Risk Reduction and Cost Synergy in Mergers and Acquisitions
via Supply Chain Network Integration

Zugang Liu
Department of Business and Economics
Pennsylvania State University, Hazleton, PA, 08071, USA
zugangliu@psu.edu

Anna Nagurney
Department of Finance and Operations Management
Isenberg School of Management
University of Massachusetts, Amherst, MA, 01003, USA
nagurney@gbfin.umass.edu

July 2010; Revised March 2011


Abstract
The economic and financial collapse of 2008 and 2009 due to the credit crisis in the U.S. with global ramifications impacted dramatically the landscape for mergers and acquisitions (M&As). It is anticipated that, if there is a new merger wave, then a larger percentage of M&A deals will be more strategic than those in the wave that ended prior to this crisis, with special attention given to the increasingly risk-averse environment. In addition, it is expected that firms will seek to take advantage of existing strengths, be they in a particular region or in terms of distribution networks.

This paper provides a methodological framework to enable decision-makers involved in M&As to quantify the potential gains through supply chain network integration in terms of risk reduction and cost synergy. In particular, we develop new pre-merger and post-merger network models that capture the economic activities of firms who seek to determine their expected total cost and risk-minimizing product flows subject to demand satisfaction. We utilize a mean-
variance approach to capture the risk associated with cost uncertainty. In addition, we propose three new synergy measures: the expected total cost synergy, the relative risk reduction synergy, and the absolute risk reduction synergy for the assessment of the potential strategic advantages. We illustrate the analytical framework with two sets of numerical examples which yield interesting managerial implications. The first set of examples demonstrates how the uncertainty surrounding the costs influences both the cost synergy and the risk reduction achievable through M&As. The second set of examples shows that decision-makers with distinct concerns should focus on specific synergy measures.

**Keywords:** mergers and acquisitions, mean variance approach, supply chains, network integration

**Introduction**

Mergers and acquisitions (M&As) are a financial mechanism and fundamental building block of corporate management by which, respectively, firms merge to create a new firm or entity or a firm acquires another firm, which is, typically, smaller in size. According to Kusstatscher and Cooper (2005) there have been five major waves of identifiable merger and acquisition activity with the First Wave (1898 -- 1902) consisting of an increase in horizontal mergers that resulted in many U.S. industrial groups; the Second Wave (1926 -- 1939) consisting of vertical mergers with mergers involving many public utilities; the Third Wave (1969 -- 1973) having as its driving force *diversification*; the Fourth Wave (1983 -- 1986) having as its goal *efficiency*, and the Fifth Wave (1997 until 2005 or so) focusing on *globalization* with cross-border mergers being the paradigm. In 2001, over 6,000 M&A transactions occurred globally with a value of over a trillion dollars (Langabeer (2003)).

The economic and financial collapse of 2008 and 2009 due to the credit crisis in the U.S. with global ramifications (cf. Nagurney and Qiang (2009)) impacted dramatically the M&A landscape. According to The Economist (2009), in the year ending in August 2009, the value of such deals globally was just below 1.5 trillion dollars, which was 36% lower than at the same stage the year before, and 56% below at the end of August 2007, which was a year that broke preceding M&A records with approximately 4.8 trillion dollars in M&A deals transacted. It is anticipated that, if there is a new merger wave, then a larger percentage of the M&A deals will
be more strategic than those in the wave that ended in 2007, with special attention given to the increasingly risk-averse environment (The Economist (2010)). In addition, it is expected that firms will seek to take advantage of existing strengths, be they in a particular region or in terms of distribution networks.

Interestingly, according to The Economist (2010), emerging countries from Thailand to India and China have entered a period of dynamism as developed countries continue to struggle with the recession with emerging-market companies pursuing growth through M&As with a focus on acquiring brands and distribution channels. In addition, it is being reported that we can expect M&As in the healthcare, high tech, media, and energy sectors (cf. Zendrian (2010)).

The potential economic benefits and motivations of M&A have been discussed in the literature. A number of studies reported risk reductions of bank mergers due to various diversification effects (see, e.g., Hughes et al. (1999), Emmons et al. (2004), Boyd et al. (1993), Estrella (2001), Van Lelyveld and Knot (2009)). Amihud et al. (2002), on the other hand, found that the acquiring banks’ risks neither increase or decrease after mergers (see also Vallascas and Hagendorff (2011)). Wang and Reuer (2006) discussed a shareholder-based rational for firm risk reduction through mergers. The authors argued that M&As were motivated by the shareholders’ incentives to diversify the risks associated with firm-specific investments. Thijssen (2008) proposed a real option model to study the optimal and strategic timing of mergers and acquisitions between two firms where the M&As can lead to not only efficiency gains but also risk reductions. In addition, Amihud and Lev (1981) pointed out that although conglomerate mergers can reduce risks through a diversification effect they cannot create values to shareholders (see also, Levy and Sarnat (1970)). Hence, the authors argued that the motive of conglomerate mergers was due to managers’ incentives to diversify their employment risks. Moreover, Bernile and Lyandres studied the benefits of mergers along supply chains using a unique dataset of insiders’ projections of synergies (see also, Maksimovic and Phillips (2001)), and found that synergies are an important determinant of the responses of rivals, customers and suppliers, and a critical factor in explaining the market power motive for horizontal mergers.

The focus of our paper differs from the above noted studies in that here we provide an analytical framework to investigate the potential risk reduction and cost synergy created through supply chain network integration in the process of M&A. It is increasingly apparent and documented that improving supply chain integration is key to improving the likelihood of post-
merger success (Langabeer (2003), Langabeer and Seifert (2003), and Herd, Saksena, and Steger (2005)). This is understandable, since up to 80% of a firm's costs are linked to operations (Benitez and Gordon (2000)). Furthermore, the need for quantitative approaches to assess the potential for post-merger integration success is being driven by the data and the revealing statistics that have been identified in practice (Gerds, Strottman, and Jayaprakash (2010)). Especially illuminating is that empirical studies demonstrate that one out of two post-merger integration efforts fares poorly (Gerds and Schewe (2009)). In addition, in an empirical analysis of a global sample of over 45,000 data points of post-merger transactions in all significant sectors globally from services to manufacturing, significant risk factors were identified to post-merger success and several myths quantifiably negated (see Gerds, Strottmann, Jayaprakash (2010)).

