Strict Quotas or Tariffs?
Implications for Product Quality and Consumer Welfare in Differentiated Product Supply Chains

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March 2019; revised May, June, and July 2019

Transportation Research E (2019), **29**, pp 136-161.

Abstract

We introduce a supply chain network equilibrium model with differentiated products, in which firms compete on product quantities and quality. We then extend the model to include a strict quota or tariff. We establish the equivalence between the model with a strict quota and that with a tariff, when the quota constraint is tight, and the tariff corresponds to the equilibrium Lagrange multiplier associated with the constraint for the former model. Numerical examples reveal that although firms may benefit from the imposition of a quota or tariff, the welfare of consumers in the country imposing the instrument declines.

Keywords: supply chains, networks, trade policies, agricultural products, trade war, game theory

1. Introduction

Supply chain networks provide the backbone for the production, storage, and distribution of products around the globe. Due to the importance of global trade to both producers and consumers, a variety of products have been subject to trade policy instruments imposed by governments (cf. Meixell and Gargeya (2005)). Examples of trade policy instruments that have been applied on imported products ranging from food to consumer durables have included tariffs, quotas, as well as two-tiered tariffs, known as tariff rate quotas (cf. Nagurney, Besik, and Dong (2019)). The impacts of trade instruments on trade flows are well-known (cf. Nagurney, Besik, and Dong (2019) and the references therein). For example, unit tariffs will decrease product flows from production sites in countries on which they are imposed.

The determination of the effects of trade instruments on product quality, however, has been less-researched and has been the subject of debates (cf. Lutz (2005) and Nagurney and Li (2016)). As noted in Hallak (2006), growing evidence reveals that there are large differences across countries in terms of the quality of products that they produce and export (see also Schott (2004), Feenstra and Romalis (2014), and Crino and Ogliari (2015)). Furthermore, as also emphasized therein, a rich theoretical literature predicts the important role of product quality in global trade. Hence, the development of a general, integrated framework that enables the incorporation of multiple trade policy instruments and that allows rigorous scrutiny of product quality through quantitative modeling, analysis, and impact is a timely endeavor.

The need for such a framework is especially relevant given the prevalence of tariffs and quotas in the news. For example, the United States has imposed tariffs on steel. The response from the European Union was to impose quotas (cf. Meyer (2019)). Numerous tariffs were imposed by the United States on goods from China in 2018 including: food, toilet paper, hats, backpacks, beauty care products, sporting goods, home improvement items, and pet products, valued at $200 billion in Chinese imports (Shively (2018)). China then retaliated with their government deciding to impose tariffs of 5% to 10% on $60 billion worth of U.S. products. The tariffs apply to 5,207 items (Kuhn (2018)).

The research questions that this paper aims to address and answer are the following:

1. Is there any correlation between the strict quota and the unit tariff schemes? Are they equivalent under certain conditions?

2. Do firms benefit from the imposition of a specific strict quota or unit tariff?

3. What are the impacts on demands, prices, and product quality as the imposed strict

quota or unit tariff changes?

4. Do consumers at different demand markets gain more welfare from the imposition of a strict quota or a unit tariff? What is the impact on consumer welfare as the quota or tariff changes?

5. Under the imposition of a strict quota or unit tariff, in order to be more profitable in competition with other firms, how should firms adjust the locations of production facilities and demand markets in their supply chain networks?

We acknowledge that the values of quotas and tariffs, as trade policy instruments, may change over time. The number of firms in competition, as well as each firm's supply chain network, costs, and demand markets served may also change over time. Therefore, our goal in this paper is to answer the above research questions with general supply chain network models. Such models are flexible enough, as well as computable, to provide insights on a spectrum of scenarios. Answers to the above research questions are obtained through theory as well as numerical examples and sensitivity analysis and summarized in the final section of this paper.

2. Literature Review and Paper Organization

Much of the modeling research that includes the quality of products produced and traded, in the presence of trade policy instruments, has appeared in the economics literature. The theoretical research has focused on a monopoly (Krishna (1987)), or on a duopoly (Das and Donnenfeld (1989) and Herguera, Kujal, and Petrakis (2000)), or on perfect competition (Falvey (1979)). As for oligopolistic competition, researchers have, typically, assumed exogenously fixed product qualities or homogeneous goods (Leland (1979), Shapiro (1983), and Deneckere, Kovenock, and Sohn (2000)). However, it is clear that quality can be a strategic variable in firms' decision-making and also in terms of consumers differentiating among the firms (cf. Jacobson and Aaker (1987), Veldman and Gaalman (2014), and Nagurney, Besik, and Yu (2018)).

There have also been empirical studies conducted to assess the interrelationships between a spectrum of trade policies and product quality as in cheese (cf. Macieira and Grant (2014)), the steel industry (Boorstein and Feenstra (1991)), the footwear industry (Aw and Roberts (1986)), and the automobile industry (cf. Feenstra (1988) and Goldberg (1993)). Nevertheless, the existing theoretical literature has been limited in terms of the number of firms considered, as well as the number of demand markets, and has not included general transportation cost functions that include quality. Therefore, the construction of a general differentiated supply chain network model with product quality and trade policies in the form of tariffs, quotas, and also minimum quality standards merits attention.

Here we model quality in a classical way (cf. Krishna (1987), Spence (1975), and Sheshinski (1976)) as a factor that raises the willingness to pay for a unit of a product. We formulate a competitive supply chain network model in which producers have multiple production sites and seek to determine both the product flows and the quality levels of the product at the production sites so as to maximize profits. Their production costs and transportation costs are a function of both product flows and quality levels. The consumers, in turn, reflect their preferences for the firms' differentiated products through the prices that they are willing to pay at the demand markets. The demand market prices are a function of the product flows and the average product quality levels. The model includes lower and upper bounds on the quality levels at the production sites with the former being imposed by the cognizant authorities and the latter being determined by technological feasibility. To this model we then add trade policy instruments in the form of a strict quota or a tariff on a specific product in a group consisting of production sites in a country, imposed by another country. We then investigate the impacts both theoretically and numerically.

This paper is organized as follows. In Section 3, we first present the differentiated product supply chain network equilibrium model with quality, but without trade policy instruments in the form of a tariff or quota. The model generalizes the model introduced in Nagurney and Li (2014) in that the demand price functions are differentiated by producing firm. We state the governing Nash equilibrium conditions and provide the variational inequality formulation. We then generalize the model to include a quota over a group of production sites and demand markets associated with a country. Under a strict quota, both the utility functions of the competing firms, as well as their feasible sets, will depend on the strategic variables of not only the particular firm, but also on the strategies of the others. Hence, we define a Generalized Nash Equilibrium (GNE). This is the first time that such a concept has been utilized in competitive supply chain network models with quality with or without trade policy instruments. For an application of GNE to supply chain competition for storage resources in distribution centers (but without any aspects of quality or trade policy instruments), see Nagurney, Yu, and Besik (2017). For the use of GNE in disaster relief, see the work of Nagurney, Alvarez Flores, and Soylu (2016). We utilize the concept of a Variational Equilibrium to construct the variational inequality formulation. Subsequently, we introduce a differentiated product supply chain network equilibrium model with a tariff. We prove that this model coincides with the model with a strict quota where the Lagrange multiplier associated with the latter, if the strict quota constraint is tight, is precisely the imposed tariff.

Such an equivalence provides policy-makers and decision-makers with flexibility in the application of such policy instruments. In the economics literature, Bhagwati (1965) considered perfect competition (whereas we consider imperfect competition) and demonstrated that the tariff-quota equivalence occurs when such competition prevails in all markets. Fung (1989), in turn, considered a stylized oligopoly consisting of two countries with a single firm in each country and Cournot-Nash competition and also found an equivalence. In our differentiated product supply chain network model we do not limit the number of countries, nor firms in each country and also include quality to establish an equivalence between quotas and tariffs and do so in a GNE setting.

