A Supply Chain Network Game Theory Model with Product Differentiation, Outsourcing of Production and Distribution, and Quality and Price Competition

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Abstract: In this paper, we develop a supply chain network game theory model with product differentiation, possible outsourcing of production and distribution, and quality and price competition. The original firms compete with one another in terms of in-house quality levels and product flows, whereas the contractors, aiming at maximizing their own profits, engage in competition for the outsourced production and distribution in terms of prices that they charge and their quality levels. The solution of the model provides each original firm with its optimal in-house quality level as well as its optimal in-house and outsourced production and shipment quantities that minimize the total cost and the weighted cost of disrepute, associated with lower quality levels and the impact on a firm's reputation. The governing equilibrium conditions of the model are formulated as a variational inequality problem. An algorithm, which provides a discrete-time adjustment process and tracks the evolution of the product flows, quality levels, and prices over time, is proposed and convergence results given. Numerical examples are provided to illustrate how such a supply chain network game theory model can be applied in practice.

The model is relevant to products ranging from pharmaceuticals to fast fashion to high technology products.

Keywords: outsourcing, manufacturing, distribution, supply chain management, supply chain networks, product differentiation, quality competition, price competition, cost of disrepute, game theory, variational inequalities

1. Introduction

Outsourcing of production has become prevalent and has exerted an immense impact on manufacturing industries as wide-ranging as pharmaceuticals to fast fashion to high technology since the mid-1990s. One of the main arguments for the outsourcing of production, as well as distribution, is cost reduction. In addition, outsourcing, as a strategy, may enhance a firm's competitiveness by improving manufacturing efficiency, by reducing excess production capacity, and/or by diverting resources, both human and natural, to focus on a firm's core competitive competencies and advantages (Sink and Langley (1997)). In addition, depending upon the location of the outsourcing of manufacturing, firms may also obtain benefits from supportive government policies (Zhou (2007)).

Outsourcing thrives particularly in the fashion industry, as lower labor costs can always be found in various regions globally. According to the ApparelStats Report released by the AAFA (American Apparel and Footwear Association), 97.7 percent of the apparel sold in the United States in year 2011 was produced outside the US (AAFA (2012)). In addition to the garment industry, in the pharmaceutical industry, more than 40 percent of the drugs sold in the US are from foreign sources, with 80 percent of active ingredients produced mainly in China and India in 2010 (Economy In Crisis (2010)). Moreover, in the electronics industry, in the fourth quarter of year 2012, 100 percent of the 26.9 million iPhones sold by Apple were designed in California, but assembled in China (Apple (2012) and Rawson (2012)). In the automotive industry, it is forecasted that the global automotive component outsourcing market will continue to grow and will reach 855 billion US dollars by year 2016 (Yahoo (2013a)).

As the volume of outsourcing has increased, interestingly, the supply chain networks weaving the original manufacturers and the contractors are becoming increasingly complex. Firms may no longer outsource exclusively to specific contractors, and there may be contractors engaging with multiple firms, who are actually competitors. As an illustration, the US head office of Volvo has been outsourcing the production of components to companies such as Minda HUF, Visteon, Arvin Meritor, and Rico Auto. These major sources, in turn, obtain almost 100 percent of their components, ranging from the engine parts, the suspension and braking parts, to the electric parts, from Indian contractors (Klum (2007)). Moreover, in the IT industry, Apple, Compaq, Dell, Gateway, Lenovo, and Hewlett-Packard are consumers of Quanta Computer Incorporated, a Taiwan-based Chinese manufacturer of notebook computers (The New York Times (2002)), and Foxconn, another Taiwan-based manufacturer, is currently producing tablet computers for Apple, Google, Android, and Amazon (Nystedt (2010) and Topolsky (2010)).

However, parallel to the dynamism of and growth in outsourcing, issues of quality have gradually emerged. In 2003, the suspension of the license of Pan Pharmaceuticals, the world's fifth largest contract manufacturer of health supplements, due to quality failure, caused costly consequences for the original firms in terms of product recalls and credibility losses (Allen (2003)). In 2007, the international toy giant Mattel recalled 19 million outsourced toy cars because of lead paint and small, poorly designed magnets, which could harm children, if ingested (The New York Times (2007)). In 2010, the suicide scandal at Foxconn, which revealed the poor working conditions in its contract manufacturing plants, led to negative impacts on the reputation of multiple original electronic product manufacturers (McEntegart (2010)).

Moreover, the garment industry has suffered numerous recent instances of catastrophic failures in the form of quality associated with the outsourcing of production, notably, in Bangladesh. A conflagration at an unauthorized sub-contracted garment factory producing for Walmart, Sears, Disney, and other apparel corporations in Bangladesh took at least 112 lives in 2012, which was the deadliest in the history of Bangladesh (Yahoo (2012)). Only five months later, a similar illegally constructed commercial building making clothes for major European and American brands collapsed. 1,127 people were killed and 2,500 injured, which makes this the deadliest accidental structural failure in modern human history (BBC (2013) and Yahoo (2013b)).

It is clear that cost reduction due to outsourcing, including offshore outsourcing, is an extremely important point of consideration for the original manufacturing firms. However, with the increasing volume of outsourcing and the increasing complexity of the supply chain networks associated with outsourcing, it is imperative for firms to be able to rigorously assess not only the possible gains due to outsourcing but also the potential costs associated with disrepute (loss of reputation) resulting from the possible quality degradation due to outsourcing.

Outsourcing, in an era of globalization, has drawn the attention of practitioners, as well as academics and has been a topic of numerous studies in the literature of supply chain management. However, only a small portion of the supply chain literature focuses on quality related issues associated with outsourcing. Kaya and Özer (2009) studied the factors that influence quality risk in outsourcing through supply chain models with one original firm and one contractor. The paper by Xiao, Xia, and Zhang (2014) examined outsourcing decisions for two competing manufacturers who have quality improvement opportunities and product differentiation. Nagurney, Li, and Nagurney (2013) developed a pharmaceutical supply chain network model with outsourcing and quality related risk in a network with one original firm

and multiple contractors.

This paper, in contrast to the above-noted ones, focuses on the supply chain network of multiple competing original firms and the contractors. For each contractor, the number of contracted original firms is not predetermined. Some contractors may end up contracting with one or more original firms, while others may contract to none. At the same time, each original firm can also outsource the production of all of its products, along with their delivery to the demand markets, or outsource some to any number of contractors, or choose not to outsource. Furthermore, this model provides each original firm with the equilibrium in-house quality level and the equilibrium make-or-buy and contractor-selection policy, with the demand for its product being satisfied in multiple demand markets. We assume that the original firms compete in the sense of Nash (1950, 1951) - Cournot (1838). Each firm aims at determining its equilibrium in-house quality level, in-house production quantities and shipments, and outsourcing quantities, which satisfy demand requirements, so as to minimize its total cost and the weighted cost of disrepute. The contractors, in turn, are competing a la Nash (1950, 1951) - Bertrand (1883) in determining their optimal quality and price levels in order to maximize their individual total profits.

In our model, the products from different original firms are considered to be *differentiated by their brands* (see also, e.g., Nagurney et al. (2013) and Yu and Nagurney (2013)). When consumers observe a brand of a product, they consider the quality, function, and reputation of that particular brand name and the product. With outsourcing, chances are that the product was manufactured by a completely different company than the brand indicates, but the level of quality and the reputation associated with the outsourced product still remain with the "branded" original firm. If a product is recalled for a faulty part and that part was outsourced, the original firm is the one that carries the burden of correcting its damaged reputation.

In addition, although outsourcing may bring along with it quality risk, it is not the only source of potential quality problems. In-house quality failures may also occur. Therefore, we also consider in-house quality as one of the strategic decision variables for each original firm. Moreover, we also associate quality with respect to the distribution/transport activities of the products to the demand markets. We accomplish this by having the transportation/distribution functions depend explicitly on both flows and quality levels, with the assumption that the product will be delivered at the same quality level that it was produced at. Quality, as well as price, have been identified empirically as critical factors in transport mode selection for product/goods delivery (cf. Floden, Barthel, and Sorkina (2010) and Saxin, Lammgard, and Floden (2005) and the references therein).

A review of the literature on quality competition follows. As pioneers in the study of quality competition, Spence (1975), Sheshinski (1976), and Mussa and Rosen (1978) discussed firms' decisions on price and quality in a quality differentiated monopoly market with heterogeneous customers. Shortly thereafter, Dixit (1979) and Gal-or (1983) initiated the study of quantity and quality competition in an oligopolistic market with multiple firms, where several symmetric cases of oligopolistic equilibria were considered. In addition, Brekke, Siciliani, and Straume (2010) investigated the relationship between competition and quality via a spatial price-quality competition model. Nagurney and Li (2014) developed a dynamic model of Cournot-Nash oligopolistic competition with product differentiation and quality competition in a network framework. Others who have contributed to this general topic include: Ronnen (1991), Banker, Khosla, and Sinha (1998), Johnson and Myatt (2003), and Acharyya (2005).

This paper is organized as follows. In Section 2, we describe the decision-making behavior of the original firms, who compete in quantity and quality, and that of the contractors, who compete in price and quality. Then, we construct the supply chain network game theory model with product differentiation, the possible outsourcing of production and distribution, and quality and price competition. We obtain the governing equilibrium conditions, and derive the equivalent variational inequality formulation. Here, for consistency, we define and quantify the quality levels, the quality costs, and the disrepute cost in a manner similar to that in Nagurney and Li (2014) and Nagurney, Li, and Nagurney (2013). In the former paper, there was no outsourcing, whereas in the latter, there was no competition among original firms since only a single one was considered. Moreover, in this paper, the quality levels of the original firms' products are no longer assumed to be perfect as was done in the case of a single firm (Nagurney, Li, and Nagurney (2013)), but, rather, are strategic variables of the firms.

