Optimal Supply Chain Network Design and Redesign at Minimal Total Cost and with Demand Satisfaction

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Abstract:

In this paper, we propose a framework for supply chain network design and redesign that allows for the determination of the optimal levels of capacity and operational product flows associated with supply chain activities of manufacturing, storage, and distribution at minimal total cost and subject to the satisfaction of product demands. We formulate both the design and redesign problems as variational inequalities and show that the same algorithm, which exploits the underlying network structure, can be used for the solution of either problem. We illustrate the new framework with numerical examples that demonstrate the practicality and flexibility of the approach.

Keywords: Supply chain design, capacity investments, network optimization, supply chain network redesign

1. Introduction

Supply chain networks have emerged as the backbones of economic activities in the modern world. Their importance to the timely and efficient delivery of products as varied as food, energy, pharmaceuticals, clothing, computer hardware, and even toys, etc., has fueled an immense interest in their analysis on the part of both researchers and practitioners. A fundamental topic underlying supply chain networks, their formulation, analysis, and solution, is that of supply chain network design, which may be viewed as foundational to supply chain management. Indeed, the determination of an *optimal* supply chain network design poses numerous challenges, beginning with the problem conceptualization, and its rigorous formulation, to its ultimate solution.

High tech companies, including Samsung, Hewlett Packard, and IBM, as well as apparel companies from Benetton to Zara well understand the competitive advantages of careful cost control in supply chains (cf. respectively, Samsung Data Systems (2009), Lee, Billington, and Carter (1993), Swaminathan and Tayur (1999), Harle, Pich, and Van der Heyden (2001), and Dapiran (1992)). In addition, more and more companies, including Frito-Lay, Tesco, P&G, and Colgate are being recognized for their supply chain performance (see Lapide and Cottrill (2004)). Nevertheless, the analytical challenges of identifying not only the optimal capacities associated with various supply chain network activities, coupled with the optimal production quantities, storage volumes, as well as shipments are tremendous, since the possibilities of where to site manufacturing plants and distribution centers, for example, and at which capacities, may be great. Furthermore, the determination of the optimal supply chain network design (or redesign if a supply chain network already exists with some capacities) needs to be done in a rigorous manner that captures the system-wide nature of the problem.

In this paper, we provide a *system-optimization* perspective for supply chain network design that allows for the simultaneous determination of link capacities, through investments, and the product flows on various links, that is, the manufacturing, storage, distribution/shipment links, etc. A system-optimization perspective for supply chain network design, as we demonstrate, enables the modeling of the economic activities associated with a firm as a network. However, unlike a classical system-optimization formulation, with origins in transportation science (cf. Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969), Dafermos (1971), Sheffi (1985), Nagurney (1993), and Patriksson (1994)), we explicitly consider capacities on the links as design variables of the proposed networks. Furthermore, unlike much of the classical supply chain network literature (cf. Beamon (1998), Min and Zhou (2002), Handfield and Nichols Jr. (2002), and Meixell and Gargeya (2005) for surveys and Geunes and Pardalos (2003) for an annotated bibliography) in our framework we do not need discrete variables and we use continuous variables exclusively in our model formulations. In addition, we are not limited to linear costs (cf. Melkote and Daskin (2001) and the references therein); but, rather, the models can handle nonlinear and nonseparable costs, which can capture the reality of today's networks from transportation to telecommunication ones, which may be congested (cf. Nagurney and Qiang (2009)), and upon which supply chains depend for their functionality. Indeed, it is now becoming increasingly well-documented that transportation congestion has direct impacts on supply chains (see Weisbrod, Vary, and Tresz (2001) and Sankaran, Gore, and Coldwell (2005)). The solution of our models (both design and redesign), which can handle congestion, yields the optimal supply chain network topology since those links with zero optimal capacities can, in effect, be eliminated.

System-optimization models have recently been developed for supply chain network integration in the case of mergers and acquisitions (see Nagurney (2009), Nagurney and Woolley (2010), and Nagurney, Woolley, and Qiang (2009)). However, in those models, in contrast to the ones in this paper, it is assumed that the capacities on the supply chain network links are fixed and known. An alternative approach (cf. Nagurney, Dong, and Zhang (2002), Zhang, Dong, and Nagurney (2003), Zhang (2006), Cruz, Nagurney, and Wakolbinger (2006), Cruz and Wakolbinger (2008), Nagurney (2010)) considers competition among decision-makers in supply chains and uses equilibrium (as opposed to optimization) as the governing concept. In such supply chain network equilibrium models (see also Nagurney (2006) and the references therein) there are no explicit capacity link variables. The design issue in such models is, typically, handled by eliminating the links in the solution that have zero product flows (for a recent empirical application to large-scale electric power supply chains, see Liu and Nagurney (2009)).

This paper is organized as follows. In Section 2, we develop the supply chain network design model in which capacity levels and product flows are endogenous variables and establish that the optimization problem is equivalent to a variational inequality problem, with nice features for computations. The solution of the supply chain network design model yields the optimal product flows and investment capacities on the supply chain network so that the total cost is minimized and the demands at the retail outlets are satisfied. In Section 3, we propose a supply chain network redesign model, which, in a special case, collapses to the design model of Section 2. Section 4 contains numerical examples, both design and redesign ones, that illustrate and demonstrate the practicality and flexibility of the new models proposed in this paper. In Section 5, we summarize the results in this paper and present our conclusions. We also provide suggestions for future research.

2. The Supply Chain Network Design Model

In this Section, we describe the supply chain network design model. We assume that the firm is considering its possible supply chain economic activities, associated with its product, which are represented by a network. For definiteness, we consider the network topology depicted in Figure 1 but emphasize that the modeling framework developed here is not limited to such a network. Indeed, as will become apparent, what is required, to begin with, is the appropriate network topology with a top level (origin) node 1 corresponding to the firm and the bottom level (destination) nodes corresponding to the retail outlets (demand points) that the firm must supply. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is manufactured and, ultimately, delivered to the consumers at the retailers or demand markets. Hence, different supply chain network topologies to that depicted in Figure 1 correspond to distinct supply chain network problems. For example, if a product can be delivered directly to the retailers from a manufacturing plant, then there would be a direct link joining the corresponding nodes.