In this paper, we set out to construct a methodological framework to enable the assessment of potential synergies associated with post-merger integration through the explicit incorporation of risk. In the recent uncertain economic and financial climate, it is essential to quantitatively assess a priori the potential cost savings associated with a proposed merger or acquisition and the associated risk. As noted by finance professionals (see Schneeweis, Crowder, and Kazemi (2010)), the concept of risk is multi-dimensional and for many risk is simply the probability of a bad outcome. In fact, according to Steinbach (2001), a fundamental (and still debated question) is even how risk should be measured.

As discussed in Qiang, Nagurney, and Dong (2009), risk in the context of supply chains may be associated with the production/procurement processes, the transportation/shipment of the goods, and/or the demand markets. Such supply chain risks are directly reflected in firms' financial performances, and priced in the financial market. For example, Hendricks and Singhal (2010) estimated that the average stock price reaction to supply-demand mismatch announcements was approximately -6.8%. For another instance, supply chain disruptions can cause firms' equity risks to increase by 13.50% on average after the disruption announcements (Hendricks and Singhal (2005)). Cruz et al (2006) proposed an integrated framework that incorporated financial engineering and social networks in supply chain management to optimize the objective function that considered both profit and risk. For a comprehensive review of supply chain risk management models, please see Kleindorfer and Saad (2005), Tang (2006), Nagurney (2006), and Wu and Blackhurst (2009). In our paper, we aim to provide a method that can help
decision-makers and investors understand and evaluate the potential cost synergy and risk reduction achieved through mergers of supply chains. We believe that it is essential to study supply chain risk management from a holistic point of view, even in the context of mergers and acquisitions, since failure to capture the full complexity of the network may result in paradoxical behavior (see Nagurney (2010)).

In particular, in this paper, we take a mean-variance (MV) approach to the measurement of risk, which dates to the work of the Nobel laureate Markowitz (1952, 1959) and which even today (cf. Schneeweis, Crowder, and Kazemi (2010)) remains a fundamental approach to minimizing volatility. The MV approach has been increasingly used in the supply chain management literature to study decision-making under risk and uncertainty. For example, Hodder (1984), Hodder and Jucker (1985a, 1985b) and Hodder and Dincer (1986) utilized an MV framework to investigate facility location problems under various sources of uncertainty. Lau and Lau (1999) studied the return policy in a two-echelon supply chain where both the manufacturer and the retailer behaved under the MV assumption. Chen and Federgruen (2000) constructed the efficient frontier of a single echelon inventory problem using MV analysis. Gan, Sethi, and Yan (2005) used the MV approach to model a supply chain coordination problem. Kim, Cohen, and Netessine (2007) focused on performance-based contracting in after-sale supply chains where the decision-makers used an MV framework to analyze return and risk. Choi, et al. (2008) also investigated channel coordination problems and return policies of supply chains using an MV approach.

Since our focus is on mergers and acquisitions through network integration, and, specifically, through supply chain network integration, we envision a firm as a network of its economic activities consisting of manufacturing, which is conducted at the firm's plants or manufacturing facilities; distribution, which occurs between the manufacturing plants and the distribution centers, which store the product; and the ultimate transportation/shipment of the product to the retailers. Associated with each such economic activity is a link in the network with a total associated cost that depends on the flow of the product on the link. The links, be they manufacturing, shipment, or storage links have capacities on the flows. We assume, as given, the demand for the product at each retailer.

In this paper, we build upon the recent work in mergers and acquisitions of Nagurney (2009) that focused on horizontal network integration. Here, however, we develop the following
significant extension: we utilize a mean-variance approach in order to capture the risk associated with supply chain activities both prior to and post the merger/acquisition under investigation. This new modeling framework allows one to capture quantitatively the risk associated not only with the supply chain network activities but also with the merger/acquisition itself, which has been identified as being critical in practice (cf. Gerds, Strottmann, and Jayaprakash (2010)). Such risks, which we associate with the new links representing the merger/acquisition may include, for example, issues concerning the merging of different information technologies, human resource issues, and/or distinct managerial approaches and business processes. All firms, both prior and post the merger, minimize both their expected total costs and the risk, as captured through the variance of the total costs, with a suitable weight assigned to the latter. In addition, we introduce new measures for the quantification of synergy associated with M&As: the expected total cost synergy, the absolute risk synergy, and the relative risk synergy measures.

We emphasize that the new network models that we develop in this paper, although focused on mergers and acquisitions through the prism of supply chain network integration, are sufficiently general to be applied in such contexts as the assessment of M&As in a variety of network industries from transportation (airline, rail, etc.) to telecommunications, as well as energy. In addition, we believe that our network models can also be utilized in financial services.

The paper is organized as follows. In Section 2, we first develop the optimization problems faced by two separate firms each of which manages a supply chain network and minimizes both the expected total cost and the total risk in operations with an individual weight associated with its valuation of the risk. We then formulate the potential integration of the two firms through a merger/acquisition and capture the total expected costs and risks through the integration of their supply chain networks with the explicit incorporation of new links associated with the merger/acquisition. We demonstrate that all the associated system-optimization problems can be formulated and solved as variational inequality problems (see, e.g., Nagurney (1999)) with a structure that can be easily exploited for computational purposes. Such a general formulation also lays the groundwork for future game theoretic formulations.