In Section 3, we describe the algorithm, the Euler method, which yields closed form expressions, at each iteration, for the prices and the quality levels, with the product flows being solved exactly by an equilibration algorithm. The Euler method, in the context of our supply chain network game theory model, can be interpreted as a discrete-time adjustment process until the equilibrium state is achieved. Convergence results are also provided. It is applied in Section 4 to compute solutions to numerical examples, along with sensitivity analysis, in order to demonstrate the generality and the applicability of the proposed framework. In Section 5, we summarize our results and present our conclusions.

2. The Supply Chain Network Game Theory Model with Product Differentiation, Outsourcing of Production and Distribution, and Quality and Price Competition

In this Section, we develop the supply chain network game theory model with product differentiation, possible outsourcing, price and quality competition among the contractors, and quantity and quality competition among the original firms. We consider a finite number of I original firms, with a typical firm denoted by i, who compete noncooperatively. The products of the I firms are not homogeneous but, rather, are differentiated by brands. Firm i; $i = 1, \ldots, I$, is involved in the processes of in-house manufacturing and distribution of its brand name product, and may subcontract its manufacturing and distribution activities to contractors who may be located overseas. We seek to determine the optimal product flows from each firm to its demand markets, along with the prices the contractors charge the firms, and the quality levels of the in-house manufactured products and the outsourced products.

For clarity and definiteness, we consider the supply chain network topology of the I firms depicted in Figure 1. Each firm i; $i = 1, \ldots, I$, is considering in-house and outsourcing manufacturing facilities and serves the same n_R demand markets. A link from each top-tiered node i, representing original firm i, is connected to its in-house manufacturing facility node M^i. The in-house distribution activities of firm i, in turn, are represented by links connecting M^i to the demand nodes: $R_1, \ldots, R_{n_R}$.

In this model, we capture the possible outsourcing of the products from the I firms in terms of their production and delivery. As depicted in Figure 1, there are n_O contractors available to each of the I firms. Each firm may potentially contract to any of these contractors who then produce and distribute the product to the same n_R demand markets. In Figure 1, hence, there are additional links from each top-most node i; $i = 1, \ldots, I$, to the n_O contractor nodes, $O_1, \ldots, O_{n_O}$, each of which corresponds to the transaction activity of firm i with contractor j. The next set of links, which emanates from the contractor nodes to the demand markets, reflect the production and delivery of the outsourced products to the n_R demand markets.

As shown in Figure 1, the outsourced flows of different firms are represented by links with different colors, for convenience and clarity of depiction, which indicates that, in the processes of transaction and outsourcing of manufacturing and distribution, the outsourced products are still differentiated by brands. In Figure 1, we use red links to denote the outsourcing flows of Firm 1 with blue links referring to those of Firm I. The mathematical notation given below explicitly handles such options.
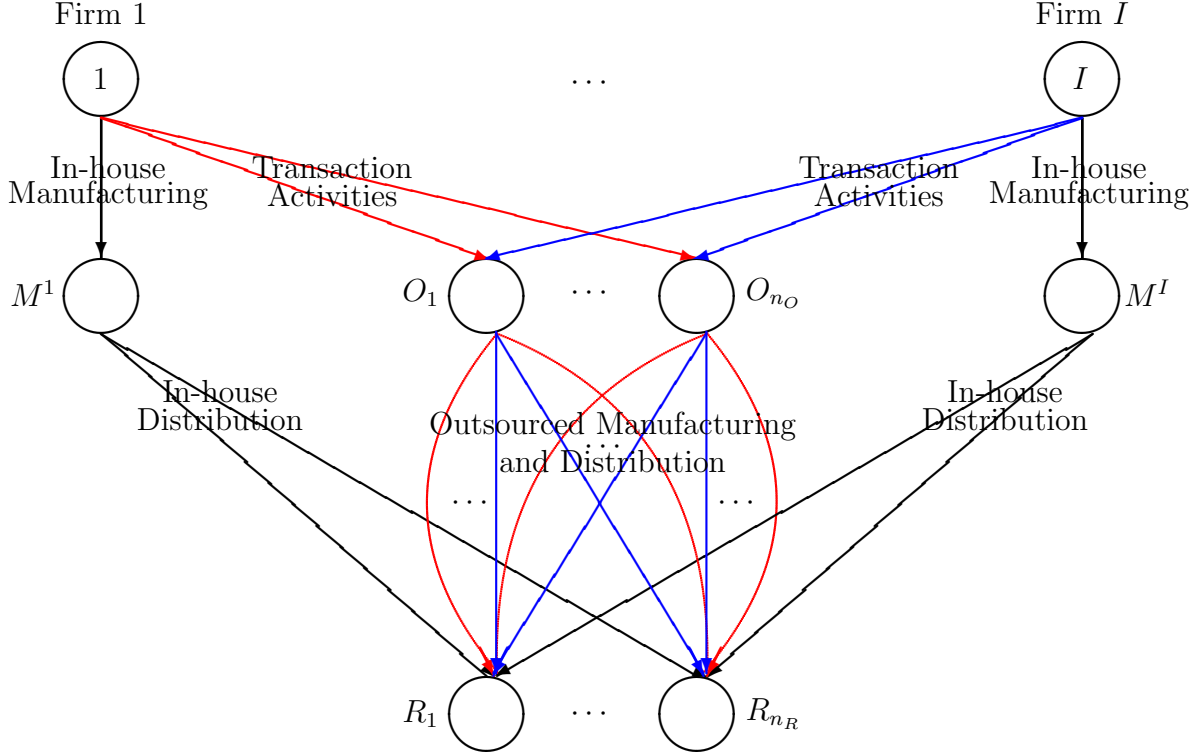
Figure 1: The Supply Chain Network Topology with Outsourcing

Let $G = [N, L]$ denote the graph consisting of nodes $[N]$ and directed links $[L]$ as in Figure 1. The top set of links consists of the manufacturing links, whether in-house or outsourcing, whereas the next set of links consists of the associated distribution links. For simplicity, we let $n = 1 + n_O$, where n_O is the number of potential contractors, denote the number of manufacturing plants, whether in-house or belonging to the contractors.

The notation for the model is given in Table 1. The vectors are assumed to be column vectors. The optimal/equilibrium solution is denoted by "*". Inn_R, In_R, In_Rn_O, In_O, In, and nn_R are equivalent to $I \times n \times n_R$, $I \times n_R$, $I \times n_R \times n_O$, $I \times n_O$, $I \times n$, and $n \times n_R$, respectively.

Given the impact of quality on production and on distribution costs, the production costs and the distribution costs are not independent of quality in our model. We express the production and distribution costs, whether in-house or associated with outsourcing, as functions that depend on both production quantities and quality levels (see, e.g., Spence (1975), Rogerson (1988), and Lederer and Rhee (1995)). These functions are assumed to be convex in quality and quantity.

Table 1: Notation for the Supply Chain Network Game Theory Model with Outsourcing

Notation	Definition
Q_{ijk}	the nonnegative amount of firm i's product produced at manufacturing plant j, whether in-house or contracted, and delivered to demand market k, where $j = 1, \ldots, n$. For firm i, we group its own $\{Q_{ijk}\}$ elements into the vector Q_i, and group all such vectors for all original firms into the vector Q, where $Q \in R_+^{Inn_R}$. All in-house quantities are grouped into the vector $Q^1 \in R_+^{In_R}$, with all outsourcing quantities into the vector $Q^2 \in R_+^{In_R n_O}$.
d_{ik}	the demand for firm i's product at demand market k; $k = 1, \ldots, n_R$.
q_i	the nonnegative quality level of firm i's product produced in-house. We group the $\{q_i\}$ elements into the vector $q^1 \in R_+^I$.
q_{ij}	the nonnegative quality level of firm i's product produced by contractor j; $j = 1, \ldots, n_O$. We group all the $\{q_{ij}\}$ elements for firm i's product into the vector $q_i^2 \in R_+^{n_O}$. For each contractor j, we group its own $\{q_{ij}\}$ elements into the vector q_j, and then group all such vectors for all contractors into the vector $q^2 \in R_+^{In_O}$.
q	We group all the q_i and q_{ij} into the vector $q \in R_+^{In}$.
π_{ijk}	the price charged by contractor j; $j = 1, \ldots, n_O$, for producing and delivering a unit of firm i's product to demand market k. We group the $\{\pi_{ijk}\}$ elements for contractor j into the vector $\pi_j \in R_+^{In_R}$, and group all such vectors for all the contractors into the vector $\pi \in R_+^{In_R n_O}$.
$f_i(Q^1, q^1)$	the total in-house production cost of firm i.
$c_i(q^1)$	the total quality cost of firm i.
$tc_{ij}(\sum_{k=1}^{n_R} Q_{i,1+j,k})$	the total transaction cost associated with firm i transacting with contractor j; $j = 1, \ldots, n_O$. The detailed definition will be given later.
$c_{ik}(Q^1, q^1)$	the total transportation cost associated with delivering firm i's product manufactured in-house to demand market k; $k = 1, \ldots, n_R$.
$sc_{ijk}(Q^2, q^2)$	the total cost of contractor j; $j = 1, \ldots, n_O$, to produce and distribute the product of firm i to demand market k.
$\hat{c}_j(q^2)$	the total quality cost faced by contractor j; $j = 1, \ldots, n_O$.
$oc_{ijk}(\pi_{ijk})$	the opportunity cost associated with pricing the product of firm i at π_{ijk}, and delivering to demand market k, by contractor j; $j = 1, \ldots, n_O$.
$q_i'(Q_i, q_i, q_i^2)$	the average quality level of firm i's product (cf. (3)).
$dc_i(q_i'(Q_i, q_i, q_i^2))$	the cost of disrepute of firm i.

