A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case

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Abstract:

In this paper, we present a new theoretical framework for the quantification of strategic advantages associated with horizontal mergers through the integration of supply chain networks. The framework is a system-optimization one in which each firm is represented as a network of economic activities associated with manufacturing, distribution, and storage and with explicit capacities imposed on the links. We present the models pre- and post- the horizontal mergers and define a measure for the quantification of the gains, if any, associated with the mergers. We illustrate the framework with several numerical examples. For simple classes of mergers we derive formulae for the strategic advantages.

Keywords: System-optimization, supply chains, network acquisitions, horizontal mergers, supply chain performance metrics, strategic advantage, synergy, transportation and logistics

1. Introduction

In this paper, we explore the strategic advantages of supply chain network integration using a system-optimization perspective. The motivation for this research is the growing interest in the construction of measures for quantifying the impacts on supply chains when firms merge. We utilize a system-optimization perspective since it is grounded in transportation science and since it enables one to graphically represent and to formulate and compute the strategic advantages associated with a variety of mergers. In this paper, we focus on horizontal mergers and we assume that the firms that merge are in the same industry. Recent examples of mergers include the merger of Kmart and Sears in the retail industry (see Knowledge@Wharton (2005)) and the merger of Molson and Coors in the beverage industry (cf. Beverage World, 2007) with Molson Coors expected to exceed its merger synergy goal of $175 million in annual savings by the end of 2007.

As noted by Langabeer (2003) the use of mergers and acquisitions (M&As) continues to grow "exponentially" with over 6,000 M&A transactions conducted world-wide in 2001, with a value of over a trillion dollars. Nevertheless, many scholars argue whether or not mergers achieve their objectives. For example, Marks and Mirvis (2001) found that fewer than 25% of all mergers achieve their stated objectives. Langabeer and Seifert (2003) determined a direct correlation between how effectively supply chains of merged firms are integrated and how successful the merger is. Furthermore, they state, based on the empirical findings in Langabeer (2003), which analyzed hundreds of mergers over the past decade, that improving supply chain integration between merging companies is the key to improving the likelihood of post-merger success.

In this paper, we envision each firm as a network of economic activities consisting of manufacturing, which is conducted at the firm's plants or manufacturing facilities; distribution, which occurs between the manufacturing plants and the distribution centers, which also store the product produced by each firm, and the ultimate distribution of the product to the retailers. Associated with each such economic activity is a link in the network with a total associated cost that depends on the flow of the product on the link. The links, be they manufacturing, shipment, or storage links have capacities on the flows. We assume, as given, the demand for the product at each retailer.

Some of the history of the evolution of network models for firms and numerous network-based economic models can be found in the book by Nagurney (1993). More recently, a variety of supply chain network equilibrium models, initiated by Nagurney, Dong, and Zhang (2002) have been developed, which focus on competition among decision-makers (such as manufacturers, distributors, and retailers) at a tier of the supply chain but cooperation between tiers. The relationships of such supply chain network equilibrium problems to transportation network equilibrium problems, which are characterized by user-optimizing behavior have also been established (cf. Nagurney, 2006a). Zhang, Dong, and Nagurney (2003) and Zhang (2006), in turn, modeled competition among supply chains in a supply chain economy context. See the book by Nagurney (2006b) for a spectrum of supply chain network equilibrium models and applications.

Here, in contrast, we provide a *system-optimization* perspective for supply chains. It is worth mentioning that the term system-optimization in transportation science was coined by Dafermos and Sparrow (1969) to correspond to Wardrop's second principle of travel behavior with user-optimization corresponding to his first principle (cf. Wardrop, 1952). A system-optimization perspective for supply chains, as we demonstrate, enables the modeling of the economic activities associated with a firm as a network, and, hence, the evaluation of the strategic advantages, often referred to as *synergy*, due to mergers (or acquisitions), in a network format. However, unlike a classical system-optimization formulation, we explicitly consider capacities on the links of the networks. As noted by Soylu et al. (2006) more and more companies now realize the strategic importance of controlling the supply chain as a whole (see also, Brown et al., 2001, for a specific corporate example). Furthermore, Min and Zhou (2002) emphasized the need to analyze the synergy obtained through both inter-functional and inter-organizational integration. Hakkinen et al. (2004) further described the integration of logistics after M&As with a review of the literature and concluded that operational issues, in general, and logistics issues, in particular, have received insufficient attention; see also Herd et al. (2005).

In particular, synergy in supply chains has been considered based on mixed integer linear programming models by Soylu et al. (2006), who focused on energy systems, and by Xu (2007), who developed multiperiod supply chain planning models with an emphasis on the distribution aspects, and investigated market regions, distribution configurations, as well as product characteristics and planning horizons. Juga (1996) earlier considered network synergy in logistics with a specific case study but did not present any mathematical models. Nijkamp and Reggiani (1998), in turn, investigated network synergies with links as productive factors. Gupta and Gerchak (2002) presented a model to capture operational efficiency and focused on production efficiency associated with mergers and acquisitions whereas Alptekinoglu and Tang (2005), subsequently, focused on distribution efficiency associated with mergers and acquisitions. The latter authors also established that their model was a convex nonlinear programming problem.

This paper is organized as follows. In Section 2, we develop the system-optimization problems, which are nonlinear programming problems with network structure, faced by the two firms whose merger we wish to evaluate. We present both the pre-merger optimization problem, which we refer to as the baseline case, or Case 0, and the system-optimization problems associated with three distinct horizontal mergers:

Case 1: the firms merge and retailers associated with either original firm can now get the product from any manufacturing plant but still use their original distribution centers;

Case 2: the firms merge and the retailers can obtain the product from any distribution center but the manufacturers deal with their original distribution centers, and

Case 3: the firms merge and the retailers can obtain the product produced at any of the manufacturing facilities and distributed by any of the distribution centers.

We represent the underlying associated networks before and after the mergers and demonstrate that the solution of all the system-optimization problems can be obtained by solving a variational inequality problem, with a structure that can be easily exploited for computational purposes. Notably, our framework incorporates manufacturing/production activities, as well as distribution and storage activities both prior to and post the merger (or acquisition).

In Section 3, we present a measure of strategic advantage, which allows one to evaluate the gains, if any, associated with the above-described horizontal mergers. In Section 4, we provide a spectrum of examples, in which we compute the strategic advantage associated with mergers in the case of many different scenarios. In Section 5, we summarize the results in this paper and present our conclusions.

2. The Pre- and Post-Horizontal Mergers Supply Chain Network Models

In this Section, we describe the supply chain network models prior and post the horizontal mergers. We consider two firms, denoted by Firm A and Firm B, who are integrated post the merger. We assume that each firm produces the same homogeneous product since the focus here is on horizontal mergers in the same industry. The explicit horizontal mergers that we model and evaluate are:

Case 1: Firms A and B merge and share manufacturing plants only;

Case 2: Firms A and B merge and share distributions centers only;

Case 3: Firms A and B merge and share manufacturing plants and distribution centers.

