Spatial Price Equilibrium with Information Asymmetry in Quality and Minimum Quality Standards

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Abstract: A spatial price equilibrium model with information asymmetry in quality is developed in both static and dynamic versions. Producers at the supply markets are aware of the quality of their products, whereas consumers, located at the demand markets, are aware only of the average quality of the products that are shipped to their demand markets. Minimum quality standards are also captured in order to assess the impacts of such policy interventions. We establish qualitative results, in the form of existence, uniqueness, and stability analysis. An algorithm is proposed, along with a convergence proof. It is then utilized to compute solutions to a spectrum of spatial price equilibrium numerical examples in order to explore the impacts of information asymmetry under different scenarios. The numerical examples, which are of quite general functional forms, reveal that, as the number of supply markets increases, the "anonymizing" effect leads to a decrease in the average quality. On the other hand, as the number of demand markets increases, the pressure to improve quality increases, and the average quality increases. Finally, we demonstrate that, after the imposition of minimum quality standards, the average quality at the demand markets increases and the prices also increase.

Key words: spatial price equilibrium, quality competition, information asymmetry, average quality, minimal quality standards, variational inequalities, projected dynamical systems

1. Introduction

In the Network Economy, products produced in one part of our globe may be transported across continents and even oceans to consumers at demand markets thousands of miles away. Demanding consumers have come to expect fresh produce in all seasons, fuel and energy to power their vehicles, homes, and equipment, upon demand, pharmaceutical products when needed, and clothing, high technology, and even toys upon request. At the same time, despite the great distances that may be involved, quality is what we, as consumers, seek in the food that we eat, the clothes that we wear, the toys that our children play with, the latest high tech products that we crave, the life-saving and prolonging medicines that those in need take, the cars that we drive, the planes that we fly in, the homes that we live in and the appliances that we use, and, of course, the air that we breathe. Indeed, as noted by Nagurney and Li (2014a), quality is emerging as an important characteristic in numerous products, ranging from food (see, e.g., Marsden (2004), Trienekens and Zuurbier (2008)) to pharmaceuticals (see Masoumi, Yu, and Nagurney (2012) and Bennett and Yin (2013)) to durable manufactured products such as automobiles (see Shank and Govindarajan (1994)) to high tech products, including microprocessors (see Goettler and Gordon (2011)), and even services associated with the Internet (cf. Kruse (2010) and Nagurney et al. (2013a)).

Given the great distances that may separate supply markets from demand markets, there may exist information asymmetry when it comes to the quality of certain products, especially those that are considered to be, more or less, homogeneous and are not differentiated by their brands. Moreover, some of the recent shortcomings in product quality, which have even resulted in illnesses as well as in deaths, have drawn increasing attention to information asymmetry. Notable examples of serious quality product shortcomings around the world have ranged from the adulteration of milk and infant formula in China (See Yang et al. (2009)) to the heparin adulteration, also in China (cf. BioPharma Today (2009)), which led to a pharmaceutical identity crisis, to substandard medicines in developing countries (see Bate and Boateng (2007) and Gaudiano et al. (2007)) to food-borne illnesses in the US and in Europe (see, respectively, Jaslow (2013) and European Food Safety Authority and European Centre for Disease Prevention and Control (2013)), to name just a few.

Markets with asymmetric information in terms of product quality have been studied by many notable economists, including the 2001 Nobel laureates Akerlof (1970), Spence (1973, 1975), and Stiglitz (1987). Leland (1979) further argued that such markets may benefit from minimum quality standards. However, information asymmetry in a spatial context has been less explored research-wise.

In this paper, we develop a spatial price equilibrium model with information asymmetry in quality in that the producers at the supply markets are aware of their product quality whereas consumers at the demand markets are only aware of the average quality of the products. Examples of products that can be modelled this way include: fresh produce, oil and fuel, rice and grains, wood, generic medicines, and other non-branded products and commodities, etc.

Our framework builds on spatial price equilibrium modeling dating to the classical work of Samuelson (1952) and Takayama and Judge (1964, 1971), but uses a variational inequality approach (cf. Nagurney (1999)) to include the critical quality dimension and an expanded set of equilibrium conditions followed by projected dynamical systems theory (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996a)) to describe the dynamic adjustment process and the associated stability analysis. Until this work, projected dynamical systems theory had been used by Nagurney, Takayama, and Zhang (1995a, b) and Nagurney and Zhang (1996b) to formulate, analyze, and solve dynamic spatial price equilibrium problems but without quality elements or information asymmetry. For a survey of spatial price equilibrium models see Labys and Yang (1997) and for a related survey with a focus on spatial economic location, see Kilkenny and Thisse (1999). For a spectrum of static and dynamic supply chain network models, see the book by Nagurney (2006).

In particular, we note that our framework is applicable to agricultural industries (cf. Thompson (1989)), such as eggs (cf. Judge (1956)), potatoes (Howard (1984)), beef (Sohn (1970)), cereal grains (Ruijs et al. (2001)), soybeans (Barraza De La Cruz, Pizzolato, and Barraza de La Cruz (2010)), and dairy (Bishop, Pratt, and Novakovic (1994)). Moreover, spatial price equilibrium models have been applied to the forestry sector (Hieu and Harrison (2011)). Such perfectly competitive models are also relevant to the mineral ore and energy industries (see Hwang et al. (1994), Labys and Yang (1991), and Labys (1999)), in particular, to the coal (Newcomb and Fan (1980)), aluminum (Newcomb, Reynolds, and Masbruch (1990)), and natural gas (Irwin and Yang (1996)) sectors. Interestingly, spatial price equilibrium models have also been developed for predator-prey networks and food webs (see Nagurney and Nagurney (2011)).

In this paper, we define quality as "the degree to which a specific product conforms to a design or specification." Shewhart (1931), Juran (1951), Levitt (1972), Gilmore (1974), Crosby (1979), Deming (1990), and Chase and Aquilano (1992), most of whom are operations management scholars, are the major advocates of this conformance-to-specification definition of quality. This definition makes quality relatively straightforward to quantify, which is essential for firms and researchers who need to measure it, manage it, model it, compare it across time, and to also make associated decisions (Shewhart (1931)). In addition, with notice that consumers' needs and desires for a product are actually governed by specific requirements and these can be correctly translated to a specification by, for example, engineers (Oliver (1981)), this feature makes the conformance-to-specification definition quite general. Beyond consumers' needs, the specification of a product can also include both international and domestic standards (Yip (1989)).

The related literature on quality and imperfect spatial competition has focused on product differentiation (cf. Nagurney and Li (2014a,b) and the references therein). In such models, unlike the models in this paper, there are a finite number of profit-maximizing producers, who compete in an oligopolistic manner. Various network oligopoly models, concentrating on individual industry sectors, have also been developed by Nagurney, Li, and Nagurney (2013), who examined the impacts of outsourcing of pharmaceutical production, and by Yu and Nagurney (2013), who focused on supply chain competition for fresh produce. The book by Nagurney et al. (2013b) describes and synthesizes a plethora of supply chain network models in which a key product characteristic of perishability. For additional references to supply chain models with information asymmetry, see Nagurney and Li (2014b). We also note the book by Kogan and Tapiero (2007), which contains supply chain models with quality and information asymmetry consisting of a single supplier and buyer.

For an overview of quality from the perspective of operations management as well as economics, see Lederer and Rhee (1995) (see also, e.g., Spence (1975)). We recognize the classics of the traditional quality management literature, notably, the books by Juran (1951), Feigenbaum (1956), and Deming (1990), with a critical survey to that date given in Kolesar (1993). Additional references can be found in Nagurney and Li (2014a,b).

We also include in our framework minimum quality standards, which are policy tools to protect consumers. Minimum quality standards are especially relevant to industries in which spatial price equilibrium models are applicable, notably, agricultural products (including food), as well as energy and mineral ores. Indeed, as noted in Metzger (1988), page 1: *minimum quality standards have been applied to such diverse items as medical drugs, auto safety and fuel efficiency features, electrical appliances, foods, clothing, and cosmetics.*

The novelty of the contributions in this paper are as follows:

1. We introduce the important spatial dimension using a network construct in the case of perfect competition and information asymmetry in quality.

2. We construct rigorous static (equilibrium) and dynamic models, without and with mini-

mum quality standards, an important policy instrument.

3. We provide a qualitative analysis of the equilibrium product shipments and average quality level pattern, in particular, existence and uniqueness results, as well as stability analysis results.

4. We propose an effective computational procedure, along with conditions for convergence. Our model can handle nonlinear functions and as many supply markets and demand markets as mandated by the specific application.

5. Our numerical examples illustrate the impact of the imposition of quality standards, as well as the impact of an increasing number of supply markets or demand markets on the equilibrium product shipments, the average quality levels at the demand markets, and on the incurred prices.

This paper is organized as follows. In Section 2, we develop the spatial price equilibrium model with information asymmetry in quality and then extend it to include minimum quality standards at the supply markets. We present a unified variational inequality for the integration of the two models. The projected dynamical systems model is, subsequently, constructed. It captures the dynamic adjustment process for the evolution of the product shipments and quality levels over time until the equilibrium point, equivalently, stationary point, is achieved. In Section 3, we present qualitative properties of the variational inequality formulation of the integrated model in product shipments and quality levels, in the form of existence and uniqueness results. We then provide stability analysis results for the projected dynamical system. Notably, the set of stationary points of the latter corresponds to the set of solutions to the former.

In Section 4, we present the algorithm, accompanied by convergence results. The algorithm yields closed form expressions, at each iteration, for the product shipments and the quality levels. The algorithm tracks the dynamic trajectories and approximates them until an equilibrium is achieved. In Section 5, we illustrate the model and the computational scheme through several numerical examples in order to also gain insights into the impacts of information asymmetry in quality in a spatial context. In Section 6, we summarize our results and present our conclusions.

