# Supply Chain Networks with Global Outsourcing and Quick-Response Production Under Demand and Cost Uncertainty

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**Abstract**

This paper develops a modeling and computational framework for supply chain networks with global outsourcing and quick-response production under demand and cost uncertainty. Our model considers multiple off-shore suppliers, multiple manufacturers, and multiple demand markets. Using variational inequality theory, we formulate the governing equilibrium conditions of the competing decision-makers (the manufacturers) who are faced with two-stage stochastic programming problems but who also have to cooperate with the other decision-makers (the off-shore suppliers). Our theoretical and analytical results shed light on the value of outsourcing from novel real option perspectives. Moreover, our simulation studies reveal important managerial insights regarding how demand and cost uncertainty affect the profits, the risks, as well as the global outsourcing and quick-production decisions of supply chain firms under competition.

## 1. Introduction

In the past decade, global outsourcing has become increasingly prevalent and has reshaped supply chains in almost all industries. The main benefit of offshore outsourcing is cost savings. A recent study published by PRTM management consultants reported that the average cost reduction was 17% per globalization initiative among the three hundred surveyed international firms (Cohen et al. (2008)). However, the supply chain firms that are involved in global outsourcing are exposed to various risks such as, the demand risk, the supply disruption risk, the exchange rate risk, etc. A study conducted by The Economist Magazine (The Economist Intelligence Unit (2009)) showed that, among these risks, demand uncertainty is ranked as the top risk factor by 500 global company executives with responsibility for risk management. Moreover, the long lead time in global outsourcing, ranging from three months to nine months (Walker (1999), CNN Tech (2004), Sen (2008)), further amplifies the demand risk since precisely predicting demands months in advance is extremely difficult, if not impossible.

Demand risk can, nevertheless, be mitigated by increasing the flexibility and the responsiveness of supply chains. A well-known case is Zara, the Spanish apparel retailer, which achieves great flexibility by using onshore quick-response production to manufacture 70% to 85% of its products. As a result, Zara is able to reduce the lead time to only three weeks which helps it to quickly respond to demand and to be able to reduce both markdowns and lost sales (CNN Tech (2004), Colye et al. (2008), Cachon and Swinney (2009)). Another example of a fast-response industry is the toy industry, which has grown to $50 billion, and with supply chains that span the globe. As noted by Johnson (2006), toys are one of the oldest consumer products, with a volatile demand, and rapid change and uncertainty, with companies such as Mattel exploiting both in-house manufacturing capacity as well as outsourcing opportunities, especially in emerging economies.

Due to such success, supply chain flexibility and responsiveness have drawn increasing attention from international firms. The PRTM supply chain trend survey noted that the enhancement of supply chain flexibility is expected to overtake product quality and customer service as the top focus of global supply chain firms (Cohen et al. (2008)).

Another important risk factor faced by firms with both global outsourcing options and quick-response production capabilities is production cost uncertainty. Note that, at the time the firms make outsourcing decisions, the quick-response production cost may still be uncertain, and may not be revealed until months afterwards. Such a decision-making environment is not only relevant to apparel and toys, but also to consumer electronics, personal computers, and seasonal merchandise, including merchandise associated with special events and holidays (see Walker (1999)). The focus of this paper, hence, is to develop an analytical framework for supply chain firms who are faced with offshore-outsourcing and in-house quick-response production decisions under demand and cost uncertainty. Our analytical model, theoretical results, and accompanying simulation analysis reveal important managerial insights for global supply chain managers.

Supply chain outsourcing has been the theme of many studies in the literature. Kouvelis and

Milner (2002) proposed a two-stage model to investigate a firm's outsourcing and capacity decisions with the interplay of demand and supply uncertainty. Lee et al. (2002) considered outsourcing decisions in advanced planning and scheduling in manufacturing supply chains. Yang et al. (2007) studied a sourcing problem where a firm is faced with stochastic demand and is allowed to order from multiple suppliers with random yields. Liu and Nagurney (2011) focused on the impacts of exchange rate risk and competition intensity on offshore outsourcing decisions of firms with different risk attitudes. Nagurney et al. (2011) studied supply chain network design for critical needs products with outsourcing options. For more research regarding global supply chain design and outsourcing decisions, we refer the reader to the review by Meixell and Gargeya (2005).

A number of studies have considered supply chain outsourcing decisions under cost uncertainty from a real option perspective. Datta (2005) pointed out that, if a firm has the capability to outsource certain activities, the firm holds a real call option, the strike price of which is the cost of producing those activities in-house. Alvarez and Stenbacka (2007) used a real option approach to analyze a firm's optimal outsourcing level and optimal organization mode. Jiang et al. (2008) proposed a real option approach to evaluate the outsourcing contract under cost uncertainty from the vendors' perspective. For additional research that uses option valuation models to study production and sourcing decisions, we refer the reader to the literature review by Cohen and Mallik (1997).

Quick-response production, in turn, has drawn increasing attention from researchers. Upton (1995) provided empirical evidence of the connections between quick-response manufacturing and various technological and managerial characteristics of manufacturing plants. Yang and Wee (2001) studied a quick-response production strategy for market demand with continuous deterioration in stock. Barnes-Schuster et al. (2002) examined how a supplier and a retailer in a supply chain can coordinate, through supply chain contracts with options, in order to quickly respond to market demand. Cachon and Swinney (2009) considered pricing and purchasing decisions of a retailer with a quick-response program in the presence of strategic consumers. For more studies regarding the benefits of quick-response production, see Fisher and Raman (1996), Eppen and Iyer (1997), Iyer and Bergen (1997), Suri (1998), Fisher et al. (2001), Jones et al. (2001), Petruzzi and Dada (2001), Nagurney and Yu (2011a, b), and Cachon and Swinney (2011)).

Our paper differs from the above-mentioned ones in that we study the global outsourcing and quick-response production decisions of supply chain firms from a network perspective in which we allow multiple suppliers, multiple manufacturers, and multiple demand markets to interact under both demand and cost uncertainty. In particular, each manufacturer maximizes his own expected profit through a two-stage stochastic programming problem, while competing with the other manufarturers, but cooperates with the off-shore suppliers in the first stage. We express the governing equilibrium conditions of the entire supply chain network using a variational inequality formulation. To the best of our knowledge, this paper is the first modeling effort that captures the behavior of multiple, competing decision-makers who are faced with two-stage stochastic programming problems, but also have to cooperate with other decision-makers. In addition, we present theoretical and analytical results that explain the value of outsourcing from new real options perspectives that are distinct from those in the literature. We then use our modeling framework, theory, and simulation studies to investigate the following questions:

(1). How does demand uncertainty affect supply chain firms' decisions regarding outsourcing, in-house production, and sales under competition?

(2). How does demand uncertainty affect supply chain firms' profits and risks under competition?

(3). How does the prevalence of the quick-response in-house production affect supply chain firms' decisions, profits, and risks under demand uncertainty?

(4). How does cost uncertainty affect supply chain firms' decisions regarding outsourcing, in-house production, and sales under competition?

(5). How does cost uncertainty affect supply chain firms' profits and risks under competition?

Our model provides new real option interpretations for outsourcing decisions which indicate that the outsourcing cost that the manufacturers without quick-response capability are willing to pay will *increase* as the uncertainty of demand gets *higher*; and that the outsourcing cost the manufacturers with quick-response capability are willing to pay will *decrease* as the uncertainty of the quick-response production cost gets *higher*. Our simulation studies show that manufacturers with quick-response production capability have higher average profits and lower risks than manufacturers without such capability. However, the probability that manufacturers with quick-response production have higher profits than manufacturers without quick-response production ranges from 0.2 to 0.6 at different demand uncertainty levels. In particular, we find that *manufacturers without quick-response production are more profitable when the demand turns out to be at normal levels while manufacturers with such capability are more profitable when the demand is unexpectedly high or low.*

Our results also show that, as the prevalence of quick-response production increases among manufacturers, the quick-response production quantity of each manufacturer will decrease while the outsourcing quantity of each manufacturer will increase. In addition, as the prevalence of quick-response production increases, the profit gap and the risk gap between manufacturers with and without such capability will become smaller. Moreover, we find that, as the uncertainty of the cost of quick-response production increases, the manufacturers with such capability will increase their quick-response production and will reduce outsourcing. As a consequence, such manufacturers will become increasingly more profitable than manufacturers without quick-response production capabilities.

It is also worth emphasizing that our results reveal that manufacturers without in-house quick-response production will be *indirectly* and *negatively* affected by the uncertainty of the cost of quick-response production through market competition. For example, when the quick-response production cost turns out to be lower than expected, the profits of manufacturers without quick-response production will be greatly reduced due to the competition with manufacturers with such capabilities. Furthermore, as the quick-response cost uncertainty increases, the risk of manufacturers without quick-response production capability will also increase since they are indirectly exposed to the cost uncertainty through market competition.

This paper is organized as follows: In Section 2, we develop the supply chain network model with outsourcing and quick-response production under demand and cost uncertainty. We model heterogenous decision-makers in supply chain networks, identify the governing equilibrium conditions, and derive the variational inequality formulation. We also provide some interesting analytical results. In Section 3, we identify the model's qualitative properties, and propose a computational procedure. In Section 4, we use simulation analysis to study the impacts of demand and cost uncertainty on the profits, the risks, and the decisions of supply chain firms under competition. Section 5 highlights the managerial insights and concludes the paper.

## 2. The Global Supply Chain Network Model with Cost and Demand Uncertainty

The model that we develop is based on variational inequality theory. Recall that the finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^n$, such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{1}$$

where $F$ is a given continuous function from $\mathcal{K}$ to $R^n$, $\mathcal{K}$ is a given closed, convex set, and $< \cdot, \cdot >$ denotes the inner product in $n$-dimensional Euclidean space.

The variational inequality formulation allows for a unified treatment of equilibrium and optimization problems, and is closely related to many mathematical programming problems, such as: constrained and unconstrained optimization problems, fixed point problems, and complementarity problems. For an introduction to finite-dimensional variational inequality theory, we refer the reader to the book by Nagurney (1999). Variational inequality models allow one to analyze complex network equilibrium problems in supply chains, transportation, finance, and electric power (see, for example, Dong et al. (2005), Cruz and Wakolbinger (2008), Nagurney and Ke (2006), and Liu and Nagurney (2009)).

The majority of the notation used throughout our model presentation is given in Table 1. All vectors throughout this paper are assumed to be column vectors unless otherwise noted. The equilibrium solution is denoted by "*".

We now develop the supply chain network equilibrium model with global outsourcing and quick-response production under demand and cost uncertainty. The network structure and the timeline are shown in Figure 1. Our model considers a two stage network, with $I$ suppliers, $J$ manufacturers, and $M$ demand markets. In the first stage, each manufacturer $j$ decides how much to order from the suppliers, that is, he determines his $v_j^i$s, but does this before the information regarding the demand ($\omega \in \Omega$) and the fast-response in-house production cost ($\pi \in \Pi$) is revealed. The links between the suppliers $\{i\}$ and the manufacturers $\{j\}$ in Stage 1 represent these outsourcing decisions, with the

Table 1: Notation for the Global Supply Chain Network Model

| Notation | Definition |
|---|---|
| $\omega \in \Omega$ | the demand scenarios |
| $\pi \in \Pi$ | the quick-response production cost scenarios |
| $\Theta_\omega$ | the vector of demand factors in Scenario $\omega$ with component for Market $m$ denoted by $\theta_{\omega m}$ |
| $\Phi_\pi$ | the vector of cost factors in Scenario $\pi$ with component for Manufacturer $j$ denoted by $\phi_{\pi j}$ |
| $v^i_j$ | the outsourcing quantity from Manufacturer $j$ to Supplier $i$. We group the $v^i_j$s into the vector $V$. |
| $V^i$ | the total production of Supplier $i$, that is, $\sum_{j=1}^J v^i_j$. |
| $u^j_{\omega\pi}$ | the quick-response production quantity of Manufacturer $j$ in the intersection of Scenarios $\omega$ and $\pi$. We group the $u^j_{\omega\pi}$s in Scenario $\omega$ and $\pi$ into the vector $U_{\omega\pi}$, and group the $u^j_{\omega\pi}$s into the vector $U$. |
| $y^{jm}_{\omega\pi}$ | the sales of Manufacturer $j$ in Market $m$ in the intersection of Scenarios $\omega$ and $\pi$. We group the $y^{jm}_{\omega\pi}$s in Scenario $\omega$ and $\pi$ into the vector $Y_{\omega\pi}$, group the $y^{jm}_{\omega\pi}$s in Scenario $\omega$ into the vector $Y_\omega$, and group all $Y_\omega$s into the vector $Y$. |
| $Y^m_{\omega\pi}$ | the total sales of all manufacturers in Market $m$ in the intersection of Scenarios $\omega$ and $\pi$, that is, $\sum_{j=1}^J y^{jm}_{\omega\pi}$. |
| $CAP_i$ | the production capacity of Supplier $i$ |
| $CAP_j$ | the quick-response production capacity of Manufacturer $j$ |
| $\rho^j_m(\theta_\omega, Y^m_{\omega\pi})$ | the inverse demand function for Manufacturer $j$'s product in Market $m$ in demand scenario $\omega$ and $\pi$ |
| $c_j(\phi_\pi, u^j_{\omega\pi})$ | the quick-response in-house production cost function of Manufacturer $j$ in cost scenario $\pi$ and demand scenario $\omega$ |
| $c_i(V^i)$ | the production cost function of Supplier $i$ |
| $h^i_j$ | the unit transportation and transaction cost between Supplier $i$ and Manufacturer $j$ |
| $\rho^{i*}_j$ | the equilibrium product price in the transaction between Supplier $i$ and Manufacturer $j$ |

associated flows given by the $v^i_j$s.