We also provide constructs for quantifying consumer welfare in the presence or absence of tariffs or quotas in differentiated product supply chain networks with quality. Through simple illustrative examples in Section 3, we show that the imposition of a tariff or quota may adversely affect both the quality of products as well as the consumer welfare. Nagurney, Besik, and Dong (2019) proposed a spatial price equilibrium model with a tariff rate quota, which is a two-tiered tariff, allowing for a quota to be exceeded, but under a higher tariff. That perfectly competitive model did not require a GNE formulation as does the oligopoly model with strict quota in this paper. Moreover, in that work quality was not even considered. The paper of Li, Nagurney, and Yu (2018), in turn, proposed a spatial price equilibrium model with consumer learning and quality, but there were no trade policy instruments incorporated. The model in this paper is the first supply chain network equilibrium model with quality and trade policy instruments in the form of a tariff or strict quota. Here we consider unit tariffs; for ad valorem tariffs and a spatial price equilibrium model, see Nagurney, Nicholson, and Bishop (1996). Dong and Kouvelis (2019) recently proposed a tariff model based on the Lu and Van Mieghem (2009) newsvendor network model in order to ascertain the basis for interpreting tariff impacts at different stages of the supply chain, either at the input level or the finished goods level. However, that work did not consider the quality of the products nor strict quotas, as our framework in this paper does. Moreover, our model is not limited to a fixed number of firms, production sites, or number of demand markets.

In Section 4, we propose an effective algorithm, which is then applied in Section 5 to a series of numerical examples. The examples are focused on the agricultural product of soybeans. The examples explore the impacts of the imposition of a strict quota or a tariff by China on soybeans that are produced in the United States on equilibrium soybean flows, product quality, prices, firm profits, and consumer welfare. We consider multiple scenarios, including a disruption at a production side, as well as the addition of a demand market. The numerical examples yield interesting results for decision-makers and policy makers.

In Section 6, we summarize the results and present our conclusions, along with suggestions for future research.

3. The Differentiated Product Supply Chain Network Equilibrium Models with Quality

In this Section, we construct the differentiated product supply chain network equilibrium models in which the firms compete in product quantities and quality levels. In Section 3.1, we consider the case without trade interventions in the form of strict quotas or tariffs. We then, in Section 3.2, extend the model to include such trade instruments and establish their equivalence in Section 3.3. The consumer welfare formula is presented in Section 3.4, with illustrative examples given in Section 3.5.

3.1 The Differentiated Product Supply Chain Network Equilibrium Model without Trade Interventions

The firms produce a product, which is substitutable, but differentiated by firm. In the supply chain network economy (cf. Figure 1), there are I firms, corresponding to the top-tier nodes, with a typical firm denoted by i, which compete with one another in a noncooperative manner in the production and distribution of the products, and on quality.

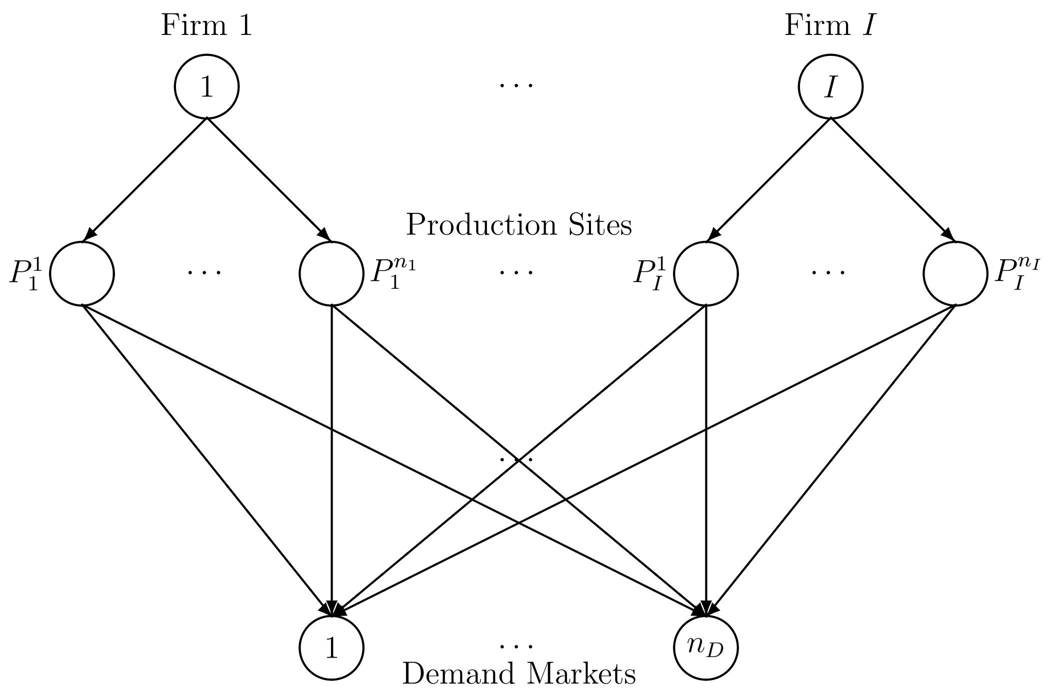

Figure 1: The Differentiated Product Supply Chain Network Topology

Each firm i has, at its disposal, n_i production sites, corresponding to the middle tier nodes in Figure 1, which can be located in the same country as the firm or be in different countries. The production sites of firm i at the middle tier are denoted by $P_i^1, \ldots, P_i^{n_i}$, respectively, with a typical site denoted by P_i^j. The firms determine the quantities to produce at each of their sites which are then transported to the n_D demand markets, corresponding to the bottom nodes in Figure 1. A typical demand market is denoted, without loss of generality, by k; $k = 1, \ldots, n_D$. In addition, the firms must determine the quality level of the product at each of their production sites, which can differ from site to site. At the demand markets, consumers signal their preferences through the prices that they are willing to pay for the products, which are differentiated by firm (although they are substitutes). Since a product may be produced at one or more sites, a firm's product at a demand market is characterized by its average quality. We assume that the quality is preserved (with an associated cost) in the distribution process.

A link in Figure 1 joining a firm node with one of its production site nodes corresponds to the production/manufacturing activity, whereas a link joining a production site node with a demand market node corresponds to the activity of distribution. Note that, in the case of agricultural production, production sites would correspond to farms.

The notation for the model is given in Table 1.

The production output at firm i's production site P_i^j and the demand for the product at each demand market k must satisfy, respectively, the conservation of flow equations (1) and (2):

$$s_{ij} = \sum_{k=1}^{n_D} Q_{ijk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{1}$$

$$d_{ik} = \sum_{j=1}^{n_i} Q_{ijk}, \quad k = 1, \ldots, n_D. \tag{2}$$

According to (1), the output produced at a firm's production site is equal to the sum of the product amounts distributed to the demand markets from that site, and, according to (2), the quantity of a product produced by a firm and consumed at a demand market is equal to the sum of the amounts shipped by the firm from its production sites to that demand market.

