A Network Economic Game Theory Model of a Service-Oriented Internet with Price and Quality Competition in Both Content and Network Provision

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Abstract. One key challenge in the current Internet is the inefficiency of the mechanisms by which technology is deployed and the business and economic models surrounding these processes. Customers' demands are driving the Internet and telecommunication networks towards the provision of quality-based end-to-end services, which need a richer family of performance guarantees. We believe that novel insights into future Internet structures can be obtained from taking into account the associated economic models and equilibrium conditions among providers. This paper develops both a basic and a general network economic game theory model of a quality-based service-oriented Internet to study the competition among the service providers (both content and network ones). We derive the governing equilibrium conditions and provide the equivalent variational inequality (VI) formulations. An algorithm is proposed, which yields closed-form expressions, at each iteration, for the prices and quality levels. In order to illustrate the modeling framework and the algorithm, we present computed solutions to numerical examples. The results show the generality of the proposed network economic model for a future Internet.

Key words: oligopolies, service-oriented Internet, price competition, quality competition, Bertand-Nash equilibrium, network economics, game theory, variational inequalities

1. Introduction

The current Internet has enabled numerous distributed applications and services. However, providers generally face many challenges in determining technical and economic solutions to providing services (see Wolf et al. (2012)). Key challenges are how to price and bill these services and how to establish economic relationships with other providers that are necessary to provide end-to-end services. Equilibrium models for the Internet generally assume basic economic relationships and consider price as the only factor that affects users' demand (see Laffont et al. (2003), Zhang et al. (2010), and Caron, Kesidis, and Altman (2010)). However, in new paradigms for the Internet and even in the case of supply chain networks, price is not the only factor and Quality of Service (QoS), i.e., the ability to provide different priorities to applications, users, or data flows, comes into play (see Hu and Qiang (2013), Nagurney et al. (2013), Nagurney and Li (2013), Nagurney, Li, and Nagurney (2013)).

Emerging technologies and applications have pushed the capabilities required of the Internet beyond what the current infrastructure can provide. To address these limitations, the networking research community has taken up the task of designing new architectures for the future Internet, with accompanying proper economic pricing mechanisms in order to make them manageable (see Wolf et al. (2012)). The future Internet needs to live up to the diversified requirements of next-generation applications and new users' requirements comprising mobility, security, and flexibility. Zhang et al. (2010) point out that economic relationships are far more mysterious than the underlying technology, as the business relationships that give rise to observed connections are mostly hidden from view. Our knowledge drops even further when we face services offered over a new paradigm with the ability to create new functionality that lets users choose winners and losers. In fact, economic complexity in designing the Next Generation Internet (NGI) is advancing the role of pricing models (see Jain, Durresi, and Paul (2011) and Wolf et al. (2012)).

The NGI is expected to be service-oriented with each provider offering specific services. In the Internet of services with comparable functionalities, but varying quality levels, services are available at different costs in the service marketplace, so that users can decide which services from which service provider to select (Wolf et al. (2012)). NGI, typically, includes multi-tier service providers. For example, a content service provider (CSP) is a website that handles the distribution of online content such as blogs, videos, music or files, whereas a network service provider (NSP) is an entity that provides network access or long-haul network transport. These offer equal or rather similar services at different QoS levels and different costs. Hereafter, please note that we use "Content Provider (CP)" instead of content service provider, and "Network Provider (NP)" in place of network service provider, for simplicity, and the fact that any provider offers a service, which can be either a content or a network service.

Various economic models have been studied for the future Internet. Zhang et al. (2010) proposed a two-stage Stackelberg game with Cournot and Bertrand competition. The price of a service offered by the content provider is determined as a function of the user's demand and the network access price. The network providers charge CPs by maximizing their profit as a function of market share and the CPs' marginal cost. Nagurney et al. (2013) modeled a service-oriented network using variational inequality theory in an oligopoly market of NPs and CPs. The proposed model, when solved, yields the service volumes and the quality level Nash (1950, 1951) equilibrium for services offered by the content providers. Nagurney and Wolf (2013) developed a game theory model of a service-oriented Internet in which profit-maximizing service providers provide substitutable (but not identical) services and compete with the quantities of services in a Cournot-Nash manner, whereas the network transport providers, which transport the services to the users at the demand markets, and are also profit-maximizers, compete with prices in Bertrand fashion and on quality.

Another body of work has studied two-sided payment effects in the future Internet and evaluated neutral vs. non-neutral networks in a market in which the network providers collect a fee from both the users and the content providers. Laffont et al. (2003) modeled Bertrand competition between NPs in a network with two network providers, multi-content providers, and heterogeneous users. A new pricing mechanism "off-net cost pricing principle" was proposed to find the optimum price to charge users and content providers. They analyzed the impact of an access charge on welfare and profit. The outcomes showed that the access charge determines the allocation of communication costs and affects the level of traffic. Hermalin and Katz (2007) modeled the simultaneous choice of network providers for charging households and content providers, when the NP is able to offer several levels vs. one level of service quality. They concluded that restricting the network provider to supply one level of quality has more negative outcomes. Musacchio, Schwartz, and Walrand (2011) investigated a two-sided market where CPs and NPs invest jointly in the network infrastructure and share the revenue. Users' demand is determined as a function of product of CPs' and NPs' investment (can be assumed as their quality) and decreases exponentially if the price goes up. Economides and Tag (2012) also investigated what price network providers should charge users and content providers in order to maximize profits. Their analysis showed that the NP and the users are better-off while the CPs and the social surplus are always worse-off under network freedom (a non-neutral network).

In this paper, we focus on the development of game theory models in equilibrium settings. We consider an Internet with a service-oriented architecture, in which content and network providers interact and compete in prices and quality of services. Our methodology is inspired, in the first part, by Altman, Legout, and Xu (2011) and, in the second part, by El Azouzi, Altman, and Wynter (2003). Altman, Legout, and Xu (2011) studied the effect of side payments, while taking into account the different levels of quality offered by a network provider in the Internet with one CP, one NP, and one market of users. Our basic model completes their model by including the quality of both providers into the demand function and assuming a production cost function for the content provider. El Azouzi, Altman, and Wynter (2003) modeled an oligopoly market of content providers and one network provider in a bi-criteria Nash equilibrium competition between content providers. Their model restricts the network to one network provider and quality for only the content providers' service. Therefore, it cannot reveal the competition among the network providers for users. Our model overcomes these limitations by including multiple providers, multiple users (demand markets), and demands as a function of the prices and quality levels of all providers. On top of that, our proposed model presents a general framework for modeling alternative cost functions and demand functions associated with the services and the demand markets.

Our contributions to the literature are:

• We include quality levels, in addition to prices, for both network and content providers, as they engage in competition for users at the demand markets.

• Consumers have more choices in that they can select network and content providers.

• We handle heterogeneity in the providers' cost functions and in the users' demands and do not limit ourselves to linear demand functions.

• We provide a natural underlying set of adjustment processes until the equilibrium; equivalently, the stationary point, is achieved.

• The theoretical framework is supported by a rigorous algorithm that is well-suited for implementation.

• We perform sensitivity analysis in order to investigate the impact of the transfer prices on the providers' prices, quality levels, and their utilities, which reflect their profits.

The structure of this paper is as follows. In Section 2, a basic model of a service-oriented Internet and its analysis are presented. A game theory model of service providers (CPs and NPs) is then constructed and analyzed in Section 3 to show the competitive behavior of content and network providers in prices and quality of services and their interactions with the users at the demand markets. This model extends the work of Nagurney et al. (2013) and Nagurney and Wolf (2013) in that quality of both content and of network provision is captured. In addition, we allow for side payments and utilize direct demand functions (rather than their inverses). We demonstrate that the Nash equilibrium conditions are equivalent to the solution of variational inequality problems. We then present, in Section 4, continuous-time adjustment processes for the providers as a projected dynamical system (Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)), along with an algorithm, which provides a time discretization of the continuous-time adjustment processes. The algorithm is the Euler method and yields closed form expressions for the price and quality of each provider. It is the applied to compute solutions to several examples in Section 5, accompanied by sensitivity analysis, in order to provide insights into the network economics. We summarize and present our conclusions in Section 6.

2. The Basic Model

In this section, a basic model is presented for illustration purposes. Figure 1 shows the structure of the content flows and Figure 2 depicts the structure of the financial payments in a basic (preliminary) model of a quality-based service-oriented Internet, which consists of a single content provider, CP_1, a single network provider, NP_1, and one demand market (user) u_1. For simplicity, a user refers to a market of users.

Figure 1

The network provider and the content provider determine the equilibrium price and quality for their services offered to the user. According to Figure 2, the network provider charges the user a price p_{s_1} for transferring a unit of content while maintaining the quality at q_{s_1}. The user is also charged by the content provider a price p_{c_1} for each content of quality q_{c_1} that he receives through the network provider.

Figure 2

We consider a usage base price, rather than a flat rate price, for both network and content provision since we are modelling a service-oriented Internet in which all providers offer different services at various prices and quality. The user signals his preferences via a demand function d_{111} (1), for the content produced by CP_1 and transferred by NP_1, which depends on the price and the quality of both network and content provision, as follows:

$$d_{111} = d_0 - \alpha p_{s_1} - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1}. \tag{1}$$

The α, β, γ, and δ are all ≥ 0. d_0 is the demand at zero usage based on the price and the best effort service delivery (i.e., $q_{s_1} = q_{c_1} = 0$). Based on this demand function, the user will request more service as the price goes down or the quality increases in network and content provision. The α and β reflect the sensitivity of the user to the network and content provider's prices, respectively.

We consider different price sensitivity for content and network provider charges according to the assumption that there is an intrinsic value in the network besides the services offered by the content providers; otherwise, α and β would be equal. The γ and δ illustrate the effect of the quality of service of the network and the content providers on the user's demand. In this simple, illustrative service-oriented Internet model, the network provider also charges the content provider a transfer price p_{t_1} per unit of content transfers for the right to access end users. By charging a transfer price p_{t_1} we have a two-sided market. We also assume that the demand function is monotonically decreasing in price but increasing in quality.

The quality of the network, q_{s_1}, can be defined by various metrics such as latency, jitter, or bandwidth. Latency is a measure of the delay that the traffic experiences as it traverses a network and jitter is defined as the variation in that delay. Bandwidth is measured as the amount of data that can pass through a point in a network over time (see Smith and Garcia-Luna-Aceves (2008)). Here, we define the quality as the "expected delay," which is computed by the Kleinrock function (see Altman, Legout, and Xu (2011)) as the reciprocal of the square root of delay:

$$q_{s_1} = \frac{1}{\sqrt{Delay}} = \sqrt{b(d, q_{s_1}) - d_{111}}, \tag{2}$$

where $b(d_{111}, q_{s_1})$ is the total bandwidth of the network and is a function of demand and quality, that is:

$$b(d_{111}, q_{s_1}) = d_{111} + q_{s_1}^2. \tag{3}$$

Therefore, the greater the demand at higher quality, the larger the amount of bandwidth used. The network provider incurs a cost of transferring the demand while supporting q_{s_1} for data shipment, denoted by CS_1. We assume a convex, continuous, and differentiable transfer function for NP_1:

$$CS_1 = CS_1(d_{111}, q_{s_1}) = R\big(d_{111} + q_{s_1}^2\big), \tag{4}$$

where R is the unit cost of bandwidth. The quality of content provided can be specified for a specific domain of content, e.g., video streaming. In this case, quality is defined as the quality of videos produced by the content provider and CP_1's production cost, CC_1, is a convex and continuous function of quality of service:

$$CC_1 = CC_1(q_{c_1}) = Kq_{c_1}^2. \tag{5}$$

Our model is different from the model of Altman, Legout, and Xu (2011) since we introduce quality and a cost function for content provision. Based on the network structure, the user demand would be equal to the content provider's supply and the network provider's shipments. We assume that there is competition between the noncooperatively competing CP_1 and NP_1 and we seek to determine the Nash equilibrium price and quality that maximize their respective utilities. The network provider's income in a two-sided market would be the summation of the revenue of transferring services from the content provider to the user and providing Internet access for users.

Let $\mathscr{S}_{CP}$ denote the price and quality strategies of CP_1 where $\mathscr{S}_{CP} \equiv \{(p_{c_1}, q_{c_1}) \mid p_{c_1} \geq 0 \text{ and } q_{c_1} \geq 0\}$. The utility of the content provider, U_{CP_1}, which corresponds to his profits, is the difference between his revenue and his cost, and is given by:

$$U_{CP_1} = U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{c_1} - p_{t_1})d_{111} - CC_1 = (p_{c_1} - p_{t_1})d_{111} - Kq_{c_1}^2. \tag{6}$$

Let $\mathscr{S}_{NP}$ denote the price and quality strategies of NP_1 where $\mathscr{S}_{NP} \equiv \{(p_{s_1}, q_{s_1}) \mid p_{s_1} \geq 0 \text{ and } q_{s_1} \geq 0\}$. The utility of the network provider, U_{NP_1}, represents his profits and also is the difference between his revenue and his cost:

$$U_{NP_1} = U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{s_1} + p_{t_1})d_{111} - CS_1 = (p_{s_1} + p_{t_1} - R)d_{111} - Rq_{s_1}^2. \tag{7}$$

Here, since the basic model builds on the model of Altman, Kegout, and Xu (2011), and to enable the subsequent analytics in Section 2.2, we assume that the demand function is linear as in (1). In Section 3, we relax this assumption in our general model.