Although the products of the I firms are differentiated, in the processes of manufacturing and delivery, common factors, such as resource and technology, may be utilized. Hence, in this model, we allow in-house production cost and distribution cost functions to depend on the vectors of Q^1 and q^1. For the same reason, the total outsourcing production and distribution cost functions are assumed to be functions depending on the vectors of Q^2 and q^2, and the quality cost functions of the original firms and those of the contractors also depend on the vector of q^1 and of q^2, respectively.

In addition, the transaction costs between the original firms and the contractors are also considered. Transaction cost is the "cost in making contract" (Coase (1937)), and includes the costs that occur in the processes of evaluating suppliers, negotiation, the monitoring of and the enforcing of contracts in order to ensure the quality, as widely applied in the outsourcing literature (cf. Heshmati (2003), Liu and Nagurney (2013), and Nagurney, Li, and Nagurney (2013)). Through the evaluation processes of the contractors and the transaction costs, the production, distribution, and quality cost information of each potential contractor are assumed to be known by the original firms.

2.1 The Behavior of the Original Firms and Their Optimality Conditions

Recall that the quality level of firm i's product produced in-house is denoted by q_i, where $i = 1, \ldots, I$, and the quality level of firm i's product produced by contractor j is denoted by q_{ij}, where $j = 1, \ldots, n_O$. Both vary from a 0 percent defect-free level to a 100 percent defect-free level, so that, respectively,

$$0 \leq q_{ij} \leq q^U, \quad i = 1, \ldots, I; j = 1, \ldots, n_O, \tag{1}$$

$$0 \leq q_i \leq q^U, \quad i = 1, \ldots, I, \tag{2}$$

where q^U is the value representing perfect quality level associated with the 100% defect-free level.

The average quality level of firm i's product is, hence, an average quality level determined by the in-house quality level, the in-house product flows, the quality levels of the contractors, and the outsourced product flows. Thus, the average quality level for firm i's product, both in-house and outsourced, can be expressed as

$$q_i'(Q_i, q_i, q_i^2) = \frac{\sum_{k=1}^{n_R} \sum_{j=2}^{n} Q_{ijk} q_{i,j-1} + \sum_{k=1}^{n_R} Q_{i1k} q_i}{\sum_{k=1}^{n_R} d_{ik}}, \quad i = 1, \ldots, I. \tag{3}$$

Following Nagurney, Li, and Nagurney (2013), we assume that the disrepute cost of firm i, $dc_i(q_i'(Q_i, q_i, q_i^2))$, is a monotonically decreasing function of the average quality level.

Here, however, we no longer assume as was done in Nagurney, Li, and Nagurney (2013), who considered only a single original firm, that the original firms produce their branded products with perfect quality. Hence, the average quality level in (3) is a generalization of the corresponding one in that paper.

Each original firm i selects the product flows Q_i and the in-house quality level q_i, whereas each contractor j, who competes for contracts in quality and price, selects its outsourcing quality level vector q_j and outsourcing price vector π_j.

Firm i's utility function is denoted by U_i^1, where $i = 1, \ldots, I$. The objective of original firm i is to maximize its utility (cf. (4) below) represented by minus its total costs that include the production cost, the quality cost, the transportation costs, the payments to the contractors, the transaction costs, along with the weighted cost of disrepute, with the non-negative term ω_i denoting the weight that firm i imposes on the disrepute cost function. Note that, the production and transportation cost functions only capture the costs of production and delivery, and depend on both the quantities and the quality levels. However, the quality cost is the cost associated with quality management and reflects the "cost incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved" (ASQC (1971) and BS (1990)), and is above the costs of production and delivery activities. Thus, the production and the transportation costs and the quality cost are entirely different costs, and they do not overlap.

Hence, firm i seeks to

$$\text{Maximize}_{Q_i, q_i} \quad U_i^1 = -f_i(Q^1, q^1) - c_i(q^1) - \sum_{k=1}^{n_R} c_{ik}(Q^1, q^1) - \sum_{j=1}^{n_O} \sum_{k=1}^{n_R} \pi_{ijk}^* Q_{i,1+j,k}$$

$$- \sum_{j=1}^{n_O} tc_{ij}\left(\sum_{k=1}^{n_R} Q_{i,1+j,k}\right) - \omega_i dc_i(q_i'(Q_i, q_i, q_i^{2*})) \tag{4}$$

subject to:

$$\sum_{j=1}^{n} Q_{ijk} = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{5}$$

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n; k = 1, \ldots, n_R, \tag{6}$$

and (2).

Note that, according to (4), the prices and the contractors' quality levels are evaluated at their equilibrium values.

We assume that all the cost functions in (4) are continuous, continuously differentiable, and convex. The original firms compete in a noncooperative in the sense of Nash (1950,

1951) with each one trying to maximize its own utility. The strategic variables for each original firm i are all the in-house and the outsourcing flows produced and shipped by firm i and its in-house quality level.

We define the feasible set K_i as $K_i = \{(Q_i, q_i) | Q_i \in R_+^{nn_R}$ with (5) satisfied and q_i satisfying (2)}. All $K_i; i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\mathcal{K}^1 \equiv \Pi_{i=1}^I K_i$.

Definition 1: Supply Chain Network Cournot-Nash Equilibrium with Product Differentiation and Outsourcing of Production and Distribution

An in-house and outsourced product flow pattern and in-house quality level $(Q^*, q^{1^*}) \in \mathcal{K}^1$ is said to constitute a Cournot-Nash equilibrium if for each firm $i; i = 1, \ldots, I$,

$$U_i^1(Q_i^*, \hat{Q}_i^*, q_i^*, \hat{q}_i^*, q^{2^*}, \pi_i^*) \geq U_i^1(Q_i, \hat{Q}_i^*, q_i, \hat{q}_i^*, q^{2^*}, \pi_i^*), \quad \forall (Q_i, q_i) \in K_i, \tag{7}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$

$$\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

According to (7), a Cournot-Nash equilibrium is established if no firm can unilaterally improve upon its utility by selecting an alternative vector of in-house or outsourced product flows and quality level.

Variational Inequality Formulations

Next, we derive the variational inequality formulation of the Cournot-Nash equilibrium with product differentiation and outsourcing according to Definition 1 (see Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), Liu and Nagurney (2009), and Cruz, Nagurney, and Wakolbinger (2006)) in the following theorem.

Theorem 1

Assume that, for each firm $i; i = 1, \ldots, I$, the utility function $U_i^1(Q, q^1, q^{2^*}, \pi_i^*)$ is concave with respect to its variables Q_i and q_i, and is continuous and continuously differentiable. Then $(Q^*, q^{1^*}) \in \mathcal{K}^1$ is a Counot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^I \sum_{h=1}^n \sum_{m=1}^{n_R} \frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial Q_{ihm}} \times (Q_{ihm} - Q_{ihm}^*) - \sum_{i=1}^I \frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial q_i} \times (q_i - q_i^*) \geq 0,$$

$$\forall (Q, q^1) \in \mathcal{K}^1, \tag{8}$$

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with notice that: for $h = 1$; $i = 1, \ldots, I$; $m = 1, \ldots, n_R$:

$$-\frac{\partial U_i^1}{\partial Q_{ihm}} = \left[\frac{\partial f_i}{\partial Q_{ihm}} + \sum_{k=1}^{n_R}\frac{\partial c_{ik}}{\partial Q_{ihm}} + \omega_i\frac{\partial dc_i}{\partial q_i'}\frac{\partial q_i'}{\partial Q_{ihm}}\right],$$

for $h = 2, \ldots, n$; $i = 1, \ldots, I$; $m = 1, \ldots, n_R$:

$$-\frac{\partial U_i^1}{\partial Q_{ihm}} = \left[\pi_{i,h-1,m}^* + \frac{\partial tc_{i,h-1}}{\partial Q_{ihm}} + \omega_i\frac{\partial dc_i}{\partial q_i'}\frac{\partial q_i'}{\partial Q_{ihm}}\right],$$

for $i = 1, \ldots, I$:

$$-\frac{\partial U_i^1}{\partial q_i} = \left[\frac{\partial f_i}{\partial q_i} + \frac{\partial c_i}{\partial q_i} + \sum_{k=1}^{n_R}\frac{\partial c_{ik}}{\partial q_i} + \omega_i\frac{\partial dc_i}{\partial q_i'}\frac{\partial q_i'}{\partial q_i}\right].$$

For additional background on the variational inequality problem, we refer the reader to the book by Nagurney (1999).

2.2 The Behavior of the Contractors and Their Optimality Conditions

The objective of contractor j; $j = 1, \ldots, n_O$, is profit maximization. Their revenues are obtained from the purchasing activities of the original firms, while their costs are the costs of production and distribution, the total quality costs, and the opportunity costs. Nagurney, Li, and Nagurney (2013) also utilized opportunity costs on the contractors' side with the definition of opportunity cost being "the loss of potential gain from other alternatives when one alternative is chosen" (New Oxford American Dictionary, 2010). Mankiw (2011) emphasized that general opportunity cost functions include both explicit and implicit costs. Explicit opportunity costs require monetary payment, and include possible anticipated regulatory costs, wage expenses, and the opportunity cost of capital (see Porteus (1986)). Implicit opportunity costs are those that do not require payment, but still need to be monetized to the decision-maker for decision-making. They can include the time and effort put in (see Payne, Bettman, and Luce (1996)), and the profit that the decision-maker could have earned, if he had made other choices (Sandoval-Chavez and Beruvides (1998)).

