Spatial Price Equilibrium, Perishable Products, and Trade Policies in the Covid-19 Pandemic

Anna Nagurney

Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts, Amherst, Massachusetts 01003

December 2020; revised February 2021
Accepted for publication in the Special Issue of the *Montes Taurus Journal of Pure and Applied Mathematics*
Dedicated to Professor Themistocles M. Rassias on the occasion of his 70th Birthday

Abstract: The Covid-19 pandemic is a major healthcare disaster of global proportions, and has resulted in immense suffering and loss of life, along with disruptions in economic and social activities. Governments of many nations in the midst of the pandemic have been instituting a variety of trade policies, including tariffs and quotas, on products deemed important to their citizenry. Many of the products, such as food items, medicines, and even PPEs are perishable products. In this paper, we construct the first multiproduct spatial price equilibrium model, in both static and dynamic versions, that captures product perishability, and trade policies in the form of tariffs and quotas. The static model is formulated as a variational inequality problem and the dynamics studied using a projected dynamical system. Theoretical results are presented along with a series of increasingly complex numerical examples, the solutions to which are computed using a proposed algorithm, for which convergence results are also given. The results illustrate the importance of having a rigorous theoretical and algorithmic framework to assess the impacts of trade policies, before their implementation.

Key words: spatial price equilibrium, perishable products, trade policies, Covid-19 pandemic, variational inequalities, projected dynamical systems

1. Introduction

The Covid-19 pandemic, which was declared by the World Health Organization on March 11, 2020, has impacted every corner of the globe economically and socially and has caused tremendous loss of life and suffering (World Health Organization (2020a)). Now, with progress on Covid-19 vaccines, and with approval by several countries, along with the distribution and administering of the vaccine commencing, there is hope (Krouse, Hopkins, and Wilde Mathews (2020)).

In the midst of the pandemic, there have been numerous supply chain disruptions, affecting food products to Personal Protective Equipment (PPE) due, for example, to shortages of labor as well as raw materials, transportation bottlenecks, and even major changes in the demand landscape as more people work from home (see Nagurney et al. (2020)). Many governments, concerned about various product shortages, have been instituting trade policies with the goal of protecting their citizenry. Global Trade Alert (GTA) reports that, as of April 25, 2020, 122 new export bans in more than 75 countries including the United States, China, and the European Union (EU) were imposed on medical supplies including antibiotics, and face masks. In addition, many countries modified the tariffs on various products, including food items (Evenett (2020), Pelc (2020), Global Trade Alert (2020)). Belarus has imposed temporary restrictions on exports of food products such as onions and garlic due to the pandemic, whereas Greece instituted temporary prohibition of exports of medicinal products (vaccines and medicines) that are or might be in short supply due to the Covid-19 pandemic (World Trade Organization (2020b)). China temporarily decreased import tariffs on several types of products such as medical supplies, raw materials, agricultural products, and meat (ITC MACMAP (2020)).

Interestingly, many of the products that have had trade policies imposed or modified during the pandemic are perishable products, including foods such as fruits and vegetables, and meat and dairy products, as well as various medicines, such as antibiotics. Indeed, transport of perishable food products, for example, even under the best of conditions (cf. Yu and Nagurney (2013)), and the same for vaccines and pharmaceuticals (cf. Nagurney et al. (2013)), given the distances that may be involved from producing regions (the supply markets) to consuming regions (the demand markets), may result in losses. Furthermore, face masks, a form of PPE, and essential to the safety of healthcare workers, also deteriorate over time, as was discovered in the US National Strategic Stockpile, which had not replenished N95 masks since 2009 (Reinhard and Brown (2020)), when the H1N1 flu epidemic struck. The Covid-19 pandemic has vividly revealed the global nature of trade and the associated vulnerabilities.

In this paper, inspired by the Covid-19 pandemic and the imposition of different trade policies on perishable products, we construct a multiproduct spatial price equilibrium model that allows for the quantification of the impacts of tariffs and quotas on perishable product flows and on supply market prices and demand market prices. We present the governing equilibrium conditions, derive the variational inequality formulation, and also detail the underlying dynamics based on the theory of projected dynamical systems (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996a)). We also propose an algorithm, which provides a discretization of the continuous time adjustment process until a steady state is achieved.

This work adds to the literature on spatial price equilibrium problems, which originated in the classical work of Samuelson (1952) and Takayama and Judge (1964, 1971). The literature has especially benefited from the methodology of variational inequalities, with some of the first major modeling extensions that used variational inequalities being due to Florian and Los (1982), Dafermos and Nagurney (1984, 1987), Dafermos (1986), Nagurney, Takayama, and Zhang (1995a, b) and Nagurney and Zhang (1996b). We utilize in our modeling framework arc multipliers, which were proposed by Thore (1986) and applied for spatial price equilibrium problems by Nagurney (1988) and Nagurney and Aronson (1989) but without the inclusion of any trade policies.

Recent contributions to the modeling, analysis, and solution of spatial price equilibrium problems have included, among others, the work of Nagurney, Li, and Nagurney (2014), who considered the introduction of quality into spatial price equilibrium models and that of Nagurney, Besik, and Dong (2019), who demonstrated how tariff rate quotas, along with tariffs and quotas, could be incorporated into spatial price equilibrium models, using the theory of variational inequalities. However, these contributions did not handle perishability of products, a feature that is quite relevant in the pandemic and imposed trade policies. Furthermore, they were single commodity/product models. The spatial price equilibrium framework, as noted by Nagurney, Li, and Nagurney (2014), is relevant to agricultural industries (cf. Thompson (1989)), including eggs (cf. Judge (1956)), potatoes (Howard (1984)), beef (Sohn (1970)), cereal grains (Ruijs et al. (2001)), soybeans (Barraza De La Cruz, Pizzolato, and Barraza de La Cruz (2010)), and dairy (Bishop, Pratt, and Novakovic (1994)).

It is important to recognize the breadth and depth of both theory and applications of variational inequalities. For outstanding edited volumes with applications ranging from finance to supply chains and cybersecurity and even game theory in the Covid-19 pandemic, please the books by Pardalos, Rassias, and Khan (2010), Rassias, Floudas, and Butenko (2014), Kalyagin, Pardalos, and Rassias (2014), Daras and Rassias (2015, 2017), and Rassias and Pardalos (2021).

This paper is organized as follows. In Section 2, first the static multiproduct spatial price equilibrium model for perishable products with tariffs and quotas is presented. The governing equilibrium conditions are stated, followed by the derivation of the variational inequality formulation. An alternative variational inequality is then given, which is in multiproduct shipment variables only. This enables an effective and easy to implement computational scheme. Existence of a solution to both variational inequalities is guaranteed. We then proceed to describe a dynamic adjustment process for the evolution of the multiproduct shipments. We construct the associated projected dynamical system and present several qualitative results under appropriate monotonicity conditions. In Section 3, the Euler method is recalled and its realization in the context of the new model detailed. It has the notable feature that, at each iteration, the multiproduct shipments can be determined using closed form expressions. We provide convergence results accompanied by a series of numerical examples of increasing complexity. Section 4 presents a summary of the results, along with the conclusions, and highlights several directions for future research.

2. Multiproduct Spatial Price Equilibrium for Perishable Products in the Presence of Trade Policies

In this Section, we introduce the multiproduct spatial price equilibrium model for perishable products in the presence of trade policies such as tariffs and quotas. Recall that spatial price equilibrium models are an example of perfect competition. We first, in Section 2.1, introduce the static model, and provide the governing equilibrium conditions, along with alternative variational inequality formulations. In Section 2.2, we describe the underlying dynamics associated with the multiproduct trade volumes (shipments) and present the projected dynamical systems model whose set of stationary points corresponds to the set of solutions of the variational inequality problem governing the static spatial price equilibrium model.

