Supply Chain Performance Assessment and Supplier and Component Importance Identification in a General Competitive Multitiered Supply Chain Network Model

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Abstract

In this paper, we develop a multitiered competitive supply chain network game theory model, which includes the supplier tier. The firms are differentiated by brands and can produce their own components, as reflected by their capacities, and/or obtain components from one or more suppliers, who also are capacitated. The firms compete in a Cournot-Nash fashion, whereas the suppliers compete a la Bertrand since firms are sensitive to prices. All decision-makers seek to maximize their profits with consumers reflecting their preferences through the demand price functions associated with the demand markets for the firms' products. We construct supply chain network performance measures for the full supply chain and the individual firm levels that assess the efficiency of the supply chain or firm, respectively, and also allow for the identification and ranking of the importance of suppliers as well as the components of suppliers with respect to the full supply chain or individual firm. The framework is illustrated through a series of numerical supply chain network examples.

Keywords: supply chains, networks, suppliers, game theory, performance assessment, importance indicators

1. Introduction

Suppliers are critical in providing essential components and resources for finished goods in today's globalized supply chain networks. The number of components comprising a finished product may be small or immense as in aircraft manufacturing and other complex high-tech products. Even in the case of simpler products, such as bread, ingredients may travel across the globe as inputs into production processes. Suppliers are also decision-makers and they compete with one another to provide components to downstream manufacturing firms. When suppliers are faced with disruptions, whether due to man-made activities or errors, natural disasters, unforeseen events, or even terrorist attacks, the ramifications and effects may propagate through a supply chain or multiple supply chains. Hence, capturing supplier behavior is essential in modeling the full scope of supply chain network competition and in identifying the importance of a supplier and the components that he provides to the firms.

There are many vivid examples of supplier failures, due to natural disasters, and associated supply chain disruptions. A classic example is the Royal Philips Electronics cell phone chip manufacturing plant fire, due to a lightning strike on March 17, 2000, and subsequent water and smoke damage, which adversely affected Ericsson, which, unlike Nokia, did not have a backup, and suffered a second quarter operating loss in 2000 of $200 million in its mobile phone division (cf. [18]). The Fukushima triple disaster on March 11, 2011 in Japan resulted in shortages of memory chips, automotive sensors, silicon wafers, and even certain colors of automotive paints, because of the affected suppliers (see [14]). The worst floods in 50 years that followed in October 2011 in Thailand impacted both Apple and Toyota supply chains, since Thailand is the world's largest producer of computer hard disk drives and also a big automotive manufacturing hub ([47]). However, not all supplier shortcomings need be due to disasters. Boeing, facing challenges with its 787 Dreamliner supply chain design and numerous delays, ended up having to buy two suppliers for $2.4 billion because the units were underperforming in the chain ([42]).

Although there has been extensive research on multitiered supply chain network equilibrium problems, beginning with the work of Nagurney, Dong, and Zhang [24]; see, e.g., [8, 9, 17, 20, 22, 36, 37, 44], there has been less work done on integrating suppliers and their behavior into general multitiered supply chain network equilibrium frameworks. Also, since there has been a dearth of general supply chain network models with suppliers, the identification of which suppliers are important in a supply chain has received less research attention, although it is a very important issue in practice (see [5]). Some examples, nevertheless, include the work of Liu and Nagurney [16], who developed an integrated supply chain network equilibrium model with fuel suppliers focusing on the electric power industry in New England, and that of Liu and Cruz [15] who modeled supply chains with credit trade and financial risk. However, those papers did not identify which suppliers or the components that they produce are the most important from a supply chain network efficiency perspective.

As noted in [39], most supply disruption studies have focused on a local point of view, in the form of a single-supplier problem (see, e.g., [12, 33]) or a two-supplier problem ([34]). Very few research papers have examined supply chain risk management with multiple decision-makers (cf. [43]). We believe that it is imperative to formulate and solve supply chains from a system-wide holistic perspective and to include both supplier and firm decision-makers in the supply chain network tiers. Indeed, such an approach has also been argued by Wu, Blackhurst, and Chidambaram [46].

In this paper, we develop a multitiered competitive supply chain network game theory model, which includes the supplier tier. The firms are differentiated by brands and can produce their own components, as reflected by their capacities, and/or obtain components from one or more suppliers, who also are capacitated. The firms compete in a Cournot-Nash fashion, whereas the suppliers compete a la Bertrand since firms are sensitive to prices. All decision-makers seek to maximize their profits with consumers reflecting their preferences through the demand price functions associated with the demand markets for the firms' products.

Our contributions to the literature are twofold: 1. the development of a general multitiered competitive supply chain network equilibrium model with suppliers and firms that includes capacities and constraints to capture the production activities and 2. the construction of supply chain network performance measures, on the full supply chain and on the individual firm levels, that assess the efficiency of the supply chain or firm, respectively, and also allow for the identification and ranking of the importance of suppliers as well as the components of suppliers with respect to the full supply chain or individual firm. The supply chain network performance measure is inspired by our work on network performance assessment in a variety of network systems ranging from transportation to the Internet (see [28, 29] and the references therein) as well as in supply chains (cf. [7, 38, 39]) but with the addition of the supplier tier, which is the focus here.

Our framework adds to the growing literature on supply chain disruptions (cf. [3, 4, 29, 45]) by providing metrics that allow individual firms, industry overseers or regulators, and/or government policy-makers to identify the importance of suppliers and the components that they produce for various product supply chains. To the best of our knowledge, such a list of importance indicators that we provide in terms of supplier importance, and their components, at firm and full supply chain levels, has not been constructed before in a general, holistic supply chain network model with competition.

The paper is organized as follows. In Section 2, we develop the new supply chain network model, describe the behavior of the firms and the suppliers, identify the governing equilibrium conditions, and provide the variational inequality formulation. In Section 3, we propose the supply chain network performance measures at the full supply chain and individual firm levels, and define the supplier and supplier component importance indicators. In Section 4, we describe an algorithm, which is then applied in Section 5 to compute solutions to numerical supply chain network examples to illustrate the model and methodology and how the performance measures and the supplier and component importance indicators can be applied in practice. We summarize our results and present our conclusions in Section 6.

2. The Multitiered Supply Chain Network Game Theory Model with Suppliers

In this section, we develop a multitiered supply chain network game theory model with suppliers and firms that procure components from the suppliers for their products, which are differentiated by brand. We consider a supply chain network consisting of I firms, with a typical firm denoted by i, n_S suppliers, with a typical supplier denoted by j, and a total of n_R demand markets, with a typical demand market denoted by k.

The firms compete noncooperatively, and each firm corresponds to an individual brand representing the product that it produces. We assume that product i, which is the product produced by firm i, requires n_{l^i} different components, and the total number of different components required by the I products is n_l. We allow for the situation that each supplier may be able to produce a variety of components for each firm.

The I firms are involved in the processes of assembling the products using the components needed, transporting the products to the demand markets, and, possibly, producing one or more of the components of the products. The suppliers, in turn, are involved in the processes of producing and delivering the components of the products to the firms. Both in-house and contracted component production activities are captured in the model. The capacity/ability of production is also considered.

The network topology G of the problem is depicted in Figure 1, where G consists of the set of nodes N and the set of links L, so that $G = [N, L]$. Firm i's network topology; $i = 1, \ldots, I$, is denoted by G_i. G_i consists of the sets of nodes and links that represent the economic activities associated with firm i and its suppliers. In Figure 1, the first two sets of links from the top nodes are links corresponding to distinct supplier components. The links from the top-tiered nodes $j; j = 1, \ldots, n_S$, representing the suppliers, are connected to the associated manufacturing nodes, denoted by nodes $1, \ldots, n_l$. These links represent the manufacturing activities of the suppliers. The next set of links that emanate from $1, \ldots, n_l$ to the firms, denoted by nodes $1, \ldots, I$, reflects the transportation of the components to the associated firms. In addition, the links that connect nodes $1^i, \ldots, n^i_{l^i}$, which are firm i's component manufacturing nodes, and firm i are the manufacturing links of firm i for producing its components.

The rest of the links in Figure 1 are links corresponding to the finished products. The link connecting firm i and node i', which is the assembly node of firm i, represents the activity of assembling firm i's product using the components needed, which may be produced by firm i, the suppliers, or both. Finally, the links joining nodes $1', \ldots, I'$ with demand market nodes $1, \ldots, n_R$ correspond to the transportation of the products to the demand markets.

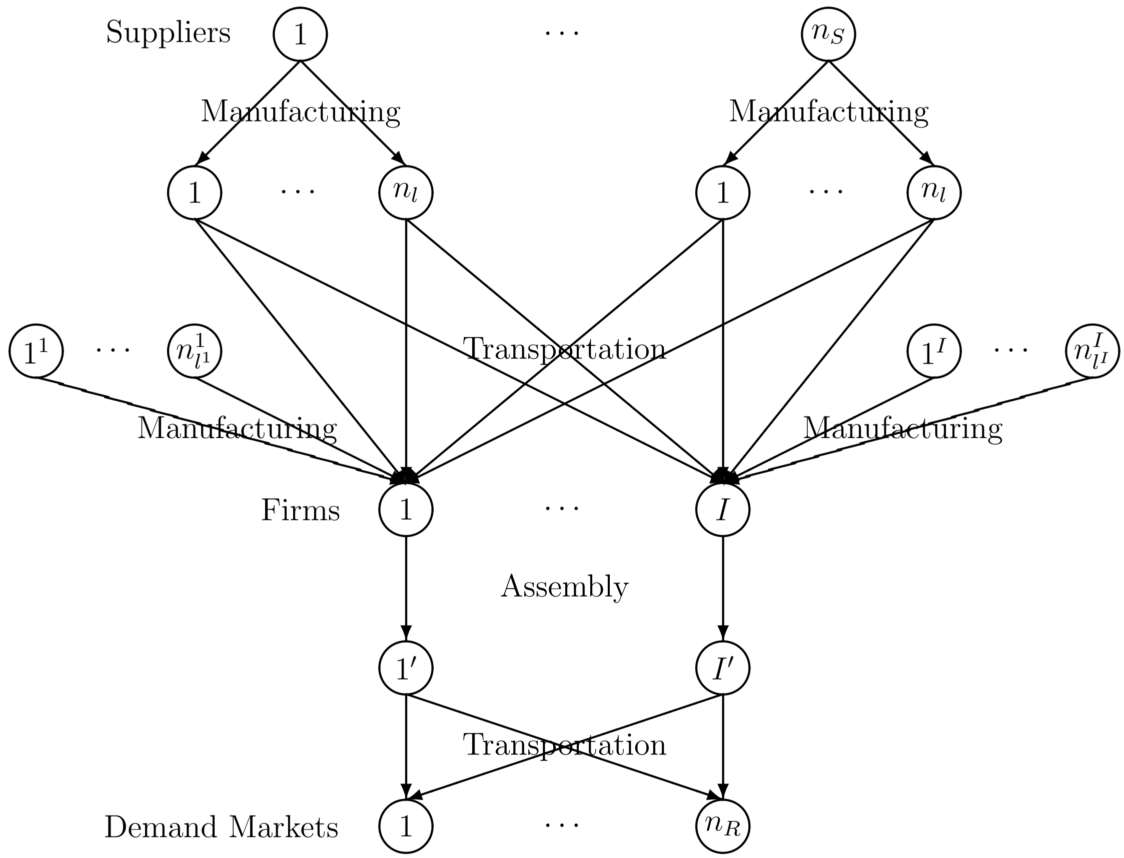

Figure 1: The Multitiered Supply Chain Network Topology

In this paper, we seek to determine the optimal component production quantities, both by the firms and by the suppliers, the optimal product shipments from the firms to the demand markets, and the prices that the suppliers charge the firms for producing and delivering their components. The firms compete noncooperatively under the Cournot-Nash equilibrium concept in product shipments and component production quantities, while the suppliers compete in Bertrand fashion in the prices that they charge the firms.