2. Spatial Price Equilibrium with Asymmetric Information in Quality

In this Section, we first develop the spatial price equilibrium model with asymmetric information in quality and derive the variational inequality formulation. We then demonstrate how the model can be modified to include policies in the form of minimum quality standards and present a unified version integrating both the models. We also describe the underlying dynamics associated with the product shipments as well as the quality levels and present the projected dynamical systems model whose set of stationary points corresponds to the set of solutions of the variational inequality problem governing the integrated model.

Please refer to Figure 1 for the underlying network structure of the spatial price equilibrium problem (SPEP). We focus, here, on the bipartite problem, for clarity and definiteness.

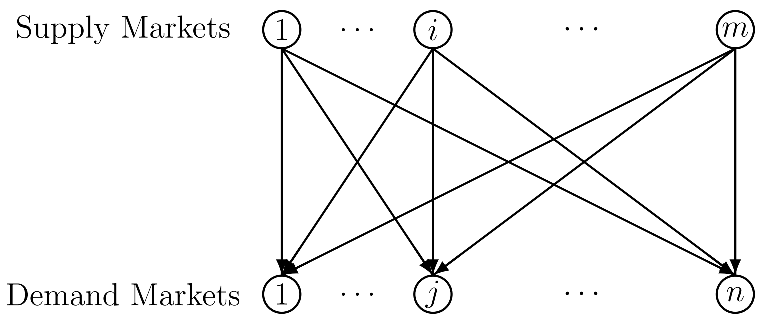


Figure 1: The bipartite network structure of the spatial price equilibrium problem

We assume that there are m supply markets and n demand markets that are, generally, spatially separated. The product that is produced and that is consumed is homogeneous in that the consumers at the demand markets do not differentiate by point of origin. Let s_i denote the nonnegative product output (supply) produced at supply market i and let d_j denote the nonnegative demand for the product consumed at demand market j. Let Q_{ij} denote the nonnegative shipment of the product from supply market i to demand market j. We group the supplies into the vector $s \in R^m_+$, the demands into the vector $d \in R^n_+$, and the product shipments into the vector $Q \in R^{mn}_+$. Here q_i denotes the quality level, or, simply, the quality, of product i, which is produced at supply market i. We group the quality levels of all the supply markets into the vector $q \in R^m_+$. All vectors here are assumed to be column vectors.

2.1 The Equilibrium Model

We first develop the static model. The following conservation of flow equations must hold:

$$s_i = \sum_{j=1}^{n} Q_{ij}, \quad i = 1, \ldots, m; \tag{1}$$

$$d_j = \sum_{i=1}^{m} Q_{ij}, \quad j = 1, \ldots, n; \tag{2}$$

$$Q_{ij} \geq 0, \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{3}$$

and, since the quality levels must also be nonnegative, we must have that

$$q_i \geq 0, \quad i = 1, \ldots, m. \tag{4}$$

Hence, according to (1), the supply of the product at each supply market is equal to the sum of the amounts of the product shipped to all the demand markets, and, according to (2), the quantity of the product consumed at a demand market is equal to the sum of the amounts of the product shipped from all the supply markets to that demand market. Both the shipment quantities and the quality levels must be nonnegative, as in (3) and (4), respectively. We define the feasible set $\mathcal{K}^1 \equiv \{(s, d, Q, q) | (1) - (4) \text{ holds}\}$.

We associate with each supply market i a supply price π_i, and allow for the general situation where the supply price at i may depend upon the entire supply pattern and on its quality level, that is,

$$\pi_i = \pi_i(s, q_i), \quad i = 1, \ldots, m. \tag{5}$$

The supply price function for each supply market i in (5) is assumed to be monotonically increasing in the s_i and q_i variables.

As noted earlier, consumers, located at the demand markets, which are spatially separated, respond not only to the quantities available of the product but also to their *average* quality, where the average quality of the product at demand market j, denoted by $\hat{q}_j$, is given by the expression:

$$\hat{q}_j = \frac{\sum_{i=1}^{m} q_i Q_{ij}}{\sum_{i=1}^{m} Q_{ij}}, \quad j = 1, \ldots, n. \tag{6}$$

We utilize average quality as proposed by the Nobel laureate George Akerlof (see Akerlof (1970)) and further developed by the Nobel laureates J.E. Stiglitz (1987), and Spence (1975). However, since we are dealing with a more general spatial network model, we provide the expression for average quality given by (6). Average quality is a reasonable statistic for quality since in our perfectly competitive framework there are many producers/sellers in each supply market and they are price-takers. Quality, hence, is not directly observable in our model by the buyers/consumers at the demand markets but consumers can estimate the quality of the product by the average quality of the product in the demand markets (cf.

Akerlof (1970), Stiglitz (1975), Metzger (1988), Barbieri (2013), Nagurney and Li (2014b)). Of course, price information, as per below, also implies a certain level of quality. Moreover, average quality can be conveyed among consumers through word of mouth, their own consumption experiences, advertising, etc.

We group the average quality at all the demand markets into the vector $\hat{q} \in R^n_+$. We assume that there is a positive demand at each demand market; otherwise, we remove that demand node from the bipartite network. Hence, the denominator in (6) is positive for each demand market. Nagurney, Li, and Nagurney (2013) utilized a variant of (4) to assess the average quality of pharmaceutical products in the case of outsourcing but assumed that the demands at the demand markets were fixed and known, rather than elastic, as we do here.

We denote the demand price at a demand market j by ρ_j and we allow the demand price for a product at a demand market to depend, in general, upon the entire demand pattern, as well as on the average quality values at all the demand markets, that is,

$$\rho_j = \rho_j(d, \hat{q}), \quad j = 1, \ldots, n. \tag{7}$$

The generality of the expression in (7) enables modeling and application flexibility. Each demand price function is, typically, monotonically decreasing in demand at its demand market but increasing in the average quality. Akerlof (1970) utilized (but in an aspatial context) demand functions that were a function of the price and the average quality. Note that the information asymmetry in our model is similar to that in Akerlof (1970) and Leland (1979), in that the producers at the supply markets are aware of the quality of their product, as expressed by the supply price functions (5), but consumers at the demand markets are aware only of the average quality, as expressed by the demand price functions (7). However, our model captures the crucial spatial dimension, which is especially relevant in the Network Economy.

In our model, we assume that, the increase in consumers' perception of average quality increases the benefits of the product to them. Therefore, consumers are willing to pay more for a product with a higher average quality, and, hence, at the demand markets, prices increase as the average quality increases. Also, as noted in Nagurney and Li (2014a), Kaya and Özer (2009) and Kaya (2011) used demand price functions in price and quality variables, which were increasing functions of quality, and of the form: $q = a - bp + e + \varepsilon$, where q is the demand and e is the quality level. Xie et al. (2011), in turn, utilized a demand price function of the form $D = a + \alpha x - \beta p$, which extended the function used by Banker, Khosla, and Sinha (1998). Anderson and Palma (2001) captured the utility of each consumer u expressed as $u = q - p + \varepsilon$ in their research on asymmetric oligopolies. Note that, in our framework, we do not limit ourselves to linear demand price functions. Moreover, we allow the demand price of a product to depend not only on its demand but also on those of the other products: the same holds for the dependence of the prices on the product average quality levels.

According to an interview with the Nobel laureate Michael Spence (see Shah (2012)), with informational asymmetry significant quality differences between the supply and demand sides are manifested. Moreover, *product differentiation disappears, prices will reflect the average quality rather than differential quality*, as revealed through our demand price functions (7).

Let c_{ij} denote the unit transportation cost associated with shipping the product from supply market i to demand market j, where the transportation cost is given by the function:

$$c_{ij} = c_{ij}(Q), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{8}$$

Here, we assume that the unit transportation cost also includes, as appropriate, any unit transaction cost. Examples of transaction costs may be tariffs and taxes. For the sake of modeling generality, we let the unit transportation costs depend, in general, upon the entire product shipment pattern and assume that the transportation cost functions are monotonically increasing.

In addition, we also have opportunity costs associated with the supply markets, which are functions of the quality of the product at the supply market. We denote the opportunity cost function at supply market i by OC_i and we have that

$$OC_i = OC_i(q_i), \quad i = 1, \ldots, m. \tag{9}$$

Leland (1979), building on the work of Akerlof (1970) in quality, utilized opportunity cost functions as in (9) and also related them to the supply price functions at the markets. This we do, as well, in our extension of the supply price equilibrium conditions to include quality, under asymmetric information.

We assume that all the supply price, demand price, unit transportation cost, and opportunity cost functions are continuous. Opportunity costs were also used in an imperfectly competitive network model in Nagurney, Li, and Nagurney (2013).

We are now ready to state the spatial price equilibrium conditions, which are a generalization of the well-known spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) (see also Nagurney (1999)), to include quality.

Definition 1: Spatial Price Equilibrium Conditions with Information Asymmetry in Quality

We say that a supply, product shipment, demand, and quality pattern $(s^*, Q^*, d^*, q^*) \in \mathcal{K}^1$ *is a spatial equilibrium with information asymmetry in quality if it satisfies the following conditions: for each pair of supply and demand markets* (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$\pi_i(s^*, q_i^*) + c_{ij}(Q^*) \begin{cases} = \rho_j(d^*, \hat{q}^*), & \text{if } Q_{ij}^* > 0, \\ \geq \rho_j(d^*, \hat{q}^*), & \text{if } Q_{ij}^* = 0, \end{cases} \tag{10}$$

and for each supply market i; $i = 1, \ldots, m$:

$$OC_i(q_i^*) \begin{cases} = \pi_i(s^*, q_i^*), & \text{if } q_i^* > 0, \\ \geq \pi_i(s^*, q_i^*), & \text{if } q_i^* = 0. \end{cases} \tag{11}$$

According to (10), there is a positive quantity of the product shipped from a supply market to a demand market, in equilibrium, if the supply price at the originating supply market plus the unit transportation cost is equal to the demand price at the demand market. If the supply price plus the unit transportation cost exceeds that demand price, then there will be no trade of the product between the pair of supply and demand markets for that product. According to (11), the equilibrium quality of the product produced at a supply market is positive if the opportunity cost at the supply market is equal to the supply price. If the opportunity cost at the supply market exceeds the supply price at that market, then the equilibrium quality at that supply market will be zero.