In Section 3, we propose three measures that can be used to evaluate the potential strategic advantages that can be achieved, using different perspectives, in a merger or acquisition. In Section 4, we provide two sets of simulation examples. The first set of examples studies how the cost uncertainty affects the possible expected cost synergy and risk reduction through the
integration of supply chain networks. The second set of examples, in turn, applies the three measures provided in Section 4 to evaluate the gains from the merger/acquisition from various perspectives, and shows that decision-makers with different concerns may reach distinct conclusions regarding the benefits achieved through supply chain network integration. In Section 5, we provide managerial insights and present our conclusions.

2. The Pre- and Post-Merger Supply Chain Network Models with Risk

In this section, we present the supply chain network models prior to and post the merger. We consider two firms, denoted by Firm A and Firm B, whose supply chains are integrated post the merger. The firms consider both the expected total cost and the risk associated with their operations. Since our paper focuses on the expected cost synergy and risk reduction generated through supply chain network integration we assume that the firms produce substitutable products the demands of which are known and fixed.

As mentioned in the Introduction, we assume that the two firms own their individual manufacturing facilities/plants, and distribution centers, and that each firm seeks to determine the optimal production quantities at its manufacturing plants, the optimal quantities of product shipped from the manufacturing plants to its distribution centers, and the optimal quantity of product shipped from the distribution centers to the retailers. Each firm is assumed to minimize its expected total cost and the total risk of its operations, subject to the demand being satisfied at the retail outlets. In particular, as discussed in the Introduction, our paper utilizes the mean-variance approach where the variance of the total cost is used to proxy the risk (see also Luenberger (1998)).

In Section 2.1, we consider the pre-merger case where each firm's optimization problem is solved individually. In Section 2.2, we formulate the post-merger model where the manufacturing plants and the distribution centers are shared through the integration of the firms' supply chain networks.
2.1 The Pre-Merger Supply Chain Network Model(s) with Risk

We now formulate the optimization problem faced by each of the two firms. We assume that firm $i; i=A, B,$ operates $n_M^i$ manufacturing facilities/plants; $n_D^i$ distribution centers, and serves $n_R^i$ retailers. We let $G_i = [N_i, L_i]$ for $i = A, B$ denote the graph consisting of nodes and directed links representing the economic activities associated with each firm $i$. We also let $G^0 = [N^0, L^0] = \bigcup_{i=A,B} [N_i, L_i]$.

In each of the networks in Figure 1 the links connecting the top-tiered nodes $i$ and the manufacturing nodes of the respective firm $i$, $M_1^i, ..., M_{n_M^i}^i$, represent the manufacturing activities. The links joining the manufacturing nodes, with the distribution center nodes of each firm, $D_1^i, ..., D_{n_D^i}^i$, correspond to the shipment links between the manufacturing plants and the distribution centers. The links connecting nodes $D_1^i, ..., D_{n_D^i}^i$ with nodes $D_1^j, ..., D_{n_D^j}^j$ for $i = A, B$ correspond to the storage activities. Finally, the shipment activities between the distribution
centers and retailers are represented by the links joining the nodes $D^{i}_{1,2}, \ldots, D^{i}_{n_{R},2}$ with the retail nodes: $R^{i}_{1}, \ldots, R^{i}_{n_{R}}$.

Without any loss in generality, we denote the links in Figure 1 by $a, b$, etc., and the total cost on link $a$ by $\hat{c}_a$. We use $d_{R^{i}_{k}}$ to denote the fixed demand for the product at retailer $R^{i}_{k}$ associated with firm $I_{i} = A, B; k = 1, \ldots, n_{R}$. Let $x_{p}$ denote the nonnegative flow of the product on path $p$ connecting (origin) node $i$ with a (destination) retail node of firm $i; i = A, B$. Then the following conservation of flow equations must hold for each firm $i$:

$$
\sum_{p \in P^{i}_{R_{i}}} x_{p} = d_{R^{i}_{k}}, \quad i = A, B; k = 1, \ldots, n^{i}_{R},
$$

(1)

where $P^{0}_{R_{i}}$ denotes the set of paths joining node $i$ with retail node $R^{i}_{k}$. Hence, the demand at each retail node must be satisfied by the product flows destined to that node.

We use $f_{a}$ to denote the product flow on link $a$. The following conservation of flow equations must also be satisfied:

$$
f_{a} = \sum_{p \in P_{i}} x_{p} \delta_{ap}, \quad \forall a \in L_{i}; i = A, B,
$$

(2)

where $\delta_{ap} = 1$ if link $a$ is contained in path $p$ and $\delta_{ap} = 0$, otherwise. Here $P_{i}$ denotes the set of all paths in firm $i$’s network in Figure 1, that is, $P_{i} = \bigcup_{k=1, \ldots, n_{R}} P^{0}_{R_{i}}$. Note that (2) means that the flow on a link is equal to the sum of the flows on paths that contain that link. We also have that the path flows must be nonnegative, that is,

$$
x_{p} \geq 0, \quad \forall p \in P_{i}; i = A, B.
$$

(3)

We assume that the total cost on a link is the function of the flow of the product on the link; see, for example, Nagurney (2006) and the references therein. Moreover, we allow the total costs to be influenced by uncertainty factors. In particular, the total cost on link $a$, $\hat{c}_a$, takes the form:

$$
\hat{c}_a = \hat{c}_a(f, \omega_{a}) = \omega_{a} \hat{h}_a f_{a} + h_{a} f_{a}, \quad \forall a \in L_{i}; i = A, B,
$$

(4)

where $\omega_{a}$ denotes the exogenous random variable affecting the total cost of link $a$. We allow $\omega_{a}$ to follow any distribution, and permit the $\omega_{a}$’s of different links to be correlated with one another.
The $\omega_a$s can represent various factors of uncertainty, such as, for example, those associated with foreign exchange rates, the production disruption frequencies, and/or the energy and material prices. Note that in (4), $\hat{h}_af_a$, represents that part of the total cost that is subject to the variation of $\omega_a$, whereas $h_af_a$ denotes that part of the total cost that is independent of $\omega_a$. Furthermore, we assume that there are nonnegative capacities on the links with the capacity on link $a$ denoted by $u_a, \forall a$.