The notation for the model is given in Table 1. The vectors are assumed to be column vectors. The equilibrium solution is denoted by "*".

Table 1: Notation for the Multitiered Supply Chain Network Game Theory Model with Suppliers

Notation	Definition
Q^S_{jil}	the nonnegative amount of firm i's component l produced by supplier j; $j = 1,\ldots,n_S$; $i = 1,\ldots,I$; $l = 1,\ldots,n_{l^i}$. For firm i, we group its $\{Q^S_{jil}\}$ elements into the vector $Q^S_i \in R^{n_S n_{l^i}}_+$. All the $\{Q^S_{jil}\}$ elements are grouped into the vector $Q^S \in R^{n_S \sum_{i=1}^I n_{l^i}}_+$.
CAP^S_{jil}	the capacity of supplier j for producing firm i's component l.
Q^F_{il}	the nonnegative amount of firm i's component l produced by firm i itself. For firm i, we group its $\{Q^F_{il}\}$ elements into the vector $Q^F_i \in R^{n_{l^i}}_+$, and group all such vectors into the vector $Q^F \in R^{\sum_{i=1}^I n_{l^i}}_+$.
CAP^F_{il}	the capacity of firm i for producing its component l.
Q_{ik}	the nonnegative shipment of firm i's product from firm i to demand market k; $k = 1,\ldots,n_R$. For firm i, we group its $\{Q_{ik}\}$ elements into the vector $Q_i \in R^{n_R}_+$, and group all such vectors into the vector $Q \in R^{In_R}_+$.
π_{jil}	the price charged by supplier j for producing one unit of firm i's component l. For supplier j, we group its $\{\pi_{jil}\}$ elements into the vector $\pi_j \in R^{\sum_{i=1}^I n_{l^i}}_+$, and group all such vectors into the vector $\pi \in R^{n_S \sum_{i=1}^I n_{l^i}}_+$.
d_{ik}	the demand for firm i's product at demand market k. We group all $\{d_{ik}\}$ elements into the vector $d \in R^{In_R}_+$.
θ_{il}	the amount of component l needed by firm i to produce one unit product i.
$f^S_{jl}(Q^S)$	supplier j's production cost for producing component l; $l = 1,\ldots,n_l$.
$tc^S_{jil}(Q^S)$	supplier j's transportation cost for shipping firm i's component l.
$oc_j(\pi)$	supplier j's opportunity cost.
$f_i(Q)$	firm i's cost for assembling its product using the components needed.
$f^F_{il}(Q^F)$	firm i's production cost for producing its component l.
$tc^F_{ik}(Q)$	firm i's transportation cost for shipping its product to demand market k.
$c_{ijl}(Q^S)$	the transaction cost paid by firm i for transacting with supplier j for its component l.
$\rho_{ik}(d)$	the demand price for firm i's product at demand market k.

2.1 The Behavior of the Firms and Their Optimality Conditions

Given the prices π^* of the components charged by the suppliers, the objective of firm i; $i = 1, \ldots, I$, is to maximize its utility/profit U_i^F, which is the difference between its total revenue and its total cost. The total cost includes the assembly cost, the production cost, the transportation costs, the payments to the suppliers, and the transaction costs. As noted in Table 1, the assembly cost functions, the production cost functions, the transportation cost functions, and the demand price functions are general functions in vectors of quantities, which capture the competition among firms for resources.

Hence, firm i seeks to

$$\text{Maximize}_{Q_i, Q_i^F, Q_i^S} \quad U_i^F = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - f_i(Q) - \sum_{l=1}^{n_{l^i}} f_{il}^F(Q^F) - \sum_{k=1}^{n_R} tc_{ik}^F(Q)$$

$$- \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} \pi_{jil}^* Q_{jil}^S - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} c_{ijl}(Q^S) \tag{1a}$$

subject to:

$$Q_{ik} = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{2}$$

$$\sum_{k=1}^{n_R} Q_{ik} \theta_{il} \leq \sum_{j=1}^{n_S} Q_{jil}^S + Q_{il}^F, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{3}$$

$$Q_{ik} \geq 0, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{4}$$

$$CAP_{jil}^S \geq Q_{jil}^S \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{5}$$

$$CAP_{il}^F \geq Q_{il}^F \geq 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{6}$$

We assume that all the cost functions and the demand price functions in (1) are continuous and continuously differentiable. The cost functions are convex, and the demand price functions are monotone decreasing. According to constraint (2), the product shipment from a firm to a demand market should be equal to the quantity of the firm's product consumed at that demand market. Constraint (3) captures the material requirements in the assembly process. Constraint (4) is the nonnegativity constraint for product shipments. Constraints (5) and (6) indicate that the component production quantities should be nonnegative and limited by the associated capacities, which can capture the abilities of producing. If a supplier or a firm is not capable of producing a certain component, the associated capacity would be 0.

In light of (2), we can define the demand price function $\hat{\rho}_{ik}$ in product shipments of the firms, so that $\hat{\rho}_{ik}(Q) \equiv \rho_{ik}(d); i = 1, \ldots, I, k = 1, \ldots, n_R$. Therefore, (1a) is equivalent to:

$$\text{Maximize}_{Q_i, Q_i^F, Q_i^S} \quad U_i^F = \sum_{k=1}^{n_R} \hat{\rho}_{ik}(Q) Q_{ik} - f_i(Q) - \sum_{l=1}^{n_{li}} f_{il}^F(Q^F) - \sum_{k=1}^{n_R} tc_{ik}^F(Q)$$

$$- \sum_{j=1}^{n_S} \sum_{l=1}^{n_{li}} \pi_{jil}^* Q_{jil}^S - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{li}} c_{ijl}(Q^S). \tag{1b}$$

The firms compete in the sense of Nash (1950, 1951). The strategic variables for each firm i are the product shipments to the demand markets, the in-house component production quantities, and the contracted component production quantities produced by the suppliers.

We define the feasible set $\overline{K}_i^F$ as $\overline{K}_i^F \equiv \{(Q_i, Q_i^F, Q_i^S) | (3)$ - (6) are satisfied$\}$. All $\overline{K}_i^F$; $i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\overline{\mathcal{K}}^F \equiv \Pi_{i=1}^I \overline{K}_i^F$.

Definition 1: A Cournot-Nash Equilibrium

A product shipment, in-house component production, and contracted component production pattern $(Q^*, Q^{F^*}, Q^{S^*}) \in \overline{\mathcal{K}}^F$ is said to constitute a Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$,

$$U_i^F(Q_i^*, \hat{Q}_i^*, Q_i^{F^*}, \hat{Q}_i^{F^*}, Q_i^{S^*}, \hat{Q}_i^{S^*}, \pi^*) \geq U_i^F(Q_i, \hat{Q}_i^*, Q_i^F, \hat{Q}_i^{F^*}, Q_i^S, \hat{Q}_i^{S^*}, \pi^*),$$

$$\forall (Q_i, Q_i^F, Q_i^S) \in \overline{K}_i^F, \tag{7}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$
$$\hat{Q}_i^{F^*} \equiv (Q_1^{F^*}, \ldots, Q_{i-1}^{F^*}, Q_{i+1}^{F^*}, \ldots, Q_I^{F^*}),$$
$$\hat{Q}_i^{S^*} \equiv (Q_1^{S^*}, \ldots, Q_{i-1}^{S^*}, Q_{i+1}^{S^*}, \ldots, Q_I^{S^*}).$$

According to (7), a Cournot-Nash equilibrium is established if no firm can unilaterally improve upon its profit by selecting an alternative vector of product shipments, in-house component production quantities, and contracted component production quantities produced by the suppliers.

2.1.1 Variational Inequality Formulations

We now derive the variational inequality formulation of the Cournot-Nash equilibrium (see [6, 11, 31, 32]) in the following theorem.

Theorem 1

Assume that, for each firm i; $i = 1, \ldots, I$, the utility function $U_i^F(Q, Q^F, Q^S, \pi^*)$ is concave with respect to its variables in Q_i, Q_i^F, and Q_i^S, and is continuous and continuously differentiable. Then $(Q^*, Q^{F^*}, Q^{S^*}) \in \overline{\mathcal{K}}^F$ is a Counot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$
-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*) - \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})
$$

$$
-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*}) \geq 0, \quad \forall (Q, Q^F, Q^S) \in \overline{\mathcal{K}}^F, \tag{8}
$$

with notice that: for $i = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$
-\frac{\partial U_i^F}{\partial Q_{ik}} = \left[\frac{\partial f_i(Q)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q)}{\partial Q_{ik}} Q_{ih} - \hat{\rho}_{ik}(Q) \right],
$$

for $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$
-\frac{\partial U_i^F}{\partial Q_{il}^F} = \left[\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^F)}{\partial Q_{il}^F} \right],
$$

for $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$
-\frac{\partial U_i^F}{\partial Q_{jil}^S} = \left[\pi_{jil}^* + \sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^S)}{\partial Q_{jil}^S} \right],
$$

or, equivalently, $(Q^*, Q^{F^*}, Q^{S^*}, \lambda^*) \in \mathcal{K}^F$ is a vector of the equilibrium product shipment, in-house component production, contracted component production pattern, and Lagrange multipliers if and only if it satisfies the variational inequality

$$
\sum_{i=1}^{I}\sum_{k=1}^{n_R} \left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q^*)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}} Q_{ih}^* - \hat{\rho}_{ik}(Q^*) + \sum_{l=1}^{n_{l^i}} \lambda_{il}^* \theta_{il} \right] \times (Q_{ik} - Q_{ik}^*)
$$

$$
+ \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \left[\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^{F^*})}{\partial Q_{il}^F} - \lambda_{il}^* \right] \times (Q_{il}^F - Q_{il}^{F^*})
$$

$$
+ \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \left[\pi_{jil}^* + \sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^{S^*})}{\partial Q_{jil}^S} - \lambda_{il}^* \right] \times (Q_{jil}^S - Q_{jil}^{S^*})
$$

$$
+ \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \left[\sum_{j=1}^{n_S} Q_{jil}^{S^*} + Q_{il}^{F^*} - \sum_{k=1}^{n_R} Q_{ik}^* \theta_{il} \right] \times (\lambda_{il} - \lambda_{il}^*) \geq 0, \quad \forall (Q, Q^F, Q^S, \lambda) \in \mathcal{K}^F, \tag{9}
$$

where $\mathcal{K}^F \equiv \Pi_{i=1}^I K_i^F$ and $K_i^F \equiv \{(Q_i, Q_i^F, Q_i^S, \lambda_i) | \lambda_i \geq 0$ with (4) - (6) satisfied$\}$. λ_i is the n_{l^i}-dimensional vector with component l being the element λ_{il} corresponding to the Lagrange multiplier associated with the (i,l)-th constraint (3). Both the above-defined feasible sets are convex.