As mentioned in the Introduction, the formalism that we utilize is that of *system-optimization*, where each of the firms is assumed to own its manufacturing facilities/plants, and its distribution centers, and each firm seeks to determine the optimal production of the product at each of its manufacturing plants and the optimal quantities shipped to the distribution centers, where the product is stored, and, finally, shipped to the retail outlets. We assume that each firm seeks to minimize the total costs associated with the production, storage, and distribution activities, subject to the demand being satisfied at the retail outlets.

In Section 2.1, we formulate the pre-merger system-optimization problem associated with each of the firms individually and together prior to the merger and we consider this as the baseline case, Case 0. In Section 2.2, we formulate the three post-merger models, corresponding to Case 1, Case 2, and Case 3, respectively.

2.1 The Pre-Merger Supply Chain Network Model(s)

We formulate the optimization problem faced by Firm A and Firm B as follows. We assume that each firm is represented as a network of its economic activities, as depicted in Figure 1. Each firm i; $i = A, B$, has n_M^i manufacturing facilities/plants; n_D^i distribution centers, and serves n_R^i retail outlets. We let $G_i = [N_i, L_i]$ for $i = A, B$ denote the graph

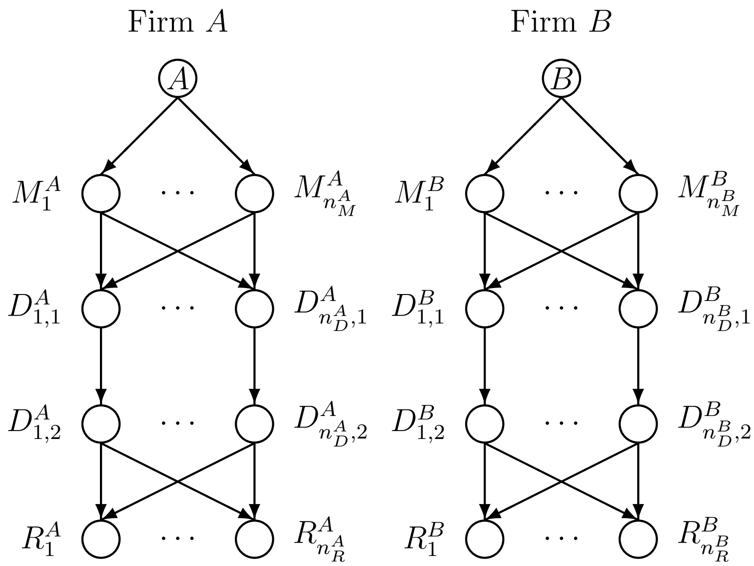

Figure 1: Case 0: Firms A and B Prior to Horizontal Merger

consisting of nodes and directed links representing the economic activities associated with each firm i. We also let $G^0 = [N^0, L^0] \equiv \cup_{i=A,B}[N_i, L_i]$. The links from the top-tiered nodes i; $i = A, B$ in each network in Figure 1 are connected to the manufacturing nodes of the respective firm i, which are denoted, respectively, by: $M_1^i, \ldots, M_{n_M^i}^i$, and these links represent the manufacturing links.

The links from the manufacturing nodes, in turn, are connected to the distribution center nodes of each firm i; $i = A, B$, which are denoted by $D_{1,1}^i, \ldots, D_{n_D{}^i,1}^i$. These links correspond to the shipment links between the manufacturing plants and the distribution centers where the product is stored. The links joining nodes $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$ with nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = A, B$ correspond to the storage links. Finally, there are shipment links joining the nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = A, B$ with the retail outlet nodes: $R_1^i, \ldots, R_{n_R^i}^i$ for each firm $i = A, B$. Note that each firm i has its individual retail outlets where it sells the product, as depicted in Figure 1.

We assume that associated with each link (cf. Figure 1) of the network corresponding to each firm i; $i = A, B$ is a total cost. We denote, without any loss in generality, the links by

a, b, etc., and the total cost on a link a by $\hat{c}_a$. The demands for the product are assumed as given and are associated with each firm and retailer pair. Let $d_{R^i_k}$ denote the demand for the product at retailer R^i_k associated with firm i; $i = A, B$; $k = 1, \ldots, n^i_R$. Let x_p denote the nonnegative flow of the product on path p joining (origin) node i with a (destination) retailer node of firm i; $i = A, B$. Then the following conservation of flow equations must hold for each firm i:

$$\sum_{p \in P^0_{R^i_k}} x_p = d_{R^i_k}, \quad i = A, B; k = 1, \ldots, n^i_R, \tag{1}$$

where $P^0_{R^i_k}$ denotes the set of paths connecting (origin) node i with (destination) retail node R^i_k.

In addition, we let f_a denote the flow of the product on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^0} x_p \delta_{ap}, \quad \forall p \in P^0, \tag{2}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise. Here P^0 denotes the set of *all* paths in Figure 1, that is, $P^0 = \cup_{i=A,B;k=1,\ldots,n^i_R} P^0_{R^i_k}$. Obviously, since here we consider the two firms prior to any merger the paths associated with a given firm have no links in common with paths of the other firm. This changes when the horizontal mergers occur, in which case the number of paths and the sets of paths also change, as do the number of links and the sets of links, as we demonstrate in Section 2.2.

Of course, we also have that the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^0. \tag{3}$$

The total cost on a link, be it a manufacturing/production link, a shipment link, or a storage link is assumed to be a function of the flow of the product on the link; see, for example, Nagurney (2006b) and the references therein.

Hence, we may write that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L^0. \tag{4}$$

We assume that the total cost on each link is convex, is continuously differentiable, and has a bounded second order partial derivative. We assume the same for all links that are added post mergers, as well. Such conditions will guarantee convergence of the proposed algorithm.

Furthermore, we assume that there are nonnegative capacities on the links with the capacity on link a denoted by u_a, $\forall a$. This is very reasonable since the manufacturing plants, the shipment links, as well as the distribution centers, which serve also as the storage facilities can be expected to have capacities, in practice.

The total cost associated with the economic activities of both firms prior to the merger is minimized when the following system-optimization problem is solved:

$$\text{Minimize} \quad \sum_{a \in L^0} \hat{c}_a(f_a) \tag{5}$$

subject to: constraints (1) – (3) and

$$f_a \leq u_a, \quad \forall a \in L^0. \tag{6}$$

Clearly, the solution of the above optimization problem will minimize the total costs associated with each firm individually as well as both firms together since they are prior to the merger independent and share no manufacturing facilities or distribution facilities or retail outlets. Observe that this problem is, as is well-known in the transportation literature (cf. Beckmann, McGuire, and Winsten, 1956; Dafermos and Sparrow, 1969), a *system-optimization* problem but in *capacitated* form; see also Patriksson (1994) and Nagurney (2000) and the references therein. Under the above imposed assumptions, the optimization problem is a convex optimization problem. If we further assume that the feasible set underlying the problem represented by the constraints (1) – (3) and (6) is non-empty, then it follows from the standard theory of nonlinear programming (cf. Bazaraa, Sherali, and Shetty, 1993) that the optimal solution, denoted by $f^* \equiv \{f_a^*\}, a \in L^0$, exists.

