})$	transportation/transaction cost incurred by manufacturer i using plant m in transacting with retailer j with marginal transaction cost denoted by $\frac{\partial c_{imj}(q_{imj})}{\partial q_{imj}}$
h	J-dimensional vector of the retailers' supplies of the product with component j denoted by h_j, with $h_j \equiv \sum_{i=1}^{I} \sum_{m=1}^{M} q_{imj}$
$c_j(h) \equiv c_j(Q^1)$	operating cost of retailer j with marginal operating cost with respect to h_j denoted by $\frac{\partial c_j}{\partial h_j}$ and the marginal operating cost with respect to q_{imj} denoted by $\frac{\partial c_j(Q^1)}{\partial q_{imj}}$
$c^t_{jk}(q^t_{jk})$	the transportation/transaction cost associated with the transaction between retailer j and demand market k via transportation/transaction t
$\hat{c}^t_{jk}(Q^2)$	unit transportation/transaction cost incurred by consumers at demand market k in transacting with retailer j via mode t
$\rho_{3k}(d)$	demand market price function at demand market k

Node im in the second tier corresponds to manufacturer i's plant m, with the second tier of nodes enumerated as: $11, \ldots, IM$. We assume that each manufacturer seeks to determine his optimal production portfolio across his manufacturing plants and his sales allocations of the product to the retailers in order to maximize his own profit. We also assume that each manufacturer seeks to minimize the total emissions associated with production and transportation to the retailers.

Retailers, which are represented by the third-tiered nodes in Figure 1, function as intermediaries. The nodes corresponding to the retailers are enumerated as: $1, \ldots, j, \ldots, J$ with node j corresponding to retailer j. They purchase the product from the manufacturers and sell the product to the consumers at the different demand markets. We assume that the retailers compete with one another in a noncooperative manner. Also, we assume that the retailers are assumed to be multicriteria decision-makers with environmental concerns and they also seek to minimize the emissions associated with transacting (which can include transportation) with the consumers as well as in operating their retail outlets.

The bottom-tiered nodes in Figure 1 represent the demand markets, which can be distinguished from one another by their geographic locations or the type of associated consumers such as whether they correspond, for example, to businesses or to households. There are K bottom-tiered nodes with node k corresponding to demand market k.

The retailers need to cover the direct costs and to decide which transportation/transaction modes should be used and how much product should be delivered. The structure of the network in Figure 1 guarantees that the conservation of flow equations associated with the production and distribution are satisfied. The flows on the links joining the manufacturers in Figure 1 to the plant nodes are respectively: $q_{11}, \ldots, q_{im}, \ldots, q_{IM}$; the flows on the links from the plant nodes to the retailer nodes are given, respectively, by the components of the vector Q^1, whereas the flows on the links joining the retailer nodes with the demand markets are given by the respective components of the vector: Q^2.

Of course, if a particular manufacturer does not own M manufacturing plants, then the corresponding links (and nodes) can just be removed from the supply chain network in Figure 1 and the notation reduced accordingly. Similarly, if a mode of transportation/transaction is not available for a retailer/demand market pair, then the corresponding link may be removed from the supply chain network in Figure 1 and the notation changed accordingly. On the other hand, multiple modes of transportation/transaction from the plants to the retailers can easily be added as links to the supply chain network in Figure 1 joining the plant nodes with the retailer nodes (with an associated increase in notation).

We now describe the behavior of the manufacturers, the retailers, and the consumers at the demand markets. We then state the equilibrium conditions of the supply chain network and provide the variational inequality formulation.

Multicriteria Decision-Making Behavior of the Manufacturers and Their Optimality Conditions

Let ρ^*_{1imj} denote the unit price charged by manufacturer i for the transaction with retailer j for the product produced at plant m. ρ^*_{1imj} is an endogenous variable and can be determined once the complete supply chain network equilibrium model is solved. Since we have assumed that each individual manufacturer i; $i = 1, \ldots, I$, is a profit maximizer, the profit-maximization objective function of manufacturer i can be expressed as follows:

$$\text{Maximize} \quad \sum_{m=1}^{M} \sum_{j=1}^{J} \rho^*_{1imj} q_{imj} - \sum_{m=1}^{M} f_{im}(q_m) - \sum_{m=1}^{M} \sum_{j=1}^{J} c_{imj}(q_{imj}). \tag{1a}$$

The first term in the objective function $(1a)$ represents the revenue and the next two terms represent the production cost and transportation/transaction costs, respectively.

In addition, we assume that manufacturer i is concerned with the total amount of emissions generated both in production of the product at the various manufacturing plants as well as in transportation of the product to the various retailers. Letting e_{im} denote the amount of emissions generated per unit of product produced at plant m of manufacturer i, and e_{imj} the amount of emissions generated in transporting the product from plant m of manufacturer i to retailer j, we have that the second objective function of manufacturer i is given by:

$$\text{Minimize} \sum_{m=1}^{M} e_{im} q_{im} + \sum_{m=1}^{M} \sum_{j=1}^{J} e_{imj} q_{imj}. \tag{1b}$$

We assign now a nonnegative weight of α_i to the emissions-generation criterion $(1b)$ with the weight associated with profit maximization (cf. $(1a)$) being set equal to 1. Thus, we can construct a *value function* for each manufacturer using a constant additive weight value function (see e.g., Nagurney and Dong (2002), Nagurney and Toyasaki (2003), and the references therein). Consequently, the multicriteria decision-making problem for manufacturer i is transformed into:

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{j=1}^{J}\rho_{1imj}^{*}q_{imj}-\sum_{m=1}^{M}f_{im}(q_m)-\sum_{m=1}^{M}\sum_{j=1}^{J}c_{imj}(q_{imj})-\alpha_i(\sum_{m=1}^{M}e_{im}q_{im}+\sum_{m=1}^{M}\sum_{j=1}^{J}e_{imj}q_{imj}) \tag{1c}$$

subject to:

$$\sum_{j=1}^{J}q_{imj}=q_{im}, \quad m=1,\ldots,M, \tag{2}$$

$$q_{imj}\geq 0, \quad m=1,\ldots,M; j=1,\ldots,J. \tag{3}$$

Conservation of flow equation (2) states that the amount of product produced at a particular plant of a manufacturer is equal to the amount of product transacted by the manufacturer from that plant with all the retailers (and this holds for each of the manufacturing plants). Expression (3) guarantees that the quantities of the product produced at the various manufacturing plants are nonnegative.

We assume that the production cost and the transportation cost functions for each manufacturer are continuously differentiable and convex (cf. (1c), subject to (2) and (3)), and that the manufacturers compete in a noncooperative manner in the sense of Nash (1950, 1951). The optimality conditions for all manufacturers simultaneously, under the above assumptions (see also Gabay and Moulin (1980), Bazaraa, Sherali, and Shetty (1993), and Nagurney (1999)), coincide with the solution of the following variational inequality: determine $(q^*, Q^{1*}) \in \mathcal{K}^1$ satisfying

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q_m^*)}{\partial q_{im}}+\alpha_i e_{im}\right]\times[q_{im}-q_{im}^*]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\frac{\partial c_{imj}(q_{imj}^*)}{\partial q_{imj}}+\alpha_i e_{imj}-\rho_{1imj}^*\right]\times[q_{imj}-q_{imj}^*]\geq 0, \quad \forall (q,Q^1)\in\mathcal{K}^1, \tag{4}$$

where $\mathcal{K}^1 \equiv \{(q,Q^1)|(q,Q^1)\in R_+^{IM+IMJ}$ and (2) holds$\}$.

