Quantification of International Trade Network Performance Under Disruptions to Supply, Transportation, and Demand Capacity, and Exchange Rates in Disasters

Anna Nagurney and Dana Hassani
Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts, Amherst, Massachusetts 01003
and
Oleg Nivievskyi and Pavlo Martyshev
Center for Food and Land Use Research
Kyiv School of Economics
Mykoly Shpaka St. 3
Kyiv, Ukraine 02000

Revised March 2024

Accepted in: Dynamics of Disasters - From Natural Phenomena to Human Activity, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, S. Pickl, C. Vogiatzis, Editors, Springer Nature Switzerland AG.

Abstract: Both sudden-onset and slow-onset disasters are causing disruptions to global trade, impacting the availability and affordability of commodities from agricultural to mineral ones. In this paper, we develop a multicommodity international trade network equilibrium model under disaster scenarios with distinct probabilities of the occurrence of the disasters and their impacts on the capacities associated with production, transportation, and consumption. The disaster scenarios can also affect the exchange rates. We state the governing equilibrium conditions and derive the variational inequality formulation in commodity path flow variables and Lagrange multipliers associated with the capacity constraints. For each disaster scenario, we construct an international trade network performance measure, followed by a unified performance measure that includes all the disasters and their probabilities. Robustness is then quantified as the difference between the network performance under no disruptions and the unified performance measure. An international trade network component performance indicator is also given to assess the impacts of the complete removal of trade network supply markets, demand markets, and/or transportation routes. The modeling framework is then illustrated through a series of numerical examples, motivated by Russia's war on Ukraine. The work is of relevance to decision-makers and policy-makers.

Key words: networks, international trade, disasters, disruptions, robustness, variational inequalities

1. Introduction

Disasters, both sudden-onset and slow-onset ones, are transforming the landscapes for global trade, generating shocks on the supply side, on transportation, as well as on the demand side. The COVID-19 pandemic, as a disaster not limited to time nor space, vividly demonstrated the impacts of the shutdowns of manufacturing plants and food processing plants and the challenges associated with the harvesting of agricultural products, as well as the reduction in freight service capabilities, plus the changes in the demand for many products as workers, who could, worked from home. Many suffered from the illness, and millions perished (cf. Nagurney (2023a)). Droughts on multiple continents have exacerbated food insecurity and rising hunger as crops wither. For example, the Horn of Africa is experiencing its worst ever recorded drought over five rainfall seasons (World Meteorological Organization (2023)), pushing more than 3 million of the area's residents to extreme levels of food insecurity and also resulting in the displacement of millions of people (Cassidy (2022)). Also, the drought near the Panama Canal, a major trade thoroughfare, has dramatically reduced the capacity of the number of vessels, resulting in bottlenecks and congestion (Li and Nguyen (2023)) with serious implications for international trade. In the US, the California wildfires, further exacerbated by climate change and droughts (Escriva-Bou et al. (2022)), have resulted in billions of dollars of yearly damage to the agricultural industry, specifically to the globally famous wine industry of California (Zakowski et al. (2023)). While California is home to only around 4% of the farms in the US, the state generates around 11% of US agricultural value (Vankin (2023)).

Earthquakes in early February 2023 devastated parts of Turkey, including agricultural lands, and have affected the country's GDP (see Food and Agricultural Organization of the United Nations (2023)). The major earthquake hitting Morocco in September 2023 resulted in numerous deaths and injuries in an agrarian province that was already struggling with a drought (Metz (2023)). Floods in the US (cf. PBS (2023)) and in Greece have damaged farmlands and crops (Nikas (2023)). In early September 2023, Greece experienced a four-day storm causing deadly flooding that deluged the arable land in the Thessaly Plain, an area that provides around one-sixth of Greece's agricultural output, with about 15% of the annual production of Greek grains (NASA Earth Observatory (2023), Prousalis and Papadimas (2023)). Greece regularly deals with wildfires in the summer, which also damage arable land, and the flooding has further complicated the disastrous situation in the Thessaly area, where about 90% of the population used to be engaged in the agricultural sector, contributing 8 billion euros to the Greek economy (Michalopoulos (2023)). In mid-September 2023, the Mediterranean storm Daniel and the consequent heavy rains resulted in two dams collapsing, with the flooding in eastern Libya causing the deaths of more than 11,000. Over 10,000 people remain still missing, and more than 40,000 are displaced (Magdy and Murad (2023), Mroue (2023)).

In addition to the numerous disasters, some intensified because of climate change, increasing global strife and violence, as well as wars, notably the full-scale invasion of Ukraine by Russia on February 24, 2022, have led to growing suffering and loss of life, as well as major impacts on economies and trade. Farmers in Ukraine, for example, have had to contend with the mining of their agricultural lands, with the withdrawal of Russia from the Black Sea Grain Initiative in mid-July 2023, affecting the efficient and secure transport of grain on the Black Sea (cf. Nagurney (2023b)). Many agricultural storage facilities have been destroyed in this war as well as port infrastructure. The impacts of this war are affecting the supplies of agricultural commodities as well as their prices, and food security (cf. Nagurney et al. (2023)). Disasters are increasing migratory flows and are changing commodity demand patterns (Migration Trade Portal (2021)). Clearly, disasters have caused and will continue to cause, in an era of climate change and global uncertainty, disruptions to the production, transportation, and consumption of many commodities that are needed for the sustenance and sustainability of humanity.

In this paper, we construct a multicommodity international trade network equilibrium model under disruptions in disaster scenarios affecting the capacity to produce on the supply side at the supply markets, the capacity to transport the commodities via one or more routes joining the supply and demand markets, as well as the capacity to consume the commodities at the demand markets. In addition, the model captures possible disruptions to exchange rates associated with the international trade of commodities. Each disaster scenario has associated with it a probability of occurrence. The probabilities are for mutually exclusive and collectively exhaustive cases of scenarios.

2. Literature Review, Contributions, and Organization of the Paper

The intellectual foundations for our modeling framework are based on the classical work of Samuelson (1952) and Takayama and Judge (1964, 1971) in spatial price equilibrium modeling but with the benefit of the variational inequality methodology (cf. Dafermos (1980), Florian and Los (1982), Dafermos and Nagurney (1984), Nagurney (1999), Yu and Nagurney (2013), Nagurney, Besik, and Dong (2019), Nagurney and Besik (2022)). Specifically, in this paper, we build upon the recent work of Nagurney et al. (2023, 2024) and Nagurney and Besedina (2023), which introduced exchange rates in a rigorous manner in general multicommodity spatial price equilibrium models for international trade. Prior to these works, the only spatial price equilibrium model with exchange rates was that of Devadoss and Sabala (2020), and it required assumptions to guarantee an optimization reformulation.

In this paper, we extend the work of Nagurney et al. (2024) in several significant ways:

1. We allow for not only capacities on the supplies and on the transportation routes but also at the demand markets.

2. The Nagurney et al. (2024) paper focused on agricultural commodities and, hence, the capacities at the supply markets and transportation routes were over all the commodities. Here, we impose separate bounds on individual commodities.

3. In this paper, we introduce distinct disaster scenarios, with associated probabilities and capacities and exchange rates.

4. Each disruption scenario yields a distinct variational inequality problem.

5. We construct an international trade network performance measure under each specific disaster scenario disruption and then propose a unified international trade network performance measure. We also provide the measure assuming no disruptions.

6. A robustness measure is then constructed using the unified international trade performance measure and the performance of the international trade network in the absence of disruptions.

7. An importance indicator is proposed to allow for the quantification of network components, specifically, the supply markets, the demand markets, the transportation routes, or a combination thereof, and their ranking. This is the first time that such an assessment tool has been constructed in the context of multicommodity international trade.

8. The framework is applied to Russia's war on Ukraine under different scenarios.

The research in this paper adds to the growing, broad literature on network vulnerability (see, e.g., Grubesic et al. (2008), Qiang and Nagurney (2008), Dinh et al. (2012), Nagurney and Qiang (2012), Butenko, Pasiliao, Shylo (2014), and the references therein), which has included transportation network vulnerability (see, e.g., Jiang et al. (2021)) as well as that of supply chains (see, e.g., Wu and Blackhurst (2009)). For a plethora of models and applications, see the book by Nagurney and Qiang (2009). For a summary of network robustness and resilience measures focusing on optimization approaches please refer to Sharkey et al. (2020). The work here, in contrast, is focused on an equilibium network model in the setting of international trade. The work in this paper is inspired, in part, by that of Nagurney and

Qiang (2012), who introduced a bicriteria indicator for critical needs products in a supply chain network optimization context under supply and demand disruptions. Here, however, we propose an equilibrium model, and not an optimization one. In addition, our model is not limited to a single commodity or product. We utilize a spatial price equilibrium framework since spatial price equilibrium models have had wide application to numerous economic sectors, from agriculture to minerals, and such commodities are very important to societies and economies. For example, as noted in Nagurney, Li, and Nagurney (2014), spatial price equilibrium models have been applied to various agricultural sectors (cf. Thompson (1989)) for commodities such as: eggs (see Judge (1956)), potatoes (Howard (1984)), beef (Sohn (1970)), cereal grains (Ruijs et al. (2001)), as well as dairy (Bishop, Pratt, and Novakovic (1994)). Examples of spatial price equilibrium applications to mineral ore as well as energy industries have been studied by Hwang et al. (1994), Labys and Yang (1991), Labys (1999), Newcomb and Fan (1980), Newcomb, Reynolds, and Masbruch (1990), and Irwin and Yang (1996).

As mentioned earlier, the model in this paper draws on the research of Nagurney et al. (2024). In addition, we note the work of Qiang, Nagurney, and Dong (2009), which introduced a multitiered supply chain network equilibrium model with supply-side risk and demand-side risk, as well as uncertainty in transportation and other costs. The authors also constructed a network performance measure, focusing on the supply chain network, which we build upon here to propose a network performance measure in the case of multicommodity international trade and exchange rates. We remark that the Nagurney et al. (2024) paper has also inspired the work of Passacantando and Raciti (2024), who used the model therein to construct a random variational inequality in which exchange rates are random variables, under certain assumptions. The authors also proposed a measure for route vulnerability in terms of impacts on trade flows. Aleskerov and Tkachev (2024) considered models of trade networks of food, rare earth compounds, and oil under deep uncertainty with scenarios of different consequences and the identification of the most vulnerable countries (see, also, Aleskerov et al. (2022)).

The work in this paper adds to the literature on disaster management in terms of mitigation and recovery since the international trade network performance measure under no disruptions and the associated international trade network component importance indicator (see, for example, Nagurney and Qiang (2009)) can be applied to ascertain where investments should take place to reduce the impacts of disruptions on international trade. A plethora of papers on the dynamics of disasters, relevant models, algorithms, and applications can be found in the edited volumes of Kotsireas, Nagurney, and Pardalos (2018) and Kotsireas et al. (2021). We recognize the existence of disaster scenarios that happen simultaneously, often referred to as compounding disasters, including hybrid threats such as disasters due to natural hazards and human attacks at the same time (see Fytopoulos and Pardalos (2023) and Jasper (2023)).