In this model, the contractors' opportunity costs are functions of the prices that they charge the firms, as in Table 1. If the prices charged are too low, they may not recover all the costs of the contractors', whereas if they are too high, the firms may select another contractor. In our model, these are the only costs that depend on the prices that the contractors charge the firms. Hence, there is no double counting. We note that the concept of opportunity cost (cf. Mankiw (2011)) is very relevant to both economics and operations research. It has been emphasized in competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004). Gan and Litvinov (2003), addressed an energy

application, and also constructed opportunity cost functions that are functions of prices as we consider here.

In this model, each contractor has, as its strategic variables, its quality levels for producing and distributing the original firms' products, and the prices that charges the firms. We denote the utility of each contractor j by U_j^2, with $j = 1, \ldots, n_O$, and note that it represents the profit. Hence, each contractor j; $j = 1, \ldots, n_O$, seeks to:

$$\text{Maximize}_{q_j, \pi_j} \quad U_j^2 = \sum_{k=1}^{n_R} \sum_{i=1}^{I} \pi_{ijk} Q^*_{i,1+j,k} - \sum_{k=1}^{n_R} \sum_{i=1}^{I} sc_{ijk}(Q^{2*}, q^2) - \hat{c}_j(q^2) - \sum_{k=1}^{n_R} \sum_{i=1}^{I} oc_{ijk}(\pi_{ijk}) \tag{9}$$

subject to:

$$\pi_{ijk} \geq 0, \quad j = 1, \ldots, n_O; k = 1, \ldots, n_R, \tag{10}$$

and (1) for each j. are at their equilibrium values.

According to (9), the original firms' outputs are evaluated at the equilibrium, since the contractors do not control these variables, and, hence, must respond to these outputs.

We assume that the cost functions in each contractor's utility function are continuous, continuously differentiable, and convex. The contractors compete in a noncooperative in the sense of Nash (1950, 1951), with each one trying to maximize its own profits.

We define the feasible sets $K_j \equiv \{(q_j, \pi_j) | q_j$ satisfies (1) and π_j satisfies (10) for $j\}$, $\mathcal{K}^2 \equiv \Pi_{j=1}^{n_O} K_j$, and $\mathcal{K} \equiv \mathcal{K}^1 \times \mathcal{K}^2$. All the above-defined feasible sets are convex.

Definition 2: A Bertrand-Nash Equilibrium with Price and Quality Competition

A quality level and price pattern $(q^{2*}, \pi^*) \in \mathcal{K}^2$ is said to constitute a Bertrand-Nash equilibrium if for each contractor j; $j = 1, \ldots, n_O$,

$$U_j^2(Q^{2*}, q_j^*, \hat{q}_j^*, \pi_j^*, \hat{\pi}_j^*) \geq U_j^2(Q^{2*}, q_j, \hat{q}_j^*, \pi_j, \hat{\pi}_j^*), \quad \forall (q_j, \pi_j) \in K_j, \tag{11}$$

where

$$\hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_{n_O}^*),$$

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_O}^*).$$

According to (11), a Bertrand-Nash equilibrium is established if no contractor can unilaterally improve upon its profits by selecting an alternative vector of quality levels or prices charged to the original firms.

Variational Inequality Formulations

Next, we present the variational inequality formulation of the Bertrand-Nash equilibrium according to Definition 2 (see, Bertrand (1883), Nash (1950, 1951), Gabay and Moulin (1980), Nagurney (2006)) in the following theorem.

Theorem 2

Assume that, for each contractor j; $j = 1, \ldots, n_O$, the profit function $U_j^2(Q^{2*}, q^2, \pi)$ is concave with respect to the variables π_j and q_j, and is continuous and continuously differentiable. Then $(q^{2*}, \pi^*) \in \mathcal{K}^2$ is a Bertrand-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{l=1}^{I}\sum_{j=1}^{n_O} \frac{\partial U_j^2(Q^{2*}, q^{2*}, \pi^*)}{\partial q_{lj}} \times (q_{lj} - q_{lj}^*) - \sum_{l=1}^{I}\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \frac{\partial U_j^2(Q^{2*}, q^{2*}, \pi^*)}{\partial \pi_{ljk}} \times (\pi_{ljk} - \pi_{ljk}^*) \geq 0,$$

$$\forall (q^2, \pi) \in \mathcal{K}^2. \tag{12}$$

with notice that: for $j = 1, \ldots, n_O$; $l = 1, \ldots, I$:

$$-\frac{\partial U_j^2}{\partial q_{lj}} = \sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial sc_{ijk}}{\partial q_{lj}} + \frac{\partial \hat{c}_j}{\partial q_{lj}},$$

and for $j = 1, \ldots, n_O$; $l = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$-\frac{\partial U_j^2}{\partial \pi_{ljk}} = \frac{\partial oc_{ljk}}{\partial \pi_{ljk}} - Q_{l,1+j,k}^*.$$

2.3 The Equilibrium Conditions for the Supply Chain Network with Product Differentiation, Outsourcing of Production and Distribution, and Quality Competition

In equilibrium, the optimality conditions for all contractors and the optimality conditions for all the original firms must hold simultaneously, according to the definition below.

Definition 3: Supply Chain Network Equilibrium with Product Differentiation, Outsourcing of Production and Distribution, and Quality and Price Competition

The equilibrium state of the supply chain network with product differentiation, outsourcing of production and distribution, and quality and price competition is one where both variational inequalities (8) and (12) hold simultaneously.

Theorem 3

The equilibrium conditions governing the supply chain network model with product differentiation, outsourcing of production and distribution, and quality competition are equivalent to the solution of the variational inequality problem: determine $(Q^*, q^{1*}, q^{2*}, \pi^*) \in \mathcal{K}$, such that:

$$-\sum_{i=1}^{I}\sum_{h=1}^{n}\sum_{m=1}^{n_R} \frac{\partial U_i^1(Q^*, q^{1*}, q^{2*}, \pi_i^*)}{\partial Q_{ihm}} \times (Q_{ihm} - Q_{ihm}^*) - \sum_{i=1}^{I} \frac{\partial U_i^1(Q^*, q^{1*}, q^{2*}, \pi_i^*)}{\partial q_i} \times (q_i - q_i^*)$$

$$-\sum_{l=1}^{I}\sum_{j=1}^{n_O} \frac{\partial U_j^2(Q^{2*}, q^{2*}, \pi^*)}{\partial q_{lj}} \times (q_{lj} - q_{lj}^*) - \sum_{l=1}^{I}\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \frac{\partial U_j^2(Q^{2*}, q^{2*}, \pi^*)}{\partial \pi_{ljk}} \times (\pi_{ljk} - \pi_{ljk}^*) \geq 0,$$

$$\forall (Q, q^1, q^2, \pi) \in \mathcal{K}. \tag{13}$$

Proof: Summation of variational inequalities (8) and (12) yields variational inequality (13). A solution to variational inequality (13) satisfies the sum of (8) and (12) and, hence, is an equilibrium according to Definition 3.□

Variational inequality (13) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{14}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in Ω-dimensional Euclidean space, where $\Omega = Inn_R + I + In_O + In_O n_R$. Indeed, if we define the column vectors $X \equiv (Q, q^1, q^2, \pi)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X), F_4(X))$, such that:

$$F_1(X) = \left[\frac{\partial U_i^1(Q, q^1, q^2, \pi_i)}{\partial Q_{ihm}}; h = 1, \ldots, n; i = 1, \ldots, I; m = 1, \ldots, n_R \right],$$

$$F_2(X) = \left[\frac{\partial U_i^1(Q, q^1, q^2, \pi_i)}{\partial q_i}; i = 1, \ldots, I \right],$$

$$F_3(X) = \left[\frac{\partial U_j^2(Q^2, q^2, \pi)}{\partial q_{lj}}; l = 1, \ldots, I; j = 1, \ldots, n_O \right],$$

$$F_4(X) = \left[\frac{\partial U_j^2(Q^2, q^2, \pi)}{\partial \pi_{ljk}}; l = 1, \ldots, I; j = 1, \ldots, n_O; k = 1, \ldots, n_R \right], \tag{15}$$

and $\mathcal{K} \equiv \mathcal{K}$ then (13) can be re-expressed as (14).

3. Algorithm

The algorithm that we employed for the computation of the solution for the supply chain network game theory model is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, recall that at iteration τ of the Euler method (see also Nagurney and Zhang (1996)), one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{16}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (14).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other game theory models can be found in Nagurney, Dupuis, and Zhang (1994), and Cruz, Nagurney, and Wakolbinger (2006), Nagurney (2010), and Nagurney and Li (2014).

Note that, at each iteration τ, $X^{\tau+1}$ in (16) is actually the solution to the following strictly convex quadratic programming problem:

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X \rangle - \langle X^\tau - a_\tau F(X^\tau), X \rangle. \tag{17}$$

As for solving (17), in order to obtain the values of the product flows at each iteration τ, we can apply the exact equilibration algorithm (Dafermos and Sparrow (1969)), which has been applied to many different applications of networks with special structure (cf. Nagurney (1999) and Nagurney and Zhang (1996)). See also Nagurney and Zhang (1997) for an application to fixed demand traffic network equilibrium problems.