Multicriteria Decision-Making Behavior of the Retailers and Their Optimality Conditions

The retailers, in turn, are involved in transactions both with the manufacturers and with the consumers at demand markets.

It is reasonable to assume that the total amount of product sold by a retailer j; $j = 1, \dots, J$, is equal to the total amount of the product that he purchased from the manufacturers and that was produced via the different manufacturing plants available to the manufacturers. This assumption can be expressed as the following conservation of flow equations:

$$\sum_{k=1}^{K} \sum_{t=1}^{T} q_{jk}^{t} = \sum_{i=1}^{I} \sum_{m=1}^{M} q_{imj}, \quad j = 1, \dots, J. \tag{5}$$

Let ρ_{2jk}^{t*} denote the price charged by retailer j to demand market k via transportation/transaction mode t. This price is determined endogenously in the model once the entire network equilibrium problem is solved. As noted above, it is assumed that each retailer seeks to maximize his own profit. Hence, the profit-maximization objective function faced by retailer j may be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{K} \sum_{t=1}^{T} \rho_{2jk}^{t*} q_{jk}^{t} - c_j(Q^1) - \sum_{i=1}^{I} \sum_{m=1}^{M} \rho_{1imj}^{*} q_{imj} - \sum_{k=1}^{K} \sum_{t=1}^{T} c_{jk}^{t}(q_{jk}^{t}). \tag{6a}$$

The first term in (6a) denotes the revenue of retailer j; the second term denotes the operating cost of the retailer, and the third term denotes the payments for the product to the various manufacturers. The last term in (6a) denotes the transportation/transaction costs. Note that here we have assumed imperfect competition in terms of the operating cost but, of course, if the operating cost functions c_j; $j = 1, \dots, J$ depend only on the product handled by j (and not also on the product handled by the other retailers), then the the dependence of these functions on Q^1 can be simplified accordingly (and this is a special case of the model). The latter would reflect perfect competition.

In addition, for notational convenience, we let

$$h_j \equiv \sum_{i=1}^{I} \sum_{m=1}^{M} q_{imj}, \quad j = 1, \dots, J. \tag{7}$$

As defined in Table 1, the operating cost of retailer j, c_j, is a function of the total product inflows to the retailer, that is:

$$c_j(h) \equiv c_j(Q^1), \quad j = 1, \ldots, J. \tag{8}$$

Hence, his marginal cost with respect to h_j is equal to the marginal cost with respect to q_{imj}:

$$\frac{\partial c_j(h)}{\partial h_j} \equiv \frac{\partial c_j(Q^1)}{\partial q_{imj}}, \quad j = 1, \ldots, J; \quad m = 1, \ldots, M. \tag{9}$$

In addition, we assume that each retailer seeks to minimize the emissions associated with managing his retail outlet and with transacting with consumers at the demand markets. Let e_j denote the amount of emissions generated by the retailer j; $j = 1, \ldots, J$, and let e^t_{jk} denote the amount of emissions per unit of product transacted between k and j via t, for $j = 1, \ldots, J$; $k = 1, \ldots, K$, and $t = 1, \ldots, T$. Then we have that the second objective function of retailer j is given by:

$$\text{Minimize} \quad e_j h_j + \sum_{k=1}^{K} \sum_{t=1}^{T} e^t_{jk} q^t_{jk}. \tag{6b}$$

We associate the nonnegative weight β_j with the environmental objective (criterion) function (6b) and we construct retailer j's multicriteria decision-making problem, given by:

$$\text{Maximize} \quad \sum_{k=1}^{K} \sum_{t=1}^{T} \rho^{t*}_{2jk} q^t_{jk} - c_j(Q^1) - \sum_{i=1}^{I} \sum_{m=1}^{M} \rho^*_{1imj} q_{imj} - \sum_{k=1}^{K} \sum_{t=1}^{T} c^t_{jk}(q^t_{jk}) \tag{6c}$$

$$-\beta_j (e_j h_j + \sum_{k=1}^{K} \sum_{t=1}^{T} e^t_{jk} q^t_{jk})$$

subject to (7) and:

$$\sum_{k=1}^{K} \sum_{t=1}^{T} q^t_{jk} = \sum_{i=1}^{I} \sum_{m=1}^{M} q_{imj} \tag{10}$$

$$q_{imj} \geq 0, \quad i = 1, \ldots, I, \quad m = 1, \ldots, M, \tag{11}$$

$$q^t_{jk} \geq 0, \quad k = 1, \ldots, K; t = 1, \ldots, T. \tag{12}$$

We assume that the transaction costs and the operating costs (cf. (6a)) are all continuously differentiable and convex, and that the retailers compete in a noncooperative manner.

Hence, the optimality conditions for all retailers, simultaneously, under the above assumptions (see also Dafermos and Nagurney (1987) and Nagurney, Dong, and Zhang (2002)), can be expressed as the following variational inequality: determine $(h^*, Q^{2*}, Q^{1*}) \in \mathcal{K}^3$ such that

$$\sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j}+\beta_j e_j\right]\times[h_j-h_j^*]+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{jk}^t(q_{jk}^{t*})}{\partial q_{jk}^t}+\beta_j e_{jk}^t-\rho_{2jk}^{t*}\right]\times[q_{jk}^t-q_{jk}^{t*}]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\rho_{1imj}^*\right]\times[q_{imj}-q_{imj}^*]\geq 0, \qquad \forall(h,Q^1,Q^2,)\in\mathcal{K}^3, \tag{13}$$

where $\mathcal{K}^3 \equiv \{(h,Q^2,Q^1)|(h,Q^2,Q^1)\in R_+^{J(1+TK+IM)}\text{and (7) and (10) hold}\}$.

Equilibrium Conditions for the Demand Markets

At each demand market k; $k=1,\ldots,K$, the following conservation of flow equation must be satisfied:

$$d_k=\sum_{j=1}^{J}\sum_{t=1}^{T}q_{jk}^t. \tag{14}$$

We also assume that the consumers at the demand markets may be environmentally-conscious in choosing their modes of transaction with the retailer with an associated non-negative weight of η_k for demand market k. Since the demand market price functions are given, the market equilibrium conditions at demand market k then take the form: for each retailer j; $j=1,...,J$ and transportation/transaction mode t; $t=1,...,T$:

$$\rho_{2jk}^{t*}+\hat{c}_{jk}^t(Q^{2*})+\eta_k e_{jk}^t\begin{cases}=\rho_{3k}(d^*), & \text{if} \quad q_{jk}^{t*}>0,\\ \geq\rho_{3k}(d^*), & \text{if} \quad q_{jk}^{t*}=0.\end{cases} \tag{15}$$

Nagurney and Toyasaki (2003) (see also Nagurney and Toyasaki (2005)) considered similar demand market equilibrium conditions but in the case in which the demand functions, rather than the demand price functions as above, were given.