We assume that in the supply chain network topology there exists one path (or more) joining node 1 with each destination node. This assumption for the supply chain network design model guarantees that the demand at each retailer will be met. The solution of the model will then yield the optimal product flows and capacity investments at minimum total cost with demand satisfaction. Note that the supply chain network schematic, as in Figure 1, provides the foundation upon which the optimal supply chain network design will be determined. Those links that will have zero capacities in the optimal solution can, in effect, be, subsequently, eliminated.

In particular, as depicted in Figure 1, we assume that the firm is considering n_M manufacturing facilities/plants; n_D distribution centers, but must serve the n_R retail outlets with respective demands given by: d_{R_1}, d_{R_2}, ..., $d_{R_{n_R}}$. The links from the top-tiered node 1 are connected to the possible manufacturing nodes of the firm, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$, and these links represent the manufacturing links. The links from the manufacturing nodes, in turn, are connected to the possible distribution center nodes of the firm, and are denoted by $D_{1,1}, \ldots, D_{n_D,1}$. These links correspond to the possible shipment links between the manufacturing plants and the distribution centers where the product will be stored. The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links. Finally, there are possible shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the retail outlet (demand point) nodes: $R_1, \ldots, R_{n_R}$.

We denote the supply chain network consisting of the graph $G = [N, L]$, where N denotes the set of nodes and L the set of links. Note that G represents the topology of the full supply chain network possibilities (as in Figure 1, for example). The ultimate solution of the complete model will yield the optimal supply chain network design.

As mentioned in the Introduction, the formalism that we utilize is that of *system-optimization*, where the firm wishes to determine which manufacturing plants it should operate and at what level; the same for the distribution centers. In addition, the firm seeks to determine the capacity levels of the shipment links as well. We assume that the firm seeks to minimize the total costs associated with the production, storage, and distribution activities, along with the total investment outlays to achieve the activity levels as given by the capacities on the various links, subject to the demand being satisfied.

We assume that associated with each link (cf. Figure 1) of the network is a total cost that reflects the total cost of operating the particular supply chain activity, that is, the manufacturing of the product, the shipment of the product, the storage of the product, etc., over the time horizon underlying the design problem. We denote, without any loss in generality, the links by a, b, etc., and the total cost on a link a by $\hat{c}_a$. Hence, the presentation of the model is, in effect, not restricted to the topology in Figure 1.

A path p in the network (see, e.g., Figure 1) joining node 1, which is the origin node, to a retail node, which is a destination node, represents the activities and their sequence associated with producing the product and having it, ultimately, delivered to the consumers. Let w_k denote the pair of origin/destination nodes $(1, R_k)$ and let P_{w_k} denote the set of paths, which represent alternative associated possible supply chain network processes, joining $(1, R_k)$. P then denotes the set of all paths joining node 1 to the retail nodes.

Let x_p represent the nonnegative flow of the product on path p joining origin node 1 with a destination node of the firm. We group the path flows into the vector x.

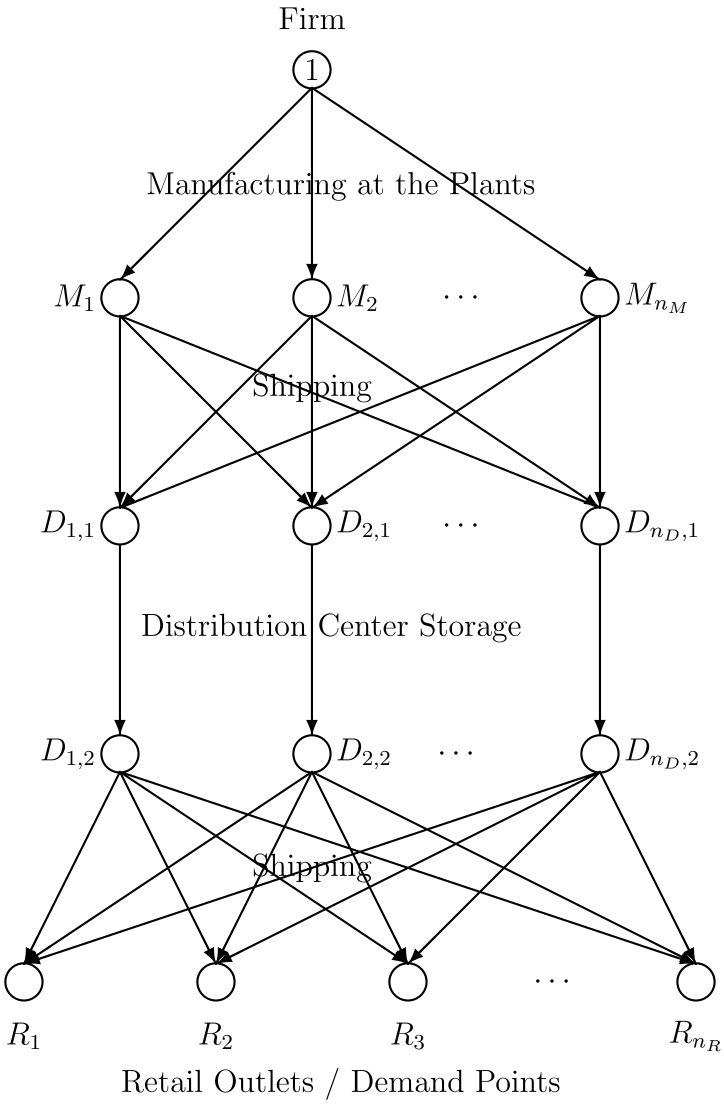

Figure 1: Supply Chain Network Topology

Then the following conservation of flow equations must hold:

$$\sum_{p \in P_{w_k}} x_p = d_{w_k}, \quad k = 1, \ldots, n_R. \tag{1}$$

In other words, the demand for the product must be satisfied at each retail outlet supplied by the firm. In addition, we let f_a denote the flow of the product on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link. We group the links flows into the vector f.

Of course, we also have that the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P, \tag{3}$$

since the product will be produced in nonnegative quantities.