We let $\mathcal{K}^0$ denote the set where $\mathcal{K}^0 \equiv \{f | \exists x \geq 0, \text{ and } (1) - (3) \text{ and } (6) \text{ hold}\}$, where f is the vector of link flows and x the vector of path flows.

Also, we associate the Lagrange multiplier β_a with constraint (6) for link a and we denote the associated optimal Lagrange multiplier by β_a^*. This term may also be interpreted as the price or value of an additional unit of capacity on link a.

We now state the following result in which we provide a variational inequality formulation of the problem.

Theorem 1

The vector of link flows $f^* \in \mathcal{K}^0$ is an optimal solution to problem (5), subject to (1) through (3) and (6), if and only if it satisfies the following variational inequality problem with the vector of nonnegative Lagrange multipliers β^*:

$$\sum_{a \in L^0} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \beta_a^* \right] \times [f_a - f_a^*] + \sum_{a \in L^0} [u_a - f_a^*] \times [\beta_a - \beta_a^*] \geq 0, \quad \forall f \in \mathcal{K}^0, \quad \forall \beta_a \geq 0, \forall a \in L^0. \tag{7}$$

Proof: See Bertsekas and Tsitsiklis (1989).

The above variational inequality problem is of the form considered by Bertsekas and Tsitsiklis (1989) and others (see, e.g., Nagurney and Dong, 2002) and variational inequality (7) can be easily solved using the modified projection method (also sometimes referred to as the extragradient method). The elegance of this computational procedure in the context of variational inequality (7) lies in that it allows one to utilize algorithms for the solution of the *uncapacitated* system-optimization problem (for which numerous algorithms exist in the transportation science literature) with a straightforward update procedure at each iteration to obtain the Lagrange multipliers. To solve the former problem we will utilize in Section 4 the well-known equilibration algorithm (system-optimization version) of Dafermos and Sparrow (1969), which has been widely applied (see also, e.g., Nagurney, 1993, 2000). Indeed, we will see that the variational inequalities governing the supply chain networks post-mergers will also be of the form (7) and, hence, also amenable to solution via the modified projection method (cf. Korpelevich, 1977; Nagurney, 1993). Recall that the modified projection method is guaranteed to converge to a solution of a variational inequality problem, provided that the function that enters the variational inequality problem is monotone and Lipschitz continuous (see, e.g., Nagurney, 1993) and that a solution exists.

Once we have solved problem (7) we have the solution f^* which minimizes the total cost (cf. (5)) in the supply chain networks associated with the two firms. We denote this total cost given by $\sum_{a \in L^0} \hat{c}_a(f_a^*)$ as TC^0 and we use this total cost value as a baseline from which to compute the strategic advantage, discussed in Section 3, associated with horizontal mergers that we describe below.

2.2 The Horizontal Merger Supply Chain Network Models

In this Section, we consider three post-merger cases. In Case 1, the firms merge and retailers associated with either original firm can now get the product from any manufacturing plant but still use their original distribution centers. In Case 2, the firms merge and the retailers can obtain the product from any distribution center but the manufacturers deal with their original distribution centers, and in Case 3, the firms merge and the retailers can obtain the product produced at any of the manufacturing facilities and distributed by any of the distribution centers. Case 1 is depicted graphically in Figure 2, whereas Case 2 is depicted in Figure 3, and Case 3 in Figure 4.

Section 2.2.1: Case 1

In Case 1, we add to the network G^0 depicted in Figure 1 a supersource node 0 and links joining node 0 to nodes $i = A, B$ to reflect the merger of the two firms. We also add new links joining each manufacturing node of each firm with the distribution center nodes of the other firm, as depicted in Figure 2. We denote the new network topology in Figure 2 by $\mathrm{G}^1 = [N^1, L^1]$ where $N^1 = N^0 \cup \text{node}\, 0$ and $L^1 = L^0 \cup \text{the additional links}$.

We assume, for simplicity, that the total costs associated with merging are negligible (since the focus here is on the operational aspects associated with production, storage, and transportation/logistics) and, therefore, the total costs associated with the links emanating from node 0 are equal to zero. Of course, if one wishes to include the costs associated with merging the two firms in this manner then one can easily include total cost functions of the form given by (4) for these links.

In addition, we assume that the new links emanating from the manufacturing nodes to the distribution center nodes have associated total cost functions of the form given by (4).

Let x_p denote the flow of the product on path p joining (origin) node 0 with a (destination)

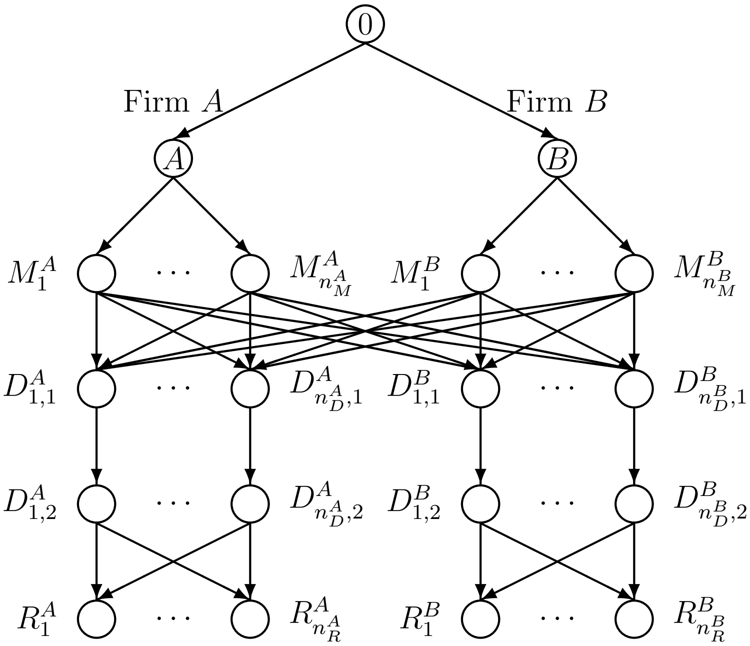

Figure 2: Case 1: Firms A and B Merge: Retailers Associated with Either Firm A or Firm B Can Now Get the Product Produced at Any Manufacturing Plant but Each Retailer is Supplied by Each Firm's Original Distribution Centers

retailer node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^1_{R^i_k}} x_p = d_{R^i_k}, \quad i = A, B; k = 1, \ldots, n^i_R, \tag{8}$$

where $P^1_{R^i_k}$ denotes the set of paths connecting node 0 with retail node R^i_k. Due to the merger, the retail outlets can obtain the product from any manufacturer. The set $P^1 \equiv \cup_{i=A,B;k=1,\ldots,n^i_R} P^1_{R^i_k}$.