In addition, the product shipments must be nonnegative, that is:

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_D. \tag{3}$$

Table 1: Notation for the Supply Chain Network Models with Product Differentiation (with and without Tariffs or Quotas)

Notation	Definition
Q_{ijk}	the nonnegative amount of firm i's product produced at production site P_i^j and shipped to demand market k. The $\{Q_{ijk}\}$ elements for all j and k are grouped into the vector $Q_i \in R_+^{n_i n_D}$. We then further group the Q_i; $i = 1, \dots, I$, into the vector $Q \in R_+^{\sum_{i=1}^I n_i n_D}$.
s_{ij}	the nonnegative product output produced by firm i at its site P_i^j. We group the production outputs for each i; $i = 1, \dots, I$, into the vector $s_i \in R_+^{n_i}$. We then further group all such vectors into the vector $s \in R_+^{\sum_{i=1}^I n_i}$.
q_{ij}	the quality level, or, simply, the quality, of product i, which is produced by firm i at its site P_i^j. The quality levels of each firm i; $i = 1, \dots, I$, the $\{q_{ij}\}$, are grouped into the vector $q_i \in R_+^{n_i}$. Then the quality levels of all firms are grouped into the vector $q \in R_+^{\sum_{i=1}^I n_i}$.
$\bar{q}_{ij}$	the upper bound on the quality of firm i's product produced at site P_i^j, with $i = 1, \dots, I$; $j = 1, \dots, n_i, \forall i$.
$\underline{q}_{ij}$	the minimum quality standard at production site P_i^j; $i = 1, \dots, I$; $j = 1, \dots, n_i, \forall i$, assumed to be nonnegative.
d_{ik}	the demand for firm i's product at demand market k, with d_{ik} assumed to be greater than zero. We group the demands for firm i's product for each $i = 1, \dots, I$, into the vector $d_i \in R_+^{n_D}$ and then group the demands for all i into the vector $d \in R_+^{In_D}$.
$\hat{q}_{ik}$	the average quality of firm i's product at demand market k; $i = 1, \dots, I$; $k = 1, \dots, n_D$, where $\hat{q}_{ik} = \frac{\sum_{j=1}^{n_i} q_{ij} Q_{ijk}}{d_{ik}}$. We group the average quality levels of all firms at all the demand markets into the vector $\hat{q} \in R_+^{In_D}$.
$\bar{Q}$	the strict quota defined for production sites in a particular country over which the quota is imposed for the product by another country to its demand markets.
λ	the Lagrange multiplier associated with the quota constraint.
$f_{ij}(s, q)$	the production cost at firm i's site P_i^j.
$\hat{c}_{ijk}(Q, q)$	the total transportation cost associated with distributing firm i's product, produced at site P_i^j, to demand market k.
$\rho_{ik}(d, \hat{q})$	the demand price function for firm i's product at demand market k.

The quality levels, in turn, must meet or exceed the nonnegative minimum quality standards (MQSs) at the production sites, but they cannot exceed their respective upper bounds of quality:

$$\bar{q}_{ij} \geq q_{ij} \geq \underline{q}_{ij}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{4}$$

It is reasonable to assume upper bounds on the quality levels due to physical/technological limitations. The MQSs are imposed by regulators (or can even be self-selected by producers) and, if there are no such MQSs that the corresponding quality lower bound is set equal to zero.

In view of (1), we can redefine the production cost functions (cf. Table 1) in terms of product shipments and quality, that is,

$$\hat{f}_{ij} = \hat{f}_{ij}(Q, q) \equiv f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{5}$$

and, in view of (2), and the definition of the average quality in Table 1, we can redefine the demand price functions in terms of quantities and average qualities, as follows:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(Q, q) \equiv \rho_{ik}(d, \hat{q}), \quad k = 1, \ldots, n_D. \tag{6}$$

We assume that the production cost and the transportation cost functions are convex and continuously differentiable and that the demand price functions are monotonically decreasing in demands, monotonically increasing in average quality, and continuously differentiable.

The strategic variables of firm i are its product shipments Q_i and its quality levels q_i, with the profit/utility U_i of firm i; $i = 1, \ldots, I$, given by the difference between its total revenue and its total costs:

$$U_i = \sum_{k=1}^{n_D} \hat{\rho}_{ik}(Q, q) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} \hat{f}_{ij}(Q, q) - \sum_{k=1}^{n_D} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q). \tag{7}$$

The first term in (7) represents the revenue of firm i; the second term represents its total production cost and the last term in (7) represents the firm's total transportation costs. The transportation cost functions depend on both quantities and quality levels and were also utilized previously by Nagurney and Wolf (2014), but for Internet applications and not supply chains. Nagurney and Li (2014) did utilize such transportation cost functions in supply chains but not in a differentiated model as we do here. Such transportation cost functions imply that the quality is preserved during the transportation process in contrast to supply chain network models in which there can be quality deterioration associated with movement down paths of a supply chain as in, for example, Nagurney et al. (2013), Yu and Nagurney (2013), and in Nagurney, Besik, and Yu (2018). Hence, all functions in the objective function (7) depend on both product quantities as well as product quality levels. Moreover, the functions corresponding to a particular firm can also, hence, in general, depend on the product shipments and quality levels of the other firms. This feature enhances the modeling of competition in that firms may also compete for resources in production and distribution and, of course, compete for consumers on the demand side.

In view of (7), we may write the profit as a function of the product shipment pattern and quality levels, that is,

$$U = U(Q, q), \tag{8}$$

where U is the I-dimensional vector with components: $\{U_1, \ldots, U_I\}$.

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, q_i)|(3) \text{ and } (4) \text{ hold}\}$ and define $K \equiv \prod_{i=1}^{I} K^i$.

We consider Cournot (1838) - Nash (1950, 1951) competition, in which the I firms produce and distribute their product in a noncooperative manner, each one trying to maximize its own profit. We seek to determine a product shipment and quality level pattern $(Q^*, q^*) \in K$ for which the I firms will be in a state of equilibrium as defined below.

Definition 1: A Differentiated Product Supply Chain Network Equilibrium with Quality

A product shipment and quality level pattern $(Q^*, q^*) \in K$ is said to constitute a differentiated product supply chain network equilibrium with quality if for each firm i; $i = 1, \ldots, I$,

$$U_i(Q_i^*, Q_{-i}^*, q_i^*, q_{-i}^*) \geq U_i(Q_i, Q_{-i}^*, q_i, q_{-i}^*), \quad \forall (Q_i, q_i) \in K^i, \tag{9}$$

where

$$Q_{-i}^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*) \quad \textit{and} \quad q_{-i}^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

According to (9), a differentiated product supply chain equilibrium is established if no firm can unilaterally improve upon its profits by choosing an alternative vector of product shipments and quality levels of its product.

The Variational Inequality Formulation

In the following theorem, we present the variational inequality (VI) formulation of the above differentiated product supply chain network equilibrium.

Theorem 1: Variational Inequality Formulation of the Differentiated Product Supply Chain Network Equilibrium Model with Quality

Assume that for each firm i; $i = 1, \ldots, I$, the profit function $U_i(Q,q)$ is concave with respect to the variables in Q_i and q_i, and is continuous and continuously differentiable. Then the product shipment and quality pattern $(Q^*, q^*) \in K$ is a differentiated product supply chain network equilibrium with quality according to Definition 1 if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_D}\frac{\partial U_i(Q^*,q^*)}{\partial Q_{ijk}}\times(Q_{ijk}-Q^*_{ijk})-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\frac{\partial U_i(Q^*,q^*)}{\partial q_{ij}}\times(q_{ij}-q^*_{ij})\geq 0, \quad \forall (Q,q)\in K, \tag{10}$$

that is,

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_D}\left[-\hat{\rho}_{ik}(Q^*,q^*)-\sum_{l=1}^{n_D}\frac{\partial\hat{\rho}_{il}(Q^*,q^*)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q^*_{ihl}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial Q_{ijk}}+\sum_{h=1}^{n_i}\sum_{l=1}^{n_D}\frac{\partial\hat{c}_{ihl}(Q^*,q^*)}{\partial Q_{ijk}}\right]$$
$$\times(Q_{ijk}-Q^*_{ijk})$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\left[-\sum_{k=1}^{n_D}\frac{\partial\hat{\rho}_{ik}(Q^*,q^*)}{\partial q_{ij}}\sum_{h=1}^{n_i}Q^*_{ihk}+\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^*,q^*)}{\partial q_{ij}}+\sum_{h=1}^{n_i}\sum_{k=1}^{n_D}\frac{\partial\hat{c}_{ihk}(Q^*,q^*)}{\partial q_{ij}}\right]$$
$$\times(q_{ij}-q^*_{ij})\geq 0, \quad \forall (Q,q)\in K. \tag{11}$$

Proof: Follows the same arguments as in the proof of Theorem 1 in Nagurney and Li (2014) for a supply chain network model without product differentiation. $\square$

Variational inequality (11) (cf. (10)) is now put into standard form (Nagurney (1999)): determine $X^* \in \mathcal{K} \subset R^N$ such that:

$$\langle F(X^*), X - X^*\rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space. $F(X)$ is a given continuous function such that $F(X) : X \to \mathcal{K} \subset R^N$. $\mathcal{K}$ is a closed and convex set.