We now establish the variational inequality formulation of the above spatial price equilibrium conditions.

Theorem 1: Variational Inequality Formulation of Spatial Price Equilibrium with Information Asymmetry in Quality

A supply, product shipment, demand, and quality pattern $(s^*, Q^*, d^*, q^*) \in \mathcal{K}^1$ *is a spatial price equilibrium with information asymmetry in quality according to Definition 1 if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m} \pi_i(s^*, q_i^*) \times (s_i - s_i^*) + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}(Q^*) \times (Q_{ij} - Q_{ij}^*) - \sum_{j=1}^{n} \rho_j(d^*, \hat{q}^*) \times (d_j - d_j^*)$$

$$+ \sum_{i=1}^{m} (OC_i(q_i^*) - \pi_i(s^*, q_i^*)) \times (q_i - q_i^*) \geq 0, \quad \forall (s, Q, d, q) \in \mathcal{K}^1. \tag{12}$$

Proof: Please see the Appendix.

We now demonstrate how policy interventions in the form of minimum quality standards will change the above equilibrium conditions. Specifically, assume that a regulator (or regulators, since the supply markets may be in different regions or even nations) imposes minimum quality standards at each supply market, denoted by $\underline{q}_i$, with $\underline{q}_i$ positive for $i = 1, \ldots, m$, so that, now, instead of (4) being satisfied, we must have that

$$q_i \geq \underline{q}_i; \quad i = 1, \ldots, m. \tag{13}$$

Also, define a new feasible set $\mathcal{K}^2 \equiv \{(s, Q, d, q) | (1) - (3) \, \text{and} \, (13) \, \text{hold}\}$.

Definition 2: Spatial Price Equilibrium with Information Asymmetry in Quality and Minimum Quality Standards

In the case of the imposition of minimum positive quality standards, then $(s^*, Q^*, d^*, q^*) \in \mathcal{K}^2$ *is a spatial price equilibrium if (10) holds and (11) is modified to: for each supply market* i*;* $i = 1, \ldots, m$*:*

$$OC_i(q_i^*) \begin{cases} = \pi_i(s^*, q_i^*), & \text{if} \quad q_i^* > \underline{q}_i, \\ \geq \pi_i(s^*, q_i^*), & \text{if} \quad q_i^* = \underline{q}_i. \end{cases} \tag{14}$$

The variational inequality formulation corresponding to an equilibrium satisfying Definition 2 has the same structure as variational inequality (12) but the feasible set is now $\mathcal{K}^2$ rather than $\mathcal{K}^1$. The proof follows using similar arguments as those used in the proof of Theorem 1 in the Appendix. Hence, we have the following

Theorem 2: Variational Inequality Formulation of Spatial Price Equilibrium with Information Asymmetry in Quality and Minimum Quality Standards

A supply, product shipment, demand, and quality pattern $(s^*, Q^*, d^*, q^*) \in \mathcal{K}^2$ *is a spatial price equilibrium with information asymmetry in quality and minimum quality standards according to Definition 2 if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m} \pi_i(s^*, q_i^*) \times (s_i - s_i^*) + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}(Q^*) \times (Q_{ij} - Q_{ij}^*) - \sum_{j=1}^{n} \rho_j(d^*, \hat{q}^*) \times (d_j - d_j^*)$$

$$+ \sum_{i=1}^{m} (OC_i(q_i^*) - \pi_i(s^*, q_i^*)) \times (q_i - q_i^*) \geq 0, \quad \forall (s, Q, d, q) \in \mathcal{K}^2. \tag{15}$$

We now provide an alternative variational inequality to that of (12) and of (15) in which the variables are product shipment and quality levels only. Such a formulation will allow for a more transparent identification of the evolution of the product shipments and quality levels over time via a projected dynamical system (PDS). In addition, we will utilize such a PDS to construct a computational procedure, which will provide us with a discretization of the continuous-time adjustment processes provided by the PDS.

Specifically, we first define supply price functions and demand price functions, denoted, respectively, by $\hat{\pi}_i(Q, q_i)$ for $i = 1, \ldots, m$, and by $\hat{\rho}_j(Q, q)$ for $j = 1, \ldots, n$, that are functions of product shipments and quality levels exclusively. This can be done because of constraints (1) and (2) and expression (6). Hence, we have:

$$\hat{\pi}_i = \hat{\pi}_i(Q, q_i) \equiv \pi_i(s, q_i), \quad i = 1, \ldots, m \tag{16}$$

and

$$\hat{\rho}_j = \hat{\rho}_j(Q, q) \equiv \rho_j(d, \hat{q}), \quad j = 1, \ldots, n. \tag{17}$$

Also, we can unify the equilibrium conditions (11) and (14) by redefining $\underline{q}_i$; $i = 1, \ldots, m$, as taking on nonnegative values so that, if $\underline{q}_i$ is equal to zero, then there is no minimum standard at supply market i, and, if $\underline{q}_i$ is positive, then there is. Thus, we now have

$$q_i \geq \underline{q}_i, \quad i = 1, \ldots, m. \tag{18}$$

Of course, a minimum quality standard may also be decided upon by the producers at a supply market and may not need to be imposed by a regulatory authority.

We define the feasible set $\mathcal{K}^3 \equiv \{(Q, q) | Q \in R^{mn}_+ \text{ and } (18) \text{ holds}\}$.

The following equilibrium conditions now integrate those in Definitions 1 and 2:

Definition 3: Integrated Spatial Price Equilibrium with Information Asymmetry in Quality

We say that a product shipment and quality pattern $(Q^*, q^*) \in \mathcal{K}^3$ is an integrated spatial equilibrium with information asymmetry in quality if it satisfies the following conditions: for each pair of supply and demand markets (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$\hat{\pi}_i(Q^*, q_i^*) + c_{ij}(Q^*) \begin{cases} = \hat{\rho}_j(Q^*, q^*), & \text{if } \; Q_{ij}^* > 0, \\ \geq \hat{\rho}_j(Q^*, q^*), & \text{if } \; Q_{ij}^* = 0, \end{cases} \tag{19}$$

and for each supply market i; $i = 1, \ldots, m$:

$$OC_i(q_i^*) \begin{cases} = \hat{\pi}_i(Q^*, q_i^*), & \text{if } \; q_i^* > \underline{q}_i, \\ \geq \hat{\pi}_i(Q^*, q_i^*), & \text{if } \; q_i^* = \underline{q}_i. \end{cases} \tag{20}$$

The following Theorem is immediate.

Theorem 3: Variational Inequality Formulation of Integrated Spatial Price Equilibrium with Information Asymmetry

A product shipment and quality level pattern $(Q^*, q^*) \in \mathcal{K}^3$ is an integrated spatial price equilibrium with information asymmetry in quality according to Definition 3 if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\hat{\pi}_i(Q^*, q_i^*) + c_{ij}(Q^*) - \hat{\rho}_j(Q^*, q^*)) \times (Q_{ij} - Q_{ij}^*)$$

$$\sum_{i=1}^{m}(OC_i(q_i^*) - \hat{\pi}_i(Q^*, q_i^*)) \times (q_i - q_i^*) \geq 0, \quad \forall (Q, q) \in \mathcal{K}^3. \tag{21}$$

We now put variational inequality (21) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{22}$$

where $\mathcal{K}$ is the feasible set, which must be closed and convex. The vector X is an N-dimensional vector, as is $F(X)$, with $F(X)$ being continuous and given, and maps X from $\mathcal{K}$ into R^N. $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. We define the vector $X \equiv (Q, q)$ and the vector $F(X) \equiv (F^1(X), F^2(X))$ with $F^1(X)$ consisting of components $F^1_{ij}(X) = \hat{\pi}_i(Q, q_i) + c_{ij}(Q) - \hat{\rho}_j(Q, q)$, $i = 1, \ldots, m$; $j = 1, \ldots, n$ and $F^2(X)$ consisting of components: $F^2_i(X) = OC_i(q_i) - \hat{\pi}_i(Q, q_i)$; $i = 1, \ldots, m$. Here $N = mn + m$. Also, we define the feasible set $\mathcal{K} \equiv \mathcal{K}^3$. Then, variational inequality (21) can be placed into standard form (22).

For additional background on the variational inequality problem, we refer the reader to the book by Nagurney (1999).