2.1 The Analysis of Two-Sided Pricing in the Basic Model

In this game, the two noncooperative agents, CP_1 and NP_1, seek to maximize their individual utilities with respect to their prices and quality. CP_1 maximizes his utility with respect to p_{c_1} and q_{c_1}:

$$\text{Maximize} \quad U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{c_1} - p_{t_1})d_{111} - Kq_{c_1}^2. \tag{8}$$

NP_1 also maximizes his utility but with respect to p_{s_1}, and q_{s_1}:

$$\text{Maximize} \quad U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{s_1} + p_{t_1} - R)d_{111} - Rq_{s_1}^2, \tag{9}$$

with all the prices and the quality levels being nonnegative.

Although the network provider needs to determine the transfer price, p_{t_1}, to charge the content provider, he cannot maximize his utility with respect to p_{t_1} simultaneously with p_{s_1}. Note that the utilities are linear functions of p_{t_1} (with the same derivatives with respect to p_{t_1} but different sign), so that if p_{t_1} is under the control of one of the providers, it would simply be set at an extreme value and, subsequently, lead to zero demand and zero income (see Kesidis (2012) and Altman, Legout, and Xu (2011)). As a result, we need to fix the p_{t_1} and maximize both U_{NP_1} and U_{CP_1} regarding the 4-tuple $(p_{s_1}, q_{s_1}, p_{c_1}, \text{and}\, q_{c_1})$. However, a subsequent and important question would be how large the side payment should be and whether NP_1 can get any benefit by charging CP_1. To overcome this issue, after optimizing the utility of CP_1 and NP_1, we check whether NP_1's profit is strictly increasing in p_{t_1} at $p_{t_1} = 0$ and under what conditions.

Definition 1: Nash Equilibrium in Prices and Quality

A price and quality level pattern $(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1}) \in \mathscr{S}_{CP} \times \mathscr{S}_{NP}$ *is said to constitute a Nash equilibrium if:*

$$U_{CP_1}(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1}) = \max_{(p_{c_1}, q_{c_1}) \in \mathscr{S}_{CP}} U_{CP_1}(p_{c_1}, q_{c_1}, p^*_{s_1}, q^*_{s_1}), \tag{10}$$

$$U_{NP_1}(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1}) = \max_{(p_{s_1}, q_{s_1}) \in \mathscr{S}_{NP}} U_{NP_1}(p^*_{c_1}, q^*_{c_1}, p_{s_1}, q_{s_1}). \tag{11}$$

Theorem 1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

Assume that the content provider's profit function, $U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$*, is concave with respect to the variables* (p_{c_1}, q_{c_1}) *and is continuous and continuously differentiable. Assume, also, that for the network provider's profit function,* $U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$*, is concave with respect to the variables* (p_{s_1}, q_{s_1}) *and is continuous and continuously differentiable.*

Then $(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1}) \in \mathscr{S}_{CP} \times \mathscr{S}_{NP}$ *is a Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality problem:*

$$-\frac{\partial U_{CP_1}(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1})}{\partial p_{c_1}} \times (p_{c_1} - p^*_{c_1}) - \frac{\partial U_{CP_1}(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1})}{\partial q_{c_1}} \times (q_{c_1} - q^*_{c_1})$$

$$-\frac{\partial U_{NP_1}(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1})}{\partial p_{s_1}} \times (p_{s_1} - p^*_{s_1}) - \frac{\partial U_{NP_1}(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1})}{\partial q_{s_1}} \times (q_{s_1} - q^*_{s_1}) \geq 0,$$

$$\forall (p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) \in \mathscr{S}_{CP} \times \mathscr{S}_{NP}, \tag{12}$$

or, equivalently, the variational inequality problem:

$$(-d_{111} + \beta(p^*_{c_1} - p_{t_1})) \times (p_{c_1} - p^*_{c_1}) + (2Kq^*_{c_1} + \delta(p_{t_1} - p^*_{c_1})) \times (q_{c_1} - q^*_{c_1})$$

$$+(-d_{111}+\alpha(p^*_{s_1}+p_{t_1}-R))\times(p_{s_1}-p^*_{s_1})+(2Rq^*_{s_1}+\gamma(R-p^*_{s_1}-p_{t_1}))\times(q_{s_1}-q^*_{s_1})\geq 0,$$

$$\forall(p_{c_1},q_{c_1},p_{s_1},q_{s_1})\in\mathscr{S}_{CP}\times\mathscr{S}_{NP}, \tag{13}$$

where d_{111} in (13) is evaluated at $(p^*_{c_1},q^*_{c_1},p^*_{s_1},q^*_{s_1})$.

Proof: (12) follows directly from Gabay and Moulin (1980) and Dafermos and Nagurney (1987). In order to obtain (13) from (12), we note that:

$$-\frac{\partial U_{CP_1}(p^*_{c_1},q^*_{c_1},p^*_{s_1},q^*_{s_1})}{\partial p_{c_1}}=-d_{111}+\beta(p^*_{c_1}-p_{t_1}), \tag{14}$$

$$-\frac{\partial U_{CP_1}(p^*_{c_1},q^*_{c_1},p^*_{s_1},q^*_{s_1})}{\partial q_{c_1}}=2Kq^*_{c_1}+\delta(p_{t_1}-p^*_{c_1}). \tag{15}$$

Similarly, we note that

$$-\frac{\partial U_{NP_1}(p^*_{c_1},q^*_{c_1},p^*_{s_1},q^*_{c_1})}{\partial p_{s_1}}=-d_{111}+\alpha(p^*_{s_1}+p_{t_1}-R), \tag{16}$$

$$-\frac{\partial U_{NP_1}(p^*_{c_1},q^*_{c_1},p^*_{s_1},q^*_{s_1})}{\partial q_{s_1}}=2Rq^*_{s_1}+\gamma(R-p^*_{s_1}-p_{t_1}). \tag{17}$$

Making the substitutions for the marginal utilities in (12) given by (14) – (17) yields variational inequality (13).□

Theorem 2: Uniqueness of the Nash Equilibrium Satisfying Variational Inequality (12)

The Nash equilibrium $(p^*_{c_1},q^*_{c_1},p^*_{s_1},q^*_{s_1})\in\mathscr{S}_{CP}\times\mathscr{S}_{NP}$ satisfying variational inequality (12) is unique, if the function F is strictly monotone over the feasible set $\mathscr{S}_{CP}\times\mathscr{S}_{NP}$, under our imposed assumptions (see Nagurney (1999)) with the function F consisting of minus the marginal utility functions of the providers w.r.t their price and quality variables.

We now provide some insights as to under what conditions F for the simple model will be strictly monotone. We note that Jacobian of F, since $F=-\nabla U(p_{c_1},q_{c_1},p_{s_1},q_{s_1})$, in view of the demand function, the revenue functions, and the cost functions, is given by:

$$\nabla F = \begin{pmatrix} -\frac{\partial^2 U_{CP_1}}{\partial p_{c_1}^2} & -\frac{\partial^2 U_{CP_1}}{\partial q_{c_1}\partial p_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial p_{s_1}\partial p_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial q_{s_1}\partial p_{c_1}} \\ -\frac{\partial^2 U_{CP_1}}{\partial p_{c_1}\partial q_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial q_{c_1}^2} & -\frac{\partial^2 U_{CP_1}}{\partial p_{s_1}\partial q_{c_1}} & -\frac{\partial^2 U_{CP_1}}{\partial q_{s_1}\partial q_{c_1}} \\ -\frac{\partial^2 U_{NP_1}}{\partial p_{c_1}\partial p_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial q_{c_1}\partial p_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial p_{s_1}^2} & -\frac{\partial^2 U_{NP_1}}{\partial q_{s_1}\partial p_{s_1}} \\ -\frac{\partial^2 U_{NP_1}}{\partial p_{c_1}\partial q_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial q_{c_1}\partial q_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial p_{s_1}\partial q_{s_1}} & -\frac{\partial^2 U_{NP_1}}{\partial q_{s_1}^2} \end{pmatrix} = \begin{pmatrix} 2\beta & -\delta & \alpha & -\gamma \\ -\delta & 2K & 0 & 0 \\ \beta & -\delta & 2\alpha & -\gamma \\ 0 & 0 & -\gamma & 2R \end{pmatrix}. \quad (18)$$

We know that if ∇F is positive-definite, then F is strictly monotone for this model and the solution to variational inequality (12) is unique. Of course, if the Jacobian is strictly diagonally dominant then it will be positive-definite.

Theorem 3

The network provider, NP_1, will benefit from charging the content provider, CP_1, if $4\alpha R > \gamma^2$ and the user is more sensitive to the price that NP_1 charges him than the price that CP_1 charges him. In other words, if $4\alpha R - \gamma^2 > 0$, and $\alpha > \beta$, then NP_1 would set a positive p_{t_1} to increase his profit.

Proof: According to the Nash equilibrium, the best response of NP_1 and CP_1 can be found when the derivatives $\frac{\partial U_{NP_1}}{\partial p_{s_1}}$, $\frac{\partial U_{NP_1}}{\partial q_{s_1}}$, $\frac{\partial U_{CP_1}}{\partial p_{c_1}}$, and $\frac{\partial U_{CP_1}}{\partial q_{c_1}}$ are all zero, under the assumption that the associated variables are all positive. Then, we will have:

$$p_{s_1} = \frac{d_0 - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1} - \alpha(p_{t_1} - R)}{2\alpha}, \quad (19)$$

$$q_{s_1} = \frac{\gamma(p_{s_1} + p_{t_1} - R)}{2R}, \quad (20)$$

$$p_{c_1} = \frac{d_0 - \alpha p_{s_1} + \gamma q_{s_1} + \delta q_{c_1} + \beta p_{t_1}}{2\beta}, \quad (21)$$

$$q_{c_1} = \frac{\delta(p_{c_1} - p_{t_1})}{2K}. \quad (22)$$

By substituting (22) into (21) and then substituting the resultant equation and (20) into (19), at the Nash equilibrium, the following expressions are obtained:

$$p_{s_1}^* = \mathrm{Max}\{0, \frac{2RK\beta[d_0 - R\alpha - (\beta - \alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)} + R - p_{t_1}\}, \quad (23)$$

$$q^*_{s_1} = \text{Max}\{0, \frac{K\gamma\beta[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)}\}, \tag{24}$$

$$p^*_{c_1} = \text{Max}\{0, \frac{2RK\alpha[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)} + p_{t_1}\}, \tag{25}$$

$$q^*_{c_1} = \text{Max}\{0, \frac{R\delta\alpha[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)}\}, \tag{26}$$

$$d_{111} = \text{Max}\{0, \frac{2RK\alpha\beta[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]}{\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)}\}. \tag{27}$$

Hence, the utilities of the network and content providers are:

$$U_{NP_1} = \frac{RK^2\beta^2(4R\alpha - \gamma^2)[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]^2}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}, \tag{28}$$

$$U_{CP_1} = \frac{KR^2\alpha^2(4K\beta - \delta^2)[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]^2}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}. \tag{29}$$

We now have the utility functions based on p_{t_1}. To determine whether NP_1 should charge CP_1 or not, we obtain the derivative of U_{NP_1} w.r.t p_{t_1} and check if it is increasing when $p_{t_1} = 0$.

$$\frac{\partial U_{NP_1}}{\partial p_{t_1}} = (\alpha - \beta)[d_0 - R\alpha - (\beta-\alpha)p_{t_1}]\frac{2RK^2\beta^2(4\alpha R - \gamma^2)}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}. \tag{30}$$

When $p_{t_1} = 0$, $\frac{\partial U_{NP_1}}{\partial p_{t_1}}$ would be:

$$(\alpha - \beta)[d_0 - R\alpha]\frac{2RK^2\beta^2(4\alpha R - \gamma^2)}{[\alpha R(4\beta K - \delta^2) + \beta K(2\alpha R - \gamma^2)]^2}. \tag{31}$$

With the assumption of a large d_0, $\frac{\partial U_{NP_1}}{\partial p_{t_1}}$ is positive if $4\alpha R - \gamma^2 > 0$ and $\alpha > \beta$. □

3. The Network Economic Game Theory Model of Price and Quality Competition in a Service-Oriented Internet

In this section, we develop a network economic game theory model for a multi-provider service-oriented network with heterogeneous markets of users. The network structure of the problem, which depicts the direction of the content flows, is given in Figure 3. See Figure 4 for a graphic depiction of the financial payments in this general model. We assume m content providers, a typical one denoted by CP_i; $\{i = 1,\ldots,m\}$, n network providers, denoted by NP_j; $\{j = 1,\ldots,n\}$, and o

markets of users, denoted by u_k; $\{k = 1,\dots,o\}$. These providers compete under the Nash concept of noncooperative behavior to set their prices and quality levels so as to maximize their utilities, which are in the form of profits.

Figure 3

Figure 4

To receive a unit of content service from CP_i with quality q_{c_i}, which is transmitted by NP_j with quality q_{s_j}, a user pays p_{c_i} and p_{s_j} to the CP_i and NP_j, respectively. The content providers also pay the network providers for transferring their content to the users. Each network provider NP_j has a fixed transmission fee p_{t_j} that he charges the CPs per unit of content. We group the p_{t_j}, p_{s_j}, q_{s_j}, p_{c_i}, and q_{c_i} for $i = 1,\dots,m$; $j = 1,\dots,n$, into vectors p_t, p_s, q_s, p_c, and q_c, respectively.

