A System-Optimization Model for Multiclass Human Migration with Migration Costs and Regulations Inspired by the Covid-19 Pandemic

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Abstract

In the last several decades, the main causes of human migration have included: poverty, war and political strife, climate change, tsunamis, earthquakes, as well as economic and educational possibilities. In this paper, we present a system-optimized network model for multiple migration classes with migration costs and regulations inspired by the Covid-19 pandemic. We derive the variational inequality formulation associated with the system-optimization problem which consists of maximizing the total societal welfare. Lagrange analysis is also performed in order to obtain a precise evaluation of the multiclass human migration phenomenon. This work adds to the literature on system-optimization of human migration in the presence of regulations and with the explicit inclusion of migration costs.

Keywords: human migration, regulations, Covid-19 pandemic, nonlinear optimization, variational inequalities, Lagrange theory

1 Introduction

The International Organization for Migration (IOM) defines a migrant as any person who is moving or has moved across an international border or within a state regardless of legal status, whether the movement is voluntary or involuntary, what the causes for the movement are, and what the length of the stay is ([15]).

According to the United Nations High Commissioner for Refugees ([39]), by the end of 2019 the number of people forcibly displaced due to war, conflict, persecution, human rights violations, poverty and economic inequality but also climate change and natural disasters, had grown to 79.5 million. That number reveals an

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increase from the number of 2018 displaced people of 70.8 million and it is the highest number on record according to available data.

The vulnerability of millions of international migrants may be exacerbated in crisis situations, as actually is the case now with the COVID-19 (COrona VIrus Disease 2019) pandemic (see [35], [40]). The infectious respiratory disease emerged in Wuhan, China, and rapidly spread around the world, posing enormous health, economic, environmental, and social challenges to the entire human population (see [5], [12]). More specifically, the COVID-19 pandemic has also affected global mobility in the form of blockages, restrictions, and travel disruptions, as risk mitigation measures are being implemented by numerous countries (see [11]). Indeed, the International Organization for Migration (IOM) reports that between 11 March 2020, when the World Health Organization (WHO) declared COVID-19 a pandemic, and 8 June 2020, the total number of movement restrictions implemented around the world has increased to more than 65,000 (see [18]). This pandemic has highlighted even more how effectively managing migration flows must be a priority for all governments in the world.

Managing human migration in the age of "super-diversity" (see [37]) means focusing on the diversified flow of migration classes that differ in legal status, country of origin, or length of stay, and may include highly skilled workers. As a consequence, identifying policies and regulations to address the variety of migration flows is essential. As an example, this may include regulations in order to reduce the vulnerability of (irregular) migrant workers (see [38]).

The United Nations, Department of Economic and Social Affairs, emphasizes that economic and social factors are the main reasons why people migrate but, on the other hand, if supported by appropriate policies, migration can contribute to inclusive and sustainable economic growth and development in both origin and destination nodes (International Migration Policies: Data Booklet, 2017). The Organization for Economic Co-operation and Development (OECD) countries in response to the COVID-19 pandemic worked on the development of short-term policy responses and longer-term challenges to migration management (see [34]). Furthermore, migration interactions in all dimensions of economic and social development will be the key to achieving the 2030 Sustainable Development Goals (SDGs) adopted by the member states of the United Nations. In the 2030 Agenda, 9 out of the 17 goals contain targets and indicators that are related to migration or mobility (see [16]).

1.1 Literature Review and Our Contributions

In the literature the migration topic has been widely addressed. Nagurney (see [27]) presents a human migration model based on networks and establishes the equilibrium conditions, which are characterized by a quadratic programming problem. Subsequently, Nagurney and Pan (see [33]) establish the relationship between projected dynamical systems and evolutionary variational inequalities to model the dynamic adjustment of a socio-economic process in the context of human migration.

Bulavsky and Kalashnikov (see [1]), in turn, introduce conjectural variations equilibria, in which the influence coefficients of each agent affects the structure of the Nash equilibrium. Isac, Bulavsky, and Kalashnikov (see [19]), later examine a model in which the potential migration groups take into account not only the current difference between the utility function values at the destination and the original locations, but also the possible variations in the utility values implied by the change of population volume due to the migration flow. In this case, they consider not perfect competition but a generalized Cournot-type model. Kalashnikov and Kalashnykova (see [20]) propose a model in which the conjectural variations coefficients depend on the total population at the destination and of its group's fraction. Moreover, the authors characterize the equilibrium with a solution of an appropriate variational inequality problem. In [6] Cojocaru formulates the human migration problem in terms of a transportation network and applies the double-layer dynamics theory. Cui and Bai (see [9]) present a mathematical model in which the population density varies when the spatial movement of individuals is a function of the departure and arrival locations. They apply the theories of positive operators and positive semigroups and then study the asymptotic behavior of solutions of migration epidemic models as time goes to infinity.