The firms consider both costs and risks in their operations using a mean-variance framework and each seeks to minimize its expected total cost and the valuation of its risk. The optimization problem faced by firm $i; i = A, B$, can be expressed as:

$$\text{Minimize } \sum_{a \in L_i} E(\hat{c}_a(f_a, \omega_a)) + \alpha_i V(\sum_{a \in L_i} \hat{c}_a(f_a, \omega_a))$$  \hspace{1cm} (5)

subject to: constraints (1) -- (3) and

$$f_a \leq u_a, \quad \forall a \in L_i,$$  \hspace{1cm} (6)

where the first term in the objective function (5) denotes the expected total cost; $\alpha_i$ denotes the risk aversion factor of firm $i$; and $V(\sum_{a \in L_i} \hat{c}_a(f_a, \omega_a))$ represents the variance of the total cost.

Note that we can substitute (4) into (5), to obtain the equivalent optimization problem:

$$\text{Minimize } \sum_{a \in L_i} E(\hat{c}_a(f_a, \omega_a)) + \alpha_i V(\sum_{a \in L_i} \hat{c}_a(f_a, \omega_a))$$  \hspace{1cm} (7)

subject to: constraints (1) -- (3) and

$$f_a \leq u_a, \quad \forall a \in L_i.$$  \hspace{1cm} (8)

We assume that the objective function in (7) is convex and that the individual terms are continuously differentiable. This optimization problem is a constrained, convex nonlinear programming problem. According to the standard theory of nonlinear programming (cf. Bazaraa, Sherali, and Shetty, 1993) if the feasible set of the problem represented by the constraints (1) -- (3) and (6) is non-empty, then the optimal solution, denoted by $f^* = \{ f^*_a \}, a \in L_i$, exists.

We define $K_i \equiv \{ f | \exists x \geq 0, \text{ and (1) - (3) and (6) hold} \},$ where $f$ is the vector of link flows and $x$ the vector of path flows. Also, we let $\beta_a$ denote the Lagrange multiplier associated with
constraint (6) for link \( a \). This term may also be interpreted as the price or value of an additional unit of capacity on link \( a \). The associated optimal Lagrange multiplier is represented by \( \beta^*_a \).

We now state the following result in which we provide a variational inequality formulation of the problem.

**Theorem 1**

The vector of link flows of firm \( i, f^* \in K_i; i = A, B \), is an optimal solution to problem (5), subject to (1) through (3) and (6), if and only if it satisfies the following variational inequality problem with the vector of optimal nonnegative Lagrange multipliers \( \beta^* \):

\[
\sum_{a \in l_i} \left[ E(\omega_a)\hat{h}_a + h_a + \alpha_i \frac{\partial V}{\partial f_a} + \beta^*_a \right] \times [f_a - f^*_a] + \sum_{a \in l_i} [u_a - f^*_a] \times [\beta_a - \beta^*_a] \geq 0,
\]

\[\forall f \in K_i, \quad \forall f \geq 0, \forall a \in L_i, \quad (9)\]

**Proof:** See Bertsekas and Tsitsiklis (1989) page 287.

We can easily solve variational inequality (9) using the modified projection method (also sometimes referred to as the extragradient method). The modified projection method has been used to solve large practical network problems (see, for example, Liu and Nagurney (2009)). For a complete description of the modified projection method, see Nagurney (1999). The subproblems (in flows) induced by this method can be solved by the well-known equilibration algorithm (system-optimization version) of Dafermos and Sparrow (1969), which has been widely applied and which exploits the underlying network structure of the problem (see also, e.g., Nagurney (2006)). The subproblems (in Lagrange multipliers), in turn, can be determined at each iteration using explicit formulae since they are constrained only to be nonnegative. Recall that the modified projection method is guaranteed to converge to a solution of a variational inequality problem, provided that the function that enters the variational inequality problem is monotone and Lipschitz continuous and that a solution exists. Since it is easy to verify that the function that enters variational inequality (9) satisfies the conditions of monotonicity and Lipschitz continuity, the modified projection method is always guaranteed to converge for our model.

In particular, we solve problem (9) to obtain the solution \( f^* \) that minimizes the objective function (5) associated with firm \( i \). We then define the expected total cost of the two firms, \( A \) and \( B \), denoted by \( TC^0 \), as:
\begin{equation}
TC^0 \equiv \sum_{a \in L_A} E(\hat{c}_a(f^*_a, \omega_a)) + \sum_{a \in L_B} E(\hat{c}_a(f^*_a, \omega_a))
\end{equation}

and we define the total risk of the two firms, denoted by $TR^0$, as:

\begin{equation}
TR^0 \equiv V(\sum_{a \in L_A} \hat{c}_a(f^*_a, \omega_a) + \sum_{a \in L_B} \hat{c}_a(f^*_a, \omega_a)).
\end{equation}

We use the values of $TC^0$ and $TR^0$ as benchmarks to compute the strategic advantages, as discussed in Section 3, below.

### 2.2: The Post-Merger Supply Chain Network Model with Risk

We now consider the post-merger supply chain network where Firms $A$ and $B$ merge and the retailers can receive the product made at any manufacturing plant and shipped from any distribution center. For simplicity, we refer to this model as the post-merger one, but it is also applicable in the case of an acquisition with minor modifications which we will discuss in the end of this section.