Furthermore, we denote the nonnegative capacity on link a by u_a, $\forall a \in L$. Note that the link capacities are variables in the supply chain network design model. We group the capacities into the vector u. The flow on each link cannot exceed the imposed capacity on the link, which, in turn, must be nonnegative. Hence, the following constraints must also be satisfied:

$$f_a \leq u_a, \quad \forall a \in L, \tag{4}$$

$$0 \leq u_a, \quad \forall a \in L. \tag{5}$$

The total cost on a link, be it a manufacturing/production link, a shipment link, or a storage link is assumed, for the sake of generality, to be a function of the flow of the product on all the links; see, for example, Nagurney (2006) and the references therein. Hence, we may write that

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \tag{6}$$

We assume that the total cost on each link is convex, continuously differentiable, and has a bounded second order partial derivative. Convex functions enable the modeling of congestion, which, as was mentioned in the Introduction, is a growing problem for supply chains especially in terms of transportation and distribution.

We denote the total investment cost of adding capacity u_a on link a by $\hat{\pi}_a$, $\forall a \in L$, and we assume that

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L, \tag{7}$$

that is, the total cost associated with capacity level u_a on link a is a function of the capacity level on the link. We assume that these functions are also convex, continuously differentiable, and have bounded second order partial derivatives. Such conditions on the total cost functions will guarantee convergence of the proposed algorithm.

The supply chain network design optimization problem faced by the firm can be expressed as follows. The firm seeks to determine the optimal levels of product processed on each supply chain network link coupled with the optimal levels of capacity investments in its supply chain network activities subject to the minimization of the total cost where the total cost includes the total cost of operating the various links and the total cost of capacity investments. Hence, the firm must solve the following problem:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f) + \sum_{a \in L} \hat{\pi}_a(u_a) \tag{8}$$

subject to: constraints (1) – (5).

Clearly, the solution of the above optimization problem will yield the product flows and the link capacities that minimize the total costs associated with the supply chain network design faced by the firm. Under the above imposed assumptions, the optimization problem is a convex optimization problem.

We associate the Lagrange multiplier λ_a with constraint (4) for link a and we denote the associated optimal Lagrange multiplier by λ_a^*. This term may also be interpreted as the price or value of an additional unit of capacity on link a. We group the Lagrange multipliers into the vector λ.

Let K^1 denote the feasible set such that

$$K^1 \equiv \{(f, u, \lambda) | \exists x \geq 0, \text{ and } (1) - (3), \text{ and } (5) \text{ hold, and } \lambda \geq 0\}.$$

We now state the following result in which we provide a variational inequality formulation of the problem.

Theorem 1

The optimization problem (8) subject to constraints: (1) – (5) is equivalent to the variational inequality problem: determine the vector of link flows, link capacities, and Lagrange multipliers $(f^*, u^*, \lambda^*) \in K^1$*, such that:*

$$\sum_{a\in L}\left[\sum_{b\in L}\frac{\partial \hat{c}_b(f^*)}{\partial f_a}+\lambda_a^*\right]\times[f_a-f_a^*]+\sum_{a\in L}\left[\frac{\partial\hat{\pi}_a(u_a^*)}{\partial u_a}-\lambda_a^*\right]\times[u_a-u_a^*]$$

$$+\sum_{a\in L}[u_a^*-f_a^*]\times[\lambda_a-\lambda_a^*]\geq 0, \quad \forall (f,u,\lambda)\in K^1. \tag{9}$$

Proof: The optimization problem (8) subject to constraints (1) – (5) is a separable constrained convex optimization problem in which the first part of the objective function (8) depends only upon the flows whereas the second part depends only upon the capacity investments. In addition, the flows must satisfy constraints (1) – (3) whereas the investments must satisfy constraints (5) with (4) corresponding to the joint constraints that are complicating constraints and that both the flow and capacity vectors must satisfy. The problem is, hence, precisely of the structure of the general separable constrained convex optimization problem considered in Bertsekas and Tsitsiklis (1989), page 287. Therein, the equivalent variational inequality formulation is given in which the complexifying joint constraints, of the form (4), are incorporated into the function that enters the variational inequality. The conclusion follows by adapting the notation of our model to the general construct.

The variational inequality (9) can be put into standard form (see Nagurney (1993)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X-X^*\rangle \geq 0, \quad \forall X\in\mathcal{K}, \tag{10}$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product in n-dimensional Euclidean space. Indeed, if we define the column vectors: $X \equiv (f,u,\lambda)$ and $F(X)\equiv(F_1(X),F_2(X),F_3(X))$, where

$$F_1(X)=\left[\sum_{b\in L}\frac{\partial\hat{c}_b(f)}{\partial f_a}+\lambda_a; a\in L\right], F_2(X)=\left[\frac{\partial\hat{\pi}_a(u_a)}{\partial u_a}-\lambda_a; a\in L\right], F_3(X)=[u_a-f_a; a\in L],$$

and $\mathcal{K} \equiv K^1$ then (9) can be re-expressed as (10).

Variational inequality (9) can be easily solved using the modified projection method (also sometimes referred to as the extragradient method). The elegance of this computational procedure in the context of variational inequality (9) lies in that it allows one to utilize algorithms for the solution of the *uncapacitated* system-optimization problem (for which numerous algorithms exist in the transportation science literature) with straightforward update procedures at each iteration to obtain the link capacities and the Lagrange multipliers. To solve the former problem we utilize in Section 4 the well-known equilibration algorithm (system-optimization version) of Dafermos and Sparrow (1969), which has been widely applied (see also, e.g., Nagurney (1993, 2006)). Recall that the modified projection method (cf. Korpelevich (1977)) is guaranteed to converge to a solution of a variational inequality problem, provided that the function that enters the variational inequality problem is monotone and Lipschitz continuous (conditions that are satisfied under the above imposed assumptions on the cost functions) and that a solution exists.

Once we have solved problem (9) we have the solution (f^*, u^*) that minimizes the total cost (cf. (8)) associated with the design of the supply chain network.

For completeness, we now establish both monotonicity of $F(X)$ above as well as Lipschitz continuity.