Proof: For a given firm i, under the imposed assumptions, (8) holds if and only if (see Bertsekas and Tsitsiklis (1989) page 287) the following holds:

$$\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}}+\sum_{h=1}^{n_R}\frac{\partial tc_{ih}^F(Q^*)}{\partial Q_{ik}}-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}}Q_{ih}^*-\hat{\rho}_{ik}(Q^*)+\sum_{l=1}^{n_{li}}\lambda_{il}^*\theta_{il}\right]\times(Q_{ik}-Q_{ik}^*)$$
$$+\sum_{l=1}^{n_{li}}\left[\sum_{m=1}^{n_{li}}\frac{\partial f_{im}^F(Q^{F^*})}{\partial Q_{il}^F}\right]\times(Q_{il}^F-Q_{il}^{F^*})$$
$$+\sum_{j=1}^{n_S}\sum_{l=1}^{n_{li}}\left[\pi_{jil}^*+\sum_{g=1}^{n_S}\sum_{m=1}^{n_{li}}\frac{\partial c_{igm}(Q^{S^*})}{\partial Q_{jil}^S}-\lambda_{il}^*\right]\times(Q_{jil}^S-Q_{jil}^{S^*})$$
$$+\sum_{l=1}^{n_{li}}\left[\sum_{j=1}^{n_S}Q_{jil}^{S^*}+Q_{il}^{F^*}-\sum_{k=1}^{n_R}Q_{ik}^*\theta_{il}\right]\times(\lambda_{il}-\lambda_{il}^*)\geq 0,\quad \forall (Q_i,Q_i^F,Q_i^S,\lambda_i)\in K_i^F. \tag{10}$$

Variational inequality (10) holds for each firm i; $i = 1, \ldots, I$, and, hence, the summation of (10) yields variational inequality (9).□

For additional background on the variational inequality problem, please refer to the book by Nagurney [19].

2.2 The Behavior of the Suppliers and Their Optimality Conditions

Opportunity costs of the suppliers are included in this model. The opportunity cost is "the loss of potential gain from other alternatives when one alternative is chosen" ([41]). It can include the time and effort put in (see [35]), and the profit that the decision-maker could have earned, if he had made other choices ([40]).

The suppliers' opportunity costs are functions of the prices that they charge the firms for producing and delivering the components, as in Table 1. The suppliers may not be able to recover their costs if the prices that they charge are too low. If the prices are too high, the suppliers may lose the contracts. Here, we capture the opportunity cost of a supplier with a general function that depends on the vector of prices, since the opportunity cost of a supplier may also be affected by the prices charged by the other suppliers (see also [26]).

Given the Q^{S^*} determined by the firms, the objective of supplier j; $j = 1, \ldots, n_S$, is to maximize its total profit, denoted by U_j^S. Its revenue is obtained from the payments of the firms, while its costs are the costs of production and delivery, and the opportunity cost. The strategic variables of a supplier are the prices that it charges the firms.

The decision-making problem for supplier j is the following:

$$\text{Maximize}_{\pi_j} \quad U_j^S = \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} \pi_{jil} Q_{jil}^{S^*} - \sum_{l=1}^{n_l} f_{jl}^S(Q^{S^*}) - \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} tc_{jil}^S(Q^{S^*}) - oc_j(\pi) \tag{11}$$

subject to:

$$\pi_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{12}$$

We assume that the cost functions of each supplier are continuous, continuously differentiable, and convex.

The suppliers compete in a noncooperative in the sense of Nash (1950, 1951), with each one trying to maximize its own profit. We define the feasible sets $K_j^S \equiv \{\pi_j | \pi_j \in R_+^{\sum_{i=1}^{I} n_{l^i}}\}$, $\mathcal{K}^S \equiv \Pi_{j=1}^{n_S} K_j^S$, and $\overline{\mathcal{K}} \equiv \overline{\mathcal{K}}^F \times \mathcal{K}^S$. All the above-defined feasible sets are convex.

Definition 2: A Bertrand-Nash Equilibrium

A price pattern $\pi^* \in \mathcal{K}^S$ is said to constitute a Bertrand-Nash equilibrium if for each supplier j; $j = 1, \ldots, n_S$,

$$U_j^S(Q^{S^*}, \pi_j^*, \hat{\pi}_j^*) \geq U_j^S(Q^{S^*}, \pi_j, \hat{\pi}_j^*), \quad \forall \pi_j \in K_j^S, \tag{13}$$

where

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_S}^*).$$

According to (13), a Bertrand-Nash equilibrium is established if no supplier can unilaterally improve upon its profit by selecting an alternative vector of prices charged to the firms.

2.2.1 Variational Inequality Formulations

The variational inequality formulation of the Bertrand-Nash equilibrium according to Definition 2 (see [1, 11, 20, 31, 32]) is given in the following theorem.

Theorem 2

Assume that, for each supplier j; $j = 1, \ldots, n_S$, the profit function $U_j^S(Q^{S^*}, \pi)$ is concave with respect to the variables in π_j, and is continuous and continuously differentiable. Then $\pi^* \in \mathcal{K}^S$ is a Bertrand-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_j^S(Q^{S^*}, \pi^*)}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) \geq 0,$$

$$\forall \pi \in \mathcal{K}^S, \tag{14}$$

with notice that: for $j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{li}$:

$$-\frac{\partial U_j^S}{\partial \pi_{jil}} = \frac{\partial oc_j(\pi)}{\partial \pi_{jil}} - Q_{jil}^{S^*}.$$

2.3 The Equilibrium Conditions for the Multitiered Supply Chain Network with Suppliers

In equilibrium, the optimality conditions for all firms and the optimality conditions for all suppliers must hold simultaneously, according to the definition below.

Definition 3: Multitiered Supply Chain Network Equilibrium with Suppliers

The equilibrium state of the multitiered supply chain network with suppliers is one where both variational inequalities (8) (or (9)) and (14) hold simultaneously.

Theorem 3

The equilibrium conditions governing the multitiered supply chain network model with suppliers are equivalent to the solution of the variational inequality problem: determine $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*) \in \overline{\mathcal{K}}$, such that:

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*) - \sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_j^S(Q^{S^*}, \pi^*)}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) \geq 0, \quad \forall (Q, Q^F, Q^S, \pi) \in \overline{\mathcal{K}}, \tag{15}$$

or, equivalently: determine $(Q^*, Q^{F^*}, Q^{S^*}, \lambda^*, \pi^*) \in \mathcal{K}$*, such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}}+\sum_{h=1}^{n_R}\frac{\partial tc_{ih}^F(Q^*)}{\partial Q_{ik}}-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}}Q_{ih}^*-\hat{\rho}_{ik}(Q^*)+\sum_{l=1}^{n_{l^i}}\lambda_{il}^*\theta_{il}\right]\times(Q_{ik}-Q_{ik}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f_{im}^F(Q^{F^*})}{\partial Q_{il}^F}-\lambda_{il}^*\right]\times(Q_{il}^F-Q_{il}^{F^*})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi_{jil}^*+\sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{igm}(Q^{S^*})}{\partial Q_{jil}^S}-\lambda_{il}^*\right]\times(Q_{jil}^S-Q_{jil}^{S^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S}Q_{jil}^{S^*}+Q_{il}^{F^*}-\sum_{k=1}^{n_R}Q_{ik}^*\theta_{il}\right]\times(\lambda_{il}-\lambda_{il}^*)$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\frac{\partial oc_j(\pi^*)}{\partial \pi_{jil}}-Q_{jil}^{S^*}\right]\times(\pi_{jil}-\pi_{jil}^*)\geq 0, \quad \forall (Q,Q^F,Q^S,\lambda,\pi)\in\mathcal{K}, \tag{16}$$

where $\mathcal{K} \equiv \mathcal{K}^F \times \mathcal{K}^S$.

Proof: Summation of variational inequalities (8) (or (9)) and (14) yields variational inequality (15) (or (16)). A solution to variational inequality (15) (or (16)) satisfies the sum of (8) (or (9)) and (14) and, hence, is an equilibrium according to Definition 3.□

We now put variational inequality (16) into standard form (cf. [19]): determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X): X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in the N-dimensional Euclidean space, $N = In_R + 2n_S \sum_{i=1}^{I} n_{l^i} + 2\sum_{i=1}^{I} n_{l^i}$, and $\mathcal{K}$ is closed and convex. We define the vector $X \equiv (Q, Q^F, Q^S, \lambda, \pi)$ and the vector $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$, such that:

$$F^1(X)=\left[\frac{\partial f_i(Q)}{\partial Q_{ik}}+\sum_{h=1}^{n_R}\frac{\partial tc_{ih}^F(Q)}{\partial Q_{ik}}-\sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q)}{\partial Q_{ik}}Q_{ih}-\hat{\rho}_{ik}(Q)+\sum_{l=1}^{n_{l^i}}\lambda_{il}\theta_{il};\right.$$

$$\left. i=1,\ldots,I; k=1,\ldots,n_R\right], \tag{18a}$$

$$F^2(X)=\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f_{im}^F(Q^F)}{\partial Q_{il}^F}-\lambda_{il}; i=1,\ldots,I; l=1,\ldots,n_{l^i}\right], \tag{18b}$$

$$F^3(X)=\left[\pi_{jil}+\sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{igm}(Q^S)}{\partial Q_{jil}^S}-\lambda_{il}; j=1,\ldots,n_S; i=1,\ldots,I; l=1,\ldots,n_{l^i}\right], \tag{18c}$$

$$F^4(X) = \left[\sum_{j=1}^{n_S} Q^S_{jil} + Q^F_{il} - \sum_{k=1}^{n_R} Q_{ik}\theta_{il}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \tag{18d}$$

$$F^5(X) = \left[\frac{\partial oc_j(\pi)}{\partial \pi_{jil}} - Q^S_{jil}; j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right]. \tag{18e}$$

Similarly, we also put variational inequality (15) into standard form: determine $Y^* \in \overline{\mathcal{K}}$ where Y is a vector in R^M, $G(Y)$ is a continuous function such that $G(Y) : Y \mapsto \overline{\mathcal{K}} \subset R^M$, and

$$\langle G(Y^*), Y - Y^* \rangle \geq 0, \quad \forall Y \in \overline{\mathcal{K}}, \tag{19}$$

where $M = In_R + \sum_{i=1}^I n_{l^i} + 2n_S \sum_{i=1}^I n_{l^i}$, and $\overline{\mathcal{K}}$ is closed and convex. We define $Y \equiv (Q, Q^F, Q^S, \pi)$, $G(Y) \equiv (-\frac{\partial U_i^F}{\partial Q_{ik}}, -\frac{\partial U_i^F}{\partial Q^F_{il}}, -\frac{\partial U_i^F}{\partial Q^S_{jil}}, -\frac{\partial U_j^S}{\partial \pi_{jil}}); j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$. Hence, (15) can be put into standard form (19).

The equilibrium solution $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)$ to (19) and the $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)$ in the equilibrium solution to (17) are equivalent for this multitiered supply chain network problem with suppliers. In addition to $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)$, the equilibrium solution to (17) also contains the equilibrium Lagrange multipliers (λ^*).