In the post-merger case, we add a supersource node 0 to the network $G^0$ depicted in Figure 1. We also connect node 0 with nodes $i = A, B$ to reflect the merger of the two firms. We further add new links connecting each manufacturing node of each firm with the distribution center nodes of the other firm, and add new links connecting each distribution center node of each firm with the retailers associated with the other firm, as depicted in Figure 2. We denote the new network topology in Figure 2 by $G^1 = [N^1, L^1]$ where $N^1 = N^0 \cup$ node 0 and $L^1 = L^0 \cup$ the additional links.

In addition, we assume that the total cost functions associated with the added new links also take the form given by (4). Note that the expected total costs and the risks associated with the merger itself can be incorporated into the functions of the merger links, i.e., the links connecting node 0 with nodes $i = A, B$. Such costs and risks may arise from such issues as the financing of the merger, the merging of different information technologies, human resource issues, and/or distinct managerial approaches and business processes. In Section 4, we present sets of examples to demonstrate how these merger links affect the overall expected costs and risks of a specific merger.
Let $x_p$ denote the flow of the product on path $p$ connecting node 0 with a retailer node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^i_{R_k}} x_p = d_{R_k}^i, \quad i = A, B; k = 1, \ldots, n^i_R$$

(12)

where $P^i_{R_k}$ denotes the set of paths joining node 0 with retail node $R_k^i$. The set $P^i \equiv \bigcup_{i=A,B;k=1,\ldots,n^i_R} P^i_{R_k}$.

In addition, as before, we let $f_a$ denote the flow of the product on link $a$, and we must have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in p_a^i} x_p \delta_{ap}, \quad \forall a \in L^i.$$  

(13)

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^i.$$  

(14)

The optimization problem associated with the post-merger firm which minimizes the expected total cost and the total risk subject to the demand for the product being satisfied at the retailers, is, thus, given by:
Minimize $\sum_{a \in \mathcal{L}} E(\hat{c}_a(f_a, \omega_a)) + \alpha V(\sum_{a \in \mathcal{L}} \hat{c}_a(f_a, \omega_a))$ (15)

subject to: constraints (12) -- (14) and

\[ f_a \leq u_a, \quad \forall a \in \mathcal{L}. \] (16)

We can substitute (4) into (15) to obtain the equivalent optimization problem:

Minimize $\sum_{a \in \mathcal{L}} E(\omega_a)\hat{h}_a f_a + \sum_{a \in \mathcal{L}} h_a f_a + \alpha V(\sum_{a \in \mathcal{L}} \omega_a \hat{h}_a f_a)$ (17)

subject to: constraints (12) -- (14) and

\[ f_a \leq u_a, \quad \forall a \in \mathcal{L}. \] (18)

where $\alpha$ is the risk aversion factor, and is specified differently in the case of mergers and in the case of acquisitions. We will explain how $\alpha$ should be determined in the discussion in the end of this section. The above optimization problem can also be solved as a variational inequality problem akin to (9) where now $a \in \mathcal{L}$, and the vectors: $f$, $x$, and $\beta$ have identical definitions as before, but are re-dimensioned accordingly. In addition, the set $K_i$ is replaced by $K^i \equiv \{f \mid \exists x \geq 0, \text{ and (12) - (14) and (16) hold}\}$. Hence, one can apply the modified projection problem to compute the solution to the variational inequality problem governing the post-merger network. Finally, we can compute the expected total cost associated with the merger, $TC^i$, which is defined as:

\[ TC^i \equiv \sum_{a \in \mathcal{L}} E(\hat{c}_a(f_a^*, \omega_a)), \] (19)

and, the total risk associated with the merger, $TR^i$, which is defined as:

\[ TR^i \equiv V(\sum_{a \in \mathcal{L}} \hat{c}_a(f_a^*, \omega_a)), \] (20)

We now discuss how our model can handle mergers and acquisitions differently. First, we note that, in the case of an acquisition, we can expect the acquiring firm to impose its valuation of risk on the integrated network link activities, whereas in the case of a merger, the risk aversion factor may be obtained after some negotiations between the two firms that merge. We, thus, assume that, in the case of an acquisition, the risk aversion factor, $\alpha = \alpha_i$, with $i$ being an acquiring firm and, in the case of a merger, $\alpha = \frac{\alpha_A + \alpha_B}{2}$ being reasonable factors. Secondly, in general, the various types of risk associated with the network integration processes (e.g. human
resource issues, distinct managerial approaches, etc.) and associated with the operations of the acquired firm in the post-merger network are expected to be greater in the case of an acquisition than in a mutually agreed upon merger. Hence, in the case of an acquisition, we expect that the cost and risk parameters associated with the links connecting Node 0 and Nodes A and B are higher than those in the case of mergers, and that the cost and risk parameters of the links associated with the acquired firms increase from their pre-merger levels.

In the next section, we discuss how we utilize the expected total costs and the total risks to compute various synergy measures to evaluate the strategic advantages associated with a merger or an acquisition.

3. Measuring the Strategic Advantage Associated with Mergers and Acquisitions under Risk

In this section, we provide three measures for the evaluation of the strategic advantage associated with mergers/acquisitions through supply chain network integration from different perspectives. The measures that we propose to capture the gains, if any, are as follows:

The Expected Total Cost Synergy

\[ S_{TC} \equiv \left( \frac{TC^0 - TC^i}{TC^0} \right) \times 100\% , \]  

The Absolute Risk Synergy

\[ S_{TR} \equiv \left( \frac{TR^0 - TR^i}{TR^0} \right) \times 100\% , \]  

The Relative Risk Synergy

\[ S_{CV} \equiv \left( \frac{CV^0 - CV^1}{CV^0} \right) \times 100\% , \]  

where \( CV^0 \) and \( CV^1 \) denote the coefficient of variation of the total cost for, respectively, the pre-merger and the post-merger networks, and are defined as follows:

\[ CV^0 \equiv \frac{\sqrt{TR^0}}{TC^0} , \]

\[ CV^1 \equiv \frac{\sqrt{TR^i}}{TC^i} , \]
Note that $CV^0$ and $CV^1$ represent the volatilities of the expected total costs of the pre-
and post-merger networks, respectively.