For the differentiated product supply chain network equilibrium model without trade interventions, we define $X \equiv (Q,q)$ and $F(X) \equiv (F^1(X), F^2(X))$ with $F^1_{ijk}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}$; $i = 1,\ldots,I$; $j = 1,\ldots,n_i$; $k = 1,\ldots,n_D$, and $F^2_{ij}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial q_{ij}}$; $i = 1,\ldots,I$; $j = 1,\ldots,n_i$. Also, we define the feasible set $\mathcal{K} \equiv K$, and let $N = In_in_D + In_i$. Then, variational inequality (11) (cf. (10)) can be put into the above standard form.

For further background on the variational inequality problem and supply chain network problems, we refer the reader to the books by Nagurney (2006) and Nagurney and Li (2016). We emphasize the above model (even without the inclusion of any trade policies) generalizes the model of Nagurney and Li (2014) in that the demand price functions are differentiated by producing firm; that is, the consumers at a demand market display their preferences accordingly.

3.2 The Differentiated Product Supply Chain Network Equilibrium Models with a Strict Quota or Tariff

Using the supply chain network model in Section 3.1 as a basis, we now construct extensions that incorporate trade instruments such as a strict quota or tariff.

3.2.1 The Differentiated Product Supply Chain Network Equilibrium Model with a Strict Quota

We first consider the inclusion of a strict quota. Specifically, we consider a country imposing a strict quota on the product flows from another country. We denote the quota by $\bar{Q}$ and we group the production sites $\{j\}$ in the country on which the quota is imposed on into the set O and the demand markets $\{k\}$ in the country imposing the quota into the set D. We then define the group $\mathcal{G}$ consisting of all these production site and demand market pairs (j,k).

Under a strict quota regime, the following additional constraint must be satisfied:

$$\sum_{i=1}^{I} \sum_{(j,k)\in\mathcal{G}} Q_{ijk} \le \bar{Q}. \tag{12}$$

Observe that constraint (12) is a common or *shared* constraint associated with firms having production sites in the country on which the quota is imposed. Hence, we will need to expand the original feasible set K applied in the model in Section 3.1. We define the set $\mathcal{S}$ as:

$$\mathcal{S} \equiv \{Q|(12)\,\text{holds}\}. \tag{13}$$

Note that, the utility function (7) of a firm i depends on not only its own strategies, but also on those of the other firms. The feasible set of a firm i does as well, which now corresponds to $K_i \cap \mathcal{S}$. Therefore, the governing equilibrium concept is no longer that of Nash Equilibrium, as was the case for the model in Section 3.1, but, rather, is that of a Generalized Nash Equilibrium (GNE).

Definition 2: A Differentiated Product Supply Chain Generalized Nash Network Equilibrium with Quality

A product shipment and quality level pattern $(Q^*, q^*) \in K \cap \mathcal{S}$ is a differentiated product supply chain Generalized Nash Network Equilibrium with quality if for each firm i; $i = 1, \ldots, I$,

$$U_i(Q_i^*, Q_{-i}^*, q_i^*, q_{-i}^*) \geq U_i(Q_i, Q_{-i}^*, q_i, q_{-i}^*), \quad \forall (Q_i, q_i) \in K^i \cap \mathcal{S}. \tag{14}$$

As noted in Nagurney et al. (2017), for a supply chain network problem but without quality and in the absence of any trade instruments, a refinement of the Generalized Nash Equilibrium is a *variational equilibrium* and it is a specific type of GNE (see Facchinei and Kanzow (2010) and Kulkarni and Shanbhag (2012)). In particular, in a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the shared/coupling constraints are all the same. This implies that the firms whose production sites are affected by the quota share a common perception of the strict quota in order to respect it.

Specifically, we have the following definition:

Definition 3: Variational Equilibrium

A vector $(Q^*, q^*) \in K \cap \mathcal{S}$ is said to be a variational equilibrium of the above Generalized Nash Network Equilibrium if it is a solution of the variational inequality

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{n_D} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) - \sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0,$$
$$\forall (Q, q) \in K \cap \mathcal{S}. \tag{15}$$

A solution to variational inequality (15) is guaranteed to exist, under the assumption that the demand for products would be finite, since then the feasible set $K \cap \mathcal{S}$ is compact, and the function that enters the variational inequality (15) is continuous (cf. Kinderlehrer and Stampacchia (1980)).

Moreover, with a variational inequality formulation of the supply chain network model with strict quotas, we can avail ourselves of algorithmic schemes which are more highly developed than those for quasivariational inequalities (cf. Bensoussan and Lions (1978) and Baiocchi and Capelo (1984)), which have been used as a formalism for GNEs (Facchinei and Kanzow (2010)).

We define a new feasible set $\mathcal{K}$ which consists of $(Q,q) \in K$ and $\lambda \in R^1_+$. We now provide a variational inequality which utilizes a Lagrange multiplier λ associated with the strict quota constraint (12). Utilizing the results in Nagurney (2018) and Gossler et al. (2019), we have the following corollary.

Corollary 1: Alternative Variational Inequality Formulation of the Differentiated Product Supply Chain Network Equilibrium Model with Quality and a Strict Quota

An alternative variational inequality to the one (15) is:

$$-\sum_{i=1}^{I}\sum_{(j,k)\notin\mathcal{G}}\frac{\partial U_i(Q^*,q^*)}{\partial Q_{ijk}}\times(Q_{ijk}-Q^*_{ijk})+\sum_{i=1}^{I}\sum_{(j,k)\in\mathcal{G}}(-\frac{\partial U_i(Q^*,q^*)}{\partial Q_{ijk}}+\lambda^*)\times(Q_{ijk}-Q^*_{ijk})$$

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\frac{\partial U_i(Q^*,q^*)}{\partial q_{ij}}\times(q_{ij}-q^*_{ij})$$

$$+(\bar{Q}-\sum_{i=1}^{I}\sum_{(j,k)\in\mathcal{G}}Q^*_{ijk})\times(\lambda-\lambda^*)\geq 0,\quad \forall (Q,q,\lambda)\in\mathcal{K}, \tag{16}$$

where $(Q^*,q^*,\lambda^*)\in\mathcal{K}$.

Observe that the variational inequality (16) is defined on a feasible set with special structure, including box-type constraints, a feature that will yield closed form expressions for the product flows, the quality levels, and the Lagrange multiplier, at each iteration of the algorithmic scheme that we propose in the next section.

Now, we put variational inequality (16) into standard form (cf. Section 3.1). For the differentiated product supply chain network equilibrium model with a strict quota, define $X \equiv (Q,q,\lambda)$ and $F(X) \equiv (F^3(X), F^4(X), F^5(X), F^6(X))$ with $F^3_{ijk} \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}; i = 1,\ldots,I; (j,k)\notin\mathcal{G}$, $F^4_{ijk}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}+\lambda; i=1,\ldots,I; (j,k)\in\mathcal{G}$, $F^5_{ij}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial q_{ij}}; i = 1,\ldots,I; j=1,\ldots,n_i$, and $F^6(X)\equiv \bar{Q}-\sum_{i=1}^{I}\sum_{(j,k)\in\mathcal{G}}Q_{ijk}$. Also, we define the feasible set $\mathcal{K}\equiv\mathcal{K}$, and let $N = In_in_D + In_i + 1$. Then, variational inequality (16) can be put into the standard form (cf. Section 2.1).

3.2.2 The Differentiated Product Supply Chain Network Equilibrium Model with a Tariff

We now introduce the tariff model based on the model in Section 3.1. We consider a group $\mathcal{G}$ as in the strict quota model in Section 3.2.1, but, rather than imposing a strict quota, we impose a unit tariff τ^* on the production sites in the country subject to the trade policy instrument.