The rest of the paper is organized as follows. The multicommodity international trade model under disruptions is constructed in Section 3, where the equilibrium conditions are stated and reformulated as a variational inequality problem in commodity path flows and associated Lagrange multipliers for each disruption scenario. In Section 4, the unified international trade network performance measure and the robustness measure of the international trade network under capacity disruptions in disasters are defined. These measures are relevant to the quantification of an international trade network's performance subject to disruptions and to its robustness. In Section 5, the algorithmic framework with which the numerical examples in Section 6 are solved is detailed. The algorithm we use is the modified projection method of Korpelevich (1977). Since we establish a variational inequality formulation with commodity shipments over paths and Lagrange multipliers associated with capacity constraints under disruptions, the algorithmic scheme results in closed-form expressions for the variables at each iteration. The explicit expressions are provided for completeness and easy reproducibility. A series of numerical examples drawn from Russia's war on Ukraine and on an international trade network for wheat and corn are solved using the presented algorithm in Section 6. Therein, calculations of the various measures, including the importance identification of different trade routes, are also provided with accompanying insights and analysis. Finally, Section 7 summarizes and concludes the paper.

3. The International Trade Model Under Disruptions

In this Section, we construct the international trade network equilibrium model under disruptions. There are m supply markets involved in the production of K commodities. A typical supply market is denoted by i and a typical commodity by k. There are n demand markets in the international trade network, with a typical demand market denoted by j. P routes join each pair of supply and demand markets with a typical route denoted by r. Note that a route r connecting a pair of supply and demand markets need not correspond to the same route r joining another pair of markets. The network representation of the model is depicted in Figure 1.

There are ω different disruption scenarios that can affect the capacities of production of the multiple commodities, their transport, and/or the demand for them. We define the disruption scenario set $\Xi^1 \equiv \{\xi_1, \xi_2, \ldots, \xi_\omega\}$ and we denote by ξ^0 the scenario where there is no

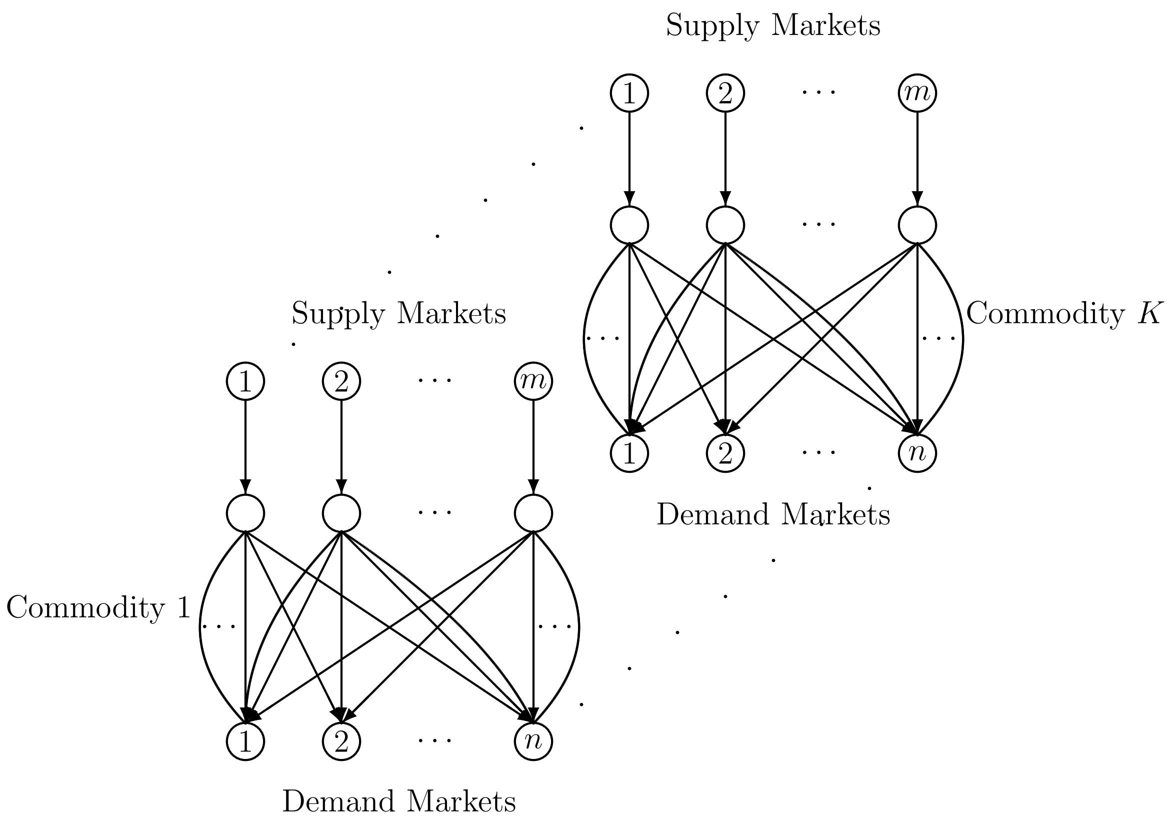

Figure 1: The Multicommodity International Trade Network

disruption. Note that the network in Figure 1 corresponds to the case where no disruption happens, i.e., ξ^0. The network capacities are updated according to each disaster scenario in Ξ^1. Of course, if trade is not possible between a pair of supply and demand markets for a commodity on a link then one can set the associated capacity to 0.00. Furthermore, we associate a probability with each disruption scenario defined, respectively, as: $p_{\xi_1}, p_{\xi_2}, \ldots, p_{\xi_\omega}$, with p_{ξ^0} denoting the probability that no disruption occurs. These disruption scenarios are assumed to be independent.

In this paper, as done in Qiang and Nagurney (2012), but in the context of a supply chain network optimization model for critical needs products in disasters, we utilize discrete probabilities to study disruption scenarios. As noted in that paper, in the case of the majority of relevant disruptions and, in particular, those due to natural hazards, there is a lack of historical data that would enable the generation of more detailed probabilities, as in the case of continuous probabilities. Discrete probabilities, which might make use of experts' subjective judgment, provide a decent baseline.

The basic model notation is given in Table 1. All vectors are column vectors.

Table 1: Notation for the Multicommodity International Trade Network Model

Notation	Parameter Definition
$u_i^{s^k \xi_l}$	upper bound on supply of commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$.
$u_{ijr}^{Q^k \xi_l}$	upper bound on transport of commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ on route r; $r = 1, \ldots, P$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$.
$u_j^{d^k \xi_l}$	upper bound on the demand of commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$, under disaster scenario ξ_l; $l = 1, \ldots, \omega$. We group all the upper bounds for all the disaster scenarios into the vector u.
$e_{ij}^{\xi_l}$	exchange rate from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ and disaster scenario ξ_l; $l = 1, \ldots, \omega$. We group the exchange rates for disaster scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $e^{\xi_l} \in R_+^{mn}$ and then group all the exchange rates for all the disaster scenarios into the vector $e \in R_+^{mn\omega}$.
Notation	**Variable Definition**
$s_i^{k\xi_l}$	the supply of the commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$. We group all the supplies at disaster scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $s^{\xi_l} \in R_+^{Km}$, and then group all the supplies for all the disaster scenarios into the vector $s \in R_+^{Km\omega}$.
$d_j^{k\xi_l}$	the demand for the commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$. We group all the demands at disaster scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $d^{\xi_l} \in R_+^{Kn}$, and then group all the demands for all the disaster scenarios into the vector $d \in R_+^{Kn\omega}$.
$Q_{ijr}^{k\xi_l}$	the shipment of the commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$, to demand market j; $j = 1, \ldots, n$, on route r; $r = 1, \ldots, P$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$. We group all the commodity shipments at disaster scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $Q^{\xi_l} \in R_+^{KmnP}$, and then group all the commodity shipments into the vector $Q \in R_+^{KmnP\omega}$.
Notation	**Function Definition**
$\pi_i^k(s^{\xi_l})$	the supply price function for commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$.
$\rho_j^k(d^{\xi_l})$	the demand price function for commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$.
$c_{ijr}^k(Q^{\xi_l})$	the unit transportation cost associated with shipping the commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ via route r; $r = 1, \ldots, P$ under disaster scenario ξ_l; $l = 1, \ldots, \omega$.

The Conservation of Flow Equations

We now state the conservation of flow equations.

The quantity of commodity k produced at supply market i under each disaster scenario ξ_l must be equal to the sum of the shipments of the commodity from i to all the demand markets in the disaster scenario over all the routes; that is:

$$s_i^{k\xi_l} = \sum_{j=1}^{n} \sum_{r=1}^{P} Q_{ijr}^{k\xi_l}, \quad k = 1, \ldots, K; i = 1, \ldots, m; l = 1, \ldots, \omega. \tag{1}$$

In addition, the demand for each commodity k at each demand market j must be equal to the sum of the shipments of the commodity to demand market j over all the routes in each disaster scenario ξ_l; that is:

$$d_j^{k\xi_l} = \sum_{i=1}^{m} \sum_{r=1}^{P} Q_{ijr}^{k\xi_l}, \quad k = 1, \ldots, K; j = 1, \ldots, n; l = 1, \ldots, \omega. \tag{2}$$

Also, all the commodity shipments must be nonnegative in all the disaster scenarios; that is:

$$Q_{ijr}^{k\xi_l} \geq 0, \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, P; l = 1, \ldots, \omega. \tag{3}$$

We have the following capacity constraints to capture disruptions under different disaster scenarios.

The supply (production output) of commodity k at supply market i under disaster scenario ξ_l cannot exceed the associated supply capacity, that is:

$$s_i^{k\xi_l} \leq u_i^{s^k\xi_l}, \quad k = 1, \ldots, K; i = 1, \ldots, m; l = 1, \ldots, \omega. \tag{4a}$$

or, equivalently, by using (1):

$$\sum_{j=1}^{n} \sum_{r=1}^{P} Q_{ijr}^{k\xi_l} \leq u_i^{s^k\xi_l}, \quad k = 1, \ldots, K; i = 1, \ldots, m; l = 1, \ldots, \omega \tag{4b}$$

We associate the Lagrange multiplier $\lambda_i^{s^k\xi_l}$ with constraint (4b) for $k = 1, \ldots, K; i = 1, \ldots, m; l = 1, \ldots, \omega$. We group these Lagrange multipliers for each disaster scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $\lambda_+^{s\xi_l} \in R_+^{Km}$.