Furthermore, we can determine the values for the in-house and the outsourced quality variables explicitly according to the following closed form expressions:

for each original firm i; $i = 1, \ldots, I$:

$$q_i^{\tau+1} = \min\{q^U, \max\{0, q_i^\tau + a_\tau(\frac{\partial f_i(Q^{1\tau}, q^{1\tau})}{\partial q_i} + \frac{\partial c_i(q^{1\tau})}{\partial q_i} + \sum_{k=1}^{n_R} \frac{\partial c_{ik}(Q^{1\tau}, q^{1\tau})}{\partial q_i}$$

$$+\omega_i \frac{\partial dc_i(q_i'^\tau)}{\partial q_i'} \frac{\partial q_i'(Q_i^\tau, q_i^\tau, q_i^{2\tau})}{\partial q_i})\}\}; \tag{18}$$

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and for the contractor and firm pairs: $l = 1, \ldots, I; j = 1, \ldots, n_O$:

$$q_{lj}^{\tau+1} = \min\{q^U, \max\{0, q_{lj}^\tau + a_\tau(\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial sc_{ijk}(Q^{2\tau}, q^{2\tau})}{\partial q_{lj}} + \frac{\partial \hat{c}_j(q^{2\tau})}{\partial q_{lj}})\}\}. \qquad (19)$$

Also, we have the following explicit formulae for the outsourced product prices: for $l = 1, \ldots, I; j = 1, \ldots, n_O; k = 1, \ldots, n_R$:

$$\pi_{ljk}^{\tau+1} = \max\{0, \pi_{ljk}^\tau + a_\tau(\frac{\partial oc_{ljk}(\pi_{ljk}^\tau)}{\partial \pi_{ljk}} - Q_{l,1+j,k}^\tau)\}. \qquad (20)$$

We now provide the convergence result. The proof follows using similar arguments as those in Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 4

In the supply chain network game theory model with product differentiation, outsourcing of production and distribution, and quality competition, let $F(X) = -\nabla U(Q, q^1, q^2, \pi)$, where we group all $U_i^1; i = 1, \ldots, I$, and $U_j^2; j = 1, \ldots, n_O$, into the vector $U(Q, q^1, q^2, \pi)$, be strongly monotone. Also, assume that F is uniformly Lipschitz continuous. Then there exists a unique equilibrium product flow, quality level, and price pattern $(Q^*, q^{1^*}, q^{2^*}, \pi^*) \in \mathcal{K}$, and any sequence generated by the Euler method as given by (16) above, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty, a_\tau > 0, a_\tau \to 0, as \tau \to \infty$ converges to $(Q^*, q^{1^*}, q^{2^*}, \pi^*)$.

Note that convergence also holds if $F(X)$ is strictly monotone (cf. Theorem 8.6 in Nagurney and Zhang (1996)) provided that the price iterates are bounded. We know that the product flow iterates as well as the quality level iterates will be bounded due to the constraints. Clearly, in practice, contractors cannot charge unbounded prices for production and delivery. Hence, we can also expect the existence of a solution, given the continuity of the functions that make up $F(X)$, under less restrictive conditions that that of strong monotonicity.

The Euler method, as outlined above for our model, can be interpreted as a discrete-time adjustment process in which each iteration reflects a time step. The original firms determine, at each time step, their optimal production (and shipment) outputs and quality levels, whereas the contractors, at each time step (iteration), compute their optimal quality levels and the prices that they charge. The process evolves over time until the equilibrium product flows, quality levels, and contractor prices are achieved, at which point no one has any incentive to switch their strategies.

4. Numerical Examples

In this Section, we present numerical supply chain network examples for which we apply the Euler method, as outlined in Section 3, to compute the equilibrium solutions. We present a spectrum of examples, accompanied by sensitivity analysis.

The supply chain network topology of the numerical examples is given in Figure 2. There are two original firms, both of which are located in North America. Their products are substitutes but are differentiated by brands in the two demand markets, R_1 and R_2. Demand Market 1 is in North America, whereas Demand Market 2 is in Asia. We use different colors to denote the outsourcing links of different original firms, with red links denoting the outsourcing links of Firm 1 and blue links denoting those of Firm 2.

Each original firm has one in-house manufacturing plant and two potential contractors. Contractor 1 and Contractor 2 are located in North America and Asia, respectively. Each firm must satisfy the demands for its product at the two demand markets. The demands for Firm 1's product at R_1 and at R_2 are 50 and 100, respectively. The demands for Firm 2's product at R_1 and at R_2 are 75 and 150.

For the computation of solutions to the numerical examples, we implemented the Euler method, as discussed in Section 3, using Matlab. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive product flow, quality level, and price is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialize the algorithm by equally distributing the product flows among the paths joining the firm top-node to the demand market, by setting the quality levels equal to 1 and the prices equal to 0.

Example 1

The data are as follows.

The production cost functions at the in-house manufacturing plants are:

$$f_1(Q^1, q^1) = (Q_{111} + Q_{112})^2 + 1.5(Q_{111} + Q_{112}) + 2(Q_{211} + Q_{212}) + .2q_1(Q_{111} + Q_{112}),$$

$$f_2(Q^1, q^1) = 2(Q_{211} + Q_{212})^2 + .5(Q_{211} + Q_{212}) + (Q_{111} + Q_{112}) + .1q_2(Q_{211} + Q_{212}).$$

The total transportation cost functions for the in-house manufactured products are:

$$c_{11}(Q_{111}) = Q_{111}^2 + 5Q_{111}, \quad c_{12}(Q_{112}) = 2.5Q_{112}^2 + 10Q_{112},$$

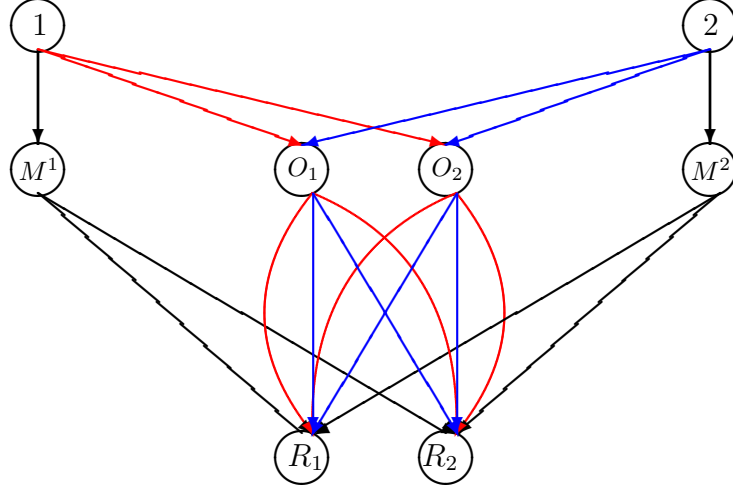$$c_{21}(Q_{211}) = .5Q_{211}^2 + 3Q_{211}, \quad c_{22}(Q_{212}) = 2Q_{212}^2 + 5Q_{212}.$$

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Figure 2: Supply Chain Network Topology for the Numerical Examples

The in-house total quality cost functions for the two original firms are given by:

$$c_1(q_1) = (q_1 - 80)^2 + 10, \quad c_2(q_2) = (q_2 - 85)^2 + 20.$$

The transaction cost functions are:

$$tc_{11}(Q_{121} + Q_{122}) = .5(Q_{121} + Q_{122})^2 + 2(Q_{121} + Q_{122}) + 100,$$

$$tc_{12}(Q_{131} + Q_{132}) = .7(Q_{131} + Q_{132})^2 + .5(Q_{131} + Q_{132}) + 150,$$

$$tc_{21}(Q_{221} + Q_{222}) = .5(Q_{221} + Q_{222})^2 + 3(Q_{221} + Q_{222}) + 75,$$

$$tc_{22}(Q_{221} + Q_{222}) = .75(Q_{231} + Q_{232})^2 + .5(Q_{231} + Q_{232}) + 100.$$

The contractors' total cost functions of production and distribution are:

$$sc_{111}(Q_{121}, q_{11}) = .5Q_{121}q_{11}, \quad sc_{112}(Q_{122}, q_{11}) = .5Q_{122}q_{11},$$

$$sc_{121}(Q_{131}, q_{12}) = .5Q_{131}q_{12}, \quad sc_{122}(Q_{132}, q_{12}) = .5Q_{132}q_{12},$$

$$sc_{211}(Q_{221}, q_{21}) = .3Q_{221}q_{21}, \quad sc_{212}(Q_{222}, q_{21}) = .3Q_{222}q_{21},$$

$$sc_{221}(Q_{231}, q_{22}) = .25Q_{231}q_{22}, \quad sc_{222}(Q_{232}, q_{22}) = .25Q_{232}q_{22}.$$

The total quality cost functions of the contractors are:

$$\hat{c}_1(q_{11}, q_{21}) = (q_{11} - 75)^2 + (q_{21} - 75)^2 + 15,$$

$$\hat{c}_2(q_{12}, q_{22}) = 1.5(q_{12} - 75)^2 + 1.5(q_{22} - 75)^2 + 20.$$

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The contractors' opportunity cost functions are:

$$oc_{111}(\pi_{111}) = (\pi_{111} - 10)^2, \quad oc_{121}(\pi_{121}) = .5(\pi_{121} - 5)^2,$$

$$oc_{112}(\pi_{112}) = .5(\pi_{112} - 5)^2, \quad oc_{122}(\pi_{122}) = (\pi_{122} - 15)^2,$$

$$oc_{211}(\pi_{211}) = 2(\pi_{211} - 20)^2, \quad oc_{221}(\pi_{221}) = .5(\pi_{221} - 5)^2,$$

$$oc_{212}(\pi_{212}) = .5(\pi_{212} - 5)^2, \quad oc_{222}(\pi_{222}) = (\pi_{222} - 15)^2.$$

The original firms' disrepute cost functions are:

$$dc_1(q_1') = 100 - q_1', \quad dc_2(q_2') = 100 - q_2',$$

where
$$q_1' = \frac{Q_{121}q_{11} + Q_{131}q_{12} + Q_{111}q_1 + Q_{122}q_{11} + Q_{132}q_{12} + Q_{112}q_1}{d_{11} + d_{12}},$$
and
$$q_2' = \frac{Q_{221}q_{21} + Q_{231}q_{22} + Q_{211}q_2 + Q_{222}q_{21} + Q_{232}q_{22} + Q_{212}q_2}{d_{21} + d_{22}}.$$

ω_1 and ω_2 are 1. q^U is 100.