The users are heterogeneous in their demands and signal their preferences through a demand function d_{ijk} for the content produced by content provider i and transmitted by NP_j to demand market k:

$$d_{ijk} = d_{ijk}(p_c, q_c, p_s, q_s), \quad \forall i, j, k. \tag{32}$$

In this game theory model, the demand d_{ijk} does not only depend on the price and quality of CP_i and NP_j, but also on the prices and quality levels of the other content and network providers as a result of competition among the providers. Moreover, unlike the specialized, illustrative model in Section 2, the demand functions above need not be linear, as in (1), and in the work of Altman, Legout, and Xu (2011) and El Azouzi, Altman, and Wynter (2003).

Herein, if p_{s_j} and p_{c_i} (q_{s_j}, and q_{c_i}) decrease (increase), d_{ijk} naturally goes up, but it decreases if the price (quality) of the other providers decreases (increases).

We now describe the behavior of the content providers.

Each content provider CP_i produces distinct (but substitutable) content of specific quality q_{c_i}, and sells at a unit price of p_{c_i}. The total supply of CP_i, SCP_i, is given by:

$$SCP_i = \sum_{j=1}^{n} \sum_{k=1}^{o} d_{ijk}, \quad i = 1,\dots,m. \tag{33}$$

Each CP_i has a production cost, CC_i, which is a function of his supply and his quality of service:

$$CC_i = CC_i(SCP_i, q_{c_i}), \quad i = 1,\dots,m. \tag{34}$$

We assume that the production cost functions are convex, continuous, and continuously differentiable functions.

We assume that the content providers are profit-maximizers, where the profit or utility of CP_i; $i = 1,\ldots,m$, which is the difference between his total revenue and his total cost, is given by the expression:

$$U_{CP_i} = U_{CP_i}(p_c,q_c,p_s,q_s) = \sum_{j=1}^{n}(p_{c_i} - p_{t_j})\sum_{k=1}^{o} d_{ijk} - CC_i. \tag{35}$$

Let $\mathscr{K}_i^1$ denote the feasible set corresponding to CP_i, where $\mathscr{K}_i^1 \equiv \{(p_{c_i},q_{c_i}) \mid p_{c_i} \geq 0 \text{ and } q_{c_i} \geq 0\}$.

We now describe the behavior of the network providers.

A network provider NP_j; $j = 1,\ldots,n$, is distinguishable by means of his quality q_{s_j}, the fee p_{t_j} that he charges each content provider to transfer one unit of content to the users, and the fee p_{s_j} that he charges users to transfer them one unit of content. By charging p_{t_j}, we have a two-sided market. Here, as in Section 2, the p_{t_j}s are assumed to be an exogenous parameter in this multi-provider model. We assume that all content providers are connected to all network providers and, subsequently, to all users. The total amount of content of services transported by NP_j, TNP_j, is given by:

$$TNP_j = \sum_{i=1}^{m}\sum_{k=1}^{o} d_{ijk}, \quad j = 1,\ldots,n. \tag{36}$$

NP_j incurs the cost, CS_j, of maintaining his network based on the offered quality and the total traffic passing through his bandwidth:

$$CS_j = CS_j(TNP_j, q_{s_j}), \quad j = 1,\ldots,n. \tag{37}$$

Similar cost functions were used in Altman, Legout, and Xu (2011), where it was noted that the (transport) network provider has to cover the costs of operating the backbone, the last mile, upgrades, etc. We also assume that these cost functions are convex, continuous, and continuously differentiable functions. The utility of NP_j; $j = 1,\ldots,n$ is defined as the difference between his income and his cost, that is:

$$U_{NP_j} = U_{NP_j}(p_c,q_c,p_s,q_s) = (p_{s_j} + p_{t_j})TNP_j - CS_j. \tag{38}$$

Let $\mathscr{K}_j^2$ denote the feasible set corresponding to NP_j, where $\mathscr{K}_j^2 \equiv \{(p_{s_j},q_{s_j}) \mid p_{s_j} \geq 0 \text{ and } q_{s_j} \geq 0\}$.

We now consider the Nash equilibrium that captures the providers' behavior.

Definition 2: Nash Equilibrium in Price and Quality

A price and quality level pattern $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathscr{K}^3 \equiv \prod_{i=1}^m \mathscr{K}_i^1 \times \prod_{j=1}^n \mathscr{K}_j^2$*, is said to constitute a Nash equilibrium if for each content provider* CP_i*;* $i = 1, \ldots, m$*:*

$$U_{CP_i}(p_{c_i}^*, \hat{p_{c_i}^*}, q_{c_i}^*, \hat{q_{c_i}^*}, p_s^*, q_s^*) \geq U_{CP_i}(p_{c_i}, \hat{p_{c_i}^*}, q_{c_i}, \hat{q_{c_i}^*}, p_s^*, q_s^*), \quad \forall (p_{c_i}, q_{c_i}) \in \mathscr{K}_i^1, \tag{39}$$

where

$$\hat{p_{c_i}^*} \equiv (p_{c_1}^*, \ldots, p_{c_{i-1}}^*, p_{c_{i+1}}^*, \ldots, p_{c_m}^*) \text{ and } \hat{q_{c_i}^*} \equiv (q_{c_1}^*, \ldots, q_{c_{i-1}}^*, q_{c_{i+1}}^*, \ldots, q_{c_m}^*), \tag{40}$$

and if for each network provider NP_j*;* $j = 1, \ldots, n$*:*

$$U_{NP_j}(p_c^*, q_c^*, p_{s_j}^*, \hat{p_{s_j}^*}, q_{s_j}^*, \hat{q_{s_j}^*}) \geq U_{NP_j}(p_{s_j}, p_c^*, q_c^*, \hat{p_{s_j}^*}, q_{s_j}, \hat{q_{s_j}^*}), \quad \forall (p_{s_j}, q_{s_j}) \in \mathscr{K}_j^2, \tag{41}$$

where

$$\hat{p_{s_j}^*} \equiv (p_{s_1}^*, \ldots, p_{s_{j-1}}^*, p_{s_{j+1}}^*, \ldots, p_{s_n}^*) \text{ and } \hat{q_{s_j}^*} \equiv (q_{s_1}^*, \ldots, q_{s_{j-1}}^*, q_{s_{j+1}}^*, \ldots, q_{s_n}^*). \tag{42}$$

According to (39) and (41), a Nash equilibrium is established if no provider can unilaterally improve upon his profits by selecting an alternative vector of quality levels and prices.

Theorem 4: Variational Inequality Formulations of Nash Equilibrium for the Service-Oriented Internet

Assume that the provider utility functions are concave, continuous, and continuously differentiable. Then $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathscr{K}^3$ *is a Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^m \frac{\partial U_{CP_i}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial p_{c_i}} \times (p_{c_i} - p_{c_i}^*) - \sum_{i=1}^m \frac{\partial U_{CP_i}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial q_{c_i}} \times (q_{c_i} - q_{c_i}^*)$$

$$-\sum_{j=1}^n \frac{\partial U_{NP_j}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial p_{s_j}} \times (p_{s_j} - p_{s_j}^*) - \sum_{j=1}^n \frac{\partial U_{NP_j}(p_c^*, q_c^*, p_s^*, q_s^*)}{\partial q_{s_j}} \times (q_{s_j} - q_{s_j}^*) \geq 0,$$

$$\forall (p_c, q_c, p_s, q_s) \in \mathscr{K}^3, \tag{43}$$

or, equivalently,

$$\sum_{i=1}^{m}\left[-\sum_{j=1}^{n}\sum_{k=1}^{o}d_{ijk}-\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial p_{c_i}}\times(p^*_{c_i}-p_{t_j})+\frac{\partial CC_i(SCP_i,q^*_{c_i})}{\partial SCP_i}\cdot\frac{\partial SCP_i}{\partial p_{c_i}}\right]\times(p_{c_i}-p^*_{c_i})$$

$$+\sum_{i=1}^{m}\left[-\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial q_{c_i}}\times(p^*_{c_i}-p_{t_j})+\frac{\partial CC_i(SCP_i,q^*_{c_i})}{\partial q_{c_i}}\right]\times(q_{c_i}-q^*_{c_i})$$

$$+\sum_{j=1}^{n}\left[-\sum_{i=1}^{m}\sum_{k=1}^{o}d_{ijk}-\sum_{i=1}^{m}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial p_{s_j}}\times(p^*_{s_j}+p_{t_j})+\frac{\partial CS_j(TNP_j,q^*_{s_j})}{\partial TNP_j}\cdot\frac{\partial TNP_j}{\partial p_{s_j}}\right]\times(p_{s_j}-p^*_{s_j})$$

$$+\sum_{j=1}^{n}\left[-\sum_{i=1}^{m}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial q_{s_j}}\times(p^*_{s_j}+p_{t_j})+\frac{\partial CS_j(TNP_j,q^*_{s_j})}{\partial q_{s_j}}\right]\times(q_{s_j}-q^*_{s_j})\geq 0,$$

$$\forall(p_c,q_c,p_s,q_s)\in\mathscr{K}^3. \qquad (44)$$

Variational inequality (44) can be put into standard form (see Nagurney (1999)): determine $X^*\in\mathscr{K}^3$ such that:

$$\langle F(X^*),X-X^*\rangle\geq 0,\qquad \forall X\in\mathscr{K}, \qquad (45)$$

where $F(X)$ is a continuous function such that $F(X):X\mapsto\mathscr{K}\subset R^N$, and $\mathscr{K}$ is a closed and convex set. The term $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space. We define $X\equiv(p_c,q_c,p_s,q_s)$, and $F(X)\equiv(F_{p_c},F_{q_c},F_{p_s},F_{q_s})$. The specific components of F are given by: for $i=1,\ldots,m;\ j=1,\ldots,n$:

$$F_{p_{c_i}}=\frac{\partial CC_i(SCP_i,q_{c_i})}{\partial SCP_i}\cdot\frac{\partial SCP_i}{\partial p_{c_i}}-\sum_{j=1}^{n}\sum_{k=1}^{o}d_{ijk}-\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial p_{c_i}}\times(p_{c_i}-p_{t_j}), \qquad (46)$$

$$F_{q_{c_i}}=\frac{\partial CC_i(SCP_i,q_{c_i})}{\partial q_{c_i}}-\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial q_{c_i}}\times(p_{c_i}-p_{t_j}), \qquad (47)$$

$$F_{p_{s_j}}=\frac{\partial CS_j(TNP_j,q_{s_j})}{\partial TNP_j}\cdot\frac{\partial TNP_j}{\partial p_{s_j}}-\sum_{i=1}^{m}\sum_{k=1}^{o}d_{ijk}-\sum_{i=1}^{m}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial p_{s_j}}\times(p_{s_j}+p_{t_j}), \qquad (48)$$

$$F_{q_{s_j}}=\frac{\partial CS_j(TNP_j,q_{s_j})}{\partial q_{s_j}}-\sum_{i=1}^{m}\sum_{k=1}^{o}\frac{\partial d_{ijk}}{\partial q_{s_j}}\times(p_{s_j}+p_{t_j}), \qquad (49)$$

where $\mathscr{K}=\mathscr{K}^3$ and $N=2m+2n$.

4. The Algorithm

In this section, we recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993) (see also Nagurney and Zhang (1996)). The general iterative scheme was designed to estimate the stationary points of the projected dynamical system

$$\dot{X} = \Pi_{\mathscr{K}}(X, -F(X)), \tag{50}$$

where

$$\Pi_{\mathscr{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{(P_{\mathscr{K}}(X - \delta F(X)) - X)}{\delta}, \tag{51}$$

and $P_{\mathscr{K}}$ is the projection on the feasible set $\mathscr{K}$ with F being the function that enters the variational inequality problem (45). Equivalently, in view of the results in Dupuis and Nagurney (1993), the general iterative scheme also estimates solutions to variational inequality (45), since the stationary points of (50) coincide with the solutions to (45).

Specifically, the context of our network economic game theory model, the projected dynamical system (50) takes on the form: for each content provider CP_i; $i = 1, \ldots, m$:

$$\dot{p}_{c_i} = \begin{cases} \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}, & \text{if} \quad p_{c_i} > 0 \\ \max\{0, \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial p_{c_i}}\}, & \text{if} \quad p_{c_i} = 0. \end{cases} \tag{52}$$

and

$$\dot{q}_{c_i} = \begin{cases} \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}, & \text{if} \quad q_{c_i} > 0 \\ \max\{0, \frac{\partial U_{CP_i}(p_c, q_c, p_s, q_s)}{\partial q_{c_i}}\}, & \text{if} \quad q_{c_i} = 0. \end{cases} \tag{53}$$

Similarly, we have that for each network provider NP_j; $j = 1, \ldots, n$:

$$\dot{p}_{s_j} = \begin{cases} \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial p_{s_j}}, & \text{if} \quad p_{s_j} > 0 \\ \max\{0, \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial p_{s_j}}\}, & \text{if} \quad p_{s_j} = 0. \end{cases} \tag{54}$$

and

$$\dot{q}_{s_j} = \begin{cases} \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial q_{s_j}}, & \text{if} \quad q_{s_j} > 0 \\ \max\{0, \frac{\partial U_{NP_j}(p_c, q_c, p_s, q_s)}{\partial q_{s_j}}\}, & \text{if} \quad q_{s_j} = 0. \end{cases} \tag{55}$$

The continuous-time adjustment processes (52)–(55) provide a natural underlying dynamics for the behavior of the competing providers until an equilibrium (stationary point) is achieved. For example, both (52) and (54) reveal that the rate of change of the price that a provider charges is equal to the marginal utility of the provider with respect to that price, if the price is positive. However, to ensure that the nonnegativity assumption on the prices is met, if the price is at the boundary, that is, it is zero, then the rate of change is equal to the projection. Similarly, (53) and (55) reveal that the rate of change of the quality levels of the providers is equal to the marginal utility of the provider with respect to the quality level. Again, the projection operation guarantees that the nonnegativity assumption on the quality levels is also satisfied.