2.2 The Projected Dynamical System Model

We now propose a dynamic adjustment process for the evolution of the product shipments and product quality levels under information asymmetry. Observe that, for a current product shipment and quality level pattern at time t, $X(t) = (Q(t), q(t))$, $-F^1_{ij}(X(t)) = \hat{\rho}_j(Q(t), q(t)) - c_{ij}(Q(t)) - \hat{\pi}_i(Q(t), q_i(t))$ is the excess price between demand market j and supply market i and $-F^2_i(X(t)) = \hat{\pi}_i(Q(t), q_i(t)) - OC_i(q_i(t))$ is the difference between the supply price and the opportunity cost at supply market i. In our framework, the rate of change of the product shipment between a supply and demand market pair (i, j) is in proportion to $-F^1_{ij}(X)$, as long as the product shipment Q_{ij} is positive, that is, when $Q_{ij} > 0$,

$$\dot{Q}_{ij} = \hat{\rho}_j(Q, q) - c_{ij}(Q) - \hat{\pi}_i(Q, q_i), \tag{23}$$

where $\dot{Q}_{ij}$ denotes the rate of change of Q_{ij}. However, when $Q_{ij} = 0$, the nonnegativity condition (3) forces the product shipment Q_{ij} to remain zero when $\hat{\rho}_j(Q, q) - c_{ij}(Q) - \hat{\pi}_i(Q, q_i) \leq 0$. Hence, in this case, we are only guaranteed of having possible increases in the shipment. Namely, when $Q_{ij} = 0$,

$$\dot{Q}_{ij} = \max\{0, \hat{\rho}_j(Q, q) - c_{ij}(Q) - \hat{\pi}_i(Q, q_i)\}. \tag{24}$$

We can write (23) and (24) compactly as:

$$\dot{Q}_{ij} = \begin{cases} \hat{\rho}_j(Q, q) - c_{ij}(Q) - \hat{\pi}_i(Q, q_i), & \text{if} \quad Q_{ij} > 0 \\ \max\{0, \hat{\rho}_j(Q, q) - c_{ij}(Q) - \hat{\pi}_i(Q, q_i)\}, & \text{if} \quad Q_{ij} = 0. \end{cases} \tag{25}$$

As for the quality levels, when $q_i > \underline{q}_i$, then

$$\dot{q}_i = \hat{\pi}_i(Q, q_i) - OC_i(q_i), \tag{26}$$

where $\dot{q}_i$ denotes the rate of change of q_i; otherwise:

$$\dot{q}_i = \max\{\underline{q}_i, \hat{\pi}_i(Q(t), q_i(t)) - OC_i(Q_i(t))\}, \tag{27}$$

since q_i cannot be lower than $\underline{q}_i$ according to (27) (and the feasible set $\mathcal{K}^3$).

Combining (26) and (27), we obtain:

$$\dot{q}_i = \begin{cases} \hat{\pi}_i(Q(t), q_i(t)) - OC_i(Q_i(t)), & \text{if} \quad q_i > \underline{q}_i \\ \max\{0, \hat{\pi}_i(Q(t), q_i(t)) - OC_i(q_i(t)), & \text{if} \quad q_i = \underline{q}_i. \end{cases} \tag{28}$$

Applying (25) to all supply and demand market pairs (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and applying (28) to all supply markets i; $i = 1, \ldots, m$, and combining the resultants, yields the following pertinent ordinary differential equation (ODE) for the adjustment processes of the product shipments and quality levels, in vector form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{29}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{30}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P_{\mathcal{K}}(X) = \text{argmin}_{z\in\mathcal{K}}\|X - z\|, \tag{31}$$

and where $\|\cdot\| = \langle x, x\rangle$.

We now interpret the ODE (29) in the context of the integrated spatial model with information asymmetry in quality. First, note that ODE (29) ensures that the production shipments are always nonnegative and the quality levels never go below the imposed lower bounds (which are never negative by assumption). ODE (29), however, retains the interpretation that if X at time t lies in the interior of $\mathcal{K}$, then the rate at which X changes is greatest when the vector field $-F(X)$ is greatest. Moreover, when the vector field $-F(X)$ pushes X to the boundary of the feasible set $\mathcal{K}$, then the projection $\Pi_{\mathcal{K}}$ ensures that X stays within $\mathcal{K}$.

Dupuis and Nagurney (1993) developed the fundamental theory with regards to existence and uniqueness of projected dynamical systems as defined by (29). We cite the following theorem from that paper.

Theorem 4

X^* solves the variational inequality problem (22), equivalently, (21), if and only if it is a stationary point of the ODE (29), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{32}$$

This theorem demonstrates that the necessary and sufficient condition for a product shipment and quality level pattern $X^* = (Q^*, q^*)$ to be an integrated spatial price equilibrium with information asymmetry in quality, according to Definition 3, is that $X^* = (Q^*, q^*)$ is a stationary point of the adjustment process defined by ODE (29), that is, X^* is the point at which $\dot{X} = 0$. We refer to (29) as PDS $(F, \mathcal{K})$.

3. Qualitative Properties

We now investigate whether and under what conditions the adjustment process defined by ODE (29) approaches a spatial price equilibrium with information asymmetry in quality. Lipschitz continuity of $F(X)$ (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996a)) guarantees the existence of a unique solution to (33), where we have that $X^0(t)$ satisfies ODE (29) with initial shipment and quality level pattern (Q^0, q^0). In other words,

$X^0(t)$ solves the initial value problem (IVP)

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0, \tag{33}$$

with $X^0(0) = X^0$.

We first establish existence and uniqueness results for the solution of variational inequality (22), equivalently, (21). We then provide some stability analysis results.

We know, from the standard theory of variational inequalities (cf. Nagurney (1999) and the references therein), that if the Jacobian of $F(X)$, denoted by $\nabla F(X)$, is positive-definite, then $F(X)$ is strictly monotone, and the solution to variational inequality (22) is unique, if it exists. Recall that $F(X)$ is strictly monotone if the following holds:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \quad X^1 \neq X^2; \tag{34a}$$

$F(X)$ is monotone if:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}; \tag{34b}$$

$F(X)$ is strongly monotone if:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \alpha \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{34c}$$

where α is some positive constant.

For our model, the Jacobian matrix $\nabla F(X)$, which is $N \times N$, can be partitioned as:

$$\nabla F(X) = \begin{pmatrix} \nabla F^{11}(X) & \nabla F^{12}(X) \\ \nabla F^{21}(X) & \nabla F^{22}(X) \end{pmatrix}, \tag{35}$$

where

$$\nabla F^{11}(X) \equiv \begin{pmatrix} \frac{\partial(\hat{\pi}_1(Q,q_1)+c_{11}(Q)-\hat{\rho}_1(Q,q))}{\partial Q_{11}} & \cdots & \frac{\partial(\hat{\pi}_1(Q,q_1)+c_{11}(Q)-\hat{\rho}_1(Q,q))}{\partial Q_{mn}} \\ \vdots & & \vdots \\ \frac{\partial(\hat{\pi}_m(Q,q_m)+c_{mn}(Q)-\hat{\rho}_n(Q,q))}{\partial Q_{11}} & \cdots & \frac{\partial(\hat{\pi}_m(Q,q)+c_{mn}(Q)-\hat{\rho}_n(Q,q))}{\partial Q_{mn}} \end{pmatrix},$$

$$\nabla F^{12}(X) \equiv \begin{pmatrix} \frac{\partial(\hat{\pi}_1(Q,q_1)+c_{11}(Q)-\hat{\rho}_1(Q,q))}{\partial q_1} & \cdots & \frac{\partial(\hat{\pi}_1(Q,q_1)+c_{11}(Q)-\hat{\rho}_1(Q,q))}{\partial q_m} \\ \vdots & & \vdots \\ \frac{\partial(\hat{\pi}_m(Q,q_m)+c_{mn}(Q)-\hat{\rho}_n(Q,q))}{\partial q_1} & \cdots & \frac{\partial(\hat{\pi}_m(Q,q_m)+c_{mn}(Q)-\hat{\rho}_n(Q,q))}{\partial q_m} \end{pmatrix},$$

$$\nabla F^{21}(X) \equiv \begin{pmatrix} \frac{\partial(-\hat{\pi}_1(Q,q_1))}{\partial Q_{11}} & \cdots & \frac{\partial(-\hat{\pi}_1(Q,q_1))}{\partial Q_{mn}} \\ \vdots & & \vdots \\ \frac{\partial(-\hat{\pi}_m(Q,q_m))}{\partial Q_{11}} & \cdots & \frac{\partial(-\hat{\pi}_m(Q,q_m))}{\partial Q_{mn}} \end{pmatrix},$$

$$\nabla F^{22}(X) \equiv \begin{pmatrix} \frac{\partial(OC_1(q_1)-\hat{\pi}_1(Q,q_1))}{\partial q_1} & \cdots & \frac{\partial(OC_1(q_1)-\hat{\pi}_1(Q,q_1))}{\partial q_m} \\ \vdots & & \vdots \\ \frac{\partial(OC_m(q_m)-\hat{\pi}_m(Q,q_m))}{\partial q_1} & \cdots & \frac{\partial(OC_m(q_m)-\hat{\pi}_m(Q,q_m))}{\partial q_m} \end{pmatrix}.$$

In constructing $\nabla F^{21}(X)$ we have made some algebraic simplifications by observing that the opportunity costs do not depend on the product shipments.

Assumption 1

Suppose that for our integrated spatial price equilibrium model there exists a sufficiently large B, such that for any supply and demand market pair (i, j):

$$F^1_{ij}(X) = \hat{\pi}_i(Q, q_i) + c_{ij}(Q) - \hat{\rho}_j(Q, q) > 0, \tag{36}$$

for all shipment patterns Q with $Q_{ij} \geq B$ and that there exists a sufficiently large $\bar{B}$, such that for any supply market i:

$$F^2_i = OC_i(q_i) - \hat{\pi}_i(Q, q_i) > 0, \tag{37}$$

for all quality level patterns q with $q_i \geq \bar{B} \geq \underline{q}_i$.

We now provide an existence result, whose proof can be established using similar arguments as the proof of Proposition 6.1 in Nagurney and Zhang (1996a) for the spatial price equilibrium problem without any quality variables.

Proposition 1: Existence

Any integrated spatial price equilibrium problem with information asymmetry in quality, as described in Section 2, that satisfies Assumption 1 possesses at least one equilibrium shipment and quality level pattern.

We now present the uniqueness result, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Proposition 2: Uniqueness

Suppose that F is strictly monotone at any solution (equilibrium point) of the variational inequality problem (VIP) defined in (22). Then the VIP has at most one equilibrium point.

Of course, if $F(X)$ is strongly monotone, a property that would hold if $\nabla F(X)$ were strongly positive-definite over $\mathcal{K}$, both existence and uniqueness of a solution X^* to (22) (equivalently, to (21)) would be guaranteed.

The following Theorem is a natural extension/adaptation and integration of Theorems 3.5, 3.6, and 3.7 in Nagurney (1999) (see also Nagurney and Zhang (1996b) and Zhang and Nagurney (1995)) to the more general spatial price equilibrium model with information asymmetry in quality. Definitions of global attractors and global exponential stability can be found in these references.