In [41] the authors make a comparison between human migration and wealth distribution. They present a model with equations for the population density and for the wealth distribution. It is based on perturbation methods and on the spectral properties of the linearized operators. The authors prove that, in the absence of cross diffusion terms, the dynamics of solutions can be described by traveling wave solutions of the corresponding reaction diffusion systems of equations. They also show the persistence of such solutions for sufficiently small cross diffusion coefficients.

Causa, Jadamba, and Raciti (see [3]), in turn, include uncertainty in the utility functions, in the migration cost functions, and in the populations. In [8] the authors present a model for the spreading of innovations in prehistoric times which is governed by human movements. They consider a spatial network where the diffusion of innovations changes in time, when the agents change their positions and also propose a stochastic simulation approach.

Nagurney and Daniele (see [29]) present the first multiclass human migration network model, with alternative conservation of flow equations and additional constraints to capture distinct types of regulations. They also conduct a Lagrange analysis. That model, in contrast to the model in this paper, is a user-optimized one. Nagurney, Daniele, and Nagurney (see [32]) construct a general, multiclass, multipath human migration model with governmentally imposed regulations associated with refugees. Further, they provide qualitative properties and then establish, via a supernetwork transformation, that the model(s) are isomorphic to traffic network equilibrium models with fixed demands. This paper is notable since the model allows for migration paths to consist of more than a single link. It is also a user-optimized model.

Nagurney, Daniele, and Cappello (see [30]) prove that, although migrants behave in a user-optimized manner, it is possible to achieve a system-optimum for a multiclass human migration network by using policy interventions in the form of subsidies. Such policy interventions allow governmental decision-makers to moderate the flow of migrants, while enhancing societal welfare. Their model is the first to apply a system-optimized perspective to human migration. That model, however, does not include migration costs between origin and destination nodes, as our new model in this paper does. Nagurney, Daniele, and Cappello (see [31]) introduce capacities on population locations and demonstrate that the results of their previous paper hold; that is, that governments can achieve system-optimization through the use of appropriate subsidies of population locations. Then, migrants behaving individually and selfishly in a user-optimized manner, upon appropriate subsidization, will reallocate themselves in a manner that is optimal from a system (societal) perspective.

It is also worth mentioning that some network equilibrium models of human migration have even been applied to the migration of animals in ecology (see [26] and [24]). Rahmati and Tularam (see [14]) provide a critical review of human migration models and note that migration is related to economic factors such as available opportunities and constraints in rural and urban areas, job access, and labor absorption in different localities.

In this paper, we extend the system-optimized models of human migration noted above to include novel utility functions, migration costs, and more general regulations. Specifically, in the objective function we take into account the changes in the utility functions of the multiple classes caused by the migratory flows and policies adopted by governments. Further, in determining the optimal flows, we consider the government policies a priori, thanks to a suitable coefficient influence vector w. Finally, we include the capacities and the regulations of the flows in a single formulation. Our aim is to find a system-optimized solution, which is a social optimum, in that an organization, such as the United Nations, maximizes the attractiveness of the origin countries, which for an individual origin is given by the sum of its utility and its expected increment of utility value, with respect to the destination one, for each migration class and each pair of countries (or locations). We also provide an equivalent formulation of the variational inequality by means of Lagrange theory. Several numerical examples are presented and analyzed.

2 Presentation of the Model

As in [2], we consider a network consisting of n nodes, that are countries or, more generally, locations, and H classes of the population. As depicted in Fig.1, the n locations are both origin nodes (where migrants are initially located) and destination nodes (where the migrants may be interested in migrating to).