The Euler method converges in 255 iterations and yields the following equilibrium solution.

The computed product flows are:

$$Q_{111}^* = 13.64, \quad Q_{121}^* = 26.87, \quad Q_{131}^* = 9.49, \quad Q_{112}^* = 9.34, \quad Q_{122}^* = 42.85,$$

$$Q_{132}^* = 47.81, \quad Q_{211}^* = 16.54, \quad Q_{221}^* = 47.31, \quad Q_{231}^* = 11.16, \quad Q_{212}^* = 12.65,$$

$$Q_{222}^* = 62.90, \quad Q_{232}^* = 74.45.$$

The computed quality levels of the original firms and the contractors are:

$$q_1^* = 77.78, \quad q_2^* = 83.61, \quad q_{11}^* = 57.57, \quad q_{12}^* = 65.45, \quad q_{21}^* = 58.47, \quad q_{22}^* = 67.87.$$

The equilibrium prices are:

$$\pi_{111}^* = 23.44, \quad \pi_{112}^* = 47.85, \quad \pi_{121}^* = 14.49, \quad \pi_{122}^* = 38.91,$$

$$\pi_{211}^* = 31.83, \quad \pi_{212}^* = 67.90, \quad \pi_{221}^* = 16.16, \quad \pi_{222}^* = 52.23.$$

Notice that, although the North American contractor produces at a lower quality and at a higher price at equilibrium, it produces and distributes more than the off-shore contractor

to R_1, who is located in North America. This happens for two reasons. First, because of the fixed demands, no pressure for quality improvement is imposed from the demand side. Secondly, as reflected in the transaction costs with the North America contractor, firms are willing to outsource more to this contractor. The Asian contractor, who produces at higher quality levels and at lower prices at equilibrium, produces and distributes more to R_2. This happens because the contractor who charges lower prices and produces at higher quality levels is highly preferable in the demand market, R_2, with the larger demand for which the original firms need to outsource more.

The total costs of the original firms' are, respectively, 11,419.90 and 24,573.94, with their incurred disrepute costs being 36.32 and 34.69. The profits of the contractors are 567.84 and 440.92. The values of q'_1 and q'_2 are, respectively, 63.68 and 65.31.

We conducted sensitivity analysis by varying the weights that the firms impose on their disrepute costs, ω, which is the vector of ω_i; $i = 1, 2$, with $\omega = (0, 0), (1000, 1000), (2000, 2000)$, $(3000, 3000), (4000, 4000), (5000, 5000)$.

We display the equilibrium product flows and the equilibrium quality levels, both the in-house and the outsourced ones, and the average quality levels, in Figure 3, with the equilibrium prices changed by each contractor, the disrepute cost, and the total cost of each original firm displayed in Figure 4.

We note that, as the weights of the disrepute costs increase, there is more pressure for firms to improve quality. Thus, all the quality levels increase. In addition, because the in-house activities are more capable of guaranteeing higher quality, the outputs of both firms are shifted in-house as the weights increase. As a result, the outsourcing prices decrease (see (12)). Moreover, as shown in Figure 4, as expected, the values of the incurred disrepute costs decrease as ω increases, but the total costs of the original firms increase.

Example 2

In Example 2, both firms consider quality levels as variables affecting their in-house transportation costs. Recall, as mentioned in the Introduction, we assume here that the costs reflect that the transportation activity will deliver the products at the same quality levels as they were produced at. The transportation cost functions of the original firms, hence, now depend on in-house quality levels as follows:

$$c_{11}(Q_{111}, q_1) = Q_{111}^2 + 1.5Q_{111}q_1, \quad c_{12}(Q_{112}, q_1) = 2.5Q_{112}^2 + 2Q_{112}q_1,$$

$$c_{21}(Q_{211}, q_2) = .5Q_{211}^2 + 3Q_{211}q_2, \quad c_{22}(Q_{212}, q_2) = 2Q_{212}^2 + 2Q_{212}q_2.$$
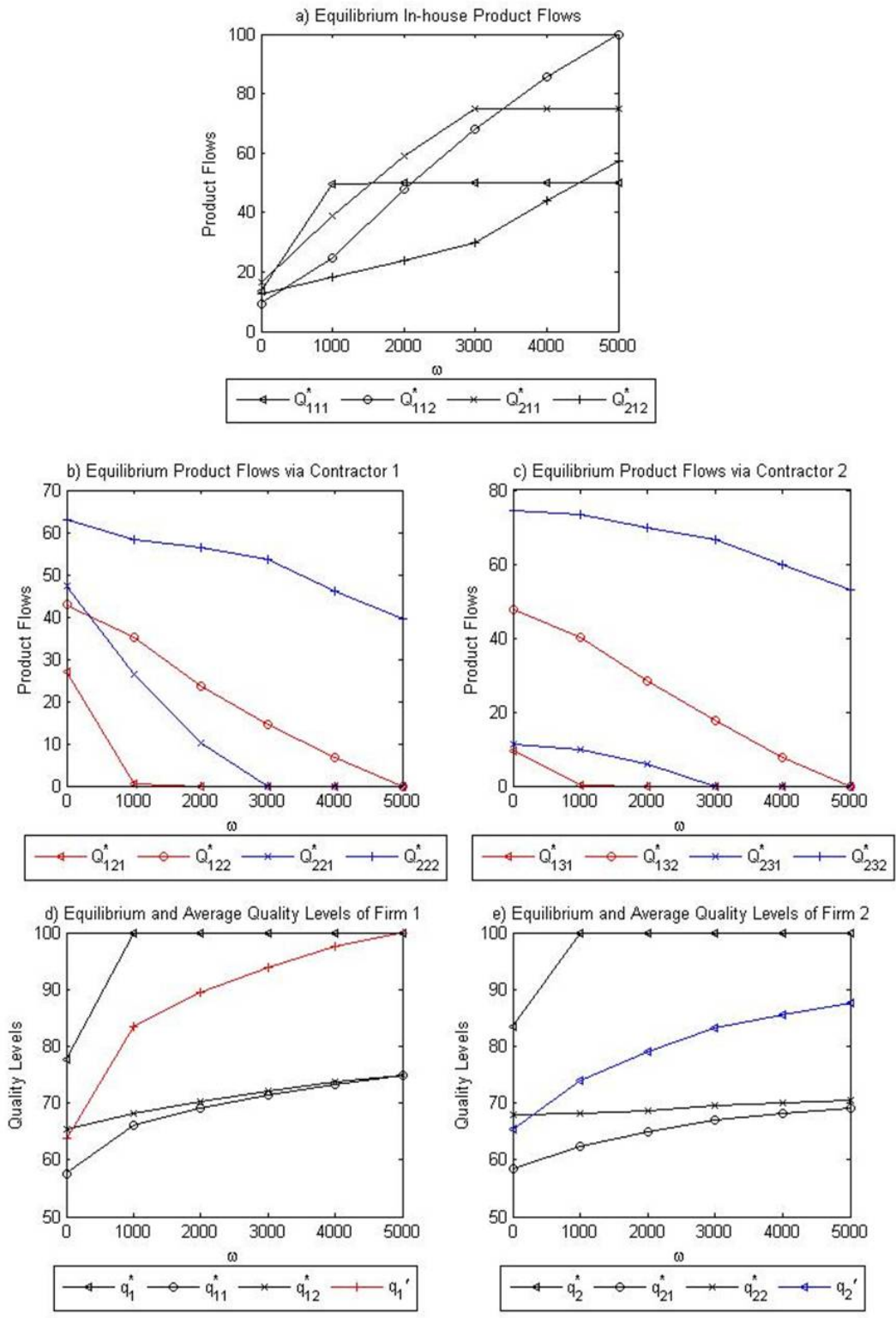
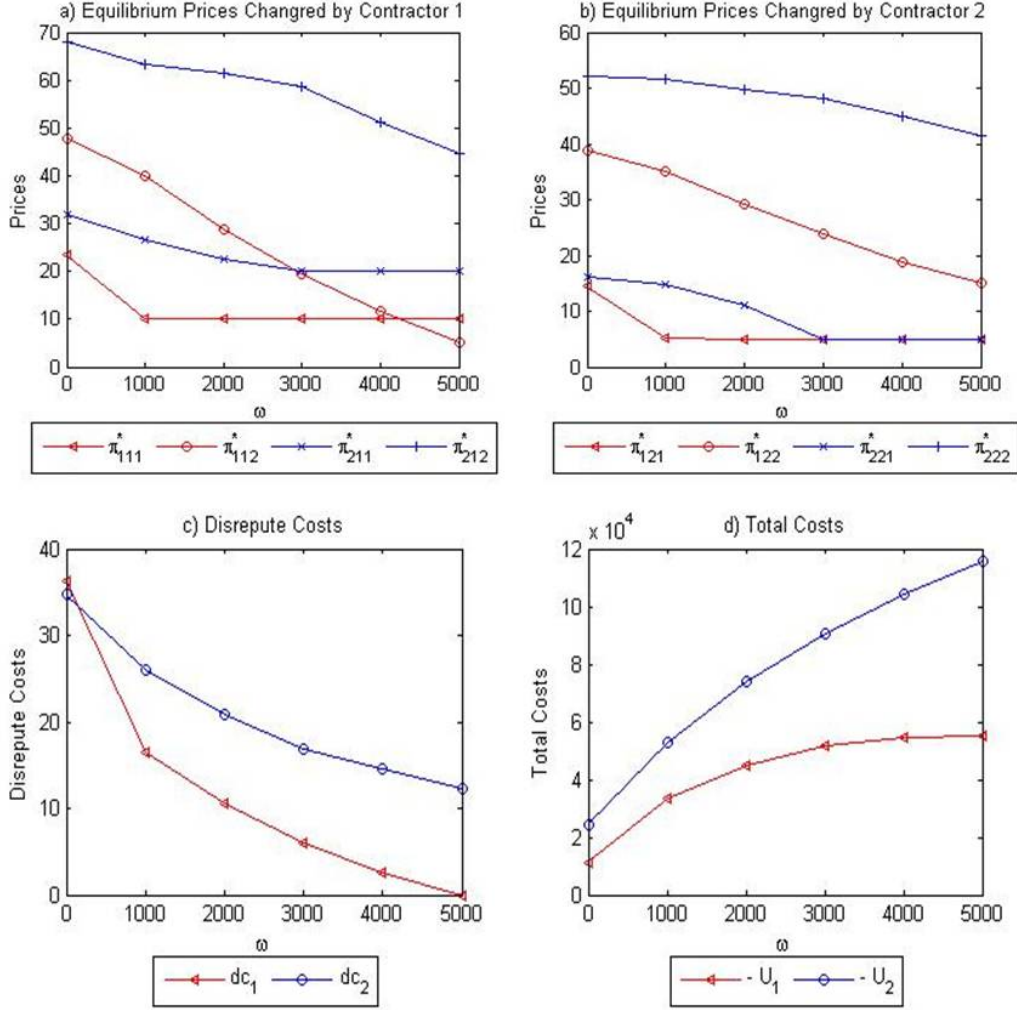Figure 3: Equilibrium Product Flows and Quality Levels as ω Increases for Example 1