Please refer to Figure 1 for the underlying network structure of the spatial price equilibrium problem (SPEP). We focus, here, on the bipartite problem, for clarity and definiteness.

We assume that there are m supply markets and n demand markets that are spatially separated and involved, respectively, in the production and consumption of J products. The product that is produced and consumed is homogeneous in that the consumers at the

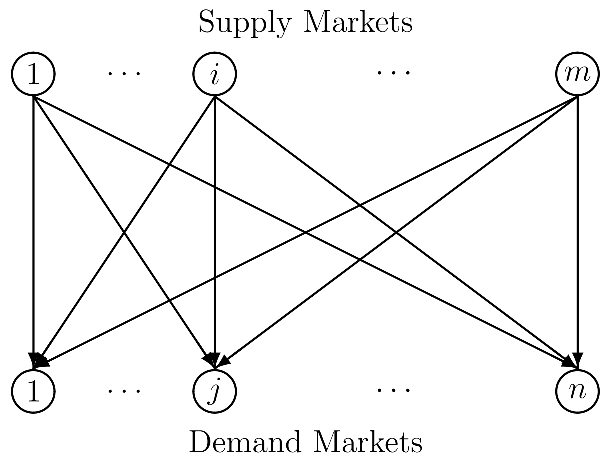

Figure 1: The Bipartite Network Structure of the Multiproduct Spatial Price Equilibrium Problem

demand markets do not differentiate by point of origin. Each supply market corresponds to a specific country and the same for each demand market. We denote a typical supply market by i; a typical demand market by j, and a typical product by k. Let s_i^k denote the nonnegative product output (supply) of product k produced at supply market i and let d_j^k denote the nonnegative demand for product k consumed at demand market j. Let Q_{ij}^k denote the nonnegative shipment of product k from supply market i to demand market j. The supplies are grouped into the vector $s \in R_+^{Jm}$; the demands into the vector $d \in R_+^{Jn}$, and the product shipments into the vector $Q \in R_+^{Jmn}$. Associated with each link (i,j) joining supply market i with demand market j is an arc multiplier for product k, α_{ij}^k, which lies in the range: $(0,1]$. The arc multipliers capture the perishability of the product as it moves from an origin node to a destination node. For example, if $\alpha_{ij}^k = .98$, this means that .98 of the amount of flow of product k that begins the transit on the link (i,j) actually arrives. All vectors here are assumed to be column vectors.

2.1 The Static Multiproduct Spatial Price Equilibrium Model

We first develop the multiproduct static model. The conservation of flow equations are as follows:

$$s_i^k = \sum_{j=1}^{n} Q_{ij}^k, \quad k = 1, \ldots, J; i = 1, \ldots, m; \tag{1}$$

$$d_j^k = \sum_{i=1}^{m} \alpha_{ij}^k Q_{ij}^k, \quad k = 1, \ldots, J; j = 1, \ldots, n. \tag{2}$$

We also must have nonnegativivity of the product shipments; that is:

$$Q_{ij}^k \geq 0, \quad k = 1, \ldots, J; i = 1, \ldots, m; j = 1, \ldots, n. \tag{3}$$

According to (1), the supply of the product k at each supply market is equal to the sum of the amounts of the product shipped to all the demand markets. (2), in turn, states that the quantity of product k consumed at a demand market is equal to the sum of the amounts of the product that actually arrive at the demand market. On the other hand, (3) guarantees the nonnegativity of the product shipments.

Associated with each supply market i and product k is a supply price π_i^k. We consider the general situation where the supply price at i of k may depend upon the entire supply pattern, that is,

$$\pi_i^k = \pi_i^k(s), \quad k = 1, \ldots, J; i = 1, \ldots, m. \tag{4}$$

The demand price at a demand market j for product k is denoted by ρ_j^k and, for the sake of generality, we let the demand price for a product at a demand market to depend, in general, upon the entire demand pattern:

$$\rho_j^k = \rho_j^k(d), \quad k = 1, \ldots, J; j = 1, \ldots, n. \tag{5}$$

Let c_{ij}^k denote the unit transportation cost associated with shipping product k from supply market i to demand market j, where the transportation cost is given by the function:

$$c_{ij}^k = c_{ij}^k(Q), \quad k = 1, \ldots, J; i = 1, \ldots, m; j = 1, \ldots, n. \tag{6}$$

The form of the functions in (6) can capture, for example, congestion.

We assume that all the supply price, demand price, and unit transportation cost functions are continuous and that the supply price functions and the unit transportation cost functions are monotone increasing, whereas the demand price functions are monotone decreasing (see also Nagurney (1999)).

Associated with supply market i, demand market j, and product k is a unit tariff τ_{ij}^k, which is imposed by the country associated with demand market j on product k produced at supply market i (which recall also corresponds to a country). Of course, if there is no such unit tariff, then the τ_{ij}^k is set equal to zero. Furthermore, we introduce a quota $\bar{M}_{ij}^k$ on product k produced at supply market i and destined for demand market j.

We are now ready to state the multiproduct spatial price equilibrium conditions, which are a generalization of the well-known spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) (see also Nagurney (1999)), to include perishability as well as trade policy instruments, inspired by the Covid-19 pandemic. We note that Thore (1986) was the first to use arc multipliers in a spatial equilibrium model but it had separable price functions, fixed unit transportation costs, and assumed a single commodity. Moreover, no trade policy instruments were incorporated. Nagurney and Aronson (1989) utilized variational inequality theory for their formulation of a multiperiod spatial price equilibrium model with gains and losses with arc multipliers, but the model was single commodity and without trade policies. Nagurney, Besik, and Nagurney (2019) introduced a spatial price equilibrium model with tariffs and quotas but it also was a single commodity one and without perishability of the products.

We define the feasible set $\mathcal{K}^1 \equiv \{(s,Q,d) | (1), (2), \text{ and } (3) \text{ hold and } Q^k_{ij} \leq \bar{M}^k_{ij}, \forall k, i, j\}$.

Definition 1: Multiproduct Spatial Price Equilibrium Conditions with Tariffs and Quotas for Perishable Products

A multiproduct supply, product shipment, and demand pattern $(s^*, Q^*, d^*) \in \mathcal{K}^1$ *is a spatial price equilibrium in the presence of unit tariffs and quotas if it satisfies the following conditions: for each product* k; $k = 1, \ldots, J$, *and for each pair of supply and demand markets* (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$\pi^k_i(s^*) + c^k_{ij}(Q^*) + \tau^k_{ij} \begin{cases} \leq \alpha^k_{ij}\rho^k_j(d^*), & \text{if } \; Q^{k*}_{ij} = \bar{M}^k_{ij}, \\ = \alpha^k_{ij}\rho^k_j(d^*), & \text{if } \; 0 < Q^{k*}_{ij} < \bar{M}^k_{ij}, \\ \geq \alpha^k_{ij}\rho^k_j(d^*), & \text{if } \; Q^{k*}_{ij} = 0. \end{cases} \tag{7}$$

According to (7), if there is a positive quantity of the product shipped from a supply market to a demand market, and this volume is not at the imposed quota, then, in equilibrium, the value of the surviving volume at the demand market must cover both the supply market price and the unit transportation cost (cf. Thore (1986)) and the unit tariff. If the volume is at the imposed quota, then the value at the demand market of the product can exceed the supply price plus the unit transportation cost and the tariff. If the supply price plus the unit transportation cost plus the unit tariff exceeds the value of the product at the demand market, then there will be no trade of the product between the pair of supply and demand markets for that product.