The interpretation of conditions (15) is as follows: consumers at a demand market will purchase the product from a retailer via a transportation/transaction mode, provided that the purchase price plus the unit transportation/transaction cost plus the marginal cost of emissions associated with that transaction is equal to the price that the consumers are willing to pay at that demand market. If the purchase price plus the unit transportation/transaction cost plus the marginal cost of emissions associated with that transaction exceeds the price the consumers are willing to pay, then there will be no transaction between that retailer and demand market via that transportation/transaction mode. The equivalent variational inequality governing all the demand markets takes the form: determine $(Q^{2*}, d^*) \in \mathcal{K}^4$, such that

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\rho_{2jk}^{t*} + \hat{c}_{jk}^{t}(Q^{2*}) + \eta_k e_{jk}^{t}\right] \times [q_{jk}^{t} - q_{jk}^{t*}] - \sum_{k=1}^{K}\rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0, \quad \forall (Q^2, d) \in \mathcal{K}^4, \tag{16}$$

where $\mathcal{K}^4 \equiv \{(Q^2, d) | (Q^2, d) \in R_+^{K(JT+1)} \text{and (14) holds}\}$.

The Equilibrium Conditions for the Supply Chain Network with Manufacturing Plants and Environmental Concerns

In equilibrium, the optimality conditions for all the manufacturers, the optimality conditions for all the retailers, and the equilibrium conditions for all the demand markets must be simultaneously satisfied so that no decision-maker has any incentive to alter his transactions.

Definition 1: Supply Chain Network Equilibrium with Manufacturing Plants and Environmental Concerns

The equilibrium state of the supply chain network with manufacturing plants and environmental concerns is one where the product flows between the tiers of the network coincide and the product flows and prices satisfy the sum of conditions (4), (13), and (16).

We now state and prove:

Theorem 1: Variational Inequality Formulation of the Supply Chain Network Equilibrium with Manufacturing Plants and Environmental Concerns

The equilibrium conditions governing the supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in \mathcal{K}^5$ *satisfying:*

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q_m^*)}{\partial q_{im}} + \alpha_i e_{im}\right] \times [q_{im} - q_{im}^*] + \sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j} + \beta_j e_j\right] \times [h_j - h_j^*]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\frac{\partial c_{imj}(q^*_{imj})}{\partial q_{imj}}+\alpha_i e_{imj}\right]\times[q_{imj}-q^*_{imj}]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c^t_{jk}(q^{t*}_{jk})}{\partial q^t_{jk}}+\hat{c}^t_{jk}(Q^{2*})+(\beta_j+\eta_k)e^t_{jk}\right]\times[q^t_{jk}-q^{t*}_{jk}]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times[d_k-d^*_k]\geq 0,$$

$$\forall (q,h,Q^1,Q^2,d)\in\mathcal{K}^5, \tag{17}$$

where

$$\mathcal{K}^5\equiv\{(q,h,Q^1,Q^2,d)|(q,h,Q^1,Q^2,d)\in R_+^{IM+J+IMJ+TJK+K} \textit{and } (2),\ (5),\ \textit{ and } (7) \textit{ hold}\}.$$

Proof: We first prove that an equilibrium according to Definition 1 coincides with the solution of variational inequality (17). Indeed, summation of (4), (13), and (16), after algebraic simplifications, yields (17).

We now prove the converse, that is, a solution to variational inequality (17) satisfies the sum of conditions (4), (13), and (16), and is, therefore, a supply chain network equilibrium pattern according to Definition 1.

First, we add the term $\rho^*_{1imj}-\rho^*_{1imj}$ to the first term in the third summand expression in (17). Then, we add the term $\rho^{t*}_{2jk}-\rho^{t*}_{2jk}$ to the first term in the fourth summand expression in (17). Since these terms are all equal to zero, they do not change (17). Hence, we obtain the following inequality:

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q^*_m)}{\partial q_{im}}+\alpha_i e_{im}\right]\times[q_{im}-q^*_{im}]+\sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j}+\beta_j e_j\right]\times[h_j-h^*_j]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\frac{\partial c_{imj}(q^*_{imj})}{\partial q_{imj}}+\alpha_i e_{imj}+\rho^*_{1imj}-\rho^*_{1imj}\right]\times[q_{imj}-q^*_{imj}]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c^t_{jk}(q^{t*}_{jk})}{\partial q^t_{jk}}+\hat{c}^t_{jk}(q^{t*}_{jk})+(\beta_j+\eta_k)e^t_{jk}+\rho^{t*}_{2jk}-\rho^{t*}_{2jk}\right]\times[q^t_{jk}-q^{t*}_{jk}]$$

$$-\sum_{k=1}^{K}\rho_{3k}(d^*)\times[d_k-d^*_k]\geq 0,\quad \forall (q,h,Q^1,Q^2,d)\in\mathcal{K}^5, \tag{18}$$

which can be rewritten as:

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q_m^*)}{\partial q_{im}}+\alpha_i e_{im}\right]\times[q_{im}-q_{im}^*]+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{g=1}^{G}\left[\frac{\partial c_{imj}(q_{imj}^*)}{\partial q_{imj}}-\rho_{1imj}^*+\alpha_i e_{imj}\right]\times[q_{imj}-q_{imj}^*]$$

$$+\sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j}+\beta_j e_j\right]\times[h_j-h_j^*]+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{jk}^t(q_{jk}^{t*})}{\partial q_{jk}^t}-\rho_{2jk}^{t*}+\beta_j e_{jk}^t\right]\times[q_{jk}^t-q_{jk}^{t*}]$$

$$+\sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{i=1}^{I}\left[\rho_{1imj}^*\right]\times[q_{imj}-q_{imj}^*]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\rho_{2jk}^{t*}+\hat{c}_{jk}^t(q_{jk}^{t*})+\eta_k e_{jk}^t\right]\times[q_{jk}^t-q_{jk}^{t*}]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times[d_k-d_k^*]\geq 0,$$

$$\forall(q,h,Q^1,Q^2,d)\in\mathcal{K}^5. \tag{19}$$

Clearly, (19) is the sum of the optimality conditions (4) and (13), and the equilibrium conditions (16), and is, hence, according to Definition 1 a supply chain network equilibrium. □

Remark

Note that, in the above model, we have assumed that the various decision-makers are environmentally conscious (to a certain degree) depending upon the weights that they assign to the respective environmental criteria denoted by α_i; $i=1,\ldots,I$ for the manfacturers; by β_j; $j=1,\ldots,J$ for the retailers, and by η_k; $k=1,\ldots,K$ for the consumers at the respective demand markets. These weights are associated with the environmental emissions generated in production, transportation/transaction, and the operation of the retail outlets as the product "moves" through the supply chain, driven by the demand for the product at the demand markets. This implies (assuming all weights are not identically equal to zero), environmentally-conscious decision-makers. It is worth emphasizing that the weights can also be interpreted as *taxes*, for example, *carbon* taxes (cf. Wu et al. (2006) and Nagurney, Liu, and Woolley (2006)), which would be assigned by a governmental authority. Such a framework was devised by Wu et al. (2006) in the case of electric power supply chains. However, in that model, the carbon emissions only occurred in the production of electric power using alternative power-generation plants, which could utilize different forms of energy (renewable or not, for example). Hence, the carbon taxes were only associated with the manufacturers and the power-generating plants. In the case of the supply chain network model in this paper, in contrast, pollution can be emitted not only at the production stage, but also in the transportation of the product, as well as during the operation of the retail outlets. In order to construct sustainable supply chains, it is essential to have a system-wide view of pollution generation.