The first measure, $S_{TC}$, quantifies the expected total cost savings obtained by the merger;
the second measure, $S_{TR}$, represents the reduction of the absolute risk achieved through the
merger; and the third measure, $S_{CV}$, reflects the reduction of the relative risk through the merger.
In Section 4, we demonstrate how these measures can be applied to determine the synergies
achieved by the merger in terms of cost savings and risk reduction. In particular, the second set
of examples in Section 4 shows that whether a merger is beneficial may depend on whether cost
synergy or risk reduction is more important to the firm's stakeholders.

4. Numerical Examples

In this Section, we present two sets of numerical examples for which we compute the
strategic advantage measures provided in Section 3. In particular, the first set of examples
examines how the uncertainty of link costs affects the cost synergy and the risk reduction
achieved through the merger. The second set of examples, in turn, compares the three measures
that evaluate the merger gains from different perspectives, and shows that decision-makers with
different concerns may reach distinct conclusions regarding the benefits achieved through a
merger.

Numerical Example Set 1

In this set, we considered Firm $A$ and Firm $B$, each of which has two manufacturing
plants: $M^i_1$ and $M^i_2$; $i = A, B$. In addition, each firm has a single distribution center which
receives the product from the manufacturing plants, and provides storage and distribution
services. Finally, each firm serves two retailers, denoted by $R^i_1$ and $R^i_2$ for $i = A, B$. A graphical
depiction of the pre-merger supply chain networks associated with the two firms is given in
Figure 3. Figure 4, in turn, depicts the network after the two firms have merged.

We utilized the modified projection method, embedded with the equilibration algorithm,
as discussed in Section 2, to compute the solutions to the problems. The algorithm was
implemented in Matlab.
We assume that the risk-aversion factor $\alpha_i=1$, $i=A, B$. We also assumed that $COV$, the covariance matrix of the random cost factors, the $\omega$s, takes the form:

$$COV = \sigma^2 I,$$

where $I$ is a $24 \times 24$ identity matrix, and $\sigma^2$ represents the magnitude of the variance.

Figure 3: The Pre-Merger Supply Chain Network Topology for the Numerical Examples

Figure 4: The Post-Merger Supply Chain Network Topology for the Numerical Examples
Table 1: Definition of Links and Associated Total Cost Functions for the Numerical Examples

<table>
<thead>
<tr>
<th>Link $a$</th>
<th>From Node</th>
<th>To Node</th>
<th>$\hat{c}_a(f_x, \omega_a)$</th>
<th>$E(\omega_a)$</th>
<th>Marginal Generalized Link Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A$</td>
<td>$M_1^a$</td>
<td>$\omega_1 f_1^1 + f_1^1$</td>
<td>$E(\omega_1) = 1$</td>
<td>$8\sigma_1^2 f_1^1 + 3$</td>
</tr>
<tr>
<td>2</td>
<td>$A$</td>
<td>$M_2^a$</td>
<td>$\omega_2 f_2^2 + f_2^2$</td>
<td>$E(\omega_2) = 1$</td>
<td>$32\sigma_2^2 f_2^2 + 5$</td>
</tr>
<tr>
<td>3</td>
<td>$M_1^a$</td>
<td>$D_{13}^1$</td>
<td>$\omega_3 f_3 + f_3$</td>
<td>$E(\omega_3) = 1$</td>
<td>$2\sigma_3^2 f_3 + 2$</td>
</tr>
<tr>
<td>4</td>
<td>$M_2^a$</td>
<td>$D_{13}^1$</td>
<td>$\omega_4 f_4 + f_4$</td>
<td>$E(\omega_4) = 1$</td>
<td>$2\sigma_4^2 f_4 + 2$</td>
</tr>
<tr>
<td>5</td>
<td>$D_{13}^1$</td>
<td>$D_{13}^1$</td>
<td>$\omega_5 f_5 + f_5$</td>
<td>$E(\omega_5) = 1$</td>
<td>$2\sigma_5^2 f_5 + 2$</td>
</tr>
<tr>
<td>6</td>
<td>$D_{12}^1$</td>
<td>$R_1^4$</td>
<td>$\omega_6 f_6 + f_6$</td>
<td>$E(\omega_6) = 1$</td>
<td>$2\sigma_6^2 f_6 + 2$</td>
</tr>
<tr>
<td>7</td>
<td>$D_{13}^1$</td>
<td>$R_2^4$</td>
<td>$\omega_7 f_7 + f_7$</td>
<td>$E(\omega_7) = 1$</td>
<td>$2\sigma_7^2 f_7 + 2$</td>
</tr>
<tr>
<td>8</td>
<td>$B$</td>
<td>$M_1^b$</td>
<td>$\omega_8 f_8 + f_8$</td>
<td>$E(\omega_8) = 1$</td>
<td>$8\sigma_8^2 f_8 + 3$</td>
</tr>
<tr>
<td>9</td>
<td>$B$</td>
<td>$M_2^b$</td>
<td>$\omega_9 f_9 + f_9$</td>
<td>$E(\omega_9) = 1$</td>
<td>$32\sigma_9^2 f_9 + 5$</td>
</tr>
<tr>
<td>10</td>
<td>$M_1^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{10} f_{10} + f_{10}$</td>
<td>$E(\omega_{10}) = 1$</td>
<td>$2\sigma_{10}^2 f_{10} + 2$</td>
</tr>
<tr>
<td>11</td>
<td>$M_2^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{11} f_{11} + f_{11}$</td>
<td>$E(\omega_{11}) = 1$</td>
<td>$2\sigma_{11}^2 f_{11} + 2$</td>
</tr>
<tr>
<td>12</td>
<td>$D_{13}^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{12} f_{12} + f_{12}$</td>
<td>$E(\omega_{12}) = 1$</td>
<td>$2\sigma_{12}^2 f_{12} + 2$</td>
</tr>
<tr>
<td>13</td>
<td>$D_{12}^b$</td>
<td>$R_1^b$</td>
<td>$\omega_{13} f_{13} + f_{13}$</td>
<td>$E(\omega_{13}) = 1$</td>
<td>$2\sigma_{13}^2 f_{13} + 2$</td>
</tr>
<tr>
<td>14</td>
<td>$D_{13}^b$</td>
<td>$R_2^b$</td>
<td>$\omega_{14} f_{14} + f_{14}$</td>
<td>$E(\omega_{14}) = 1$</td>
<td>$2\sigma_{14}^2 f_{14} + 2$</td>
</tr>
<tr>
<td>15</td>
<td>$M_1^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{15} f_{15} + f_{15}$</td>
<td>$E(\omega_{15}) = 1$</td>
<td>$2\sigma_{15}^2 f_{15} + 2$</td>
</tr>
<tr>
<td>16</td>
<td>$M_2^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{16} f_{16} + f_{16}$</td>
<td>$E(\omega_{16}) = 1$</td>
<td>$2\sigma_{16}^2 f_{16} + 2$</td>
</tr>
<tr>
<td>17</td>
<td>$M_1^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{17} f_{17} + f_{17}$</td>
<td>$E(\omega_{17}) = 1$</td>
<td>$2\sigma_{17}^2 f_{17} + 2$</td>
</tr>
<tr>
<td>18</td>
<td>$M_2^b$</td>
<td>$D_{13}^b$</td>
<td>$\omega_{18} f_{18} + f_{18}$</td>
<td>$E(\omega_{18}) = 1$</td>
<td>$2\sigma_{18}^2 f_{18} + 2$</td>
</tr>
<tr>
<td>19</td>
<td>$D_{12}^b$</td>
<td>$R_1^b$</td>
<td>$\omega_{19} f_{19} + f_{19}$</td>
<td>$E(\omega_{19}) = 1$</td>
<td>$2\sigma_{19}^2 f_{19} + 2$</td>
</tr>
<tr>
<td>20</td>
<td>$D_{12}^b$</td>
<td>$R_2^b$</td>
<td>$\omega_{20} f_{20} + f_{20}$</td>
<td>$E(\omega_{20}) = 1$</td>
<td>$2\sigma_{20}^2 f_{20} + 2$</td>
</tr>
<tr>
<td>21</td>
<td>$D_{12}^b$</td>
<td>$R_2^b$</td>
<td>$\omega_{21} f_{21} + f_{21}$</td>
<td>$E(\omega_{21}) = 1$</td>
<td>$2\sigma_{21}^2 f_{21} + 2$</td>
</tr>
<tr>
<td>22</td>
<td>$D_{12}^b$</td>
<td>$R_2^b$</td>
<td>$\omega_{22} f_{22} + f_{22}$</td>
<td>$E(\omega_{22}) = 1$</td>
<td>$2\sigma_{22}^2 f_{22} + 2$</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
<td>A</td>
<td>--</td>
<td>--</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>B</td>
<td>--</td>
<td>--</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 5: The Expected Total Costs of the Pre-Merger and the Post-Merger Networks