The utility functions $\hat{U}_i$; $i = 1, \ldots, I$, then take the form

$$\hat{U}_i = U_i - \sum_{i=1}^{I} \sum_{(j,k)\in\mathcal{G}} \tau^* Q_{ijk}, \tag{17}$$

with U_i; $i = 1, \ldots, I$, as in (7).

The definition of an equilibrium then follows according to (9) but with $\hat{U}_i$ substituted for the U_i; $i = 1, \ldots, I$. The following Theorem is immediate from Theorem 1:

Theorem 2: Variational Inequality Formulation of the Differentiated Product Model Supply Chain Network Equilibrium Model with Quality and a Tariff

Under the same assumptions as in Theorem 1, a product shipment and quality pattern $(Q^*, q^*) \in K$ is a differentiated product supply chain network equilibrium according to Definition 1 with $\hat{U}_i$ replacing U_i for $i = 1, \ldots, I$, if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \sum_{(j,k)\notin\mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) + \sum_{i=1}^{I} \sum_{(j,k)\in\mathcal{G}} \left(-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \tau^*\right) \times (Q_{ijk} - Q^*_{ijk})$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij}) \geq 0, \quad \forall (Q, q) \in K. \tag{18}$$

We now put variational inequality (18) into standard form (cf. Section 3.1). For the differentiated product supply chain network equilibrium model with a tariff, define $X \equiv (Q, q)$ and $F(X) \equiv (F^7(X), F^8(X), F^9(X))$ with $F^7_{ijk}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}; i = 1, \ldots, I; (j,k) \notin \mathcal{G}$, $F^8_{ijk}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}} + \tau^*; i = 1, \ldots, I; (j,k) \in \mathcal{G}$, and $F^9_{ij}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial q_{ij}}; i = 1, \ldots, I; j = 1, \ldots, n_i$. We also define the feasible set $\mathcal{K} \equiv K$, and let $N = In_in_D + In_i$. Then, variational inequality (18) can be put into the standard form (cf. Section 3.1).

3.3 Relationships Between the Model with a Strict Quota and the Model with a Tariff

In this Subsection, we explore the relationships between the above two models with trade policy instruments.

Specifically, we are interested in the case where $\lambda^* > 0$ in the strict quota model, which occurs when the strict quota is reached. Below, we first establish that a solution to the variational inequality (16) governing the strict quota model also satisfies the variational inequality (18) governing the model with a tariff where the tariff $\tau^* = \lambda^*$.

From variational inequality (16) we know that for $(Q^*, q^*, \lambda^*) \in \mathcal{K}$:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} (-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \lambda^*) \times (Q_{ijk} - Q^*_{ijk})$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij}) \geq$$

$$(\bar{Q} - \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} Q^*_{ijk}) \times (\lambda^* - \lambda). \tag{19}$$

Setting $\lambda = \lambda^*$ in (19) and then $\tau^* = \lambda^*$ yields:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} (-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \tau^*) \times (Q_{ijk} - Q^*_{ijk})$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij}) \geq 0, \quad \forall (Q, q) \in K, \tag{20}$$

which is precisely variational inequality (18) governing the tariff model with product differentiation and quality.

We now investigate whether a solution to VI (18) governing the model with a tariff will also solve VI (16) governing the strict quota model.

Set

$$\bar{Q} \equiv \sum_{(j,k) \in \mathcal{K}} Q^*_{ijk} \tag{21}$$

with the Q^*_{ijk} in (21) as in (18), and to each side of (18) then add the term:

$$(\bar{Q} - \sum_{(j,k) \in \mathcal{G}} Q^*_{ijk}) \times (\tau - \tau^*). \tag{22}$$

This can be done since the expression in (22) is equal to 0.

The above leads to:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} (-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \tau^*) \times (Q_{ijk} - Q^*_{ijk})$$

$$
\begin{aligned}
&-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\frac{\partial U_i(Q^*,q^*)}{\partial q_{ij}}\times(q_{ij}-q_{ij}^*)+(\bar{Q}-\sum_{(j,k)\in\mathcal{G}}Q_{ijk}^*)\times(\tau-\tau^*)\\
&\geq(\bar{Q}-\sum_{(j,k)\in\mathcal{G}}Q_{ijk}^*)\times(\tau-\tau^*)=0,\quad\forall(Q,q)\in K\,\text{and}\,\tau\geq 0.
\end{aligned}
\tag{23}
$$

But (23) is precisely VI (16) if we use the notation $\lambda=\tau$.

Hence, we have established the following equivalence. When the strict quota constraint (i.e., (12)) is tight, the equilibrium pattern of the VI with the strict quota also satisfies the one with a tariff. This result requires that the tariff be imposed on the same product shipment group as the strict quota and set to the equilibrium Lagrange multiplier associated with the strict quota constraint.

The above relationship also provides a nice interpretation for the Lagrange multiplier associated with the strict quota in that it is a price or, in effect, a tariff.

3.4 Consumer Welfare with or without Tariffs or Quotas

A measure of the consumer welfare with or without tariffs or quotas is now provided. The consumer welfare associated with product i at demand market k at equilibrium with or without a strict quota, CW_{ik}, is

$$
CW_{ik}=\int_0^{d_{ik}^*}\rho_{ik}(d_{-ik}^*,d_{ik},\hat{q}^*)\,\mathrm{d}(d_{ik})-\rho_{ik}(d^*,\hat{q}^*)d_{ik}^*,\quad i=1,\ldots,I;k=1,\ldots,n_D,
\tag{24}
$$

where $d_{-ij}^*\equiv(d_{11}^*,\ldots,d_{i,j-1}^*,d_{i,j+1}^*,\ldots,d_{mn}^*)$ (cf. Spence (1975) and Wildman (1984)).

The first term in (24) calculates the maximum total price that consumers at demand market k are willing to pay for the amount of product i that satisfies their demand at equilibrium. The second term expresses the actual total price paid by consumers for their demand. The difference then measures the benefit that consumers obtain when purchasing the product, which is the consumer welfare associated with product i at demand market k at equilibrium.

3.5 Illustrative Examples

In this Subsection, we present illustrative examples to demonstrate the relevance of the models. In Section 5, we provide a more detailed numerical analysis through examples inspired by an agricultural product - that of soybeans.

We assume a simple network structure for purposes of illustrating our mathematical framework. The supply chain network topology of the illustrative examples is depicted in

Figure 2. We assume that there are two firms, Firm 1 and Firm 2, competing to sell their products at a single demand market, Demand Market 1. Firm 1 has available the production site P_1^1, whereas Firm 2 has the production site P_2^1. The production sites, P_1^1 and P_2^1, are located in different countries, with Demand Market 1 located in the same country as P_1^1. We refer to P_1^1 and the demand market as being *domestic*.

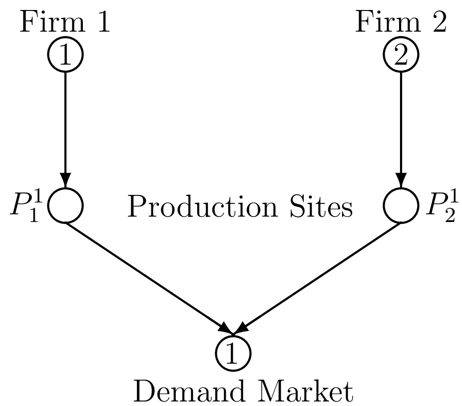

Figure 2: The Supply Chain Network Topology for the Illustrative Examples

The simplicity of the supply chain network topology allows us to immediately write down the conservation of flow equations (1) and (2) as:

$$s_{11} = d_{11} = Q_{111}, \quad s_{21} = d_{21} = Q_{211}.$$

We assume that the cost of production at a production site depends on the product flow and on the quality level. The production costs of Firm 1 and Firm 2 are:

$$\hat{f}_{11}(Q, q) = Q_{111}^2 + 3Q_{111} + q_{11},$$

$$\hat{f}_{21}(Q, q) = Q_{211}^2 + Q_{211} + 0.5q_{21}^2.$$

The total transportation cost associated with distributing a firm's product also depends on the product flow and the quality level of the products. As mentioned previously, the total transportation cost of Firm 2 is higher, since the distance between the demand market and the site P_2^1 is greater than that for the site P_1^1 and the demand market. In particular, the transportation cost functions are:

$$\hat{c}_{111}(Q, q) = Q_{111}^2 + 0.5Q_{111} + q_{11}, \quad \hat{c}_{211}(Q, q) = Q_{211}^2 + Q_{211} + 2q_{21}.$$

The average quality level expressions are:

$$\hat{q}_{11} = \frac{q_{11}Q_{111}}{Q_{111}} = q_{11}, \quad \hat{q}_{21} = \frac{q_{21}Q_{211}}{Q_{211}} = q_{21}.$$

We set the quality upper bounds as: $\bar{q}_{11} = \bar{q}_{21} = 100$. The minimum quality standards are: $\underline{q}_{11} = \underline{q}_{21} = 0.8$.