The demand for commodity k at demand market j, in turn, cannot exceed the capacity at the demand market for the commodity under the disaster scenarios as follows:

$$d_j^{k\xi_l} \leq u_j^{d^k\xi_l}, \quad k = 1, \ldots, K; j = 1, \ldots, n; l = 1, \ldots, \omega, \tag{5a}$$

or, equivalently, through the use of (2):

$$\sum_{i=1}^{m}\sum_{r=1}^{P} Q_{ijr}^{k\xi_l} \leq u_j^{d^k\xi_l}, \quad k=1,\ldots,K; j=1,\ldots,n; l=1,\ldots,\omega, \tag{5b}$$

We associate the Lagrange multiplier $\lambda_j^{d^k\xi_l}$ with constraint (5b) for each $k=1,\ldots,K; j=1,\ldots,n; l=1,\ldots,\omega$ and we group these Lagrange multipliers for each disaster scenario ξ_l; $l=1,\ldots,\omega$ into the vector $\lambda^{d\xi_l} \in R_+^{Kn}$.

In addition, the volume of commodity k that can be transported from supply market i to demand market j on route r under disaster scenario ξ_l cannot exceed the capacity of that route:

$$Q_{ijr}^{k\xi_l} \leq u_{ijr}^{Q^k\xi_l}, \quad k=1,\ldots,K; i=1,\ldots,m; j=1,\ldots,n; r=1,\ldots,P; l=1,\ldots,\omega. \tag{6}$$

We now associate the Lagrange multiplier $\lambda_{ijr}^{Q^k\xi_l}$ for each $k=1,\ldots,K; i=1,\ldots,m; j=1,\ldots,n; r=1,\ldots,P; l=1,\ldots,\omega$ in constraint (6) and we group these Lagrange multipliers for each disaster scenario ξ_l; $l=1,\ldots,\omega$ into the vector $\lambda^{Q\xi_l} \in R_+^{KmnP}$.

Re-Expression of the Supply Price and Demand Price Functions

We can redefine the supply price functions and the demand price functions given in Table 1 in terms of the commodity shipments in order to be able to state the equilibrium conditions and the governing variational inequality problem in commodity shipment variables and Lagrange multiplier variables. Of course, the supplies and the demands can be calculated from the commodity shipments using (1) and (2), respectively.

Specifically, the commodity supply price functions π_i^k, for all k, i, and disaster scenario ξ_l, due to the conservation of flow equations (1), may be redefined as follows:

$$\tilde{\pi}_i^k(Q^{\xi_l}) \equiv \pi_i^k(s^{\xi_l}), \quad k=1,\ldots,K; i=1,\ldots,m; l=1,\ldots,\omega. \tag{7}$$

Also, due to (2), we can construct new commodity demand price functions $\tilde{\rho}_j^k$, for all k, j, under disaster scenario ξ_l, such that:

$$\tilde{\rho}_j^k(Q^{\xi_l}) \equiv \rho_j^k(d^{\xi_l}), \quad k=1,\ldots,K; j=1,\ldots,n; l=1,\ldots,\omega. \tag{8}$$

It is worth noting the supply price, demand price, and unit transportation cost functions need not be separable or linear.

The supply price, demand price, and unit transportation cost functions are assumed to be continuous.

Definition 1: The Multicommodity International Trade Network Equilibrium Conditions Under Capacity Disruptions in Disasters

A multicommodity shipment and Lagrange pattern $(Q^{\xi_l *}, \lambda^{s\xi_l *}, \lambda^{Q\xi_l *}, \lambda^{d\xi_l *}) \in \mathcal{K}^{\xi_l}$*, where*

$$\mathcal{K}^{\xi_l} \equiv \{(Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}, \lambda^{d\xi_l}) | (Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}, \lambda^{d\xi_l}) \in R_+^{KmnP+Km+KmnP+Kn}\}$$

is a multicommodity international trade network equilibrium under disaster scenario ξ_l*;* $l = 1, \ldots, \omega$*, if the following conditions hold: for all commodities* k*;* $k = 1, \ldots, K$*; for all supply and demand market pairs:* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*, and for all routes* r*;* $r = 1, \ldots, P$*:*

$$(\tilde{\pi}_i^k(Q^{\xi_l *}) + c_{ijr}^k(Q^{\xi_l *}))e_{ij}^{\xi_l} + \lambda_i^{s^k\xi_l *} + \lambda_{ijr}^{Q^k\xi_l *} + \lambda_j^{d^k\xi_l *} \begin{cases} = \tilde{\rho}_j^k(Q^{\xi_l *}), & \text{if } Q_{ijr}^{k\xi_l *} > 0, \\ \geq \tilde{\rho}_j^k(Q^{\xi_l *}), & \text{if } Q_{ijr}^{k\xi_l *} = 0; \end{cases} \tag{9}$$

for all commodities k*;* $k = 1, \ldots, K$*, and for all supply markets* i*;* $i = 1, \ldots, m$*:*

$$u_i^{s^k\xi_l} \begin{cases} = \sum_{j=1}^n \sum_{r=1}^P Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_i^{s^k\xi_l *} > 0, \\ \geq \sum_{j=1}^n \sum_{r=1}^P Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_i^{s^k\xi_l *} = 0; \end{cases} \tag{10}$$

for all commodities k*;* $k = 1, \ldots, K$*, and for all supply and demand markets* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*, and for all routes* r*;* $r = 1, \ldots, P$*:*

$$u_{ijr}^{Q^k\xi_l} \begin{cases} = Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_{ijr}^{Q^k\xi_l *} > 0, \\ \geq Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_{ijr}^{Q^k\xi_l *} = 0; \end{cases} \tag{11}$$

and for all commodities k*;* $k = 1, \ldots, K$*, and for all demand markets* j*;* $j = 1, \ldots, n$*, and for all routes* r*;* $r = 1, \ldots, P$*:*

$$u_j^{d^k\xi_l} \begin{cases} = \sum_{i=1}^m \sum_{r=1}^P Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_j^{d^k\xi_l *} > 0, \\ \geq \sum_{i=1}^m \sum_{r=1}^P Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_j^{d^k\xi_l *} = 0. \end{cases} \tag{12}$$

The multicommodity international trade network equilibrium conditions (9) through (12) state that, for a given disaster ξ_l; $l = 1, \ldots, \omega$, if there is a positive flow of a commodity on a route between a pair of country supply and demand markets, and the route is not at its capacity, and the production at the country supply market is not at its capacity, and the demand for the commodity at the demand market is not at its capacity, then the supply price of the commodity at the country supply market plus the unit transportation cost associated with transporting the commodity on the route, multiplied by the exchange rate between the two countries is equal to the demand price of the commodity at the country demand market. On the other hand, if the route is at its capacity, and/or the supply is at its capacity at the country supply market, and/or the demand is at its capacity, and the flow of the commodity on a route is positive, then the demand price of the commodity at the country demand market is greater than or equal to the sum of commodity supply price and its unit transportation cost multiplied by the disaster-relevant exchange rate, with the sum of the corresponding Lagrange multipliers equal to the nonnegative difference. If the flow of a commodity is equal to zero on a route, then the country demand market price of the commodity is less than or equal to the country supply market price plus the unit transportation cost multiplied by the disaster scenario exchange rate plus the Lagrange multipliers.

Along with the Lagrange multipliers corresponding to capacity constraints in (10), (11), and (12), the equilibrium conditions (9) expand the classical spatial price equilibrium conditions of Samuelson (1952) and Takayama and Judge (1971) to include exchange rates, and limited transportation, production, and consumption capacity. Furthermore, the underlying supply price, demand price, and unit transportation cost functions in our model need not be separable (nor symmetric), and the unit transportation cost functions are flow-dependent.

Theorem 1: Variational Inequality Formulation of the Multicommodity International Trade Network Equilibrium Conditions Under Capacity Disruptions in Disasters

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_l*}, \lambda^{s\xi_l*}, \lambda^{Q\xi_l*}, \lambda^{d\xi_l*}) \in \mathcal{K}^{\xi_l}$ *is a multicommodity international trade network equilibrium under capacity disruptions in disasters, according to Definition 1, if and only if it satisfies the variational inequality:*

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[(\tilde{\pi}_i^k(Q^{\xi_l*})+c_{ijr}^k(Q^{\xi_l*}))e_{ij}^{\xi_l}+\lambda_i^{s^k\xi_l*}+\lambda_{ijr}^{Q^k\xi_l*}+\lambda_j^{d^k\xi_l*}-\tilde{\rho}_j^k(Q^{\xi_l*})\right]\times(Q_{ijr}^{k\xi_l}-Q_{ijr}^{k\xi_l*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\left[u_i^{s^k\xi_l}-\sum_{j=1}^{n}\sum_{r=1}^{P}Q_{ijr}^{k\xi_l*}\right]\times(\lambda_i^{s^k\xi_l}-\lambda_i^{s^k\xi_l*})+\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[u_{ijr}^{Q^k\xi_l}-Q_{ijr}^{k\xi_l*}\right]\times(\lambda_{ijr}^{Q^k\xi_l}-\lambda_{ijr}^{Q^k\xi_l*})$$

$$+\sum_{k=1}^{K}\sum_{j=1}^{n}\left[u_j^{d^k\xi_l}-\sum_{i=1}^{m}\sum_{r=1}^{P}Q_{ijr}^{k\xi_l*}\right]\times(\lambda_j^{d^k\xi_l}-\lambda_j^{d^k\xi_l*})\geq 0,\quad \forall(Q^{\xi_l},\lambda^{s\xi_l},\lambda^{Q\xi_l},\lambda^{d\xi_l})\in\mathcal{K}^{\xi_l}. \quad (13)$$

Proof: First, necessity is established; i.e., if $(Q^{\xi_l*}, \lambda^{s\xi_l*}, \lambda^{Q\xi_l*}, \lambda^{d\xi_l*}) \in \mathcal{K}^{\xi_l}$ satisfies equilibrium conditions (9) through (12), then it also satisfies variational inequality (13). From the equilibrium conditions, for an equilibrium commodity shipment and Lagrange multiplier pattern, and for fixed k, i, j, r, we have that:

$$\left[(\tilde{\pi}_i^k(Q^{\xi_l *}) + c_{ijr}^k(Q^{\xi_l *}))e_{ij}^{\xi_l} + \lambda_i^{s^k\xi_l *} + \lambda_{ijr}^{Q^k\xi_l *} + \lambda_j^{d^k\xi_l *} - \tilde{\rho}_j^k(Q^{\xi_l *})\right] \times (Q_{ijr}^{k\xi_l} - Q_{ijr}^{k\xi_l *}) \geq 0, \forall Q_{ijr}^{k\xi_l} \geq 0, \tag{14}$$

since if $Q_{ijr}^{k\xi_l *} > 0$, then the left-hand side in (14) is zero, so (14) holds. Since $Q_{ijr}^{k\xi_l} \geq Q_{ijr}^{k\xi_l *}$, if $Q_{ijr}^{k\xi_l *} = 0$, then the left-hand side expression is nonnegative, and (14) holds. Because (14) is true for any k, i, j, r, its summation over all these indices results in:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[(\tilde{\pi}_i^k(Q^{\xi_l *})+c_{ijr}^k(Q^{\xi_l *}))e_{ij}^{\xi_l}+\lambda_i^{s^k\xi_l *}+\lambda_{ijr}^{Q^k\xi_l *}+\lambda_j^{d^k\xi_l *}-\tilde{\rho}_j^k(Q^{\xi_l *})\right]\times(Q_{ijr}^{k\xi_l}-Q_{ijr}^{k\xi_l *}) \geq 0,$$

$$\forall Q^{\xi_l} \in R_+^{KmnP}. \tag{15}$$

Also, from equilibrium conditions (10), for a fixed k, i, we know that:

$$\left[u_i^{s^k\xi_l} - \sum_{j=1}^{n}\sum_{r=1}^{P} Q_{ijr}^{k\xi_l *}\right] \times (\lambda_i^{s^k\xi_l} - \lambda_i^{s^k\xi_l *}) \geq 0, \quad \forall \lambda_i^{s^k\xi_l} \geq 0. \tag{16}$$