Theorem 2

The function $F(X)$ as defined following (10) is monotone, that is,

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{11}$$

Proof: Expanding (11), we obtain:

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle$$

$$= \sum_{a \in L} \left[(\sum_{b \in L} \frac{\partial \hat{c}_b(f^1)}{\partial f_a} + \lambda_a^1) - (\sum_{b \in L} \frac{\partial \hat{c}_b(f^2)}{\partial f_a} + \lambda_a^2) \right] \times \left[f_a^1 - f_a^2 \right]$$

$$+\sum_{a\in L}\left[(\frac{\partial\hat{\pi}_a(u_a^1)}{u_a}-\lambda_a^1)-(\frac{\partial\hat{\pi}_a(u_a^2)}{\partial u_a}-\lambda_a^2)\right]\times\left[u_a^1-u_a^2\right]$$

$$+\sum_{a\in L}\left[(u_a^1-f_a^1)-(u_a^2-f_a^2)\right]\times\left[\lambda_a^1-\lambda_a^2\right]$$

$$=\sum_{a\in L}\left[\sum_{b\in L}\frac{\partial\hat{c}_b(f^1)}{\partial f_a}-\sum_{b\in L}\frac{\partial\hat{c}_b(f^2)}{\partial f_a}\right]\times\left[f_a^1-f_a^2\right]$$

$$+\sum_{a\in L}\left[\frac{\partial\hat{\pi}_a(u_a^1)}{\partial u_a}-\frac{\partial\hat{\pi}_a(u_a^2)}{\partial u_a}\right]\times\left[u_a^1-u_a^2\right]. \tag{12}$$

But the expression in (12) is greater than or equal to zero, since we have assumed that both the total cost functions are convex and continuously differentiable. Hence the result has been established.

Theorem 3

The function $F(X)$ as defined following (10) is Lipschitz continuous, that is,

$$\|F(X^1)-F(X^2)\|\leq\|X^1-X^2\|,\quad\forall X^1,X^2\in\mathcal{K}. \tag{13}$$

Proof: Since we have assumed that the $\hat{c}_a(f)$ functions and the $\hat{\pi}_a(u_a)$ functions have bounded second-order derivatives for all links $a\in L$, the result is direct by applying a mid-value theorem from calculus to the function F that enters the above variational inequality.

We would like to emphasize that the design of supply chain networks may also be viewed as a class of problems related to the broader set of problems in network design, but with specific features. For an excellent overview of optimization problems, including network design problems, faced in the context of telecommunications and transportation, see Migdalas (2006). For an earlier review of network design and transportation planning, see Magnanti and Wong (1984). We emphasize that we believe that the determination of (continuous) investment capacities provides a greater flexibility than assuming fixed capacity sizes and associated fixed charge costs that would yield mixed integer programming formulations. For a more recent survey, see Yang and Bell (1998). For a framework for network design in which links are constructed or not but in which there is no explicit congestion and capacity determination, see Drezner and Wesolowsky (2003). In this paper, we focus on supply chain network models and their variational inequality formulations that are particularly suited for analysis and computations. Furthermore, these models can be used as the basic framework for the development of numerous extensions, including game theoretic ones, as well as applications.

3. The Supply Chain Network Redesign Model with Existing Capacities

In Section 2, we developed the supply chain network design model, for which, at the onset, the link capacities are unknown and need to be determined, along with the optimal product flows. Here, in contrast, we consider the problem of redesigning an *existing* supply chain network, in which some of the links $a \in L$ already have positive capacity. We denote the existing capacities on the links by $\bar{u}_a$, $\forall a \in L$. In the redesign problem, one may need to offload or sell some of the existing capacity in order to determine the minimal total cost solution. This is handled in the redesign model by allowing the u_as to take on negative values, if appropriate.

In the redesign supply chain network problem the objective function (8) is still valid, as are the constraints (1) through (3). However, instead of constraints (4), we now have that:

$$f_a \leq \bar{u}_a + u_a, \quad \forall a \in L. \tag{14}$$

Also, instead of constraints (5) we now have that:

$$-\bar{u}_a \leq u_a, \quad \forall a \in L. \tag{15}$$

Note that, unlike in the supply chain network design model in Section 2, where the u_a variables have to be nonnegative, in the redesign model (cf. (15)) the values of u_a, $a \in L$, may take on negative values. However, since the link flows are nonnegative, the u_as are bounded from below, according to (15).

Observe that if all the $\bar{u}_a = 0$, for all links $a \in L$, then the redesign model collapses to the design model in Section 2, since we begin with no capacities on the links. Both models, however, work from a supply chain network topology, as in Figure 1, for example.

We associate now the Lagrange multiplier β_a with constraint (14) for link a and we denote the optimal Lagrange multiplier by β_a^*, $\forall a \in L$. We group the former Lagrange multipliers into the vector β.

Let K^2 denote the feasible set such that

$$K^2 \equiv \{(f,u,\beta)|\exists x \geq 0, \text{ and } (1)-(3), \text{ and } (15) \text{ hold, and } \beta \geq 0\},$$

where recall that f is the vector of link flows; u we refer to, w.l.o.g., as the vector of link capacities, and x is the vector of path flows.

Under the same assumptions as for the model in Section 2, we then have the following result, which is immediate:

Theorem 4

The optimization problem (8) subject to constraints: (1) – (3), (14), and (15) is equivalent to the solution of the variational inequality problem: determine the vector of link flows, link capacities, and Lagrange multipliers $(f^*, u^*, \beta^*) \in K^2$, such that:

$$\sum_{a \in L} \left[\sum_{b \in L} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \beta_a^* \right] \times [f_a - f_a^*] + \sum_{a \in L} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \beta_a^* \right] \times [u_a - u_a^*]$$
$$+ \sum_{a \in L} [\bar{u}_a + u_a^* - f_a^*] \times [\beta_a - \beta_a^*] \geq 0, \quad \forall (f,u,\beta) \in K^2. \tag{16}$$

Variational inequality (16) can also be put into standard form (10) if we define now the column vectors: $X \equiv (f, u, \beta)$ and $F(X) \equiv (\hat{F}_1(X), \hat{F}_2(X), \hat{F}_3(X))$, where

$$\hat{F}_1(X) = \left[\sum_{b \in L} \frac{\partial \hat{c}_b(f)}{\partial f_a} + \beta_a; a \in L \right], \hat{F}_2(X) = \left[\frac{\partial \hat{\pi}_a(u_a)}{\partial u_a} - \beta_a; a \in L \right], \hat{F}_3(X) = [\bar{u}_a + u_a - f_a; a \in L],$$

and $\mathcal{K} \equiv K^2$.

Note that the function F, as defined following (16), in the above variational inequality is also monotone and Lipschitz continuous under the assumptions of convexity of the total cost functions and that they have bounded second order partial derivatives.