We now describe how to recover the prices associated with the first and third tiers of nodes in the supply chain network. Clearly, the components of the vector ρ_3^* can be directly obtained from the solution to variational inequality (17). We now describe how to recover the prices ρ_{1imj}^*, for all i, m, j, and ρ_{2jk}^{t*} for all j, k, t, from the solution of variational inequality (17). The prices associated with the retailers can be obtained by setting (cf. (15)) $\rho_{2jk}^{t*} = \rho_{3k}^* - \eta_k e_{jk}^t - \hat{c}_{jk}^t(Q^{2*})$ for any j, t, k such that $q_{sk}^{t*} > 0$. The top-tiered prices, in turn, can be recovered by setting (cf. (4)) $\rho_{1imj}^* = \frac{\partial f_{im}(q_m^*)}{\partial q_{imj}} + \frac{\partial c_{imj}(q_{imj}^*)}{\partial q_{imj}} + \alpha_i e_{imj}$ for any i, m, j such that $q_{imj}^* > 0$.

In this paper, we have focused on the development of a supply chain network model with a view towards sustainability in which the weights (equivalently, taxes) are known/assigned a priori. In order to achieve a particular environmental goal (see also Nagurney (2000)), for example, in the case of a bound on the total emissions in the entire supply chain, one could conduct simulations associated with the different weights in order to achieve the desired policy result. An interesting extension would be to construct a model in which the weights/taxes are endogenous, as was done in the case of electric power supply chains and carbon taxes by Nagurney, Liu, and Woolley (2006). However, as also discussed therein, the transportation network equilibrium reformulation may be lost for the full supply chain (although still exploited computationally during the iterative algorithmic process).

3. The Transportation Network Equilibrium Model with Elastic Demands

In this Section, we recall the transportation network equilibrium model with elastic demands, due to Dafermos (1982), in which the travel disutility functions are assumed known and given. In Section 4, we establish that the supply chain network model in Section 2 can be reformulated as such a transportation network equilibrium problem but over a specially constructed network topology.

We consider a network $\mathcal{G}$ with the set of links L with n_L elements, the set of paths P with n_P elements, and the set of origin/destination (O/D) pairs W with n_W elements. We denote the set of paths joining O/D pair w by P_w. Links are denoted by a, b, etc; paths by p, q, etc., and O/D pairs by w_1, w_2, etc.

We denote the flow on path p by x_p and the flow on link a by f_a. The user travel cost on a link a is denoted by c_a and the user travel cost on a path p by C_p. We denote the travel demand associated with traveling between O/D pair w by d_w and the travel disutility by λ_w.

The link flows are related to the path flows through the following conservation of flow equations:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{20}$$

where $\delta_{ap} = 1$ if link a is contained in path p, and $\delta_{ap} = 0$, otherwise. Hence, the flow on a link is equal to the sum of the flows on paths that contain that link.

The user costs on paths are related to user costs on links through the following equations:

$$C_p = \sum_{a \in L} c_a \delta_{ap}, \quad \forall p \in P, \tag{21}$$

that is, the user cost on a path is equal to the sum of user costs on links that make up the path.

For the sake of generality, we allow the user cost on a link to depend upon the entire vector of link flows, denoted by f, so that

$$c_a = c_a(f), \quad \forall a \in L. \tag{22}$$

We have the following conservation of flow equations:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w. \tag{23}$$

Also, we assume, as given, travel disutility functions, such that

$$\lambda_w = \lambda_w(d), \quad \forall w, \tag{24}$$

where d is the vector of travel demands with travel demand associated with O/D pair w being denoted by d_w.

Definition 2: Transportation Network Equilibrium

In equilibrium, the following conditions must hold for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) - \lambda_w(d^*) \begin{cases} = 0, & \text{if} \quad x_p^* > 0, \\ \geq 0, & \text{if} \quad x_p^* = 0. \end{cases} \tag{25}$$

The interpretation of conditions (25) is as follows: only those paths connecting an O/D pair are used that have minimal travel costs and those costs are equal to the travel disutility associated with traveling between that O/D pair. As proved in Dafermos (1982), the transportation network equilibrium conditions (25) are equivalent to the following variational inequality in path flows: determine $(x^*, d^*) \in \mathcal{K}^6$ such that

$$\sum_{w \in W} \sum_{p \in P_w} C_p(x^*) \times \left[x_p - x_p^*\right] - \sum_{w \in W} \lambda_w(d^*) \times [d_w - d_w^*] \geq 0, \quad \forall (x,d) \in \mathcal{K}^6, \tag{26}$$

where $\mathcal{K}^6 \equiv \{(x,d) | (x,d) \in R_+^{n_P + n_W}$ and $d_w = \sum_{p \in P_w} x_p, \forall w\}$.

We now recall the equivalent variational inequality in link form due to Dafermos (1982).

Theorem 2

A link flow pattern and associated travel demand pattern is a transportation network equilibrium if and only if it satisfies the variational inequality problem: determine $(f^*, d^*) \in \mathcal{K}^7$ satisfying

$$\sum_{a \in L} c_a(f^*) \times (f_a - f_a^*) - \sum_{w \in W} \lambda_w(d^*) \times (d_w - d_w^*) \geq 0, \quad \forall (f,d) \in \mathcal{K}^7, \tag{27}$$

where $\mathcal{K}^7 \equiv \{(f,d) \in R_+^{n_L + n_W} |$ there exists an x satisfying (20) *and $d_w = \sum_{p \in P_w} x_p, \forall w\}$.*

Beckmann, McGuire, and Winsten (1956) were the first to formulate rigorously the transportation network equilibrium conditions (25) in the context of user link cost functions and travel disutility functions that admitted symmetric Jacobian matrices so that the equilibrium conditions (25) coincided with the Kuhn-Tucker optimality conditions of an appropriately constructed optimization problem. The variational inequality formulation, in turn, allows for asymmetric functions (see also, e.g., Nagurney (1999) and the references therein).

4. Transportation Network Equilibrium Reformulation of the Supply Chain Network Equilibrium Model with Manufacturing Plants and Environmental Concerns

In this Section, we show that the supply chain network equilibrium model presented in Section 2 is isomorphic to a properly configured transportation network equilibrium model through the establishment of a supernetwork equivalence of the former.

We now establish the supernetwork equivalence of the supply chain network equilibrium model to the transportation network equilibrium model with known travel disutility functions described in Section 3. This transformation allows us, as we will demonstrate in Section 5, to apply algorithms developed for the latter class of problems to solve the former.

Consider a supply chain network with manufacturing plants as discussed in Section 2 with given manufacturers: $i = 1, \ldots, I$; given manufacturing plants for each manufacturer: $m = 1, \ldots, M$; retailers: $j = 1, \ldots, J$; transportation/transaction modes: $t = 1, \ldots, T$, and demand markets: $k = 1, \ldots, K$. The supernetwork, $\mathcal{G}_{\mathcal{S}}$, of the isomorphic transportation network equilibrium model is depicted in Figure 2 and is constructed as follows.