In the second stage, the demand and cost information is first observed, and the manufacturers receive deliveries from the first stage which are represented (cf. Figure 1) by the flows on the links crossing from Stage 1 to Stage 2. The manufacturers then decide how much to produce using the fast-response productions, the $u^j_{\omega\pi}$s, which are represented by the flows on the links originating from nodes $j'$; $j' = 1', ...J'$. The manufacturers also decide how much to sell in the demand markets, that is, they determine their $y^{jm}_{\omega\pi}$s, which are represented by the flows on the links from the manufacturers to the demand markets. As noted in the Introduction, the second stage is, typically, three to nine months after the first stage. In addition, as shown in Figure 1, there are multiple demand and cost scenarios that the manufacturers need to consider.

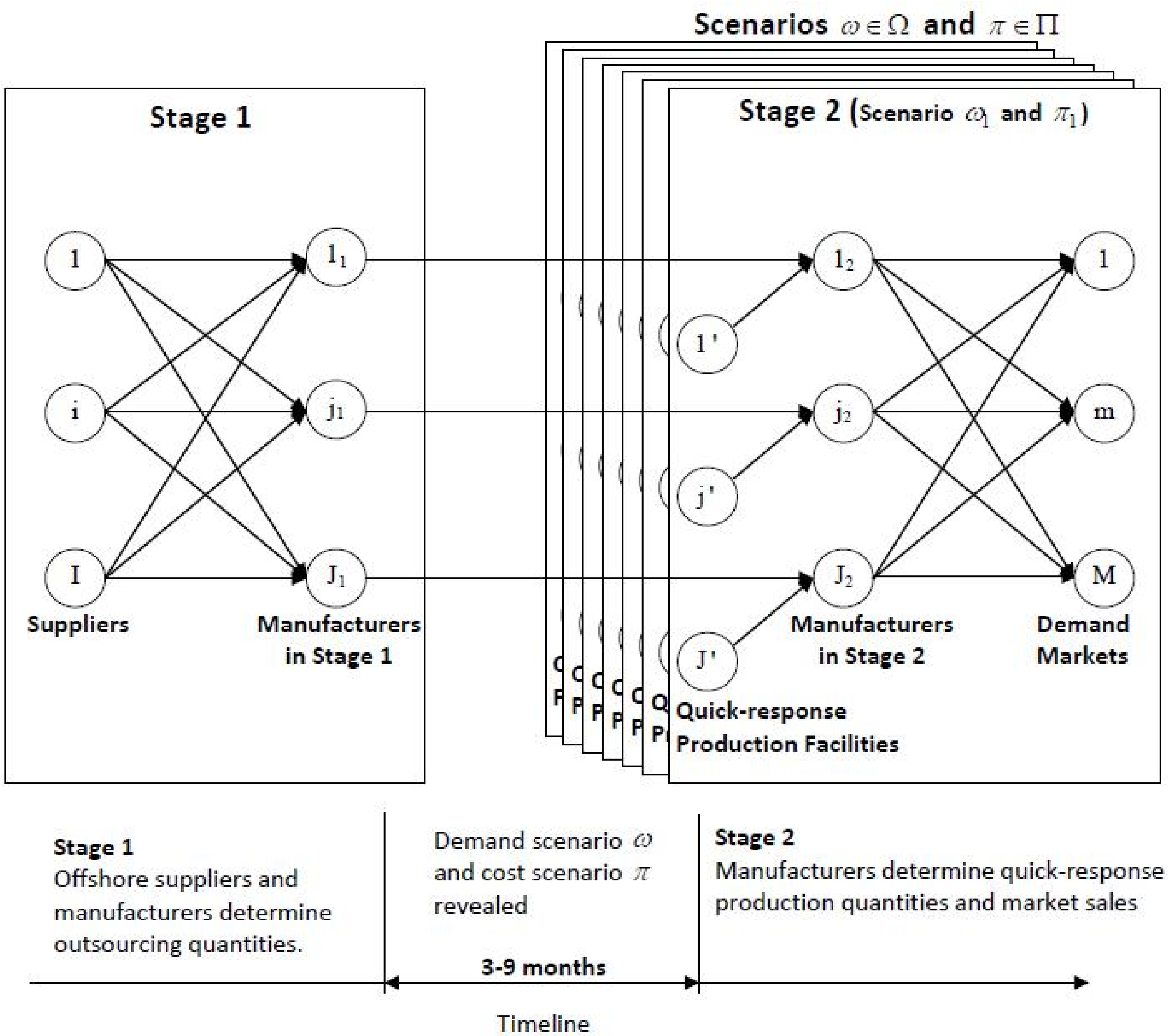


Figure 1: Supply Chain Network and Decision Timeline

Note that, if a manufacturer does not have quick-response production capacity, his in-house quick-response production capacity is equal to zero.

### 2.1 The Behavior of the Manufacturers and their Optimality Conditions

The manufacturers determine their outsourcing quantities, the $v_j^i$s, in Stage 1 and decide their quick-response productions, the $u_{\omega\pi}^j$, and their sales volumes, the $y_{\omega\pi}^{jm}$s, in Stage 2. We assume that each manufacturer maximizes his own expected profit over the scenarios. Note that each manufacturer faces a two-stage scenario-based stochastic programming problem (see, e.g., Dupacova (1996), Barbarosoglu and Arda (2004)). For the theory and applications of stochastic programming, we refer the reader to the books by Birge and Louveaux (1997) and Shapiro et al. (2009). For excellent coverage of probability theory and applications, see the book by Derman et al. (1973). The complete optimal set of decisions for manufacturer $j$ includes his optimal outsourcing quantities as well as his optimal response plan under scenario $\omega$ and $\pi$ in Stage 2. Hence, each manufacturer maximizes his expected profit across all scenarios, and, when the optimality conditions of all manufacturers hold simultaneously, an equilibrium state is reached in which no decision-maker can be better off by changing his decisions.

The optimality conditions of all manufacturers can be simultaneously expressed as a variational inequality problem. We first describe the manufacturers' decision-making problem as a two stage stochastic programming problem. We then present the variational inequality governing the equilibrium state of all the manufacturers.

*The Manufacturers' Optimization Problems*

In the first stage, the manufacturers need to determine their outsourcing quantities, that is, their $v_j^i$s. If a manufacturer orders too much from suppliers, he may end up with too many unsold products in the second stage, while, if a manufacturer orders too little from the suppliers and the demand turns out to be high in the second stage, he may have to use the more expensive quick-response in-house production. Moreover, if manufacturer $j$ does not have a quick-response production capacity (i.e., $CAP_j = 0$) he may lose sales opportunities to other manufacturers when the demand is high in the second stage. In the second stage, after the demand and cost scenarios are revealed, the manufacturer determines the quick-response production quantity, and the sales to the demand markets, $y_{\omega\pi}^{jm}$. Hence, the manufacturers need to decide their $v_j^i$s, $u_{\omega\pi}^j$s, and $y_{\omega\pi}^{jm}$s in order to maximize the expected profits in equilibrium across all scenarios. In particular, the optimization problem faced by Manufacturer $j$; $j = 1, \ldots, J$, can be expressed as a two stage stochastic programming problem as follows:

$$MAX\ E(\text{Profit}_j) = -\sum_{i=1}^{I} \rho_j^{i*} v_j^i - \sum_{i=1}^{I} h_j^i v_j^i + E[Q_{\omega\pi}^j(v_j, \Theta_\omega, \Phi_\pi)] \tag{2}$$

subject to

$$v_j^i \geq 0, \quad i = 1, ..., I, \tag{3}$$

where $\rho_j^{i*}$ is the equilibrium product price for the transaction between Supplier $i$ and Manufacturer $j$, and $v_j$ is the vector of all $v_j^i$s of manufacturer $j$. The first and second terms in the objective function (2) represent the total payout to suppliers and the transportation / transaction cost, respectively. The third term is the expected value of manufacturer $j$'s net revenue in Stage 2, $Q_{\omega\pi}^j(v_j, \Theta_\omega, \Phi_\pi)$, over all scenarios. In particular, $Q_{\omega\pi}^j(v_j, \Theta_\omega, \Phi_\pi)$ is the optimal value of the following problem:

$$MAX \text{ NetRevenue}_{j\omega\pi} = \sum_{m=1}^{M} \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m) y_{\omega\pi}^{jm} - c_j(\phi_\pi, u_{\omega\pi}^j) \tag{4}$$

subject to

$$\sum_{m=1}^{M} y_{\omega\pi}^{jm} \leq \sum_{i=1}^{I} v_j^i + u_{\omega\pi}^j, \tag{5}$$

$$u_{\omega\pi}^j \leq CAP_j. \tag{6}$$

$$u_{\omega\pi}^j \geq 0,\ y_{\omega\pi}^{jm} \geq 0, \quad m = 1, ..., M. \tag{7}$$

Note that the purchased quantities from suppliers, the $v_j^i$s, have been determined in the first stage, and cannot be changed in the second stage. The first term in the objective function (4) is the total revenue of Manufacturer $j$ from the $M$ demand markets while the second term in (4) is the in-house production cost. Since the manufacturers engage in competition through their objective functions as in (4), Manufacturer $j$'s product price in Market $m$, $\rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)$, is a function of $Y_{\omega\pi}^m$, the total sales of all the manufacturers. Note that, in Stage 2, $\theta_{m\omega}$ has been observed and becomes a constant in $\rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)$. Moreover, the purchasing costs of outsourced products do not appear in (4) since these costs have been determined in the first stage and are sunk costs in the second stage.

Constraint (5) indicates that the total sales are less than or equal to the sum of the purchased quantity and the in-house produced quantity. Constraint (6), in turn, reflects that the in-house production cannot exceed the production capacity.

Based on the standard stochastic programming theory, we can reformulate manufacturer $j$'s two-stage optimization problem as the following maximization problem:

$$MAX\ E(\text{Profit}_j) = -\sum_{i=1}^{I} \rho_j^{i*} v_j^i - \sum_{i=1}^{I} h_j^i v_j^i + \sum_{\omega\in\Omega} \sum_{\pi\in\Pi} f(\omega, \pi) \left[ \sum_{m=1}^{M} \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m) y_{\omega\pi}^{jm} - c_j(\phi_\pi, u_{\omega\pi}^j) \right] \tag{8}$$

subject to

$$\sum_{m=1}^{M} y_{\omega\pi}^{jm} \leq \sum_{i=1}^{I} v_j^i + u_{\omega\pi}^j, \quad \forall \omega \in \Omega,\ \pi \in \Pi, \tag{9}$$

$$u_{\omega\pi}^j \leq CAP_j, \quad \forall \omega \in \Omega,\ \pi \in \Pi, \tag{10}$$

$$v_j^i \geq 0, u_{\omega\pi}^j \geq 0,\ y_{\omega\pi}^{jm} \geq 0, \quad i = 1, ..., I,\ m = 1, ..., M,\ \omega \in \Omega,\ \pi \in \Pi, \tag{11}$$

where $f(\omega, \pi)$ is the joint probability of demand scenario $\omega$ and cost scenario $\pi$.

*The Optimality Conditions of All Manufacturers*

We assume that the production cost function for each manufacturer is continuously differentiable and convex (hence, it could be linear) and that the inverse demand function is continuous, continuously differentiable, and is decreasing. Also, we assume that the manufacturers compete in a noncooperative manner in the sense of Nash (1950, 1951). The optimality conditions for all manufacturers simultaneously coincide with the solution of the following variational inequality (cf. Nagurney (1999), Bazaraa et al. (1993), Gabay and Moulin (1980)): Determine $(V^*, U^*, Y^*) \in \mathcal{K}^1$ satisfying:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}[\rho_j^{i*} + h_j^i] \times \left[v_j^i - v_j^{i*}\right] + \sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J} f(\omega,\pi)\frac{\partial c_j(\phi_\pi, u_{\omega\pi}^{j*})}{\partial u_{\omega\pi}^j} \times \left[u_{\omega\pi}^j - u_{\omega\pi}^{j*}\right]$$

$$-\sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J}\sum_{m=1}^{M} f(\omega,\pi)[\rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^{m*}) + \frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^{m*})}{\partial Y_{\omega\pi}^m} y_{\omega\pi}^{jm*}] \times \left[y_{\omega\pi}^{jm} - y_{\omega\pi}^{jm*}\right] \geq 0,$$

$$\forall (V,U,Y) \in \mathcal{K}^1, \tag{12}$$

where $\mathcal{K}^1 \equiv ((V,U,Y)|(V,U,Y) \in R_+^{IJ+|\Omega||\Pi|(J+JM)}$ and (9) and (10) hold).

Note that, if variational inequality (12) is satisfied, then the optimality conditions of all manufacturers are satisfied simultaneously and, thus, an equilibrium state is reached.

**Lemma 1** *Suppose that variational inequality (12) holds. Then the optimality conditions of the second stage problem (4) for all manufacturers are simultaneously satisfied in each scenario.*

**Proof:** See the Appendix.

Lemma 1 indicates that, if variational inequality (12) holds, then the manufacturers also achieve equilibrium in each possible scenario in Stage 2.

### 2.2 The Behavior of the Offshore Suppliers and their Optimality Conditions

The offshore suppliers only transact with the manufacturers in the first stage, and do not need to consider the scenarios in the second stage. The optimization problem faced by Supplier $i$; $i = 1, ..., I$, can be expressed as follows:

$$MAX \text{ Profit}_i = \sum_{j=1}^{J} \rho_j^{i*} v_j^i - c_i(V^i) \tag{13}$$

subject to

$$\sum_{j=1}^{J} v_j^i \leq CAP_i, \tag{14}$$

$$v_j^i \geq 0, \quad \forall j.$$

We assume that the production cost function for each offshore supplier is continuously differentiable and convex (hence, it could also be linear), and that the offshore suppliers also compete in a noncooperative manner in the sense of Nash (1950, 1951). The optimality conditions for all offshore suppliers simultaneously coincide with the solution of the following variational inequality: Determine $V^* \in \mathcal{K}^2$ satisfying:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_i(V^{i*})}{\partial v_j^i} - \rho_j^{i*}] \times \left[v_j^i - v_j^{i*}\right] \geq 0,$$

$$\forall V \in \mathcal{K}^2, \tag{15}$$

where $\mathcal{K}^2 \equiv (V|V \in R_+^{IJ}$ and (14) hold).