We now establish the variational inequality formulation of the above multiproductspatial price equilibrium conditions.

Theorem 1: Variational Inequality Formulation of Multiproduct Spatial Price Equilibrium with Tariffs and Quotas for Perishable Products

A multiproduct supply, product shipment, and demand pattern $(s^*, Q^*, d^*) \in \mathcal{K}^1$ *is a spatial price equilibrium with tariffs and quotas for perishable products according to Definition 1 if and only if it satisfies the variational inequality problem:*

$$\sum_{k=1}^{J}\sum_{i=1}^{m}\pi_i^k(s^*)\times(s_i-s_i^*)+\sum_{k=1}^{J}\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^k(Q^*)+\tau_{ij}^k)\times(Q_{ij}^k-Q_{ij}^{k*})-\sum_{k=1}^{J}\sum_{j=1}^{n}\rho_j^k(d^*)\times(d_j^k-d_j^{k*})\geq 0,$$

$$\forall (s,Q,d)\in\mathcal{K}^1. \tag{8}$$

Proof: We first establish necessity, that is, if $(s^*, Q^*, d^*) \in \mathcal{K}^1$ satisfies the multiproduct spatial price equilibrium conditions according to Definition 1, then it also satisfies variational inequality (8).

Note that, for a fixed product k and a fixed pair of supply and demand markets (i, j), (7) implies that

$$(\pi_i^k(s^*)+c_{ij}^k(Q^*)+\tau_{ij}^k-\alpha_{ij}^k\rho_j^k(d^*))\times(Q_{ij}^k-Q_{ij}^{k*})\geq 0,\ \forall Q_{ij}^k,\ \text{such that } 0\leq Q_{ij}^k\leq \bar{M}_{ij}^k. \tag{9}$$

Indeed, since, if $\bar{M}_{ij}^k > Q_{ij}^{k*} > 0$, we know, from the equilibrium conditions, that the expression to the left of the multiplication sign in (9) will be identically zero, so (9) holds true; also, if $Q_{ij}^{k*} = 0$, then the expression preceding and following the multiplication sign in (9) will be nonnegative and, hence, the product is also nonnegative and (9) holds true for this case, as well. Finally, if $Q_{ij}^{k*} = \bar{M}_{ij}^k$, then (9) also holds true. Summing now (9) over all products k; over all supply markets i, and over all demand markets j, we obtain:

$$\sum_{k=1}^{J}\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i^k(s^*)+c_{ij}^k(Q^*)+\tau_{ij}^k-\alpha_{ij}^k\rho_j^k(d^*))\times(Q_{ij}^k-Q_{ij}^{k*})\geq 0,$$

$$\forall Q_{ij}^k,\ \text{such that } 0\leq Q_{ij}^k\leq\bar{M}_{ij}^k,\forall i,j,k. \tag{10}$$

Rewriting (10) as:

$$\sum_{k=1}^{J}\sum_{i=1}^{m}\pi_i^k(s^*)\times\sum_{j=1}^{n}(Q_{ij}^k-Q_{ij}^{k*})+\sum_{k=1}^{J}\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^k(Q^*)+\tau_{ij}^k)\times(Q_{ij}^k-Q_{ij}^{k*})$$

$$-\sum_{k=1}^{J}\sum_{j=1}^{n}\rho_j^k(d^*)\times(\sum_{i=1}^{m}\alpha_{ij}^kQ_{ij}^k-\sum_{i=1}^{m}\alpha_{ij}^kQ_{ij}^{k*})\geq 0,\quad \forall Q_{ij}^k,\ \text{such that } 0\leq Q_{ij}^k\leq\bar{M}_{ij}^k,\forall i,j,k, \tag{11}$$

and then simplifying (11) by using the supply and demand conservation of flow equations (1) and (2) yields:

$$\sum_{k=1}^{J}\sum_{i=1}^{m}\pi_i^k(s^*)\times(s_i^k-s_i^{k*})+\sum_{k=1}^{J}\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^k(Q^*)+\tau_{ij}^k)\times(Q_{ij}^k-Q_{ij}^{k*})-\sum_{k=1}^{J}\sum_{j=1}^{n}\rho_j^k(d^*)\times(d_j^k-d_j^{k*})\geq 0, \tag{12}$$

$$\forall (s,Q,d)\in\mathcal{K}^1.$$

Note that (12) corresponds to variational inequality (8).

We now establish sufficiency, that is, if $(s^*,Q^*,d^*)\in\mathcal{K}^1$ satisfies variational inequality (8) then it also satisfies the multiproduct spatial price equilibrium conditions (7).

We expand variational inequality (8), with the use of the conservation of flow equations (1) and (2), which yields:

$$\sum_{k=1}^{J}\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i^k(s^*)+c_{ij}^k(Q^*)+\tau_{ij}^k-\alpha_{ij}^k\rho_j^k(d^*))\times(Q_{ij}^k-Q_{ij}^{k*})\geq 0,$$

$$\forall Q\in R_+^{Jmn},\text{ such that } Q_{ij}^k\leq\bar{M}_{ij}^k, \forall i,j,k. \tag{13}$$

Let $Q_{ij}^k=Q_{ij}^{k*}$, $\forall(i,j)\neq(h,l)$; $k\neq o$, and substitute into (13). The resultant is:

$$(\pi_h^o(s^*)+c_{hl}^o(Q^*)+\tau_{hl}^o-\alpha_{hl}^o\rho_l(d^*))\times(Q_{hl}^o-Q_{hl}^{o*})\geq 0,\quad \forall Q_{hl}^o,\text{ such that }\bar{M}_{hl}^o\geq Q_{hl}\geq 0. \tag{14}$$

But (14) implies that, if $Q_{hl}^{o*}=0$, then $(\pi_h^o(s^*)+c_{hl}^o(Q^*)+\tau_{hl}^o-\alpha_{hl}^o\rho_l^o(d^*))\geq 0$, and, if $\bar{M}_{hl}^o>Q_{hl}^{o*}>0$, then, for (14) to hold, $(\pi_h^o(s^*)+c_{hl}^o(Q^*)+\tau_{hl}^o-\alpha_{hl}^o\rho_l^o(d^*))=0$. Finally, (14) implies that for $Q_{hl}^{o*}=\bar{M}_{hl}^o$, then $(\pi_h^o(s^*)+c_{hl}^o(Q^*)+\tau_{hl}^o-\alpha_{hl}^o\rho_l^o(d^*))\leq 0$. Since these results hold for any pair (h,l) and any product o, we can conclude that the equilibrium conditions (7) are satisfied by the multiproduct shipment pattern satisfying (13).

The proof is complete. □

We now provide an alternative variational inequality to that of (8) in which the variables are exclusively multiproduct shipments. Such a formulation enables a more direct determination of the evolution of the product shipments over time via a projected dynamical system (PDS). The constructed PDS will then be used to propose an algorithm, which corresponds to a discretization of the continuous-time adjustment processes provided by the PDS.