The demand price functions for the products of Firm 1 and Firm 2 at the demand market, are, in turn, functions of the average quality levels, $\hat{q}_{11}$, $\hat{q}_{22}$, and the product flows, as follows:

$$\hat{\rho}_{11}(Q, q) = -(Q_{111} + Q_{211}) + 0.5q_{11} + 20,$$

$$\hat{\rho}_{21}(Q, q) = -(Q_{211} + Q_{111}) + q_{21} + 25.$$

In the following subsections, we first solve variational inequality (11) governing the differentiated supply chain network equilibrium model with quality but without any trade policy instruments. Then, we incorporate a strict quota, and further demonstrate the equivalence to the model with a tariff, with the tariff corresponding to the equilibrium Lagrange multiplier associated with the solution of the former model.

The parameters in the cost and demand functions are reasonable for the size and the location of the hypothetical firms.

3.5.1 Illustrative Example without Trade Interventions

In this example, a strict quota or tariff is not considered, and we solve variational inequality (11). According to (11), it is reasonable to make the assumption that $Q^*_{111} > 0$, $Q^*_{211} > 0$, $\bar{q}_{11} > q^*_{11} > \underline{q}_{11}$, and $\bar{q}_{21} > q^*_{21} > \underline{q}_{21}$. Therefore, we have the following expressions, all of which are equal to 0:

$$\frac{\partial \hat{f}_{11}(Q^*, q^*)}{\partial Q_{111}} + \frac{\partial \hat{c}_{111}(Q^*, q^*)}{\partial Q_{111}} - \hat{\rho}_{11}(Q^*, q^*) - \frac{\partial \hat{\rho}_{11}(Q^*, q^*)}{\partial Q_{111}} Q^*_{111} = 0, \tag{25}$$

$$\frac{\partial \hat{f}_{11}(Q^*, q^*)}{\partial q_{11}} + \frac{\partial \hat{c}_{111}(Q^*, q^*)}{\partial q_{11}} - \frac{\partial \hat{\rho}_{11}(Q^*, q)}{\partial q_{11}} Q^*_{111} = 0, \tag{26}$$

$$\frac{\partial \hat{f}_{21}(Q^*, q^*)}{\partial Q_{211}} + \frac{\partial \hat{c}_{211}(Q^*, q^*)}{\partial Q_{211}} - \hat{\rho}_{21}(Q^*, q^*) - \frac{\partial \hat{\rho}_{21}(Q^*, q^*)}{\partial Q_{211}} Q^*_{211} = 0, \tag{27}$$

$$\frac{\partial \hat{f}_{21}(Q^*, q^*)}{\partial q_{21}} + \frac{\partial \hat{c}_{211}(Q^*, q^*)}{\partial q_{21}} - \frac{\partial \hat{\rho}_{21}(Q^*, q^*)}{\partial q_{21}} Q^*_{211} = 0. \tag{28}$$

Inserting the corresponding functions into the above equations (25) – (28), we obtain the following system of equations:

$$6Q^*_{111} + Q^*_{211} - 0.5q^*_{11} = 16.5,$$

$$0.5Q^*_{111} = 2,$$

$$6Q^*_{211} + Q^*_{111} - q^*_{21} = 23,$$

$$Q^*_{211} - q^*_{21} = 2,$$

with solution:

$$Q^*_{111} = 4.00, \quad Q^*_{211} = 3.40, \quad q^*_{11} = \hat{q}_{11} = 21.80, \qquad q^*_{21} = \hat{q}_{21} = 1.40.$$

Furthermore, the demand prices at equilibrium, in dollars, are: $\rho_{11} = 23.50$ and $\rho_{21} = 19.00$. The profits of the firms, in dollars, are: $U_1 = 4.40$ and $U_2 = 30.90$. It is evident that, even though the quality of Firm 2 is lower and it sells less at the demand market, it enjoys a higher profit than Firm 1. This is caused by the fact that the price of Firm 2's product is lower. The consumer welfare associated with the two firms' products is, respectively, $CW_{11} = 8.00$ and $CW_{21} = 5.78$.

3.5.2 An Illustrative Example with a Strict Quota and Tariff Equivalence

We now use, as a baseline, the illustrative example in Subsection 3.5.1, and impose a strict quota on the product flow from the nondomestic production site. Subsequently, we demonstrate the theoretical result of Section 3.3 of the equivalence of the equilibrium solutions to the model with a strict quota and the model with a tariff, over the same group, through this example, with the tariff for the latter being set to the equilibrium Lagrange multiplier of the former. The group consists of the production site P^1_2 of Firm 2 and Demand Market 1, since the country that the demand market is located in imposes a quota on the products from the country that the production site P^1_2 of Firm 2 is located in. Hence, the strict quota $\bar{Q}$ is imposed only on the product flow in this group. We set the strict quota $\bar{Q} = 3$ and, consequently, the strict quota constraint in (12) becomes:

$$Q_{211} \leq 3. \tag{29}$$

Similar to the expressions in (25) – (28), the variational inequality formulation in (16) yields the following expressions, which are all equal 0, since it is reasonable that the equilibrium product flows will be positive and the quality levels will not be at the boundaries:

$$\frac{\partial \hat{f}_{11}(Q^*, q^*)}{\partial Q_{111}} + \frac{\partial \hat{c}_{111}(Q^*, q^*)}{\partial Q_{111}} - \hat{\rho}_{11}(Q^*, q^*) - \frac{\partial \hat{\rho}_{11}(Q^*, q^*)}{\partial Q_{111}} Q^*_{111} = 0, \tag{30}$$

$$\frac{\partial \hat{f}_{11}(Q^*, q^*)}{\partial q_{11}} + \frac{\partial \hat{c}_{111}(Q^*, q^*)}{\partial q_{11}} - \frac{\partial \hat{\rho}_{11}(Q^*, q^*)}{\partial q_{11}} Q^*_{111} = 0, \tag{31}$$

$$\frac{\partial \hat{f}_{21}(Q^*, q^*)}{\partial Q_{211}} + \frac{\partial \hat{c}_{211}(Q^*, q^*)}{\partial Q_{211}} - \hat{\rho}_{21}(Q^*, q^*) - \frac{\partial \hat{\rho}_{21}(Q^*, q^*)}{\partial Q_{211}} Q^*_{211} + \lambda^* = 0, \tag{32}$$

$$\frac{\partial \hat{f}_{21}(Q^*, q^*)}{\partial q_{21}} + \frac{\partial \hat{c}_{211}(Q^*, q^*)}{\partial q_{21}} - \frac{\partial \hat{\rho}_{21}(Q^*, q^*)}{\partial q_{21}} Q^*_{211} = 0, \tag{33}$$

$$\bar{Q} - Q^*_{211} = 0. \tag{34}$$

Notice that the expressions are very similar to (25) - (28). However, now, in (32), the Lagrange multiplier associated with the strict quota constraint (29) is added and (34) is a new linear equation.