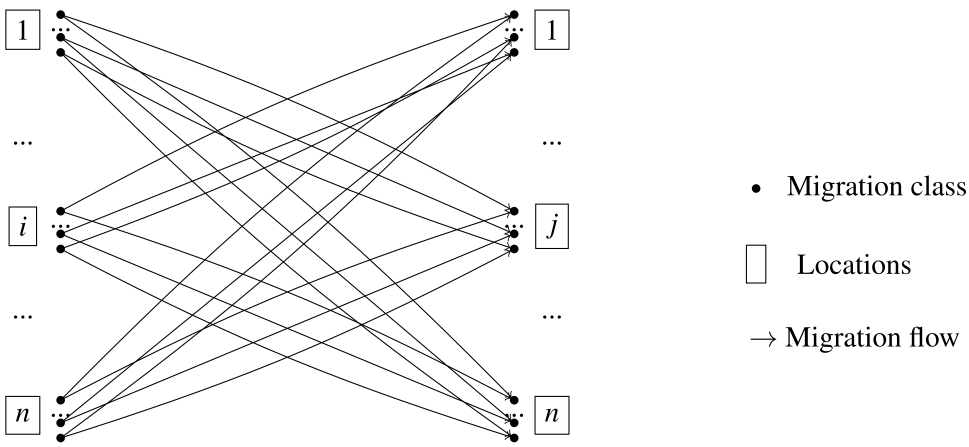

Figure 1: Network structure of the multiclass human migration model

We assume that at each location i; $i = 1,\dots,n$, there is an initial fixed population of the general class k, denoted by $\bar{p}_i^k$. We denote the nonnegative population of migrant class k at node i by p_i^k and by f_{ij}^k we denote the nonnegative migration flow out of the node i, and into the node j of the network, with $i \neq j$. It means that if a volume of population of a typical class decides to migrate, then the destination node differs from the origin one; otherwise, it remains in the same origin node. Hence, the volume of population of each class k at each node i, after the migration takes place, is given by the following conservation of flow equations:

$$p_i^k = \bar{p}_i^k - \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ij}^k + \sum_{\substack{j=1\\ j\neq i}}^{n} f_{ji}^k, \quad i = 1,...,n; \quad k = 1,\dots,H. \tag{1}$$

We consider that the sum of the flows out of each node in the network, must not exceed the initial population in that node; in other words:

$$\sum_{\substack{j=1\\ j\neq i}}^{n} f_{ij}^k \leq \bar{p}_i^k, \qquad i = 1,\dots,n; \;\; k = 1,\dots,H. \tag{2}$$

We group the populations for all the migration classes k, in each location i, into the vector population p and the migration flows of each migration class k, from each origin node i to each destination node j into the vector flow f; namely,

$$p = (p_i^k)_{\substack{i=1,\dots,n\\ k=1,\dots,H}} \in \mathbb{R}_+^{nH}, \; f = (f_{ij}^k)_{\substack{i,j=1,\dots,n,\, i\neq j\\ k=1,\dots,H}} \in \mathbb{R}_+^{n(n-1)H}. \tag{3}$$

Furthermore, as in [31], we introduce the population capacity constraint for each node in the network. Here, unlike [31], we consider that the total population at each node i must not exceed the capacity, as follows:

$$\sum_{k=1}^{H} p_i^k \leq cap_i, \quad i = 1,\dots,n, \tag{4}$$

where cap_i, for all nodes i, is the population capacity. The aforementioned constraints ensure that all the nodes are not overpopulated.

We assume that the sum of the capacities is greater than the total population of the network of all the classes of migrants.

We now introduce the origin and destination utility functions; specifically, u_i^k and v_j^k, which capture the attractiveness from an economic and/or political and/or social point of view of the origin node i and of the destination node j, respectively, as perceived by a single individual of the migration class k. In other words, such functions reflect the liveability of each node of the network as perceived by an individual of the migration class k; hence, these functions are expressed in a different way depending on whether the node is analyzed as an origin or as a destination one. We assume that the u_i^k and v_j^k are functions of the entire population vector: $u_i^k = u_i^k(p)$ and $v_j^k = v_j^k(p)$, $i, j = 1, \dots, n$; $k = 1, \dots, H$. Such functions are assumed to be continuously differentiable and concave.

To interpret the concavity condition on the utility functions in terms of applications, we assume that, without loss of generality, there is a population threshold at which utility functions stop growing. The excess of population leads to a decrease in the economic growth of the node (see [21]) due to an increase in pollution, competition for jobs, housing, etc. (see [23]).