Figure 6: The Absolute Risks of the Pre-Merger and the Post-Merger Networks
Table 1 defines the links on the networks, and the link cost functions and the marginal generalized link cost functions associated with the various supply chain activities of manufacturing, shipping/distribution, and storage. In particular, the marginal generalized link cost function in Table 1 is the derivative of the objective function (17) with respect to each link.
flow. In our examples, since the random factors are independent and the risk aversion factor $\alpha_i$ is equal to 1, the marginal generalized function of link $a$ is equal to $2\hat{h}_a^2 \sigma^2 f_a + E(\omega_a)\hat{h}_a + h_a$. In the first set of examples, since we assumed that the total cost and the total risk of the merger process are negligible, the total cost of the merger links (emanating from node 0) are assumed to be zero. The capacities on all the links in all the examples were set to: $u_a = 40, \forall a \in L_1$. The demands at the retailers were: $d_{R_1} = 10, d_{R_2} = 10$, and $d_{R_1^a} = 10, d_{R_2^a} = 10$.

Note that in Table 1 each firm has two manufacturing facilities with different cost factors (refer to links 1, 2 and to links 8, 9). Such cost structure can reflect the case where each firm has an offshore production facility with a lower cost factor and a domestic production plant with a higher cost factor. In the simulation examples, we vary $\sigma^2$ from 0.01 to 0.1 to show how the costs and the risks of the pre-merger and post-merger networks change as the uncertainty increases.

The results of the examples in this set are given in Figures 5, 6, 7, and 8.

Figure 5 shows that as the variance of the link cost uncertainty factors, $\sigma^2$, increases, the expected total costs of both the pre-merger and the post-merger networks will increase. In addition, the total cost of the post-merger network is consistently lower than that of the pre-merger network.

Figure 6, in turn, shows that as the variance of the link cost uncertainty factors, $\sigma^2$, increases, the total absolute risks of both networks represented by the variances of total costs both increase. In addition, the total risk of the post-merger network is always lower than that of the pre-merger network. Moreover, Figure 6 also shows that the total risk of the post-merger network increases less quickly than that of the pre-merger network, which makes the gap between the total risks of the two networks become larger as the link cost variance increases.

Figure 7 exhibits the trend of the relative risks of the two networks where the relative risks are represented by the volatilities (coefficient of variation) of the expected total costs. Figure 7 shows that as the variance of the link cost uncertainty factors, $\sigma^2$, increases the total cost volatilities of both networks increase. We can also observe that the relative risk of the post-merger network is always lower than that of the pre-merger network. Moreover, the relative risk of the post-merger network increases less quickly than that of the pre-merger network which
makes the gap between the relative risks of the two networks wider as the link cost uncertainty, $\sigma^2$, increases.