However, for computations, we need a time-discretization. Specifically, iteration τ of the Euler method is given by:

$$X^{\tau+1} = P_{\mathscr{K}}(X^{\tau} - a_{\tau}F(X^{\tau})). \tag{56}$$

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network based problems, similar to those constructed in Nagurney and Zhang (1996) and the references therein.

Explicit Formulae for the Euler Method Applied to Variational Inequality (45) with $F(X)$ Defined by (46) – (49)

The elegance of this procedure for the computation of solutions to our network economic model of the service-oriented Internet can be seen in the following explicit formulae. Indeed, (56) for the network economic game theory problem governed by variational inequality (45) yields the following closed form expressions, at each iteration, for the price and quality levels of each content and network provider $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$p_{c_i}^{\tau+1} = \max\Big\{0, p_{c_i}^{\tau} + a_\tau(\sum_{j=1}^{n}\sum_{k=1}^{o} d_{ijk} + \sum_{j=1}^{n}\sum_{k=1}^{o} \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i}^{\tau} - p_{t_j}) - \frac{\partial CC_i(SCP_i, q_{c_i}^{\tau})}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}})\Big\}, \tag{57}$$

$$q_{c_i}^{\tau+1} = \max\Big\{0, q_{c_i}^{\tau} + a_\tau(\sum_{j=1}^{n}\sum_{k=1}^{o} \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i}^{\tau} - p_{t_j}) - \frac{\partial CC_i(SCP_i, q_{c_i}^{\tau})}{\partial q_{c_i}})\Big\}, \tag{58}$$

$$p_{s_j}^{\tau+1} = \max\Big\{0, p_{s_j}^{\tau} + a_\tau(\sum_{i=1}^{m}\sum_{k=1}^{o} d_{ijk} + \sum_{i=1}^{m}\sum_{k=1}^{o} \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j}^{\tau} + p_{t_j}) - \frac{\partial CS_j(TNP_j, q_{s_j}^{\tau})}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}})\Big\}, \tag{59}$$

$$q_{s_j}^{\tau+1} = \max\Big\{0, q_{s_j}^{\tau} + a_\tau(\sum_{i=1}^{m}\sum_{k=1}^{o} \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j}^{\tau} + p_{t_j}) - \frac{\partial CS_j(TNP_j, q_{s_j}^{\tau})}{\partial q_{s_j}})\Big\}. \tag{60}$$

Notice that all the functions to the left of the equal signs in (57) - (60) are evaluated at their respective variables computed at the τ-th iteration.

We now provide the convergence result. The proof is direct from Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 5: Convergence

In our service-oriented Internet model, assume that $F(X) = -\nabla U(p_c, q_c, p_s, q_s)$ is strongly monotone. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathcal{K}^3$ and any sequence generated by the Euler method as given by (57) – (60), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to $(p_c^*, q_c^*, p_s^*, q_s^*)$.

5. Numerical Examples and Sensitivity Analysis

We implemented the Euler method to compute solutions to service-oriented Internet network problems using Matlab programming. For the computations we utilized a DELL XPS Series laptop with an Intel Core Duo processor with 3 GB RAM. The algorithm was considered to have converged if, at a given iteration, the absolute value of the difference of each price and each quality level differed from its respective value at the preceding iteration by no more than $\varepsilon = 10^{-6}$. The sequence $\{a_\tau\}$ was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We initialized the algorithm by setting $p^0_{c_i} = q^0_{c_i} = p^0_{s_j} = q^0_{s_j} = 0.00, \forall i, j$.

Example 1

In this example, we have two content providers, CP_1 and CP_2, one network provider, NP_1, and one market of users, u_1 (see Figure 5).

Figure 5

The demand functions are as below:

$$d_{111} = 100 - 2.8p_{s_1} - 2.1p_{c_1} + 1.3p_{c_2} + 1.62q_{s_1} + 1.63q_{c_1} - .42q_{c_2},$$

$$d_{211} = 112 - 2.8p_{s_1} + 1.3p_{c_1} - 2.7p_{c_2} + 1.62q_{s_1} - .42q_{c_1} + 1.58q_{c_2}.$$

The cost functions of the content providers, CP_1 and CP_2, are:

$$CC_1 = 1.7q^2_{c_1},$$

$$CC_2 = 2.4q^2_{c_2}.$$

The cost function of the network provider, NP_1, is:

$$CS_1 = 2.2(d_{111} + d_{211} + q^2_{s_1}).$$

The utilities of the content providers are:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - CC_1, \qquad U_{CP_2} = (p_{c_2} - p_{t_1})d_{211} - CC_2.$$

The utility of the network provider is:

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{211}) - CS_1.$$

Here, $p_{t_1} = 33$.

The Jacobian of $F(X)$ =-$\nabla U(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, denoted by $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, is

$$J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}) = \begin{pmatrix} 4.2 & -1.63 & -1.3 & .42 & 2.8 & -1.62 \\ -1.63 & 3.4 & 0 & 0 & 0 & 0 \\ -1.3 & .42 & 4.5 & -1.58 & 2.8 & -1.62 \\ 0 & 0 & -1.58 & 4.8 & 0 & 0 \\ .8 & -1.21 & 1.4 & -1.16 & 11.2 & -3.24 \\ 0 & 0 & 0 & 0 & -3.24 & 4.4 \end{pmatrix}.$$

Since the symmetric part of $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1})$, $(J + J^T)/2$, has only positive eigenvalues, which are: 1.54, 2.80, 3.11, 4.65, 6.89, and 13.51, the $F(X)$ in Example 1 is strongly monotone since $\nabla F(X)$, as above, is positive-definite. Thus, according to Theorem 5, there exists a unique equilibrium, which, according to Theorem 3.7 in Nagurney and Zhang (1996) is also globally exponentially stable for the utility gradient process.

The Euler method required 1922 iterations and 12.79 CPU seconds for convergence. The computed equilibrium solution is:

$$p^*_{c_1} = 75.68, \quad p^*_{c_2} = 63.62, \quad p^*_{s_1} = 0,$$

$$q^*_{c_1} = 20.46, \quad q^*_{c_2} = 10.08, \quad q^*_{s_1} = 22.68,$$

with incurred demands of:

$$d_{111} = 89.64, \quad d_{211} = 82.68.$$

The utility of NP_1 is 4175.73, that of CP_1 is 3114.25, and that of CP_2 is 2288.16. It is interesting that the network provider NP_1 is better off by not charging the user, that is, $p^*_{s_1} = 0$, and only charges the CPs for transferring the content to the user. Meanwhile, the users' demand for services offered by CP_1 is higher ($d_{111} > d_{211}$) in comparison with that of CP_2, since CP_1 provides content services at a higher quality ($q^*_{c_1} > q^*_{c_2}$).

Example 2

The network topology of Example 2 is given in Figure 6. We have one content provider, CP_1, two network providers, NP_1 and NP_2, and one market of users, u_1.

Figure 6

The demand functions are:

$$d_{111} = 100 - 1.8p_{s_1} + .5p_{s_2} - 1.83p_{c_1} + 1.59q_{s_1} - .6q_{s_2} + 1.24q_{c_1},$$

$$d_{121} = 100 + .5p_{s_1} - 1.5p_{s_2} - 1.83p_{c_1} - .6q_{s_1} + 1.84q_{s_2} + 1.24q_{c_1}.$$

The network providers' cost functions are:

$$CS_1 = 1.7(d_{111} + q_{s_1}^2),$$

$$CS_2 = 1.8(d_{121} + q_{s_2}^2).$$

The cost function of CP_1 is:

$$CC_1 = 1.84\left[d_{111} + d_{121} + q_{c_1}^2\right].$$

The utility function of CP_1 is:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} + (p_{c_1} - p_{t_2})d_{121} - CC_1.$$

The utility functions of the network providers are:

$$U_{NP_1} = (p_{s_1} + p_{t_1})d_{111} - CS_1, \qquad U_{NP_2} = (p_{s_2} + p_{t_2})d_{121} - CS_2.$$

We set $p_{t_1} = p_{t_2} = 0$.

The Jacobian of -$\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, denoted by $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, is

$$J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2}) = \begin{pmatrix} 7.32 & -2.48 & 1.3 & -.99 & 1 & -1.24 \\ -2.48 & 3.68 & 0 & 0 & 0 & 0 \\ 1.83 & -1.24 & 3.6 & -1.59 & -1 & .6 \\ 0 & 0 & -1.59 & 3.4 & 0 & 0 \\ 1.83 & -1.24 & -.5 & .6 & 3 & -1.84 \\ 0 & 0 & 0 & 0 & -1.84 & 3.6 \end{pmatrix}.$$

Since the symmetric part of $J(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, $(J + J^T)/2$, has only positive eigenvalues, which are: 9.44, 5.78, 3.5, 2.57, 1.4, and 1.87, we know that the $F(X)$ in Example 2 is strongly monotone. Hence, we can conclude that the equilibrium solution is unique.

The equilibrium solution was achieved after 2931 iterations of the Euler method and 18.58 seconds of CPU time:

$$p^*_{c_1} = 29.19, \quad p^*_{s_1} = 27.66, \quad p^*_{s_2} = 37.38,$$

$$q^*_{c_1} = 18.43, \quad q^*_{s_1} = 12.14, \quad q^*_{s_2} = 18.18,$$

with incurred demands of:

$$d_{111} = 46.72, \quad d_{121} = 53.37.$$

The utilities of NP_1 and NP_2 are 962.58, and 1303.77, respectively, and the utility of CP_1 is 2112.75. Note that NP_2 offers his services at a higher quality, but at a higher price than NP_1.

Example 3

The network topology of this example is depicted in Figure 7. We have two content providers, two network providers, and three markets of users.

Figure 7

The demand functions are:

$$d_{111} = 112 - 2.1p_{s_1} + .6p_{s_2} - 1.85p_{c_1} + .5p_{c_2} + .64q_{s_1} - .04q_{s_2} + .76q_{c_1} - .4q_{c_2},$$

$$d_{112} = 100 - 2.2p_{s_1} + .6p_{s_2} - 2.3p_{c_1} + .5p_{c_2} + .7q_{s_1} - .4q_{s_2} + .61q_{c_1} - .4q_{c_2},$$

$$d_{113} = 95 - .2p_{s_1} + .6p_{s_2} - 2.2p_{c_1} + .5p_{c_2} + .1q_{s_1} - .4q_{s_2} + .66q_{c_1} - .4q_{c_2},$$

$$d_{121} = 112 + .6p_{s_1} - .2p_{s_2} - 1.85p_{c_1} + .5p_{c_2} - .4q_{s_1} + .1q_{s_2} + .76q_{c_1} - .4q_{c_2},$$

$$d_{122} = 100 + .6p_{s_1} - 2p_{s_2} - 2.3p_{c_1} + .5p_{c_2} - .4q_{s_1} + .9q_{s_2} + .61q_{c_1} - .4q_{c_2},$$

$$d_{123} = 95 + .06p_{s_1} - 2.3p_{s_2} - 2.2p_{c_1} + .5p_{c_2} - .04q_{s_1} + .68q_{s_2} + .66q_{c_1} - .4q_{c_2},$$

$$d_{211} = 99 - 2.1p_{s_1} + .06p_{s_2} + .5p_{c_1} - 1.85p_{c_2} + .64q_{s_1} - .04q_{s_2} - .4q_{c_1} + .76q_{c_2},$$

$$d_{212} = 110 - 2.2p_{s_1} + .6p_{s_2} + .5p_{c_1} - 2.3p_{c_2} + .7q_{s_1} - .4q_{s_2} - .4q_{c_1} + .61q_{c_2},$$

$$d_{213} = 115 - .2p_{s_1} + .6p_{s_2} + .5p_{c_1} - 2.2p_{c_2} + .1q_{s_1} - .4q_{s_2} - .4q_{c_1} + .66q_{c_2},$$

$$d_{221} = 99 + .6p_{s_1} - .2p_{s_2} + .5p_{c_1} - 1.85p_{c_2} - .4q_{s_1} + .1q_{s_2} - .4q_{c_1} + .76q_{c_2},$$

$$d_{222} = 110 + .6p_{s_1} - 2p_{s_2} + .5p_{c_1} - 2.3p_{c_2} - .4q_{s_1} + .9q_{s_2} - .4q_{c_1} + .61q_{c_2},$$

$$d_{223} = 115 + .06p_{s_1} - 2.3p_{s_2} + .5p_{c_1} - 2.2p_{c_2} - .04q_{s_1} + .68q_{s_2} - .4q_{c_1} + .66q_{c_2}.$$

The network providers' cost functions are:

$$CS_1 = 1.2(d_{111} + d_{112} + d_{113} + d_{211} + d_{212} + d_{213} + q_{s_1}^2),$$

$$CS_2 = 3.2(d_{121} + d_{122} + d_{123} + d_{221} + d_{222} + d_{223} + q_{s_2}^2).$$

The cost functions of the content providers are:

$$CC_1 = 2.7q_{c_1}^2, \qquad CC_2 = 3.1q_{c_2}^2.$$

The utility functions of the content providers are:

$$U_{CP_1} = (p_{c_1} - p_{t_1})(d_{111} + d_{112} + d_{113}) + (p_{c_1} - p_{t_2})(d_{121} + d_{122} + d_{123}) - CC_1,$$

$$U_{CP_2} = (p_{c_2} - p_{t_1})(d_{211} + d_{212} + d_{213}) + (p_{c_2} - p_{t_2})(d_{221} + d_{222} + d_{223}) - CC_2.$$

The utility functions of the network providers are:

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{112} + d_{113} + d_{211} + d_{212} + d_{213}) - CS_1,$$

$$U_{NP_2} = (p_{s_2} + p_{t_2})(d_{121} + d_{122} + d_{123} + d_{221} + d_{222} + d_{223}) - CS_2.$$

We set $p_{t_1} = 23$ and $p_{t_2} = 21$.