Summing (16) over all indices k, i, results in:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\left[u_i^{s^k\xi_l} - \sum_{j=1}^{n}\sum_{r=1}^{P} Q_{ijr}^{k\xi_l *}\right] \times (\lambda_i^{s^k\xi_l} - \lambda_i^{s^k\xi_l *}) \geq 0, \quad \forall \lambda^{s\xi_l} \in R_+^{Km}. \tag{17}$$

Plus, from equilibrium conditions (11), for fixed k, i, j, r, it follows that:

$$\left[u_{ijr}^{Q^k\xi_l} - Q_{ijr}^{k\xi_l *}\right] \times (\lambda_{ijr}^{Q^k\xi_l} - \lambda_{ijr}^{Q^k\xi_l *}) \geq 0, \quad \forall \lambda_{ijr}^{Q^k\xi_l} \geq 0. \tag{18}$$

Summing (18) over all indices k, i, j, r, gives us the following:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[u_{ijr}^{Q^k\xi_l} - Q_{ijr}^{k\xi_l *}\right] \times (\lambda_{ijr}^{Q^k\xi_l} - \lambda_{ijr}^{Q^k\xi_l *}) \geq 0, \quad \forall \lambda^{Q\xi_l} \in R_+^{KmnP}. \tag{19}$$

And, from equilibrium conditions (12), we have that, for fixed k, j:

$$\left[u_j^{d^k\xi_l} - \sum_{i=1}^{m}\sum_{r=1}^{P} Q_{ijr}^{k\xi_l *}\right] \times (\lambda_j^{d^k\xi_l} - \lambda_j^{d^k\xi_l *}) \geq 0, \quad \forall \lambda_j^{d^k\xi_l} \geq 0. \tag{20}$$

Summing (20) over all indices k, j, results in the following:

$$\sum_{k=1}^{K}\sum_{j=1}^{n}\left[u_j^{d^k\xi_l} - \sum_{i=1}^{m}\sum_{r=1}^{P} Q_{ijr}^{k\xi_l *}\right] \times (\lambda_j^{d^k\xi_l} - \lambda_j^{d^k\xi_l *}) \geq 0, \quad \forall \lambda^{d\xi_l} \in R_+^{Kn}. \tag{21}$$

Adding (15), (17), (19), and (21) yields variational inequality (13). Therefore, necessity has been established.

We now turn to establishing sufficiency. Setting $\lambda_i^{s^k\xi_l} = \lambda_i^{s^k\xi_l *}$ for all k, i; $\lambda_{ijr}^{Q^k\xi_l} = \lambda_{ijr}^{Q^k\xi_l *}$ for all k, i, j, r; $\lambda_j^{d^k\xi_l} = \lambda_j^{d^k\xi_l *}$ for all k, j; and $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l *}$ for all k, i, j, r, except for $k = \tilde{k}$, $i = \tilde{i}$, $j = \tilde{j}$, and $r = \tilde{r}$, and plugging the resultants into (13), reduces the variational inequality (13) to:

$$\left[(\tilde{\pi}_{\tilde{i}}^{\tilde{k}}(Q^{\xi_l *}) + c_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}}(Q^{\xi_l *}))e_{\tilde{i}\tilde{j}}^{\xi_l} + \lambda_{\tilde{i}}^{s^{\tilde{k}}\xi_l *} + \lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q^{\tilde{k}}\xi_l *} + \lambda_{\tilde{j}}^{d^{\tilde{k}}\xi_l *} - \tilde{\rho}_{\tilde{j}}^{\tilde{k}}(Q^{\xi_l *})\right] \times (Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l} - Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l *}) \geq 0, \forall Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l} \geq 0. \tag{22}$$

from which it follows that the equilibrium conditions (9) hold.

Now, setting $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l *}$ for all k, i, j, r; $\lambda_{ijr}^{Q^k\xi_l} = \lambda_{ijr}^{Q^k\xi_l *}$ for all k, i, j, r; $\lambda_j^{d^k\xi_l} = \lambda_j^{d^k\xi_l *}$ for all k, j; and $\lambda_i^{s^k\xi_l} = \lambda_i^{s^k\xi_l *}$ for all k, i, except for $k = \tilde{k}$, $i = \tilde{i}$, and substituting the resultants into (13), reduces the variational inequality (13) to:

$$\left[u_{\tilde{i}}^{s^{\tilde{k}}\xi_l} - \sum_{j=1}^{n}\sum_{r=1}^{P} Q_{\tilde{i}jr}^{\tilde{k}\xi_l *}\right] \times (\lambda_{\tilde{i}}^{s^{\tilde{k}}\xi_l} - \lambda_{\tilde{i}}^{s^{\tilde{k}}\xi_l *}) \geq 0, \quad \forall \lambda_{\tilde{i}}^{s^{\tilde{k}}\xi_l} \geq 0. \tag{23}$$

from which it follows that the equilibrium conditions (10) must hold.

And, setting $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l *}$ for all k, i, j, r; $\lambda_i^{s^k\xi_l} = \lambda_i^{s^k\xi_l *}$ for all k, i; $\lambda_j^{d^k\xi_l} = \lambda_j^{d^k\xi_l *}$ for all k, j; and $\lambda_{ijr}^{Q^k\xi_l} = \lambda_{ijr}^{Q^k\xi_l *}$ for all k, i, j, r, except for $k = \tilde{k}$, $i = \tilde{i}$, $j = \tilde{j}$, and $r = \tilde{r}$, and substituting the resultants into (13), reduces the variational inequality (13) to:

$$\left[u_{\tilde{i}\tilde{j}\tilde{r}}^{Q^{\tilde{k}}\xi_l} - Q_{\tilde{i}\tilde{j}\tilde{r}}^{\tilde{k}\xi_l *}\right] \times (\lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q^{\tilde{k}}\xi_l} - \lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q^{\tilde{k}}\xi_l *}) \geq 0, \quad \forall \lambda_{\tilde{i}\tilde{j}\tilde{r}}^{Q^{\tilde{k}}\xi_l} \geq 0, \tag{24}$$

from which it follows that the equilibrium conditions (11) must hold.

Finally, setting $Q_{ijr}^{k\xi_l} = Q_{ijr}^{k\xi_l *}$ for all k, i, j, r; $\lambda_i^{s^k\xi_l} = \lambda_i^{s^k\xi_l *}$ for all k, i; $\lambda_{ijr}^{Q^k\xi_l} = \lambda_{ijr}^{Q^k\xi_l *}$ for all k, i, j, r; and $\lambda_j^{d^k\xi_l} = \lambda_j^{d^k\xi_l *}$ for all k, j, except for $k = \tilde{k}$, $j = \tilde{j}$, and substituting the resultants into (13), reduces the variational inequality (13) to:

$$\left[u_{\tilde{j}}^{d^{\tilde{k}}\xi_l} - \sum_{i=1}^{m}\sum_{r=1}^{P} Q_{i\tilde{j}r}^{\tilde{k}\xi_l *}\right] \times (\lambda_{\tilde{j}}^{d^{\tilde{k}}\xi_l} - \lambda_{\tilde{j}}^{d^{\tilde{k}}\xi_l *}) \geq 0, \quad \forall \lambda_{\tilde{j}}^{d^{\tilde{k}}\xi_l} \geq 0, \tag{25}$$

and, hence, equilibrium conditions (12) must hold. Sufficiency has also been established. □

Variational inequality (13) is now put into standard form (cf. Nagurney (1999)), VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{26}$$

with F being a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, where $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

We define $X \equiv (Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}, \lambda^{d\xi_l})$, $\mathcal{K} \equiv \mathcal{K}^{\xi_l}$, and $\mathcal{N} \equiv KmnP + Km + KmnP + Kn$. Plus, $F(X) \equiv (F_1(X), F_2(X), F_3(X), F_4(X))$ where $F_1(X)$ is comprised of the elements: $\left[(\tilde{\pi}_i^k(Q^{\xi_l}) + c_{ijr}^k(Q^{\xi_l}))e_{ij}^{\xi_l} + \lambda_i^{s^k\xi_l} + \lambda_{ijr}^{Q^k\xi_l} + \lambda_j^{d^k\xi_l} - \tilde{\rho}_j^k(Q^{\xi_l})\right], \forall k, i, j, r$, and the components of $F_2(X)$ are: $\left[u_i^{s^k\xi_l} - \sum_{j=1}^n \sum_{r=1}^P Q_{ijr}^{k\xi_l}\right], \forall k, i$; $F_3(X)$ consists of the components: $\left[u_{ijr}^{Q^k\xi_l} - Q_{ijr}^{k\xi_l}\right], \forall k, i, j, r$, and $F_4(X)$ is comprised of the elements: $\left[u_j^{d^k\xi_l} - \sum_{i=1}^m \sum_{r=1}^P Q_{ijr}^{k\xi_l}\right], \forall k, j$.

Clearly, variational inequality (13) can be put into standard form (26).

4. Assessing Performance of International Trade Networks Under Capacity and Exchange Rate Disruptions

In this Section, we introduce international trade network performance measures under individual disruptions, as well as a unified measure. In addition, we propose a robustness measure, followed by importance indicators of international trade network components (supply markets, demand markets, transportation routes, or a combination thereof). Since the demand markets may lie in different countries, we convert the demand market prices at the equilibrium into that of a common currency, here the currency being the US dollar. The demand market prices in the US currency are denoted by $\hat{\rho}_j^k(Q)$ for $k = 1, \ldots, K$; $j = 1, \ldots, n$.

Definition 2: International Trade Network Performance Indicator Under Capacity and Exchange Rate Disruption ξ_l

For an international trade network $G = [N, L]$, where N is the set of nodes and L is the set of links, as depicted in Figure 1, and, given the underlying multicommodity supply price, unit transportation cost, and demand price functions, and exchange rates and capacities associated with disaster scenario ξ_l, we define the performance $\mathcal{E}^{\xi_l}$ as follows:

$$\mathcal{E}^{\xi_l}(G, \tilde{\pi}, c, \tilde{\rho}, u^{\xi_l}, e^{\xi_l}) = \frac{1}{Kn} \sum_{k=1}^K \sum_{j=1}^n \frac{d_j^{k\xi_l *}}{\hat{\rho}_j^k(Q^{\xi_l *})}, \tag{27}$$

where the demands and the incurred demand market prices are obtained through the solution of variational inequality (13) for the problem.