4. Numerical Examples

In this Section, we present numerical examples. In Section 4.1 we consider supply chain network design problems and in Section 4.2 redesign problems. In Section 4.3, we present additional redesign examples in which the underlying total cost functions are linear in the total investment cost functions and then linear in all the total cost functions. Recall that linear functions are both convex and concave.

The modified projected method for both the design and the redesign problems governed, respectively, by variational inequalities (9) and (16) was implemented in FORTRAN. A Unix system at the University of Massachusetts Amherst was used for all the computations. We set $\alpha = .1$ in the modified projection method and set the convergence tolerance $\epsilon = 10^{-5}$. For additional algorithmic details please see Nagurney and Qiang (2009) and the references therein.

We initialized the algorithm by equally distributing the demand at each retail outlet among all the paths joining the firm node 1 to the retail outlet. All other variables, that is, the link capacities and the Lagrange multipliers, were initialized to zero.

The supply chain network topology for all the examples is as depicted in Figure 2 with the links defined by numbers as in Figure 2. The numerical examples, hence, consisted of a firm faced with 3 possible manufacturing plants, 2 distribution centers, and had to supply the 3 retail outlets.

The data for the specific examples along with the solutions are reported in the corresponding tables below.

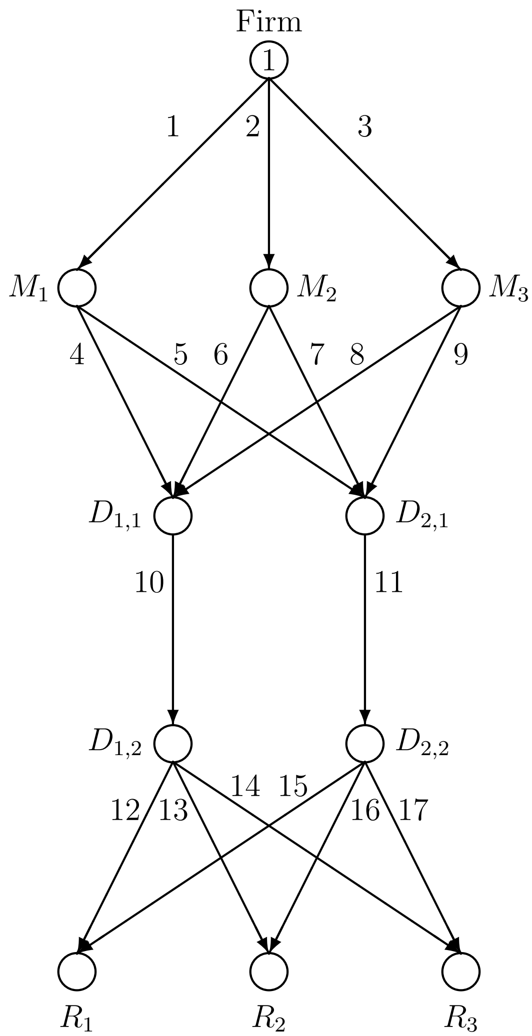

Figure 2: The Supply Chain Network Topology $G = [N, L]$ for the Examples

4.1: Supply Chain Network Design Examples

Example 1

In the first numerical example the demands were:

$$d_{R_1} = 45, \quad d_{R_2} = 35, \quad d_{R_3} = 5.$$

The total cost functions were as reported in Table 1 where we also provide the computed solution using the modified projection method. The modified projection method required 497 iterations for convergence for this example.

Hence, from the solution in Table 1 we can see that link 14, which corresponds to the shipment link joining the first distribution center to retailer R_3 has zero capacity and, thus, also

Table 1: Total Cost Functions and Solution for Example 1

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	29.08	29.08	30.08
2	$.5f_2^2 + f_2$	$2.5u_2^2 + u_2$	24.29	24.29	122.44
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	31.63	31.63	65.26
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	16.68	16.68	34.37
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	12.40	12.40	63.98
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	8.65	8.65	9.65
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	15.64	15.64	16.64
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	18.94	18.94	57.83
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	12.69	12.69	53.75
10	$.5f_{10}^2 + 2f_{10}$	$u_{10}^2 + 5u_{10}$	44.28	44.28	93.55
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	40.72	40.72	43.72
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	25.34	25.34	26.34
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	18.94	18.94	19.94
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	0.00	0.00	4.57
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	19.66	19.66	20.66
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	16.06	16.06	33.12
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + _{17}$	5.00	5.00	6.00

zero flow. It can, thus, in effect, be eliminated from the supply chain network design. Please refer to Figure 3 for the optimal supply chain network design topology for this example.

The value of the objective function (cf. (8)) at the computed optimal solution for this problem, which reflects the minimal total cost, was: 16,125.65.

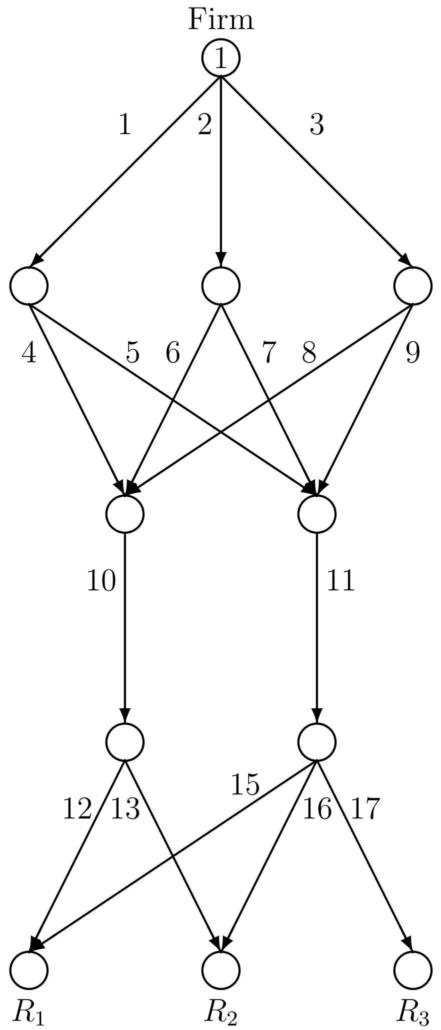

Figure 3: Optimal Supply Chain Network Topology for Example 1

Table 2: Total Cost Functions and Solution for Example 2

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	29.28	29.28	30.28
2	$.5f_2^2 + f_2$	$2.5u_2^2 + u_2$	23.78	23.78	119.91
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	31.93	31.93	65.87
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	19.01	19.01	39.01
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	10.28	10.28	53.39
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	13.73	13.73	14.73
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	10.05	10.05	11.05
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	21.77	21.77	66.31
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	10.17	10.17	43.66
10	$.5f_{10}^2 + 2f_{10}$	$5u_{10}$	54.50	54.50	5.00
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	30.50	30.50	33.50
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	29.58	29.58	30.58
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	23.18	23.18	24.18
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	1.74	1.74	11.96
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	15.42	15.42	16.42
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	11.82	11.82	24.64
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	3.26	3.26	4.26