In addition, as before, we let f_a denote the flow of the product on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^1} x_p \delta_{ap}, \quad \forall p \in P^1. \tag{9}$$

Of course, we also have that the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^1. \tag{10}$$

The optimization problem associated with this horizontal merger which minimizes the total cost subject to the demand for the product being satisfied at the retailers is given by:

$$\text{Minimize} \quad \sum_{a \in L^1} \hat{c}_a(f_a) \tag{11}$$

subject to: constraints (8) – (10) and

$$f_a \leq u_a, \quad \forall a \in L^1. \tag{12}$$

Clearly, the solution to this problem can also be obtained as a solution to a variational inequality problem akin to (7) where now $a \in L^1$, and the vectors: f, f^*, x, and β have identical definitions as before, but are re-dimensioned accordingly. Finally, the set $\mathcal{K}^0$ is replaced by $\mathcal{K}^1 \equiv \{f | \exists x \geq 0, \text{ and } (8){-}(10) \text{ and } (12) \text{ hold}\}$. Hence, one can apply the modified projection problem to compute the solution to the variational inequality problem governing Case 1, as well. The optimal solution for Case 1 has an associated total cost given by $\sum_{a \in L^1} \hat{c}_a(f_a^*)$ which we denote by TC^1.

Section 2.2.2: Case 2

We now formulate the merger associated with Case 2 in which firms A and B merge and the retailers can obtain the product from any distribution center but the manufacturers deal with their original distribution centers. Figure 3 depicts the network topology associated with this type of horizontal merger. Specifically, to network G^0 depicted in Figure 1, we add a super source node 0 and links joining node 0 with nodes $i = A, B$ to reflect the merger (as we did for Case 1). Also, we add links connecting the distribution centers of each firm to the (other) retailers. We refer to the network underlying this merger as $G^2 = [N^2, L^2]$ where $N^2 = N^1$. We associate with the new shipment links total cost functions as in (4).

We now define the feasible set $\mathcal{K}^2$ underlying the Case 2 horizontal merger problem.

Let x_p, again, denote the flow of the product on path p joining (origin) node 0 with a (destination) retailer node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^2_{R^i_k}} x_p = d_{R^i_k}, \quad i = A, B; k = 1, \ldots, n^i_R, \tag{13}$$

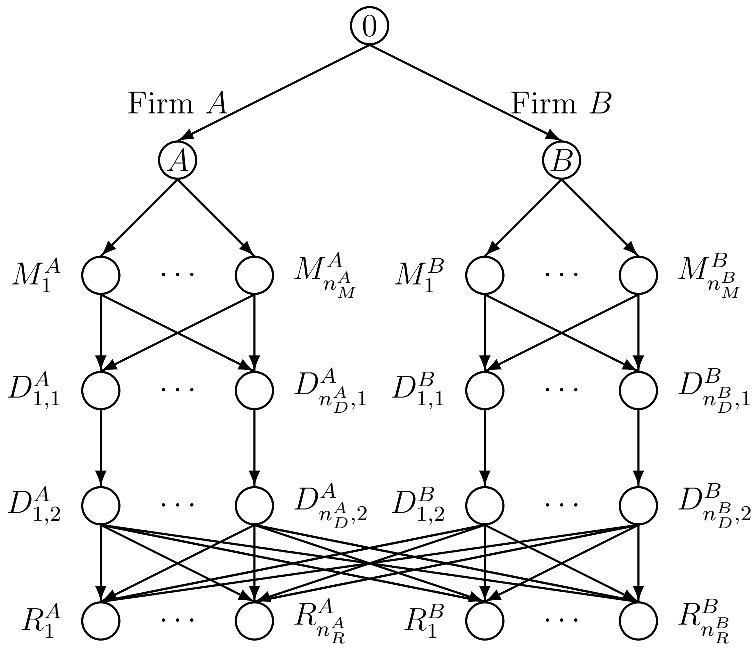

Figure 3: Case 2: Firms A and B Merge: Retailers Associated with Either Firm A or Firm B Can Now Get the Product from Any Distributor but Each Manufacturer of Each Original Firm Deals with its Original Distributor

where $P^2_{R_k^i}$ denotes the set of paths connecting node 0 with retail node R_k^i in Figure 3. Due to the merger, the retail outlets can obtain the product from any manufacturer and any distributor. The set $P^2 \equiv \cup_{i=A,B;k=1,\ldots,n_R^i} P^2_{R_k^i}$.

In addition, as before, we let f_a denote the flow of the product on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^2} x_p \delta_{ap}, \quad \forall p \in P^2. \tag{14}$$

Of course, we also have that the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^2. \tag{15}$$

The optimization problem associated with this horizontal merger which minimizes the total cost subject to the demand for the product being satisfied at the retailers is given by:

$$\text{Minimize} \quad \sum_{a \in L^2} \hat{c}_a(f_a) \tag{16}$$

subject to: constraints (13) – (15) and

$$f_a \leq u_a, \quad \forall a \in L^2. \tag{17}$$

Clearly, the solution to this problem can also be obtained as a solution to a variational inequality problem akin to (7) where now $a \in L^2$; where the vectors: f, f^*, x, and β have identical definitions as before, but are re-dimensioned/expanded accordingly. Finally, instead of $\mathcal{K}^1$ we now have $\mathcal{K}^2 \equiv \{f | \exists x \geq 0, \text{ and } (13) - (15) \text{ and } (17) \text{ hold}\}$. One can also apply the modified projection problem to compute the solution to the variational inequality problem governing Case 2. The total cost TC^2, which is the value of the objective function (16) at the optimal solution f^* is equal to $\sum_{a \in L^2} \hat{c}_a(f_a^*)$.

Section 2.2.3: Case 3

We now formulate the merger associated with Case 3 in which firms A and B merge and the retailers can obtain the product from any manufacturer and shipped from any distribution center. Figure 4 depicts the network topology associated with this type of horizontal merger. Specifically, we retain the nodes and links associated with network G^1 depicted in Figure 2 but now the additional links connecting the distribution centers of each firm to the retailers of the other are added. We refer to the network underlying this merger as $G^3 = [N^3, L^3]$ where $N^3 = N^1$. We associate with the new shipment links total cost functions as in (4).

Let x_p, again, denote the flow of the product on path p joining (origin) node 0 with a (destination) retailer node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^3_{R^i_k}} x_p = d_{R^i_k}, \quad i = A, B; k = 1, \ldots, n^i_R, \tag{18}$$

where $P^3_{R^i_k}$ denotes the set of paths connecting node 0 with retail node R^i_k in Figure 4. Due to the merger, the retail outlets can obtain the product from any manufacturer and any distributor. The set $P^3 \equiv \cup_{i=A,B;k=1,\ldots,n^i_R} P^3_{R^i_k}$.