The Jacobian of -$\nabla U(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, denoted by $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, is

$$J = \begin{pmatrix} 25.4 & -4.06 & -3 & 2.4 & 3.24 & -.6 & 3.24 & -.84 \\ -4.06 & 5.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 2.4 & 25.4 & -4.06 & 3.24 & -.6 & 3.24 & -.84 \\ 0 & 0 & -4.06 & 6.2 & 0 & 0 & 0 & 0 \\ 4.85 & -.83 & 4.85 & -.83 & 18 & -2.88 & -2.52 & 1.68 \\ 0 & 0 & 0 & 0 & -2.88 & 2.4 & 0 & 0 \\ 4.85 & -.83 & 4.85 & -.83 & -2.52 & 1.68 & 18 & -3.36 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.36 & 6.4 \end{pmatrix}.$$

The symmetric part of $J(p_{c_1}, q_{c_1}, p_{c_2}, q_{c_2}, p_{s_1}, q_{s_1}, p_{s_2}, q_{s_2})$, $(J + J^T)/2$, has only positive eigenvalues, which are: 1.85, 4.46, 5.42, 5.48, 10.71, 21.47, 28.25, and 29.56. Hence, the $F(X)$ in Example 3 is also strongly monotone and we know that the equilibrium solution is unique.

The equilibrium solution below is achieved after 1758 iterations and 19.95 CPU seconds:

$$p^*_{c_1} = 40.57, \quad p^*_{c_2} = 41.49, \quad p^*_{s_1} = 8.76, \quad p^*_{s_2} = 5.35,$$

$$q^*_{c_1} = 13.96, \quad q^*_{c_2} = 12.76 \quad q^*_{s_1} = 36.67, \quad q^*_{s_2} = 12.15,$$

with incurred demands of:

$$d_{111} = 68.11, \quad d_{112} = 35.60, \quad d_{113} = 30.87, \quad d_{211} = 51.55, \quad d_{212} = 41.80, \quad d_{213} = 47.10,$$

$$d_{121} = 53.93, \quad d_{122} = 21.68, \quad d_{123} = 25.62, \quad d_{221} = 37.37, \quad d_{222} = 27.89, \quad d_{223} = 41.86.$$

In this example, NP_1 has a lower cost of bandwidth in comparison with that of NP_2. This can be related to the technology. NP_1 may be using advanced technology and, therefore, incurs a lower cost. Hence, NP_1 can set up his services at a higher quality ($q_{s_1} > q_{s_2}$) and absorbs a higher percentage of the total demand ($TNP_1 > TNP_2$).

Please refer to Figures 8 and 9 to view the trajectories of the prices and the quality levels generated by the Euler method at iterations 0, 40, 80, ..., 1720, and 1758.

Figure 8

Figure 9

As mentioned in Section 2.1, the transfer prices are not variables in our model. However, the value of these prices: p_{t_j}; $j = 1, \dots, n$, may impact the equilibrium values of the price and quality variables and the incurred utilities of the entities in our model. In order to make the impact of their values clearer, we provide sensitivity analysis results. For Example 1, with a single network provider, NP_1, we varied the value of p_{t_1} from 0 to 40 to determine the effect on NP_1's utility, price, and quality level, and on the two content providers', CP_1 and CP_2, utilities, prices, and quality levels. The results are reported in Figure 10.

For Example 1, by increasing the value of p_{t_1}, we found that the utility of both CPs and that of NP_1 increases. Also, the prices charged by the CPs increase while the price charged by NP_1 decreases as the value of p_{t_1} increases. On the other hand, the quality of all providers does not change considerably (cf. Figure 10). It is interesting that, when $p_{t_1} \geq 33$, the price charged by the network provider, NP_1, $p^*_{s_1} = 0$, and the utilities of both content providers remain essentially unchanged. Therefore, in this case, the best value of p_{t_1} for all entities would be 33. Hence, in this example, all providers benefit with a positive p_{t_1}.

Figure 10

For Examples 2 and 3, in which we have two network providers, two kinds of sensitivity analyses were performed. The results for the first sensitivity analysis are reported in Figure 11. For the first sensitivity analysis, the value of both p_{t_1} and p_{t_2} increase simultaneously from 0 to 40. As can be seen from the results in Figure 11, the utilities of all providers decrease with increasing values of the p_{t_j}s.

Figure 11

For the second sensitivity analysis in this set, we let $p_{t_1} + p_{t_2} = 40$, so that p_{t_1} starts at 40 and decreases to 0 while p_{t_2} starts at 0 and increases to 40. This transfer pricing scheme illustrates the case where the two network providers charge the content providers differently. The results are reported in Figure 12. We determine that the total utility of providers computed as the sum of the NPs' and the CPs' utilities, which correspond to their profits, is maximized when both network providers charge equally (cf. Figure 12). By examining other values for the sum of p_{t_1} and p_{t_2}, with $n = 30$, $n = 50$, and $n = 60$, we reach the conclusion, computationally, that for a pricing scheme of $p_{t_1} + p_{t_2} = n$ the optimal total utility of all providers is obtained when $p_{t_1} = p_{t_2} = n/2$ for n as above.

Figure 12

By performing sensitivity analysis, interesting results have been observed. First, in a market with a monopolistic network provider all providers can increase their utility with a positive value of p_{t_1}. When we have multiple network providers, all providers achieve a higher utility by not charging content providers. On the other hand, if the network providers are allowed to charge content providers (lack of neutrality regulations), the social welfare or summation of all providers' utilities would be maximized if the network providers charge equally. We obtained such conclusions based on the results for Examples 2 and 3. Nevertheless, as mentioned in Musacchio and Kim (2009), Njoroge et al. (2010), Altman, Caron, and Kesidis (2010), Musacchio, Schwartx, and Walrand (2009), and Economides and Tag (2012), the overall effect of implementing network neutrality regulations (e.g., having the p_{t_j}s be zero) may still be both positive and negative depending on the parameter values and the model structure. This further emphasizes the importance of a computational framework to investigate the impacts of different values of transfer prices and their impacts, along with any other sensitivity analysis that may be desired.

Example 4

In this example, there are 4 content providers, 3 network providers, and 5 markets of user (Figure 13). Here, there are $4 \times 3 \times 5 = 60$ demand functions and 7 profit functions for the providers.

Figure 13

The demand functions for demand market k for content from content provider i that is transferred by network provider j has the following form:

$$\begin{aligned} d_{ijk} =& d^0_{ik} - \beta_{ik} p_{c_i} + \sum_{f=1, f\neq i}^{m} \left(\acute{\beta}_{fk} p_{c_f}\right) \\ = & \quad - \alpha_{jk} p_{s_j} + \sum_{l=1, l\neq j}^{n} \left(\acute{\alpha}_{lk} p_{s_l}\right) \\ = & \quad + \delta_{ik} q_{c_i} - \sum_{f=1, f\neq i}^{m} \left(\acute{\delta}_{fk} q_{c_f}\right) \\ = & \quad + \gamma_{jk} q_{s_j} - \sum_{l=1, l\neq j}^{n} \left(\acute{\gamma}_{lk} q_{s_l}\right), \quad \forall i, j, k. \end{aligned}$$

The parameters for the demand functions are given in Table 1.

Table 1

The cost function for network provider j has the following form:

$$CS_j = \sigma_j \left(\sum_{i=1}^{m} \sum_{k=1}^{o} d_{ijk} + {q_{s_j}}^2\right), \quad \forall j,$$

where $\sigma_1 = 1.2$, $\sigma_2 = 3.2$, and $\sigma_3 = 2.5$.

Also, the cost function for content provider i is given by:

$$CC_i = \kappa_i ({q_{c_i}}^2), \quad \forall i,$$

where $\kappa_1 = 2.7$, $\kappa_2 = 3.1$, $\kappa_3 = 2.9$, and $\kappa_4 = 3.2$.

The utility of each provider is the difference of its revenue and cost. The transfer price for network providers are:

$$p_{t_1} = 10, \quad p_{t_2} = 14, \quad p_{t_3} = 13.$$

The utility functions are:

$$
\begin{aligned}
U_{NP_1} =&6.12p_{c_1}+6.984p_{c_2}+7.356p_{c_3}+6.492p_{c_4}+46.8p_{s_1}-9.6p_{s_2}\\
&=-9.6p_{s_3}+1.62q_{c_1}+1.884q_{c_2}+1.776q_{c_3}+1.356q_{c_4}-11.616q_{s_1}+9.6q_{s_2}\\
&=+9.6q_{s_3}-1.2{q_{s_1}}^2-(p_{s_1}+10)\times\big(5.1p_{c_1}+5.82p_{c_2}+6.13p_{c_3}+5.41p_{c_4}\\
&=+39p_{s_1}-8p_{s_2}-8p_{s_3}+1.35q_{c_1}+1.57q_{c_2}+1.48q_{c_3}+1.13q_{c_4}-9.68q_{s_1}\\
&=+8q_{s_2}+8q_{s_3}-2020\big)-2424,
\end{aligned}
$$

$$
\begin{aligned}
U_{NP_2} =& 16.32p_{c_1}+18.624p_{c_2}+19.616p_{c_3}+17.312p_{c_4}-25.6p_{s_1}+134.656p_{s_2}\\
&-25.6p_{s_3}+4.32q_{c_1}+5.0244q_{c_2}+4.736q_{c_3}+3.616q_{c_4}+25.6q_{s_1}-21.76q_{s_2}\\
&+25.6q_{s_3}-3.2{q_{s_2}}^2-(p_{s_2}+14)\times\big(5.1p_{c_1}+5.82p_{c_2}+6.13p_{c_3}+5.41p_{c_4}\\
&-8p_{s_1}+42.08p_{s_2}-8p_{s_3}+1.35q_{c_1}+1.57q_{c_2}+1.48q_{c_3}+1.13q_{c_4}+8q_{s_1}\\
&-6.8q_{s_2}+8q_{s_3}-2020\big)-6464,
\end{aligned}
$$

$$
\begin{aligned}
U_{NP_3} =& 12.75p_{c_1}+14.55p_{c_2}+15.325p_{c_3}+13.525p_{c_4}-20p_{s_1}-20p_{s_2}\\
&+98.8p_{s_3}+3.375q_{c_1}+3.925q_{c_2}+3.7q_{c_3}+2.825q_{c_4}+20q_{s_1}+20q_{s_2}\\
&-32.1q_{s_3}-2.5{q_{s_3}}^2-(p_{s_3}+13)\times\big(5.1p_{c_1}+5.82p_{c_2}+6.13p_{c_3}+5.41p_{c_4}\\
&-8p_{s_1}-8p_{s_2}+39.52p_{s_3}+1.35q_{c_1}+1.57q_{c_2}+1.48q_{c_3}+1.13q_{c_4}+8q_{s_1}\\
&+8q_{s_2}-12.84q_{s_3}-2020\big)-5050,
\end{aligned}
$$

$$
\begin{aligned}
U_{CP_1} = {} & (p_{c_1} - 10) * (1.5p_{c_2} - 9.6p_{c_1} + 1.5p_{c_3} + 1.5p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3} \\
& + 3.15q_{c_1} - 1.5q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 510) - 2.7{q_{c_1}}^2 \\
& + (p_{c_1} - 14) \times \left(1.5p_{c_2} - 9.6p_{c_1} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3}\right. \\
& \left. + 3.15q_{c_1} - 1.5q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 510\right) \\
& + (p_{c_1} - 13) \times \left(1.5p_{c_2} - 9.6p_{c_1} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3} + 3.15q_{c_1}\right. \\
& \left. - 1.5q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21q_{s_3} + 510\right),
\end{aligned}
$$

$$
\begin{aligned}
U_{CP_2} = {} & (p_{c_2} - 10) \times \left(1.5p_{c_1} - 10.32p_{c_2} + 1.5p_{c_3} + 1.5p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3}\right. \\
& \left. - 1.5q_{c_1} + 2.93q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 491\right) - 3.1{q_{c_2}}^2 \\
& + (p_{c_2} - 14) \times \left(1.5p_{c_1} - 10.32p_{c_2} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3}\right. \\
& \left. - 1.5q_{c_1} + 2.93q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 491\right) \\
& + (p_{c_2} - 13) \times \left(1.5p_{c_1} - 10.32p_{c_2} + 1.5p_{c_3} + 1.5p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3}\right. \\
& \left. - 1.5q_{c_1} + 2.93q_{c_2} - 1.5q_{c_3} - 1.5q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21q_{s_3} + 491\right),
\end{aligned}
$$