Example 2

Example 2 had the identical total cost data as in Example 1 except that we reduced the total cost associated with capacity investment in the first distribution center so that there was no nonlinear term (see data for link 10 in Table 2). The complete data and solution are given in Table 2, for easy reference. The modified projection method converged to this solution in 1,461 iterations.

The value of the objective function (see (8)) was now: 13,718.87, which is significantly lower than in Example 1. It is interesting to note that with the reduction in total cost for the first distribution center the shipment link 14 now had positive capacity and positive product flow. In fact, the capacities and the product flows increased on all the shipment links emanating from the first distribution center (see solution for links 12, 13, and 14 in Table 2) as compared to the analogous solutions for Example 1. On the other hand, the capacities and product flows on the shipment links emanating from the second distribution center (see data for links 15, 16, and 17 in Table 2) now all decreased, relative to their solution values in Example 1.

Hence, the final supply chain network topology for this example was as in Figure 2.

Example 3

Example 3 had the same data as Example 2 but now we reduced the total costs associated with the first and second manufacturing plants by eliminating the nonlinear terms in the total cost functions associated with the capacity investments. The functions and the solution are given in Table 3, for completeness. The total cost was now: 10,726.48, a significant reduction from the value in Example 2. The algorithm converged to this solution in 1,432 iterations.

Interestingly, although the total costs were reduced at the first and the second manufacturing plants (see the data for links 1 and 2 in Table 3) the product flow increased on link 2 (in fact, it almost doubled in size), but decreased on link 1 and on link 3, associated with the third manufacturing plant, relative to the respective flows in Example 2. This demonstrates that one needs to capture the entire supply chain since local effects/changes as in the case of a reduction in total cost may have non-local impacts. The final supply chain network topology for this example was, again, as in Figure 2.

Table 3: Total Cost Functions and Solution for Example 3

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$f_1^2 + 2f_1$	u_1	20.91	20.91	1.00
2	$.5f_2^2 + f_2$	u_2	45.18	45.18	1.00
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	18.91	18.91	39.82
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	14.74	14.74	30.49
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	6.16	6.16	32.82
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	23.79	23.79	24.79
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	21.39	21.39	22.39
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	14.79	14.79	45.11
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	4.21	4.21	19.83
10	$.5f_{10}^2 + 2f_{10}$	$5u_{10}$	53.23	53.23	5.00
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	31.77	31.77	34.77
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	29.10	29.10	30.10
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	22.70	22.70	23.70
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	1.44	1.44	10.75
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	15.90	15.90	16.90
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	12.30	12.30	25.60
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	3.56	3.56	4.56

Table 4: Total Cost Functions, Initial Capacities, and Solution for Example 4

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	β_a^*
1	$f_1^2 + 2f_1$	u_1	40.00	16.94	-23.06	1.00
2	$.5f_2^2 + f_2$	u_2	40.00	33.11	-6.89	1.00
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	40.00	34.96	-1.00	0.00
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	30.00	8.82	-0.50	0.00
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	30.00	8.11	-0.40	0.00
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	30.00	14.12	-1.00	0.00
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	30.00	18.99	-1.00	0.00
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	30.00	27.39	-0.33	0.00
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	30.00	7.57	-0.75	0.00
10	$.5f_{10}^2 + 2f_{10}$	$5u_{10}$	50.00	50.33	0.33	5.00
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	50.00	34.67	-3.00	0.00
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	30.00	30.69	0.70	1.70
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	30.00	18.38	-1.00	0.00
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	30.00	1.26	-1.25	0.00
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	30.00	14.31	-1.00	0.00
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	30.00	16.62	-0.50	0.00
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	30.00	3.74	-1.00	0.00

4.2 Supply Chain Network Redesign Examples

In this Section, we solved supply chain network redesign problems, governed by variational inequality (16).

Example 4

Example 4 had the same data as Example 3 except that now we had positive initial link capacities, $\bar{u}_a$, for links $a \in L$. The complete data is given in Table 4, along with the computed solution. The modified projection method required 1,462 iterations for convergence to this solution.

The total cost was now: 6,545.78. Observe that since there was extra capacity on almost all of the links the optimal solution had almost all optimal values for the link capacities at negative value. This means that, given the data, including the total cost functions and the

Table 5: Total Cost Functions, Initial Capacities, and Solution for Example 5

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	β_a^*
1	$f_1^2 + 2f_1$	u_1	40.00	41.24	1.24	1.00
2	$.5f_2^2 + f_2$	u_2	40.00	73.90	33.90	1.00
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	40.00	54.86	14.86	31.72
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	30.00	24.27	-0.50	0.00
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	30.00	16.97	-0.40	0.00
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	30.00	37.14	7.14	8.14
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	30.00	36.76	6.76	7.77
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	30.00	39.93	9.93	30.80
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	30.00	14.92	-0.75	0.00
10	$.5f_{10}^2 + 2f_{10}$	$5u_{10}$	50.00	101.34	51.34	5.00
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	50.00	68.66	18.66	21.66
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	30.00	56.75	26.75	27.75
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	30.00	38.35	8.35	9.35
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	30.00	6.25	-1.25	0.00
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	30.00	33.25	3.25	4.25
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	30.00	31.65	1.65	4.30
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	30.00	3.75	-1.00	0.00

demands, that the firm should sell off its extra capacity in order to minimize total cost and to satisfy the demands at the three retail outlets.