### 2.3 The Equilibrium Conditions of the Global Supply Chain Network

In equilibrium, the optimality conditions for all offshore suppliers and the optimality conditions for all manufacturers must hold simultaneously. Also, the shipments that the suppliers ship to the manufacturers must be equal to the shipments that the manufacturers accept from the suppliers; hence, these two tiers of decision-makers cooperate (see also Nagurney (2006) and the references therein).

**Definition 1: Two-Stage Supply Chain Network Equilibrium**

*The equilibrium state of the supply chain network is one where the sum of (12) and (15) is satisfied, so that no decision-maker has any incentive to alter his decisions.*

**Theorem 1: Variational Inequality Formulation**

*The equilibrium conditions governing the two stage supply chain network model are equivalent to the solution of the variational inequality problem given by: Determine* $(V^*, U^*, Y^*) \in \mathcal{K}^3$ *satisfying:*

$$\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_i(V^{i*})}{\partial v_j^i} + h_j^i] \times \left[v_j^i - v_j^{i*}\right] + \sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J} f(\omega,\pi)\frac{\partial c_j(\phi_\pi, u_{\omega\pi}^j)}{\partial u_{\omega\pi}^j} \times \left[u_{\omega\pi}^j - u_{\omega\pi}^{j*}\right]$$

$$-\sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J}\sum_{m=1}^{M} f(\omega,\pi)[\rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^{m*}) + \frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^{m*})}{\partial Y_{\omega\pi}^m} y_{\omega\pi}^{jm*}] \times \left[y_{\omega\pi}^{jm} - y_{\omega\pi}^{jm*}\right] \geq 0,$$

$$\forall (V, U, Y) \in \mathcal{K}^3, \tag{16}$$

*where* $\mathcal{K}^3 \equiv ((V,U,Y)|(V,U,Y) \in R_+^{IJ+|\Omega||\Pi|(J+JM)}$ *and* (9), (10), *and* (14) *hold*).

**Proof:** See the Appendix.

The variational inequality problem (16) can be rewritten in standard form as follows: Determine $X^* \in \mathcal{K}$ satisfying

$$\left\langle F(X^*)^T, X - X^*\right\rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where $X \equiv (V, U, Y)^T$, $\mathcal{K} \equiv \mathcal{K}^3$,

$$F(X) \equiv (F^V_{ij}, F^U_{j\omega\pi}, F^Y_{jm\omega\pi}), \tag{18}$$

with the functional terms $(F^V_{ij}, F^U_{j\omega\pi}, F^Y_{jm\omega\pi})$ preceding the multiplication signs in (16), and with indices $i = 1, \ldots, I$; $j = 1, \ldots, J$; $m = 1, \ldots, M$; $\omega \in \Omega$; and $\pi \in \Pi$. Here $< \cdot, \cdot >$ denotes the inner product in $\mathcal{H}$-dimensional Euclidean space where $\mathcal{H} = IJ + |\Omega||\Pi|(J + JM)$.

We now provide certain analytical results for a simple case where there is a single supplier ($I = 1$), a single manufacturer ($J = 1$), and a single demand market ($M = 1$). In Propositions 1 and 2 we establish connections between the value of outsourcing and real call and put options. Recall that a call option gives the option holder the right, but not the obligation, to purchase the underlying asset (e.g., a stock) at a pre-determined price (strike price) before/on a future expiration day. The payoff function of a call option on the expiration day is as follows:

$$\text{payoff} = MAX(0, S - K), \tag{19}$$

where $S$ is the stock price in the market on the expiration day and $K$ is the strike price. If $S$ is higher than $K$ the option holder can exercise the option and the payoff is $S - K$, while if $S$ is lower than $K$, the option holder can let the option expire without doing anything, in which case the payoff is zero. The option premium is the price paid by the option holder in the beginning to obtain the option.

A put option, on the other hand, gives the option holder the right but not the obligation to sell the underlying asset at a pre-determined price (strike price, $K$) before/on a future expiration day. The payoff function of a put option on the expiration day is as follows:

$$\text{payoff} = MAX(0, K - S). \tag{20}$$

If $S$ is lower than $K$, the option holder can exercise the option and the payoff is $K - S$, while, if $S$ is higher than $K$, the option holder can let the option expire without doing anything in which case the payoff is zero. For additional theory and applications regarding financial and real options see the book by Ross et al. (2009).

Next, we present two Propositions. We use $\lambda^j_{\omega\pi}$s to denote Lagrange multipliers associated with constraints (9). Note that since $\lambda^j_{\omega\pi}$ is the shadow price associated with constraint (9), it represents the marginal value (the value of an additional unit) of the product for Manufacturer $j$ under scenarios $\omega$ and $\pi$ in Stage 2. We first assume that the in-house production cost is deterministic, and focus on the random demand factor. Thus, for notational simplicity, we can suppress the indices, $i$, $j$, $m$, and $\pi$. In particular, we assume that the random demand factor is additive in the inverse demand function as follows:

$$\rho(\theta_\omega, y) = a + \theta_\omega - b \times y, \tag{21}$$

where $a + \theta_\omega > 0 \ \forall \omega \in \Omega$.

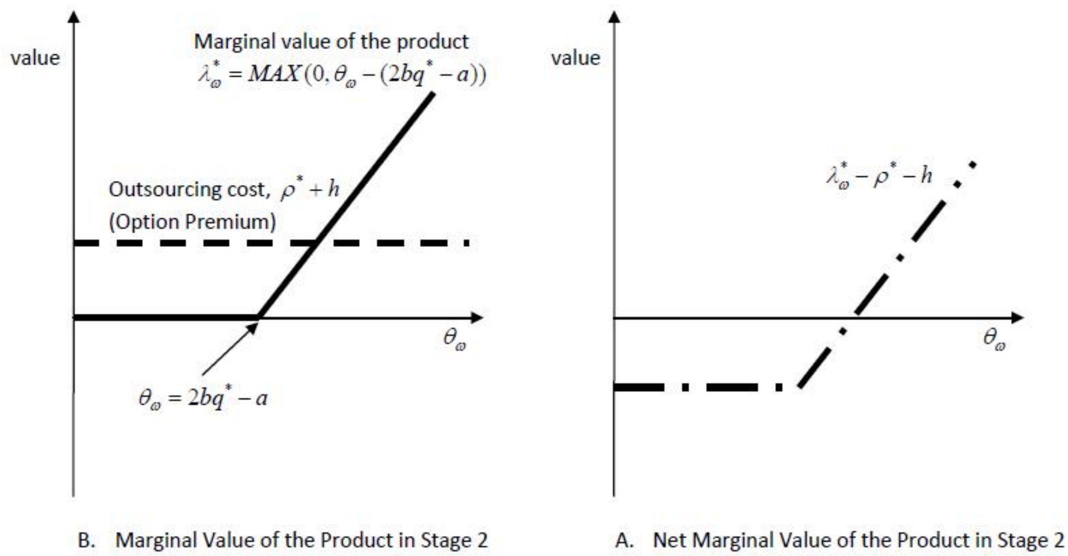


Figure 2: Marginal Value of the Product in Stage 2 with Uncertain Demand.

**Proposition 1**

*Suppose that the manufacturer's capacity for fast-response in-house production is zero and that the manufacturer's outsourcing activity is positive ($v^* > 0$). The marginal value of the product in the second stage resembles the payoff of a real call option on the random demand factor, $\theta_\omega$, with strike price $K = 2bv^* - a$, that is, $\lambda^*_\omega = MAX(0, \theta_\omega - (2bv^* - a))$. Moreover, in the first stage the outsourcing cost the manufacturer is willing to pay, $\rho^* + h$, is equal to the expected value of this real call option, $\sum_{\omega\in\Omega} f(\omega)\lambda^*_\omega$.*

**Proof**: See the Appendix.

Note that the real option relationship derived in Proposition 1 is distinct from the real option analysis in the outsourcing research literature. In the outsourcing literature, a real option usually refers to the choice and timing of outsourcing versus in-house production under varying costs. However, in the problem discussed in Proposition 1, we show another real option perspective for offshore outsourcing. In particular, in Proposition 1, the manufacturer has decided to outsource in the first stage, and the real option here refers to the manufacturer's capability to sell more of the product when the demand is high, which relies on the level of availability of the product provided by the outsourcing order placed in Stage 1. In other words, if the manufacturer orders more in the first stage, he has the option to sell more products and can avoid stockout when the demand turns out to be high. Of course, if the demand turns out to be low and there are unsold products, the manufacturer loses the premium of the option which is the outsourcing price of the product. An illustration of this relationship is presented in Figure 2. In Figure 2.A, we can see that, if the random demand factor $\theta_\omega$ is higher than $2bv^* - a$, then the marginal value of the product, $\lambda_\omega$ in Stage 2, has a positive value and linearly increases as $\theta_\omega$ increases, whereas if $\theta_\omega$ is lower than

$2bv^* - a$, $\lambda_\omega$ is constantly zero. However, the unit outsourcing cost (the option premium), $\rho^* + h$, is flat since it has been determined before $\theta_\omega$ is revealed. Figure 2.B, in turn, shows the net marginal value of the product across demand scenarios.

**Remark**

Such a real option analogy can help one to understand the value of global outsourcing from another angle. For example, we have known that the value of a call option increases as the uncertainty of the underlying factor increases, which indicates that, in our case, if the manufacturer has no quick-response production capability the unit outsourcing price the manufacturer is willing to pay will *increase* as the uncertainty of demand gets *higher*. In Section 4, we extend the analysis to more complex and general cases through the use of simulation studies to analyze multiple heterogenous manufacturers' behaviors, their interactions, profits, and the risks with/without quick-response capacities and under competition.

We now focus on the random cost factor, and assume that the demand is deterministic. Thus, for notational simplicity, we can suppress the indices, $i$, $j$, $m$, and $\omega$. In particular, we assume that the random cost factor is additive in the manufacturer's unit cost function as follows:

$$c_j(\phi_\pi, u_\pi) = (c_j + \phi_\pi) \times u_\pi. \tag{22}$$

We assume that the inverse demand function takes the form:

$$\rho(y) = a - b \times y. \tag{23}$$

**Proposition 2**

*Suppose that the manufacturer's capacity for fast-response in-house production is sufficiently large and that the manufacturer's outsourcing activity is positive ($v^* > 0$). The marginal value of the outsourcing product in the second stage resembles the payoff of the short position of a real put option on the random cost factor, $\phi_\pi$, with strike price $K = a - 2bv^* - c_j$, plus a constant, that is, $\lambda^*_\pi = (a - 2bv^*) - MAX(0, (a - 2bv^* - c_j) - \phi_\pi)$. Moreover, in the first stage, the outsourcing cost the manufacturer is willing to pay is equal to the expected payoff of such position, $\sum_{\pi \in \Pi} f(\pi)\lambda^*_\pi$.*

**Proof**: See the Appendix.

Proposition 2 reveals anther interesting real option perspective for offshore outsourcing. In Proposition 2, the real option refers to the manufacturer's capability to use in-house quick-response production to make less costly products when the cost factor, $\phi_\pi$, turns out to be lower than the normal level. Of course, if $\phi_\pi$ turns out to be high, the manufacturer may not use the in-house production. However, in the first stage, when the manufacturer outsources products from suppliers it gives up such an option in exchange for a certain outsourcing cost, which implies a short position of the option. An illustration of this relationship is presented in Figure 3. In Figure 3.A, we can see that, if the random cost factor $\phi_\pi$ is higher than $2bv^* - a - c_j$, then the marginal value of the

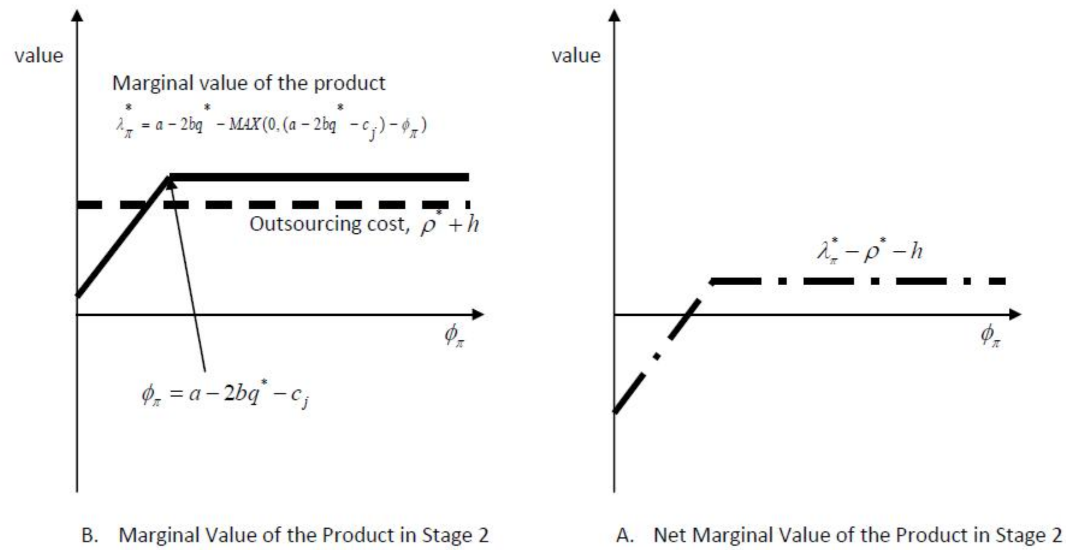


Figure 3: Marginal Value of the Product in Stage 2 with Uncertain In-House Production Cost.

product, $\lambda_\pi^*$, in Stage 2, is equal to $a - 2bv^*$, whereas, if $\phi_\pi$ is lower than $2bv^* - a - c_j$, $\lambda_\pi^*$ is equal to $c_j + \phi_\pi$. However, the unit outsourcing cost, $\rho^* + h$, is flat since it has been determined before $\phi_\pi$ is revealed. Figure 3.B, in turn, shows the net marginal value of the product across cost scenarios.