For each individual of class k, we introduce the net utility function, which is given by the difference between the individual origin and destination utility functions. Hence, the total net utility functions for the class k, for all $k = 1, \dots, H$, with respect to the route from i to j are defined as:

$$U_{ij}^k(p, f) = (u_i^k(p) - v_j^k(p)) \times f_{ij}^k, \quad i, j = 1, \dots, n; k = 1, \dots, H, \tag{5}$$

and are assumed to be continuously differentiable and concave.

Let $c_{ij}^k(f)$ and $C_{ij}^k(f)$ denote the unit migration cost function and the total migration cost function between locations i and j, respectively. We have:

$$C_{ij}^k(f) = c_{ij}^k(f) \times f_{ij}^k, \quad i, j = 1, \dots, n; \; k = 1, \dots, H. \tag{6}$$

Such costs are assumed to be convex and continuously differentiable. This assumption is justified by the concept of *diminishing marginal utility* without requiring utility functions, according to which, roughly speaking, averages are better than the extremes.

In our model we assume that there is a government or an organization of governments, such as the United Nations, whose interest is to guarantee the respect of the right to choose migration and at the same time a high level of welfare for each individual living in each location node, in order to improve the quality of life. Each node in the network differs in terms of migration policies and ideologies, how it is experienced by a class, and how ready it is to integrate immigrants (see [13]).

It is reasonable to suppose that the migration policies that are adopted in the various nodes of the network by governments influence the migration routes. Such policies can be, for example, inclusive or not, and depend more generally on choices based on the economic, social, and/or political features of the network nodes.

Hence, we introduce for each possible migration route from node i to node j, the influence coefficients w_{ij}^{k-} and w_{ij}^{k+}, which allow us to take into account the migration policies implemented in nodes i and j as a consequence of the utility changes when individuals of the class k choose to migrate from node i towards node j, respectively. We assume that such influence coefficients range in the interval $[-1,1]$. When the influence coefficient value is close to the upper bound 1, it is indicative of a more inclusive policy. In other words, these coefficients in the objective function will be related to the changes of the utility caused by the migration flows.

Therefore, considering any origin node i and any other node j in the network, the possible variations of the utility functions u_i^k and v_j^k and the subsequent policies undertaken by the governments in the aforementioned nodes will be considered in determining the optimal flows, respectively, through the following terms:

$$\delta_i^-(p,f) = \sum_{k=1}^{H} \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} w_{ij}^{k-} f_{ij}^k \right) \times \frac{\partial u_i^k(p)}{\partial p_i^k,}, \quad i=1,\dots,n, \tag{7}$$

and

$$\delta_j^+(p,f) = \sum_{k=1}^{H} \left(\sum_{\substack{i=1 \\ i\neq j}}^{n} w_{ij}^{k+} f_{ij}^k \right) \times \frac{\partial v_j^k(p)}{\partial p_j^k,}, \quad j=1,\dots,n. \tag{8}$$

Remark 2.1 *Governments hope to minimize or maximize the terms* (7) *and* (8) *since they represent, respectively, a deficit or a surplus to the starting utility functions depending on both the sign of the derivatives of the utility functions (which we assume to be concave) and the sign of the influence coefficients, that are the variation in attractiveness in terms of welfare, quality of life, and so on of a node with respect to the population and the adopted migration policies.*

In Table 1 we summarize the notation adopted for the model. Let $\mathbb{K}$ denote the feasible set such that:

$$\mathbb{K} \;=\; \left\{ (p,f) \in \mathbb{R}^{n^2H} | (1),(2),(3),(4) \text{ hold} \right\}. \tag{9}$$

2.1 Regulations

As mentioned in the introduction, a global emergency situation, such as the COVID-19 pandemic, highlights the importance of assessing and analyzing the management of human migration, in the event that flow regulations are applied. For this reason, in our model we introduce, as in [29], the flow regulations in terms of constraints.

Suppose that the typical destination node j applies a restriction R_j on the flows of some classes k coming from some nodes i of the network, which we will group

Symbol	Definition
$\bar{p}_i^k$	Initial population of class k in location i
p_i^k	Population at location i of class k
f_{ij}^k	Migration flow from i to j of class k
$v_j^k(p)$	Destination utility function of location j as perceived by an individual of class k
$u_i^k(p)$	Origin utility function of location i as perceived by an individual of class k
U_{ij}^k	Total net utility function for class k with respect to the route from i to j
$c_{ij}^k(f)$	Unit migration cost from i to j for an individual of class k
$C_{ij}^k(f)$	Total migration cost from i to j for class k
$w_{ij}^{k\pm} \in [-1,1]$	Policy influence coefficients