According to (27), the international trade network under disaster disruption ξ_l, is said to perform better if, on the average, the trade network can handle higher commodity demands at lower prices. The latter captures affordability, whereas the former – availability of the commodities. Such a measure is useful for numerous commodities, including agricultural ones necessary for food security, various raw materials, including minerals, needed for high-tech products, lumber for construction, and so on.

We are now ready to state the Unified International Trade Network Performance Measure.

Definition 3: Unified International Trade Network Performance Measure

The performance indicator $\mathcal{E}$ for an international trade network under disruption set Ξ and with associated probabilities $p_{\xi_1}, p_{\xi_2}, \ldots, p_{\xi_\omega}$, respectively, is defined as:

$$\mathcal{E} = \sum_{l=1}^{\omega} \mathcal{E}^{\xi_l} p_{\xi_l}. \tag{28}$$

We let $\mathcal{E}^0$ be the performance of the international trade network under its original (not disrupted) upper bounds/capacities and original exchange rates, such that:

$$\mathcal{E}^0(G, \tilde{\pi}, c, \tilde{\rho}, u^0, e^0) = \frac{1}{Kn} \sum_{k=1}^{K} \sum_{j=1}^{n} \frac{d_j^{k*}}{\hat{\rho}_j^k(Q^*)}, \tag{29}$$

where u^0 denotes the vector of original capacities not under disruptions and e^0 denotes the vector of exchange rates, also, not under disruptions.

We can also refer to both the expressions in (28) and (29) as "efficiency" measures. An international trade network is viewed as performing better (more efficiently) if, on the average, it can handle a greater volume of commodities at lower prices.

Robustness is another relevant measure that helps to quantify how a system performs subject to disruptions. We define the robustness measure $\mathcal{R}$ as follows.

Definition 4: Robustness of an International Trade Network Under Disruptions

The robustness, $\mathcal{R}$, of an international trade network under capacity and exchange rate disruptions is calculated as:

$$\mathcal{R} = \mathcal{E}^0 - \mathcal{E}. \tag{30}$$

According to the above definition, an international trade network is more robust if, under disruptions, its performance lies close to its performance in the absence of disruptions; that is, the closer the value of $\mathcal{R}$ is to 0.00, the more robust to disruptions the international trade network is.

Qiang, Nagurney, and Dong (2009) constructed a supply chain network equilibrium model under random demands and also proposed a robustness measure relative to their supply chain network performance measure. They noted that since their supply chain measure is based on the network equilibrium model, a network that is considered as being robust, according to their measure, is also "resilient" if its performance, after experiencing the disruption(s), is close to the "original value." *Resilience* is another concept much discussed in this era of climate change and many disasters, but it may be challenging to quantify. As noted in Qiang, Nagurney, and Dong (2009), McCarthy (2007) defined resilience "... as the ability of a system to recover from adversity, either back to its original state or an adjusted state based on new requirements, ...". Since our international trade network performance measure is based on the international trade network equilibrium model with capacities and exchange rates, an international trade network that is considered to be robust, according to our measure, is also resilient if its performance, after the disruption(s), is close to the "original value." This is in alignment with Hansson and Helgesson (2003), who proposed that robustness can be treated as a special case of resilience.

We are now ready to define an international trade network component importance indicator, I, which can assess the importance of a supply market, a demand market, a transportation route, or a combination thereof. Recall that there are m supply markets, n demand markets, and P transportation routes connecting each pair of supply and demand markets. Here, we are interested in a complete disruption (removal) of such trade network components. Construction and calculation of the importance measures allow decision-makers and policy-makers to rank the various network trade components.

Definition 5: Importance Indicator of an International Trade Network Component

The importance indicator of an international trade network component g where g can correspond to a supply market, a demand market, or a transportation route, or a combination thereof is defined as:

$$I(g) \equiv \frac{\mathcal{E}(G, \tilde{\pi}, c, \tilde{\rho}, u^0, e^0) - \mathcal{E}(G - g, \tilde{\pi}, c, \tilde{\rho}, u^0, e^0)}{\mathcal{E}(G, \tilde{\pi}, c, \tilde{\rho}, u^0, e^0)}, \tag{31}$$

where $G - g$ denotes the graph with the component g no longer functioning.

In practice, cf. Figure 1, to evaluate the importance of a supply market, we solve for the international trade network performance by removing the supply market link and the transportation route links associated with the supply market. To evaluate the importance of a demand market, we excise all the links that terminate in that demand market. Finally, to determine the efficiency of the network without a specific transportation route, we excise that network link. Note that the international trade network component importance indicator (31) quantifies the relative efficiency/performance drop of the trade network when the component is no longer available.

Similar importance indicators, but in the context of different network systems, including transportation and financial networks, can be found in the book by Nagurney and Qiang (2009). Li and Nagurney (2017), more recently, constructed a supplier and component importance measure in a competitive multitiered supply chain network model that also emphasizes the average "demand to price" for the network system.

We remark that the importance indicator is for multicommodity international trade and, given the generality of its statement in (31), one can also assess the loss of a supply market, demand market, or transportation route for a specific commodity.

5. The Computational Procedure

The computational procedure that we implement and apply in Section 6 to compute solutions to a series of numerical examples is the modified projection method of Korpelevich (1977). For easy reference, its statement is now recalled. An iteration is denoted by τ.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{32}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{33}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The convergence of this modified projection method is guaranteed if the function $F(X)$ that enters the variational inequality problem (26) is monotone and Lipschitz continuous.

Recall that the function $F(X)$ is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{34}$$

Also, $F(X)$ is Lipschitz continuous, if there exists a Lipschitz constant, $\eta > 0$, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{35}$$

Since the feasible set $\mathcal{K}^{\xi_l}$; $l = 1, \ldots, \omega$, underlying the multicommodity international agricultural trade network equilibrium model with capacities, under disruptions, is that of the nonnegative orthant, the solution of each of the subproblems in (32) and (33) can be obtained via closed-form expressions. We now provide the explicit formulae for the iterates generated in (32); similar ones can be readily obtained for (33).

Explicit Formulae at Iteration τ for the Multicommodity Shipments in Step 1

The closed-form expressions for the multicommodity shipments in (32) for the solution of variational inequality (13) are:

$$\bar{Q}^{k\xi_l\tau}_{ijr} = \max\{0, Q^{k\xi_l\tau-1}_{ijr} + \beta(\tilde{\rho}^k_j(Q^{\xi_l\tau-1}) - (\tilde{\pi}^k_i(Q^{\xi_l\tau-1}) + c^k_{ijr}(Q^{\xi_l\tau-1}))e^{\xi_l}_{ij} - \lambda^{s^k\xi_l\tau-1}_i - \lambda^{Q^k\xi_l\tau-1}_{ijr} - \lambda^{s^k\xi_l\tau-1}_j)\},$$

$$\forall k, i, j, r. \tag{36}$$

Explicit Formulae at Iteration τ for the Multicommodity Supply Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the supply capacity Lagrange multipliers for (32) for variational inequality (13) are:

$$\bar{\lambda}^{s^k\xi_l\tau}_i = \max\{0, \lambda^{s^k\xi_l\tau-1}_i + \beta(\sum_{j=1}^{n}\sum_{r=1}^{P} Q^{k\xi_l\tau-1}_{ijr} - u^{s^k\xi_l}_i)\}, \quad \forall k, i. \tag{37}$$

Explicit Formulae at Iteration τ for the Multicommodity Transportation Route Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the transportation capacity Lagrange multipliers for (32) for variational inequality (13) are:

$$\bar{\lambda}_{ijr}^{Q^k\xi_l\tau} = \max\{0, \lambda_{ijr}^{Q^k\xi_l\tau-1} + \beta(Q_{ijr}^{k\xi_l\tau-1} - u_{ijr}^{Q^k\xi_l})\}, \quad \forall k, i, j, r. \tag{38}$$

Explicit Formulae at Iteration τ for the Multicommodity Demand Capacity Lagrange Multipliers in Step 1

The closed-form expressions for the demand capacity Lagrange multipliers for (32) for variational inequality (13) are:

$$\bar{\lambda}_{j}^{d^k\xi_l\tau} = \max\{0, \lambda_{j}^{d^k\xi_l\tau-1} + \beta(\sum_{i=1}^{m}\sum_{r=1}^{P} Q_{ijr}^{k\xi_l\tau-1} - u_{j}^{d^k\xi_l})\}, \quad \forall k, j. \tag{39}$$

6. Numerical Examples

In this Section, we present numerical examples to illustrate our modeling framework.

The data for the examples are based on Ukraine, and are influenced by Russia's war on Ukraine. First, we present an example that is a slightly altered version of Example 6 in Nagurney et al. (2024) and use the notation proposed in this paper. The example consists of two commodities: wheat and corn, with the supply market being Ukraine and the demand markets being Lebanon and Egypt, as the baseline example (without disruptions). We set different capacities at the supply markets and the transportation routes since the previous paper constructed a model in which the commodity supplies and transportation route flows had constraints across the commodities, unlike the model here. There are two transportation routes connecting Ukraine with Lebanon and two transportation routes connecting Ukraine with Egypt, as depicted in Figure 2. As noted in Nagurney et al. (2024), the first route corresponds to export through a Black Sea port in Ukraine, such as the port of Odesa, and the second route corresponds to the transportation of grains via barge, rail, or truck through the western borders of Ukraine to Romania, and then from a Romanian port on the Black Sea, such as the port of Constanta. We denote the wheat commodity by 1 and that of corn by 2. The computed equilibrium pattern is that in Nagurney et al. (2024) since the respective constraints are not tight. For completeness, we report the input data here, as well as the equilibrium commodity flows and prices. We then calculate the international trade

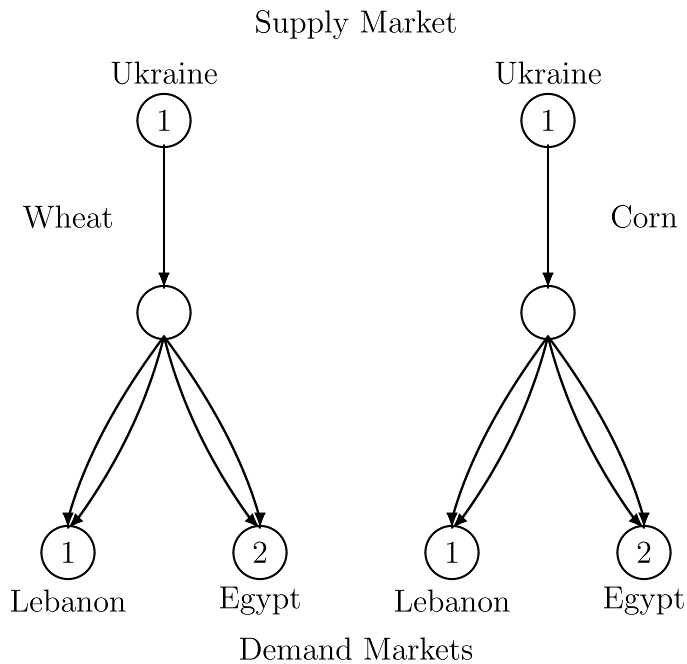

Figure 2: The International Trade Network for the Numerical Examples

network performance/efficiency measure $\mathcal{E}^0$ and also the importance of the transportation routes (and their ranking).