We define multiproduct supply price functions and demand price functions, denoted, respectively, by $\hat{\pi}_i^k(Q)$ for $k=1,\ldots,J$; $i=1,\ldots,m$, and by $\hat{\rho}_j^k(Q)$ for $k=1,\ldots,J$;

$j = 1, \ldots, n$, that are functions of the multiproduct shipments. This can be done because of constraints (1) and (2). We, therefore, have:

$$\hat{\pi}_i^k = \hat{\pi}_i^k(Q) \equiv \pi_i^k(s), \quad k = 1, \ldots, J; i = 1, \ldots, m, \tag{15}$$

and

$$\hat{\rho}_j^k = \hat{\rho}_j^k(Q) \equiv \rho_j^k(d), \quad k = 1, \ldots, J; j = 1, \ldots, n. \tag{16}$$

We define the feasible set $\mathcal{K}^2 \equiv \{Q | Q \in R_+^{Jmn}, \text{ and } Q_{ij}^k \leq \bar{M}_{ij}^k, \forall k, i, j\}$.

The following corollary is immediate.

Corollary 1: Alternative Variational Inequality Formulation of the Multiproduct Spatial Price Equilibrium with Tariffs and Quotas for Perishable Products

A multiproduct shipment pattern $Q^* \in \mathcal{K}^2$ is a spatial price equilibrium with tariffs and quotas for perishable products according to Definition 1 if and only if it satisfies the variational inequality problem:

$$\sum_{k=1}^{J} \sum_{i=1}^{m} \sum_{j=1}^{n} (\hat{\pi}_i^k(Q^*) + c_{ij}^k(Q^*) + \tau_{ij}^k - \alpha_{ij}^k \hat{\rho}_j^k(Q^*)) \times (Q_{ij}^k - Q_{ij}^{k*}), \quad \forall Q \in \mathcal{K}^2. \tag{17}$$

We now put variational inequality (17) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

where $\mathcal{K}$ is the feasible set, which must be closed and convex. The vector X is an N-dimensional vector, as is $F(X)$, with $F(X)$ being continuous and given, and maps X from $\mathcal{K}$ into R^N. $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. We define the vector $X \equiv Q$ and the vector $F(X)$ with components $F_{ij}^k(X) = \hat{\pi}_i^k(Q) + c_{ij}^k(Q) + \tau_{ij}^k - \alpha_{ij}^k \hat{\rho}_j^k(Q)$; $k = 1, \ldots, J$; $i = 1, \ldots, m$; $j = 1, \ldots, n$. Here $N = Jmn$. Also, we define the feasible set $\mathcal{K} \equiv \mathcal{K}^2$. Then, variational inequality (17) can be placed into standard form (18).

Remark

Since the feasible sets $\mathcal{K}^1$ and $\mathcal{K}^2$ are compact and the functions that enter the respective variational inequalities are assumed to be continuous, existence of a solution to both variational inequality (8) and variational inequality (17) is guaranteed from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)). It also follows from the classical theory that if $F(X)$ in (19) is strictly monotone, that is,

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \quad X^1 \neq X^2, \tag{19}$$

then the solution X^* *which recall is equal to Q^* is unique.

For additional background on the variational inequality problem, we refer the reader to the book by Nagurney (1999).

2.2 The Projected Dynamical System Multiproduct Spatial Price Model

We now propose a dynamic adjustment process for the evolution of the multiproduct shipments. Note that, for a current vector of multiproduct shipments at time t, $X(t) = Q(t)$, $-F^1_{ij}(X(t)) = \alpha^k_i \hat{\rho}^k_j(Q(t)) - c^k_{ij}(Q(t)) - \tau^k_{ij} - \hat{\pi}^k_i(Q(t))$ is the excess value of the product between demand market j and supply market i. In our framework, the rate of change of the product shipment of product k between a supply and demand market pair (i, j), which is denoted by $\dot{Q}^k_{ij}$, is in proportion to $-F^k_{ij}(X)$, as long as the product shipment Q^k_{ij} is positive, and not at its quota; that is, when $\bar{M}^k_{ij} > Q^k_{ij} > 0$:

$$\dot{Q}^k_{ij} = \alpha^k_{ij} \hat{\rho}^k_j(Q) - c^k_{ij}(Q) - \tau^k_{ij} - \hat{\pi}^k_i(Q). \tag{20}$$

However, when Q^k_{ij} falls on the boundary, that is, is at level zero or at its imposed quota, then we have that

$$\dot{Q}^k_{ij} \; = \; \min\{\bar{M}^k_{ij}, \max\{0, \alpha^k_{ij} \hat{\rho}^k_j(Q) - c^k_{ij}(Q) - \tau^k_{ij} - \hat{\pi}^k_i(Q)\}\}. \tag{21}$$

We can write (20) and (21) compactly as:

$$\dot{Q}^k_{ij} = \begin{cases} \alpha^k_{ij} \hat{\rho}^k_j(Q) - c^k_{ij}(Q) - \tau^k_{ij} - \hat{\pi}^k_i(Q), & \text{if} \quad 0 < Q_{ij} < \bar{M}^k_{ij} \\ \min\{\bar{M}^k_{ij}, \max\{0, \alpha^k_{ij} \hat{\rho}^k_j(Q) - c^k_{ij}(Q) - \tau^k_{ij} - \hat{\pi}^k_i(Q)\}\}, & \text{otherwise.} \end{cases} \tag{22}$$

Applying (22) to all products k; $k = 1, \ldots, J$; and all supply and demand market pairs (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, yields the following pertinent ordinary differential equation (ODE) for the adjustment processes of the multiproduct shipments, in vector form:

$$\dot{X} \; = \; \Pi_{\mathcal{K}}(X, -F(X)), \tag{23}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) \; = \; \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{24}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P_{\mathcal{K}}(X) \, = \, \text{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{25}$$

and where $\|\cdot\| = \langle x, x \rangle$. Observe that (23) has a disctinuoys righthand side, which is in cotrast to classical dynamical systems (cf. Hirsch, Smale, and Devaney (2013)).

We provide the interpretation of the ODE (23) in the context of the multiproduct spatial model for perishable products with tariffs and quotas. Observe that the ODE (23) guarantees that the product shipments are always nonnegative and that they never go above the imposed quotas. ODE (23), nevertheless, retains the interpretation that if X at time t lies in the interior of $\mathcal{K}$, then the rate at which X changes is greatest when the vector field $-F(X)$ is greatest. Furthermore, when the vector field $-F(X)$ pushes X to the boundary of the feasible set $\mathcal{K}$, then the projection $\Pi_{\mathcal{K}}$ ensures that X stays within $\mathcal{K}$.

Dupuis and Nagurney (1993) developed the basic theory with regards to existence and uniqueness of projected dynamical systems as defined by (23). We cite the following theorem from that paper.

Theorem 2

X^* solves the variational inequality problem (18), equivalently, (17), if and only if it is a stationary point of the ODE (23), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{26}$$

This theorem demonstrates that the necessary and sufficient condition for a multiproduct shipment pattern $X^* = Q^*$ to be a spatial price equilibrium, according to Definition 1, is that $X^* = Q^*$ is a stationary point of the adjustment process defined by ODE (23), that is, X^* is the point at which $\dot{X} = 0$. We refer to (23) as PDS $(F, \mathcal{K})$.

Lipschitz continuity of $F(X)$ (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996a)) guarantees the existence of a unique solution to (23), where we have that $X^0(t)$ satisfies ODE (23) with initial shipment and quality level pattern Q^0. In other words, $X^0(t)$ solves the initial value problem (IVP)

$$\dot{X} \, = \, \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) \, = \, X^0, \tag{27}$$

with $X^0(0) = X^0$.

Following Nagurney and Zhang (1996), the following theorem is immediate.