Table 1: Functions, parameters, and decision variables of the model

together in the set C^j of pairs (i,k) to which restrictions are imposed by j, as follows:

$$\sum_{(i,k)\in C^j} f_{ij}^k \leq R_j. \tag{10}$$

As noted in [29], the (10) restrictions, for each node j represent the most general case of flow regulations which, depending on the adopted policies, may be more specific, such as:

- restrictions for a single class $\bar{k}$ and coming from a single node in the network $\bar{i}$:

$$f_{\bar{i}j}^{\bar{k}} \leq R_j, \tag{11}$$

- restrictions for a single class $\bar{k}$

$$\sum_{(i,\bar{k})\in C^j} f_{ij}^{\bar{k}} \leq R_j, \quad \forall j, \tag{12}$$

- restrictions for every class coming from origin node $\bar{i}$

$$\sum_{(\bar{i},k)\in C^j} f_{\bar{i}j}^{k} \leq R_j, \quad \forall j. \tag{13}$$

We denote by $\mathbb{K}^1$ the feasible with the above regulations as follows:

$$\mathbb{K}^1 \quad = \quad \left\{(p,f) \in \mathbb{R}^{n^2H} |\ (10) \text{ holds}\right\}. \tag{14}$$

2.2 The Multiclass Human Migration Network System-Optimization Problem and its Variational Formulation

The multiclass human migration network system-optimization problem can be expressed as follows. The cognizant organization seeks to determine the optimal flows, as well as the optimal populations at each node in the network, subject to the convenience to remain, given by the difference between the total net utility function and the migration costs and also trying to take into account the choices of policies by the governments/organization and the potential variations of the utility function as closely as possible. As a consequence, we are dealing with a system-optimized model. Therefore, the optimization problem is constructed as follows:

$$\text{Maximize} \sum_{k=1}^{H} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[U_{ij}^{k}(p,f) - C_{ij}^{k}(f) + \delta_i^{-}(p,f) + \delta_j^{+}(p,f) \right] \tag{15}$$

subject to: constraints (1), (2), (3), (4), and (10). We introduce the feasible set for the optimization problem under regulations

$$\mathbb{K}^2 = \left\{ (p,f) \in \mathbb{R}^{n^2 H} \mid (1),(2),(3),(4), \text{ and } (10) \text{ hold} \right\}.$$

Under the above assumptions, the objective function in (15) is concave and continuously differentiable and so, using the classical variational theory (see [22] and [28]), it is easy to prove that an optimal solution for the optimization problem, denoted by $(p^*, f^*) \in \mathbb{K}^2$, satisfies the following variational inequality: find $(p^*, f^*) \in \mathbb{K}^2$, such that

$$\begin{aligned}
&-\sum_{q=1}^{H} \sum_{l=1}^{n} \left(\sum_{k=1}^{H} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\partial U_{ij}^{k}(p^*,f^*)}{\partial p_l^q} + \frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial p_l^q} + \frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial p_l^q} \right) \times (p_l^q - p_l^{q*}) \\
&-\sum_{q=1}^{H} \sum_{l=1}^{n} \sum_{\substack{s=1 \\ s \neq l}}^{n} \left(\sum_{k=1}^{H} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\partial U_{ij}^{k}(p^*,f^*)}{\partial f_{ls}^q} - \frac{\partial C_{ij}^{k}(f^*)}{\partial f_{ls}^q} + \frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial f_{ls}^q} \right. \\
&\left. + \frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial f_{ls}^q} \right) \times (f_{ls}^q - f_{ls}^{q*}) \geq 0, \qquad \forall (p,f) \in \mathbb{K}^2.
\end{aligned} \tag{16}$$

Applying the well-known results about variational inequalities in finite dimension (see [22] and [4], [7], [10] and [29]), we can find an equivalent formulation of the variational inequality using the Lagrange multipliers associated with the constraints defining the feasible set $\mathbb{K}^2$ and proving the strong duality.