Figure 4: Equilibrium Prices, Disrepute Costs, and Total Costs of the Firms as ω Increases for Example 1

The remaining data are identical to those in Example 1.

The Euler method converges in 298 iterations and yields the following equilibrium solution.

The computed product flows are:

$$Q^*_{111} = 0.00, \quad Q^*_{121} = 36.42, \quad Q^*_{131} = 13.58, \quad Q^*_{112} = 0.00, \quad Q^*_{122} = 46.42,$$

$$Q^*_{132} = 53.58, \quad Q^*_{211} = 0.00, \quad Q^*_{221} = 60.13, \quad Q^*_{231} = 14.87, \quad Q^*_{212} = 3.83,$$

$$Q^*_{222} = 65.50, \quad Q^*_{232} = 80.68.$$

The computed quality levels of the original firms and the contractors are:

$$q_1^* = 80, \quad q_2^* = 80.90, \quad q_{11}^* = 54.29, \quad q_{12}^* = 63.81, \quad q_{21}^* = 56.16, \quad q_{22}^* = 67.04.$$

The equilibrium prices are:

$$\pi_{111}^* = 28.21, \quad \pi_{112}^* = 51.42, \quad \pi_{121}^* = 18.58, \quad \pi_{122}^* = 41.79,$$

$$\pi_{211}^* = 35.03, \quad \pi_{212}^* = 70.50, \quad \pi_{221}^* = 19.87, \quad \pi_{222}^* = 55.34.$$

The total costs of the original firms are, respectively, 13,002.64 and 27,607.44, with incurred disrepute costs of 41.45 and 38.80. The profits of the contractors are, respectively, 967.96 and 656.78. The average quality levels of the original firms, q_1' and q_2', are 58.55 and 61.20.

We also conducted sensitivity analysis by varying the weights associated with the disrepute costs, ω, for $\omega = (0,0), (1000, 1000), (2000, 2000), (3000, 3000), (4000, 4000), (5000, 5000)$. We display the results of this sensitivity analysis in Figures 5 and 6.

We now discuss the results of the sensitivity analysis for Example 2. As shown in Figures 5 and 6, as the weights of the disrepute costs increase, the changing trends of all the variables and costs in Figures 5 and 6 are the same as those in Example 1, except the trends for Q_{231}^*, q_{22}^*, and π_{221}^*. As ω increases from 0 to 2000, Q_{231}^* increases. However, as ω increases further, Q_{231}^* decreases. The trends of q_{22}^* and π_{221}^* then change accordingly. The reason is as following. Because now in-house transportation costs more than before, in order to satisfy the fixed demand d_{21}, Firm 2 tends to shift more production and distribution to the contractor with a good quality level, when ω is small. This is why Q_{231}^* increases as ω increases from 0 to 2000. Nevertheless, as ω increases further, Firm 2 is under greater pressure to improve quality. Therefore, Firm 2 then shifts more production in-house, and, as a result, Q_{231}^* decreases.

For Firm 1, whose cost functions are completely distinct from those of Firm 2, it is always more cost-wise for it to improve quality by shifting product flows in-house. Thus, the changing trends of all the variables and costs of Firm 1 in Figures 5 and 6 are either monotonically increasing or decreasing.
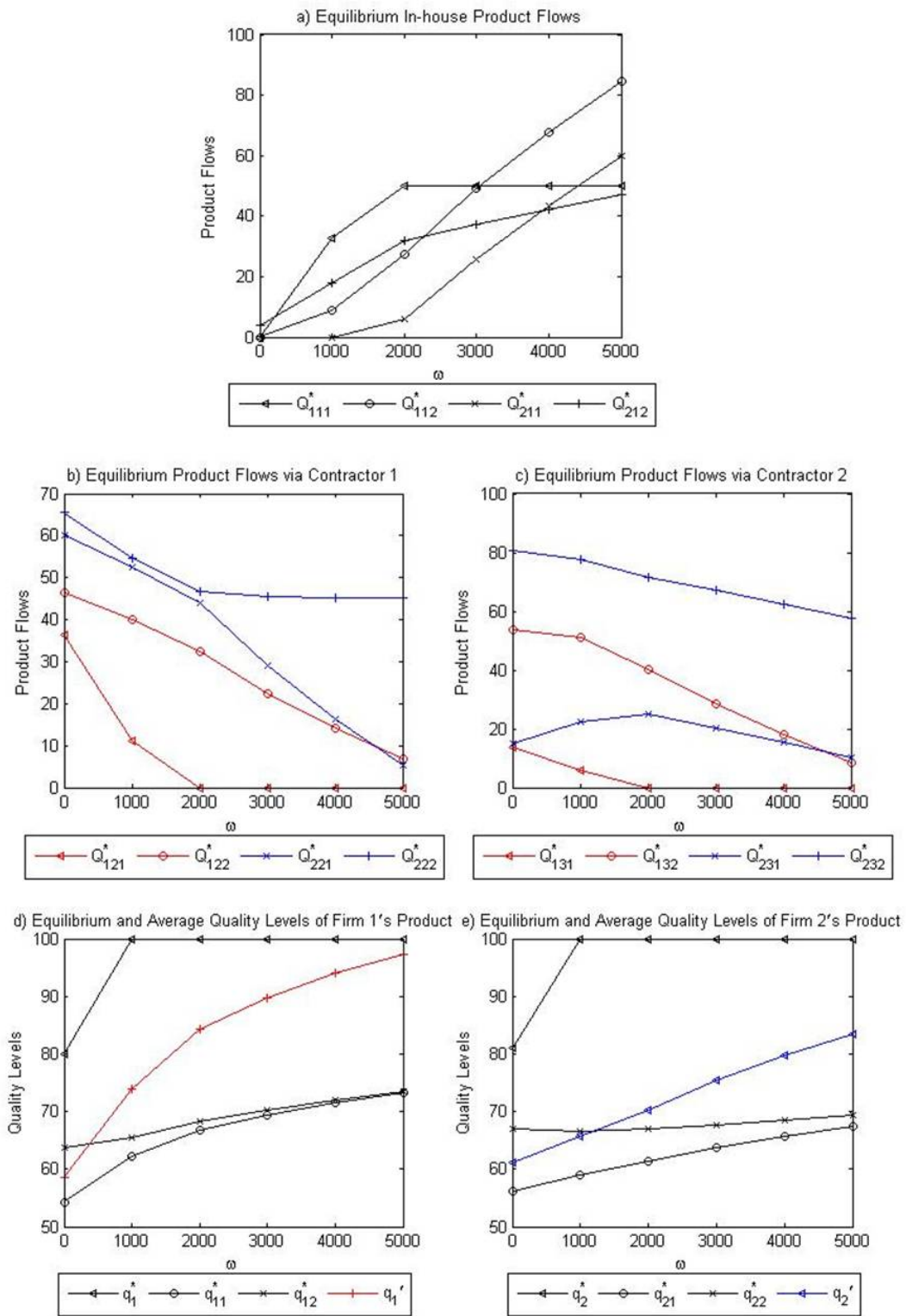
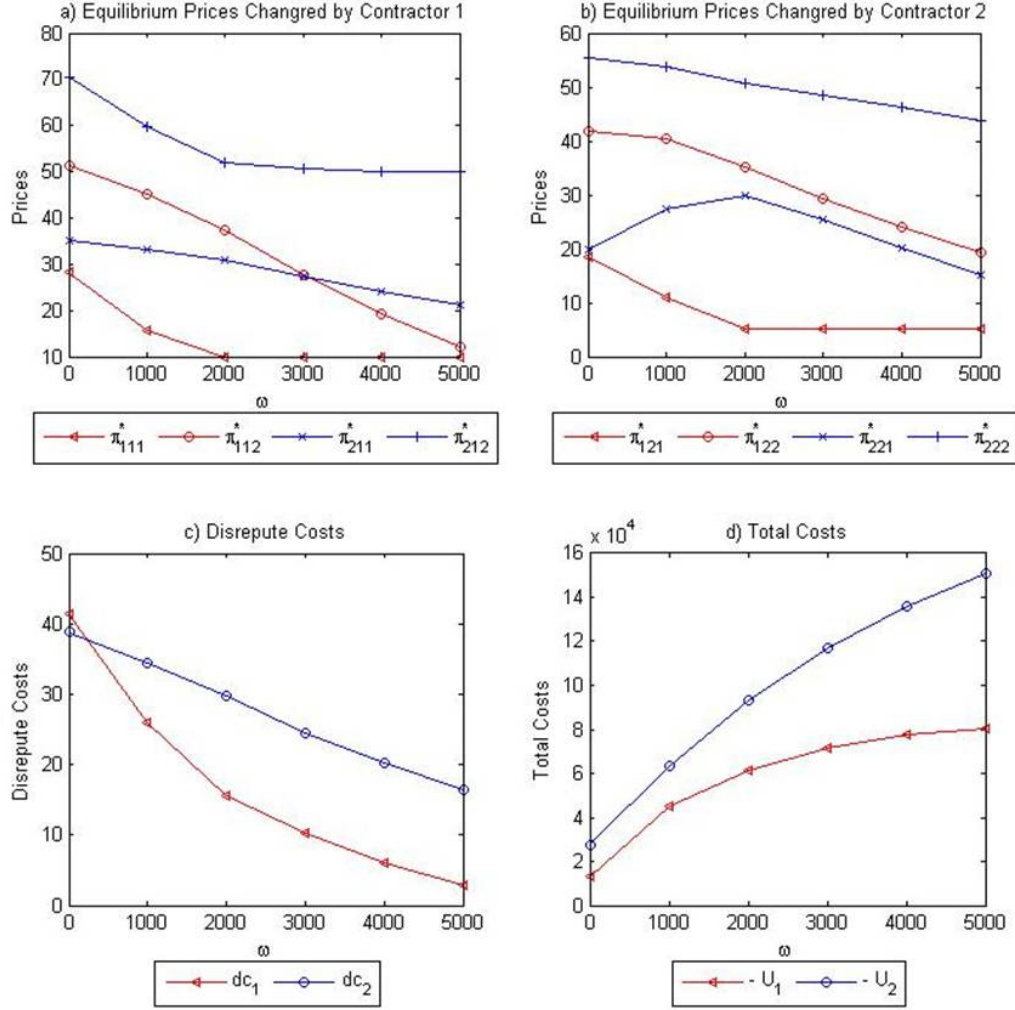Figure 5: Equilibrium Product Flows and Quality Levels as ω Increases for Example 2