Theorem 3: Stability Analysis

(i). *If $F(X)$ is monotone, then every multiproduct spatial price equilibrium for perishable products with tariffs and quotas, X^*, is a global monotone attractor for the the PDS$(F, \mathcal{K})$. If $F(X)$ is locally monotone at X^*, then it is a monotone attractor for the PDS$(F, \mathcal{K})$.*

(ii). *If $F(X)$ is strictly monotone, then the unique multiproduct spatial price equilibrium for perishable products with tariffs and quotas is a strictly global monotone attractor for the PDS$(F, \mathcal{K})$. If $F(X)$ is locally strictly monotone at X^*, then it is a strictly monotone attractor for the PDS$(F, \mathcal{K})$.*

(iii). *If $F(X)$ is strongly monotone, then the unique multiproduct spatial price equilibrium for perishable products with tariffs and quotas is globally exponentially stable for the PDS$(F, \mathcal{K})$. If $F(X)$ is locally strongly monotone at X^*, then X^* is exponentially stable.*

3. The Algorithm and Numerical Examples

The projected dynamical system (23) may be interpreted as a continuous-time adjustment process in multiproduct shipments. Nevertheless, for computational purposes, a discrete-time algorithm, which serves as an approximation to the continuous-time trajectories is essential. In Section 3.1 we present the algorithm, the Euler method, along with convergence and results and, in Section 3.2, we present several numerical examples, which are solved using the Euler method.

We recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, iteration τ of the Euler method (see also Nagurney and Zhang (1996a)) is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{28}$$

where recall that $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (18).

As established in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996a), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other spatial models can be found in Nagurney and Zhang (1996a), Nagurney, Dupuis, and Zhang (1994), Nagurney et al. (2002), Cruz (2008), Nagurney (2006, 2010), Nagurney and Yu (2012), and in Nagurney, Li, and Nagurney (2014).

Explicit Formulae for the Euler Method Applied to the Multiproduct Spatial Price Equilibrium Model for Perishable Products with Tariffs and Quotas

The algorithm yields explicit formulae for the product shipments at each iteration. Specifically, we have the following closed form expression for the product shipments $k = 1, \ldots, J$; $i = 1, \ldots, m; j = 1, \ldots, n$:

$$Q_{ij}^{k\tau+1} = \min\{\bar{M}_{ij}^k, \max\{0, Q_{ij}^{k\tau} + a_\tau(\alpha_{ij}^k \hat{\rho}_j^k(Q^\tau) - c_{ij}^k(Q^\tau) - \tau_{ij}^k - \hat{\pi}_i^k(Q^\tau))\}\}. \quad (29)$$

Expression (29) has an interpretation as a discrete-time adjustment process.

We now provide the convergence result. The proof is direct from Theorem 6.10 in Nagurney and Zhang (1996a).

Theorem 4: Convergence

If in the multiproduct spatial price equilibrium problem for perishable products with tariffs and quotas $F(X)$ is strictly monotone at any equilibrium pattern and F is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}, \quad (30)$$

where L is a positive number known as the Lipschitz constant, then there exists a unique equilibrium multiproduct shipment pattern $Q^* \in \mathcal{K}$ and any sequence generated by the Euler method as given by (28), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to Q^*.

3.2 Numerical Examples

We now present numerical examples for illustrative purposes. The Euler method was implemented in FORTRAN, and a Linux system at the University of Massachusetts Amherst used for the computations. The convergence criterion was $\epsilon = 10^{-5}$; that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each product shipment differed from its respective value at the preceding iteration by no more than ϵ.

The sequence $\{a_\tau\}$ was: $1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We initialized the algorithm by setting each product shipment, $Q_{ij}^k = 0.00$, $\forall k, i, j$.

Examples 1, 2, and 3

Examples 1, 2, and 3 consist of two supply markets and two demand markets, as illustrated in Figure 2. Supply Market 1 is in Country 1 and Supply Market 2 is in Country 2; the same for the respective demand markets. There is a single product and, therefore, we suppress the superscript notation of "1" in the functions and the variables. Example 1 serves as the baseline and, for comparison purposes, we assume no perishability of the product so all the α_{ij}s are equal to 1. There are no tariffs; hence, all the τ_{ij}s are equal to 0.00 and there are no quotas in Example 1.

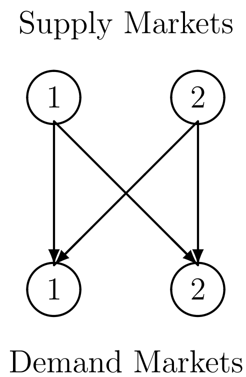

Figure 2: The Network Structure of the Spatial Price Equilibrium Example 1, 2, and 3 Problems

The supply price functions are:

$$\pi_1(s) = 5s_1 + s_2 + 2, \quad \pi_2(s) = 2s_2 + 1.5s_1 + 1.5.$$

The unit transportation cost functions are:

$$c_{11}(Q) = .01Q_{11}^2 + Q_{11} + 10, \quad c_{12}(Q) = .02Q_{12}^2 + 2Q_{12} + 13.5,$$

$$c_{21}(Q) = .03Q_{21}^2 + 3Q_{21} + 14.25, \quad c_{22}(Q) = .02Q_{22}^2 + .02Q_{22} + 11.5.$$

The demand price functions are:

$$\rho_1(d) = -2d_1 - 1.5d_2 + 380, \quad \rho_2(d) = -4d_2 - d_1 + 410.$$

The Euler method converges in 114 iterations and yields the equilibrium product shipments, along with the incurred supply prices, unit transportation costs, and the demand prices reported in Table 1.

Example 2 has the same data as Example 1 except that now Country 1 is concerned about the producers at its supply market and, hence, it imposes a unit tariff $\tau_{21} = 124$. The algorithm, again, converges in 114 iterations and yields the solution reported in Table 1. Note that now $Q^*_{21} = 0.00$; in other words, Country 2 no longer provides any of its product to consumers at the demand market in Country 1. We also performed some sensitivity analysis and observed that if the unit tariff is decreased just slightly to 123, for example, or takes on any lower value, then there is a positive volume of flow of the product from Country 2 to Country 1. Observe, also, that Country 1, in imposing such a tariff, protects the producers at its supply market since the supply price π_1 at the equilibrium increases to 208.79 from 200.12 (and the supply also increases). However, its consumers at the demand market in Country 1 now pay a higher price for the product of 260.40, as compared to a price of 227.67 in Example 1. Demand decreases at Demand Market 1 but increases at Demand Market 2.

Example 3, in turn, is constructed from Example 2 and has the same data as Example 2 except now we incorporate arc multipliers to capture perishability. The arc multipliers are:

$$\alpha_{11} = .98, \quad \alpha_{12} = .95, \quad \alpha_{21} = .95, \quad \alpha_{22} = .99.$$

The Euler method converged in 105 iterations to the solution reported in Table 1. In Example 3, the production outputs (supplies) of the perishable product increase at both supply markets and that is reasonable since the producers need to supply more of the product due to losses en route to the demand markets. The demand prices for the perishable product rise in both demand markets, signifying the importance of preserving the flow of the perishable product in transit.

Examples 4, 5, and 6

Examples 4, 5, and 6 had the network topology depicted in Figure 3. In particular, a new demand market was added. Example 4 had the same data as Example 3 but with the following new data associated with the new transportation links joining the supply markets to the new demand market:

$$c_{13}(Q) = .02Q_{13}^2 + 2Q_{13} + 14.5, \quad c_{23}(Q) = .03Q_{23}^2 + 3Q_{23} + 15.$$

Also, the demand price at Demand Market 3 corresponding to another country was as follows:

$$\rho_3(d) = -3d_3 - d_2 + 350.$$

Finally, the arc multipliers on the new links are: $\alpha_{13} = \alpha_{23} = .97$.