Theorem 5

(i). *If $F(X)$ is monotone, then every spatial price equilibrium with information asymmetry in quality, X^*, provided its existence, is a global monotone attractor for the the PDS$(F, \mathcal{K})$. If $F(X)$ is locally monotone at X^*, then it is a monotone attractor for the PDS$(F, \mathcal{K})$.*

(ii). *If $F(X)$ is strictly monotone, then there exists at most one spatial price equilibrium with information asymmetry in quality, X^*. Furthermore, provided existence, the unique spatial price equilibrium is a strictly global monotone attractor for the PDS$(F, \mathcal{K})$. If $F(X)$ is locally strictly monotone at X^*, then it is a strictly monotone attractor for the PDS$(F, \mathcal{K})$.*

(iii). *If $F(X)$ is strongly monotone, then there exists a unique spatial price equilibrium with information asymmetry in quality, which is globally exponentially stable for the PDS$(F, \mathcal{K})$. If $F(X)$ is locally strongly monotone at X^*, then X^* is exponentially stable.*

4. The Algorithm

The projected dynamical system (29) yields a continuous-time adjustment process in product shipments and in quality. However, for computational purposes, a discrete-time algorithm, which serves as an approximation to the continuous-time trajectories is needed. We now recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, iteration τ of the Euler method (see also Nagurney and Zhang (1996a)) is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{38}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ (cf. (31)) and F is the function that enters the variational inequality problem (22).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996a), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other spatial models can be found in Nagurney and Zhang (1996a), Nagurney, Dupuis, and Zhang (1994), Nagurney et al. (2002), Cruz (2008), Nagurney (2006, 2010), Nagurney and Yu (2012), Nagurney, Yu, and Qiang (2011), and in Nagurney and Li (2014a).

Explicit Formulae for the Euler Method Applied to the Integrated Spatial Price Equilibrium Model with Information Asymmetry in Quality

The elegance of this procedure for the computation of solutions to our spatial price equilibrium model can be seen in the following explicit formulae. In particular, we have the following closed form expression for the product shipments $i = 1, \dots, m; j = 1, \dots, n$:

$$Q_{ij}^{\tau+1} = \max\{0, Q_{ij}^{\tau} + a_\tau(\hat{\rho}_j(Q^\tau, q^\tau) - c_{ij}(Q^\tau) - \hat{\pi}_i(Q^\tau, q_i^\tau))\}, \tag{39}$$

and the following closed form expression for all the quality levels $i = 1, \dots, m$:

$$q_i^{\tau+1} = \max\{0, q_i^\tau + a_\tau(\hat{\pi}_i(Q^\tau, q_i^\tau) - OC_i(q_i^\tau))\}. \tag{40}$$

Expressions (39) and (40) can also be interpreted as discrete-time adjustment processes.

We now provide the convergence result. The proof is direct from Theorem 6.10 in Nagurney and Zhang (1996a).

Theorem 6

In the spatial price equilibrium problem with information asymmetry in quality let $F(X)$ be strictly monotone at any equilibrium pattern and assume that Assumption 1 is satisfied. Also, assume that F is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{41}$$

where L is a positive number known as the Lipschitz constant. Then there exists a unique equilibrium product shipment and quality level pattern $(Q^*, q^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (38), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (Q^*, q^*).

In the next Section, we apply the Euler method to compute solutions to numerical spatial price equilibrium problems with information asymmetry in quality.

5. Numerical Examples

We implemented the Euler method, as described in Section 4, in FORTRAN, using a Linux system at the University of Massachusetts Amherst. The convergence criterion was $\epsilon = 10^{-6}$; that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each product shipment and each quality level differed from its respective value at the preceding iteration by no more than ϵ.

The sequence $\{a_\tau\}$ was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We initialized the algorithm by setting each product shipment, $Q_{ij} = 1$, $\forall (i,j)$, and by setting the quality level of the product at each supply market, $q_i = 0.00$, $\forall i$.

5.1 Examples in Which the Number of Supply Markets is Increased

In the first set of examples we set out to explore the impact of an addition of a supply market. Such an experiment is relevant since additional supply markets may be added in practice due, for example, to increasing demand, the opening up of new markets, the elimination of trade barriers, new competitive entrants, etc.

The network topologies for Examples 1, 2, and 3 are depicted in Figure 2. The input data and the computed equilibrium solutions for these examples are reported, respectively, in Tables 1 and 2. The results for the average quality levels, demands, and demand prices are summarized in Table 3 and in Figure 3.

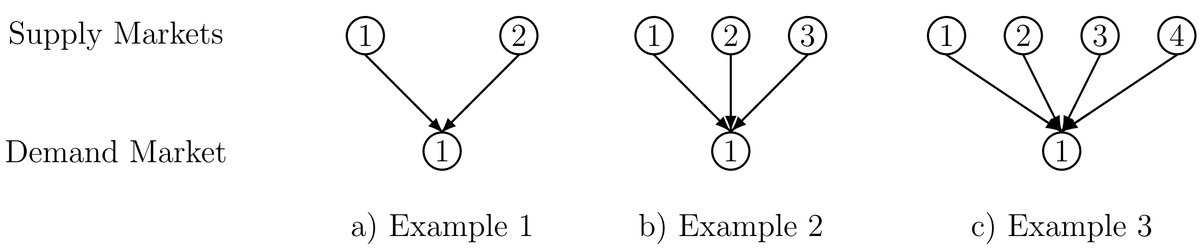


Figure 2: Network Topologies for Examples 1, 2, and 3

Example 1

The first example has the network topology given in Figure 2.a, that is, there are 2 supply markets and a single demand market. In this example, the second supply market is located further from the demand market than the first supply market.

The input data are provided in Table 1. The Euler method converged in 254 iterations to the equilibrium solution given in Table 2, which shows that the second supply market produces (and ships) more of the product to the demand market than the first supply market does but at a lower quality.

For completeness, we now demonstrate how some of the qualitative results in Section 3 can be applied to this example. Specifically, we determine $\nabla F(X)$ according to (44) and evaluate it at the equilibrium solution $X^* = (Q_{11}^*, Q_{21}^*, q_1^*, q_2^*)$, which yields:

$$\nabla F(X) = \begin{pmatrix} 7.77 & 2.17 & .58 & -.58 \\ 1.77 & 6.17 & -.42 & .42 \\ -5 & 0 & 4 & 0 \\ 0 & -2 & 0 & 9 \end{pmatrix}.$$

We note that the eigenvalues of $\frac{1}{2}(\nabla F(X) + \nabla F(X)^T)$ are: 2.8333, 5.2333, 8.7444, and 10.1290 and since these are all positive, there is a unique equilibrium and the conditions for convergence of the algorithm are also satisfied. Also, according to Theorem 5, we know that the computed X^* is globally expnentially stable.

Example 2

In the second example, the data are as in Example 1, except that we add a new supply market 3 as depicted in Figure 2.b. The new supply market is located closer to the demand market than supply market 2, but further than supply market 1.

The input data for this example are given in Table 1. The Euler method converged in 305 iterations and yielded the equilibrium solution shown in Table 2. We note that both supply markets 1 and 2 now ship less of the product to the demand market than they did in Example 1, and they each lowered the quality of the product that they produce.

Example 3

The final example in this set has the topology given in Figure 2.c. There are 4 supply markets and a single demand market. The example has the same data as Example 2 except for the new data associated with supply market 4, which are provided in Table 1.

The Euler method converged in 337 iterations and yielded the equilibrium solution given in Table 2. Here, we observe that the first three supply markets have reduced both the amounts that they ship to the demand market as well as the quality level of their product in comparison to their corresponding values in Example 2.

Table 1: Input Data for Examples 1, 2, and 3

Example	m	n	$\hat{\pi}(Q,q)$	$c(Q)$	$\hat{\rho}_1(Q,q)$	$OC(q)$
1	2	1	$\hat{\pi}_1(Q,q_1)=5Q_{11}+q_1+5$, $\hat{\pi}_2(Q,q_2)=2Q_{21}+q_2+10$.	$c_{11}(Q)=Q_{11}+15$, $c_{21}(Q)=2Q_{21}+20$.	$\hat{\rho}_1(Q,q)=-2(Q_{11}+Q_{21})+(\frac{q_1Q_{11}+q_2Q_{21}}{Q_{11}+Q_{21}})+100.$	$OC_1(q_1)=5q_1$, $OC_2(q_2)=10q_2$.
2	3	1	$\hat{\pi}_1(Q,q_1)$, $\hat{\pi}_2(Q,q_2)$, $\hat{\pi}_3(Q,q_3)=2Q_{31}+.5q_3+5$.	$c_{11}(Q)$, $c_{21}(Q)$, $c_{31}(Q)=Q_{31}+20$.	$\hat{\rho}_1(Q,q)=-2(Q_{11}+Q_{21}+Q_{31})+(\frac{q_1Q_{11}+q_2Q_{21}+q_3Q_{31}}{Q_{11}+Q_{21}+Q_{31}})+100.$	$OC_1(q_1)$, $OC_2(q_2)$, $OC_3(q_3)=5q_3$.
3	4	1	$\hat{\pi}_1(Q,q_1)$, $\hat{\pi}_2(Q,q_2)$, $\hat{\pi}_3(Q,q_3)$, $\hat{\pi}_4(Q,q_4)=Q_{41}+.7q_4+5$.	$c_{11}(Q)$, $c_{21}(Q)$, $c_{31}(Q)$, $c_{41}(Q)=2Q_{41}+10$.	$\hat{\rho}_1(Q,q)=-2(Q_{11}+Q_{21}+Q_{31}+Q_{41})+(\frac{q_1Q_{11}+q_2Q_{21}+q_3Q_{31}+q_4Q_{41}}{Q_{11}+Q_{21}+Q_{31}+Q_{41}})+100.$	$OC_1(q_1)$, $OC_2(q_2)$, $OC_3(q_3)$, $OC_4(q_4)=10q_4$.