The currency codes are: UAH for Ukrainian hryvnia, LBP for the Lebanese pound, EGP for the Egyptian pound, and USD for the United States dollar. The time horizon is one year, and the unit for the commodity shipments is tons with prices and costs associated with a ton of the specific commodity. The modified projection method was implemented in FORTRAN on a Linux system at the University of Massachusetts Amherst with a convergence tolerance of 10^{-2}. This means that the algorithm was considered to have converged to the equilibrium commodity shipment and Lagrange multiplier pattern if the absolute value of each computed variable at two successive iterations differed by no more than this value. Since we first consider the efficiency of the international trade network in a baseline setting, we suppress the superscript notation for the disruption scenarios.

The exchange rates are from early January 2022, before the full-scale invasion:

$$e_{11} = 55.0581, \quad e_{12} = .5714,$$

$$USD/UAH = 27.4619, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 15.7300.$$

The supply price functions for the commodities per ton in Ukrainian hryvnia are:

$$\pi_1^1(s) = .000136 s_1^1 + .000068 s_1^2 + 7,001.60, \quad \pi_1^2(s) = .000073 s_1^1 + .000142 s_1^2 + 6,728.20.$$

The unit transportation cost functions per ton in Ukrainian hryvnia are:

$$c^1_{111}(Q) = .000556Q^1_{111} + 2,046.80,\ c^1_{112}(Q) = .007512Q^1_{112} + 10,984.60,$$

$$c^1_{121}(Q) = .000185Q^1_{121} + 2,046.80,\ c^1_{122}(Q) = .007312Q^1_{122} + 10,984.60,$$

$$c^2_{111}(Q) = .005566Q^2_{111} + 2,046.80,\ c^2_{112}(Q) = .006812Q^2_{112} + 10,984.60,$$

$$c^2_{121}(Q) = .001259Q^2_{121} + 2,046.80,\ c^2_{122}(Q) = .007012Q^2_{122} + 10,984.60.$$

The demand price functions in the local currencies are:

$$\rho^1_1(d) = -.15d^1_1 + 602,344.00, \quad \rho^2_1(d) = -.68d^2_1 + 574,560.00,$$

$$\rho^1_2(d) = -.000475d^1_2 + 6,290.00, \quad \rho^2_2(d) = -.000758d^2_2 + 5,980.00.$$

The supply capacities, in tons, in Ukraine are: $u^{s^1}_1 = 4,000,000.00$ and $u^{s^2}_1 = 3,000,000.00$.

The transportation capacities (upper bounds), in tons, over routes are:

$$u^{Q^1}_{111} = 2,500,000.00, u^{Q^1}_{112} = 250,000.00, u^{Q^1}_{121} = 2,500,000.00, u^{Q^1}_{122} = 250,000.00,$$

$$u^{Q^2}_{111} = 2,500,000.00, u^{Q^2}_{112} = 250,000.00, u^{Q^2}_{121} = 2,500,000.00, u^{Q^2}_{122} = 250,000.00.$$

We do not include capacities on the demands since both the Lebanese and Egyptians rely heavily on these grains.

As reported in Nagurney et al. (2024), where different notation was used, the modified projection method converges to the following equilibrium commodity shipment pattern:

$$Q^{1*}_{111} = 477,085.5938, \quad Q^{1*}_{121} = 1,605,672.5000, \quad Q^{1*}_{112} = 0.0000, \quad Q^{1*}_{122} = 0.0000,$$

$$Q^{2*}_{111} = 79,128.0781, \quad Q^{2*}_{121} = 560,130.3750, \quad Q^{2*}_{112} = 0.0000, \quad Q^{2*}_{122} = 0.0000.$$

This commodity flow pattern is quite close to Ukraine's actual wheat and corn exports to Lebanon and Egypt in 2021 and the projected amounts in 2022, with the assumption that the invasion would have never occurred. Pre-war, Lebanon, on the average, imported 70% of its wheat demand and 20% of its corn demand from Ukraine, and, in the case of Egypt, these percentages were 25% and 5% of its wheat and corn demand, respectively (IndexMundi (2022a,b), TrendEconomy (2022a,b)).

The equilibrium commodity supplies are:

$$s^{1*}_1 = 2,082,758.1250, \quad s^{2*}_1 = 639,258.4375.$$

The equilibrium commodity demands are:

$$d_1^{1*} = 477,085.5938, \quad d_1^{2*} = 79,128.0781, \quad d_2^{1*} = 1,605,672.5000, \quad d_2^{2*} = 560,130.3750.$$

The incurred supply prices in Ukraine in hryvnia and in US dollars at the equilibrium are:

$$\pi_1^1(s^*) = 7,328.3252 = \$266.8542, \quad \pi_1^2(s^*) = 6,971.0166 = \$253.8432.$$

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds and in US dollars are:

$$\rho_1^1(d^*) = 530,781.1875 = \$351.0457, \quad \rho_1^2(d^*) = 520,752.9063 = \$344.4132.$$

The demand prices in Egypt in Egyptian pounds and in US dollars are:

$$\rho_2^1(d^*) = 5,527.3057 = \$351.3862, \quad \rho_2^2(d^*) = 5,555.4214 = \$353.1736.$$

These results closely resemble the actual prices in these countries before the full-scale invasion. Ukrainian farmers earned around \$270 per ton of grain at the time (Martyshev, Nivievskyi, and Bogonos (2023)). The demand prices in Lebanon and Egypt were also quite similar to the results (Breisinger et al. (2022), El Safty (2022), Galal (2022), Hamdan (2022)).

All the Lagrange multipliers are equal to: 0.0000, since the production and transportation capacities exceeded the corresponding flows. In this example, only the maritime routes are used; that is, they have positive flows.

6.1 International Trade Network Performance Calculation and the Importance of Transportation Routes

Calculation of the performance/efficiency of the international trade network, $\mathcal{E}^0$, according to formula (29), recalling that the demand market prices are reported in a common currency, and, here, we use the US dollar, yields

$$\mathcal{E}^0 = 1,936.08.$$

We now proceed to calculate the importance of the transportation routes.

The efficiency of the international trade network without the first transportation route joining Ukraine to Lebanon, which can carry both wheat and corn, is: 1,599.44, and, hence, the importance of that transportation route is: .17.

The efficiency of the network without the second route, in turn, joining Ukraine with Lebanon is 1,936.08, and, hence, the importance of that route is 0.00, and that makes sense since, given the above computational results, that route is not used.

The efficiency of the international trade network without the first route joining Ukraine with Egypt is 467.70, and, hence, its importance is .76.

Finally, the efficiency/performance of the international trade network without the second route joining Ukraine with Egypt is: 1,936.08 and, hence, its importance is 0.00.

We see that the most important transportation route is the maritime route from Ukraine to Egypt, followed by the maritime route from Ukraine to Lebanon, which is to be expected since Egypt has a population almost twenty times that of Lebanon, and its grain imports to satisfy the demand of its people is, accordingly, on a much larger scale. For additional information on the actual wheat and corn imports of the two countries, please refer to IndexMundi (2022a,b) and TrendEconomy (2022a,b). The above results reinforce the importance of the maritime routes for the efficiency of this international trade network. For example, pre-war, Ukraine used to export more than 90% of its grains via maritime freight through its Black Sea ports (Picheta et al. (2022)).

6.2 Disruption Scenarios and Robustness

We now consider several disruptions, as in wartime. Specifically, scenario ξ_1 corresponds to the supply capacities for wheat and corn in Ukraine being disrupted so that the original capacities are reduced by 50%, yielding: $u_1^{s^1\xi_1} = 2,000,000$ tons and $u_1^{s^2\xi_1} = 1,500,000$ tons. The remainder of the data is as in the above examples. This scenario corresponds to production capacity being reduced, as in the case of the destruction of land through mining, bombing, etc. The second scenario ξ_2 corresponds to the disruption of the maritime transportation routes for wheat that can use different ports on the Black Sea. The data remain as in the original (undisrupted, pre-full-scale-invasion) example, but with $u_{111}^{Q^1\xi_2} = 1,500,000$ tons and $u_{121}^{Q^1\xi_2} = 1,500,000$ tons. The third disruption scenario ξ_3 integrates the data of scenarios ξ_1 and ξ_2 in terms of capacity reductions in production and transportation, but now the supply capacities are further disrupted by 50% so that $u_1^{s^1\xi_1} = 1,000,000$ tons and $u_1^{s^2\xi_1} = 750,000$ tons. We retain the same exchange rates in all the scenarios. Recall that, in computing the international trade network performance/efficiency, we use the common currency of the US dollar, for consistency.

We report the computed multicommodity equilibrium shipments of wheat and corn for the three disruption scenarios in Table 2.

Equilibrium	Scenario		
Commodity Shipment	ξ_1	ξ_2	ξ_3
$Q_{111}^{1\xi_l}$	480,127.50000	481,216.7500	517,281.0625
$Q_{121}^{1\xi_l}$	1,519,873.1250	1,500,000.0000	482,719.0000
$Q_{121}^{1\xi_l}$	0.0000	0.0000	0.0000
$Q_{122}^{1\xi_l}$	0.0000	0.0000	0.0000
$Q_{111}^{2\xi_l}$	79,433.9766	79,509.8125	83,296.5781
$Q_{121}^{2\xi_l}$	563,800.2500	563,800.2500	586,686.3125
$Q_{121}^{2\xi_l}$	0.0000	0.0000	0.0000
$Q_{122}^{2\xi_l}$	0.0000	0.0000	0.0000

Table 2: Equilibrium Commodity Shipments Under the Disruption Scenarios

Under scenario ξ_1, all the Lagrange multipliers are equal to 0.0000 at the equilibrium except that $\lambda_1^{s^1\xi_1*} = 53.7601$. Under scenario ξ_2, all the Lagrange multipliers are equal to 0.0000. Under scenario ξ_3, all the Lagrange multipliers are equal to 0.0000 at the equilibrium except that $\lambda_1^{s^1\xi_1*} = 734.1813$. From the size and positivity of the Lagrange multiplier associated with the production/supply of wheat in Ukraine, we see the importance of preserving Ukraine's capacity in wheat production, which is very reasonable since it has been called the breadbasket of Europe, if not the world, and renowned for its rich black soil, known as chernozem.