Equilibrium Product Flows	Ex. 1	Ex. 2	Ex. 3
Q_{11}^*	23.16	31.61	32.08
Q_{12}^*	6.57	2.79	1.38
Q_{21}^*	21.72	0.00	0.00
Q_{22}^*	27.80	34.80	35.50
Supply Prices at Equilibrium	**Ex. 1**	**Ex. 2**	**Ex. 3**
$\pi_1(s^*)$	200.17	208.79	204.81
$\pi_2(s^*)$	145.13	122.70	122.70
Transportation Costs at Equilibrium Plus Tariff	**Ex. 1**	**Ex. 2**	**Ex. 3**
$c_{11}(Q^*)$	38.53	51.60	52.38
$c_{12}(Q^*)$	27.50	19.23	16.29
$c_{21}(Q^*) + \tau_{21}$	93.57	138.25	138.25
$c_{22}(Q^*)$	82.54	105.33	107.71
Demand Prices at Equilibrium	**Ex. 1**	**Ex. 2**	**Ex. 3**
$\rho_1(d^*)$	238.69	260.40	262.43
$\rho_2(d^*)$	227.67	228.03	232.74

Table 1: Equilibrium Product Shipments and Incurred Supply Prices, Transportation Costs (which Include Unit Tariff if Imposed), and Demand Prices for Examples 1, 2, and 3

The equilibrium solution is reported in Table 2. With a demand market added, the demand market prices increase at the two original demand markets and the supply market prices increase at both supply markets, as compared to their respective values in Example 3.

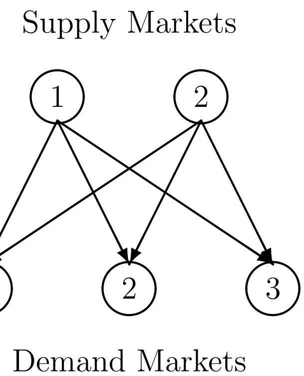

Figure 3: The Network Structure of the Spatial Price Equilibrium Example 4, 5, and 6 Problems

The Euler method converged in 139 iterations to the solution reported in Table 2.

Example 5, in turn, has the same data as that in Example 4 but now we investigate the impact of reduction in a tariff, as has been happening in the Covid-19 pandemic, especially on essential perishable products such as certain food items, medicines, and PPE. Hence, the tariff τ_{21}, which was imposed in Example 2, where it was set to 124, and retained in the examples through Example 4, is now reduced to $\tau_{21} = 10$. The Euler method converges in 154 iterations to the equilibrium solution reported in Table 2. With the reduction in the tariff, the consumers at Demand Market 1 (associated with Country 1) benefit with a reduction in the price of the product. Clearly, governmental authorities should weigh the costs and the benefits of potential trade policies not just for those involved in production at the supply markets but also the consumers at the demand markets, especially those in the countries imposing the trade policies.

Example 6 has the same data as Example 5, except that now Country 2 is concerned about the volume of shipments from its supply market to its country's demand market so it imposes a quota of $\bar{M}_{23} = 10$ on the perishable product. The Euler method now converges in 149 iterations to the equilibrium pattern reported in Table 2. The volume of product shipment from Supply Market 2 to Demand Market 3 is at the quota of 10.00. The supply market price at the first supply market increases, whereas that at the second market decreases. The demand market prices decrease at the first two demand markets but increase at the third demand market.

Examples 7, 8, and 9

In this set of examples there are two products. The network topology remains as in Figure 3, since there are two supply markets and three demand markets involved, respectively, in the production and consumption of the two products.

Example 7 has the same data as Example 6 for Product 1 but new data for Product 2 as follows.

The supply price functions associated with Product 2 are:

$$\pi_1^2(s) = 2s_1^2 + s_1^1 + 2, \quad \pi_2^2(2) = 3s_2^2 + s_2^1 + 1.$$

The unit transportation cost functions associated with Product 2 are:

$$c_{11}^2(Q) = .01Q_{11}^2 + Q_{11}^2 + 10, \quad c_{12}^2(Q) = .02Q_{12}^2 + 2Q_{12}^2 + 13.5, \quad c_{13}^2(Q) = .02Q_{13}^2 + 2Q_{13}^2 + 14.5,$$

$$c_{21}^2(Q) = .03Q_{21}^2 + 3Q_1^2 + 14.25, \quad c_{22}^2(Q) = .02Q_{22}^2 + 2Q_{22}^2 + 11.5, \quad c_{23}^2(Q) = .03Q_{23}^2 + 3Q_{23}^2 + 15.$$

The demand price functions at the demand markets for Product 2 are:

$$\rho_1^2(d) = -2d_1^2 - d_1^1 + 250, \quad \rho_2^2(d) = -3d_2^2 - d_2^1 + 200, \quad \rho_3^2(d) = -4d_3^2 - d_3^1 + 300.$$

Equilibrium Product Flows	Ex. 4	Ex. 5	Ex. 6
Q_{11}^*	28.90	22.17	20.81
Q_{12}^*	1.70	3.52	2.21
Q_{13}^*	3.04	5.62	9.38
Q_{21}^*	0.00	15.77	17.29
Q_{22}^*	31.33	27.18	28.83
Q_{23}^*	20.13	17.37	10.00
Supply Prices at Equilibrium	**Ex. 4**	**Ex. 5**	**Ex. 6**
$\pi_1(s^*)$	221.67	218.88	220.07
$\pi_2(s^*)$	154.88	169.11	162.32
Transportation Costs at Equilibrium Plus Tariff	**Ex. 4**	**Ex. 5**	**Ex. 6**
$c_{11}(Q^*)$	47.26	37.09	35.13
$c_{12}(Q^*)$	16.95	20.78	18.02
$c_{13}(Q^*)$	20.77	26.38	35.01
$c_{21}(Q^*) + \tau_{21}$	138.25	79.03	85.07
$c_{22}(Q^*)$	93.78	80.64	85.79
$c_{23}(Q^*)$	87.56	76.15	48.00
Demand Prices at Equilibrium	**Ex. 4**	**Ex. 5**	**Ex. 6**
$\rho_1(d^*)$	274.41	261.20	260.41
$\rho_2(d^*)$	251.18	252.28	250.62
$\rho_3(d^*)$	249.94	252.85	262.97

Table 2: Equilibrium Product Shipments and Incurred Supply Prices, Transportation Costs (which Include Unit Tariff if Imposed), and Demand Prices for Examples 4, 5, and 6

The additional arc multipliers associated with the links and Product 2 are:

$$\alpha_{11}^2 = .98, \quad \alpha_{12}^2 = .97, \quad \alpha_{13}^2 = .96,$$

$$\alpha_{21}^2 = .99, \quad \alpha_{22}^2 = .98, \quad \alpha_{23}^2 = .97.$$

The Euler method converges in 149 iterations to the equilibrium solution reported in Table 3. Note that now we use the superscript "1" for product 1 functions and variables.

Observe that in Example 7, the equilibrium product flows, along with the incurred functional values, for Product 1 are the same as in Example 6 since we did not modify the functions associated with Product 1 (and, hence, they do not depend on Product 2). More of Product 1 is produced at Supply Market 2, whereas more of Product 2 is produced at Supply Market 1. The demand market prices for Product 2 are substantially lower than those for Product 1 at each of the demand markets. Overall, the total demand for Product 2 is lower than that for Product 1 in the supply chain network economy.