**Remark**

This real option interpretation can help one to understand the value of global outsourcing from another angle. For example, we have known that the values of put options increase as the uncertainty of the underlying factor increases. In Proposition 2, outsourcing the production is analogous to the short position of the put option which indicates that, in our case, the unit outsourcing cost the manufacturer is willing to pay will *decrease* as the uncertainty of the quick-response production cost gets *higher*. In Section 4, we extend the analysis to additional cases through the use of simulation studies in order to analyze multiple heterogenous manufacturers' behaviors, their interactions, profits, and the risks with/without quick-response capacities, and under competition.

## 3. Qualitative Properties

In this section we provide existence results and also establish conditions under which the function $F$ that enters variational inequality (16) is monotone. Such a property is useful for establishing convergence of the algorithmic scheme that we use in our simulation study. It is important to note that monotonicity plays a role in variational inequalities similar to the role that convexity plays in optimization problems.

**Theorem 2: Existence** *If all the cost functions are continuously differentiable and the inverse demand functions are continuous and continuously differentiable then there exists a solution to* *variational inequality (16).*

**Proof:** Since the production quantities are limited by capacities, the feasible set of (16) is compact, and it is also nonempty. Under the above assumptions, $F(X)$ is continuous in (16) and, hence, according to the theory of variational inequality (cf. Nagurney (1999)), there exists a solution to variational inequality (16).

**Theorem 3: Monotonicity**

*Suppose that all the cost functions in the model are continuously differentiable and convex. Also, suppose that all inverse demand functions are continuously differentiable, decreasing, and concave (hence, it could be linear). Then the vector F that enters the variational inequality (16) as expressed in (17) is monotone, that is,*

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, X' \neq X''. \tag{24}$$

**Proof:** See the Appendix.

We provide an algorithm for the computation of solutions to variational inequality (16). In particular, we recall the modified projection method (Nagurney (1999)). The method converges to a solution of our model provided that $F(X)$ is monotone and Lipschitz continuous, and that a solution exists, which is the case for our model. The algorithm is presented in the Appendix.

## 4. Simulation Studies

In this section we investigate the impact of demand and cost uncertainty on decision-making, profitability, and risk of the various decision-makers in the supply chain network. We utilize a series of simulation studies to analyze the five questions posed in the Introduction.

In particular, we answer Questions 1 and 2 using Example 1; we answer Question 3 using Example 2, and we answer Questions 4 and 5 using Example 3.

### Simulation Example 1

In Example 1, we focus on the first and the second questions raised in the Introduction. We consider two suppliers ($I = 2$), two manufacturers ($J = 2$), and one demand market ($M = 1$). Since the purpose of this example is to study the impact of demand uncertainty on the decisions, profits, and risks of manufacturers, with and without quick-response in-house production capabilities, in the examples we assume that the two manufacturers have the same cost factors but different in-house production capability. In particular, we assume that Manufacturer 1's in-house production capacity is 10 while Manufacturer 2's in-house production capacity is 0. We use Monte Carlo simulation to generate 200 demand factor scenarios ($|\Omega| = 200$) where $\theta_\omega$ follows a normal distribution. We let $E(\theta_\omega) = 20$ and vary the standard deviation, $\sigma(\theta_\omega)$, from 0 to 10. In Example 1, we assume that the in-house production cost is deterministic and we suppress $\pi$. The parameters of Example 1 are specified in Table 2.

Table 2: Parameter Specification for Example 1

| **Notation** | **value** |
|---|---|
| $E(\theta_\omega)$ | 20 |
| $\sigma(\theta_\omega)$ | from 0 to 10 with interval = 2 |
| $CAP_i$ | 10, $i = 1, 2$ |
| $CAP_j$ | $CAP_1 = 10$; $CAP_2 = 0$ |
| $\rho^j_m(\theta_\omega, Y^m_\omega)$ | $\rho^j_m(\theta_\omega, Y^m_\omega) = \theta_\omega - \sum_{j=1}^{2} y^{jm}_\omega$, $j = 1, 2;\ m = 1$ |
| $c_j(u^j_\omega)$ | $c_j(u^j_\omega) = 13u^j_\omega + 0.2u^{j2}_\omega$, $j = 1$ |
| $c_i(V^i)$ | $c_i(V^i) = 10\sum_{j=1}^{2} v^i_j + 0.2(\sum_{j=1}^{2} v^i_j)^2$, $i = 1, 2$ |
| $h^i_j$ | $h^i_j = 0.5$, $i = 1, 2;\ j = 1, 2$ |

The results are shown in Figures 5 and 6. In particular, Figure 4 explains the details of the interplay between the two manufacturers when the standard deviation of the demand factor, $\theta_\omega$, is equal to 10. Figure 4.A shows that the in-house production of Manufacturer 1 is zero when the realized value of $\theta_\omega$ is low, and linearly increases as the realized value of of $\theta_\omega$ increases until it reaches the capacity. Note that the in-house production of Manufacturer 2 is always zero since it does not have such capability. Figure 4.B compares the different behaviors of the two manufacturers in terms of sales. We can see that when the demand factor is lower than level **a** both manufacturers increase their sales as the demand factor increases. When the demand factor is between levels **a** and **b**, Manufacturer 1's sales are flat since it does not have more product purchased from suppliers and it has not started quick-response production yet. Manufacturer 2's sales, on the other hand, increase with a slightly steeper slope from levels **a** to **b**, due to the fact that Manufacturer 2 orders products from suppliers. When the demand factor is between levels **b** and **c** we can see that both manufacturers' sales are flat since they both have sold out products ordered from suppliers and it is still not economically justifiable for Manufacturer 1 to start in-house production. When the demand factor is between levels **c** and **d** Manufacturer 2 starts to use quick-response production and its sales linearly increase until it reaches its production capacity.

Figure 4.C compares the marginal values of the product perceived by the two manufacturers at Stage 2. First, in general, marginal values of the product of both manufacturers increase as the demand factor gets higher. Second, the slopes of the product marginal values of both manufacturers change when the demand factor reaches each of the levels (**a**, **b**, **c**, **d**) where the manufacturers' sales decisions change. Recall that, without interaction with Manufacturer 2, Manufacturer 1's marginal value of the product is described in Proposition 1 and is shown in Figure 2.A. In this example, however, Manufacturer 2's marginal value of product is altered due to the interaction with Manufacturer 1.

Figure 4.D shows the profits of the two manufacturers at different demand factor values. Recall that the mean of the demand factor $\theta_\omega$ is 20. We can observe that, if the demand factor is close to the mean, Manufacturer 2's profit is higher, whereas, if the demand factor is far from the mean, Manufacturer 1's profit is higher. In addition, since the demand factor follows s normal distribution,

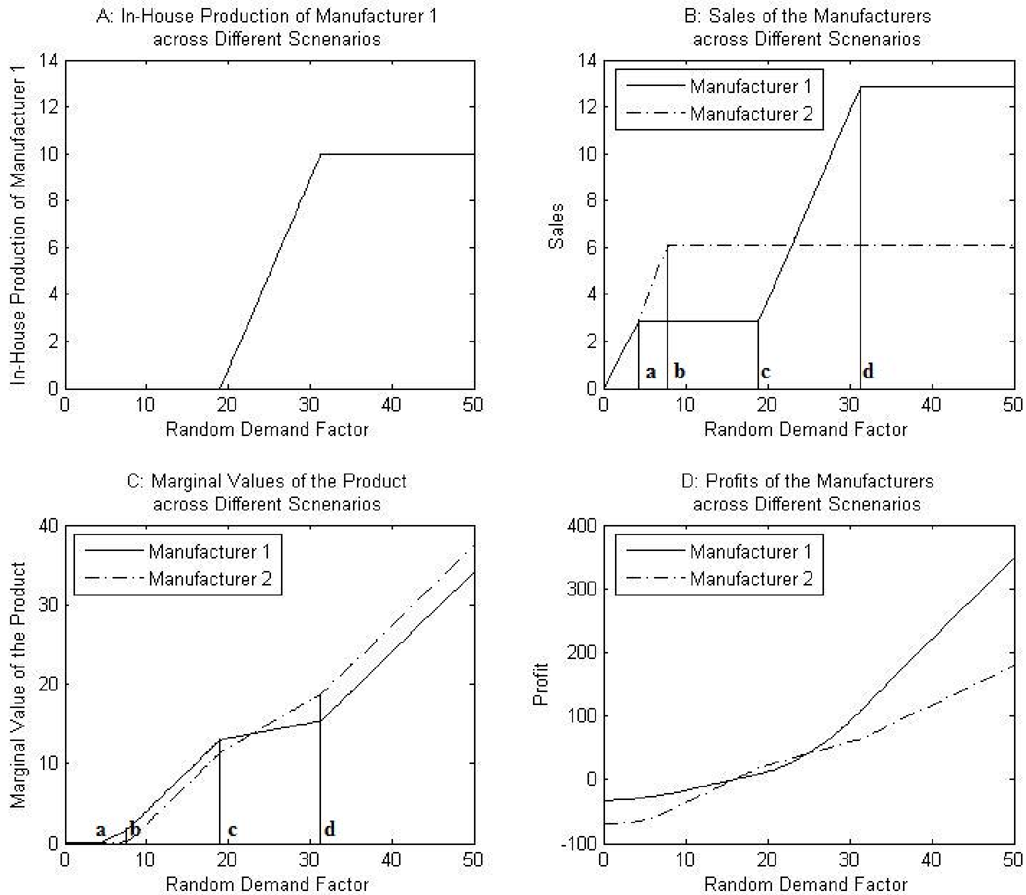


Figure 4: The Marginal Value of Products, Manufacturers' Decisions, and Manufacturers' Profits in Different Demand Scenarios

the demand factor that is closer to the mean is more likely to occur. *Hence, Figure 4.D indicates that Manufacturer 2 is more profitable when the demand turns out to be at a normal level while Manufacturer 1 is more profitable when the demand is unexpectedly high or low.*

Figure 5, in turn, compares the two manufacturers' decisions, profits, and risks as the standard deviation of the demand uncertainty factor rises from 0 to 10. Figure 5.A presents the trends of the two manufacturers' outsourcing of in-house production activities. We can see that Manufacturer 2, which does not have a quick-response production facility, increases its outsourcing quantity as demand uncertainty increases. This is consistent with the discussion following Proposition 1 which implies that as demand uncertainty increases the value of the product outsourced to suppliers will increase, and, as a consequence, the manufacturer will tend to increase its outsourcing quantity. Figure 5.A also shows that Manufacturer 2, which has quick-response in-house production capability, will increase its in-house production and will reduce its outsourcing quantity as the demand uncertainty increases. Figure 5.B compares the average profits of the two manufacturers where we can see that the profit of Manufacturer 1 increases as the demand uncertainty increases, and is consistently higher than that of Manufacturer 2. This result is expected since Manufacturer 1 has the option to use quick-response production which creates additional value to its supply chain.

It is interesting to see that, in Figure 5.C, the probability that the profit of Manufacturer 1 is higher than that of Manufacturer 2 increases from 0.2 to 0.6 as the demand uncertainty increases. This indicates that when the uncertainty is relatively low Manufacturer 2 has a greater chance to gain higher profit than Manufacturer 1 has. This result can be explained by Figure 4.D where we can see that, when the demand uncertainty factor is around the mean ($E(\theta_\omega) = 20$), Manufacturer 2 has higher profit but only beats Manufacturer 1 slightly, whereas when the demand uncertainty factor is far from the mean, Manufacturer 1 has a profit higher than that of Manufacturer 2 and the difference is much greater. Note that, with a normal distribution, most values of $\theta_\omega$ are around the mean , which increases the frequency that Manufacturer 2 beats Manufacturer 1. This situation is more pronounced when the standard deviation of $\theta_\omega$ is low so that the vast majority of the values of $\theta_\omega$ concentrate around the mean, which gives Manufacturer 2 a greater chance to obtain a higher profit.

Figure 5.D, in turn, compares the $5^{th}$ percentile profits of the two manufacturers. The $5^{th}$ percentile profit of Manufacturer 2 almost linearly decreases as the standard deviation of $\theta_\omega$ goes up, while the $5^{th}$ percentile profit of Manufacturer 1 first decreases and then becomes almost flat as the demand uncertainty increases. Note that, since the $5^{th}$ percentile profit/return is a commonly used measure for risk, Figure 5.D indicates that a fast-response production capability can significantly reduce risk and the benefit becomes greater as the demand uncertainty increases.