Table 2: Equilibrium Solutions for Examples 1, 2, and 3

	Example 1	Example 2	Example 3
Q^*	$Q^*_{11} = 7.06$, $Q^*_{21} = 9.79$.	$Q^*_{11} = 5.32$, $Q^*_{21} = 6.80$, $Q^*_{31} = 10.64$.	$Q^*_{11} = 3.90$, $Q^*_{21} = 4.37$, $Q^*_{31} = 7.44$, $Q^*_{41} = 11.11$.
q^*	$q^*_1 = 10.08$, $q^*_2 = 3.29$.	$q^*_1 = 7.90$, $q^*_2 = 2.62$, $q^*_3 = 5.84$.	$q^*_1 = 6.13$, $q^*_2 = 2.08$, $q^*_3 = 4.42$, $q^*_4 = 1.73$.

We now provide the results for the average quality levels, demands, and demand prices for this set of examples in Table 3 and Figure 3. From these numerical results, we observe that, as the number of supply markets increases, the average quality at the demand market decreases, the demand increases, and the price decreases. Akerlof (1970) observed that, as the price falls, normally the quality would also fall. One can say that there is an "anonymizing" effect here in that with more producers the consumers at the demand markets are less likely to know from which supply market the product comes. Moreover, since there is no loss in reputation of producers at specific supply markets on the supply-side, the producers at the supply markets have no incentive to maintain or to increase their product quality.

Table 3: Summary of the Results for the Average Quality Levels, Demands, and Demand Prices for Examples 1, 2, and 3

	Example 1	Example 2	Example 3
m	2	3	4
n	1	1	1
$\hat{q}_1$	6.13	5.36	3.17
d_1	16.85	22.76	26.82
ρ_1	72.44	59.83	49.54

These examples clearly exhibit the effects of information asymmetry and, as Akerlof (1970) noted on page 488: *There are many markets in which buyers use some market statistic to judge the quality of prospective purchases. In this case there is incentive for sellers to market poor quality merchandise, since the returns for good quality accrue mainly to the entire group whose statistic is affected rather than to the individual seller.* In our model, as in Akerlof's, the statistic is the average quality.

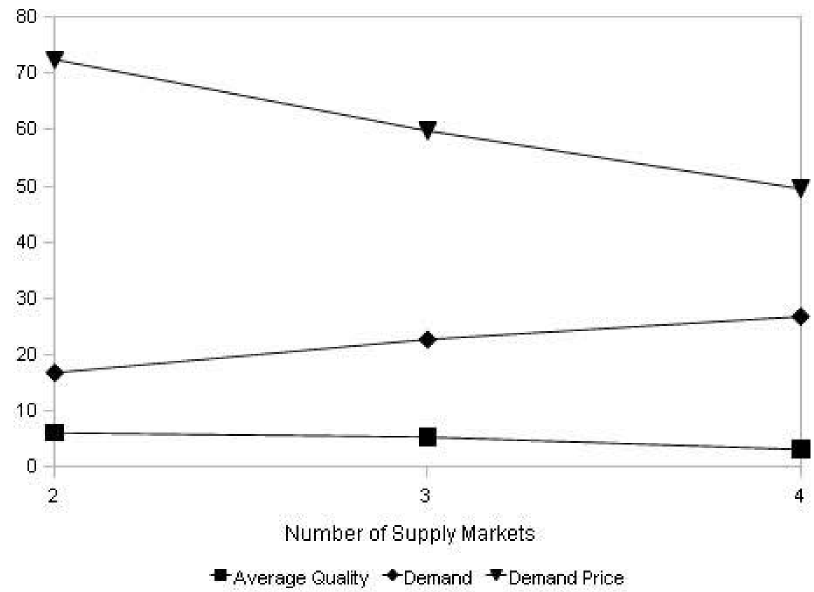


Figure 3: Impact of Additional Supply Markets on Average Quality, Demand, and Demand Price

5.2 Examples in Which the Number of Demand Markets is Increased

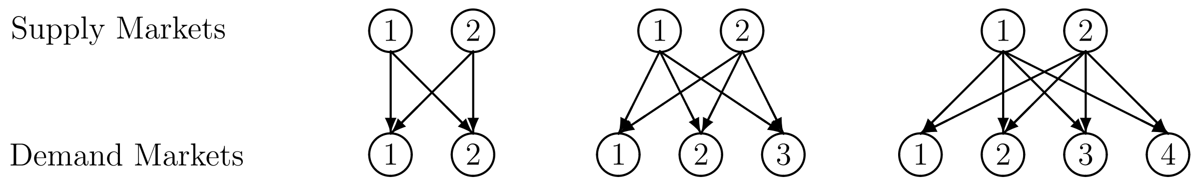
In the second set of examples we proceeded to explore the impact of the addition of demand markets. Such an experiment is valuable since new demand markets may open up and it is worthwhile to examine the impact on prices and the average quality on both existing and the new demand markets. New demand markets may arise because of demanding consumers, additional marketing, the opening up of new markets, reduction of trade restrictions, etc.

The network topologies for Examples 4, 5, and 6 are depicted in Figure 4, and the input data and the computed equilibrium solutions for these examples are reported, respectively, in Tables 4 and 5. The results for the average quality levels, demands, and demand prices are summarized in Table 6 and Figure 5.

Example 4

The network topology for Example 4 is given in Figure 4.a. This example consists of 2 supply markets and 2 demand markets. The input data are as in Example 1, except for the added data associated with the new demand market 2, which are provided in Table 4.

The Euler method required 342 iterations for convergence to the equilibrium solution shown in Table 5. Note that supply market 1 only serves demand market 1 since $Q_{12}^* = 0.00$.



a) Example 4 b) Example 5 c) Example 6

Figure 4: Network Topologies for Examples 4, 5, and 6

Example 5

Example 5 is constructed from Example 4, but it includes a new demand market 3 as depicted in Figure 4.b. New data are added as reported in Table 4.

The Euler method required 408 iterations for convergence to the solution in Table 5. Supply market 1 continues to supply only demand market 1. Also, interestingly, the quality levels of the product produced at supply market 1 and at supply market 2 have increased in comparison to the respective values for Example 4.

Example 6

Example 6 is constructed from Example 5 and has the same data except for the additional data associated with the new demand market 4. The network topology for this example is depicted in Figure 4.c. The input data are provided in Table 4.

The Euler method required 1,081 iterations for convergence to the equilibrium solution reported in Table 5.

We now provide the results of average quality levels, demands, and demand prices for the examples in this set. Please refer to Table 6. In Figure 5 the results for the first demand market are depicted. The respective results for the other demand markets follow a similar trend.

Our numerical examples indicate that, as the number of demand markets increases, the average quality at the demand markets increases. Also, as the number of demand markets increases, the demand prices at the demand markets increase, whereas the demands decrease slightly. A greater number of demand markets provides greater economic pressure on the producers at the supply markets to improve quality.

Table 4: Input Data for Examples 4, 5, and 6

Example	m	n	$\hat{\pi}(Q,q)$	$c(Q)$	$\hat{\rho}(Q,q)$	$OC(q)$
4	2	2	$\hat{\pi}_1(Q,q_1)=5(Q_{11}+Q_{12})+q_1+5,$ $\hat{\pi}_2(Q,q_2)=2(Q_{21}+Q_{22})+q_2+10.$	$c_{11}(Q),$ $c_{21}(Q),$ $c_{12}(Q)=Q_{12}+10,$ $c_{22}(Q)=2Q_{22}+10.$	$\hat{\rho}_1(Q,q)=-2(Q_{11}+Q_{21})+(\frac{q_1Q_{11}+q_2Q_{21}}{Q_{11}+Q_{21}})+100,$ $\hat{\rho}_2(Q,q)=-(Q_{12}+Q_{22})+(\frac{q_1Q_{12}+q_2Q_{22}}{Q_{12}+Q_{22}})+50.$	$OC_1(q_1),$ $OC_2(q_2).$
5	2	3	$\hat{\pi}_1(Q,q_1)=5(Q_{11}+Q_{12}+Q_{13})+q_1+5,$ $\hat{\pi}_2(Q,q_2)=2(Q_{21}+Q_{22}+Q_{23})+q_2+10.$	$c_{11}(Q),$ $c_{21}(Q),$ $c_{12}(Q),$ $c_{22}(Q),$ $c_{13}(Q)=2Q_{13}+10,$ $c_{23}(Q)=Q_{23}+5.$	$\hat{\rho}_1(Q,q)=-2(Q_{11}+Q_{21})+(\frac{q_1Q_{11}+q_2Q_{21}}{Q_{11}+Q_{21}})+100,$ $\hat{\rho}_2(Q,q),$ $\hat{\rho}_3(Q,q)=-(Q_{13}+Q_{23})+(\frac{q_1Q_{13}+q_2Q_{23}}{Q_{13}+Q_{23}})+50.$	$OC_1(q_1),$ $OC_2(q_2).$
6	2	4	$\hat{\pi}_1(Q,q_1)=5(Q_{11}+Q_{12}+Q_{13}+Q_{14})+q_1+5,$ $\hat{\pi}_2(Q,q_2)=2(Q_{21}+Q_{22}+Q_{23}+Q_{24})+q_2+10.$	$c_{11}(Q),$ $c_{21}(Q),$ $c_{12}(Q),$ $c_{22}(Q),$ $c_{13}(Q),$ $c_{23}(Q),$ $c_{14}(Q)=Q_{14}+5,$ $c_{24}(Q)=2Q_{24}+5.$	$\hat{\rho}_1(Q,q)=-2(Q_{11}+Q_{21})+(\frac{q_1Q_{11}+q_2Q_{21}}{Q_{11}+Q_{21}})+100,$ $\hat{\rho}_2(Q,q),$ $\hat{\rho}_3(Q,q),$ $\hat{\rho}_4(Q,q)=-(Q_{14}+Q_{24})+.5(\frac{q_1Q_{14}+q_2Q_{24}}{Q_{14}+Q_{24}})+60.$	$OC_1(q_1),$ $OC_2(q_2).$

Table 5: Equilibrium Solutions for Examples 4, 5, and 6

	Example 4	Example 5	Example 6
Q^*	$Q^*_{11} = 7.30$, $Q^*_{12} = 0.00$, $Q^*_{21} = 8.93$, $Q^*_{22} = 2.43$.	$Q^*_{11} = 7.60$, $Q^*_{12} = 0.00$, $Q^*_{13} = 0.00$, $Q^*_{21} = 7.89$, $Q^*_{22} = 1.16$, $Q^*_{23} = 4.23$.	$Q^*_{11} = 7.11$, $Q^*_{12} = 0.00$, $Q^*_{13} = 0.00$, $Q^*_{14} = 0.94$, $Q^*_{21} = 7.43$, $Q^*_{22} = 0.18$, $Q^*_{23} = 0.18$, $Q^*_{24} = 4.34$.
q^*	$q^*_1 = 10.38$, $q^*_2 = 3.64$.	$q^*_1 = 10.75$, $q^*_2 = 4.06$.	$q^*_1 = 11.32$, $q^*_2 = 4.38$.