Example 5

Example 5 was also a supply chain network redesign problem. The data were as in Example 4 except that now the demand at each retailer outlet doubled, that is, the demands were now:

$$d_{R_1} = 90, \quad d_{R_2} = 70, \quad d_{R_3} = 10.$$

The complete data for this example, along with its solution, are given in Table 5. The algorithm now required 1,489 iterations for convergence. The total cost was now: 26,859.99. With higher demands there are fewer links with negative capacity values, as one would expect since the demand must be satisfied.

Table 6: Total Cost Functions, Initial Capacities, and Solution for Example 6

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	β_a^*
1	$f_1^2 + 2f_1$	u_1	40.00	41.15	1.15	1.00
2	$.5f_2^2 + f_2$	u_2	40.00	73.76	33.76	1.00
3	$.5f_3^2 + f_3$	$u_3^2 + 2u_3$	40.00	55.09	15.09	32.19
4	$1.5f_4^2 + 2f_4$	$u_4^2 + u_4$	30.00	20.27	-0.50	0.00
5	$f_5^2 + 3f_5$	$2.5u_5^2 + 2u_5$	30.00	20.88	-0.40	0.00
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	30.00	33.11	3.11	4.11
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	30.00	40.64	10.64	11.64
8	$.5f_8^2 + 2f_8$	$1.5u_8^2 + u_8$	30.00	36.71	6.71	21.12
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	30.00	18.39	-0.75	0.00
10	$.5f_{10}^2 + 2f_{10}$	$u_{10}^2 + 5u_{10}$	50.00	90.09	40.09	85.18
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + 3u_{11}$	50.00	79.71	29.91	32.91
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	30.00	53.68	23.68	24.68
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	30.00	35.28	5.28	6.28
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	30.00	1.13	-1.25	0.00
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	30.00	36.32	6.32	7.32
16	$.5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	30.00	34.72	4.72	10.44
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	30.00	8.87	-1.00	0.00

Example 6

The final example in this set, Example 6, had the same data as Example 5 but now the total cost associated with investing in the first distribution center was as in Example 1. The complete data for this example, along with the computed solution, are given in Table 6. The total cost at the optimal solution was: 28,918, which is higher than in Example 5 but expected since the total cost associated at the first distribution center is now higher. The modified projection method required 1,369 iterations for convergence. Note that the same links have negative capacities as in Example 5. In addition, the solutions do not significantly differ from one another.

Note that the final supply chain redesign network topology for Examples 4 through 6 is as in Figure 2.

Table 7: Total Cost Functions, Initial Capacities, and Solution for Example 7

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	β_a^*
1	$f_1^2 + 2f_1$	u_1	40.00	34.37	-5.63	1.00
2	$.5f_2^2 + f_2$	u_2	40.00	66.46	26.46	1.00
3	$.5f_3^2 + f_3$	$2u_3$	40.00	69.17	29.17	2.00
4	$1.5f_4^2 + 2f_4$	u_4	30.00	18.46	-11.54	1.00
5	$f_5^2 + 3f_5$	$2u_5$	30.00	15.92	-14.08	2.00
6	$f_6^2 + 2f_6$	u_6	30.00	29.33	-0.67	1.00
7	$.5f_7^2 + 2f_7$	u_7	30.00	37.13	7.13	1.00
8	$.5f_8^2 + 2f_8$	u_8	30.00	54.96	24.96	1.00
9	$f_9^2 + 5f_9$	$3u_9$	30.00	14.21	-15.79	3.00
10	$.5f_{10}^2 + 2f_{10}$	$5u_{10}$	50.00	102.75	52.75	5.00
11	$f_{11}^2 + f_{11}$	$3u_{11}$	50.00	67.25	17.25	3.00
12	$.5f_{12}^2 + 2f_{12}$	u_{12}	30.00	62.40	32.40	1.00
13	$.5f_{13}^2 + 5f_{13}$	u_{13}	30.00	37.61	7.61	1.00
14	$f_{14}^2 + 7f_{14}$	$5u_{14}$	30.00	2.74	-27.26	5.00
15	$f_{15}^2 + 2f_{15}$	u_{15}	30.00	27.60	-2.40	1.00
16	$.5f_{16}^2 + 3f_{16}$	u_{16}	30.00	32.39	2.39	1.00
17	$.5f_{17}^2 + 2f_{17}$	u_{17}	30.00	7.26	-22.74	1.00

4.3 Supply Chain Network Redesign Examples with Primarily Linear Costs

In order to investigate the effect of nonlinearities on the total cost functions on the supply chain netwok redesign problem, in this set of examples, we progressively removed the nonlinear terms. The complete description of the examples and the data follow.

Example 7

This example was constructed from Example 6 and the data are reported in Table 7. Specifically, we retained the data as in Example 6 but now there were no associated nonlinear total cost terms for the capacity investment cost functions; refer to Table 7. The modified projection method converged in 2,100 iterations and the total cost now was: 25,178.30. In this example, the same links had negative capacities as in Example 6 but there were additional such links as compared to those in the solution for Example 6. Hence, now the firm could

Table 8: Total Cost Functions, Initial Capacities, and Solution for Example 8

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	f_a^*	u_a^*	β_a^*
1	$2f_1$	u_1	40.00	0.00	-40.00	0.86
2	f_2	u_2	40.00	170.00	130.00	1.00
3	f_3	$2u_3$	40.00	0.00	-40.00	1.85
4	$2f_4$	u_4	30.00	0.00	-30.00	0.86
5	$3f_5$	$2u_5$	30.00	0.00	-30.00	1.73
6	$2f_6$	u_6	30.00	0.00	-30.00	1.00
7	$2f_7$	u_7	30.00	170.00	140.00	1.00
8	$2f_8$	u_8	30.00	0.00	-30.00	0.85
9	$5f_9$	$3u_9$	30.00	0.00	-30.00	2.06
10	$2f_{10}$	$5u_{10}$	50.00	0.00	-50.00	4.71
11	f_{11}	$3u_{11}$	50.00	170.00	120.00	3.00
12	$2f_{12}$	u_{12}	30.00	0.00	-30.00	0.71
13	$5f_{13}$	u_{13}	30.00	0.00	-30.00	0.56
14	$7f_{14}$	$5u_{14}$	30.00	0.00	-30.00	0.00
15	$2f_{15}$	u_{15}	30.00	90.00	60.00	1.00
16	$3f_{16}$	u_{16}	30.00	70.00	40.00	1.00
17	$2f_{17}$	u_{17}	30.00	10.00	-20.00	1.00

benefit even more from selling off its excess capacity in order to minimize total cost and to satisfy the consumer demand.