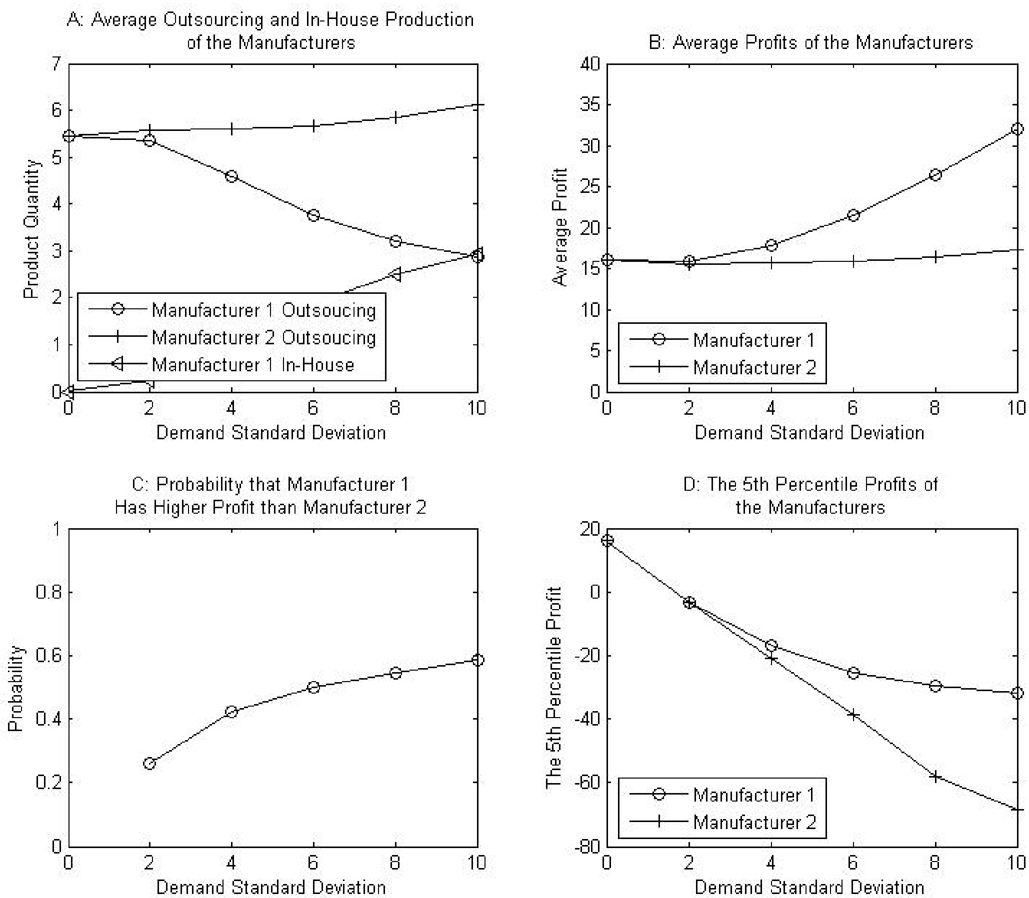


Figure 5: Manufacturers' Decisions, Profits, and Risks at Different Levels of Demand Uncertainty

**Simulation Example 2**

In Example 2, we focus on the third question raised in the beginning of this section. In particular, we consider ten suppliers ($I = 5$), ten manufacturers ($J = 5$), and a single demand market ($M = 1$). Since the purpose of this example is to study how the prevalence of the quick-response in-house production affects supply chain firms' decisions, profits, and risks under demand uncertainty, in this example, we vary the number of manufacturers that have quick-response in-house production capability from 0 to 5. We use Monte Carlo simulation to generate 200 demand factor scenarios ($|\Omega| = 200$) where $\theta_\omega$ follows a normal distribution. Similar to Example 1, we assume that the in-house production cost is deterministic and suppress $\pi$. The parameters of Example 2 are specified in Table 3. Throughout this example, we call the manufacturers with in-house production capacity **Type 1** manufacturers and the manufactures without in-house production capacity **Type 2** manufacturers.

Table 3: Parameter Specification for Example 2

| **Notation** | **value** |
|---|---|
| $E(\theta_\omega)$ | 20 |
| $\sigma(\theta_\omega)$ | 8 |
| $CAP_i$ | 10, i=1,2 |
| $CAP_j$ | 10 for Type 1 manufacturers; 0 for Type 2 manufacturers |
| $\rho^j_m(\theta_\omega, Y^m_\omega)$ | $\rho^j_m(\theta_\omega, Y^m_\omega) = \theta_\omega - \frac{\sum_{j=1}^{5} y^{jm}_\omega}{5}$, $j = 1, 5;\ m = 1$ |
| $c_j(u^j_\omega)$ | $c_j(u^j_\omega) = 13u^j_\omega + 0.2u^{j2}_\omega$, for Type 1 manufacturers |
| $c_i(V^i)$ | $c_i(V^i) = 10\sum_{j=1}^{2} v^i_j + 0.2(\sum_{j=1}^{2} v^i_j)^2$, $i = 1, ..., 5$ |
| $h^i_j$ | $h^i_j = 0.5$, $i = 1, ..., 5;\ j = 1, ..., 5$ |

Figure 6 presents the results for Example 2. Note that in Figure 6 the curves of Type 1 manufacturers do not have values when the percentage of such manufacturers is 0; and the curves of Type 2 manufacturers do not have values when the percentage of Type 1 manufacturers is 100%.

Figure 6.A shows that, as the prevalence of quick-response production among manufacturers increases, the outsourcing quantities of both Type 1 and Type 2 manufacturers increase. However, it is worth noting that the total outsourcing quantity of all manufacturers should decrease since the percentage of Type 2 manufacturers becomes lower. The quick-response production quantities of Type 1 manufacturers decrease as the percentage of Type 1 manufacturers increases. This is because of the following reason: When there are more manufacturers who have quick-response production capability the competition among those manufacturers becomes more intense and the benefit gets smaller. Therefore, the Type 1 manufacturers reduce quick-response production and turn back to outsourcing to lower costs.

Figure 6.B compares the profits of Types 1 and 2 manufacturers. The average profit of Type 1 manufacturers decreases as the percentage of Type 1 manufacturers increases, which is due to the fact the benefit of quick-response production declines as more manufacturers have such capability. From Figure 6.B we can also see that the average profit of Type 2 manufacturers is

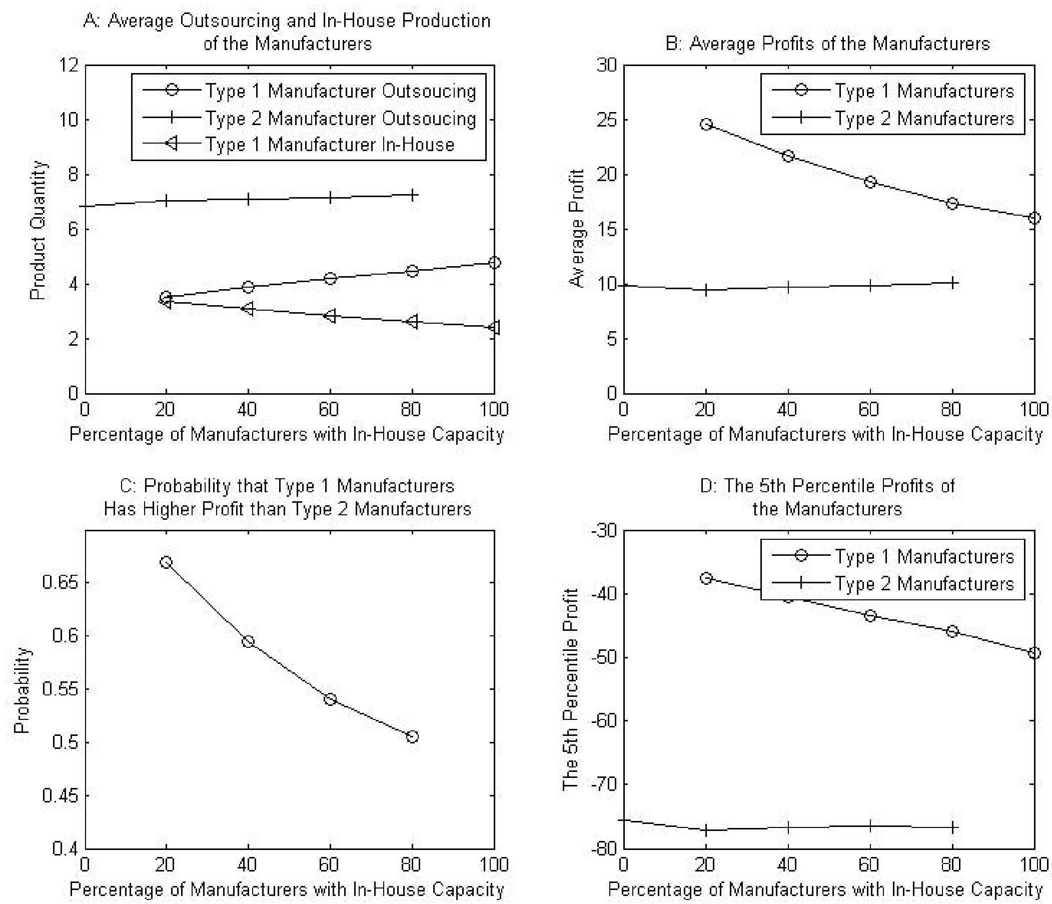


Figure 6: Manufacturers' Decisions, Profits, and Risks at Different Levels of Prevalence of Quick-response Production

always significantly lower than the average profit of Type 1 manufacturers.

Figure 6.C shows that the probability that Type 1 manufacturers beat Type manufacturers decreases from around 0.67 to around 0.5 as the percentage of Type 1 manufacturers increases from 20% to 80%, which is consistent with Figure 6.B. Figure 6.D, in turn, presents the $5^{th}$ percentile profits of Type 1 and Type 2 manufacturers. The trend is similar to that of the mean profits in Figure 6.B, which indicates that the risk of Type 1 manufacturers increases as the percentage of Type 1 manufacturers increases, and is still consistently lower than that of Type 2 manufacturers.

**Simulation Example 3**

Example 3 focuses on the last two questions raised in the Introduction. In particular, we consider two suppliers ($I = 2$), two manufacturers ($J = 2$), and one demand market ($M = 1$). Since the purpose of this example is to study the impact of the uncertainty of in-house production cost on the decisions, profits, and risks of manufacturers, with and without quick-response in-house production capability, in this example we assume that the two manufacturers have the same cost factors but different in-house production capability. Similar to the discussion for Example 1, we assume that Manufacturer 1's in-house production capacity is 10 while Manufacturer 2's in-house production capacity is 0. We use Monte Carlo simulation to generate 200 demand factor scenarios ($|\pi| = 200$) where $\phi_\pi$ follows normal distribution. We let $E(\phi_\pi) = 0$ and vary the standard deviation, $\sigma(\phi_\pi)$, from 0 to 4. In Example 3, we assume that the inverse demand function is deterministic and suppress $\omega$. The parameters of Example 3 are specified in Table 4.

Table 4: Parameter Specification for Example 3

| **Notation** | **value** |
|---|---|
| $E(\phi_\pi)$ | 0 |
| $\sigma(\phi_\pi)$ | from 0 to 4 with interval = 1 |
| $CAP_i$ | 10, i=1,2 |
| $CAP_j$ | $CAP_j = 10$; $j = 1, 2$ |
| $\rho^j_m(Y^m_\pi)$ | $\rho^j_m(Y^m_\pi) = 20 - \sum_{j=1}^{2} y^{jm}_\pi$, $j = 1, 2;\ m = 1$ |
| $c_j(u^j_\pi)$ | $c_j(u^j_\pi) = (13 + \phi_\pi)u^j_\pi + 0.2u^{j2}_\pi$, $j = 1$ |
| $c_i(V^i)$ | $c_i(V^i) = 10\sum_{j=1}^{2} v^i_j + 0.2(\sum_{j=1}^{2} v^i_j)^2$, $i = 1, 2$ |
| $h^i_j$ | $h^i_j = 0.5$, $i = 1, 2;\ j = 1, 2$ |

The results are shown in Figures 8 and 9. Similar to Example 1, we first explain the details of the interplay between the two manufacturers in Figure 7 where the standard deviation of the cost is equal to 4. We then use Figure 8 to compare the trends of the decisions, profits, and risks of the two manufacturers as the standard deviation of cost increases from 0 to 4.

Figure 7.A shows that the in-house production of Manufacturer 1 is zero when the quick-response production cost turns out to be high and increases as the production cost decreases. As a result, in Figure 7.B, we can see that the sales of Manufacturer 1 are flat when the cost factor is high and increases as the in-house production cost becomes lower. Figure 7.C compares the marginal value

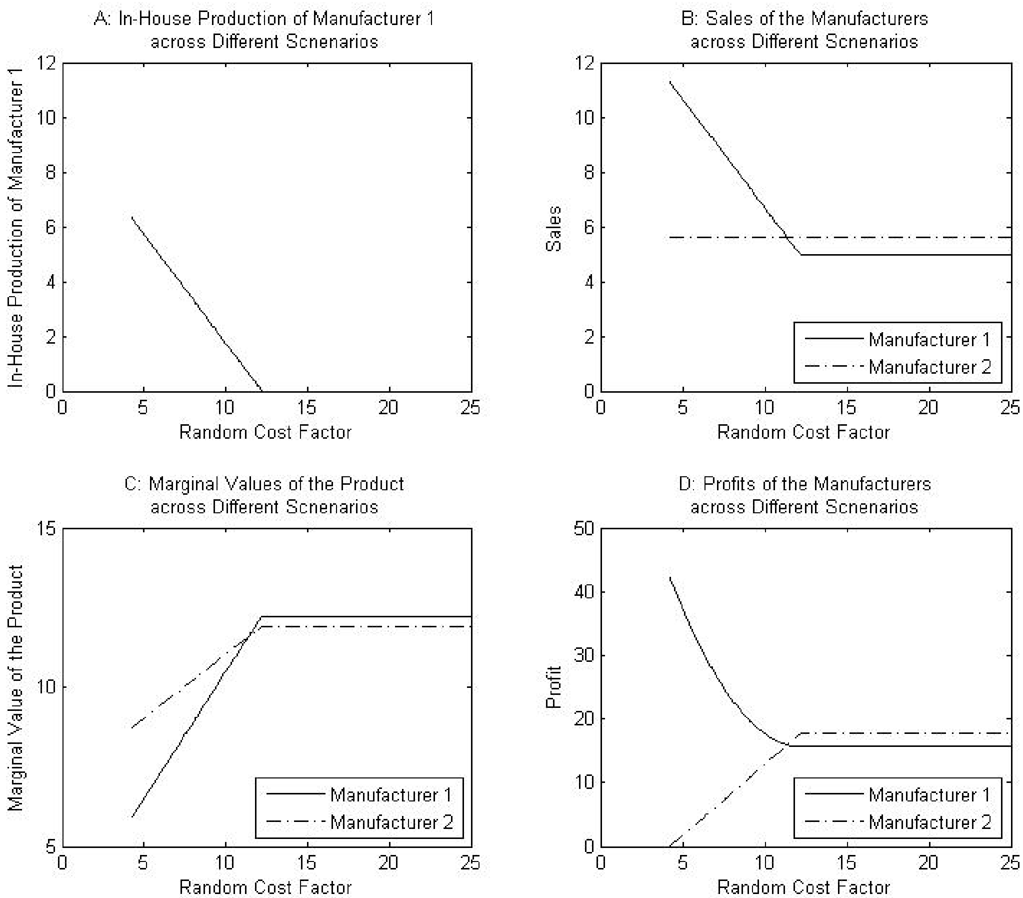


Figure 7: The Marginal Value of Products, Manufacturers' Decisions, and Manufacturers' Profits in Different Cost Scenarios

of the supplier's product perceived by the two manufacturers in Stage 2. The marginal value of the product is higher for Manufacturer 1 than for Manufacturer 2 when the cost factor is high while the marginal value for Manufacturer 1 decreases faster than that for Manufacturer 2 when the cost becomes lower. Such a difference is due to the utilization of in-house production by Manufacturer 2 when the cost is lower.