It consists of six tiers of nodes with the origin node 0 at the top or first tier and the destination nodes at the sixth or bottom tier. Specifically, $\mathcal{G}_{\mathcal{S}}$ consists of a single origin node 0 at the first tier, and K destination nodes at the bottom tier, denoted, respectively, by: $z_1, \ldots, z_K$. There are K O/D pairs in $\mathcal{G}_{\mathcal{S}}$ denoted by $w_1 = (0, z_1)$, …, $w_k = (0, z_k)$,…, $w_K = (0, z_K)$. Node 0 is connected to each second-tiered node x_i; $i = 1, \ldots, I$ by a single link. Each second-tiered node x_i, in turn, is connected to each third-tiered node x_{im}; $i = 1, \ldots, I$; $m = 1, \ldots, M$ by a single link, and each third-tiered node is then connected to each fourth-tiered node y_j; $j = 1, \ldots, J$ by a single link. Each fourth-tiered node y_j is connected to the corresponding fifth-tiered node $y_{j'}$ by a single link. Finally, each fifth-tiered node $y_{j'}$ is connected to each destination node z_k; $k = 1, \ldots, K$ at the sixth tier by T parallel links.

Hence, in $\mathcal{G}_{\mathcal{S}}$, there are $I + IM + 2J + K + 1$ nodes; $I + IM + IMJ + J + JTK$ links,

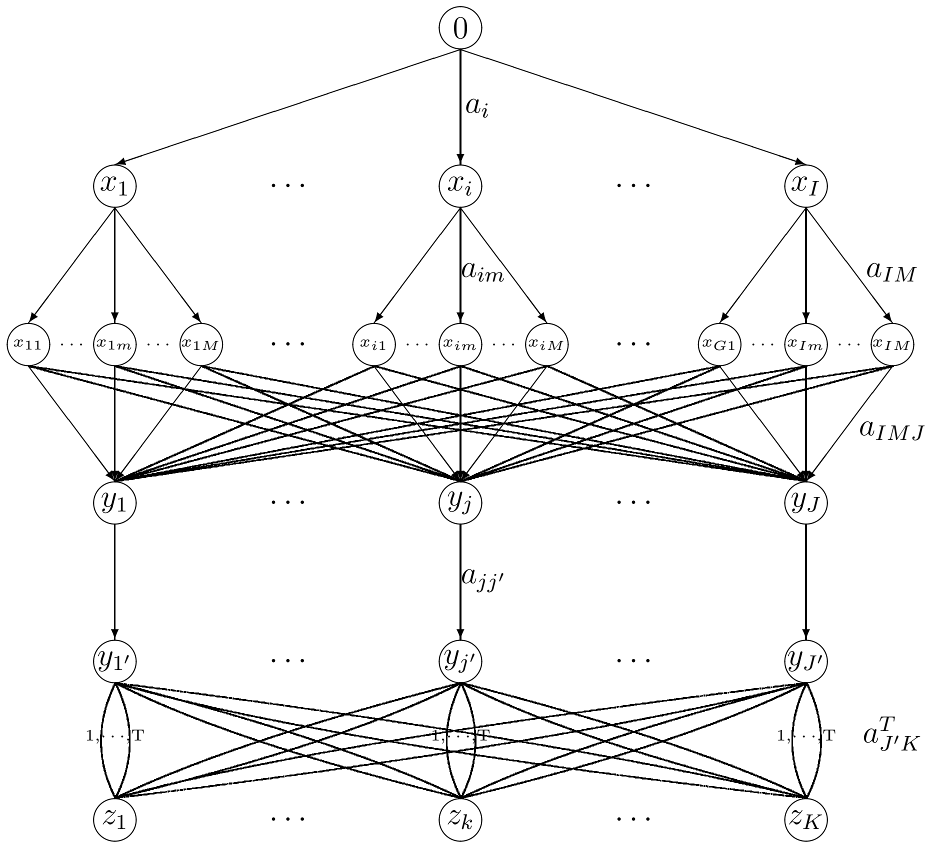

Figure 2: The $\mathcal{G}_{\mathcal{S}}$ Supernetwork Representation of Supply Chain Network Equilibrium with Manufacturing Plants

K O/D pairs, and $IMJTK$ paths. We now define the link and link flow notation. Let a_i denote the link from node 0 to node x_i with associated link flow f_{a_i}, for $i = 1, \ldots, I$. Let a_{im} denote the link from node x_i to node x_{im} with link flow $f_{a_{im}}$ for $i = 1, \ldots, I$; $m = 1, \ldots, M$. Also, let a_{imj} denote the link from node x_{im} to node y_j with associated link flow $f_{a_{imj}}$ for $i = 1, \ldots, I$; $m = 1, \ldots, M$, and $j = 1, \ldots, J$. Let $a_{jj'}$ denote the link connecting node y_j with node $y_{j'}$ with associated link flow $f_{a_{jj'}}$ for $jj' = 11', \ldots, JJ'$. Finally, let $a^t_{j'k}$ denote the t-th link joining node $y_{j'}$ with node z_k for $j' = 1', \ldots, J'$; $t = 1, \ldots, T$, and $k = 1, \ldots, K$ and with associated link flow $f_{a^t_{j'k}}$. We group the link flows into the vectors as follows: we group the $\{f_{a_i}\}$ into the vector f^1; the $\{f_{a_{im}}\}$ into the vector f^2, the $\{f_{a_{imj}}\}$ into the vector f^3; the $\{f_{a_{jj'}}\}$ into the vector f^4, and the $\{f_{a^t_{j'k}}\}$ into the vector f^5.

Thus, a typical path connecting O/D pair $w_k = (0, z_k)$, is denoted by $p^t_{imjj'k}$ and consists of five links: $a_i, a_{im}, a_{imj}, a_{jj'}$, and $a^t_{j'k}$. The associated flow on the path is denoted by $x_{p^t_{imjj'k}}$. Finally, we let d_{w_k} be the demand associated with O/D pair w_k where λ_{w_k} denotes the travel disutility for w_k.

Note that the following conservation of flow equations must hold on the network $\mathcal{G}_{\mathcal{S}}$:

$$f_{a_i} = \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{j'=1}^{J'} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad i = 1, \ldots, I, \tag{28}$$

$$f_{a_{im}} = \sum_{j=1}^{J} \sum_{j'=1'}^{J'} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad i = 1, \ldots, I; m = 1, \ldots, M, \tag{29}$$

$$f_{a_{imj}} = \sum_{j'=1'}^{J'} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad i = 1, \ldots, I; m = 1, \ldots, M; j = 1, \ldots, J, \tag{30}$$

$$f_{a_{jj'}} = \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad jj' = 11', \ldots, JJ', \tag{31}$$

$$f_{a^t_{j'k}} = \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{j=1}^{J} x_{p^t_{imjj'k}}, \quad j' = 1', \ldots, J'; t = 1, \ldots, T; k = 1, \ldots, K. \tag{32}$$

Also, we have that

$$d_{w_k} = \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{jj'=11'}^{JJ'} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad k = 1, \ldots, K. \tag{33}$$

If all path flows are nonnegative and (28)–(33) are satisfied, the feasible path flow pattern induces a feasible link flow pattern.