By inserting the appropriate functions into (30) – (33), with the strict quota $\bar{Q} = 3$, we obtain the following system of equations:

$$6Q^*_{111} + Q^*_{211} - 0.5q^*_{11} = 16.5,$$

$$0.5Q^*_{111} = 2,$$

$$6Q^*_{211} + Q^*_{111} - q^*_{21} + \lambda^* = 23,$$

$$Q^*_{211} - q^*_{21} = 2,$$

$$Q^*_{211} = 3.$$

The equilibrium product flows, quality levels, and the Lagrange multiplier are, hence:

$$Q^*_{111} = 4.00, \quad Q^*_{211} = 3.00, \quad q^*_{11} = \hat{q}_{11} = 21.00, \quad q^*_{21} = \hat{q}_{21} = 1.00,$$

$$\lambda^* = 2.00.$$

The demand prices at equilibrium of Firm 1 and Firm 2 are: $\rho_{11} = 23.50$ and $\rho_{21} = 19.00$ and are the same as in the example without the strict quota. The profits of the firms are now: $U_1 = 6.00$ and $U_2 = 24.50$. With the strict quota imposed, the quality levels of the products, as well as the average quality at the demand market, decrease from their respective values when there is no imposed quota. In addition, the consumer welfare associated with the firms' products is now: $CW_{11} = 8.00$ and $CW_{21} = 4.50$. The value of CW_{21} is lower than the corresponding one for the example without a strict quota.

Further, using the result obtained in Section 3.3, provided that $\lambda^* = 2.00$ is the assigned tariff τ^* in the tariff model, the above equilibrium product flows and equilibrium quality levels then solve VI (18).

Hence, the above numerical example demonstrates that a strict quota or tariff may adversely affect both the quality of the products and the consumer welfare, which is clearly not good for consumers. However, the profit of the domestic firm increases, whereas that of the firm with the nondomestic production site decreases.

4. The Algorithm

The algorithm that we utilize to compute the solutions for the differentiated product supply chain network equilibrium models with quality is the modified projection method (see Korpelevich (1977) and Nagurney (1999)). This algorithm is guaranteed to converge if the function F that enters the standard form of the variational inequality (cf. Section 3.1) satisfies monotonicity and Lipschitz continuity (see Nagurney (1999)) and that a solution exists.

Recall that the function $F(X)$ is said to be monotone, if

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K},$$

and that the function $F(X)$ is Lipschitz continuous, if there exists a constant $L > 0$, known as the Lipschitz constant, such that

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}.$$

The statement of the modified projection method is as follows, with t denoting an iteration counter:

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $t := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{35}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{36}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t+1$ and go to Step 1.

Steps 1 and 2 of the modified projection method (cf. (35) and (36)) yield explicit formulae for the computation of the product flows, quality levels, and the Lagrange multiplier, i.e., the equivalent tariff, for the three models developed in Section 2. In particular, at each iteration of the algorithm, we have the following explicit formulae for the three models, respectively, for Step 1. The corresponding explicit formulae for Step 2 are similar in form.

Explicit Formulae for the Differentiated Product Supply Chain Network Equilibrium Model Variables without Trade Interventions in Step 1 of the Modified Projection Method

$$\bar{Q}_{ijk}^{t+1} = \max\{0, Q_{ijk}^t + \beta(\hat{\rho}_{ik}(Q^t, q^t) + \sum_{l=1}^{n_D} \frac{\partial \hat{\rho}_{il}(Q^t, q^t)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial Q_{ijk}}$$

$$-\sum_{h=1}^{n_i} \sum_{l=1}^{n_D} \frac{\partial \hat{c}_{ihl}(Q^t, q^t)}{\partial Q_{ijk}})\}, \quad i = 1, \dots, I; j = 1, \dots, n_i; k = 1, \dots, n_D, \tag{37}$$

$$\bar{q}_{ij}^{t+1} = \max\{\underline{q_{ij}}, \min\{\bar{q}_{ij}, q_{ij}^t + \beta(\sum_{k=1}^{n_D} \frac{\partial \hat{\rho}_{ik}(Q^t, q^t)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial q_{ij}}$$

$$-\sum_{h=1}^{n_i} \sum_{k=1}^{n_D} \frac{\partial \hat{c}_{ihk}(Q^t, q^t)}{\partial q_{ij}})\}\}, \quad i = 1, \dots, I; j = 1, \dots, n_i. \tag{38}$$

Explicit Formulae for the Differentiated Product Supply Chain Network Equilibrium Model Variables with a Strict Quota in Step 1 of the Modified Projection Method

$$\bar{Q}_{ijk}^{t+1} = \max\{0, Q_{ijk}^t + \beta(\hat{\rho}_{ik}(Q^t, q^t) + \sum_{l=1}^{n_D} \frac{\partial \hat{\rho}_{il}(Q^t, q^t)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial Q_{ijk}}$$

$$-\sum_{h=1}^{n_i} \sum_{l=1}^{n_D} \frac{\partial \hat{c}_{ihl}(Q^t, q^t)}{\partial Q_{ijk}} - \lambda^t)\}, \quad i = 1, \dots, I; j = 1, \dots, n_i; k = 1, \dots, n_D; (j,k) \in \mathcal{G}, \tag{39}$$

$$\bar{\lambda}^{t+1} = \max\{0, \lambda^t + \beta(\sum_{i=1}^{I} \sum_{(j,k) \in G} Q_{ijk}^t - \bar{Q})\}. \tag{40}$$

The explicit formulae for the product shipments, $\bar{Q}_{ijk}^{t+1}; i = 1, \dots, I; j = 1, \dots, n_i; k = 1, \dots, n_D; (j,k) \notin \mathcal{G}$, are the same as in (37), with the explicit formulae for product quality, $\bar{q}_{ij}^{t+1}; i = 1, \dots, I; j = 1, \dots, n_i$, the same as in (38).

Explicit Formulae for the Differentiated Product Supply Chain Network Equilibrium Model Variables with a Tariff in Step 1 of the Modified Projection Method

$$\bar{Q}_{ijk}^{t+1} = \max\{0, Q_{ijk}^t + \beta(\hat{\rho}_{ik}(Q^t, q^t) + \sum_{l=1}^{n_D} \frac{\partial \hat{\rho}_{il}(Q^t, q^t)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial Q_{ijk}}$$

$$- \sum_{h=1}^{n_i} \sum_{l=1}^{n_D} \frac{\partial \hat{c}_{ihl}(Q^t, q^t)}{\partial Q_{ijk}} - \tau^*)\}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_D; (j, k) \in \mathcal{G}. \quad (41)$$

The explicit formulae for the product shipments, $\bar{Q}_{ijk}^{t+1}; i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_D; (j, k) \notin \mathcal{G}$, and for product quality, $\bar{q}_{ij}^{t+1}; i = 1, \ldots, I; j = 1, \ldots, n_i$, are the same as in (37) and (38), respectively.

5. Numerical Examples

In this Section, we focus on the supply chain network of soybeans, an agricultural product that has a great impact on today's agricultural trade. Soybeans were discovered and domesticated in China over 3000 years ago, with the United States being a leader in producing, consuming, and exporting soybeans globally (Song, Xu, and Marchant (2004)). In the United States, soybean production and export have become essential parts of the agricultural economy, with soybeans ranked second among crops in farm value in 2005 (Ash, Livesey, and Dohlman (2006)). Lundgren (2018) reports that, in 2018, soybean production in the United States reached 5.11 billion bushels with an export of 2.13 billion bushels.

China, in turn, is the largest importer of soybeans due to its rapidly increasing population size (Brown (2012)). The consumption of soybeans in China, in 2017, was reported to be 112.18 million tons, but the domestic production volume was only 13 million tons (Wood (2018)). Due to this huge gap, China has to rely heavily on soybeans imported from foreign countries, such as the United States, Brazil, and Argentina.

In 2018, the trade war between China and the United States escalated, with the Chinese government imposing quotas and tariffs on the soybeans exported from the United States in retaliation (Wong and Koty (2019)). According to Appelbaum (2018), this created an opportunity for other large soybean exporters, such as Brazil and Argentina. In 2017, Brazil exported 53.8 million tons of soybeans to China, corresponding to 75% of its production volume (Zhou et al. (2018)). Shane (2018) claims that the Chinese importer, Hebei Power Sea Feed Technology, bought thousands of tons of soybeans for animal feed from Brazil instead of the United States in 2018.