Table 6: Summary of the Results for the Average Quality Levels, Demands, and Demand Prices for Examples 1, 4, 5, and 6

	Example 1	Example 4	Example 5	Example 6
m	2	2	2	2
n	1	2	3	4
$\hat{q}$	$\hat{q}_1 = 6.13$.	$\hat{q}_1 = 6.67$, $\hat{q}_2 = 3.64$.	$\hat{q}_1 = 7.35$, $\hat{q}_2 = 4.06$, $\hat{q}_3 = 4.06$.	$\hat{q}_1 = 7.77$, $\hat{q}_2 = 4.38$, $\hat{q}_3 = 4.38$, $\hat{q}_4 = 5.62$.
d	$d_1 = 16.85$.	$d_1 = 16.23$, $d_2 = 2.43$.	$d_1 = 15.49$, $d_2 = 1.16$, $d_3 = 4.23$.	$d_1 = 14.54$, $d_2 = 0.18$, $d_3 = 2.77$, $d_4 = 5.29$.
ρ	$\rho_1 = 72.44$.	$\rho_1 = 74.21$, $\rho_2 = 51.21$.	$\rho_1 = 76.37$, $\rho_2 = 52.91$, $\rho_3 = 49.83$.	$\rho_1 = 78.70$, $\rho_2 = 54.20$, $\rho_3 = 51.61$, $\rho_4 = 57.53$.

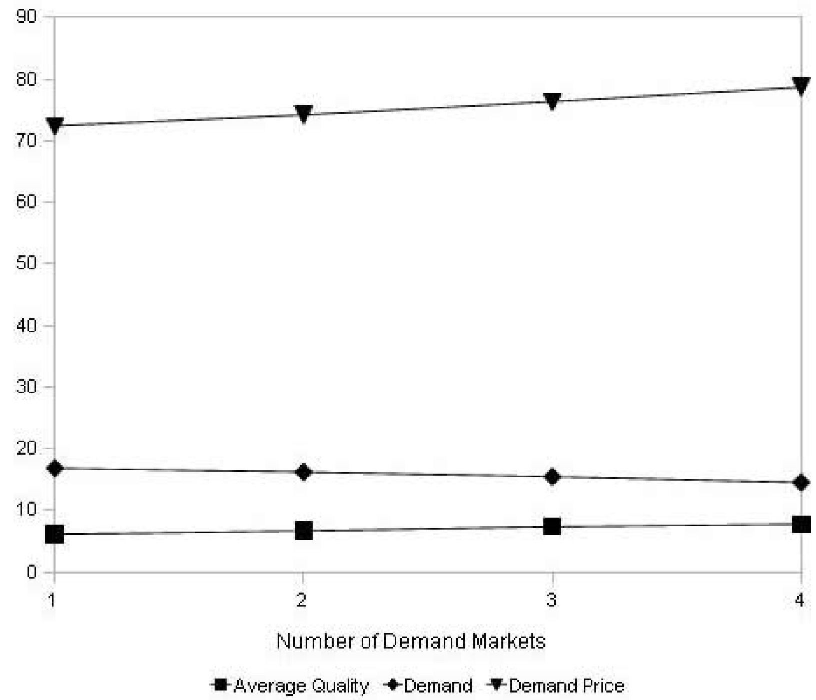


Figure 5: Impact of Additional Demand Markets on Average Quality, Demand, and Demand Price at Demand Market 1

5.3 Examples in Which Minimum Quality Standards Are Imposed

In the last set of numerical examples, we imposed minimum quality standards. Specifically, for each of the preceding examples, we set the minimum quality standard at all supply markets equal to the maximum equilibrium quality level at the supply markets computed for the corresponding example without imposed minimum quality standards.

The results for these examples are reported in Table 7. The results for the average quality levels, demands, and demand prices for all 12 examples are summarized in Table 8 in order to show the impact of the minimum quality standards.

Example 7

This example is constructed from Example 1, except that now we have that:

$$\underline{q}_1 = \underline{q}_2 = 10.80.$$

The Euler method converged in 218 iterations to the equilibrium solution given in Table 7.

An interesting question is whether the imposition of minimum quality standards has affected the stability results as obtained for Example 1. We determine $\nabla F(X)$ according to (44) for Example 7 (which recall was constructed from Example 1) and evaluate it at the equilibrium solution $X^* = (Q_{11}^*, Q_{21}^*, q_1^*, q_2^*)$, which yields:

$$\nabla F(X) = \begin{pmatrix} 8.00 & 2.00 & -.42 & .42 \\ 2.00 & 6.00 & .44 & .46 \\ -5 & 0 & 4 & 0 \\ 0 & -2 & 0 & 9 \end{pmatrix}.$$

We note that the eigenvalues of $\frac{1}{2}(\nabla F(X) + \nabla F(X)^T)$ are: 2.2520, 5.4962, 9.1440, and 10.1077. Since these eigenvalues are all positive, there is a unique equilibrium and we know from Theorem 5, that the computed X^* in the case of minimum quality standards, is globally exponentially stable. Hence, the same type of stability holds for this example with minimum quality standards, as without.

Example 8

This example is constructed from Example 2, except that now we impose the minimum quality standard (which was the highest achieved equilibrium quality level in Example 2):

$$\underline{q}_1 = \underline{q}_2 = 7.90.$$

The Euler method converged in 302 iterations and yielded the equilibrium solution provided in Table 7. Interestingly, the quality level of the product at the first supply market exceeds the imposed minimum quality standard.

Example 9

Example 9 is constructed from Example 3 with the imposition of the following minimum quality standard (which was the highest computed equilibrium quality level in Example 3):

$$\underline{q}_1 = \underline{q}_2 = 6.13.$$

The Euler method converged in 332 iterations and yielded the equilibrium solution reported in Table 7.

Example 10

Example 10 has the same data as Example 4 except for the imposition of the following minimum quality standards:

$$\underline{q}_1 = \underline{q}_2 = 10.38.$$

The Euler method required 202 iterations for convergence to the equilibrium solution shown in Table 7.

Example 11

Example 11 is built from Example 5 and, hence, has the same data as Example 5 except for the inclusion of the following minimum quality standards:

$$\underline{q}_1 = \underline{q}_2 = 10.75.$$

The Euler method required 482 iterations for convergence to the equilibrium solution in Table 7.

Example 12

In our final example, we proceeded as in the preceding 5 examples. We constructed Example 12 from Example 6, using its data with the inclusion of minimum quality standards, which were set as follows:

$$\underline{q}_1 = 11.32, \quad \underline{q}_2 = 11.32.$$

The value 11.32 was selected since it is the highest equilibrium quality obtained at a supply market for Example 6.

The Euler method required 950 iterations for convergence to the equilibrium solution.

As revealed in Tables 2 and 5, when no minimum quality standards are imposed, supply market 1 is always the one that is able to produce at the highest quality level. However, in Examples 7 through 12, with minimum quality standards set to the highest quality levels of supply market 1, the other supply markets are forced to produce higher quality at higher costs. Under this circumstance, in order to break-even, some products, which were originally produced by supply markets 2, 3, and/or 4, are given up to supply market 1. With more products to produce, supply market 1 produces at even higher quality (cf. (11)) in the cases with minimum quality standards (cf. Table 7) than in the cases without (cf. Tables 2 and 5).

In Table 8, a summary of the results for all 12 examples is provided in order to demonstrate the impacts of minimum quality standards. The examples are grouped without and with the corresponding minimum quality standards. As depicted in Table 3, after imposing minimum quality standards, the average quality at the demand markets increases, and the prices also increase.