In addition, as before, we let f_a denote the flow of the product on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^3} x_p \delta_{ap}, \quad \forall p \in P^3. \tag{19}$$

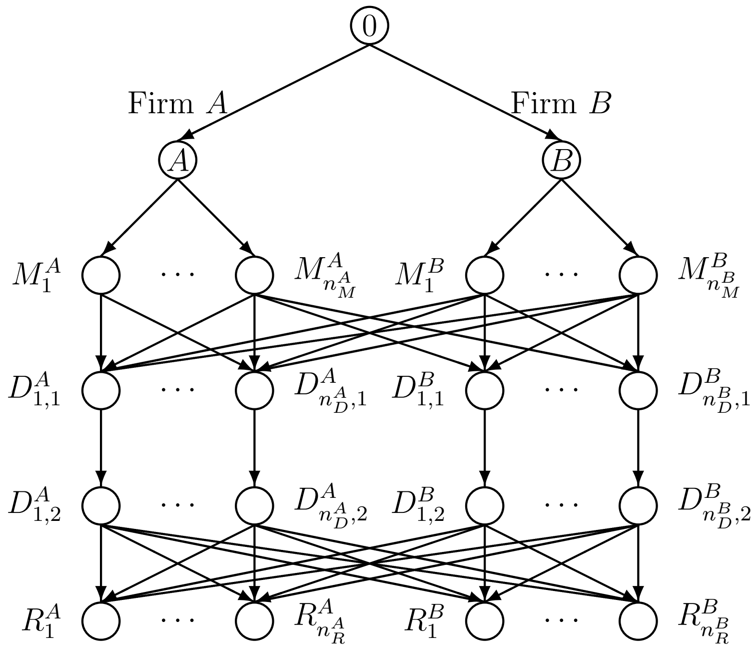

Figure 4: Case 3: Firms A and B Merge: Retailers Associated with Either Firm A or Firm B Can Now Get the Product Produced at Any Manufacturing Plant and Distributed and Stored by Any Distribution Center

Of course, we also have that the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^3. \tag{20}$$

The optimization problem associated with this horizontal merger which minimizes the total cost subject to the demand for the product being satisfied at the retailers is given by:

$$\text{Minimize} \quad \sum_{a \in L^3} \hat{c}_a(f_a) \tag{21}$$

subject to: constraints (18) – (20) and

$$f_a \leq u_a, \quad \forall a \in L^3. \tag{22}$$

Clearly, the solution to this problem can also be obtained as a solution to a variational inequality problem akin to (7) where now $a \in L^3$; where the vectors: f, f^*, x, and β have identical definitions as before, but are re-dimensioned/expanded accordingly. Finally, instead of $\mathcal{K}^1$ we now have $\mathcal{K}^3 \equiv \{f | \exists x \geq 0, \text{ and } (18) - (20) \text{ and } (22) \text{ hold}\}$. One can also apply the modified projection problem to compute the solution to the variational inequality problem governing Case 3. The total cost TC^3, which is the value of the objective function (21) at the optimal solution f^* is equal to $\sum_{a \in L^3} \hat{c}_a(f_a^*)$. In the next section, we discuss how we utilize the total costs: TC^0, TC^1, TC^2, and TC^3 to determine the strategic advantage (or synergy) associated with the respective horizontal mergers associated with Cases 1, 2, and 3.

Hence, we have a unified framework for the formulation of the system-optimization problems associated with the supply chain networks pre- and post- the merger of the two firms A and B.

3. Measuring the Strategic Advantage Associated with Horizontal Mergers

In this Section, we provide a measure for quantifying the strategic advantage associated with horizontal mergers.

The measure that we utilize to capture the gains, if any, associated with a horizontal merger Case i; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0} \right] \times 100\%, \tag{23}$$

where recall that TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i. Note that $\mathcal{S}^i$; $i = 1, 2, 3$ may also be interpreted as *synergy*. For example, Xu (2007) in the context of her MILP (mixed integer linear) programming models associated with the evaluation of mergers with a focus on distribution used a similar measure. Here, however, the total costs are based on the supply chain network models developed in Section 2. These models include manufacturing, distribution, as well as storage of the product that is produced by the two firms both pre- and post- the mergers.

In the case of simple, stylized examples one may be able to derive explicit formulae for $\mathcal{S}^i$. For example, if both firms A and B pre-merger have a single manufacturing plant, a single distribution center, and a single retailer, and identical demands at the retailers, given by d, and assuming that the total costs on each link $a \in L^0$ are given by: $\hat{c}_a = g f_a^2 + h f_a$, with $g > 0$ and $h > 0$, and the capacities on the links are not less than the demand d then it is straightforward to determine (cf. Figure 1) that: $TC^0 = 8[gd^2 + hd]$. Assume now that new links are added to construct Case 1, Case 2, and Case 3 accordingly, where we assume that the total costs on the new links are all identically equal to zero and their capacities are greater than or equal to the demand d. Then, since the addition of the new zero cost links creates new paths, and new S-O flow solutions, we obtain that $TC^1 = TC^2 = 6[gd^2 + hd]$ and $TC^3 = 4[gd^2 + hd]$. It follows that:

$$\mathcal{S}^1 = \mathcal{S}^2 = 25\%, \quad \mathcal{S}^3 = 50\%.$$

A slightly more general case would be as above but now we assume that the manufacturing link, denoted by $a \in L^0$ of either firm has an identical cost of the form: $g_a f_a^2 + h_a f_a$; the first shipment link b of either firm has a total cost of the form: $g_b f_b^2 + h_b f_b$; the storage link, denoted, for simplicity, by c, of either firm has a total cost of the form: $g_c f_c^2 + h_c f_c$, and, finally, the total cost associated with each bottom shipment link has a total cost given by: $g_d f_d^2 + h_d f_d$, where we assume that g_a, g_b, g_c, and $g_d > 0$ and h_a, h_b, h_c, and $h_d > 0$. Then, one can, also, easily derive the following total cost formulae from which the strategic advantages can then be determined according to (23), assuming that, as above, the total costs associated with the new, cross-linkage shipment links associated with the respective mergers are all identically equal to zero:

$$TC^0 = 2[g_a d^2 + h_a d] + 2[g_b d^2 + h_b d] + 2[g_c d^2 + h_c d] + 2[g_d d^2 + h_d d],$$

$$TC^1 = 2[g_a d^2 + h_a d] + 2[g_c d^2 + h_c d] + 2[g_d d^2 + h_d d],$$

$$TC^2 = 2[g_a d^2 + h_a d] + 2[g_b d^2 + h_b d] + 2[g_c d^2 + h_c d],$$

$$TC^3 = 2[g_a d^2 + h_a d] + 2[g_c d^2 + h_c d].$$

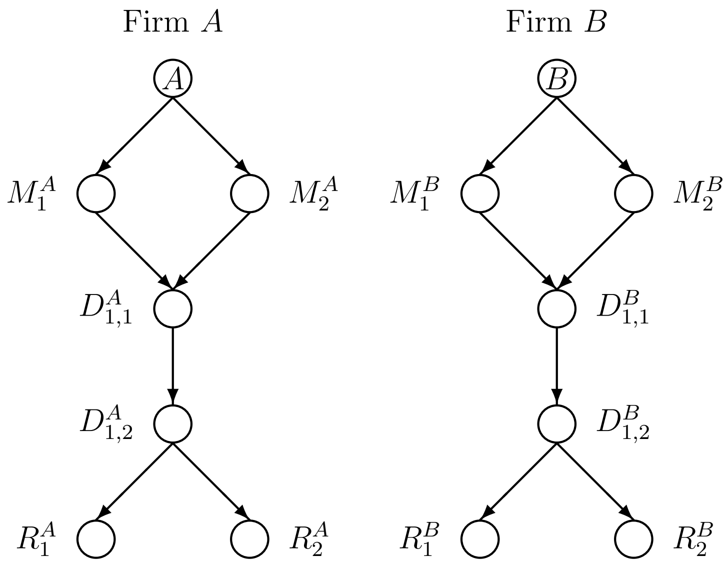

Figure 5: Case 0 Network Topology for the Numerical Examples

4. Numerical Examples

In this Section, we present five numerical examples for which we compute the strategic advantage measure as in (23) for the different Cases.