Indeed, variational inequality (16) can be rewritten as a minimization problem, since, if we denote by $V(p,f)$ the left-hand side of (16), then we have:

$$V(p,f) \geq 0 \text{ in } \mathbb{K}^2 \text{ and } \min_{\mathbb{K}^2} V(p,f) = V(p^*,f^*) = 0.$$

Now, denoting by $\lambda^1 \in \mathbb{R}_+^{nH}$, $\lambda^2 \in \mathbb{R}_+^{n(n-1)H}$, $\varepsilon \in \mathbb{R}^{nH}$, $\nu \in \mathbb{R}_+^{nH}$, and $\mu, \gamma \in \mathbb{R}_+^n$, the Lagrange multiplier vectors associated with the nonnegativity constraints (3), and constraints (1), (2), (4), and (10), respectively, we can consider the following

Lagrange function:

$$
\begin{aligned}
&\mathscr{L}(p,f,\lambda^1,\lambda^2,\varepsilon,\nu,\mu,\gamma)=V(p,f)+\sum_{i=1}^{n}\sum_{k=1}^{H}\lambda_i^{k1}(-p_i^k)\\
&+\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{H}\lambda_{ij}^{k2}(-f_{ij}^k)+\sum_{i=1}^{n}\sum_{k=1}^{H}\varepsilon_{ik}\left(p_i^k-\bar{p}_i^k+\sum_{\substack{j=1\\j\neq i}}^{n}f_{ij}^k-\sum_{\substack{j=1\\j\neq i}}^{n}f_{ji}^k\right) \qquad (17)\\
&+\sum_{i=1}^{n}\sum_{k=1}^{H}\nu_{ik}\left(\sum_{\substack{j=1\\j\neq i}}^{n}f_{ij}^k-\bar{p}_i^k\right)+\sum_{i=1}^{n}\mu_i\left(\sum_{k=1}^{H}p_i^k-cap_i\right)+\sum_{j=1}^{n}\gamma_j\left(\sum_{(i,j)\in C^j}f_{ij}^k-R_j\right).
\end{aligned}
$$

Making use of the Lagrange theory, if (p^*,f^*) is a solution to variational inequality (16), we are able to prove that the following KKT conditions (18)-(19) hold and vice versa. Moreover, we show that strong duality (22) holds.

Theorem 2.1 *The Lagrange multipliers in (17) do exist and, for all* $i,\ j=1,\dots,n$, *and* $k=1,\dots,H$, *the following conditions hold true:*

$$
\overline{\lambda}_i^{k1}(-p_i^{k*})=0,\quad \overline{\lambda}_{ij}^{k2}(-f_{ij}^{k*})=0,\quad \overline{\nu}_{ik}\left(\sum_{\substack{j=1\\j\neq i}}^{n}f_{ij}^{k*}-\bar{p}_i^k\right)=0, \qquad (18)
$$

$$
\overline{\mu}_i\left(\sum_{k=1}^{H}p_i^{k*}-cap_i\right)=0,\quad \overline{\gamma}_j\left(\sum_{(i,j)\in C^j}f_{ij}^{k^*}-R_j\right)=0, \qquad (19)
$$

$$
\frac{\partial U_{ij}^k(p^*,f^*)}{\partial p_i^k}+\frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial p_i^k}+\frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial p_i^k}-\overline{\lambda}_i^{k1}+\overline{\varepsilon}_{ik}+\overline{\mu}_i=0, \qquad (20)
$$

$$
\frac{\partial U_{ij}^k(p^*,f^*)}{\partial f_{ij}^k}-\frac{\partial C_{ij}^k(f^*)}{\partial f_{ij}^k}+\frac{\partial \delta_i^{k+}(p^*,f^*)}{\partial f_{ij}^k}+\frac{\partial \delta_i^{k-}(p^*,f^*)}{\partial f_{ij}^k}-\overline{\lambda}_{ij}^{k2}+\overline{\varepsilon}_{ik}+\overline{\gamma}_j=0, \qquad (21)
$$

where $\overline{\lambda}^1$, $\overline{\lambda}^2$, $\overline{\varepsilon}$, $\overline{\nu}$, $\overline{\mu}$, $\overline{\gamma}$ *are the optimal Lagrange multiplier vectors. Moreover, the strong duality also holds true; namely:*

$$
\begin{aligned}
V(p^*,f^*) &= \min_{\mathbb{K}^2} V(p,f) \qquad (22)\\
&= \max_{\substack{\lambda^1\in\mathbb{R}_+^{nH},\lambda^2\in\mathbb{R}_+^{2nH}\\ \varepsilon\in\mathbb{R}^{nH},\nu\in\mathbb{R}^{nH},\mu,\gamma\in\mathbb{R}_+^n}}\ \min_{(p,f)\in\mathbb{R}^{nH+2nH}} \mathscr{L}(p,f,\lambda^1,\lambda^2,\nu,\mu,\gamma).
\end{aligned}
$$

Proof. See Theorem 3.1 in [4].