We can construct a feasible link flow pattern for $\mathcal{G}_{\mathcal{S}}$ based on the corresponding feasible supply chain flow pattern in the supply chain network model, $(q, h, Q^1, Q^2, d) \in \mathcal{K}^5$, in the following way:

$$q_i \equiv f_{a_i}, \quad i = 1, \ldots, I, \tag{34}$$

$$q_{im} \equiv f_{a_{im}}, \quad i = 1, \ldots, I; m = 1, \ldots, M, \tag{35}$$

$$q_{imj} \equiv f_{a_{imj}}, \quad i = 1, \ldots, I; m = 1, \ldots, M; j = 1, \ldots, J, \tag{36}$$

$$h_j \equiv f_{a_{jj'}}, \quad jj' = 11', \ldots, JJ', \tag{37}$$

$$q^t_{jk} = f_{a^t_{j'k}}, \quad j = 1, \ldots, J; j' = 1', \ldots, J'; t = 1, \ldots, T; k = 1, \ldots, K, \tag{38}$$

$$d_k = \sum_{j=1}^{J} \sum_{t=1}^{T} q^t_{jk}, \quad k = 1, \ldots, K. \tag{39}$$

Observe that although q_i is not explicitly stated in the model in Section 2, it is inferred in that

$$q_i = \sum_{m=1}^{M} q_{im}, \quad i = 1, \ldots, I, \tag{40}$$

and simply represents the total amount of product produced by manufacturer i.

Note that if (q, Q^1, h, Q^2, d) is feasible then the link flow and demand pattern constructed according to (34)–(39) is also feasible and the corresponding path flow pattern which induces this link flow (and demand) pattern is also feasible.

We now assign user (travel) costs on the links of the network $\mathcal{G}_{\mathcal{S}}$ as follows: with each link a_i we assign a user cost c_{a_i} defined by

$$c_{a_i} \equiv 0, \quad i = 1, \ldots, I, \tag{41}$$

$$c_{a_{im}} \equiv \frac{\partial f_{im}}{\partial q_{im}} + \alpha_i e_{im}, \quad i = 1, \ldots, I; m = 1, \ldots, M, \tag{42}$$

with each link a_{imj} we assign a user cost $c_{a_{imj}}$ defined by:

$$c_{a_{imj}} \equiv \frac{\partial c_{imj}}{\partial q_{imj}} + \alpha_i e_{imj}, \quad i = 1, \ldots, I; m = 1, \ldots, M; j = 1, \ldots, J, \tag{43}$$

with each link jj' we assign a user cost defined by

$$c_{a_{jj'}} \equiv \frac{\partial c_j}{\partial h_j} + \beta_j e_j, \quad jj' = 11', \ldots, JJ'. \tag{44}$$

Finally, for each link $a^t_{j'k}$ we assign a user cost defined by

$$c_{a^t_{j'k}} \equiv \frac{\partial c^t_{jk}}{\partial q^t_{jk}} + \hat{c}^t_{jk} + (\beta_j + \eta_k)e^t_{jk}, \quad j' = j = 1, \ldots, J; t = 1, \ldots, T; k = 1, \ldots, K. \tag{45}$$

Then a user of path $p^t_{imjj'k}$, for $i = 1, \ldots, I$; $m = 1, \ldots, M$; $jj' = 11', \ldots, JJ'$; $t = 1, \ldots, T$; $k = 1, \ldots, K$, on network $\mathcal{G}_{\mathcal{S}}$ in Figure 2 experiences a path travel cost $C_{p^t_{imjj'k}}$ given by

$$C_{p^t_{imjj'k}} = c_{a_i} + c_{a_{im}} + c_{a_{imj}} + c_{a_{jj'}} + c_{a^t_{j'k}}$$
$$= \frac{\partial f_{im}}{\partial q_{im}} + \alpha_i e_{im} + \frac{\partial c_{imj}}{\partial q_{imj}} + \alpha_i e_{imj} + \frac{\partial c_j}{\partial h_j} + \beta_j e_j + \frac{\partial c^t_{jk}}{\partial q^t_{jk}} + \hat{c}^t_{jk} + (\beta_j + \eta_k)e^t_{jk}. \tag{46}$$

Also, we assign the (travel) demands associated with the O/D pairs as follows:

$$d_{w_k} \equiv d_k, \quad k = 1, \ldots, K, \tag{47}$$

and the (travel) disutilities:

$$\lambda_{w_k} \equiv \rho_{3k}, \quad k = 1, \ldots, K. \tag{48}$$

Consequently, the equilibrium conditions (25) for the transportation network equilibrium model on the network $\mathcal{G}_{\mathcal{S}}$ state that for every O/D pair w_k and every path connecting the O/D pair w_k:

$$C_{p^t_{imjj'k}} - \lambda_{w_k}$$
$$= \frac{\partial f_{im}}{\partial q_{im}} + \alpha_i e_{im} + \frac{\partial c_{imj}}{\partial q_{imj}} + \alpha_i e_{imj} + \frac{\partial c_j}{\partial h_j} + \beta_j e_j + \frac{\partial c^t_{jk}}{\partial q^t_{jk}} + \hat{c}^t_{jk} + (\beta_j + \eta_k)e^t_{jk} - \lambda_{w_k} \begin{cases} = 0, & \text{if} \quad x^*_{p^t_{imjj'k}} > 0, \\ \geq 0, & \text{if} \quad x^*_{p^t_{imjj'k}} = 0. \end{cases} \tag{49}$$

We now show that the variational inequality formulation of the equilibrium conditions (49) in link form as in (27) is equivalent to the variational inequality (17) governing the supply chain network equilibrium with manufacturing plants and environmental concerns.

For the transportation network equilibrium problem on $\mathcal{G}_{\mathcal{S}}$, according to Theorem 2, we have that a link flow and travel disutility pattern $(f^*, d^*) \in \mathcal{K}^7$ is an equilibrium (according to (49)), if and only if it satisfies the variational inequality:

$$\sum_{i=1}^{I} c_{a_i}(f^{1*}) \times (f_{a_i} - f^*_{a_i}) + \sum_{i=1}^{I}\sum_{m=1}^{M} c_{a_{im}}(f^{2*}) \times (f_{a_{im}} - f^*_{a_{im}}) + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J} c_{a_{imj}}(f^{3*}) \times (f_{a_{imj}} - f^*_{a_{imj}})$$

$$+ \sum_{jj'=11'}^{JJ'} c_{a_{jj'}}(f^{4*}) \times (f_{a_{jj'}} - f^*_{a_{jj'}}) + \sum_{j'=1'}^{J'}\sum_{k=1}^{K}\sum_{t=1}^{T} c_{a^t_{j'k}}(f^{5*}) \times (f_{a^t_{j'k}} - f^*_{a^t_{j'k}})$$

$$- \sum_{k=1}^{K} \lambda_{w_k}(d^*) \times (d_{w_k} - d^*_{w_k}) \geq 0, \quad \forall (f, d) \in \mathcal{K}^7. \tag{50}$$

After the substitution of (34)–(45) and (47)–(48) into (50), we have the following variational inequality: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in \mathcal{K}^5$ satisfying:

$$\sum_{i=1}^{I}\sum_{m=1}^{M}[\frac{\partial f_{im}(q_i^*)}{\partial q_{im}} + \alpha_i e_{im}] \times [q_{im} - q^*_{im}] + \sum_{j=1}^{J}[\frac{\partial c_j(h^*)}{\partial h_j} + \beta_j e_j] \times [h_j - h^*_j]$$

$$+ \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}[\frac{\partial c_{imj}(q^*_{imj})}{\partial q_{imj}} + \alpha_i e_{imj}] \times [q_{imj} - q^*_{imj}]$$

$$+ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}[\frac{\partial c^t_{jk}(q^{t*}_{jk})}{\partial q^t_{jk}} + \hat{c}^t_{jk}(Q^{2*}) + (\beta_j + \eta_k)e^t_{jk}] \times [q^t_{jk} - q^{t*}_{jk}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d^*_k] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d) \in \mathcal{K}^5. \tag{51}$$

Variational inequality (51) is precisely variational inequality (17) governing the supply chain network equilibrium. Hence, we have the following result:

Theorem 3

A solution $(q^*, h^*, Q^{1^*}, Q^{2^*}, d^*) \in \mathcal{K}^5$ of the variational inequality (17) governing the supply chain network equilibrium coincides with the (via (34)–(45) and (47)–(48)) feasible link flow and travel demand pattern for the supernetwork $\mathcal{G}_{\mathcal{S}}$ constructed above and satisfies variational inequality (50). Hence, it is a transportation network equilibrium according to Theorem 2.