We consider Firm A and Firm B, each of which has two manufacturing plants: M_1^i and M_2^i; $i = A, B$. In addition, each firm has a single distribution center to which the product is shipped from the manufacturing plants and stored. Finally, once stored, the product is shipped to the two retailers associated with each firm and denoted by R_1^i and R_2^i for $i = A, B$. A graphical depiction of the supply chain networks associated with the two firms pre-merger and representing Case 0 is given in Figure 5. Figure 6 depicts the Case 1 horizontal merger; Figure 7 depicts the Case 2 horizontal merger, and Figure 8 depicts the Case 3 horizontal merger of these two firms.

We utilized the modified projection method, embedded with the equilibration algorithm, as discussed in Section 2, in order to compute the solutions to the problems. We implemented the algorithm in FORTAN and utilized a Unix system at the University of Massachusetts for the computations.

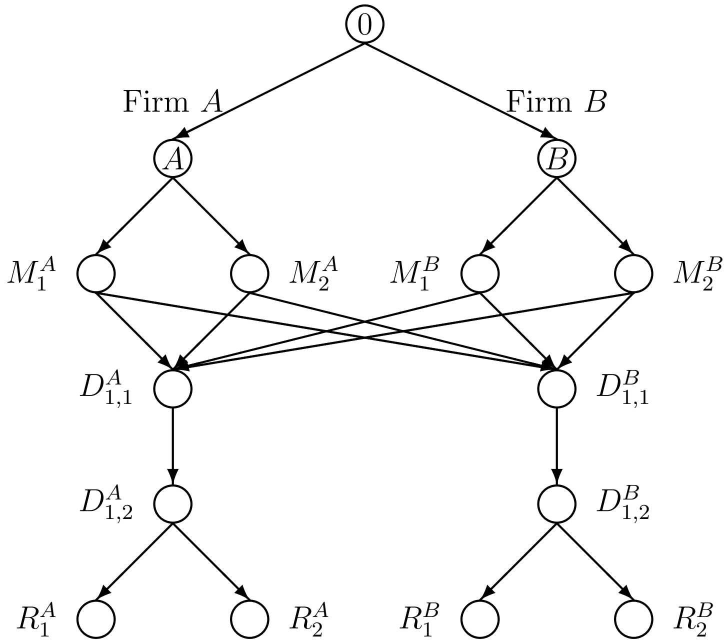

Figure 6: Case 1 Network Topology for the Numerical Examples

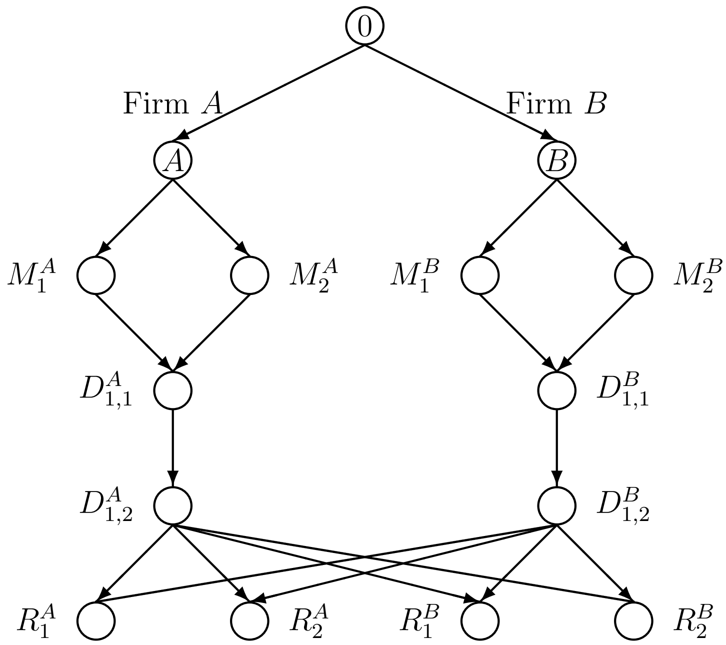

Figure 7: Case 2 Network Topology for the Numerical Examples

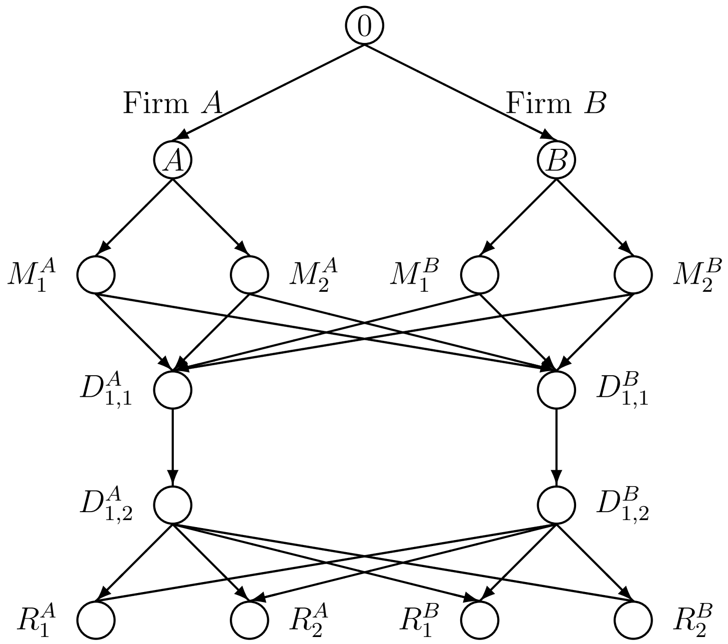

Figure 8: Case 3 Network Topology for the Numerical Examples

In Table 1, we define the links on the various networks, and the total link cost functions associated with the various supply chain activities of manufacturing, shipping/distribution, and storage. Since, as mentioned earlier, the merger links (emanating from node 0) are not assumed to have associated total costs, we do not introduce cost functions for those links. The capacities on all the links in all the examples (see (6), (12), (17), and (22)) were set to: $u_a = 15$ for all links a.

The demands at the retailers, except were noted, were: $d_{R_1^A} = 5$, $d_{R_2^A} = 5$, and $d_{R_1^B} = 5$, $d_{R_2^B} = 5$.

Below we provide additional details concerning the particular examples.