2.4. Qualitative Properties

We now present some qualitative properties of the solution to variational inequalities (17) and (19), equivalently, (16) and (15). In particular, we provide the existence result and the uniqueness result.

In a supply chain network with suppliers, it is reasonable to expect that the price charged by each supplier j for producing one unit of firm i's component l, π_{jil}, is bounded by a sufficiently large value, since, in practice, each supplier cannot charge unbounded prices to the firms. Therefore, the following assumption is not unreasonable:

Assumption 1

Suppose that in our supply chain network model with suppliers there exists a sufficiently large Π*, such that,*

$$\pi_{jil} \leq \Pi, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{20}$$

With this assumption, we have the following existence result.

Theorem 4: Existence

With Assumption 1 satisfied, there exists at least one solution to variational inequalities (17) and (19); equivalently, (16) and (15).

Proof: We first prove that there exists at least one solution to variational inequality (19) (cf. (15)). Due to constraint (3), the product quantities Q_{ik}; $i = 1, \ldots, I; k = 1, \ldots, n_S$ are bounded, since the components quantities are nonnegative and capacitated (cf. (5) and (6)). Therefore, with Assumption 1, the feasible set of variational inequality (19) is bounded. Since the cost functions and the demand price functions are continuously differentiable, and the feasible set is convex and compact, the existence of a solution to (19) is then guaranteed (cf. [13] and Theorem 1.5 in [19]). Since (19) and (17) (cf. (16)) are equivalent (Theorem 3 in [25]), the existence of (17) is guaranteed. □

Theorem 5: Uniqueness

If Assumption 1 is satisfied, the equilibrium product shipment, in-house component production, contracted component production, and suppliers' price pattern $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)$ *in variational inequality (19), equivalently, (17), is unique under the following conditions:*

(i) one of the two families of convex functions $f_i(Q)$; $i = 1, \ldots, I$, *and* $tc^F_{ik}(Q)$; $k = 1 \ldots n_R$, *is strictly convex in* Q_{ik};
(ii) the $f^F_{il}(Q^F)$; $i = 1, \ldots, I, l = 1, \ldots n_{l^i}$, *are strictly convex in* Q^F_{il};
(iii) the $c_{ijl}(Q^S)$; $j = 1, \ldots, n_S$, $i = 1, \ldots, I, l = 1, \ldots n_{l^i}$, *are strictly convex in* Q^S_{jil};
(iv) the $oc_j(\pi)$; $j = 1, \ldots, n_S$, *are strictly convex in* π_{jil};
(v) the $\rho_{ik}(d)$; $i = 1, \ldots, I, k = 1, \ldots n_R$, *are strictly monotone decreasing of* d_{ik}.

Proof: Assume the above conditions. Then the negative utility functions, $-U^F_i$ and $-U^S_j$; $\forall i = 1, \ldots, I, j = 1, \ldots, n_S$, are strictly convex in associated variables (cf. (1b), (11), and Theorems 1 and 2). Therefore,

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[(-\frac{\partial U^F_i(Q', Q^{F'}, Q^{S'}, \pi')}{\partial Q_{ik}}) - (-\frac{\partial U^F_i(Q'', Q^{F''}, Q^{S''}, \pi'')}{\partial Q_{ik}})\right] \times (Q'_{ik} - Q''_{ik})$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[(-\frac{\partial U^F_i(Q', Q^{F'}, Q^{S'}, \pi')}{\partial Q^F_{il}}) - (-\frac{\partial U^F_i(Q'', Q^{F''}, Q^{S''}, \pi'')}{\partial Q^F_{il}})\right] \times (Q^{F'}_{il} - Q^{F''}_{il})$$
$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[(-\frac{\partial U^F_i(Q', Q^{F'}, Q^{S'}, \pi')}{\partial Q^S_{jil}}) - (-\frac{\partial U^F_i(Q'', Q^{F''}, Q^{S''}, \pi'')}{\partial Q^S_{jil}})\right] \times (Q^{S'}_{jil} - Q^{S''}_{jil})$$
$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[(-\frac{\partial U^S_j(Q^{S'}, \pi')}{\partial \pi_{jil}}) - (-\frac{\partial U^S_j(Q^{S''}, \pi'')}{\partial \pi_{jil}})\right] \times (\pi'_{jil} - \pi''_{jil}) > 0,$$

$$\forall (Q', Q^{F'}, Q^{S'}, \pi'), (Q'', Q^{F''}, Q^{S''}, \pi'') \in \overline{\mathcal{K}}, \quad (Q', Q^{F'}, Q^{S'}, \pi') \neq (Q'', Q^{F''}, Q^{S''}, \pi''), \tag{21}$$

that is,

$$\langle G(Y') - G(Y''), Y' - Y'' \rangle > 0, \quad \forall Y', Y'' \in \overline{\mathcal{K}}, Y' \neq Y'', \tag{22}$$

where $Y' = (Q', Q^{F'}, Q^{S'}, \pi')$, $Y'' = (Q'', Q^{F''}, Q^{S''}, \pi'')$. (22) proves that $G(Y)$ is strictly monotone. Under the existence (Theorem 4) and the strict monotonicity, the proof of uniqueness follows the standard variational inequality theory (cf. [13]). □

Theorem 6: Lipschitz Continuity

The function that enters the variational inequality problem (17) is Lipschitz continuous, that is,

$$\| F(X') - F(X'') \| \leq L \| X' - X'' \|, \quad \forall X', X'' \in \mathcal{K}, \; \textit{where } L > 0. \tag{23}$$

Proof: Since we have assumed that all the cost functions have bounded second-order partial derivatives, and the demand price functions have bounded first-order and second-order partial derivatives, the result is direct by applying a mid-value theorem from calculus to the $F(X)$ that enters variational inequality (17). □

3. Supply Chain Network Performance Measures

We now present the supply chain network performance measure for the whole competitive supply chain network G and that for the supply chain network of each individual firm i; $i = 1, \ldots, I$, under competition. Such measures capture the efficiency of the supply chains in that the higher the demand to price ratios normalized over associated firm and demand market pairs, the higher the efficiency. Hence, a supply chain network is deemed to perform better if it can satisfy higher demands, on the average, relative to the product prices.

Definition 4.1: The Supply Chain Network Performance Measure for the Whole Competitive Supply Chain Network *G*

The supply chain network performance/efficiency measure, $\mathcal{E}(G)$, for a given competitive supply chain network topology G and the equilibrium demand vector d^*, is defined as follows:

$$\mathcal{E} = \mathcal{E}(G) = \frac{\sum_{i=1}^{I} \sum_{k=1}^{n_R} \frac{d^*_{ik}}{\rho_{ik}(d^*)}}{I \times n_R}, \tag{24}$$

where recall that I is the number of firms and n_R is the number of demand markets in the competitive supply chain network, and d^*_{ik} and $\rho_{ik}(d^*)$ denote the equilibrium demand and the equilibrium price, respectively, associated with firm i and demand market k.

Definition 4.2: The Supply Chain Network Performance Measure for an Individual Firm under Competition

The supply chain network performance/efficiency measure, $\mathcal{E}_i(G_i)$, for the supply chain network topology of a given firm i, G_i, under competition and the equilibrium demand vector d^*, is defined as:

$$\mathcal{E}_i = \mathcal{E}_i(G_i) = \frac{\sum_{k=1}^{n_R} \frac{d^*_{ik}}{\rho_{ik}(d^*)}}{n_R}, \quad i = 1, \ldots, I. \tag{25}$$

3.1 The Importance of Supply Chain Network Suppliers and Their Components

With our supply chain network performance/efficiency measures, we are ready to investigate the importance of suppliers and their components, which correspond to nodes in our supply chain, for the whole competitive supply chain network and for each individual firm under competition. The importance is determined by studying the impact of the suppliers and the components on the supply chain efficiency through their removal.

We define the importance of a supplier for the whole competitive supply chain network as follows:

Definition 5.1: Importance of a Supplier for the Whole Competitive Supply Chain Network G

The importance of a supplier j, corresponding to a supplier node $j \in G$, $I(j)$, for the whole competitive supply chain network, is measured by the relative supply chain network efficiency drop after j is removed from the whole supply chain:

$$I(j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G-j)}{\mathcal{E}(G)}, \quad j = 1, \ldots, n_S, \tag{26}$$

where $G - j$ is the resulting supply chain after supplier j is removed from the competitive supply chain network G.

The upper bound of the importance of a supplier is 1. The higher the value, the more important a supplier is to the supply chain.

We also can construct using an adaptation of (26) a robustness-type measure for the whole competitive supply chain by evaluating how the supply chain is impacted if *all* the suppliers are eliminated due to a major disruption. One may recall the triple disaster in

Fukushima, Japan in March 2011 as an illustration of such an event. Specifically, we let:

$$I(\sum_{j=1}^{n_S} j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - \sum_{j=1}^{n_S} j)}{\mathcal{E}(G)}, \tag{27}$$

measure how the whole supply chain can respond if all of its suppliers are unavailable.

The importance of a supplier for the supply chain network of an individual firm under competition is defined as follows:

Definition 5.2: Importance of a Supplier for the Supply Chain Network of an Individual Firm under Competition

The importance of a supplier j, corresponding to a supplier node $j \in G_i$, $I_i(j)$, for the supply chain network of a given firm i under competition, is measured by the relative supply chain network efficiency drop after j is removed from G_i:

$$I_i(j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; j = 1, \ldots, n_S. \tag{28}$$

The corresponding robustness measure for the supply chain of firm i if all the suppliers are eliminated is:

$$I_i(\sum_{j=1}^{n_S} j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - \sum_{j=1}^{n_S} j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I. \tag{29}$$

In addition, we define the importance of a supplier's component for the whole competitive supply chain network as follows:

Definition 5.3: Importance of a Supplier's Component for the Whole Competitive Supply Chain Network G

The importance of a supplier j's component l_j; $l_j = 1_j, \ldots, n_{l_j}$, corresponding to j's component node $l_j \in G$, $I(l_j)$, for the whole competitive supply chain network, is measured by the relative supply chain network efficiency drop after l_j is removed from G:

$$I(l_j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - l_j)}{\mathcal{E}(G)}, \quad j = 1, \ldots, n_S; l_j = 1_j, \ldots, n_{l_j}. \tag{30}$$

where $G - l_j$ is the resulting supply chain after supplier j's component l_j is removed from the whole competitive supply chain network.

The corresponding robustness measure for the whole competitive supply chain network if all suppliers' component l_j; $l_j = 1_j, \ldots, n_{lj}$, are eliminated is:

$$I(\sum_{j=1}^{n_S} l_j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - \sum_{j=1}^{n_S} l_j)}{\mathcal{E}(G)}, \quad l_j = 1_j, \ldots, n_{lj}. \tag{31}$$

The importance of a supplier's component for the supply chain network of an individual firm is defined as:

Definition 5.4: Importance of a Supplier's Component for the Supply Chain Network of an Individual Firm under Competition

The importance of supplier j's component l_j; $l_j = 1_j, \ldots, n_{lj}$, corresponding to a component node $l_j \in G_i$, $I_i(l_j)$, for the supply chain network of a given firm i under competition, is measured by the relative supply chain network efficiency drop after l_j is removed from G_i:

$$I_i(l_j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - l_j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; j = 1, \ldots, n_S; l_j = 1_j, \ldots, n_{lj}. \tag{32}$$

The corresponding robustness measure for the supply chain network of firm i if all suppliers' component l_j, $l_j = 1_j, \ldots, n_{lj}$, are eliminated is:

$$I_i(\sum_{j=1}^{n_S} l_j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - \sum_{j=1}^{n_S} l_j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; l_j = 1_j, \ldots, n_{lj}. \tag{33}$$

Note that, in removing a supplier node, we also remove all the links emanating from the node, and the subsequent component nodes and links. Similarly, in removing a component node of a supplier, we remove from the supply chain network topology that node and the links that emanate to and from the node.