Figure 7.D presents results regarding the trends of the profits of the two interacting manufacturers. We can see that, when the cost is high Manufacturer 2's profit is higher than Manufacturer 1, since Manufacturer 2 orders more products from suppliers in Stage 1, and makes more sales (see Figure 7.B). However, as the in-house production cost factor declines, Manufacturer 1 enjoys a lower production cost and the profit of Manufacturer 1 increases. The profit of Manufacturer 2, however, decreases as the in-house production cost decreases due to the market interaction between the manufacturers where the increasing sales from Manufacturer 1 reduce the profit margin of Manufacturer 2. Note that since Manufacturer 2 does not have in-house production capacity the changing in-house production cost would not affect its profit if Manufacturer 1 did not exist.

Figure 8, in turn, compares the two manufacturers' decisions, profits, and risks as the standard deviation of the in-house production cost rises from 0 to 4. Figure 8.A shows that, as the cost uncertainty increases, Manufacturer 1's in-house production increases and the outsourcing quantity decreases since the quick-response production cost variations provide more opportunity for Manufacturer 1. This is also consistent with the discussion following Proposition 2, which predicts that the value of outsourcing decreases as the uncertainty of the quick-response production cost increases.

Figure 8.B shows that Manufacturer 1 becomes increasingly more profitable than Manufacturer 2 as the cost uncertainty increases. This is also due to the fact that an increase in the cost uncertainty gives more opportunity for low cost production to Manufacturer 1.

Figure 8.C, however, shows that the median profit of Manufacturer 1 is lower than that of Manufacturer 2, and that the gap becomes larger as the cost uncertainty increases. This seeming contraction can be explained by Figure 7.D, which shows that, when the production cost is high, Manufacturer 2 beats Manufacturer 1 marginally, while when the production is low, Manufacturer 1 beats Manufacturer 2 by a large amount. As a result, although Manufacturer 2 can earn higher profits in more scenarios, Manufacturer 1 has a higher average profit across all scenarios.

Figure 8.D compares the risks ($5^{th}$ percentile profits) of the two manufacturers. Similar to results in other examples, the risk of Manufacturer 1 is always lower than that of Manufacturer 2. Moreover, we can see that the $5^{th}$ percentile profit of Manufacturer 2 decreases from around 16 to around 4 when the cost uncertainty increases even though Manufacturer 2 does not have any in-house production capacity. This is because Manufacturer 2 is indirectly exposed to the production cost uncertainty due to the competition with Manufacturer 1.

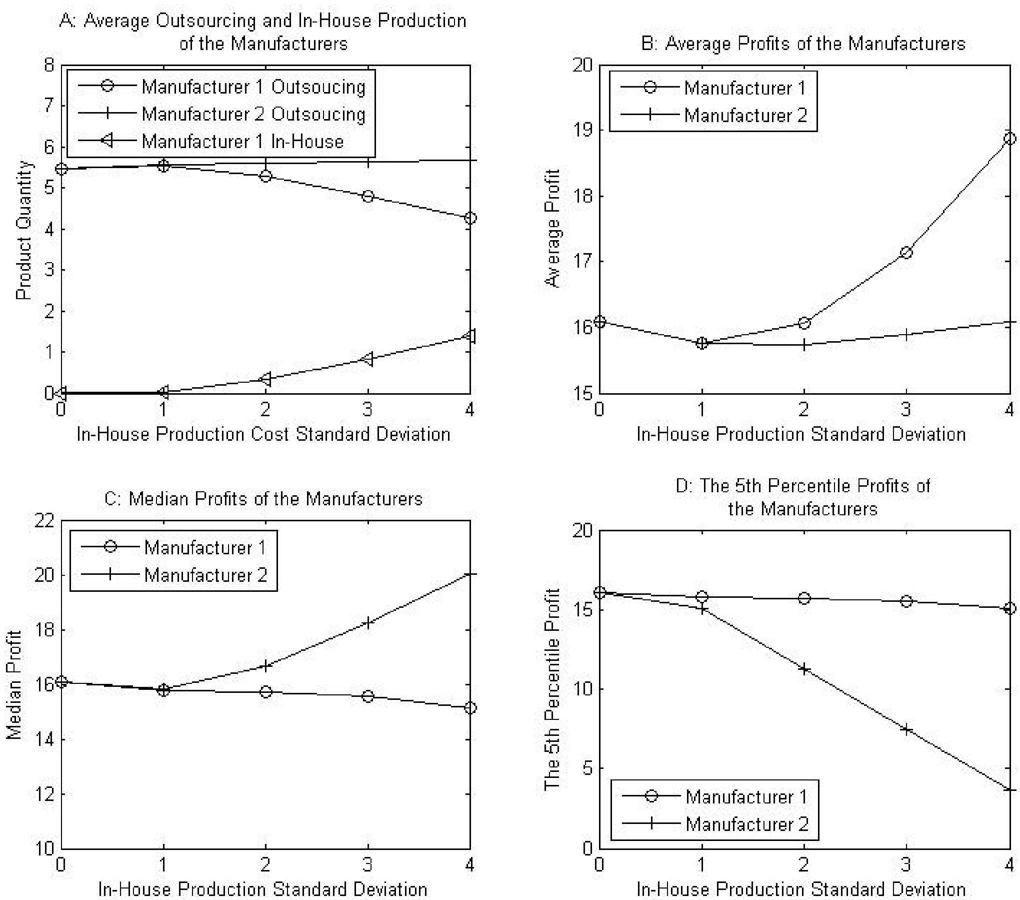


Figure 8: Manufacturers' Decisions, Profits, and Risks at Different Levels of Uncertainty of Quick-response Production Cost

## 5. Managerial Insights and Conclusion

This paper studied the impact of demand and cost uncertainty on the profits, risks, and decisions of a network of supply chain firms. In particular, we developed a variational inequality model that considers heterogenous supply chain firms' decision-making and determines the equilibrium of the supply chain network. We also established interesting theoretical and analytical results, provided important qualitative properties for the model, and presented an algorithm that was guaranteed to converge, under reasonable assumptions. We utilized a series of computational examples to answer several questions regarding supply chain firms' profitability, risk, and decision-making under demand and cost uncertainty.

Our analytical results revealed new real option interpretations for outsourcing decisions, which suggest that the outsourcing cost the manufacturers without quick-response production capability are willing to pay will increase when the demand uncertainty increases; and that the unit outsourcing cost the manufacturer with quick-response production is willing to pay will decrease when the uncertainty of the quick-response production cost increases. Our first simulation example showed that manufacturers with quick-response production capability have higher average profits and lower risks than manufacturers without quick-response production capability. However, the probability that manufacturers with quick-response production have higher profits than manufacturers without such capacity ranges from 0.2 to 0.6 at different demand uncertainty levels. In particular, we found that manufacturers without quick-response production are more profitable when the demand turns out to be at normal levels while manufacturers with such capability are more profitable when the demand is unexpectedly high and unexpectedly low. Our results also show that as the prevalence of quick-response production increases among manufacturers the quick-response production quantity of each manufacturer with such capability will decrease while the outsourcing quantity of each manufacturer will increase.

Our second simulation example indicates that, as the prevalence of quick-response production increases, the profit gap and the risk gap between manufacturers, with and without such capability, will become smaller. Our third example shows that, as the cost of quick-response production increases, the manufacturers with such capability will increase their quick-response production levels and will reduce outsourcing. Moreover, they become increasingly more profitable than manufacturers without quick-response production.

It is also worth noting that the third example indicates that manufacturers without in-house quick-response production will be indirectly affected by the uncertainty of the cost of quick-response production through competition with manufacturers who have such capability. For example, when the quick-response production cost turns out to be lower than expected, the profits of manufacturers without the quick-response production will be greatly reduced due to the competition with manufacturers with such capabilities. As the quick-response cost uncertainty increases, the risk of manufacturers without quick-response production will also increase since they are indirectly exposed to the cost uncertainty through market competition.

Our results reveal important managerial insights for supply chain decision-makers who are faced with decisions regarding outsourcing and quick-response production under demand and cost uncertainty. First, for manufacturers who do not have quick-response production capability, rising demand uncertainty will increase the value of outsourcing. However, for risk-neutral decision-makers who have quick-response production capability, rising cost uncertainty will reduce the value of outsourcing. Second, manufacturers with quick-response production can expect higher average profit and lower risk than their competitors who do not have such capability. However, these manufacturers may not have a higher chance to beat their competitors in terms of profit when the demand uncertainty is low. Moreover, they may have lower profits if the demand turns out to be at normal levels. Third, the prevalence of quick-response production will reduce the benefit. Fourth, manufacturers without quick-response capability should understand that they can still be indirectly and negatively affected by the cost variations of quick-response production through market competition.

**Acknowledgments**

This paper is dedicated to Professor Cyrus Derman, whose contributions have inspired generations to advance both theory and applications.

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## Appendix

**Proof of Lemma 1:**

In each scenario $\omega$ and $\pi$, the optimality conditions (4) of all manufacturers coincide with the solution of the following variational inequality ((cf. Nagurney (1999), Bazaraa et al. (1993), Gabay and Moulin (1980)): Determine $(U^*_{\omega\pi}, Y^*_{\omega\pi}) \in \mathcal{K}^4$ satisfying:

$$\sum_{j=1}^{J} \frac{\partial c_j(\phi_\pi, u^j_{\omega\pi})}{\partial u^j_{\omega\pi}} \times \left[u^j_{\omega\pi} - u^{j*}_{\omega\pi}\right]$$

$$-\sum_{j=1}^{J}\sum_{m=1}^{M} [\rho^j_m(\theta_{m\omega}, Y^{m*}_{\omega\pi}) + \frac{\partial \rho^j_m(\theta_{m\omega}, Y^{m*}_{\omega\pi})}{\partial Y^m_{\omega\pi}} y^{jm*}_{\omega\pi}] \times \left[y^{jm}_{\omega\pi} - y^{jm*}_{\omega\pi}\right] \geq 0,$$

$$\forall (U_{\omega\pi}, Y_{\omega\pi}) \in \mathcal{K}^4, \tag{25}$$

where $\mathcal{K}^4 \equiv ((U_{\omega\pi}, Y_{\omega\pi}) | (U_{\omega\pi}, Y_{\omega\pi}) \in R^{J+JM}_+$ and (5) and (6) hold).