Table 1: Definition of Links and Associated Total Cost Functions for the Numerical Examples

Link a	From Node	To Node	Ex. 1: $\hat{c}_a(f_a)$	Ex. 2: $\hat{c}_a(f_a)$	Ex. 3,4: $\hat{c}_a(f_a)$	Ex. 5: $\hat{c}_a(f_a)$
1	A	M_1^A	$f_1^2+2f_1$	$f_1^2+2f_1$	$.5f_1^2+f_1$	$.5f_1^2+f_1$
2	A	M_2^A	$f_2^2+2f_2$	$f_2^2+2f_2$	$.5f_2^2+f_2$	$.5f_2^2+f_2$
3	M_1^A	$D_{1,1}^A$	$f_3^2+2f_3$	$f_3^2+2f_3$	$f_3^2+2f_3$	$f_3^2+2f_3$
4	M_2^A	$D_{1,1}^A$	$f_4^2+2f_4$	$f_4^2+2f_4$	$f_4^2+2f_4$	$f_4^2+2f_4$
5	$D_{1,1}^A$	$D_{1,2}^A$	$f_5^2+2f_5$	$.5f_5^2+f_5$	$.5f_5^2+f_5$	$.5f_5^2+f_5$
6	$D_{1,2}^A$	R_1^A	$f_6^2+2f_6$	$f_6^2+2f_6$	$f_6^2+2f_6$	$f_6^2+2f_6$
7	$D_{1,2}^A$	R_2^A	$f_7^2+2f_7$	$f_7^2+2f_7$	$f_7^2+2f_7$	$f_7^2+2f_7$
8	B	M_1^B	$f_8^2+2f_8$	$f_8^2+2f_8$	$f_8^2+2f_8$	$f_8^2+2f_8$
9	B	M_2^B	$f_9^2+2f_9$	$f_9^2+2f_9$	$f_9^2+2f_9$	$f_9^2+2f_9$
10	M_1^B	$D_{1,1}^B$	$f_{10}^2+2f_{10}$	$f_{10}^2+2f_{10}$	$f_{10}^2+2f_{10}$	$f_{10}^2+2f_{10}$
11	M_2^B	$D_{1,1}^B$	$f_{11}^2+2f_{11}$	$f_{11}^2+2f_{11}$	$f_{11}^2+2f_{11}$	$f_{11}^2+2f_{11}$
12	$D_{1,1}^B$	$D_{1,2}^B$	$f_{12}^2+2f_{12}$	$.5f_{12}^2+f_{12}$	$.5f_{11}^2+f_{11}$	$.5f_{11}^2+f_{11}$
13	$D_{1,2}^B$	R_1^B	$f_{13}^2+2f_{13}$	$f_{13}^2+2f_{13}$	$f_{13}^2+2f_{13}$	$f_{13}^2+2f_{13}$
14	$D_{1,2}^B$	R_2^B	$f_{14}^2+2f_{14}$	$f_{14}^2+2f_{14}$	$f_{14}^2+2f_{14}$	$f_{14}^2+2f_{14}$
15	M_1^A	$D_{1,1}^B$	$f_{15}^2+2f_{15}$	$f_{15}^2+2f_{15}$	$f_{15}^2+2f_{15}$	0.00
16	M_2^A	$D_{1,1}^B$	$f_{16}^2+2f_{16}$	$f_{16}^2+2f_{16}$	$f_{16}^2+2f_{16}$	0.00
17	M_1^B	$D_{1,1}^A$	$f_{17}^2+2f_{17}$	$f_{17}^2+2f_{17}$	$f_{17}^2+2f_{17}$	0.00
18	M_2^B	$D_{1,1}^A$	$f_{18}^2+2f_{18}$	$f_{18}^2+2f_{18}$	$f_{18}^2+2f_{18}$	0.00
19	$D_{1,2}^A$	R_1^B	$f_{19}^2+2f_{19}$	$f_{19}^2+2f_{19}$	$f_{19}^2+2f_{19}$	0.00
20	$D_{1,2}^A$	R_2^B	$f_{20}^2+2f_{20}$	$f_{20}^2+2f_{20}$	$f_{20}^2+2f_{20}$	0.00
21	$D_{1,2}^B$	R_1^A	$f_{21}^2+2f_{21}$	$f_{21}^2+2f_{21}$	$f_{21}^2+2f_{21}$	0.00
22	$D_{1,2}^B$	R_2^A	$f_{22}^2+2f_{22}$	$f_{22}^2+2f_{22}$	$f_2^2+2f_{22}$	0.00

Example 1

Example 1 served as the baseline example and the total cost functions for the links are reported in Table 1 with the total costs associated with each of the three merger cases given in Table 2. Note that the strategic advantage or synergy approximately doubled when retailers could obtain the product from any manufacturer and from any distribution center (Case 3) relative to Cases 1 and 2 going from 7.5% to 15.1%.

Example 2

Example 2 was constructed from Example 1 as follows. The demands were the same at the retailers as in Example 1 as were the capacities and total cost functions for all links except for the total cost functions associated with the storage links, link 5 and link 12, representing the total costs associated with storing the product at the distribution centers associated with Firm A and Firm B, respectively. Rather than having these total costs be given by $\hat{c}_5 = f_5^2 + 2f_5$ and $\hat{c}_{12} = f_{12} + 2f_{12}$ as they were in Example 1, they were now reduced to: $\hat{c}_5 = .5f_5^2 + f_5$ and $\hat{c}_{12} = .5f_{12} + f_{12}$ as reported in Table 1. The strategic advantage associated with all three horizontal mergers now increased, as reported in Table 2.

Example 3

Example 3 was constructed from Example 2 and had the same data except that now we reduced the total cost associated with the manufacturing plants belonging to Firm A as given in Table 1. Specifically, we changed $\hat{c}_1 = f_1^2 + 2f_1$ and $\hat{c}_2 = f_2 + 2f_2$ to: $\hat{c}_1 = .5f_1^2 + f_1$ and $\hat{c}_2 = .5f_2 + f_2$. The computed strategic advantage for each of the three horizontal mergers is given in Table 2. These values were greater than the respective ones for Example 2, although not substantially so.

Example 4

Example 4 was identical to Example 3 except that now the demand $d_{R_1^A} = 10$, that is, the demand for the product doubled at the first retailer associated with Firm A. The total costs and the strategic advantages for the different horizontal merger cases are given in Table 2. Note that now the synergies associated with Case 1 and Case 2 mergers were lower than

Table 2: Total Costs and Strategic Advantages of the Different Merger Cases for the Numerical Examples

Example	1	2	3	4	5
TC^0	660.00	540.00	505.00	766.25	766.25
TC^1	610.00	490.00	447.80	687.92	573.80
$\mathcal{S}^1$	7.5%	9.2%	11.3%	10.2%	26.5%
TC^2	610.00	490.00	447.80	675.60	549.73
$\mathcal{S}^2$	7.5%	9.2%	11.3%	13.3%	28.2%
TC^3	560.00	432.00	389.80	581.30	320.20
$\mathcal{S}^3$	15.1%	20%	20.8%	24.1%	57.5%

those obtained for Example 3, suggesting that if the manufacturing costs of one plant are much higher than the other than these types of mergers are not as beneficial. However, the Case 3 merger yielded a strategic advantage of 24.1% which was higher than that obtained for this case in Example 3.