Under scenario ξ_1, wheat to Egypt is the most impacted commodity shipment. Both wheat and corn shipments to Lebanon and that of corn to Egypt increase but are still quite close to those under scenario ξ_0; however, the wheat commodity flow to Egypt decreases relatively much more. The disruption to the supply of corn is not impactful since Lebanon and Egypt primarily depend on wheat, which is used for bread, a staple food in both countries, to meet their people's caloric and nutritional demands. Meanwhile, the limited production of wheat causes Lebanon and Egypt to compete over Ukrainian wheat essentially, and it comes at the cost of Egypt since Lebanon is facing economic turmoil and more severe food insecurity issues, resulting in its higher dependence on importing wheat from Ukraine (World Food Programme (2023)). Under the second scenario, the disrupted transportation capacities for wheat are still high enough to satisfy the demand of both Egypt and Lebanon almost to the pre-war levels. However, note that the wheat flow to Egypt, which has a much higher demand than Lebanon, is almost at the bound. Under scenario ξ_3, the further disruption to the supply of wheat in Ukraine, again, comes at the price of Egypt's share of Ukrainian wheat; now, Lebanon imports more wheat and corn, while Egypt shifts towards replacing part of its lost wheat shipment by importing more Ukrainian corn.

We now calculate the international trade network performance under disruptions, $\mathcal{E}$, according to (28), with the assumption of the following probabilities: $p_{\xi_1} = p_{\xi_2} = .25$ and $p_{\xi_3} = .50$, where: $\mathcal{E}^{\xi_1} = 1,872.62$, $\mathcal{E}^{\xi_2} = 1,857.74$, and $\mathcal{E}^{\xi_3} = 1,163.43$. We find that, under the consideration of the three disruption scenarios, and the associated probabilities, $\mathcal{E} = 1,514.30$. Hence, the robustness $\mathcal{R}$ (cf. (30)) is equal to: 421.78 for this international trade network under such disruptions and probabilities. Clearly, under these disruption scenarios, which are quite representative of the actual scenarios as the war on Ukraine by Russia has progressed over more than 2 years, the international trade network considered here is not robust since the value of $\mathcal{R}$ is not close to 0.00.

7. Summary and Conclusions

In this paper, we constructed a multicommodity international trade network equilibrium model under capacity disruptions in disasters to the production and transportation of commodities, as well as to the demand in the consuming markets, and the exchange rates. In the model, the capacities are specific to each commodity. Each disaster scenario has an associated probability and impact on the capacities and exchange rates. The governing equilibrium conditions of the international trade network were presented under each specific disaster scenario and then formulated as a variational inequality problem. The equilibrium conditions and the derived variational inequality formulation include the Lagrange multipliers associated with production/supply, transportation, and demand capacity constraints.

An international trade network performance measure for each specific disaster scenario and a unified international trade network performance measure were proposed. The latter includes all the possible disaster scenarios and their probabilities. In addition, a robustness measure was introduced based on the unified international trade network performance measure and the performance measure of the international trade network in the absence of disruptions. Also, an international trade network importance indicator was identified, quantifying the importance of network components, such as: the supply markets, transportation routes, demand markets, or a combination thereof in the international trade network in the absence of disasters. The importance indicator enables a decision-maker to rank such network components in terms of their criticality to the international trade network's performance.

We also provided an algorithmic scheme to solve the variational inequality problem. The proposed algorithm, because of the underlying feasible set of the variational inequality problem, yields closed-form expressions for the computation of each of the variables, and these expressions are provided for easy reproducibility.

Numerical examples, drawn from the consequences of Russia's war on Ukraine, and focused on the agricultural trade of wheat and corn from Ukraine to MENA (Middle East and North Africa) countries, were solved with the proposed algorithm to feature several disaster scenarios in addition to the baseline scenario. The results for the disaster scenarios are compared to those of the baseline case to derive further insights. In addition, the international trade network performance measure is calculated for each investigated scenario, followed by the calculation of the unified international trade network performance measure and the robustness measure. We also ranked the transportation routes in the numerical examples' network using the introduced importance indicator. The results of the numerical examples demonstrate how policy-makers can use the introduced importance indicator and the performance measures to quantitatively assess the performance/efficiency of an international trade network under different disaster scenarios. Furthermore, in the context of our specific set of examples, which focuses on Russia's war on Ukraine, the results are of significant relevance to the Ukrainian government and the governments of Lebanon and Egypt.

There are many possibilities to extend the research in this paper in future research. Applying the model and the metrics to trade through the Panama Canal, for example, which is now suffering from bottlenecks because of the lingering drought, would be very interesting. Also, expanding the number of supply markets and demand markets in the international trade network considered in Section 6 would be very worthwhile. Capturing quantitatively the impact of migratory flows on demands would necessitate an expanded model but would be very topical, given increased flows of human migrants from South America to the US as well as from Africa to Europe. In addition, some other modeling efforts would be of interest. For example, the model in this paper does not account for storage; therefore, it would be interesting to extend the model to a multiperiod model. Furthermore, in cases where historical data can be obtained, analyzing disaster scenarios with continuous probabilities associated with them would be worthwhile. Moreover, the importance indicator and performance measures could be extended to assess networks in which routes can consist of multiple links and transportation modes, as in the model of Nagurney et al. (2023). In addition, with some disasters occurring simultaneously, it is important to investigate how to handle such probabilities as well as those associated with hybrid threats. We leave such possible research directions for the future.

Acknowledgments

The authors acknowledge the helpful comments and suggestions of two anonymous reviewers and those of a Co-Editor on an earlier version of this paper.

The first author is grateful to the organizers, speakers, and participants at the 6th International Conference on Dynamics of Disasters, which took place July 3-6, 2023 in Piraeus, Greece, for the inspiring talks and discussions.

The authors are thankful for the partnership between the University of Massachusetts Amherst and the Kyiv School of Economics that supported this collaboration.

References

Aleskerov, F., Dutta, S., Egorov, D., Tkachev, D., 2022. Networks under deep uncertainty. *Procedia Computer Science*, 214, 1285-1292.

Aleskerov, F., Tkachev, D., 2024. Network analysis of disasters' consequences under deep uncertainty. Accepted in *Dynamics of Disasters - From Natural Phenomena to Human Activity*, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, S. Pickl, and C. Vogiatzis, Editors, Apringer Nature Switzerland AG.

Bishop, P., Pratt, J., Novakovic, A., 1994. Using a joint-input, multi-product formulation to improve spatial price equilibrium models. Cornell University, Department of Agricultural Economics Staff Paper no. 94-06, Ithaca, New York.

Breisinger, C., Khouri, N., Glauber, J., Laborde, D., 2022. One of the world's worst economic collapses, now compounded by the Ukraine crisis: What's next for Lebanon? International Food Policy Research Institute, May 6. Available at:
https://www.ifpri.org/blog/one-worlds-worst-economic-collapses-now-compounded-ukraine-crisis-whats-next-lebanon

Butenko, S., Pasiliao, E.L., Shylo, V., Editors, 2014. *Examining Robustness and Vulnerability of Networked Systems*. IOS Press, Amsterdam, The Netherlands.

Cassidy, E., 2022. Worst drought on record parches Horn of Africa. NASA Earth Observatory. Available at:
https://earthobservatory.nasa.gov/images/150712/worst-drought-on-record-parches-horn-of-africa

Dafermos, S., 1980. Traffic equilibrium and variational inequalities. *Transportation Science*, 14(1), 42-54.

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. *Operations Research*, 32, 1069-1086.

Devadoss, S., Sabala, E., 2020. Effects of yuan-dollar exchange rate changes on world cotton markets. *Journal of Agriculture and Applied Economics*, 52(3), 420-439.

Dinh, T.N., Xuan, Y., Thai, M. T., Pardalos, P.M., Znati, T., 2012. On new approaches of assessing network vulnerability: Hardness and approximation. *IEEE/ACM Transactions on Networking*, 20(2), 609-619.

El Safty, S., 2022. Egypt's private sector wheat imports stall due to dollar shortage. Reuters, October 4. Available at:
https://www.reuters.com/markets/commodities/egypts-private-sector-wheat-imports-stall-due-dollar-shortage-2022-10-04/

Escriva-Bou, A., Medellín-Azuara, J., Hanak, E., Abatzoglou, J., Viers, J., 2022. Policy brief: Drought and California's agriculture. Public Policy Institute of California, April. Available at:
https://www.ppic.org/publication/policy-brief-drought-and-californias-agriculture/

Florian, M., Los, M., 1982. A new look at static spatial price equilibrium models. *Regional Science and Urban Economics*, 12, 579-597.

Food and Agricultural Organization of the United Nations, 2023. Türkiye earthquakes: initial assessment indicates losses of more than 20 percent in food production. March 31. Available at:
https://www.fao.org/newsroom/detail/turkiye-earthquakes-initial-assessment-indicates-losses-of-more-than-20-percent-in-food-production/en

Fytopoulos, A., Pardalos, P.M., 2023. Future threats: Creating a resilient society. In: K.P. Balomenos, A. Fytopoulos, and P.M. Pardalos, Editors, *Handbook for Management of Threats*, Springer Nature Switzerland AG, pp 1-12.

Galal, S., 2022. Monthly average prices for wheat in Egypt 2019-2022. Statista, December 16. Available at:
https://www.statista.com/statistics/1172542/monthly-average-prices-for-wheat-in-egypt/#: :text =The%20monthly%20average%20price%20for,kilogram%20as%20of%20February%202022.

Grubesic, T.H., Matisziw, T.C., Murray, A.T., Snediker, D., 2008. Comparative approaches for assessing network vulnerability. *International Regional Science Review*, 31(1), 88-112.

Hamdan, H., 2022. Lebanese fear wheat shortage amid Ukrainian crisis. Al-Monitor, March 2. Available at:

https://www.al-monitor.com/originals/2022/03/lebanese-fear-wheat-shortage-amid-ukrainian-crisis

Hansson, S.O., Helgesson, G., 2003. What is stability?” *Synthese*, 136, 219-235.

Howard, E.A., 1984. An analysis of interregional competition in the U.S. summer potato market, Master of Science dissertation, Department of Agricultural Economics, Texas A&M University, College Station, Texas.

Hwang, M.J., Yang, C.W., Kim, J., Irwin, C., 1994. Impact of environmental regulation on the optimal allocation of coal among regions in the United States. *International Journal of Environment and Pollution*, 4, 59-74.

IndexMundi, 2022a. Lebanon wheat imports by year. Available at: https://www.indexmundi.com/agriculture/?country=lb&commodity=wheat&graph=imports

IndexMundi, 2022b. Egypt wheat imports by year. Available at: https://www.indexmundi.com/agriculture/?country=eg&commodity=wheat&graph=imports

Irwin, C.L., Yang, C.W., 1996. Impact analysis of carbon taxes on the steam coal and natural gas market in the eastern United States: A linear complementarity model. *International Journal of Environment and Pollution*, 6, 57-73.