Equilibrium Product Flows	Ex. 7	Ex. 8	Ex. 9
Q_{11}^{1*}	20.81	20.66	28.68
Q_{12}^{1*}	2.21	5.19	2.04
Q_{13}^{1*}	9.38	10.12	5.00
Q_{21}^{1*}	17.29	19.32	5.00
Q_{22}^{1*}	28.83	33.52	39.36
Q_{23}^{1*}	10.00	10.00	5.00
Q_{11}^{2*}	22.59	22.03	27.96
Q_{12}^{2*}	.70	0.00	0.00
Q_{13}^{2*}	16.53	16.40	5.00
Q_{21}^{2*}	7.12	6.76	5.00
Q_{22}^{2*}	1.29	0.00	3.02
Q_{23}^{2*}	11.27	11.08	5.00
Supply Prices at Equilibrium	**Ex. 7**	**Ex. 8**	**Ex. 9**
$\pi_1^1(s^*)$	220.07	220.24	213.56
$\pi_2^1(s^*)$	162.32	153.94	119.75
$\pi_1^2(s^*)$	114.02	114.84	103.63
$\pi_2^2(s^*)$	116.15	117.37	89.41
Transportation Costs at Equilibrium Plus Tariff	**Ex. 7**	**Ex. 8**	**Ex. 9**
$c_{11}^1(Q^*)$	35.13	34.92	46.90
$c_{12}^1(Q^*)$	18.02	24.42	17.67
$c_{13}^1(Q^*)$	35.01	36.78	25.00
$c_{21}^1(Q^*) + \tau_{21}$	85.07	93.41	40.00
$c_{22}^1(Q^*)$	85.79	101.02	121.22
$c_{23}^1(Q^*)$	48.00	48.00	30.75
$c_{11}^2(Q^*)$	37.70	36.89	45.77
$c_{12}^2(Q^*)$	14.90	13.50	13.50
$c_{13}^2(Q^*)$	53.02	52.69	25.00
$c_{21}^2(Q^*)$	37.12	35.91	30.00
$c_{22}^2(Q^*)$	14.10	11.50	17.71
$c_{23}^2(Q^*)$	52.63	51.91	30.75
Demand Prices at Equilibrium	**Ex. 7**	**Ex. 8**	**Ex. 9**
$\rho_1^1(d^*)$	260.41	260.37	265.77
$\rho_2^1(d^*)$	250.62	257.54	243.40
$\rho_3^1(d^*)$	262.97	264.97	311.25
$\rho_1^2(d^*)$	154.82	154.82	152.45
$\rho_2^2(d^*)$	132.91	123.77	109.31
$\rho_3^2(d^*)$	174.00	174.51	251.70

Table 3: Equilibrium Product Shipments and Incurred Supply Prices, Transportation Costs (which Include Unit Tariff if Imposed), and Demand Prices for Examples 7, 8, and 9

In Example 8, as time passes, and consumers become aware of the availability of a second product, the functions may change. Hence, Example 8 has the same data as Example 7, except that now be modified the supply price functions and the demand price functions for Product 1 to represent the effect of Product 2 at the supply markets and the demand markets, respectively, as follows. The new supply price functions for Product 1 in Example 8 are:

$$\pi_1^1(s) = 5s_1^1 + s_1^2 + 2, \quad \pi_2^1(s) = 2s_2^1 + 1.5s_2^2 + 1.5.$$

The new demand price functions for Product 1 in Example 8 are:

$$\rho_1^1(d) = -2d_1^1 - 1.5d_1^2 + 380, \quad \rho_2^1(d) = -4d_2^1 - d_2^2 + 410, \quad \rho_3^1(d) = -3d_3^1 - d_3^2 + 350.$$

The Euler method converges in 124 iterations to the equilibrium solution in Table 3. The supply market prices and the demand market prices differ from those obtained in Example 7. Also, two of the product flows: Q_{12}^{2*} and Q_{22}^{2*} now drop to 0.00.

In Example 9, we explore the following scenario. Countries 1 and 2 are concerned about the growing pandemic and institute quotas of 5 on each of the exports of both products outside of their country. In other words, Country 1 does not have a quota on products from Supply Market 1 to Demand Market 1; similarly, Country 2 does not have a quota on its products from Supply Market 2 to Demand Market 2; all other exports are limited to a quota of 5 from a supply market to a demand market.

The Euler method converges in 71 iterations to the equilibrium solution in Table 3.

The trade policy of rather tight quotas on exports results in many of the trade flows being at the quota of 5 at the equilibrium. All the supply market prices are lower for each of the products, as compared to their respective values in Example 8. However, some of the demand market prices are higher, whereas others are lower. This shows the importance of having a rigorous theoretical and computational framework to ascertain the impacts of trade policies such as tariffs and quotas on perishable products in the pandemic, before they are applied.

The above numerical examples, although stylized, demonstrate that one can obtain both valuable quantitative as well as qualitative information regarding the impacts of trade policies in the form of tariffs and/or quotas in the case of perishable products in the Covid-19 pandemic through the use of the proposed rigorous modeling and algorithmic framework developed in this paper.

4. Summary and Conclusions

The Covid-19 pandemic has transformed the globe and the associated economic and social activities, causing immense disruptions, pain, and suffering. Although vaccines have been created through intense scientific efforts and are now being distributed, it will take a long time for many to be vaccinated given the many challenges involved. Furthermore, there is yet no cure for the disease Covid-19. Many countries in the midst of the pandemic have been imposing adding trade policies on a variety of products deemed important in the pandemic from certain food items to various medicines and medical supplies, including PPEs. Interestingly, quite a few of the products that have been the target of new trade policies from alterations in tariffs to quotas regarding exports are actually perishable products, notably, agricultural products, antibiotics, and even PPEs.

In this paper, we construct, for the first time, a multiproduct spatial price equilibrium model for perishable products with the integration of trade policies in the form of tariffs and quotas. The equilibrium conditions are shown to satisfy alternative variational inequality formulations. We also propose a dynamic version of the model, whose set of stationary points coincides with the set of solutions to a variational inequality that we derived. We present both qualitative results and outline a computational scheme, which can be interpreted as a time-discretization of the continuous time adjustment process associated with the evolution of the multiproduct shipments over time. The algorithm is implemented and a series of related and increasingly complex numerical examples solved. The results demonstrate the importance of having a rigorous framework for assessing the impacts of different trade policies in the Covid-19 pandemic, on supply market prices, demand market prices, as well as trade flows.

Further research may include extending this work to include paths of multiple links joining supply markets with demand markets, with associated arc multipliers for perishability as well as having the arc multipliers be a function of flows on the links. Conducting empirical research to parameterize the underlying model functions for specific perishable products would also be of value. We leave such research for the future.

Acknowledgments

This paper is dedicated to Professor Themistocles M. Rassias, on the occasion of his 70th birthday. Professor Rassias's scholarship, global impact, professional contributions, and many friendships and collaborations deserve celebrating! I am grateful to be able to write this paper in his honor and I thank him.

Looking forward to scientific conferences, post the pandemic, so that we can all meet again face to face to exchange our latest research and to also enjoy wonderful company and conference banquets together!

The author also acknowledges two reviewers and thanks them for their constructive comments on an earlier version of this paper.

References

Barraza De La Cruz, B.C., Pizzolato, N.D., Barraza De La Cruz, A., 2010. An application of the spatial equilibrium model to soybean production in tocantins and neighboring states in Brazil. Pesquisa Operacional, 30(2), 443-464.