4. Algorithm

We employ the Euler method for the computation of the solution for the multitiered supply chain network game theory model with suppliers. The Euler Method is induced by the general iterative scheme of Dupuis and Nagurney [10]. Specifically, recall that at iteration τ of the Euler method (see also [30]), one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{34}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (17).

As shown in [10] and [30], for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other network models can be found in [21, 25, 27].

Explicit Formulae for the Euler Method Applied to the Multitiered Supply Chain Network Game Theory Model with Suppliers

The Euler method yields, at each iteration, explicit formulae for the computation of the product shipment, in-house component production, and contracted component production pattern, the Lagrange multipliers, and the prices charged by the suppliers. In particular, we have the following closed form expressions:

for the product shipments: for $i = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$Q_{ik}^{\tau+1} = \max\{0, Q_{ik}^{\tau} + a_\tau(-\frac{\partial f_i(Q^\tau)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q^\tau)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q^\tau)}{\partial Q_{ik}} Q_{ih}^\tau + \hat{\rho}_{ik}(Q^\tau) - \sum_{l=1}^{n_{l^i}} \lambda_{il}^\tau \theta_{il})\}; \tag{35a}$$

for the in-house component production pattern: for $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$Q_{il}^{F^{\tau+1}} = \min\{CAP_{il}^F, \max\{0, Q_{il}^{F^\tau} + a_\tau(-\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^{F^\tau})}{\partial Q_{il}^F} + \lambda_{il}^\tau)\}\}; \tag{35b}$$

for the contracted component production pattern: for $j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$:

$$Q_{jil}^{S^{\tau+1}} = \min\{CAP_{jil}^S, \max\{0, Q_{jil}^{S^\tau} + a_\tau(-\pi_{jil}^\tau - \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^{S^\tau})}{\partial Q_{jil}^S} + \lambda_{il}^\tau)\}\}; \tag{35c}$$

and for the Lagrange multipliers: for $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$\lambda_{il}^{\tau+1} = \max\{0, \lambda_{il}^\tau + a_\tau(-\sum_{j=1}^{n_S} Q_{jil}^{S^\tau} - Q_{il}^{F^\tau} + \sum_{k=1}^{n_R} Q_{ik}^\tau \theta_{il})\}. \tag{35d}$$

Also, the following closed form expressions are for the prices charged by the suppliers:
for $j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$:

$$\pi_{jil}^{\tau+1} = \max\{0, \pi_{jil}^\tau + a_\tau(-\frac{\partial oc_j(\pi^\tau)}{\partial \pi_{jil}^\tau} + Q_{jil}^{S^\tau})\}. \tag{35e}$$

5. Numerical Examples

In this Section, we present numerical supply chain network examples with suppliers, which we solve via the Euler method, as described in Section 4. We implemented the Euler method using Matlab on a Lenovo Z580. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive quantities, prices, and Lagrange multipliers is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialize the algorithm by setting the product and component quantities equal to 50 and the prices and the Lagrange multipliers equal to 0.

Example 1

The supply chain network topology of Example 1 is given in Figure 2. There are two firms serving demand markets 1 and 2. The firms procure the components of their products from supplier 1. They also have the option of producing the components needed by themselves.

The product of firm 1 requires two components, which are 1^1 and 2^1. 2 units of component 1^1 and 3 units of component 2^1 are needed for producing one unit of firm 1's product. The product of firm 2 requires two components, 1^2 and 2^2. To produce one unit of firm 2's product, 2 units of component 1^2 and 2 units of component 2^2 are needed. Therefore,

$$\theta_{11} = 2, \quad \theta_{12} = 3, \quad \theta_{21} = 2, \quad \theta_{22} = 2.$$

Components 1^1 and 1^2 are the same component, which corresponds to node 1 in the second tier in Figure 2 below. Components 2^1 and 2^2 correspond to nodes 2 and 3, respectively.

The data are as follows.

The capacities of the suppliers are:

$$CAP^S_{111} = 80, \quad CAP^S_{112} = 90, \quad CAP^S_{121} = 80, \quad CAP^S_{122} = 50,$$

Thus, supplier 1 is capable of producing components 1^1, 2^1, 1^2, and 2^2 for the firms.

The firms are not capable of producing components 1^1 or 1^2, so their capacities are:

$$CAP^F_{11} = 0, \quad CAP^F_{12} = 20, \quad CAP^F_{21} = 0, \quad CAP^F_{22} = 30.$$

The supplier's production costs are:

$$f^S_{11}(Q^S_{111}, Q^S_{121}) = 2(Q^S_{111} + Q^S_{121}), \quad f^S_{12}(Q^S_{112}) = 3Q^S_{112}, \quad f^S_{13}(Q^S_{122}) = Q^S_{122}.$$

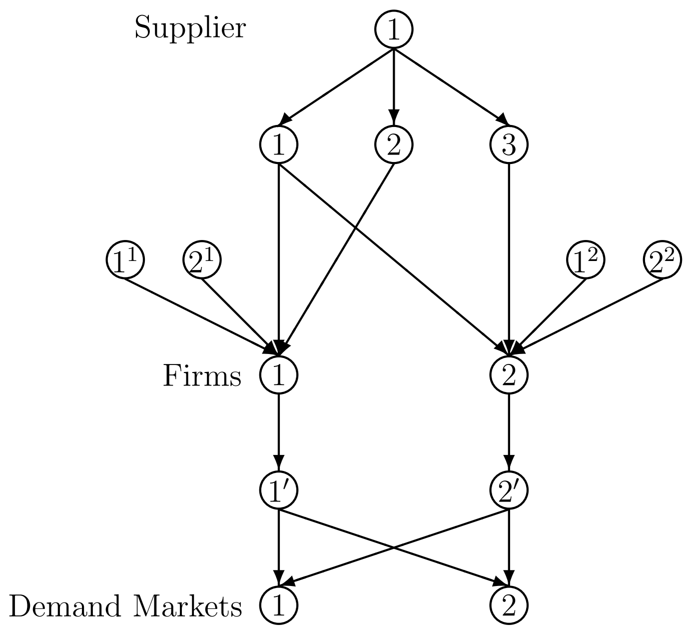

Figure 2: Example 1

The supplier's transportation costs are:

$$tc^S_{111}(Q^S_{111}, Q^S_{112}) = 0.75Q^S_{111} + 0.1Q^S_{112}, \quad tc^S_{112}(Q^S_{112}, Q^S_{111}) = 0.1Q^S_{112} + 0.05Q^S_{111},$$

$$tc^S_{121}(Q^S_{121}, Q^S_{122}) = Q^S_{121} + 0.2Q^S_{122}, \quad tc^S_{122}(Q^S_{122}, Q^S_{121}) = 0.6Q^S_{122} + 0.25Q^S_{121}.$$

The opportunity cost of the supplier is:

$$oc_1(\pi_{111}, \pi_{112}, \pi_{121}, \pi_{122}) = 0.5(\pi_{111} - 10)^2 + (\pi_{112} - 5)^2 + 0.5(\pi_{121} - 10)^2 + 0.75(\pi_{122} - 7)^2.$$

The firms' assembly costs are:

$$f_1(Q_{11}, Q_{12}, Q_{21}, Q_{22}) = 2(Q_{11} + Q_{12})^2 + 2(Q_{11} + Q_{12}) + (Q_{11} + Q_{12})(Q_{21} + Q_{22}),$$

$$f_2(Q_{11}, Q_{12}, Q_{21}, Q_{22}) = 1.5(Q_{21} + Q_{22})^2 + 2(Q_{21} + Q_{22}) + (Q_{11} + Q_{12})(Q_{21} + Q_{22}).$$

The firms' production costs for producing their components are:

$$f^F_{11}(Q^F_{11}, Q^F_{21}) = 3Q^{F^2}_{11} + Q^F_{11} + 0.5Q^F_{11}Q^F_{21}, \quad f^F_{12}(Q^F_{12}) = 2Q^{F^2}_{12} + 1.5Q^F_{12},$$

$$f^F_{21}(Q^F_{11}, Q^F_{21}) = 3Q^{F^2}_{21} + 2Q^F_{21} + 0.75Q^F_{11}Q^F_{21}, \quad f^F_{22}(Q^F_{22}) = 1.5Q^{F^2}_{22} + Q^F_{22}.$$

The firms' transportation costs for shipping their products to the demand markets are:

$$tc^F_{11}(Q_{11}, Q_{21}) = Q_{11}^2 + Q_{11} + 0.5Q_{11}Q_{21}, \quad tc^F_{12}(Q_{12}, Q_{22}) = 2Q_{12}^2 + Q_{12} + 0.5Q_{12}Q_{22},$$

$$tc^F_{21}(Q_{21}, Q_{11}) = 1.5Q_{21}^2 + Q_{21} + 0.25Q_{11}Q_{21}, \quad tc^F_{22}(Q_{12}, Q_{22}) = Q_{22}^2 + 0.5Q_{22} + 0.25Q_{12}Q_{22}.$$

The transaction costs of the firms are:

$$c_{111}(Q^S_{111}) = 0.5Q^{S^2}_{111} + 0.25Q^S_{111}, \quad c_{112}(Q^S_{112}) = 0.25Q^{S^2}_{112} + 0.3Q^S_{112},$$

$$c_{211}(Q^S_{121}) = 0.3Q^{S^2}_{121} + 0.2Q^S_{121}, \quad c_{212}(Q^S_{122}) = 0.2Q^{S^2}_{122} + 0.1Q^S_{122}.$$

The demand price functions are:

$$\rho_{11}(d_{11}, d_{21}) = -1.5d_{11} - d_{21} + 500, \quad \rho_{12}(d_{12}, d_{22}) = -2d_{12} - d_{22} + 450,$$

$$\rho_{21}(d_{11}, d_{21}) = -2d_{21} - 0.5d_{11} + 500, \quad \rho_{22}(d_{12}, d_{22}) = -2d_{22} - d_{12} + 400.$$

The Euler method converges in 380 iterations. The equilibrium product shipments are:

$$Q^*_{11} = 13.39, \quad Q^*_{12} = 4.51, \quad Q^*_{21} = 18.62, \quad Q^*_{22} = 5.87.$$

The equilibrium demands are:

$$d^*_{11} = 13.39, \quad d^*_{12} = 4.51, \quad d^*_{21} = 18.62, \quad d^*_{22} = 5.87$$

with the induced demand prices being

$$\rho_{11} = 461.30, \quad \rho_{12} = 435.11, \quad \rho_{21} = 456.07, \quad \rho_{22} = 383.75.$$

The equilibrium in-house component production pattern is:

$$Q^{F^*}_{11} = 0.00, \quad Q^{F^*}_{12} = 11.50, \quad Q^{F^*}_{21} = 0.00, \quad Q^{F^*}_{22} = 14.35.$$

The equilibrium contracted component production pattern is:

$$Q^{S^*}_{111} = 35.78, \quad Q^{S^*}_{112} = 42.18, \quad Q^{S^*}_{121} = 48.99, \quad Q^{S^*}_{122} = 34.64.$$

The equilibrium Lagrange multipliers are:

$$\lambda^*_{11} = 81.82, \quad \lambda^*_{12} = 47.48, \quad \lambda^*_{21} = 88.58 \quad \lambda^*_{22} = 44.05.$$

The equilibrium prices charged by the supplier are:

$$\pi^*_{11} = 45.78, \quad \pi^*_{12} = 26.09, \quad \pi^*_{21} = 58.99, \quad \pi^*_{22} = 30.09.$$

The profits of the firms are, respectively, 2,518.77 and 3,485.51. The profit of the supplier is 3,529.19.