On the other hand, suppose $(V^*, U^*, Y^*)$ is a solution to variational inequality (12), we can select any pair of $\bar{\omega} \in \Omega$ and $\bar{\pi} \in \Pi$, and set $q^i_j = q^{i*}_j$, $u^j_{\omega\pi} = u^{j*}_{\omega\pi}, \forall \omega \neq \bar{\omega}$ or $\pi \neq \bar{\pi}$, and $y^{jm}_{\omega\pi} = y^{jm*}_{\omega\pi}, \forall \omega \neq \bar{\omega}$ or $\pi \neq \bar{\pi}$. Variational inequality (12) then reduces to the following variational inequality.

$$\sum_{j=1}^{J} f(\bar{\omega}, \bar{\pi}) \frac{\partial c_j(\phi_\pi, u^j_{\bar{\omega}\bar{\pi}})}{\partial u^j_{\bar{\omega}\bar{\pi}}} \times \left[u^j_{\bar{\omega}\bar{\pi}} - u^{j*}_{\bar{\omega}\bar{\pi}}\right]$$

$$-\sum_{j=1}^{J}\sum_{m=1}^{M} f(\bar{\omega}, \bar{\pi}) [\rho^j_m(\theta_{m\bar{\omega}}, Y^{m*}_{\bar{\omega}\bar{\pi}}) + \frac{\partial \rho^j_m(\theta_{m\bar{\omega}}, Y^{m*}_{\bar{\omega}\bar{\pi}})}{\partial Y^m_{\bar{\omega}\bar{\pi}}} y^{jm*}_{\bar{\omega}\bar{\pi}}] \times \left[y^{jm}_{\bar{\omega}\bar{\pi}} - y^{jm*}_{\bar{\omega}\bar{\pi}}\right] \geq 0,$$

$$\forall (U_{\bar{\omega}\bar{\pi}}, Y_{\bar{\omega}\bar{\pi}}) \in \bar{\mathcal{K}}^4, \tag{26}$$

where $\bar{\mathcal{K}}^4 \equiv ((U_{\bar{\omega}\bar{\pi}}, Y_{\bar{\omega}\bar{\pi}}) | (U_{\bar{\omega}\bar{\pi}}, Y_{\bar{\omega}\bar{\pi}}) \in R^{J+JM}_+$ and (5) and (6) hold). Since $f(\bar{\omega}, \bar{\pi})$ is positive and constant, we can divide $f(\bar{\omega}, \bar{\pi})$ on both side of (26) and obtain:

$$\sum_{j=1}^{J} \frac{\partial c_j(\phi_\pi, u^j_{\bar{\omega}\bar{\pi}})}{\partial u^j_{\bar{\omega}\bar{\pi}}} \times \left[u^j_{\bar{\omega}\bar{\pi}} - u^{j*}_{\bar{\omega}\bar{\pi}}\right]$$

$$-\sum_{j=1}^{J}\sum_{m=1}^{M} [\rho^j_m(\theta_{m\bar{\omega}}, Y^{m*}_{\bar{\omega}\bar{\pi}}) + \frac{\partial \rho^j_m(\theta_{m\bar{\omega}}, Y^{m*}_{\bar{\omega}\bar{\pi}})}{\partial y^{jm}_{\bar{\omega}\bar{\pi}}} y^{jm*}_{\bar{\omega}\bar{\pi}}] \times \left[y^{jm}_{\bar{\omega}\bar{\pi}} - y^{jm*}_{\bar{\omega}\bar{\pi}}\right] \geq 0,$$

$$\forall (U_{\bar{\omega}\bar{\pi}}, Y_{\bar{\omega}\bar{\pi}}) \in \bar{\mathcal{K}}^4. \tag{27}$$

Note that since variational inequality (27) is (25) under scenario $\bar{\omega}$ and $\bar{\pi}$,$(U^*_{\bar{\omega}\bar{\pi}}, Y^*_{\bar{\omega}\bar{\pi}})$ is also a solution to (25) under scenario $\bar{\omega}$ and $\bar{\pi}$. Thus, the optimality conditions of all manufacturers are satisfied in scenario $\bar{\omega}$ and $\bar{\pi}$.

Since $\bar{\omega}$ and $\bar{\pi}$ are arbitrarily selected, we conclude that the optimality conditions of all manufacturers are satisfied in every possible individual scenario. **Q.E.D.**

**Lemma 2**

*Suppose $X^* \in R^n_+$ is a solution to the variatoinal inequality*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in R^n_+, \tag{28}$$

*where $F$ is a given continuous function from $R^n_+$ to $R^n$ with the $l^{th}$ element denoted by $f^l(X^*)$. In addition, let $x^l$ denote the $l^{th}$ element of $X$. Then the following conditions hold:*

*If $x^{l*} > 0$ then $f^l(X^*) = 0$;*

*If $x^{l*} = 0$ then $f^l(X^*) \geq 0$;*

*If $f^l(X^*) > 0$ then $x^{l*} = 0$.*

**Proof:** For an arbitrary $l$ we can construct $\hat{X} \in R^n_+$ where $\hat{x}^k = x^{k*}$, $\forall k \neq l$. Since (18) holds for all $X \in R^n_+$ it also holds for $\hat{X}$, that is,

$$\langle F(X^*)^T, \hat{X} - X^* \rangle \geq 0 \tag{29}$$

which reduces to

$$f^l(X^*) \times (x^l - x^{l*}) \geq 0, \quad \forall x^l \in R^1_+, \tag{30}$$

Since (20) holds for all $x^l \geq 0$ we can easily verify the below:

If $x^{l*} > 0$ then $f^l(X^*) = 0$;

If $x^{l*} = 0$ then $f^l(X^*) \geq 0$;

If $f^l(X^*) > 0$ then $x^{l*} = 0$.

Given that $l$ is selected arbitrarily these results hold for all the elements. **Q.E.D.**

**Proof of Theorem 1:**

Summation of inequalities (12) and (15) yields, after algebraic simplification, the variational inequality (16). We now establish the converse, that is, that a solution to variational inequality (16) satisfies the sum of conditions (12) and (15) and is, hence, an equilibrium according to Definition 1. To inequality (16) add the term $+\rho_j^{i*} - \rho_j^{i*}$ to the first set of brackets preceding the multiplication sign. The addition of such terms does not alter (16) since the value of these terms is zero. The resulting inequality can be rewritten to become equivalent to the price and material flow pattern satisfying the sum of the conditions (12) and (15). The proof is complete. **Q.E.D.**

**Proof of Proposition 1**:

First, since the production cost function for each manufacturer is continuously differentiable and convex (hence, it could be linear) and the inverse demand function is decreasing, each manufacturer's objective function (8) is concave. In addition, the constraints (9)-(11) are all linear constraints. Therefore, the KKT conditions of the problem defined by (8)-(11) are both necessary and sufficient conditions for optimal solutions. We can express the KKT conditions of all manufacturers using the following variational inequality: Determine $(V^*, U^*, Y^*, \lambda^*, \mu^*) \in \mathcal{K}^1$ satisfying:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}[\rho_j^{i*} + h_j^i - \sum_{\omega\in\Omega}\sum_{\pi\in\Pi} f(\omega,\pi)\lambda_{\omega\pi}^{j*}] \times \left[v_j^i - v_j^{i*}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J} f(\omega,\pi)[\frac{\partial c_j(\phi_\pi, u_{\omega\pi}^j)}{\partial u_{\omega\pi}^j} + \mu_{\omega\pi}^{j*} - \lambda_{\omega\pi}^{j*}] \times \left[u_{\omega\pi}^j - u_{\omega\pi}^{j*}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J}\sum_{m=1}^{M} f(\omega,\pi)[\lambda_{\omega\pi}^{j*} - \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^{m*}) - \frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^{m*})}{\partial Y_{\omega\pi}^m} y_{\omega\pi}^{jm*}] \times \left[y_{\omega\pi}^{jm} - y_{\omega\pi}^{jm*}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J} f(\omega,\pi)[\sum_{i=1}^{I} v_j^{i*} + u_{\omega\pi}^{j*} - \sum_{m=1}^{M} y_{\omega\pi}^{jm*}] \times \left[\lambda_{\omega\pi}^j - \lambda_{\omega\pi}^{j*}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{\pi\in\Pi}\sum_{j=1}^{J} f(\omega,\pi)[CAP_j - u_{\omega\pi}^{j*}] \times \left[\mu_{\omega\pi}^j - \mu_{\omega\pi}^{j*}\right] \geq 0,$$

$$\forall (V, U, Y, \lambda, \mu) \in R_+^{IJ+|\Omega||\Pi|(3J+JM)}, \tag{31}$$

where $\lambda$ and $\mu$ denote the vectors of Lagrangian multipliers associated with constraints (9) and (10), respectively. Since the KKT conditions are sufficient and necessary conditions for optimality, variational inequalities (31) and (12) are equivalent. Variational inequality (31) will be used to prove Propositions 1 and 2.

Since we now consider a single supplier, a single manufacturer without in-house production capacity and a single market with deterministic cost factor we can suppress indexes $i$, $j$, $m$, and $\pi$. After we substitute (21) into $\rho(\theta_\omega, y)$, Variational inequality (31) reduces to the following form:

$$[\rho^* + h - \sum_{\omega\in\Omega} f(\omega)\lambda_\omega^*] \times [q - v^*] + \sum_{\omega\in\Omega} f(\omega)[\lambda_\omega^* - (a + \theta_\omega - 2by_\omega^*)] \times [y_\omega - y_\omega^*]$$

$$+\sum_{\omega\in\Omega} f(\omega)[v^* - y_\omega^*] \times [\lambda_\omega - \lambda_\omega^*] \geq 0, \quad \forall (V, y, \lambda) \in R_+^{1+2|\Omega|} \tag{32}$$

We only consider the scenarios the probabilities of which are positive, that is, $f(\omega) > 0, \forall \omega \in \Omega$.

Next, we will prove $\lambda_\omega^* = MAX(0, \theta_\omega - (2bv^* - a))$ by showing that if $\theta_\omega - (2bv^* - a) > 0$ then $\lambda_\omega^* = \theta_\omega - (2bv^* - a)$ in Step 1, and if $\theta_\omega - (2bv^* - a) \leq 0$ then $\lambda_\omega^* = 0$ in Step 2.

Step 1

First note that since $\lambda^*_\omega \geq 0$ we have $v^* \geq y^*_\omega,\ \forall\omega \in \Omega$ (see Lemma 2). Hence, if $\theta_\omega - (2bv^* - a) > 0$ then $\theta_\omega - (2by^*_\omega - a) > 0,\ \forall\omega \in \Omega$, equivalently, $(a + \theta_\omega - 2by^*_\omega) > 0,\ \forall\omega \in \Omega$.

According to Lemma 2 since $y^* \geq 0$ we also have $\lambda^*_\omega \geq (a + \theta_\omega - 2by^*_\omega)$. Thus, we now have $\lambda^*_\omega > 0$.

Since $\lambda^*_\omega > 0$ according to Lemma 2 we must have

$$v^* - y^*_\omega = 0. \tag{33}$$

Given that $v^* > 0$ we have $y^*_\omega > 0$. Now, since $y^*_\omega > 0$ based on Lemma 2 we have $\lambda^*_\omega - (a + \theta_\omega - 2by^*_\omega) = 0$ which is equivalent to $\lambda^*_\omega = (a+\theta_\omega - 2by^*_\omega)$. Finally, given (33) we have $\lambda^*_\omega = a+\theta_\omega - 2bv^*$.

we have now that if $\theta_\omega - (2bv^* - a) > 0$ then $\lambda^*_\omega = a + \theta_\omega - 2bv^*$ ($\lambda^*_\omega = \theta_\omega - (2bv^* - a)$).

Step 2

Next, we show that if $\theta_\omega - (2bv^* - a) \leq 0$ then $\lambda^*_\omega = 0$. Recall that since $\lambda^*_\omega \geq 0$ we have $v^* \geq y^*_\omega,\ \forall\omega \in \Omega$ (see Lemma 2). Now, we discuss the cases for $v^* = y^*_\omega$ and $v^* > y^*_\omega$ separately.

First, if $v^* = y^*_\omega$ since $v^* > 0$ we have $y^*_\omega > 0$ which implies that $\lambda^*_\omega - (a + \theta_\omega - 2by^*_\omega) = 0$. Since $v^* = y^*_\omega$ we have $\lambda^*_\omega = (a + \theta_\omega - 2bv^*)$, equivalently, $\lambda^*_\omega = \theta_\omega - (2bv^* - a)$. Given that $\theta_\omega - (2bv^* - a) \leq 0$ and $\lambda^*_\omega \geq 0$, we have $\lambda^*_\omega = 0$.

Next, if $v^* > y^*_\omega$ we directly have $\lambda^*_\omega = 0$ from Lemma 2. Hence, we have now if $\theta_\omega - (2bv^* - a) \leq 0$ then $\lambda^*_\omega = 0$.

In summary, if $\theta_\omega - (2bv^* - a) > 0$ then $\lambda^*_\omega = \theta_\omega - (2bv^* - a))$, and if $\theta_\omega - (2bv^* - a) \leq 0$ then $\lambda^*_\omega = 0$. Therefore, we have $\lambda^*_\omega = MAX(0, \theta_\omega - (2bv^* - a))$.

Since $v^* > 0$, we have $\rho^* + h = \sum_{\omega\in\Omega} f(\omega)\lambda^*_\omega$ (see Lemma 2), which indicates that in Stage 1 the unit cost the manufacturer is willing to pay for the product is equal to the expected payoff of this real call option. **Q.E.D.**

**Proof of Proposition 2**:

Since we now consider a single supplier, a single manufacturer with sufficient in-house production capacity and a single market with deterministic demand factor we can suppress indexes $i$, $j$, $m$, and $\omega$. After we substitute (22) and (23) into (31), Variational inequality (31) reduces to the following form:

$$\begin{aligned}
&[\rho^* + h - \sum_{\pi\in\Pi} f(\pi)\lambda^*_\pi] \times [q - v^*] + \sum_{\pi\in\Pi} f(\pi)[c_j + \phi_\pi + \mu^*_\pi - \lambda^*_\pi] \times [u_\pi - u^*_\pi] \\
&+ \sum_{\pi\in\Pi} f(\pi)[\lambda^*_\pi - (a - 2b \times y^*_\pi)] \times [y_\pi - y^*_\pi] + \sum_{\pi\in\Pi} f(\pi)[v^* + u^*_\pi - y^*_\pi] \times [\lambda_\pi - \lambda^*_\pi] \\
&+ \sum_{\pi\in\Pi} f(\pi)[CAP_j - u^*_\pi] \times [\mu_\pi - \mu^*_\pi] \geq 0, \quad \forall(V, U, Y, \lambda, \mu) \in R^{1+4|\Pi|}_+,
\end{aligned} \tag{34}$$

Since this analytical analysis focuses on the case where the manufacturer's in-house production is sufficiently large, we have $CAP_j - u^*_\pi > 0,\ \forall \pi \in \Pi$. Therefore, $\mu^*_\pi = 0, \forall \pi \in \Pi$ (see Lemma 2) and variational inequality (34) can reduce to:

$$[\rho^* + h - \sum_{\pi\in\Pi} f(\pi)\lambda^*_\pi] \times [q - v^*] + \sum_{\pi\in\Pi} f(\pi)[c_j + \phi_\pi - \lambda^*_\pi] \times [u_\pi - u^*_\pi]$$
$$+ \sum_{\pi\in\Pi} f(\pi)[\lambda^*_\pi - (a - 2b \times y^*_\pi)] \times [y_\pi - y^*_\pi] + \sum_{\pi\in\Pi} f(\pi)[v^* + u^*_\pi - y^*_\pi] \times [\lambda_\pi - \lambda^*_\pi] \geq 0,$$
$$\forall (V, U, Y, \lambda) \in R_+^{1+3|\Pi|}, \tag{35}$$

We only consider the scenarios the probabilities of which are positive, that is, $f(\pi) > 0, \forall \pi \in \Pi$.