Finally, Figure 8 summarizes the three measures discussed in Section 3. First, we can see that, in this example set, all the three measures are always positive which indicates that the merger of the two networks reduces both the expected total cost and the total risk when the cost and the risk of merger links are negligible. In addition, the value of the expected total cost synergy, $S_{TC}$, is relatively low, and is below 5% while the values of the two risk reduction synergy measures, $S_{TR}$ and $S_{VC}$, are both consistently higher than 30%. Finally, we can observe that as the variance of the link cost uncertainty factors, $\sigma^2$, increases, the values of the two risk reduction synergy measures also increase while the value of the expected total cost synergy slightly decreases.

**Numerical Example Set 2**

In the second set of examples, we used the same parameters as in Set 1 except that we now assumed that the costs and risks of the merger links are not negligible. In particular, we assumed that the total cost functions of the two merger links are as follows:

\[
\hat{c}_{23}(f_{23}, \omega_{23}) = \omega_{23}f_{23}, \quad (27)
\]

\[
\hat{c}_{24}(f_{24}, \omega_{24}) = \omega_{24}f_{24}, \quad (28)
\]

where $E(\omega_{23}) = 1$, $E(\omega_{24}) = 1$, and the variance of $\omega_{23}$ and $\omega_{24}$ are equal to $\hat{\sigma}^2$. We varied $\hat{\sigma}^2$ from 0.0 to 0.8 to show how the three measures change as the risk incurred in the merger process increases. We now assumed that the variance of the uncertainty factors associated with the other links, $\hat{\sigma}^2$, is fixed and is equal to 0.1.
Figure 9: The Three Synergy Measures for Set 2

Figure 9 shows the values of the three synergy measures as the variances of the cost uncertainty factors of the merger links, $\hat{\sigma}^2$, increase. We can see that the expected total cost synergy, $S_{TC}$, is negative, which indicates that the merger of the two supply chain networks will increase the expected total cost. This is due to the fact that the cost incurred in the merger process offsets the potential savings through network integration.

Figure 9 also shows that the absolute risk synergy, $S_{TR}$, decreases as $\hat{\sigma}^2$ increases, and becomes negative when $\hat{\sigma}^2 > V^*$. This trend indicates that the reduction of absolute risk diminishes when the risk of the merger process increases, and the absolute risk actually starts to increase when $\hat{\sigma}^2 > V^*$. Moreover, we can see that the third measure, $S_{CV}$, remains positive in Figure 9, which indicates that now the relative risk or the total cost volatility is always reduced by this supply chain network merger. However, Figure 9 also shows that as $\hat{\sigma}^2$ increases, $S_{CV}$ decreases and approaches zero. This trend implies that the reduction in the relative risk or the total cost volatility becomes smaller as the risk of the merger process becomes larger.

It is interesting that in this second set of examples the three synergy measures evaluate different aspects of potential gains through the merger. Synergy measure 1 shows that the merger of the two networks does not reduce the expected total cost. However, the merger can still be beneficial to the firms' stakeholders since the total risk may be reduced through the merger.
Moreover, if $\hat{\sigma}^2 < \nu^*$, both the absolute risk and the relative risk are reduced, and if $\hat{\sigma}^2 > \nu^*$, only the relative risk is reduced while the absolute risk will increase in the post-merger network. Therefore, if the decision-maker's only concern is cost synergy this merger may not make sense. Nevertheless, if the decision-maker also cares about risk he or she will need to carefully compare different risk measures in order to correctly evaluate the potential risk reduction through the merger.

5. Managerial Insights and Conclusions

This paper focused on the potential cost synergy and risk reduction achievable through mergers/acquisitions via supply chain network integration. In particular, we developed a variational inequality modeling framework that considers the costs and the risks associated not only with the production, transportation, and storage activities in supply chain networks, but also with the merger/acquisition itself. The framework allows one to estimate the expected total cost and the total risk of the supply chain networks before and after the merger. In addition, we provided three synergy measures that can assist decision-makers in the evaluation of potential gains of M&As from different perspectives.

We then presented two sets of numerical examples to demonstrate how our modeling framework can be used to determine strategic advantages that are achievable. In particular, the first set of examples demonstrated the impact of the uncertainty of link costs on the merger gains in terms of cost synergy and risk reduction. The second set of examples, in turn, showed that decision-makers with different concerns should focus on distinct synergy measures which might not lead to the same conclusion regarding the benefits of the merger.

Our results provide interesting managerial insights for executives who are faced with M&A decisions. Our first set of examples showed that if the expected total costs and the risks of the merger are negligible, both the total cost and the total risk would be reduced through the merger. In addition, the risk reduction achieved through the merger was more prominent when the uncertainty of link costs was higher.

Our second set of examples showed that the cost and the risk of merger could have a significant impact on the total cost and the total risk of the post-merger firm, and should be carefully evaluated. Our examples also demonstrated that whether a merger makes sense economically may depend on the priority concerns of the decision-makers, and on the measures
used to evaluate the gains. For instance, a merger that could not lower the expected total cost might still be able to reduce the total risk, and, hence, be considered beneficial to the firms' stakeholders.

This research can be extended in several directions. First, it would be interesting to conduct empirical study in order to compare our model’s analyses with that of the insiders’ projections (see, e.g., Houston et al. (2001) and Bernile and Lyandres (2011)). Secondly, the model can further incorporate firms’ financial structures to study how supply chain integrations in M&As affect the values of the firms’ shareholders, debt holders, customers, and suppliers.

Acknowledgments: The authors are grateful for the helpful comments and suggestions provided during the review of our original paper.

References


The Economist, (2009), The return of the deal, September 10.