Figure 6: Equilibrium Prices, Disrepute Costs, and Total Costs of the Firms as ω Increases for Example 2

Further Comparison of Examples 1 and 2

We now compare Examples 1 and 2 with ω in both examples equal to 0. Please refer to the preceding figures. This case is interesting and informative since it represents the scenario that neither Firm 1 nor Firm 2 cares about its possible reputation loss due to its product/brand having a lower quality. After the incorporation of quality levels into both firms' in-house transportation costs, it now costs more for both firms to transport the same amounts of their products manufactured in-house and to maintain the same in-house quality levels as in Example 1, as reflected by the in-house transportation functions in Examples 1 and 2. Thus, in Example 2, the equilibrium in-house product flows are lower as compared to those in Example 1, and, in order to satisfy the demands, the equilibrium outsourced flows of both

firms are higher. The corresponding results are: the contractors charge more to the firms; the outsourcing quality levels of the contractors are lower; the contractors' total profits increase, and the firms' total costs are higher than those in Example 1.

Example 3

In Example 3, we consider the scenario that the in-house transportation from the two firms to each demand market gets much more congested than before, and each firm's in-house quantities also affect the other firm's in-house transportation costs. The total in-house transportation cost functions of the two firms now become:

$$c_{11}(Q_{111}, Q_{211}, q_1) = Q_{111}^2 + 1.5Q_{111}q_1 + 7Q_{211},$$

$$c_{12}(Q_{112}, Q_{212}.q_1) = 2.5Q_{112}^2 + 2Q_{112}q_1 + 10Q_{212},$$

$$c_{21}(Q_{211}, Q_{111}, q_2) = .5Q_{211}^2 + 3Q_{211}q_2 + 8Q_{111},$$

$$c_{22}(Q_{212}, Q_{112}, q_2) = 2Q_{212}^2 + 2Q_{212}q_2 + 10Q_{112}.$$

The remaining data are identical to those in Example 1.

The total costs of Firm 1 and Firm 2 associated with different ω values are displayed in Tables 2 and 3, respectively. Note that, in Table 2, the total cost of Firm 1 increases monotonically, whether ω_1 or ω_2 increases. The same result is inferred from Table 3 for Firm 2.

The reason is the following. As discussed for Examples 1 and 2, when $\omega_i; i = 1, 2$, increases, the in-house production quantities of firm i increase. Now, in Example 3, because of the in-house production costs (as in Example 1) and the new in-house transportation costs, the increase of firm i's in-house quantities would also increase the other firm's total cost.

According to the results in Tables 2 and 3, strategically, if a firm has to increase the weight of its own disrepute cost, it is more cost-wise to increase it before the other firm does. If the firm increases its weight at the same time as, or after the other firm does, it would incur more cost under the same disrepute cost weight.

5. Summary and Conclusions

In this paper, we developed a supply chain network game theory model with product differentiation, outsourcing of production and distribution, and price and quality competition. The original firms compete with one another in in-house quality levels and in-house and

Table 2: Total Costs of Firm 1 with Different Sets of ω_1 and ω_2

ω	$\omega_1 = 0$	$\omega_1 = 1000$	$\omega_1 = 2000$	$\omega_1 = 3000$	$\omega_1 = 4000$	$\omega_1 = 5000$
$\omega_2 = 0$	12,999.09	45,135.09	61,322.22	71,463.36	77,437.89	80,462.63
$\omega_2 = 1000$	13,218.71	45,348.05	61,535.18	71,676.32	77,650.85	80,675.60
$\omega_2 = 2000$	13,425.67	45,571.40	61,758.53	71,899.67	77,874.20	80,898.94
$\omega_2 = 3000$	13,666.29	45,812.52	61,999.65	72,140.79	78,115.32	81,114.01
$\omega_2 = 4000$	14,091.85	46,034.08	62,221.20	72,362.34	78,336.88	81,361.62
$\omega_2 = 5000$	14,091.85	46,239.00	62,426.12	72,567.26	78,541.80	81,566.54

Table 3: Total Costs of Firm 2 with Different Sets of ω_1 and ω_2

ω	$\omega_1 = 0$	$\omega_1 = 1000$	$\omega_1 = 2000$	$\omega_1 = 3000$	$\omega_1 = 4000$	$\omega_1 = 5000$
$\omega_2 = 0$	27,585.65	28,203.96	28,561.15	28,798.10	29,005.24	29,187.92
$\omega_2 = 1000$	62,896.33	63,626.00	63,983.19	64,220.14	64,427.28	64,609.96
$\omega_2 = 2000$	92,753.88	93,312.11	93,669.30	93,906.25	94,113.39	94,296.07
$\omega_2 = 3000$	116,378.40	116,981.94	117,339.13	117,576.08	117,783.22	117,965.90
$\omega_2 = 4000$	135,237.43	135,872.91	136,230.10	136,467.05	136,674.19	136,856.87
$\omega_2 = 5000$	150,231.01	150,886.51	151,243.69	151,480.65	151,687.79	151,870.47

outsouced production (and shipment) flows in order to minimize their total costs and the weighted disrepute costs. The contractors, in turn, compete in their quality levels and the prices that they charge the original firms for manufacturing and distributing the products to the demand markets. This model provides the optimal make-or-buy as well as contractor selection decisions for each original firm.

We modeled the impact of quality on in-house and outsourced production and transportation and on the reputation of each firm through the quantification of the quality levels, quality cost, and the disrepute cost, with the production and the transportation cost functions depending on both quantities and quality levels. The product quality levels and quality costs were defined and quantified based on concepts and ideas in classic quality management literature (see also Nagurney and Li (2014) and Nagurney, Li, and Nagurney (2013)). The disrepute cost, which captures the impact of quality on a firm's reputation, was formulated as a function of the average quality level of the firm.

Variational inequality theory was employed in the formulations of the equilibrium conditions of the original firms, the contractors, and the supply chain network game theory model with product differentiation, possible outsourcing of production and distribution, and quality and price competition. The algorithm adopted is the Euler method, which provides

a discrete-time adjustment process and tracks the evolution of the in-house and outsourced production (and shipment) flows, the in-house and the outsourced quality levels, and the prices over time. It also yields closed form explicit formulae at each iteration with nice features for computation for all variables except for the production/shipment ones, which are computed via an exact equilibration algorithm.

In order to demonstrate the generality of the model and the computational scheme, we then provided solutions to a series of numerical examples, accompanied by sensitivity analysis.

Future research directions may include the impact of distinct transportation modes on quality as well as the inclusion of additional tiers of supply chain decision-makers.

Acknowledgments

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