We now further discuss the interpretation of the supply chain network equilibrium conditions. These conditions define the supply chain network equilibrium in terms of paths and path flows, which, as shown above, coincide with Wardrop's (1952) first principle of user-optimization in the context of transportation networks over the network given in Figure 2. Hence, we now have an entirely new interpretation of supply network equilibrium with environmental concerns which states that only minimal cost paths will be used from the super source node 0 to any destination node. Moreover, the cost on the utilized paths for a particular O/D pair is equal to the disutility (or the demand market price) that the users are willing to pay.

In Section 5, we will show how Theorem 3 can be utilized to exploit algorithmically the theoretical results obtained above when we compute the equilibrium patterns of numerical supply chain network examples using an algorithm previously used for the computation of elastic demand transportation network equilibria. Of course, existence and uniqueness results obtained for elastic demand transportation network equilibrium models as in Dafermos (1982) as well as stability and sensitivity analysis results (see also Nagurney and Zhang (1996)) can now be transferred to sustainable supply chain networks using the formalism/equivalence established above.

5. Computations

In this Section, we provide numerical examples to demonstrate how the theoretical results in this paper can be applied in practice. We utilize the Euler method for our numerical computations. The Euler method is induced by the general iterative scheme of Dupuis and Nagurney (1993) and has been applied by Nagurney and Zhang (1996) to solve variational inequality (26) in path flows (equivalently, variational inequality (27) in link flows). Convergence results can be found in the above references.

The Euler Method

For the solution of (26), the Euler method takes the form: at iteration τ compute the path flows for paths $p \in P$ (and the travel demands) according to:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + \alpha_\tau(\lambda_w(d^\tau) - C_p(x^\tau))\}. \tag{52}$$

The simplicity of (52) lies in the explicit formula that allows for the computation of the path flows in closed form at each iteration. The demands at each iteration simply satisfy (23) and this expression can be substituted into the $\lambda_w(\cdot)$ functions.

The Euler method was implemented in FORTRAN and the computer system used was a Sun system at the University of Massachusetts at Amherst. The convergence criterion utilized was that the absolute value of the path flows between two successive iterations differed by no more than 10^{-4}. The sequence $\{\alpha_\tau\}$ in the Euler method (cf. (52)) was set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. The Euler method was initialized by setting the demands equal to 100 for each O/D pair with the path flows equally distributed. The Euler method was also used to compute solutions to electric power supply chain network examples, reformulated as transportation network equilibrium problems in Wu et al. (2006).

In all the numerical examples, the supply chain network consisted of two manufacturers, with two manufacturing plants each, two retailers, one transportation/transaction mode, and two demand markets as depicted in Figure 3. The supernetwork representation which allows for the transformation (as proved in Section 4) to a transportation network equilibrium problem is given also in Figure 3. Hence, in the numerical examples (see also Figure 2) we had that: $I = 2$, $M = 2$, $J = 2$, $J' = 2'$, $K = 2$, and $T = 1$.

The notation is presented for the examples in the form of the supply chain network equilibrium model of Section 2. The equilibrium solutions for the examples, along with the translations of the computed equilibrium link flows, and the travel demands (and disutilities) into the equilibrium supply chain flows and prices are given in Table 2.

Example 1

The data for the first numerical example is given below. In order to construct a benchmark, we assumed that all the weights associated with the environmental criteria were equal to zero, that is, we set: $\alpha_1 = \alpha_2 = 0$, $\beta_1 = \beta_2 = 0$, and $\eta_1 = \eta_2 = 0$.

The production cost functions for the manufacturers were given by:

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21},$$

$$f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

The transportation/transaction cost functions faced by the manufacturers and associated with transacting with the retailers were given by:

$$c_{imj}(q_{imj}) = .5q_{imj}^2 + 3.5q_{imj}, \quad i = 1; m = 1,2; j = 1,2;$$

$$c_{imj}(q_{imj}) = .5q_{imj}^2 + 2q_{imj}, \quad i = 2; m = 1,2; j = 1,2.$$

The operating costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand market price functions at the demand markets were:

$$\rho_{31}(d) = -d_1 + 500, \quad \rho_{32} = -d_2 + 500,$$

and the unit transportation/transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}_{jk}^1(q_{jk}^1) = q_{jk}^1 + 5, \quad j = 1,2; k = 1,2.$$

All other transportation/transaction costs were assumed to be equal to zero. We assumed that the manufacturing plants emitted pollutants where $e_{11} = e_{12} = e_{21} = e_{22} = 5$.

We utilized the supernetwork representation of this example depicted in Figure 3 with the links enumerated as in Figure 3 in order to solve the problem via the Euler method. Note that there are 13 nodes and 20 links in the supernetwork in Figure 3. Using the procedure outlined in Section 4, we defined O/D pair $w_1 = (0, z_1)$ and O/D pair $w_2 = (0, z_2)$ and we associated the O/D pair travel disutilities with the demand market price functions as in (48) and the user link travel cost functions as given in (41)–(45) (analogous constructions were done for the subsequent examples).

The Euler method converged in 56 iterations and yielded the equilibrium solution given in Table 2 (cf. also the supernetwork in Figure 3). In Table 2 we also provide the translations of the computed equilibrium pattern(s) into the supply chain network flow, demand and price notation using (34)–(40) and (47)–(48).