$$
\begin{aligned}
U_{CP_3} = {} & (p_{c_3} - 10) \times \left(1.5p_{c_1} + 1.5p_{c_2} - 10.63p_{c_3} + 1.5p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3}\right. \\
& \left. - 1.5q_{c_1} - 1.5q_{c_2} + 3.02q_{c_3} - 1.5q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 508\right) - 2.9{q_{c_3}}^2 \\
& + (p_{c_3} - 14) \times \left(1.5p_{c_1} + 1.5p_{c_2} - 10.63p_{c_3} + 1.5p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3} - 1.5q_{c_1}\right. \\
& \left. - 1.5q_{c_2} + 3.02q_{c_3} - 1.5q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 508\right) \\
& + (p_{c_3} - 13) \times \left(1.5p_{c_1} + 1.5p_{c_2} - 10.63p_{c_3} + 1.5p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3}\right. \\
& \left. - 1.5q_{c_1} - 1.5q_{c_2} + 3.02q_{c_3} - 1.5q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21q_{s_3} + 508\right),
\end{aligned}
$$

$$U_{CP_4} = (p_{c_4} - 10) \times \big(1.5p_{c_1} + 1.5p_{c_2} + 1.5p_{c_3} - 9.91p_{c_4} - 9.75p_{s_1} + 2p_{s_2} + 2p_{s_3}$$
$$- 1.5q_{c_1} - 1.5q_{c_2} - 1.5q_{c_3} + 3.37 * q_{c_4} + 2.42q_{s_1} - 2q_{s_2} - 2q_{s_3} + 511\big) - 3.2{q_{c_4}}^2$$
$$+ (p_{c_4} - 14) \times \big(1.5p_{c_1} + 1.5p_{c_2} + 1.5p_{c_3} - 9.91p_{c_4} + 2p_{s_1} - 10.52p_{s_2} + 2p_{s_3} - 1.5q_{c_1}$$
$$- 1.5q_{c_2} - 1.5q_{c_3} + 3.37q_{c_4} - 2q_{s_1} + 1.7q_{s_2} - 2q_{s_3} + 511\big)$$
$$+ (p_{c_4} - 13) \times \big(1.5p_{c_1} + 1.5p_{c_2} + 1.5p_{c_3} - 9.91p_{c_4} + 2p_{s_1} + 2p_{s_2} - 9.88p_{s_3} - 1.5q_{c_1}$$
$$- 1.5q_{c_2} - 1.5q_{c_3} + 3.37q_{c_4} - 2q_{s_1} - 2q_{s_2} + 3.21 * q_{s_3} + 511\big).$$

The Euler method required 9046 iterations and 212.56 CPU seconds for convergence. The equilibrium result is:

$$p^*_{c_1} = 32.27, \quad p^*_{c_2} = 26.37, \quad p^*_{c_3} = 27.35, \quad p^*_{c_4} = 30.51,$$

$$p^*_{s_1} = 21.77, \quad p^*_{s_2} = 0, \quad p^*_{s_3} = 5.45,$$

$$q^*_{c_1} = 34.89, \quad q^*_{c_2} = 19.90, \quad q^*_{c_3} = 23.46, \quad q^*_{c_4} = 28.71,$$

$$q^*_{s_1} = 123.32, \quad q^*_{s_2} = 11.48, \quad q^*_{s_3} = 40.95.$$

The utilities of network providers are:

$$U_{NP_1} = 18209.15, \quad U_{NP_2} = 1796.99, \quad U_{NP_2} = 5856.37.$$

The content providers' utilities are:

$$U_{CP_1} = 8666.85, \quad U_{CP_2} = 5376.46, \quad U_{CP_3} = 6101.34, \quad U_{CP_4} = 7686.85.$$

According to the result[1], NP_1 transfers almost 60 percent of total demand for all demand markets and CP_1 has the largest supply (around 30%)[2] among the content providers.

6. Summary and Conclusions

[1] $TNP_1 = 1192.41$, $TNP_2 = 205.40$, and $TNP_3 = 630.16$

[2] $SCP_1 = 574.22$, $SCP_2 = 434.53$, $SCP_3 = 478.91$, and $SCP_4 = 540.30$

In this paper, we developed a modeling and computational framework for a service-oriented Internet using game theory and variational inequality theory. First, we modeled a simple, illustrative Internet with a single content provider and a single network provider and analyzed the effect of the price that the network provider charges the content provider for data transmission. User's demand is a function of price and quality of both providers and goes up (down) as the price (quality) of the providers decreases. The analysis showed that the network provider benefits from charging the content provider if the user is more sensitive towards the network provider's fee.

We then modeled a market of multiple providers. The providers (content and network providers) are assumed to compete in an oligopolistic manner using quality and price of offered services to users as strategic variables. All providers are noncooperative and are assumed to be utility maximizers with their utilities consisting of profits. The users, in turn, reflect their preferences for the services produced by a content provider and transported by a network provider through the demand functions, which are functions of price and quality of not only that network and content provider, but also of the other providers. We also provided the equilibrium model's equivalent variational inequality formulation with nice features for computational purposes. We used the Euler method to solve numerical examples in order to illustrate the proposed model.

There are many issues in our proposed framework that are worthy of further discussion and investigation. For instance, in our model, the price mechanisms are usage-based with bandwidth-based pricing for the content or network providers. Nevertheless, we can consider a flat-rate or a two-part tariff pricing mechanism in order and compare the results. Another big debate in the future Internet is whether or not to offer short-term contracts to enable users to select between service offerings from different providers with long-term lock-ins not being the only option. Therefore, it would be interesting to study and compare both short-term and long-term contracts in the NGI structure. In some cases, the quality of one provider might be blocked by an upper bound or a lower bound. We might have capacity restrictions for data transmission on the NPs' bandwidth or be faced with content production capacity limitations for the CPs. These limitations could be added into the models as constraints and the new models formulated and solved with appropriate methods.

In addition, including uncertainty into the demand functions would enable us to capture possible forecasting errors. It would also be worthwhile to construct multiperiod network economic game theory models for a service-oriented Internet. Finally, it would be interesting to explore having capacities at the network layer as strategic variables.

We believe that our general network economic model is an important step in these directions, and it provides a good foundation to address the above issues in future research.