The existence of at least one solution to variational inequality (16) is guaranteed from the classical theory of variational analysis (see Th.3.1. in [22]), since the feasible set is compact and the function that enters the variational inequality is continuous (see [25] for additional existence results).

3 Numerical Examples

In this section we present two illustrative examples for the optimization problem (15), without and with regulations, respectively. Specifically, for the regulation model three different situations are analyzed.

For each example we consider two migration classes and two locations in the network, as depicted in Figure 2.

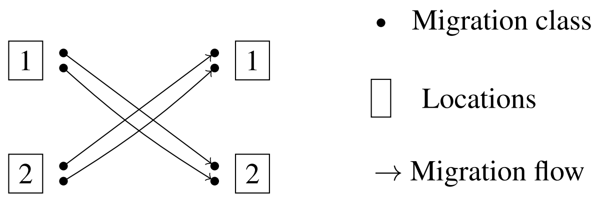

Figure 2: Network structure of 2-class human migration model.

Let us consider the data as in Table 2, that are the common data for the two examples.

We also introduce the population node capacities $\text{cap}_1 = 150$, $\text{cap}_2 = 180$ and, consequently, the capacity constraints as follows:

$$p_1^1 + p_1^2 \leq 150 \tag{23}$$

$$p_2^1 + p_2^2 \leq 180. \tag{24}$$

In the case without regulations, the feasible set $\mathbb{K}$ (9), is given by:

$$\begin{aligned} \mathbb{K} \quad = \quad & \big\{(p,f) \in \mathbb{R}_+^8 : p_1^1 = 40 - f_{12}^1 + f_{21}^1;\, p_2^1 = 30 - f_{21}^1 + f_{12}^1, \\ & p_1^2 = 25 - f_{21}^2 + f_{12}^2, p_2^2 = 15 - f_{21}^2 + f_{12}^2\, f_{12}^1 \leq 40,\ f_{21}^1 \leq 30, \\ & f_{12}^2 \leq 25,\ f_{21}^2 \leq 15;\ p_1^1 + p_1^2 \leq 150,\ p_2^1 + p_2^2 \leq 180\big\}. \end{aligned} \tag{25}$$

The solution to variational inequality (16) is:

$$(p_1^{1^*}, p_2^{1^*}, p_1^{2^*}, p_2^{2^*}, f_{12}^{1\ *}, f_{21}^{1\ *}, f_{12}^{2\ *}, f_{21}^{2\ *}) = (45.92, 24.07, 33.27, 6.72, 0, 5.92, 0, 8.27).$$

As we can see from the solution, for both migrant classes, there is no flow from node 2 to node 1.

We now introduce the regulation constraint (13), for every class coming from the origin node 2, as follows:

$$f_{21}^1 + f_{21}^2 \leq R_1. \tag{26}$$

where R_1 is the typical restriction applied by the destination node 1. We give three different values to R_1 and, then, make a comparison with the solution obtained in

Origin utility functions	$u_1^1 = -0.54p_1^1 - 0.11p_2^1 - 0.17p_1^2$
	$u_2^1 = 0.49p_2^1 - 0.39p_2^2$
	$u_1^2 = 0.26p_2^2 - 1.77p_1^2 - 0.47p_1^1$
	$u_2^2 = p_2^1 + 0.08p_1^2 + 0.64p_2^2$
Destination utility functions	$v_1^1 = 0.02p_1^1$
	$v_2^1 = 0.02p_2^2 - 0.51p_2^1$
	$v_1^2 = p_2^1$
	$v_2^2 = 0.15p_2^1 + p_2^2$
Migration costs	$c_{12}^1 = 5.54f_{12}^1$
	$c_{21}^1 = 5.08f_{21}^1$
	$c_{12}^2 = 1.72f_{12}^2$
	$c_{21}^2 = 5.00f_{21}^2$
Initial populations	$\bar{p}_1^1 = 40$
	$\bar{p}_2^1 = 30$
	$\bar{p}_1^2 = 25$
	$\bar{p}_2^2 = 15$
Influence coefficients	$w_{12}^{1-} = 0.3$
	$w_{21}^{1-} = -0.002$
	$w_{12}^{1+} = 0.26$
	$w_{21}^{1+} = 0.9$
	$w_{12}^{2-} = -0.9$
	$w_{21}^{2-} = 0.15$
	$w_{12}^{2+} = 0.4$
	$w_{21}^{2+} = 0.2$