The final supply chain network redesign topology remained as in Figure 2.

Example 8

This example was constructed from Example 7 and the data are reported in Table 8. The data were as in Example 7 but now we also eliminated the nonlinear terms associated with the total cost functions for operating the supply chain links. Please refer to Table 8 for the complete data for Example 8. The total cost at the optimal solution was: 980.00. The algorithm converged to the solution in Table 8 in 4,573 iterations. It is fascinating to observe that the final supply chain redesign network for this problem is such that only the second manufacturing plant is used, and it ships the entire production amount to the second distribution center where all the product is stored for delivery to the three retail outlets. Hence, only 6 out of the 17 possible links have positive product flows. Moreover, all the links without any product flows have their final link capacity, given by the expression: $\bar{u}_a - u_a^*$, exactly equal to zero, thus, the supply chain link can be eliminated from the network design. Please refer to Figure 4 for the optimal supply chain network redesign topology for this example.

We then conducted some additional sensitivity analysis. When the demands in Example 8 were halved, and the problem was resolved, the same result in terms of which links had optimal zero capacities and optimal zero flows held, but the corresponding link product flows were halved. Also, when we doubled the demands in Example 8, the same links, again, had positive product flows and capacities, but at double the values as in Example 8. Hence, the final supply chain network redesign topology remained as in Figure 4.

These examples illustrate the practicality and flexibility of the modeling approach and algorithm.

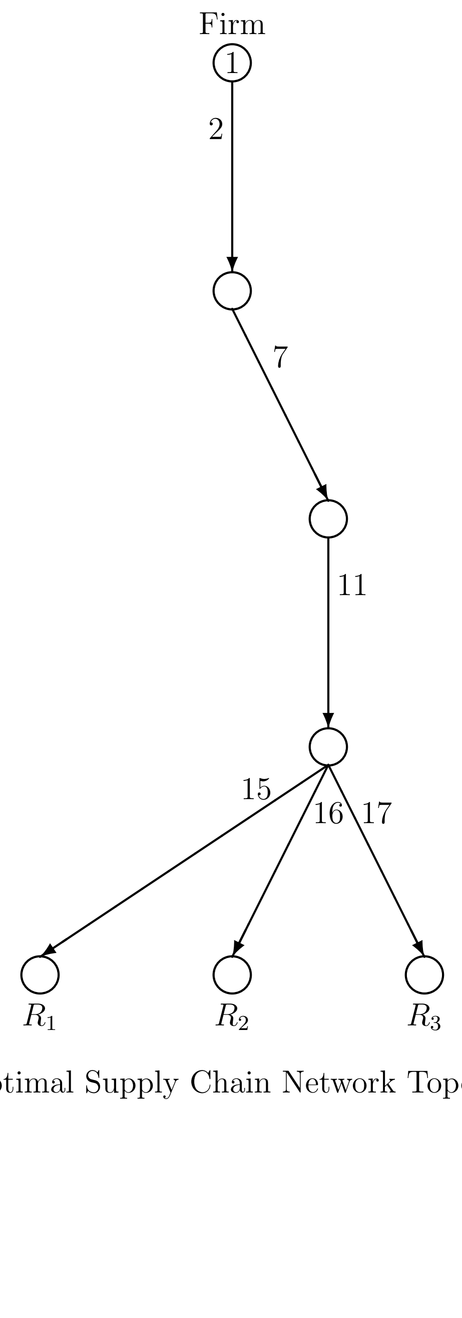

Figure 4: Optimal Supply Chain Network Topology for Example 8

5. Summary and Conclusions

In this paper, we presented a novel system-optimization approach for the representation of economic activities associated with supply chain networks, in particular, manufacturing, distribution, as well as storage, which we then utilized to formulate both the supply chain network design and redesign problems. The models do not need discrete variables plus the cost functions that can be handled are not limited to linear or separable ones.

We established that both the design and redesign optimization models with endogenous link capacity variables and product flow variables could be formulated and solved as variational inequality problems. We also investigated properties of the functions that entered the variational inequalities, which are needed for convergence of the algorithmic scheme. We illustrated the framework with numerical supply chain network examples for both the design and the redesign models. The notable feature of the variational inequality formulations and the accompanying algorithm is that the complicating model constraints are handled in a manner that allows for the full exploitation of the underlying network structure using even available network-based algorithms. The solution of the models yields the optimal supply chain network topology at minimal total cost and with the satisfaction of the demand for the product at the retailers. Those links with zero optimal capacities can, in effect, be eliminated from the final supply chain topology.

It is also worth noting that the models were presented, for definiteness, in a multitiered context with manufacturing plants at the top tier, shipment links joining the manufacturing nodes with the distribution centers and the centers with the retailers. The framework, however, as presented, can handle any proposed supply chain network topology, at the onset, and only requires that there is a single path joining the supersource node which represents the firm with each retailer outlet with positive demand. Hence, the framework can handle as many manufacturing plants (and even suppliers) and as complex of a feasible supply chain network topology as the firm wishes to investigate. Also, the algorithm (an equilibration algorithm) that we used to solve the embedded network optimization problems at each iteration of the proposed algorithm (the modified projection method) has been applied to solve large-scale transportation network and supply chain problems (cf. Liu and Nagurney (2009)) and, hence, the computational procedure should be effective also in large-scale supply chain network design and redesign settings. The numerical results that we have presented reveal the complete example solutions for the purpose of transparency and easy reproducibility.

The models in this paper can be extended in different directions and can be applied in different industrial settings. For the former, we can include multiple criteria associated with supply chain network design to incorporate, for example, risk and uncertainty, and even environmental emissions, if sustainability is the goal (see. e.g., Nagurney and Woolley (2010) and Qiang, Nagurney, and Dong (2009)). In addition, the framework can be extended to handle the production of multiple products as well as to explicitly include the global dimensions with the addition of taxes, tariffs, exchange rates, etc. We leave such extensions for future research. In terms of applications, we plan on exploring the use of this framework in the context of food supply chains, vaccine production and distribution, and pharmaceuticals.

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