We now apply the supply chain network performance measures and the supplier and component importance indicators presented in Section 3 to this example.

The supply chain network performance measure $\mathcal{E}(G)$ for the whole competitive supply chain network (cf. (24)) for Example 1 is:

$$\begin{aligned}\mathcal{E}(G) =& \frac{\frac{d_{11}}{\rho_{11}}+\frac{d_{12}}{\rho_{12}}+\frac{d_{21}}{\rho_{21}}+\frac{d_{22}}{\rho_{22}}}{I \times n_R}\\ =& \frac{\frac{13.39}{461.30}+\frac{4.51}{435.11}+\frac{18.62}{456.07}+\frac{5.87}{383.75}}{2\times 2}\\ =& 0.0239.\end{aligned}$$

The supply chain network performance measure for the supply chain network topology of firm 1 (cf. (25)) is then given by:

$$\begin{aligned}\mathcal{E}_1(G_1) =& \frac{\frac{d_{11}}{\rho_{11}}+\frac{d_{12}}{\rho_{12}}}{n_R}\\ =& \frac{\frac{13.39}{461.30}+\frac{4.51}{435.11}}{2}\\ =& 0.0197,\end{aligned}$$

and that of firm 2 is:

$$\begin{aligned}\mathcal{E}_2(G_2) =& \frac{\frac{d_{21}}{\rho_{21}}+\frac{d_{22}}{\rho_{22}}}{n_R}\\ =& \frac{\frac{18.62}{456.07}+\frac{5.87}{383.75}}{2}\\ =& 0.0281.\end{aligned}$$

Note that, in this example, only supplier 1 is able to produce components 1^1 and 1^2, which is the first component of supplier 1 (i.e., node 1 in the second tier in Figure 2), and neither of the firms can. Without supplier 1, no products of the firms can be assembled or delivered to the demand markets. Therefore,

$$\mathcal{E}(G-1)=0, \quad \mathcal{E}_1(G_1-1)=0, \quad \mathcal{E}_2(G_2-1)=0.$$

According to (26) and (28),

$$I(1)=1, \quad I_1(1)=1, \quad I_2(1)=1,$$

that is, the importance of supplier 1 for the whole competitive supply chain network, for the supply chain of firm 1, and that for the supply chain of firm 2 is 1. Without supplier 1, the supply chain network in Figure 2 will collapse.

In addition, the supply chain network performance for the supply chain without supplier 1's component 1 (i.e., node 1 in the second tier in Figure 2) is:

$$\mathcal{E}(G-1_1)=0, \quad \mathcal{E}_1(G_1-1_1)=0, \quad \mathcal{E}_2(G_2-1_1)=0,$$

and the importance of supplier 1's component 1 is:

$$I(1_1)=1, \quad I_1(1_1)=1, \quad I_2(1_1)=1.$$

Therefore, supplier 1's component 1 is the most important component compared to its components 2 (i.e., node 2 in the second tier in Figure 2) and 3 (i.e., node 3 in the second tier in Figure 2).

Now suppose that supplier 1's component 2 is removed from Figure 2. The Euler method converges in 992 iterations, and achieves the equilibrium solution shown in Table 2.

Table 2: Equilibrium Solution and Incurred Demand Prices After the Removal of Supplier 1's Component 2

Q^*	$Q^*_{11}=6.49$	$Q^*_{121}=0.17$	$Q^*_{21}=19.08$	$Q^*_{22}=6.46$
Q^{F^*}	$Q^{F^*}_{11}=0.00$	$Q^{F^*}_{12}=20.00$	$Q^{F^*}_{21}=0.00$	$Q^{F^*}_{22}=14.90$
Q^{S^*}	$Q^{S^*}_{111}=13.33$	$Q^{S^*}_{121}=51.08$	$Q^{S^*}_{122}=36.18$	
λ^*	$\lambda^*_{11}=36.92$	$\lambda^*_{12}=103.29$	$\lambda^*_{21}=91.93$	$\lambda^*_{22}=45.70$
π^*	$\pi^*_{111}=23.33$	$\pi^*_{121}=61.08$	$\pi^*_{122}=31.12$	
d^*	$d^*_{11}=6.49$	$d^*_{12}=0.17$	$d^*_{21}=19.08$	$d^*_{22}=6.46$
ρ	$\rho_{11}=471.18$	$\rho_{12}=443.19$	$\rho_{21}=458.59$	$\rho_{22}=386.91$

The profits of the firms are now 1,519.08 and 3,755.89. The profit of the supplier is 2,458.92.

The associated supply chain network performance measure values are now:

$$\mathcal{E}(G-2_1)=0.0181, \quad \mathcal{E}_1(G_1-2_1)=0.0071, \quad \mathcal{E}_2(G_2-2_1)=0.0292.$$

After supplier 1's component 3 is removed from Figure 2, in 1487 iterations, the Euler method converges to the equilibrium solution, which is presented in Table 3.

The profits of the firms are 2,724.82 and 3,043.42, and the profit of the supplier is 2,177.26.

Table 3: Equilibrium Solution and Incurred Demand Prices After the Removal of Supplier 1's Component 3

Q^*	$Q^*_{11} = 13.75$	$Q^*_{121} = 4.88$	$Q^*_{21} = 14.25$	$Q^*_{22} = 0.75$
Q^{F^*}	$Q^{F^*}_{11} = 0.00$	$Q^{F^*}_{12} = 11.94$	$Q^{F^*}_{21} = 0.00$	$Q^{F^*}_{22} = 30.00$
Q^{S^*}	$Q^{S^*}_{111} = 37.26$	$Q^{S^*}_{112} = 43.96$	$Q^{S^*}_{121} = 30.00$	
λ^*	$\lambda^*_{11} = 84.78$	$\lambda^*_{12} = 49.26$	$\lambda^*_{21} = 58.20$	$\lambda^*_{22} = 103.44$
π^*	$\pi^*_{111} = 47.26$	$\pi^*_{112} = 26.98$	$\pi^*_{121} = 40.00$	
d^*	$d^*_{11} = 13.75$	$d^*_{12} = 4.88$	$d^*_{21} = 14.25$	$d^*_{22} = 0.75$
ρ	$\rho_{11} = 465.12$	$\rho_{12} = 439.50$	$\rho_{21} = 464.62$	$\rho_{22} = 393.63$

The associated supply chain network performance measure values are now:

$$\mathcal{E}(G - 3_1) = 0.0183, \quad \mathcal{E}_1(G_1 - 3_1) = 0.0203, \quad \mathcal{E}_2(G_2 - 3_1) = 0.0163.$$

We summarize the supply chain network performance measure values in Table 4. The importance of supplier 1's components 1, 2, and 3 (cf. (30) and (32)) and their rankings, not only for the whole supply chain network but also for each firm's supply chain, are reported in Table 5.

Table 4: Supply Chain Network Performance Measure values for Example 1

	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-1_1)$	$\mathcal{E}(G-2_1)$	$\mathcal{E}(G-3_1)$
Whole Supply Chain	0.0239	0	0	0.0181	0.0183
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-1_1)$	$\mathcal{E}_i(G_i-2_1)$	$\mathcal{E}_i(G_i-3_1)$
Firm 1's Supply Chain	0.0197	0	0	0.0071	0.0203
Firm 2's Supply Chain	0.0281	0	0	0.0292	0.0163

Because supplier 1's component 2 is produced exclusively for firm 1, it is more important for firm 1 than supplier 1's component 3, but not as important as component 1. After removing it from the supply chain, firm 1's profit decreases, but firm 1's competitor, firm 2's profit, increases because of competition. The supply chain performance of firm 2's supply chain also increases after the removal. In addition, component 2 is most important for firm 1 than for firm 2 and for the whole supply chain network.

For a similar reason, since supplier 1's component 3 is made exclusively for firm 2, it is more important than supplier 1's component 2 for firm 2. After dropping component 3 from the supply chain, firm 2's profit decreases, and its competitor, firm 1's profit, increases.

Table 5: Importance and Rankings of Supplier 1's Components 1, 2, and 3 for Example 1

	Importance for the Whole Supply Chain	Ranking	Importance for Firm 1's Supply Chain	Ranking	Importance for Firm 2's Supply Chain	Ranking
Supplier 1	1		1		1	
Component 1	1	1	1	1	1	1
Component 2	0.2412	2	0.6401	2	−0.0387	3
Component 3	0.2331	3	−0.0329	3	0.4197	2

	Importance for the Whole Supply Chain	Importance for Firm 1's Supply Chain	Importance for Firm 2's Supply Chain
Supplier 1	1	1	1
Ranking	1	1	1
Component 1	1	1	1
Ranking	1	1	1
Component 2	0.2412	0.6401	−0.0387
Ranking	2	1	3
Component 3	0.2331	−0.0329	0.4197
Ranking	2	3	1

The supply chain performance of firm 1's supply chain also increases. Component 3 is most important for firm 2 than for firm 1 and for the whole supply chain.

Example 2

Example 2 is the same as Example 1 except that supplier 1 is no longer the only entity that can produce components 1^1 and 1^2. Both firms recover their capacities for producing components 1^1 and 1^2 and, hence, they are raised from 0 to 20. The capacities of the firms are now:

$$CAP^F_{11} = 20, \quad CAP^F_{12} = 20, \quad CAP^F_{21} = 20, \quad CAP^F_{22} = 30.$$

The Euler method converges in 408 iterations. The equilibrium solution is presented in Table 6.

Table 6: Equilibrium Solution and Incurred Demand Prices for Example 2

Q^*	$Q^*_{11} = 14.43$	$Q^*_{121} = 5.13$	$Q^*_{21} = 19.60$	$Q^*_{22} = 7.02$
Q^{F^*}	$Q^{F^*}_{11} = 10.23$	$Q^{F^*}_{12} = 12.50$	$Q^{F^*}_{21} = 11.28$	$Q^{F^*}_{22} = 15.47$
Q^{S^*}	$Q^{S^*}_{111} = 28.89$	$Q^{S^*}_{112} = 46.19$	$Q^{S^*}_{121} = 41.97$	$Q^{S^*}_{122} = 37.78$
λ^*	$\lambda^*_{11} = 68.04$	$\lambda^*_{12} = 51.49$	$\lambda^*_{21} = 77.35$	$\lambda^*_{22} = 47.40$
π^*	$\pi^*_{111} = 38.89$	$\pi^*_{112} = 28.10$	$\pi^*_{121} = 51.97$	$\pi^*_{122} = 32.19$
d^*	$d^*_{11} = 14.43$	$d^*_{12} = 5.13$	$d^*_{21} = 19.60$	$d^*_{22} = 7.02$
ρ	$\rho_{11} = 458.75$	$\rho_{12} = 432.72$	$\rho_{21} = 453.58$	$\rho_{22} = 380.83$

The profits of the firms are now 2,968.88 and 4,110.89, and the profit of the supplier is now 3,078.45. With recovered capacities, the profits of the firms increase, but that of the supplier decreases, compared to the corresponding values in Example 1. If there are costs for capacity increment for each firm, and if the costs are less than the associated profit increment, it is profitable for firms to recover their capacities and produce more components in-house. If not, purchasing from the supplier would be a wise choice. In Example 2, the demand prices decrease due to more demand.