Next, we will prove $\lambda^*_\pi = (a - 2bv^*) - MAX(0, (a - 2bv^* - c_j) - \phi_\pi)$ by showing that if $(a - 2bv^* - c_j) - \phi_\pi > 0$ then $\lambda^*_\pi = c_j + \phi_\pi$ in Step 1, and if $(a - 2bv^* - c_j) - \phi_\pi \leq 0$ then $\lambda^*_\pi = a - 2bv^*$ in Step 2.

Before we prove the results in Steps 1 and 2. We first show that $a - 2bv^* > 0$. Since the outsourcing activity is nonzero, i.e., $v^* > 0$, (35) implies that $\rho^* + h = \sum_{\pi\in\Pi} f(\pi)\lambda^*_\pi$ (see Lemma 2). Since the total outsourcing cost should be positive, i.e., $\rho^* + h > 0$, we have that there must exist $\bar{\pi} \in \Pi$ such that $\lambda^*_{\bar{\pi}} > 0$. If $\lambda^*_{\bar{\pi}} > 0$, (35) implies that $v^* + u^*_{\bar{\pi}} = y^*_{\bar{\pi}}$ (see Lemma 2). Since $v^* > 0$ and $u^*_{\bar{\pi}}$ is non-negative we have $y^*_{\bar{\pi}} > 0$. If $y^*_{\bar{\pi}} > 0$, (35) implies that $\lambda^*_{\bar{\pi}} - (a - 2by^*_{\bar{\pi}}) = 0$ (see Lemma 2). Since $\lambda^*_{\bar{\pi}} > 0$ we have $a - 2by^*_{\bar{\pi}} > 0$. On the other hand $v^* + u^*_{\bar{\pi}} = y^*_{\bar{\pi}}$ implies that $v^* \leq y^*_{\bar{\pi}}$. Therefore, we have $a - 2bv^* > 0$.

Step 1

We now investigate the case where $(a - 2bv^* - c_j) - \phi_\pi > 0$, or, equivalently, $a - 2bv^* > c_j + \phi_\pi$. Since (35) implies that $c_j + \phi_\pi \geq \lambda^*_\pi$ and $\lambda^*_\pi \geq a - 2by^*_\pi$ (see Lemma 2), we have $a - 2bv^* > a - 2by^*_\pi,\ \forall \pi$. Hence, $v^* < y^*_\pi$. On the other hand, (35) also implies that $v^* + u^*_\pi \geq y^*_\pi$ (see Lemma 2). Hence, we have now $u^*_\pi > 0$ which according to (35) implies that $\lambda^*_\pi = c_j + \phi_\pi$ (see Lemma 2).

Step 2

We now investigate the situation where $(a - 2bv^* - c_j) - \phi_\pi \leq 0$, or, equivalently, $a - 2bv^* \leq c_j + \phi_\pi$. We now have to discuss two cases: $u^*_\pi > 0$ and $u^*_\pi = 0$, separately.

We first prove that $u^*_\pi > 0$ cannot be possible by contraction. If $u^*_\pi > 0$, (35) implies $c_j + \phi_\pi = \lambda^*_\pi$ (see Lemma 2). Since we have assumed that the cost has to be positive $(c_j + \phi_\pi > 0)$ we have $\lambda^*_\pi > 0$. Since $\lambda^*_\pi > 0$, (35) implies that $v^* + u^*_\pi = y^*_\pi$ (see Lemma 2). Since $u^*_\pi > 0$, we have $y^*_\pi > v^*$ which implies that $a - 2bv^* > a - 2by^*_\pi$. Since we have had $c_j + \phi_\pi = \lambda^*_\pi$ and $a - 2bv^* \leq c_j + \phi_\pi$, we now have $\lambda^*_\pi > a - 2by^*_\pi$ which, based on (35), implies that $y^*_\pi = 0$ (see Lemma 2). Note that $y^*_\pi = 0$ contradicts with the relationship, $y^*_\pi > v^*$ $(v^* \geq 0)$, which we have shown previously.

We now focus on the case $u^*_\pi = 0$. If $u^*_\pi = 0$, the last term of (35) implies that $v^* \geq y^*_\pi$ (see Lemma 2). The third term of (35) also implies that $\lambda^*_\pi - (a - 2by^*_\pi) \geq 0$ (see Lemma 2). So, we now have $\lambda_\pi^* - (a - 2bv^*) \geq 0$. Note that in the beginning of the proof, we have shown that $(a - 2bv^*) > 0$. Hence, now we have $\lambda_\pi^* > 0$. If $\lambda_\pi^* > 0$, (35) implies that $v^* = y_\pi^*$ given that $u_\pi^* = 0$ (see Lemma 2). Since $v^* > 0$, we have $y_\pi^* > 0$ which implies that $\lambda_\pi^* - (a - 2by_\pi^*) = 0$ (see Lemma 2). Again, since we have proved $v^* = y_\pi^*$, we now have $\lambda_\pi^* = a - 2bv^*$.

Therefore, we have proved that if $(a - 2bv^* - c_j) - \phi_\pi > 0$ then $\lambda_\pi^* = c_j + \phi_\pi$, and if $(a - 2bv^* - c_j) - \phi_\pi \leq 0$ then $\lambda_\pi^* = a - 2bv^*$, which is $\lambda_\pi^* = (a - 2bv^*) - MAX(0, (a - 2bv^* - c_j) - \phi_\pi)$.

**Q.E.D.**

**Proof of Theorem 3**:

The Jacobian of $F(X)$ that enters (16) can be written as

$$Jacobian = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}, \tag{36}$$

where $A$ is the $IJ \times IJ$ submatrix corresponding to $F_{ij}^V$, $i = 1, \ldots, I$ and $j = 1, \ldots, J$; $B$ is the $J|\Omega||\Pi| \times J|\Omega||\Pi|$ submatrix corresponding to $F_{j\omega\pi}^U$, $j = 1, \ldots, J$; $\omega \in \Omega$; and $\pi \in \Pi$, and $C$ is the $JM|\Omega||\Pi| \times JM|\Omega||\Pi|$ submatrix corresponding to $F_{jm\omega\pi}^Y$, $j = 1, \ldots, J$, $m = 1, \ldots, M$; $\omega \in \Omega$; and $\pi \in \Pi$.

Since all cost functions are continuously differentiable and convex (hence, it can be linear), we can verify that $A$ and $B$ are positive semidefinite. Next, we will show that $C$ is also positive semidefinite. $C$ can be written as

$$C = \begin{pmatrix} f(\omega_1, \pi_1) C_{\omega_1 \pi_1}^1 & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & 0 \\ \vdots & \cdots & f(\omega, \pi) C_{\omega\pi}^m & \cdots & \vdots \\ 0 & \cdots & \cdots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & f(\omega_{|\Omega|}, \pi_{|\Pi|}) C_{\omega_{|\Omega|}\pi_{|\Pi|}}^M \end{pmatrix}, \tag{37}$$

where $C_{\omega\pi}^m$ is a $J \times J$ submatrx corresponding to $F_{jm\omega\pi}^Y$, $j = 1, \ldots, J$, for market $m$ and in scenario $\omega$ and $\pi$.

Note that the elements in $C_{\omega\pi}^m$ are $\frac{\partial F_{jm\omega\pi}^Y}{\partial y_{\omega\pi}^{lm}}$, $j = 1, \ldots, J$ and $l = 1, \ldots, J$, where $\frac{\partial F_{jm\omega\pi}^Y}{\partial y_{\omega\pi}^{lm}} = -2\frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial Y_{\omega\pi}^m} - \frac{\partial^2 \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial {Y_{\omega\pi}^m}^2} \times y_{\omega\pi}^{jm}$ if $l = j$, and $\frac{\partial F_{jm\omega\pi}^Y}{\partial y_{\omega\pi}^{lm}} = -\frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial Y_{\omega\pi}^m} - \frac{\partial^2 \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial {Y_{\omega\pi}^m}^2} \times y_{\omega\pi}^{jm}$ if $l \neq j$. Therefore, $C_{\omega\pi}^m$ can be written as the sum of three matrixes, $C_{\omega\pi}^m = C_{\omega\pi}^{m1} + C_{\omega\pi}^{m2} + C_{\omega\pi}^{m3}$, where

$$C_{\omega\pi}^{m1} = -\begin{pmatrix} \frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial Y_{\omega\pi}^m} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & 0 \\ \vdots & \cdots & \frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial Y_{\omega\pi}^m} & \cdots & \vdots \\ 0 & \cdots & \cdots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \frac{\partial \rho_m^j(\theta_{m\omega}, Y_{\omega\pi}^m)}{\partial Y_{\omega\pi}^m} \end{pmatrix}, \tag{38}$$

$$C^{m2}_{\omega\pi} = -\frac{\partial \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial Y^m_{\omega\pi}} \begin{pmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & \ddots & \cdots & \cdots & 1 \\ \vdots & \cdots & 1 & \cdots & \vdots \\ 1 & \cdots & \cdots & \ddots & 1 \\ 1 & \cdots & \cdots & 1 & 1 \end{pmatrix}, \tag{39}$$

and

$$C^{m3}_{\omega\pi} = -\frac{\partial^2 \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial {Y^m_{\omega\pi}}^2} \begin{pmatrix} y^{1m}_{\omega\pi} & y^{1m}_{\omega\pi} & \cdots & \cdots & y^{1m}_{\omega\pi} \\ y^{2m}_{\omega\pi} & \ddots & \cdots & \cdots & y^{2m}_{\omega\pi} \\ \vdots & \cdots & y^{jm}_{\omega\pi} & \cdots & \vdots \\ y^{(J-1)m}_{\omega\pi} & \cdots & \cdots & \ddots & y^{(J-1)m}_{\omega\pi} \\ y^{Jm}_{\omega\pi} & \cdots & \cdots & y^{Jm}_{\omega\pi} & y^{Jm}_{\omega\pi} \end{pmatrix}. \tag{40}$$

Since the inverse demand function $\rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})$ is a decreasing function of the total sales , $Y^m_{\omega\pi}$, $-\frac{\partial \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial Y^m_{\omega\pi}}$ is positive. Hence, $C^{m1}_{\omega\pi}$ is positive semidefinite. We can verify that the only non-zero eigenvalue of $C^{m2}_{\omega\pi}$ is $-J\frac{\partial \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial Y^m_{\omega\pi}}$. Since $-\frac{\partial \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial Y^m_{\omega\pi}}$ is positive, $C^{m2}_{\omega\pi}$ is also positive semidefinite. We can also verify that the only non-zero eigenvalue of $C^{m3}_{\omega\pi}$ is $-\frac{\partial^2 \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial {Y^m_{\omega\pi}}^2} \times \sum_{j=1}^J y^{jm}_{\omega\pi}$, which is equal to $-\frac{\partial^2 \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial {Y^m_{\omega\pi}}^2} Y^m_{\omega\pi}$. Since $\rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})$ is concave, $-\frac{\partial^2 \rho^j_m(\theta_{m\omega}, Y^m_{\omega\pi})}{\partial {Y^m_{\omega\pi}}^2} Y^m_{\omega\pi} \geq 0$. So, $C^{m3}_{\omega\pi}$ is also positive semidefinite. Since $C^{m1}_{\omega\pi}$, $C^{m2}_{\omega\pi}$, and $C^{m3}_{\omega\pi}$ are all positive semidefinite, $C^m_{\omega\pi}$ is positive semidefinite and C is positive semidefinite.

Finally, since $A$, $B$, and $C$ are all positive semidefinite, the Jacobian of $F(X)$ is positive semidefinite and $F(X)$ is monotone. **Q.E.D**

**The Computational Procedure: The Extragradient Algorithm (Modified Projection Method)**

**Step 0: Initialization**

Start with an $X^0 \in \mathcal{K}$ and select $\omega$, such that $0 < \omega \leq \frac{1}{L}$, where $L$ is the Lipschitz constant for function $F(X)$. Let $\mathcal{T} = 1$.

**Step 1: Construction and Computation**

Compute $\bar{X}^{\mathcal{T}-1}$ by solving the variational inequality subproblem:

$$\left\langle (\bar{X}^{\mathcal{T}-1} + (\omega F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}))^T, X' - \bar{X}^{\mathcal{T}-1} \right\rangle \geq 0, \quad \forall X' \in \mathcal{K}. \tag{41}$$

**Step 2: Adaptation**

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\left\langle (X^{\mathcal{T}} + (\omega F(\bar{X}^{\mathcal{T}-1}) - X^{\mathcal{T}-1}))^T, X' - X^{\mathcal{T}} \right\rangle \geq 0, \quad \forall X' \in \mathcal{K}. \tag{42}$$

**Step 3: Convergence Verification**

If $||X^{\mathcal{T}} - X^{\mathcal{T}-1}||_\infty \leq \epsilon$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$ and go to Step 1. (We set the parameter $\omega = 0.1$ and the tolerance $\epsilon = 0.00001$ for all computations of the numerical examples in Section 4.)

Note that the subproblems in Steps 1 and 2 above are separable quadratic programming problems and, hence, there are numerous algorithms that can be used to solve these embedded subproblems.