Example 5

Example 5 was constructed from Example 4 and here we considered an idealized version in that the total cost functions (cf. Table 1) associated with shipment links which are added after the respective horizontal mergers are all equal to zero. The strategic advantages were now quite significant, as Table 2 reveals. Indeed, the strategic advantage for Case 3 was now 57.5%. This example demonstrates that significant cost reductions can occur in mergers in which the costs associated with distribution between the associated plants and distribution centers and the distribution centers and retailers are very low.

5. Summary and Conclusions

In this paper, we presented a novel system-optimization approach for the representation of economic activities associated with supply chain networks, in particular, manufacturing, distribution, as well as storage, which we then utilized to quantify the strategic advantages or gains, if any, associated with the integration of supply chain networks through the horizontal merger of firms. However, unlike the classical system-optimization model in transportation science, here we presented a model with capacities on the links to represent the capacities associated with manufacturing plants, shipment/distribution routes, and storage facilities. A given firm's economic activities may be cast into this network economic form with total costs associated with the links and the demands at the retailers assumed known and given. A particular firm was assumed to be interested in determining its optimal production quantity of the product, the amounts to be shipped to its distribution centers, where the product is stored, and then shipped to the retailers, in order to satisfy the demand and to minimize the total associated costs.

We established that the system-optimization model with capacities could be formulated and solved as a variational inequality problem. We then used this framework to explore the strategic advantages that could be obtained from the integration of supply chain networks through distinct horizontal mergers of two firms and we identified three distinct cases of horizontal mergers. In particular, we presented a measure for strategic advantage or synergy and then computed the strategic advantage for different cases of horizontal mergers for five numerical examples. In addition, in the case of certain supply chain networks with special structure, we were able to obtain explicit formulae for the total costs associated with different horizontal mergers and the associated strategic advantages.

The novelty of the framework lies in the graphical depiction of distinct types of mergers, with a focus on the involved firms' supply chain networks, and the efficient and effective computation of the total costs pre- and post- the mergers, coupled with the determination of the associated strategic advantage or synergy of the particular merger. No such general framework, which includes the integration of the manufacturing, distribution, and storage activities of firms, along with capacities, and the effects on total costs associated with three distinct types of horizontal mergers has been presented before. Possible extensions of this work are the consideration of multiple product supply chains modeled as system-optimization problems with capacities, and the investigation of the associated synergies with mergers and acquisitions of such firms, as well as multiperiod models. Of course, it would also be very interesting to model not only horizontal mergers of multiproduct supply chain networks and the associated strategic advantages but, also, vertical mergers and the possible synergies. It would also be illuminating to conduct additional sensitivity analysis exercises on the models proposed in this paper and to evaluate different supply chain network configurations. Finally, it would be interesting to explore supply chain network integration synergies when particular link total cost functions are no longer convex but are, rather, concave.

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References

Alptekinoglu, A., Tang, C. S., 2005. A model for analyzing multi-channel distribution systems. European Journal of Operational Research 163, 802-824.

Bazaraa, M. S., Sherali, H. D., Shetty, C. M., 1993. Nonlinear Programming: Theory and Algorithms. John Wiley & Sons New York.

Beckmann, M. J., McGuire, C. B., Winsten, C. B., 1956. Studies in the Economics of Transportation. Yale University Press, New Haven, Connecticut.

Bertsekas, D. P., Tsitsiklis, J. N., 1989. Parallel and Distributed Computation - Numerical Methods. Prentice Hall, Englewood Cliffs, New Jersey.

Beverage World, 2007. Molson Coors Brewing Co.; http://www.beverageworld.com/content/view/33296/

Brown, G., Keegan, J., Vigus, B., Wood, K., 2001. The Kellogg company optimizes production, inventory and distribution. Interfaces 31, 1-15.

Dafermos, S. C., Sparrow, F. T., 1969. The traffic assignment problem for a general network. Journal of Research of the National Bureau of Standards 73B, 91-118.

Gupta, D., Gerchak, Y., 2002. Quantifying operational synergues in a Merger and Acquisition. Management Science 48, 517-534.

Hakkinen, L., Norrman, A., Hilmola, O.-P., Ojala, L., 2004. Logistics integration in horizontal mergers and acquisitions. The International Journal of Logistics Management 15, 27-42.

Herd, T., Saksena, A. K., Steger, T. W., 2005. Delivering merger synergy: A supply chain perspective on achieving high performance. Outlook – Point of View, Accenture, May.

Juga, J., 1996. Organizing for network synergy in logistics: A case study. International Journal of Physical Distribution & Logistics 26, 51-66.

Knowledge@Wharton, 2005. Sears-Kmart merger: Is it a tough sell?
http://knowledge.wharton.upenn.edu/article.cfm?articleis=1081

Korpelevich, G. M., 1977. The extragradient method for fnding saddle points and other problems. Matekon 13, 35-49.

Langabeer, J., 2003. An investigation of post-merger supply chain performance. Journal of Academy of Business and Economics 2, 14-25.

Langabeer, J., Seifert, D., 2003. Supply chain integration: The key to merger success (synergy). Supply Chain Management Review 7, 58-64.

Marks, M. L., Mirvis, P. H., 2001. Making mergers and acquisitions work: Strategic and psychological preparation. Academy of Management Executive 15, 80-92.

Min, H., Zhou, G. 2002. Supply chain modeling: past, present, future. Computers and Industrial Engineering 43, 231-249.

Nagurney, A., 1993. Network Economics: A Variational Inequality Approach. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2000. Sustainable Supply Chain Networks. Edward Elgar Publishing, Cheltenham, England.

Nagurney, A. 2006a. On the relationship between supply chain and transportation network equilibria: A supernetwork equivalence with computations. Transportation Research E 42, 293-316.

Nagurney, A. 2006b. Supply Chain Network Economics: Dynamics of Prices, Flows and Profits. Edward Elgar Publishing, Cheltenham, England.

Nagurney, A., Dong, J., Zhang, D., 2002. A supply chain network equilibrium model. Transportation Research E 38, 281-303.

Nijkamp, P., Reggiani, A., 1998. The Economics of Complex Spatial Systems. Elsevier, Amsterdam, The Netherlands.

Patriksson, M., 1994. The Traffic Assignment Problem – Models and Methods. VSP, Utrecht, The Netherlands.

Soylu, A., Oruc, C., Turkay, M., Fujita, K., Asakura, T., 2006. Synergy analysis of collaborative supply chain management in energy systems using multi-period MILP. European Journal of Operational Research 174, 387-403.

Wardrop, J. G., 1952. Some theoretical aspects of road traffic research. In: Proceedings of the Institution of Civil Engineers, Part II 1, 325-378.

Xu, S., 2007. Supply Chain Synergy in Mergers and Acquisitions: Strategies, Models and Key Factors, PhD dissertation, University of Massachusetts, Amherst, Massachusetts.

Zhang, D., 2006. A network economic model for supply chain vs. supply chain competition. Omega 34, 283-295.

Zhang, D., Dong, J., Nagurney, A., 2003. A supply chain network economy: modeling and qualitative analysis. In: Nagurney, A. (Ed), Innovations in Financial and Economic Networks. Edward Elgar Publishing, Cheltenham, England, pp. 197-213.