The supply chain network performance measure values and the importance of supplier 1's components 1, 2, and 3 and their rankings are reported as in Tables 7 and 8.

Table 7: Supply Chain Network Performance Measure Values for Example 2

	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-1_1)$	$\mathcal{E}(G-2_1)$	$\mathcal{E}(G-3_1)$
Whole Supply Chain	0.0262	0.0086	0.0105	0.0197	0.0195
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-1_1)$	$\mathcal{E}_i(G_i-2_1)$	$\mathcal{E}_i(G_i-3_1)$
Firm 1's Supply Chain	0.0217	0.0067	0.0106	0.0071	0.0226
Firm 2's Supply Chain	0.0308	0.0105	0.0105	0.0324	0.0163

Table 8: Importance and Rankings of Supplier 1 and its Components 1, 2, and 3 for Example 2

	Importance for the Whole Supply Chain	Ranking	Importance for Firm 1's Supply Chain	Ranking	Importance for Firm 2's Supply Chain	Ranking
Supplier 1	0.6721		0.6897		0.6598	
Component 1	0.5984	1	0.5121	2	0.6590	1
Component 2	0.2476	3	0.6721	1	−0.0505	3
Component 3	0.2586	2	−0.0438	3	0.4710	2

	Importance for the Whole Supply Chain	Importance for Firm 1's Supply Chain	Importance for Firm 2's Supply Chain
Supplier 1	0.6721	0.6897	0.6598
Ranking	2	1	3
Component 1	0.5984	0.5121	0.6590
Ranking	2	3	1
Component 2	0.2476	0.6721	−0.0505
Ranking	2	1	3
Component 3	0.2586	−0.0438	0.4710
Ranking	2	3	1

With firms' recovered capacities for producing components 1^1 and 1^2, supplier 1's component 1 is still the most important component for the whole supply chain network and for firm 2, compared to the other components. However, for firm 1's supply chain, component 2 is now the most important component.

In addition, supplier 1 is now most important for firm 1. Therefore, in the case of a disruption on the supplier's side, firm 1's supply chain will be affected the most. Moreover, components 1 and 3 are most important for firm 2, and component 2 is most important for firm 1.

Example 3

Example 3 is the same as Example 2, except that two more suppliers are now available to the firms in addition to supplier 1. The supply chain network topology of Example 3 is given in Figure 3.

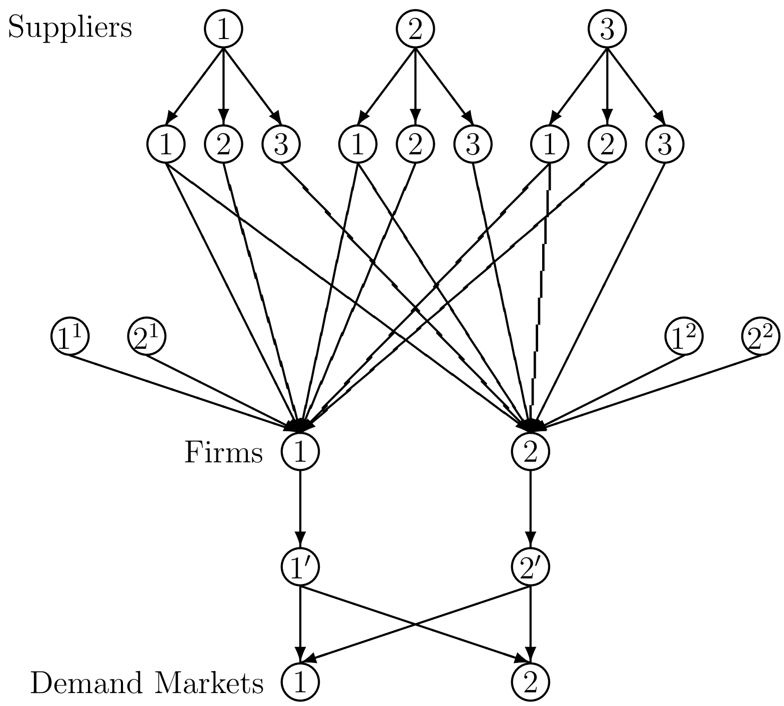

Figure 3: Example 3

The data associated with suppliers 2 and 3 are following.

The capacities of suppliers 2 and 3 are:

$$CAP_{211}^S = 60, \quad CAP_{212}^S = 70, \quad CAP_{221}^S = 50, \quad CAP_{222}^S = 60,$$

$$CAP_{311}^S = 50, \quad CAP_{312}^S = 80, \quad CAP_{321}^S = 80, \quad CAP_{322}^S = 60.$$

Hence, suppliers 2 and 3 are capable of providing components 1^1, 2^1, 1^2, and 2^2 for the firms.

The production costs of the suppliers are:

$$f_{21}^S(Q_{211}^S, Q_{221}^S) = Q_{211}^S + Q_{221}^S, \quad f_{22}^S(Q_{212}^S) = 3Q_{212}^S, \quad f_{23}^S(Q_{222}^S) = 2Q_{222}^S,$$

$$f_{31}^S(Q_{311}^S, Q_{321}^S) = 10(Q_{311}^S + Q_{321}^S), \quad f_{32}^S(Q_{312}^S) = Q_{312}^S, \quad f_{33}^S(Q_{322}^S) = 2.5Q_{322}^S.$$

The transportation costs are:

$$tc_{211}^S(Q_{211}^S, Q_{212}^S) = 0.5Q_{211}^S + 0.2Q_{212}^S, \quad tc_{212}^S(Q_{212}^S, Q_{211}^S) = 0.3Q_{212}^S + 0.1Q_{211}^S,$$

$$tc_{221}^S(Q_{221}^S, Q_{222}^S) = 0.8Q_{221}^S + 0.2Q_{222}^S, \quad tc_{222}^S(Q_{222}^S, Q_{221}^S) = 0.75Q_{222}^S + 0.1Q_{221}^S,$$

$$tc_{311}^S(Q_{311}^S, Q_{312}^S) = 0.4Q_{311}^S + 0.05Q_{312}^S, \quad tc_{312}^S(Q_{312}^S, Q_{311}^S) = 0.4Q_{312}^S + 0.2Q_{311}^S,$$

$$tc_{321}^S(Q_{321}^S, Q_{322}^S) = 0.7Q_{321}^S + 0.1Q_{322}^S, \quad tc_{322}^S(Q_{322}^S, Q_{321}^S) = 0.6Q_{322}^S + 0.1Q_{321}^S.$$

The opportunity costs are:

$$oc_2(\pi_{211}, \pi_{212}, \pi_{221}, \pi_{222}) = (\pi_{211} - 6)^2 + 0.75(\pi_{212} - 5)^2 + 0.3(\pi_{221} - 8)^2 + 0.5(\pi_{222} - 4)^2,$$

$$oc_3(\pi_{311}, \pi_{312}, \pi_{321}, \pi_{322}) = 0.5(\pi_{311} - 5)^2 + 1.5(\pi_{312} - 5)^2 + 0.5(\pi_{321} - 3)^2 + 0.5(\pi_{322} - 4)^2.$$

The transaction costs of the firms now become:

$$c_{121}(Q_{211}^S) = 0.5Q_{211}^{S^2} + Q_{211}^S, \quad c_{122}(Q_{212}^S) = 0.25Q_{212}^{S^2} + 0.3Q_{212}^S,$$

$$c_{221}(Q_{221}^S) = Q_{221}^{S^2} + 0.1Q_{221}^S, \quad c_{222}(Q_{222}^S) = Q_{222}^{S^2} + 0.5Q_{222}^S,$$

$$c_{131}(Q_{311}^S) = 0.2Q_{311}^{S^2} + 0.3Q_{311}^S, \quad c_{132}(Q_{312}^S) = 0.5Q_{312}^{S^2} + 0.2Q_{312}^S,$$

$$c_{231}(Q_{321}^S) = 0.1Q_{321}^{S^2} + 0.1Q_{321}^S, \quad c_{232}(Q_{322}^S) = 0.5Q_{322}^{S^2} + 0.1Q_{322}^S.$$

The rest of the data for firms 1 and 2 and the demand price functions are the same as in Example 2.

The Euler method converges in 563 iterations and achieves the equilibrium solution shown in Table 9.

The profits of the firms are now 4,968.67 and 5,758.13, and the profits of the suppliers are 1,375.22, 725.17, and 837.44, respectively. With more competition on the supplier's side, the prices of supplier 1 decrease, and its profit also decreases, compared to the values in Example 2. However, the profits of the firms increase. In addition, with more products made, the prices at the demand markets decrease.

The supply chain network performance measure values and the importance of the suppliers are reported in Tables 10 and 11.

Table 9: Equilibrium Solution and Incurred Demand Prices for Example 3

Q^*	$Q^*_{11} = 21.82$	$Q^*_{12} = 9.61$	$Q^*_{21} = 24.23$	$Q^*_{22} = 12.41$
Q^{F^*}	$Q^{F^*}_{11} = 5.57$	$Q^{F^*}_{12} = 9.11$	$Q^{F^*}_{21} = 6.48$	$Q^{F^*}_{22} = 12.94$
Q^{S^*}	$Q^{S^*}_{111} = 13.71$	$Q^{S^*}_{112} = 32.64$	$Q^{S^*}_{121} = 21.77$	$Q^{S^*}_{122} = 30.68$
	$Q^{S^*}_{211} = 20.45$	$Q^{S^*}_{212} = 27.98$	$Q^{S^*}_{221} = 10.07$	$Q^{S^*}_{222} = 11.78$
	$Q^{S^*}_{311} = 23.13$	$Q^{S^*}_{312} = 24.56$	$Q^{S^*}_{321} = 34.94$	$Q^{S^*}_{322} = 17.86$
λ^*	$\lambda^*_{11} = 37.68$	$\lambda^*_{12} = 37.94$	$\lambda^*_{21} = 45.03$	$\lambda^*_{22} = 39.83$
π^*	$\pi^*_{111} = 23.71$	$\pi^*_{112} = 21.32$	$\pi^*_{121} = 31.77$	$\pi^*_{122} = 27.45$
	$\pi^*_{211} = 16.23$	$\pi^*_{212} = 23.65$	$\pi^*_{221} = 24.79$	$\pi^*_{222} = 15.78$
	$\pi^*_{311} = 28.13$	$\pi^*_{312} = 13.19$	$\pi^*_{321} = 37.94$	$\pi^*_{322} = 21.86$
d^*	$d^*_{11} = 21.82$	$d^*_{12} = 9.61$	$d^*_{21} = 24.23$	$d^*_{22} = 12.41$
ρ	$\rho_{11} = 443.04$	$\rho_{12} = 418.38$	$\rho_{21} = 440.64$	$\rho_{22} = 365.58$