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Figures

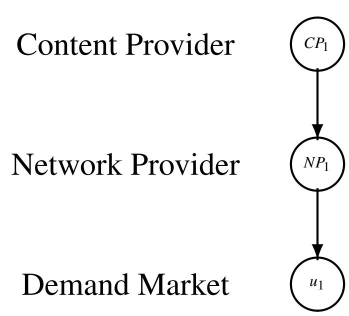

Figure 1: Network Topology for the Basic Model's Content Flow

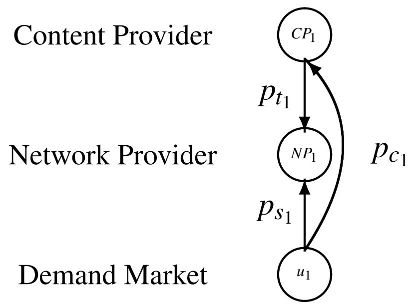

Figure 2: The Network Structure of the Basic Model's Financial Payment Flows

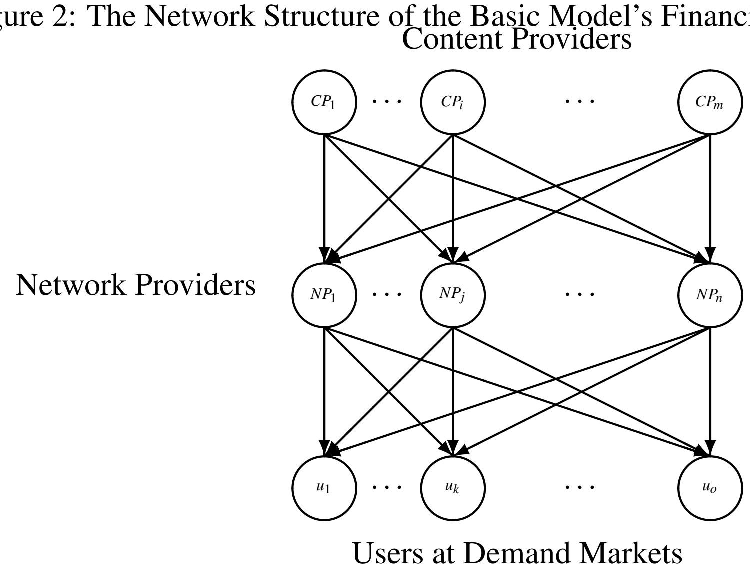

Figure 3: The Network Structure of the Multi-Provider Model's Content Flows

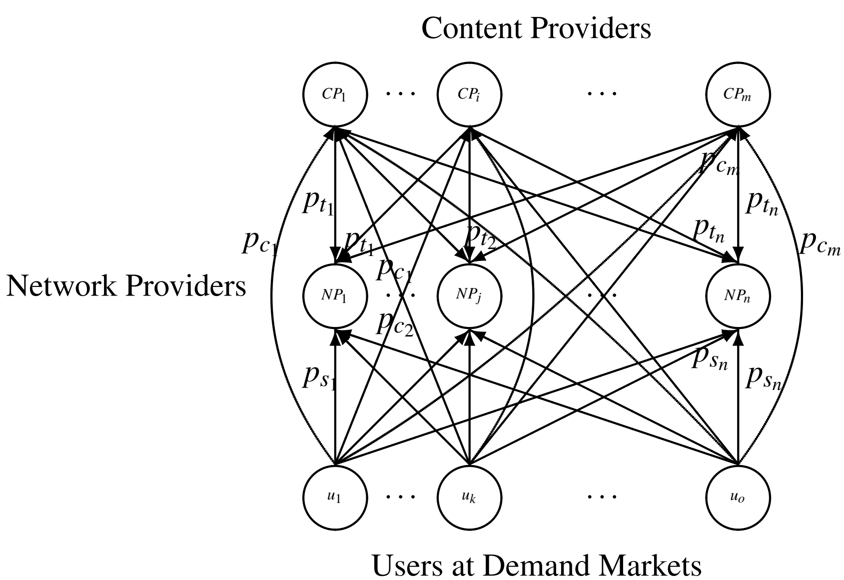

Figure 4: Graphic of the Multi-Provider Model with a Focus on Payments

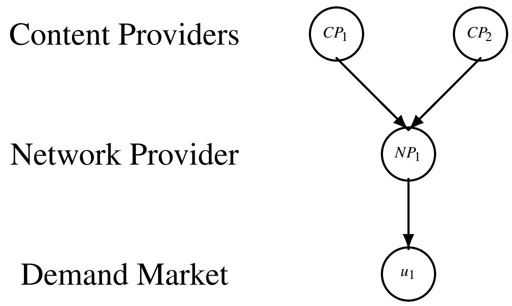

Figure 5: Network Topology of Content Flows for Example 1

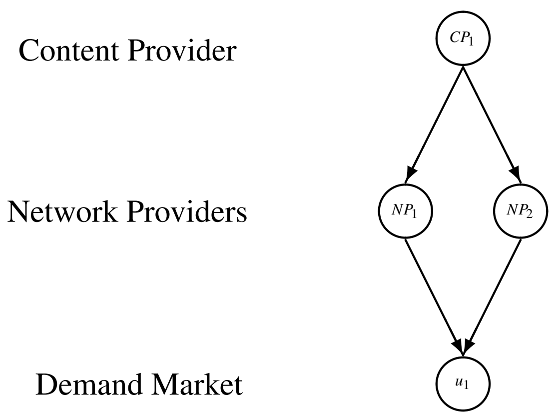

Figure 6: Network Topology of Content Flows for Example 2

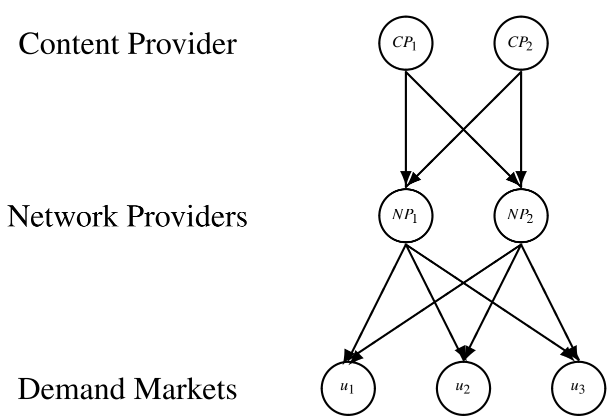

Figure 7: Network Topology of Content Flows for Example 3

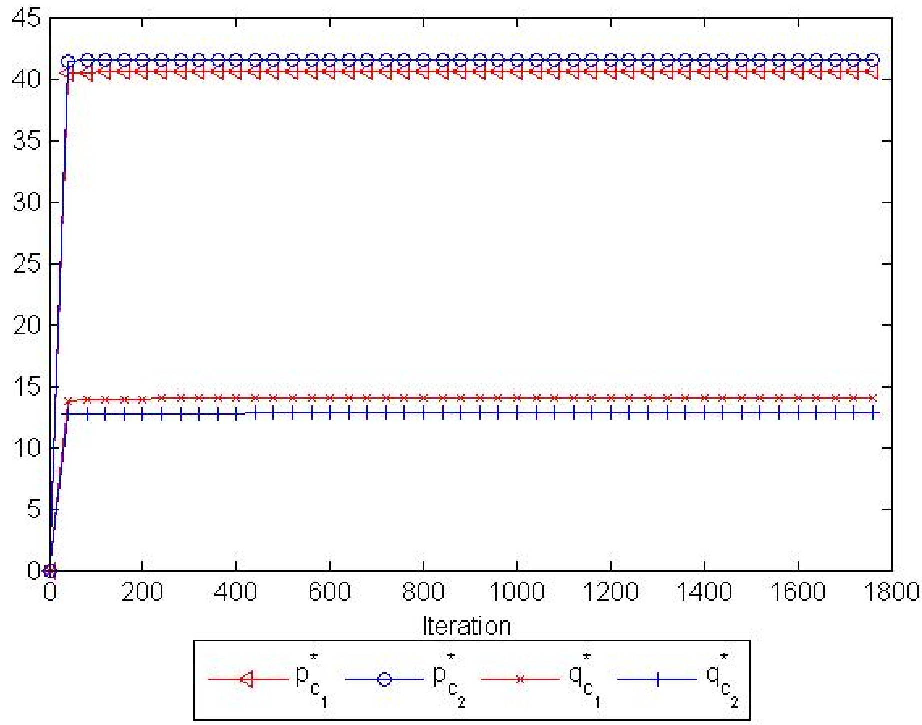

Figure 8: Prices and Quality Levels of Content Providers for Example 3

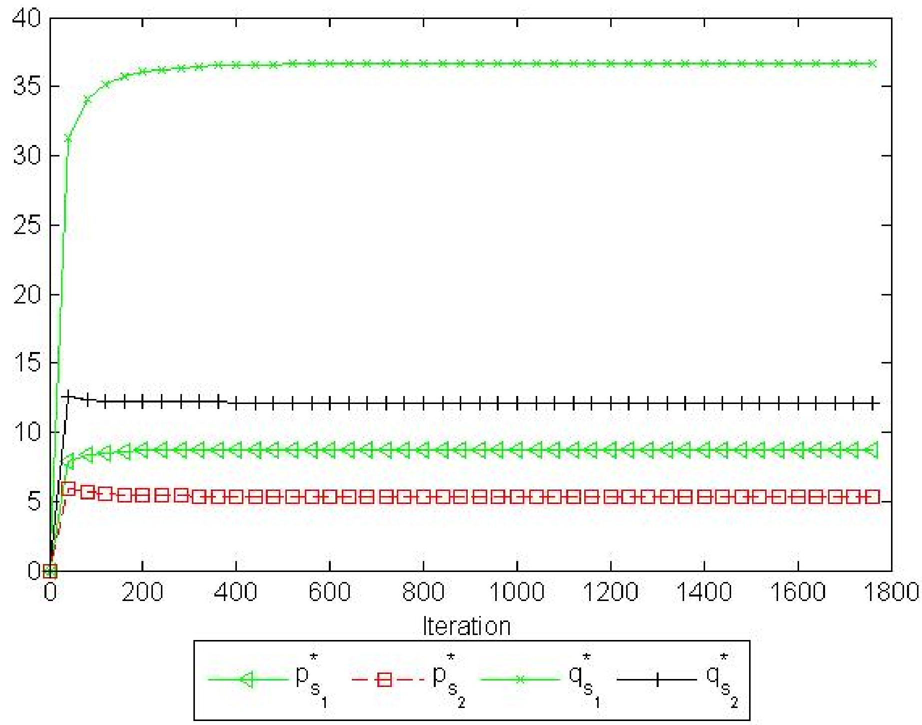

Figure 9: Prices and Quality Levels of Network Providers for Example 3

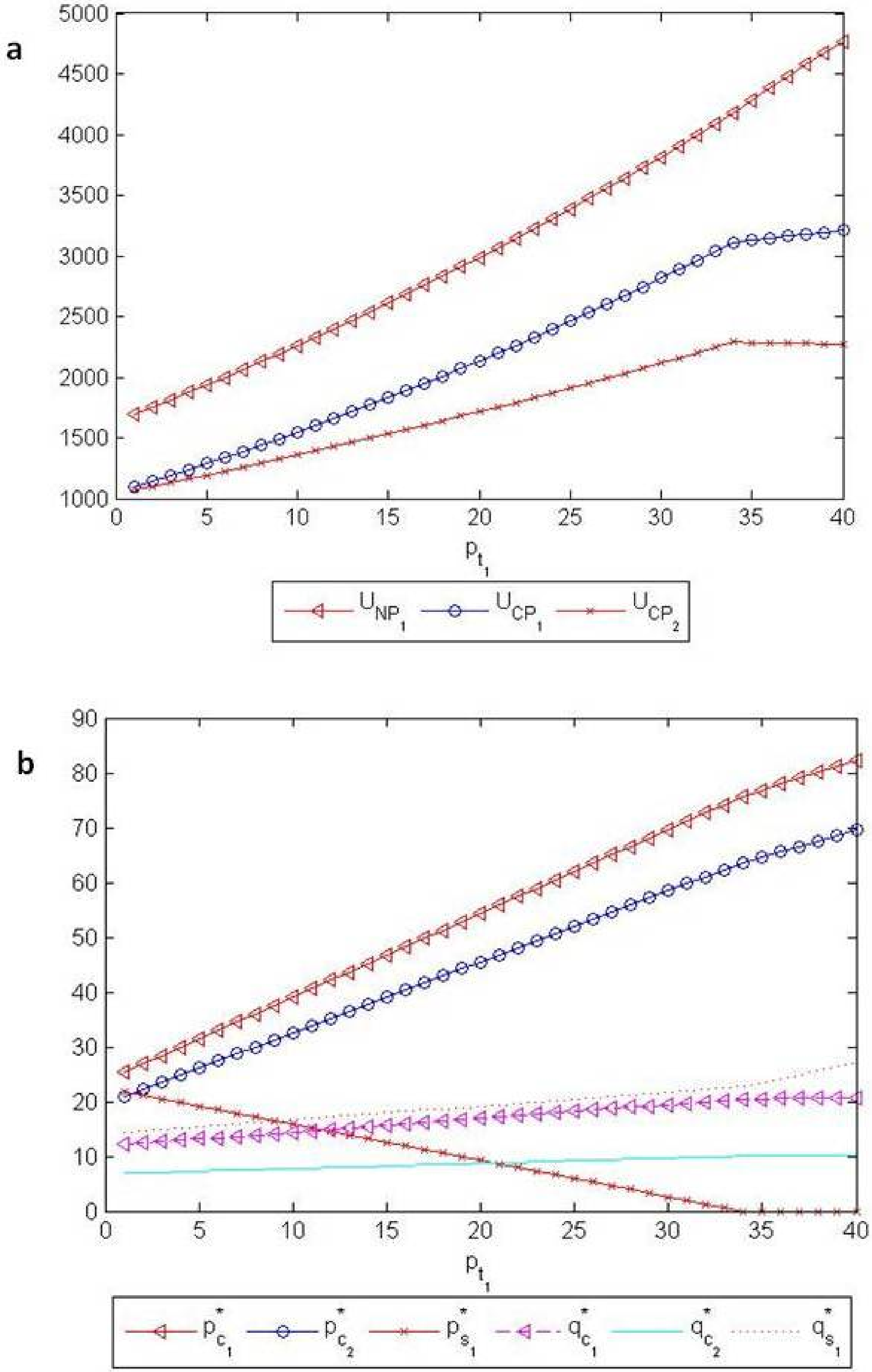

Figure 10: Effect of p_{t_1} Value on Utilities, Prices, and Quality in Example 1

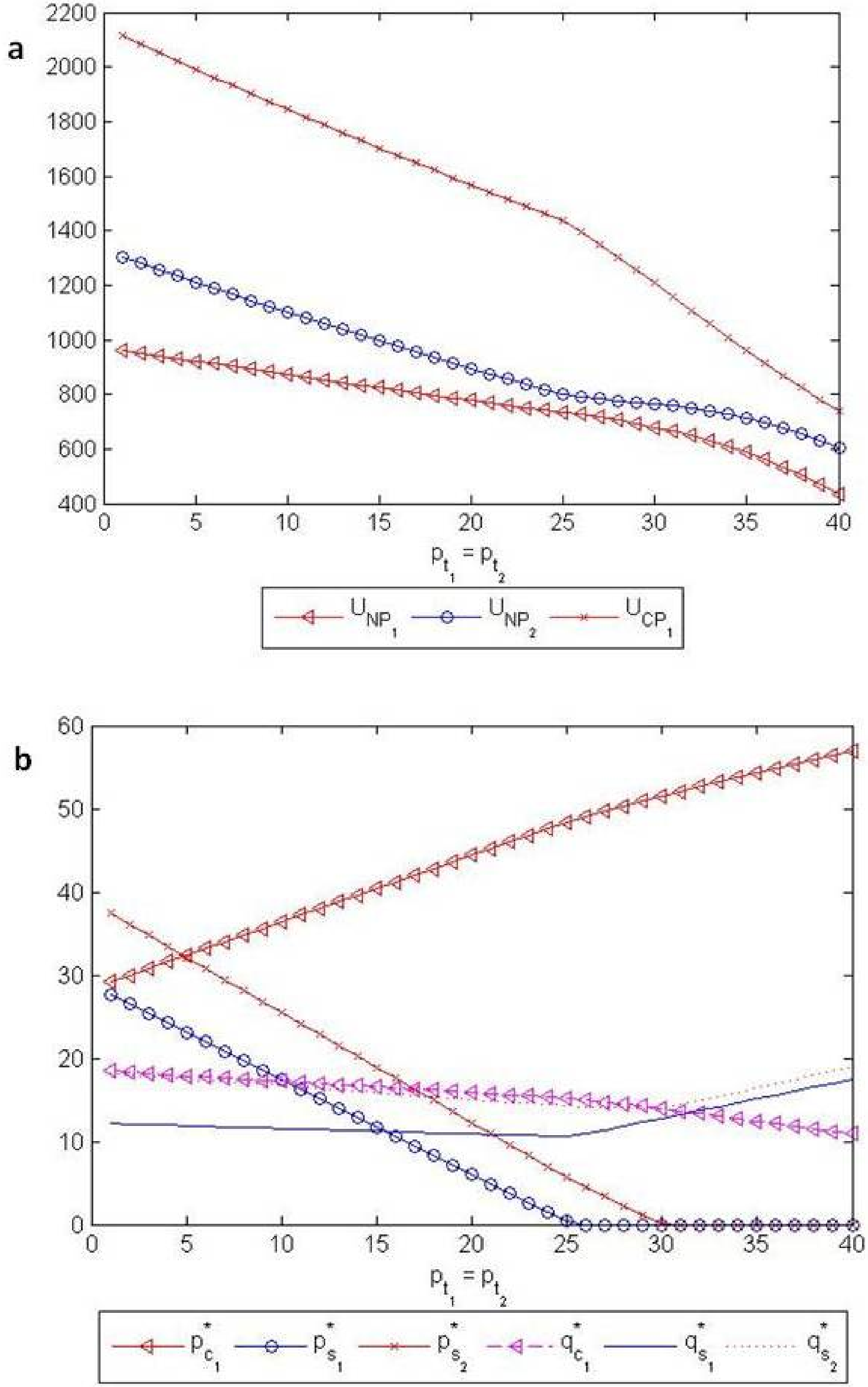

Figure 11: Effect of p_{t_1} and p_{t_2} Values on Utilities, Prices, and Quality in Example 2 with $p_{t_1} = p_{t_2}$