Table 7: Results for Examples 7 through 12

	Example 7	Example 8	Example 9	Example 10	Example 11	Example 12
Q^*	$Q^*_{11} = 7.66$, $Q^*_{21} = 9.17$.	$Q^*_{11} = 5.66$, $Q^*_{21} = 6.10$, $Q^*_{31} = 11.11$.	$Q^*_{11} = 4.24$, $Q^*_{21} = 3.97$, $Q^*_{31} = 7.98$, $Q^*_{41} = 10.90$.	$Q^*_{11} = 7.91$, $Q^*_{12} = 0.00$, $Q^*_{21} = 8.18$, $Q^*_{22} = 2.73$.	$Q^*_{11} = 8.20$, $Q^*_{12} = 0.00$, $Q^*_{13} = 0.00$, $Q^*_{21} = 7.03$, $Q^*_{22} = 1.37$, $Q^*_{23} = 4.55$.	$Q^*_{11} = 7.65$, $Q^*_{12} = 0.00$, $Q^*_{13} = 0.00$, $Q^*_{14} = 1.01$, $Q^*_{21} = 6.72$, $Q^*_{22} = 0.59$, $Q^*_{23} = 3.39$, $Q^*_{24} = 3.40$.
q^*	$q^*_1 = 10.82$, $q^*_2 = 10.80$.	$q^*_1 = 8.32$, $q^*_2 = 7.90$, $q^*_3 = 7.90$.	$q^*_1 = 6.55$, $q^*_2 = 6.13$, $q^*_3 = 6.13$, $q^*_4 = 6.13$.	$q^*_1 = 11.13$, $q^*_2 = 10.38$.	$q^*_1 = 11.50$, $q^*_2 = 10.75$.	$q^*_1 = 12.07$, $q^*_2 = 11.32$.
$\hat{q}$	$\hat{q}_1 = 10.81$.	$\hat{q}_1 = 8.00$.	$\hat{q}_1 = 6.20$.	$\hat{q}_1 = 10.75$, $\hat{q}_2 = 10.38$.	$\hat{q}_1 = 11.15$, $\hat{q}_2 = 10.75$, $\hat{q}_3 = 10.75$.	$\hat{q}_1 = 11.72$, $\hat{q}_2 = 11.32$, $\hat{q}_3 = 11.32$, $\hat{q}_4 = 11.49$.
d	$d_1 = 16.83$.	$d_1 = 22.86$.	$d_1 = 27.10$.	$d_1 = 16.09$, $d_2 = 2.73$.	$d_1 = 15.23$, $d_2 = 1.37$, $d_3 = 4.55$.	$d_1 = 14.37$, $d_2 = 0.59$, $d_3 = 3.39$, $d_4 = 4.41$.
ρ	$\rho_1 = 76.76$.	$\rho_1 = 62.28$.	$\rho_1 = 52.00$.	$\rho_1 = 78.57$, $\rho_2 = 57.65$.	$\rho_1 = 80.70$, $\rho_2 = 59.38$, $\rho_3 = 56.20$.	$\rho_1 = 82.98$, $\rho_2 = 60.73$, $\rho_3 = 57.93$, $\rho_4 = 61.34$.

Table 8: Summary of the Results for the Average Quality Levels, Demands, and Demand Prices for Examples 1 through 12

Example	m	n	$\hat{q}$	d	ρ
1	2	1	6.13	16.85	72.44
7	2	1	10.81	16.83	76.76
2	3	1	5.36	22.76	59.83
8	3	1	8.00	22.86	62.28
3	4	1	3.17	26.82	49.54
9	4	1	6.20	27.10	52.00
4	2	2	6.67, 3.64	16.23, 2.43	74.21, 51.21
10	2	2	10.75, 10.38	16.09, 2.73	78.57, 57.65
5	2	3	7.35, 4.06, 4.06	15.49, 1.16, 4.23	76.37, 52.91, 49.83
11	2	3	11.15, 10.75, 10.75	15.23, 1.37, 4.55	80.70, 59.38, 56.20
6	2	4	7.77, 4.38, 4.38, 5.62	14.54, 0.18, 2.77, 5.29	78.70, 54.20, 51.61, 57.53
12	2	4	11.72, 11.32, 11.32, 11.49	14.37, 0.59, 3.39, 4.41	82.98, 60.73, 57.93, 61.34

6. Summary and Conclusions

In this paper, we constructed both static and dynamic models of spatial competition in the case of asymmetric information in terms of product quality. The work builds on models of spatial price equilibria but incorporates significant extensions to capture the reality that producers, located at the supply markets, know the quality of the product that they produce whereas consumers, located at the demand markets, may only be aware of the average quality. Such a framework may apply to products as varied as certain food products, oil and gas, and even medicines that are produced far from points of consumption.

The methodologies that we utilized for the development of the static models was that of variational inequality theory and, for the dynamic counterpart, we utilized projected dynamical systems theory. We provided qualitative analysis in terms of conditions for existence and uniqueness of equilibria as well as stability analysis for the solutions of the associated projected dynamical system.

We proposed an algorithm, along with convergence results, and then demonstrated through a series of numerical examples how the modeling and computational framework can be used to assess the impacts of the addition of supply markets and the addition of demand markets on the demand market prices, average quality at the demand markets, and the demands. We find that as the number of supply markets increases, the average quality at the demand market decreases, the demand increases, and the price decreases. In contrast, as the number of demand markets increases, the average quality at the demand markets increases, the demand decreases, and the demand price at the demand markets increases.

In addition, we showed how the model also handles minimum quality standards and provided numerical examples that illustrated the impacts of such policy regulations on the flows, prices, and average quality. In particular, after the imposition of minimum quality standards, the average quality at the demand markets increases and the prices also increase.

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Appendix

Proof of Theorem 1:

We first establish necessity, that is, if $(s^*, Q^*, d^*, q^*) \in \mathcal{K}^1$ satisfies the spatial price equilibrium conditions according to Definition 1, then it also satisfies variational inequality (12).

Note that, for a fixed pair of supply and demand markets (i, j), (10) implies that

$$(\pi_i(s^*, q_i^*) + c_{ij}(Q^*) - \rho_j(d^*, \hat{q}^*)) \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q_{ij} \geq 0. \tag{A1}$$

Indeed, since, if $Q_{ij}^* > 0$, we know, from the equilibrium conditions, that the expression to the left of the multiplication sign in (A1) will be identically zero, so (A1) holds true; also, if $Q_{ij}^* = 0$, then the expression preceding and following the multiplication sign in (A1) will be nonnegative and, hence, the product is also nonnegative and (A1) holds true for this case, as well. Summing (A1) over all supply markets i and over all demand markets j, we obtain:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i(s^*,q_i^*)+c_{ij}(Q^*)-\rho_j(d^*,\hat{q}^*))\times(Q_{ij}-Q_{ij}^*)\geq 0, \quad \forall Q\in R_+^{mn}. \qquad (A2)$$

Rewriting (A2) as:

$$\sum_{i=1}^{m}\pi_i(s^*,q_i^*)\times\sum_{j=1}^{n}(Q_{ij}-Q_{ij}^*)+\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}(Q^*)\times(Q_{ij}-Q_{ij}^*)$$

$$-\sum_{j=1}^{n}\rho_j(d^*,\hat{q}^*))\times(\sum_{i=1}^{m}Q_{ij}-\sum_{i=1}^{m}Q_{ij}^*)\geq 0, \qquad (A3)$$

and then simplifying (A3) by using the supply and demand conservation of flow equations (1) and (2) yields:

$$\sum_{i=1}^{m}\pi_i(s^*,q_i^*)\times(s_i-s_i^*)+\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}(Q^*)\times(Q_{ij}-Q_{ij}^*)-\sum_{j=1}^{n}\rho_j(d^*,\hat{q}^*)\times(d_j-d_j^*)\geq 0. \quad (A4)$$

Analogously, it follows that if q^* satisfies (11), then:

$$(OC_i(q_i^*)-\pi_i(s^*,q_i^*))\times(q_i-q_i^*)\geq 0, \quad \forall q_i\geq 0. \qquad (A5)$$

Summing (A5) over all i, we obtain

$$\sum_{i=1}^{m}(OC_i(q_i^*)-\pi_i(s^*,q_i^*))\times(q_i-q_i^*)\geq 0, \quad \forall q\in R_+^{m}. \qquad (A6)$$

Combining now (A4) and (A6), yields variational inequality (12).

We now turn to establishing sufficiency, that is, if $(s^*,Q^*,d^*,q^*)\in\mathcal{K}^1$ satisfies variational inequality (12) then it also satisfies the spatial price equilibrium conditions (10) and (11).

We first expand variational inequality (12), with the use of the conservation of flow equations (1) and (2), to obtain:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i(s^*,q_i^*)+c_{ij}(Q^*)-\rho_j(d^*,\hat{q}^*))\times(Q_{ij}-Q_{ij}^*)\geq 0, \quad \forall Q\in R_+^{mn}. \qquad (A7)$$

Let $q_i=q_i^*$, $\forall i$, and $Q_{ij}=Q_{ij}^*$, $\forall (i,j)\neq(k,l)$, and substitute into (A7). The resultant is:

$$(\pi_k(s^*,q_k^*)-c_{kl}(Q^*)-\rho_l(d^*,\hat{q}^*))\times(Q_{kl}-Q_{kl}^*)\geq 0, \quad \forall Q_{kl}\geq 0. \qquad (A8)$$

But (A8) implies that, if $Q^*_{kl} = 0$, then $(\pi_k(s^*, q^*_k) - c_{kl}(Q^*) - \rho_l(d^*, \hat{q}^*)) \geq 0$, and, if $Q^*_{kl} > 0$, then, for (A8) to hold, $(\pi_k(s^*, q^*_k) - c_{kl}(Q^*) - \rho_l(d^*, \hat{q}^*)) = 0$. But since these results hold for any pair (k, l), we can conclude that the equilibrium conditions (10) are satisfied by the product shipment pattern satisfying (12).

Similarly, we now let $Q_{ij} = Q^*_{ij}$, $\forall (i, j)$, and $q_i = q^*_i$, $\forall i \neq k$. Substitution into (12) yields:

$$(OC_k(q^*_k) - \pi_k(s^*, q^*_k)) \times (q_k - q^*_k) \geq 0, \quad \forall q_k \geq 0. \tag{A9}$$

According to (A9), if $q^*_k = 0$, then $(OC_k(q^*_k) - \pi_k(s^*, q^*_k)) \geq 0$, since $q_k \geq 0$, and, if $q^*_k > 0$, then $(OC_k(q^*_k) - \pi_k(s^*, q^*_k)) = 0$, since $(q_k - q^*_k)$ can be positive, negative, or zero. Since these results hold for any supply market k, we know that $q^* \in R^m_+$ satisfying variational inequality (12) also satisfies equilibrium conditions (11). The proof is complete. $\square$