Bishop, P., Pratt, J., Novakovic, A., 1994. Using a joint-input, multi-product formulation to improve spatial price equilibrium models. Cornell University, Department of Agricultural Economics Staff Paper no. 94-06, Ithaca, New York.

Cruz, J.M., 2008. Dynamics of supply chain networks with corporate social responsibility through integrated environmental decision-making. European Journal of Operational Research, 184, 1005-1031.

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. Operations Research, 32, 1069-1086.

Dafermos, S., Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. Regional Science and Urban Economics, 17(2), 245-254.

Daras, N.J., Rassias, M.Th., Editors, 2015. Computation, Cryptography, and Network Security. Springer International Publishing Switzerland.

Daras, N.J., Rassias, Th.M., Editors, 2017. Operations Research, Engineering, and Cyber Security - Trends in Applied Mathematics and Technology, Springer, New York.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research, 44, 9-42.

Evenett, S.J., 2020. Flawed prescription: Export curbs on medical goods won't tackle shortages. In: COVID-19 and Trade Policy: Why Turning Inward Won't Work, R. Baldwin, S.J. Evenett, Editors, CEPR Press, London, UK, pp 49-62. Available at:
https://voxeu.org/content/covid-19-and-trade-policy-why-turning-inward-won-t-work

Florian, M., Los. M. 1982. A new look at static spatial price equilibrium models. Regional

Science and Urban Economics, 12, 579597.

Global Trade Alert, 2020. The GTA reports. Available at:
https://www.globaltradealert.org/reports/51

Hieu, P.S., Harrison, S., 2011. A review of the formulation and application of the spatial equilibrium models to analyze policy. Journal of Forestry Research, 22(4), 671-679.

Hirsch, M.W., Smale, S., Devaney, R.L., 2013. Differential Equations, Dynamical Systems, and an Introduction to Chaos, third edition. Academic Press, New York.

Howard, E.A., 1984. An analysis of interregional competition in the U.S. summer potato market, Master of Science dissertation, Department of Agricultural Economics, Texas A&M University, College Station, Texas.

ITC MACMAP, 2020. COVID-19 temporary trade measures. Available at:
https://macmap.org/en/covid19

Judge, G.G., 1956. A spatial equilibrium model for eggs. Connecticut Agricultural Experiment Station, Storrs, Connecticut.

Kalyagin, V.A., Pardalos, P.M., Rassias, Th.M., Editors 2014. Network Models in Economics and Finance. Springer, New York.

Kinderlehrer, D., Stampacchia, G., 1980. Variational Inequalities and Their Applications. Academic Press, New York.

Krouse, S., Hopkins, J.S., Wilde Mathews, A., 2020. The mass distribution of Covid-19 vaccines is under way. 'Everything has to come together.' The Wall Street Journal, December 13.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Boston, Massachusetts.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits. Cheltenham, England: Edward Elgar Publishing.

Nagurney, A., 2010. Supply chain network design under profit maximization and oligopolistic competition. Transportation Research E, 46, 281-294.

Nagurney, A., Aronson, J., 1989. A general dynamic spatial price network equilibrium model with gains and losses. Networks, 19(7), 751-769.

Nagurney, A., Besik, D., Dong, J., 2019. Tariffs and quotas in world trade: A unified variational inequality framework. European Journal of Operational Research, 275(1), 347-360.

Nagurney, A., Dupuis, P., Zhang, D., 1994. A dynamical systems approach for network oligopolies and variational inequalities. Annals of Regional Science, 28, 263-283.

Nagurney, A., Ke, K., Cruz, J., Hancock, K., Southworth, F., 2002. Dynamics of supply chains: A multilevel (logistical/informational/financial) network perspective. Environment & Planning B, 29, 795-818.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Nagurney, A., Salarpour, M., Dong, J., Dutta, P., 2020. Competition for medical supplies under stochastic demand: A Generalized Nash Equilibrium framework. In Nonlinear Analysis and Global Optimization, T.M. Rassias and P.M. Pardalos, Editors, Springer Nature Switzerland AG, in press.

Nagurney, A., Takayama, T., Zhang, D., 1995a. Massively parallel computation of spatial price equilibrium problems as dynamical systems. Journal of Economic Dynamics and Control, 18, 3-37.

Nagurney, A., Takayama, T., Zhang, D., 1995b. Projected dynamical systems modeling and computation of spatial network equilibria. Networks, 26, 69-85.

Nagurney, A., Yu, M., 2012. Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. International Journal of Production Economics, 135, 532-540.

Nagurney, A., Yu, M., Masoumi, A.H., Nagurney, L.S., 2013. Networks Against Time: Supply Chain Analytics for Perishable Products. Springer Science + Business Media, New York.

Nagurney, A., Zhang, D., 1996a. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.

Nagurney, A., Zhang, D., 1996b. Stability of spatial price equilibrium modeled as a projected dynamical system. Journal of Economic Dynamics and Control, 20, 43-63.

Pardalos, P.M., Rassias. Th.M., Khan, A.A., 2010. Nonlinear Analysis and Variational

Problems - In Honor of George Isac. Springer, New York.

Pelc, K., 2020. Can COVID-era export restrictions be deterred? Canadian Journal of Political Science, 53(2), 349-356.

Rassias, Th.M., Floudas, C.A., Butenko, S., Editors, 2014. Optimization in Science and Engineering - In Honor of the 60th Birthday of Panos M. Pardalos. Springer, New York.

Rassias, Th.M., Pardalos, P.M., 2021. Nonlinear Analysis and Global Optimization. Springer International Publishing Switzerland, in press.

Reinhart, B., Brown, E., 2020. Face masks in national stockpile have not been substantially replenished since 2009. The Washington Post, March 10.

Ruijs, A., Schweigman, C., Lutz, C., Sirpe, G., 2001. Cereal trade in developing countries: Stochastic spatial equilibrium models. Technical report, University of Groningen, The Netherlands.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review, 42, 283-303.

Sohn, H.K., 1970. A spatial equilibrium model of the beef industry in the United States, PhD Dissertation, Department of Agricultural Economics, University of Hawaii.

Takayama, T., Judge, G.G., 1964. An intertemporal price equilibrium model. Journal of Farm Economics, 46, 477-484.

Takayama, T., Judge, G.G., 1971. Spatial and Temporal Price and Allocation Models. Amsterdam, The Netherlands: North-Holland.

Thompson, R.L., 1989. Spatial and temporal price equilibrium agricultural models. In: Quantitative Methods for Market Oriented Economic Analysis Over Space and Time. W.C. Labys, T. Takayama, and N. Uri, Editors, Gower Publishing Co. Ltd., Brookfield, Vermont, 49-65.

Thore, S., 1986. Generalized network spatial equilibrium: The deterministic and the chance-constrained case. Papers of the Regional Science Association, 59, 93-102.

World Health Organization, 2020a. Rolling updates on coronavirus disease (COVID-19). Available at:
https://www.who.int/ emergencies/diseases/novel-coronavirus-2019/events-as-they-happen

World Health Organization, 2020b. Shortage of personal protective equipment endangering health workers worldwide. Available at: https://www.who.int/news-room/detail/03-03-2020-shortage-of-personal-protective-equipment-endangering-health-workers-worldwide

World Trade Organization, 2020a. Trade set to plunge as COVID-19 pandemic upends global economy.

World Trade Organization, 2020b. COVID-19: Trade and trade-related measures.

Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. European Journal of Operational Research, 224(2), 273-282.

Zhang, D., Nagurney, A., 1995. On the stability of projected dynamical systems. Journal of Optimization Theory and its Applications, 85, 97-124.