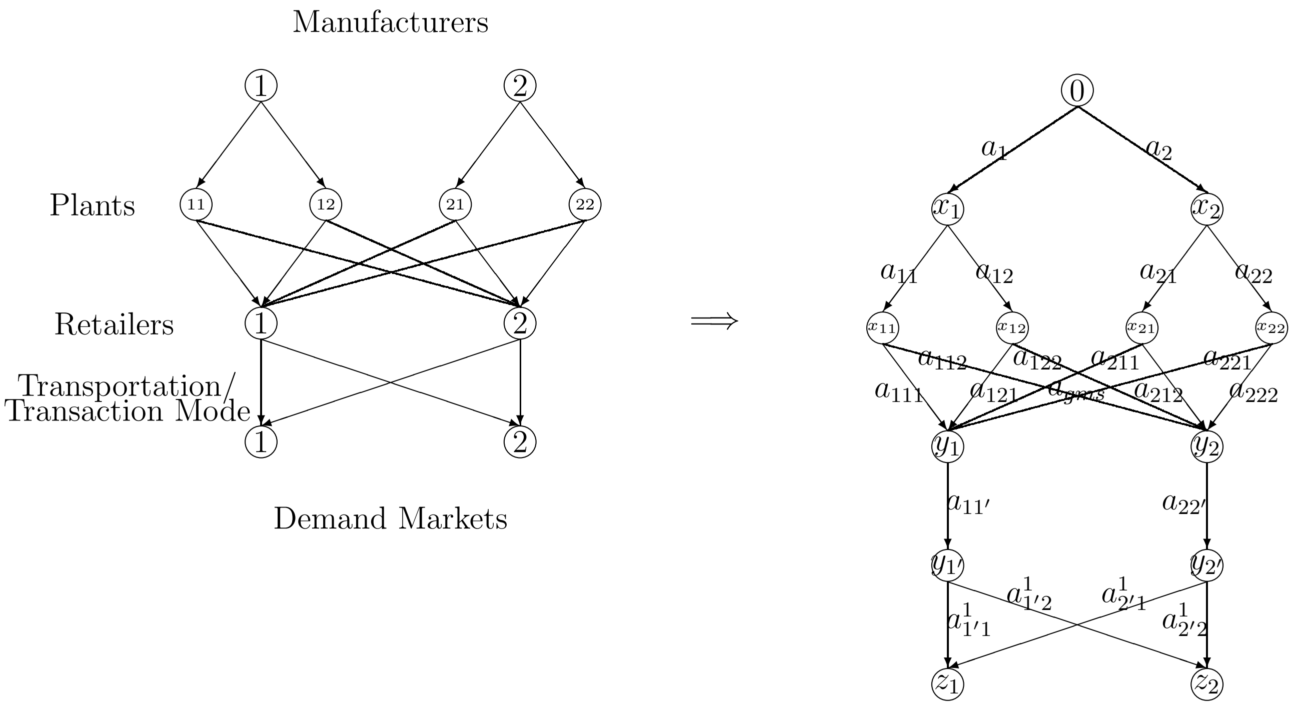

Figure 3: Supply Chain Network and Corresponding Supernetwork $\mathcal{G}_{\mathcal{S}}$ for the Numerical Examples

We don't report the path flows due to space limitations (there are eight paths connecting each O/D pair) but note that all paths connecting each O/D pair were used, that is, had positive flow and the travel costs for paths connecting each O/D pair were equal to the travel disutility for that O/D pair. The optimality/equilibrium conditions were satisfied with excellent accuracy. The total amount of emissions in this example was: $e_{11}q^*_{11} + e_{12}q^*_{12} + e_{21}q^*_{21} + e_{22}q^*_{22} = 1,089$.

Example 2

We then solved the following variant of Example 1. We kept the data identical to that in Example 1 except that we assumed now that the weights associated with the environmental criteria of the manufacturers were: $\alpha_1 = \alpha_2 = 1$, with all other weights equal to zero. The complete computed solution is now given.

The Euler method converged in 56 iterations and yielded the equilibrium link flows, travel demands and travel disutilities (cf. Figure 3) given in Table 2. Although we do not report the equilibrium path flows, due to space constraints, we note that, in this example, all paths were again used. The total emissions generated were equal to: $1,077.85$ and, hence, as expected, given that both manufacturers now associated positive weights with the environmental criteria, the total emissions were reduced, relative to the amount emitted in Example 1.

Example 3

Example 3 was constructed as follows from Example 2. The data were identical to the data in Example 2, except that we now assumed that the first retailer used a polluting mode of transportation to deliver the product to the consumers at the demand markets so that $e^1_{11} = e^1_{12} = 10$. We also assumed that the consumers were now environmentally conscious and that the weights associated with the environmental criteria at the demand markets were $\eta_1 = \eta_2 = 1$.

The Euler method converged in 67 iterations and yielded the new equilibrium pattern given in Table 2. In this example (as in Examples 1 and 2), all paths connecting each O/D pair were used, that is, they had positive equilibrium flows. The total amount of pollution emitted was now: $e_{11}q^*_{11} + e_{12}q^*_{12} + e_{21}q^*_{21} + e_{22}q^*_{22} + e^{1*}_{11}q^1_{11} + e^{1*}_{12}q^{1*}_{12} = 1,585.95$.

Example 4

In Example 4, we set out to ask the question, how high would η_1 and η_2 have to be so that the demand markets did not utilize retailer 1 at all and the associated link flows would be zero on those transportation/transaction links? We conducted simulations and found that with $\eta_1 = \eta_2 = 32$ the desired result was achieved (with $\eta_1 = \eta_2 = 30$ there were still positive flows on those polluting links).

The Euler method converged in 102 iterations with the computed equilibrium link flows, travel demands and travel disutilities given in Table 2, along with the equivalent equilibrium supply chain network flows/transactions, demands, and prices. There were four paths used (and four not used) in each O/D pair. The total amount of emissions were now: 756.70. Hence, environmentally conscious consumers could significantly reduce the environmental emissions through the economics and the underlying decision-making behavior in the supply chain network.

Table 2: Equilibrium Solutions of Examples 1, 2, 3, and 4

Equilibrium Values	Example 1	Example 2	Example 3	Example 4
$f^*_{a_1} = q^*_1$	48.17	47.68	47.17	42.04
$f^*_{a_2} = q^*_2$	169.62	167.89	166.20	109.34
$f^*_{a_{11}} = q^*_{11}$	33.37	33.03	32.69	25.87
$f^*_{a_{12}} = q^*_{12}$	14.80	14.65	14.48	16.37
$f^*_{a_{21}} = q^*_{21}$	33.71	33.37	33.02	26.17
$f^*_{a_{22}} = q^*_{22}$	135.91	134.53	133.17	83.17
$f^*_{a_{11'}} = h^*_1$	108.90	107.79	103.82	0.00
$f^*_{a_{22'}} = h^*_2$	108.90	107.79	109.54	151.58
$f^*_{a_{111}} = q^*_{111}$	16.69	16.52	15.60	0.00
$f^*_{a_{112}} = q^*_{112}$	16.69	16.52	17.09	25.87
$f^*_{a_{121}} = q^*_{121}$	7.40	7.32	6.61	0.00
$f^*_{a_{122}} = q^*_{122}$	7.40	7.32	7.87	16.37
$f^*_{a_{211}} = q^*_{211}$	16.85	16.68	15.77	0.00
$f^*_{a_{212}} = q^*_{212}$	16.85	16.68	17.25	26.17
$f^*_{a_{221}} = q^*_{221}$	67.96	67.26	65.84	0.00
$f^*_{a_{222}} = q^*_{222}$	67.96	67.26	67.33	83.17
$f^*_{a^1_{1'1}} = q^{1*}_{11}$	54.45	53.89	51.91	0.00
$f^*_{a^1_{1'2}} = q^{1*}_{12}$	54.45	53.89	51.91	0.00
$f^*_{a^1_{2'1}} = q^{1*}_{21}$	54.45	53.89	54.77	75.79
$f^*_{a^1_{2'2}} = q^{1*}_{22}$	54.45	53.89	54.77	75.79
$d^*_{w_1} = d^*_1$	108.90	107.79	106.68	75.79
$d^*_{w_2} = d^*_2$	108.90	107.79	106.68	75.79
$\lambda_{w_1} = \rho_{31}$	391.11	392.23	393.30	424.21
$\lambda_{w_2} = \rho_{32}$	391.11	392.23	393.30	424.21

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