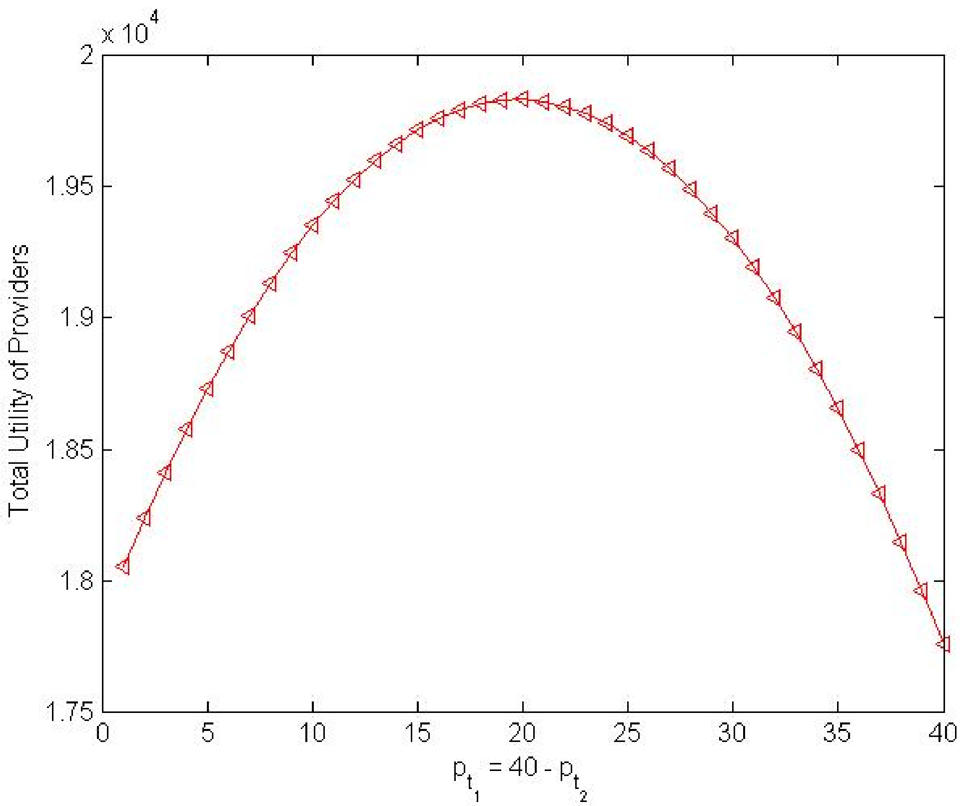

Figure 12: Effect of p_{t_1} and p_{t_2} Values on Total Utility in Example 3

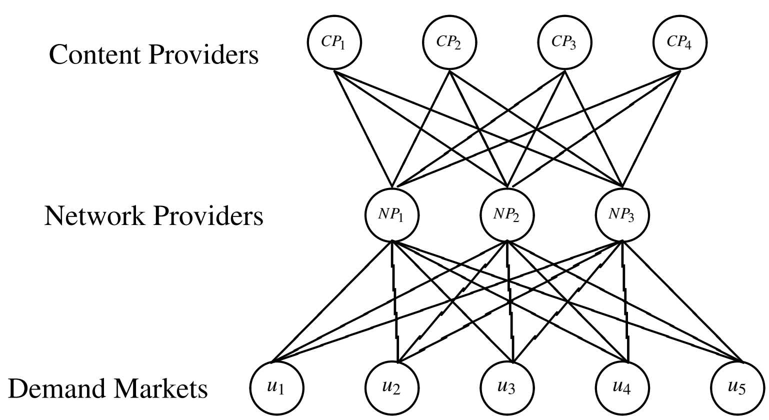

Figure 13: Network Topology of Content Flows for Example 4

Table 1: Demand Functions for Example 4

d_{ijk}	d^0_{ik}	p_{c_1}	p_{c_2}	p_{c_3}	p_{c_4}	p_{s_1}	p_{s_2}	p_{s_3}	q_{c_1}	q_{c_2}	q_{c_3}	q_{c_4}	q_{s_1}	q_{s_2}	q_{s_3}
d_{111}	112	-1.85	0.3	0.3	0.3	-2.1	0.4	0.4	0.76	-0.3	-0.3	-0.3	0.64	-0.4	-0.4
d_{211}	99	0.3	-2	0.3	0.3	-2.1	0.4	0.4	-0.3	0.56	-0.3	-0.3	0.64	-0.4	-0.4
d_{311}	100	0.3	0.3	-2.3	0.3	-2.1	0.4	0.4	-0.3	-0.3	0.61	-0.3	0.64	-0.4	-0.4
d_{411}	97	0.3	0.3	0.3	-1.99	-2.1	0.4	0.4	-0.3	-0.3	-0.3	0.71	0.64	-0.4	-0.4
d_{121}	112	1.85	0.3	0.3	0.3	0.4	-1.92	0.4	0.76	-0.3	-0.3	-0.3	-0.4	0.1	-0.4
d_{221}	99	0.3	-2	0.3	0.3	0.4	-1.92	0.4	-0.3	0.56	-0.3	-0.3	-0.4	0.1	-0.4
d_{321}	100	0.3	0.3	-2.3	0.3	0.4	-1.92	0.4	-0.3	-0.3	0.61	-0.3	-0.4	0.1	-0.4
d_{421}	97	0.3	0.3	0.3	-1.99	0.4	-1.92	0.4	-0.3	-0.3	-0.3	0.71	-0.4	0.1	-0.4
d_{131}	112	-1.85	0.3	0.3	0.3	0.4	0.4	-1.8	0.76	-0.3	-0.3	-0.3	-0.4	-0.4	0.5
d_{231}	99	0.3	-2	0.3	0.3	0.4	0.4	-1.8	-0.3	0.56	-0.3	-0.3	-0.4	-0.4	0.5
d_{331}	100	0.3	0.3	-2.3	0.3	0.4	0.4	-1.8	-0.3	-0.3	0.61	-0.3	-0.4	-0.4	0.5
d_{431}	97	0.3	0.3	0.3	-1.99	0.4	0.4	-1.8	-0.3	-0.3	-0.3	0.71	-0.4	-0.4	0.5
d_{112}	98	-1.85	0.3	0.3	0.3	-1.9	0.4	0.4	0.59	-0.3	-0.3	-0.3	0.49	-0.4	-0.4
d_{212}	100	0.3	-2.2	0.3	0.3	-1.9	0.4	0.4	-0.3	0.73	-0.3	-0.3	0.49	-0.4	-0.4
d_{312}	89	0.3	0.3	-1.89	0.3	-1.9	0.4	0.4	-0.3	-0.3	0.61	-0.3	0.49	-0.4	-0.4
d_{412}	110	0.3	0.3	0.3	-2.14	-1.9	0.4	0.4	-0.3	-0.3	-0.3	0.71	0.49	-0.4	-0.4

Continued

Table 1: Demand Functions (Continued)

d_{ijk}	d_{ik}^0	p_{c_1}	p_{c_2}	p_{c_3}	p_{c_4}	p_{s_1}	p_{s_2}	p_{s_3}	q_{c_1}	q_{c_2}	q_{c_3}	q_{c_4}	q_{s_1}	q_{s_2}	q_{s_3}
d_{122}	98	-1.85	0.3	0.3	0.3	0.4	-2.2	0.4	0.59	-0.3	-0.3	-0.3	-0.4	0.7	-0.4
d_{222}	100	0.3	-2.2	0.3	0.3	0.4	-2.2	0.4	-0.3	0.73	-0.3	-0.3	-0.4	0.7	-0.4
d_{322}	89	0.3	0.3	-1.89	0.3	0.4	-2.2	0.4	-0.3	-0.3	0.61	-0.3	-0.4	0.7	-0.4
d_{422}	110	0.3	0.3	0.3	-2.14	0.4	-2.2	0.4	-0.3	-0.3	-0.3	0.71	-0.4	0.7	-0.4
d_{132}	98	-1.85	0.3	0.3	0.3	0.4	0.4	-2	0.59	-0.3	-0.3	-0.3	-0.4	-0.4	0.65
d_{232}	100	0.3	-2.2	0.3	0.3	0.4	0.4	-2	-0.3	0.73	-0.3	-0.3	-0.4	-0.4	0.65
d_{332}	89	0.3	0.3	-1.89	0.3	0.4	0.4	-2	-0.3	-0.3	0.61	-0.3	-0.4	-0.4	0.65
d_{432}	110	0.3	0.3	0.3	-2.14	0.4	0.4	-2	-0.3	-0.3	-0.3	0.71	-0.4	-0.4	0.65
d_{113}	102	-2	0.3	0.3	0.3	-2	0.4	0.4	0.66	-0.3	-0.3	-0.3	0.3	-0.4	-0.4
d_{213}	95	0.3	-1.97	0.3	0.3	-2	0.4	0.4	-0.3	0.46	-0.3	-0.3	0.3	-0.4	-0.4
d_{313}	115	0.3	0.3	-2.2	0.3	-2	0.4	0.4	-0.3	-0.3	0.59	-0.3	0.3	-0.4	-0.4
d_{413}	97	0.3	0.3	0.3	-2.05	-2	0.4	0.4	-0.3	-0.3	-0.3	0.6	0.3	-0.4	-0.4
d_{123}	102	-2	0.3	0.3	0.3	0.4	-1.95	0.4	0.66	-0.3	-0.3	-0.3	-0.4	0.1	-0.4
d_{223}	95	0.3	-1.97	0.3	0.3	0.4	-1.95	0.4	-0.3	0.46	-0.3	-0.3	-0.4	0.1	-0.4
d_{323}	115	0.3	0.3	-2.2	0.3	0.4	-1.95	0.4	-0.3	-0.3	0.59	-0.3	-0.4	0.1	-0.4
d_{423}	97	0.3	0.3	0.3	-2.05	0.4	-1.95	0.4	-0.3	0.3	-0.3	0.6	-0.4	0.1	-0.4
d_{133}	102	-2	0.3	0.3	0.3	0.4	0.4	-2.24	0.66	-0.3	-0.3	-0.3	-0.4	-0.4	0.68
d_{233}	95	0.3	-1.97	0.3	0.3	0.4	0.4	-2.24	-0.3	0.46	-0.3	-0.3	-0.4	-0.4	0.68

Continued

Table 1: Demand Functions (Continued)

d_{ijk}	d_{ik}^0	p_{c_1}	p_{c_2}	p_{c_3}	p_{c_4}	p_{s_1}	p_{s_2}	p_{s_3}	q_{c_1}	q_{c_2}	q_{c_3}	q_{c_4}	q_{s_1}	q_{s_2}	q_{s_3}
d_{333}	115	0.3	0.3	-2.2	0.3	0.4	0.4	-2.24	-0.3	-0.3	0.59	-0.3	-0.4	-0.4	0.68
d_{433}	97	0.3	0.3	0.3	-2.05	0.4	0.4	-2.24	-0.3	-0.3	-0.3	0.6	-0.4	-0.4	0.68
d_{114}	97	-1.9	0.3	0.3	0.3	-1.9	0.4	0.4	0.49	-0.3	-0.3	-0.3	0.59	-0.4	-0.4
d_{214}	101	0.3	-2.2	0.3	0.3	-1.9	0.4	0.4	-0.3	0.7	-0.3	-0.3	0.59	-0.4	-0.4
d_{314}	90	0.3	0.3	-2	0.3	-1.9	0.4	0.4	-0.3	-0.3	0.65	-0.3	0.59	-0.4	-0.4
d_{414}	109	0.3	0.3	0.3	-1.87	-1.9	0.4	0.4	-0.3	-0.3	-0.3	0.7	0.59	-0.4	-0.4
d_{124}	97	-1.9	0.3	0.3	0.3	0.4	-2.14	0.4	0.49	-0.3	-0.3	-0.3	-0.4	0.6	-0.4
d_{224}	101	0.3	-2.2	0.3	0.3	0.4	-2.14	0.4	-0.3	0.7	-0.3	-0.3	-0.4	0.6	-0.4
d_{324}	90	0.3	0.3	-2	0.3	0.4	-2.14	0.4	-0.3	-0.3	0.65	-0.3	-0.4	0.6	-0.4
d_{424}	109	0.3	0.3	0.3	-1.87	0.4	-2.14	0.4	-0.3	-0.3	-0.3	0.7	-0.4	0.6	-0.4
d_{134}	97	-1.9	0.3	0.3	0.3	0.4	0.4	-1.89	0.49	-0.3	-0.3	-0.3	-0.4	-0.4	0.8
d_{234}	101	0.3	-2.2	0.3	0.3	0.4	0.4	-1.89	-0.3	0.7	-0.3	-0.3	-0.4	-0.4	0.8
d_{334}	90	0.3	0.3	-2	0.3	0.4	0.4	-1.89	-0.3	-0.3	0.65	-0.3	-0.4	-0.4	0.8
d_{434}	109	0.3	0.3	0.3	-1.87	0.4	0.4	-1.89	-0.3	-0.3	-0.3	0.7	-0.4	-0.4	0.8
d_{115}	101	-2	0.3	0.3	0.3	-1.85	0.4	0.4	0.65	-0.3	-0.3	-0.3	0.4	-0.4	-0.4
d_{215}	96	0.3	-1.95	0.3	0.3	-1.85	0.4	0.4	-0.3	0.48	-0.3	-0.3	0.4	-0.4	-0.4
d_{315}	114	0.3	0.3	-2.24	0.3	-1.85	0.4	0.4	-0.3	-0.3	0.56	-0.3	0.4	-0.4	-0.4
d_{415}	98	0.3	0.3	0.3	-1.86	-1.85	0.4	0.4	-0.3	-0.3	-0.3	0.65	0.4	-0.4	-0.4

Continued

Table 1: Demand Functions (Continued)

d_{ijk}	d_{ik}^0	p_{c_1}	p_{c_2}	p_{c_3}	p_{c_4}	p_{s_1}	p_{s_2}	p_{s_3}	q_{c_1}	q_{c_2}	q_{c_3}	q_{c_4}	q_{s_1}	q_{s_2}	q_{s_3}
d_{125}	101	-2	0.3	0.3	0.3	0.4	-2.31	0.4	0.65	-0.3	-0.3	-0.3	-0.4	0.2	-0.4
d_{225}	96	0.3	-1.95	0.3	0.3	0.4	-2.31	0.4	-0.3	0.48	-0.3	-0.3	-0.4	0.2	-0.4
d_{325}	114	0.3	0.3	-2.24	0.3	0.4	-2.31	0.4	-0.3	-0.3	0.56	-0.3	-0.4	0.2	-0.4
d_{425}	98	0.3	0.3	0.3	-1.86	0.4	-2.31	0.4	-0.3	-0.3	-0.3	0.65	-0.4	0.2	-0.4
d_{135}	101	-2	0.3	0.3	0.3	0.4	0.4	-1.95	0.65	-0.3	-0.3	-0.3	-0.4	-0.4	0.58
d_{235}	96	0.3	-1.95	0.3	0.3	0.4	0.4	-1.95	-0.3	0.48	-0.3	-0.3	-0.4	-0.4	0.58
d_{335}	114	0.3	0.3	-2.24	0.3	0.4	0.4	-1.95	-0.3	-0.3	0.56	-0.3	-0.4	-0.4	0.58
d_{435}	98	0.3	0.3	0.3	-1.86	0.4	0.4	-1.95	-0.3	-0.3	-0.3	0.65	-0.4	-0.4	0.58