Jasper, S., 2023. Resilience against hybrid threats: Empowered by emerging technologies: A study based on Russian invasion of Ukraine. In: K.P. Balomenos, A. Fytopoulos, and P.M. Pardalos, Editors, *Handbook for Management of Threats*, Springer Nature Switzerland AG, pp 209-226.

Jiang, Y., Wang, Y., Szeto, W.Y., Chow, A. H.F.; Nagurney, A., 2021. Probabilistic assessment of transport network vulnerability with equilibrium flows. *International Journal of Sustainable Transportation*, 15(7), 512 - 523.

Judge G.G., 1956. A spatial equilibrium model for eggs. Connecticut Agricultural Experiment Station, Storrs, Connecticut.

Korpelevich G.M., 1977. The extragradient method for finding saddle points and other problems. *Matekon*, 13, 35-49.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Editors, 2018. *Dynamics of Disasters: Algorithmic Approaches and Applications*. Springer International Publishing Switzerland.

Kotsireas, I.S., Nagurney, A., Pardalos, P.M., Tsokas, A., Editors, 2021. *Dynamics of Disasters: Impact, Risk, Resilience, and Solutions*. Springer Nature Switzerland AG.

Labys, W.C., 1999. *Modeling Mineral and Energy Markets*. Kluwer Academic Publishers, Boston, Massachusetts.

Labys, W.C., Yang, C.W., 1991. Advances in the spatial equilibrium modeling of mineral and energy issues. *International Regional Science Review*, 14, 61-94.

Li, D., Nagurney. A., 2017. Supply chain performance assessment and supplier and component importance identification in a general competitive multitiered supply chain network model. *Journal of Global Optimization*, 67, 223-250.

Li, X., Nguyen, V., 2023. The Panama Canal needs a rainy October: Impacts of Panama Canal drought on global agricultural trade. RaboResearch, August.

Magdy, S., Murad, Y., 2023. Left behind and grieving, survivors of Libya floods call for accountability. Associated Press, September 20. Available at:
https://apnews.com/article/libya-derna-dams-collapse-survivors-floods-chaos-1a2e0c06e61360580ab1277fdf3dae76

Martyshev, M., Nivievskyi, O., Bogonos, M., 2023. Regional war, global consequences: Mounting damages to Ukraine's agriculture and growing challenges for global food security. IFPRI Blog. March 27. Available at: https://www.ifpri.org/blog/regional-war-global-consequences-mounting-damages-ukraines-agriculture-and-growing-challenges

McCarthy, J.A., 2007. From protection to resilience: injecting 'Moxie' into the infrastructure security continuum. Critical Infrastructure Protection Program, Discussion Paper Series, George Mason University, Fairfax, Virginia.

Metz, S., 2023. 6 things to know about the devastating Morocco earthquake. Associated Press, September 11. Available at:
https://www.pbs.org/newshour/world/6-things-to-know-about-the-devastating-morocco-earthquake

Michalopoulos, S., 2023. After wildfires, floods deal Greek farmers the final blow. Euractiv. Available at:
https://www.euractiv.com/section/politics/news/after-wildfires-floods-deal-greek-farmers-the-final-blow/

Migration Trade Portal, 2021. Migration and trade. October 21. Available at:

https://www.migrationdataportal.org/themes/migration-and-trade

Mroue, B., 2023. Dozens of Syrians are among the missing in catastrophic floods in Libya, a war monitor says. Associated Press, September 16. Available at: https://apnews.com/article/libya-floods-syrians-killed-missing-2ed4b6a0802dbe8966254daabc2540e9

Nagurney, A., 1999. *Network Economics: A Variational Inequality Approach*, second edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2023a. *Labor and Supply Chain Networks*. Springer Nature Switzerland AG.

Nagurney, A., 2023b. Why Russia pulled out of its grain deal with Ukraine – and what that means for the global food system. *The Conversation*, July 19. Available at: https://theconversation.com/why-russia-pulled-out-of-its-grain-deal-with-ukraine-and-what-that-means-for-the-global-food-system-210046

Nagurney, A., Besedina, E., 2023. A multicommodity spatial price equilibrium model with exchange rates and non-tariff measures for agri-food international trade. *Operations Research Forum*, in press.

Nagurney, A., Besik, D, 2022. Spatial price equilibrium networks with flow-dependent arc multipliers. *Optimization Letters*, 16, 2483–2500.

Nagurney, A., Besik, D., Dong, J., 2019. Tariffs and quotas in world trade: A unified variational inequality framework. *European Journal of Operational Research*, 275(1), 347-360.

Nagurney, A., Hassani, D., Nivievskyi, O., Martyshev, P., 2023. Exchange rates and multicommodity international trade: Insights from spatial price equilibrium modeling with policy instruments via variational inequalities. *Journal of Global Optimization*, 87, 1-30.

Nagurney, A., Hassani, D., Nivievskyi, O., Martyshev, P., 2024. Multicommodity international agricultural trade network equilibrium: Competition for limited production and transportation capacity under disaster scenarios with implications for food security. *European Journal of Operational Research*, 314(3), 1127-1142.

Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. *International Journal of Production Economics*, 158, 300-313.

Nagurney, A., Qiang, Q., 2009. *Fragile Networks: Indentifying Vulnerabilities and Synergies in an Uncertain World*. John Wiley & Sons, Hoboken, New Jersey.

Nagurney, A., Qiang, Q., 2012. Fragile networks: Identifying vulnerabilities and synergies in an uncertain age. *International Transactions in Operational Research*, 19, 123–160.

NASA Earth Observatory, 2023. Greece's agricultural heartland underwater. September 8. Available at:
https://earthobservatory.nasa.gov/images/151816/greeces-agricultural-heartland-underwater

Newcomb, R.T., Fan, J., 1980. Coal market analysis issues. EPRI Report EA-1575, Palo Alto, California.

Newcomb, R.T., Reynolds, S.S., Masbruch, T.A., 1990. Changing patterns of investment decision making in world aluminum. *Resources and Energy*, 11, 261-297.

Nikas, S., 2023. Greek premier to request EU support for country's flood damage. Bloomberg, September 10. Available at:
https://www.bloomberg.com/news/articles/2023-09-10/greek-premier-to-request-eu-support-for-country-s-flood-damage#xj4y7vzkg

Passacantando, M., Raciti, F., 2024. A random variational inequality model of international agricultural supply chain with a vulnerability analysis under disaster scenarios. Accepted in *Dynamics of Disasters - From Natural Phenomena to Human Activity*, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, S.W. Pickl, and C. Vogiatzis, Editors, Springer Nature Switzerland AG.

PBS, 2023. Northeast farmers 'heartbroken' as floods devastate months of labor and crops are swept away. July 1. Available at:
https://www.pbs.org/newshour/nation/northeast-farmers-heartbroken-as-floods-devastate-months-of-labor-and-crops-are-swept-away

Picheta, R., Karadsheh, J., Gigova, R., Lister, T., 2022. Kyiv and Moscow agree deal to resume Ukraine grain exports from Black Sea ports. CNN, July 22. Available at:
https://www.cnn.com/2022/07/22/europe/ukraine-russia-grain-deal-turkey-intl/index.html

Prousalis, S., Papadimas, L., 2023. Boats, helicopters rescue hundreds after storm in Greece. Reuters. September 8. Available at:
https://www.reuters.com/business/environment/deadly-greek-rainstorm-ebbs-trail-devastation-emerges-2023-09-08/

Qiang, Q., Nagurney, A., 2008. A unified network performance measure with importance identification and the ranking of network components. *Optimization Letters*, 2, 127–142.

Qiang, Q., Nagurney, A., 2012. A bi-criteria indicator to assess supply chain network performance for critical needs under capacity and demand disruptions. *Transportation Research C*, 46(5), 801-812.

Qiang, Q., Nagurney, A., Dong, J., 2009. Modeling of supply chain risk under disruptions with performance measurement and robustness analysis. In: *Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers*. T. Wu and J. Blackhurst, Editors, Springer, London, England, pp. 91-111.

Ruijs, A., Schweigman, C., Lutz, C., Sirpe, G., 2001. Cereal trade in developing countries: Stochastic spatial equilibrium models. Technical report, University of Groningen, The Netherlands.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. *American Economic Review*, 42, 283-303.

Sharkey, T.C., Nurre Pinkley, S.G., Eisenberg, D.A., Alderson, D.L., 2020. In search of network resilience: An optimization-based view. *Networks*, 77(2), 225-254.

Sohn, H.K., 1970. *A Spatial Equilibrium Model of the Beef Industry in the United States*, PhD Dissertation, Department of Agricultural Economics, University of Hawaii.

Takayama, T., Judge, G.G., 1964. An intertemporal price equilibrium model. *Journal of Farm Economics*, 46, 477-484.

Takayama, T., Judge, G.G., 1971. *Spatial and Temporal Price and Allocation Models.* North Holland, Amsterdam, The Netherlands.

Thompson, R.L., 1989. Spatial and temporal price equilibrium agricultural models. In: *Quantitative Methods for Market Oriented economic Analysis Over Space and Time.* W.C. Labys, T. Takayama, and N. Uri, Editors, Gower Publishing Co. Ltd., Brookfield, Vermont, pp. 49-65.

TrendEconomy, 2022a. Annual international trade statistics by country (HS) - Lebanon, November 14. Available at:
https://trendeconomy.com/data/h2/Lebanon/1001

TrendEconomy, 2022b. Annual international trade statistics by country (HS) - Egypt, November 14. Available at:
https://trendeconomy.com/data/h2?commodity=1001&reporter=Egypt&trade_flow=Export, Import&partner=World&indicator=TV,YoY&time_period=2010,2011,2012,2013,2014,2015, 2016,2017,2018,2019,2020,2021

Vankin, J., 2023. How California agriculture has shaped the state and the country. California Local, May 8. Available at:
https://californialocal.com/localnews/statewide/ca/article/show/36707-california-agriculture-dairy-wheat-fruit-vegetables/#: :text=Despite%20being%20home%20to%20just,of%20the %20U.S.%20agricultural%20value.

World Food Programme, 2023. Economic crisis, currency depreciation, and unprecedented increase in food and non-food prices leave 2 million people in need of assistance in Lebanon. January 19. Available at:
https://www.wfp.org/news/economic-crisis-currency-depreciation-and-unprecedented-increase-food-and-non-food-prices

World Meteorological Organization, 2023. Catastrophic consequences of Horn of Africa drought will continue in 2023. February 17. Available at:
https://wmo.int/media/news/catastrophic-consequences-of-horn-of-africa-drought-will-continue-2023

Wu, T., Blackhurst, J., 2009. *Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers.* Springer, Berlin, Germany.

Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. *European Journal of Operational Research*, 224(2), 273-282.

Zakowski, E., Parker, L.E., Johnson, D., Aguirre, J., Ostoja, S.M., 2023. California wine grape growers need support to manage risks from wildfire and smoke. *California Agriculture*, 77(2), 40-48.