Table 10: Supply Chain Network Performance Measure Values for Example 3

	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-2)$	$\mathcal{E}(G-3)$	$\mathcal{E}(G-\sum_{j=1}^{n_S} j)$
Whole Supply Chain	0.0403	0.0334	0.0361	0.0332	0.0086
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-2)$	$\mathcal{E}_i(G_i-3)$	$\mathcal{E}_i(G_i-\sum_{j=1}^{n_S} j)$
Firm 1's Supply Chain	0.0361	0.0309	0.0303	0.0309	0.0067
Firm 2's Supply Chain	0.0445	0.0358	0.0419	0.0355	0.0105

Table 11: Importance and Rankings of Suppliers for Example 3

	Importance for the Whole Supply Chain	Ranking	Importance for Firm 1's Supply Chain	Ranking	Importance for Firm 2's Supply Chain	Ranking
Supplier 1	0.1717	2	0.1443	2	0.1939	2
Supplier 2	0.1035	3	0.1612	1	0.0566	3
Supplier 3	0.1760	1	0.1438	3	0.2021	1
All Suppliers	0.7864		0.8139		0.7641	

	Importance for the Whole Supply Chain	Importance for Firm 1's Supply Chain	Importance for Firm 2's Supply Chain
Supplier 1	0.1717	0.1443	0.1939
Ranking	2	3	1
Supplier 2	0.1035	0.1612	0.0566
Ranking	2	1	3
Supplier 3	0.1760	0.1438	0.2021
Ranking	2	3	1
All Suppliers	0.7864	0.8139	0.7641
Ranking	2	1	3

As shown in Table 11, supplier 2 is the most important supplier for firm 1's supply chain, and supplier 3 is the most important supplier for firm 2 and the whole supply chain network, compared to the other suppliers. In addition, suppliers 1 and 3 are most important for firm 2. Supplier 2 is most important for firm 1's supply chain.

The group of suppliers, including suppliers 1, 2, and 3, is most important for firm 1. If a major disaster occurs and all the suppliers are unavailable to the firms, firm 1's supply chain will be affected the most.

6. Summary and Conclusions

Supply chains provide the critical infrastructure for the production and distribution of products around the globe. In the case of many products from simple ones to high tech ones, components that comprise the product are produced by suppliers and then assembled by firms. Hence, the behavior of both suppliers and firms needs to be captured in order to be able to assess both supply chain network performance as well as vulnerabilities.

In this paper, we propose a new multitiered model consisting of competing firms, who can procure components for their products, which are represented by brands, from suppliers or can make them, as appropriate, in-house. The firms compete in terms of quantities whereas the suppliers in terms of prices charged for the components. The optimizing behavior of the decision-makers is captured and a unified variational inequality constructed, whose solution yields the equilibrium quantities of the components, produced in-house and/or contracted for, the prices charged by the suppliers, as well as the Lagrange multipliers associated with the capacities. Qualitative properties of the solution are also discussed.

The model is then used as the setting for the introduction of supply chain network performance measures for the entire supply chain network economy consisting of all the firms as well as for that of an individual firm. Importance indicators are then constructed that allow for the ranking of suppliers for the whole supply chain or that of an individual firm, as well as for the supplier components. This rigorous methodology can be used for planning purposes as well as for investment purposes. Moreover, it can be utilized as a tool for regulators since information about both individual firms as well as the entire supply chain network is revealed.

The model as well as the performance measures are then illustrated through a series of examples, the solutions of which, are computed using a proposed algorithm, with nice features for implementation.

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References

1. Bertrand, J.: Theorie mathematique de la richesse sociale. Journal des Savants, September, 499-508 (1883)

2. Bertsekas, D.P., Tsitsiklis, J.N.: Parallel and Distributed Computation - Numerical Methods. Prentice Hall, Englewood Cliffs, New Jersey (1989)

3. Chen, Z., Teng, C., Zhang, D., Sun, J. (2014). Modelling inter-supply chain competition with resource limitation and demand disruption. International Journal of Systems Science DOI: 10.1080/00207721.2014.942499

4. Chopra, S., Sodhi, M.S.: Managing risk to avoid supply-chain breakdown. MIT Sloan Management Review **46**(1), 53-61 (2004)

5. Clendon, L., Bird, L.: 5th annual survey supply chain resilience. Business Continuity Institute, Caversham, United Kingdom (2013)

6. Cournot, A.A. Researches into the Mathematical Principles of the Theory of Wealth, English translation. MacMillan, London, England (1838)

7. Cruz, J.M.: Dynamics of supply chain networks with corporate social responsibility through integrated environmental decision-making. European Journal of Operational Research **184**, 1005-1031 (2008)

8. Cruz, J.M.: The effects of network relationships on global supply chain vulnerability. In: Wu, T., Blackhurst, J. (eds.), Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers. Springer, Berlin, Germany, pp 113-140 (2011)

9. Cruz, J.M., Nagurney, A., Wakolbinger, T.: Financial engineering of the integration of global supply chain networks and social networks with risk management. Naval Research Logistics **53**, 674-696 (2006)

10. Dupuis, P., Nagurney, A.: Dynamical systems and variational inequalities. Annals of

Operations Research **44**, 9-42 (1993)

11. Gabay, D., Moulin, H.: On the uniqueness and stability of Nash equilibria in noncooperative games. In: Bensoussan, A., Kleindorfer, P., Tapiero, C.S. (eds.), Applied Stochastic Control in Econometrics and Management Science. North-Holland, Amsterdam, The Netherlands, pp 271-294 (1980)

12. Gupta, D.: The (Q, r) inventory system with an unreliable supplier. INFOR **34**, 59-76 (1996)

13. Kinderlehrer, D., Stampacchia, G.: An Introduction to Variational Inequalities and Their Applications. Academic Press, New York (1980)

14. Lee, D., Pierson, D.: Disaster in Japan exposes supply chain flaw. Los Angeles Times, April 6 (2011)

15. Liu, Z., Cruz, J.M.: Supply chain networks with corporate financial risks and trade credits under economic uncertainty. International Journal of Production Economics **137**, 55-67 (2012)

16. Liu, Z., Nagurney, A.: An integrated electric power supply chain and fuel market network framework: Theoretical modeling with empirical analysis for New England. Naval Research Logistics **56**, 600-624 (2006)

17. Liu, Z., Nagurney, A.: Supply chain networks with global outsourcing and quick-response production under demand and cost uncertainty. Annals of Operations Research **208**(1), 251-289 (2013)

18. Mukherjee, A.S.: The Spider's Strategy: Creating Networks to Avert Crisis, Create Change, and Get Ahead. FT Press, Upper Saddle River, New Jersey (2008)

19. Nagurney, A.: Network Economics: A Variational Inequality Approach. Kluwer Academic Publishers, Dordrecht, The Netherlands (1999)

20. Nagurney, A.: Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits. Edward Elgar Publishing, Cheltenham, England (2006)

21. Nagurney, A.: Formulation and analysis of horizontal mergers among oligopolistic firms with insights into the merger paradox: A supply chain network perspective. Computational Management Science **7**, 377-410 (2010)

22. Nagurney, A., Cruz, J., Dong, J., Zhang, D.: Supply chain networks, electronic commerce, and supply side and demand side risk. European Journal of Operational Research **26**, 120-142 (2005)

23. Nagurney, A., Dhanda, K.K.: Marketable pollution permits in oligopolistic markets with transaction costs. Operations Research **4**8(3), 424-435 (2000)

24. Nagurney, A., Dong, J., Zhang, D.: A supply chain network equilibrium model. Transportation Research E **38**, 281-303 (2002)

25. Nagurney, A., Dupuis, P., Zhang, D.: A dynamical systems approach for network oligopolies and variational inequalities. Annals of Regional Science **28**, 263-283 (1994)

26. Nagurney, A., Li, D.: A supply chain network game theory model with product differentiation, outsourcing of production and distribution, and quality and price competition. Annals of Operations Research **228**(1), 479-503 (2015)

27. Nagurney, A., Li, D., Nagurney, L.S.: Pharmaceutical supply chain networks with outsourcing under price and quality competition. *International Transactions in Operational Research* **20**(6), 859-888 (2013)

28. Nagurney, A., Qiang, Q.: A network efficiency measure with application to critical infrastructure networks. Journal of Global Optimization **40**, 261-275 (2008)

29. Nagurney, A., Qiang, Q.: Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World. John Wiley & Sons, Hoboken, New Jersey (2009)

30. Nagurney, A., Zhang, D.: Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts (1996)

31. Nash, J.F.: Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, USA **36**, 48-49 (1950)

32. Nash, J.F.: Noncooperative games. Annals of Mathematics **54**, 286-298 (1951)

33. Parlar, M.: Continuous review inventory problem with random supply interruptions. European Journal of Operations Research **99**, 366-385 (1997)

34. Parlar, M., Perry, D.: Inventory models of future supply uncertainty with single and multiple suppliers. *Naval Research Logistics* **43**, 191-210 (1996)

35. Payne, J.W., Bettman, J.R., Luce, M.F.: When time is money: Decision behavior under opportunity-cost time pressure. Organizational Behavior and Human Decision Processes **66**(2), 131-152 (1996)

36. Qiang, Q.: The closed-loop supply chain network with competition and design for remanufactureability. Journal of Cleaner Production **105**, 346-356 (2015)

37. Qiang, Q., Ke, K., Anderson, T., Dong, J.: The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. Omega **41**(2), 186-194 (2013)

38. Qiang, Q., Nagurney, A.: A bi-criteria indicator to assess supply chain network performance for critical needs under capacity and demand disruptions. *Transportation Research A* **46**(5), 801-812 (2012)

39. Qiang, Q., Nagurney, A., Dong, J.: Modeling of supply chain risk under disruptions with performance measurement and robustness analysis. In: Wu, T., Blackhurst, J. (eds.), Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers. Springer, Berlin, Germany, pp 91-111 (2009)

40. Sandoval-Chavez, D.A., Beruvides, M.G.: Using opportunity costs to determine the cost of quality: A case study in a continuous-process industry. The Engineering Economist **43**, 107-124 (1998)

41. Stevenson, A., Lindberg, C.A., (eds). New Oxford American Dictionary, 3rd edition, Oxford University Press, England (2010)

42. Tang, C.S., Zimmerman, J.D., Nelson, J.I.: Managing new product development and supply chain risks: The Boeing 787 case. Supply Chain Forum **10**(2), 74-86 (2009)

43. Tomlin, B.T.: On the value of mitigation and contingency strategies for managing supply chain disruption risks. Management Science **52**, 639-657 (2006)

44. Toyasaki, F., Daniele, P., Wakolbinger, T.: A variational inequality formulation of equilibrium models for end-of-life products with nonlinear constraints. European Journal of Operational Research **236**, 340-350 (2014)

45. Wu, T., Blackhurst, J. (eds.): Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers. Springer, Berlin, Germany (2009)

46. Wu, T., Blackhurst, J., Chidambaram, V.: The model for inbound supply risk analysis. Computers in Industry **57**, 350-365 (2006)

47. Yang, J.: Worst Thai floods in 50 years hit Apple, Toyota supply chain. Bloomberg October 21 (2011)