Table 2: Data for the two numerical examples

the case without regulations. In the case with regulations, the feasible set $\mathbb{K}^1$ (14) is:

$$\begin{aligned}\mathbb{K}^1 \quad &= \big\{(p,f) \in \mathbb{R}^8_+ : p^1_1 = 40 - f^1_{12} + f^1_{21};\, p^1_2 = 30 - f^1_{21} + f^1_{12},\\ &p^2_1 = 25 - f^2_{21} + f^2_{12}, p^2_2 = 15 - f^2_{21} + f^2_{12}\; f^1_{12} \leq 40,\; f^1_{21} \leq 30,\\ &f^2_{12} \leq 25,\; f^2_{21} \leq 15;\; p^1_1 + p^2_1 \leq 150,\, p^1_2 + p^2_2 \leq 180;\; f^1_{21} + f^2_{21} \leq R_1\big\}. \quad (27)\end{aligned}$$

We consider three cases:

- **Case 1,** $R_1 = 5$**:** The solution to variational inequality (16) is:

$$(p^{1*}_1, p^{1*}_2, p^{2*}_1, p^{2*}_2, f^{1\,*}_{12}, f^{1\,*}_{21}, f^{2\,*}_{12}, f^{2\,*}_{21})$$
$$= (41.21, 28.78, 28.78, 11.21, 0, 1.21, 0, 3.78).$$

- **Case 2,** $R_1 = 10$**:** The solution to variational inequality (16) is:

$$(p^{1*}_1, p^{1*}_2, p^{2*}_1, p^{2*}_2, f^{1\,*}_{12}, f^{1\,*}_{21}, f^{2\,*}_{12}, f^{2\,*}_{21})$$
$$= (43.77, 26.22, 31.22, 8.77, 0, 3.77, 0, 6.22).$$

- **Case 3,** $R_1 = 37$**:** The solution to variational inequality (16) is:

$$(p^{1*}_1, p^{1*}_2, p^{2*}_1, p^{2*}_2, f^{1\,*}_{12}, f^{1\,*}_{21}, f^{2\,*}_{12}, f^{2\,*}_{21})$$
$$= (45.91, 24.08, 33.27, 6.72, 0, 5.91, 0, 8.27).$$

As we can see from the solution, also in these three cases, for both classes of population, there is no flow from node 2 to node 1. We obtain a reduction of the flows from node 2 to 1 for every migration class in each cases in which restrictions are introduced.

Note that, when R_1 increases, which means the movement possibility in the network increases, then also the optimal flows for both classes from node 2 to node 1 increase till the values without regulations.

The variational inequalities of the optimization problems both without and with regulations were solved using the Projection-Contraction method (see [36]). The algorithm was coded using Matlab and was run on a PC with 8 GB RAM, Asus Intel (R) Core (TM) i5-10210U CPU@1.60 GHz.

4 Conclusions and Further Research

The Covid-19 pandemic, a global healthcare disaster, has dramatically influenced the movement of humans over space and time in 2020. It has, also, impacted international migration as governments institute regulations banning travel. The world has seen immense migratory flows over the past decades with migrants seeking more amenable locations for themselves and their families. The topic of human migration has assumed further attention during the pandemic.

Migration can have positive as well as negative effects on the lives of the migrants. Positive aspects include: potentially the reduction of unemployment, a better quality of life, learning about a new culture, customs, and languages, and/or economic growth of the region. On the other hand, negative effects can include: increasing competition for jobs; possibly, growth in poverty, criminality, and exploitation, as well as pollution.

In this paper, we introduced a network-based model for multiclass human migration with the objective of improving the system, that is, the society. Unlike previous system-optimization models for human migration, the new model includes migration costs as well as novel utility functions. We, nevertheless, retain regulations introduced earlier by the authors on the migratory flows. The model is studied qualitatively and numerical examples also provided.

In future research it would be interesting to compare the system-optimized solution of this model with the user-optimized one; namely, from the migrant's point of view. Furthermore, migration could be analyzed as a noncooperative game, with appropriate strategies of the players considering the levels of difficulty in reaching a new place (due, for instance, to the national regulations).

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