In this Section, we present a series of numerical examples for the differentiated product supply chain network equilibrium models for soybeans, and examine the effects of quotas and tariffs on soybean trade and quality. The data is created through a simulation study to reflect the real life prices of soybeans. The models in this section were solved via the modified projection method, as described in Section 4, which was implemented in Matlab on an OS X 10.14.1 system. The convergence tolerance was: 10^{-6}, so that the algorithm was deemed to have converged when the absolute value of the difference between each successive product shipment, quality level, and the Lagrange multiplier was less than or equal to 10^{-6}. We set β in the algorithm to .15 and initialized the algorithm with the product shipments equal to 100 and with the quality levels and the Lagrange multiplier equal to 0.

The baseline example, Example 5.1, has no quotas or tariffs imposed. In subsequent examples, we add strict quotas, tariffs, and then also consider the addition of a demand market.

Example 5.1: 2 Firms, with 3 Production Sites for the First Firm, Two Production Sites for the Second, and a Single Demand Market

The supply chain network for soybeans for Example 5.1 is depicted in Figure 3. There are two producing firms, Firm 1 and Firm 2, which are located in the United States. The firms are inspired by Cargill and Archer Daniels Midland Company (ADM), two of the largest agricultural companies in the United States with production sites in the United States and overseas (Zhou et al. (2018)). They are referred to, hence, accordingly.

Recall that the middle nodes represent production sites. Cargill owns three production sites, P_1^1, P_2^1, and P_3^1, which are located in the United States, Brazil, and Argentina, respectively. ADM's production sites are denoted by P_1^2 and P_2^2, and they are located, respectively, in the United States and Brazil. There is a single demand market, Demand Market 1, located in China. We consider a time horizon of a month and the currency is in US dollars.

The production cost functions of Cargill at its production sites, P_1^1, P_2^1, and P_3^1 are:

$$\hat{f}_{11}(Q_{111}, q_{11}) = 0.04Q_{111}^2 + 0.35Q_{111} + 0.4Q_{111}q_{11} + 0.6q_{11}^2,$$

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.05Q_{121}^2 + 0.35Q_{121} + 0.4Q_{121}q_{12} + 0.4q_{12}^2,$$

$$\hat{f}_{13}(Q_{131}, q_{13}) = 0.05Q_{131}^2 + 0.8Q_{131} + 0.4Q_{131}q_{13} + 0.4q_{13}^2.$$

The production cost functions faced by ADM at its production sites, P_1^2 and P_2^2, are:

$$\hat{f}_{21}(Q_{211}, q_{21}) = 0.06Q_{211}^2 + 0.5Q_{211} + 1.2Q_{211}q_{21} + q_{21}^2,$$

$$\hat{f}_{22}(Q_{221}, q_{22}) = 0.07Q_{221}^2 + 0.3Q_{221} + 1.3Q_{221}q_{22} + 1.5q_{22}^2.$$

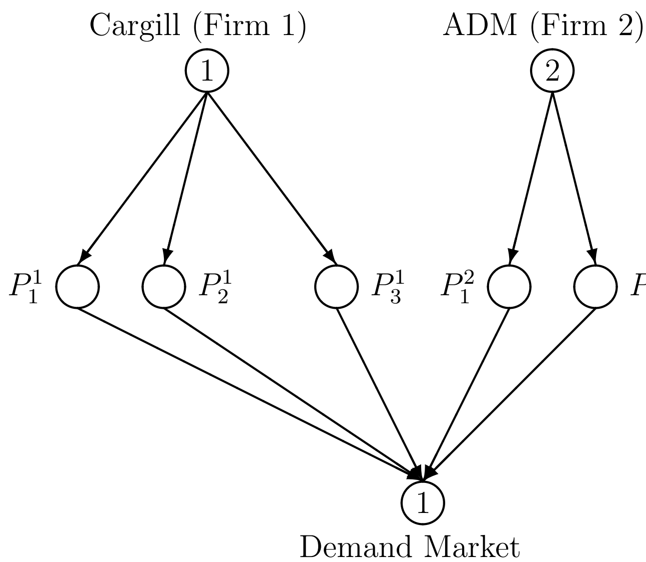

Figure 3: The Supply Chain Network Topology for Example 5.1

The total transportation cost functions associated with Cargill for shipping its soybeans to Demand Market 1 are:

$$\hat{c}_{111}(Q_{111}, q_{11}) = 0.02Q_{111}^2 + 0.2Q_{111} + 0.5q_{11}^2, \quad \hat{c}_{121}(Q_{121}, q_{12}) = 0.02Q_{121}^2 + 0.4Q_{121} + 0.8q_{12}^2,$$

$$\hat{c}_{131}(Q_{131}, q_{13}) = 0.02Q_{131}^2 + 0.5Q_{131} + 0.8q_{13}^2,$$

and ADM's total transportation cost functions are:

$$\hat{c}_{211}(Q_{211}, q_{21}) = 0.02Q_{211}^2 + 0.5Q_{211} + 0.6q_{21}^2, \quad \hat{c}_{221}(Q_{221}, q_{22}) = 0.02Q_{221}^2 + 0.4Q_{221} + 0.8q_{22}^2.$$

The demand price functions for the soybeans of Cargill and ADM at Demand Market 1 are:

$$\rho_{11}(d, \hat{q}) = 1500 - (0.3d_{11} + 0.2d_{21}) + 0.7\hat{q}_{11},$$

$$\rho_{21}(d, \hat{q}) = 1600 - (0.35d_{21} + 0.3d_{11}) + 2\hat{q}_{21},$$

with the average quality $\hat{q}_{11}$ and $\hat{q}_{21}$ being:

$$\hat{q}_{11} = \frac{Q_{111}q_{11} + Q_{121}q_{12} + Q_{131}q_{13}}{Q_{111} + Q_{121} + Q_{131}}, \quad \hat{q}_{21} = \frac{Q_{211}q_{21} + Q_{221}q_{22}}{Q_{211} + Q_{221}}.$$

Furthermore, the upper and lower bounds of quality levels are:

$$\bar{q}_{11} = \bar{q}_{12} = \bar{q}_{13} = \bar{q}_{21} = \bar{q}_{22} = 100,$$

$$\underline{q}_{11} = \underline{q}_{12} = \underline{q}_{13} = \underline{q}_{21} = \underline{q}_{22} = 10.$$

The modified projection method yielded the following equilibrium product flows of soybeans, in tons, as well as the equilibrium quality levels in Table 2. The demands for soybeans at equilibrium and the average quality are reported in Table 3.

Equilibrium Flows	**Results**	**Equilibrium Quality**	**Results**
Q^*_{111}	756.70	q^*_{11}	100.00
Q^*_{121}	591.26	q^*_{12}	73.91
Q^*_{131}	585.90	q^*_{13}	73.24
Q^*_{211}	779.32	q^*_{21}	100.00
Q^*_{221}	612.32	q^*_{22}	93.18

Table 2: Equilibrium Soybean Flows and Equilibrium Quality Levels for Example 5.1

Demand	**Results**	**Average Quality**	**Results**
d^*_{11}	1,933.86	$\hat{q}_{11}$	83.91
d^*_{21}	1,391.64	$\hat{q}_{21}$	97.00

Table 3: Equilibrium Demands and Average Quality for Example 5.1

The average quality of Cargill's soybeans is lower than that of soybeans produced by ADM. This is because, as indicated in the demand price functions, the price of ADM's soybeans is more sensitive to higher average quality at the demand market than the price of Cargill's soybeans. Therefore, ADM has a stronger incentive to provide higher quality than Cargill in Demand Market 1.

We also report the equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, the consumer welfare associated with the two firms at the demand market, and the profits of Cargill and ADM, in dollars, in Table 4.

Demand Price	**Results**	**Consumer Welfare**	**Results**	**Profits**	**Results**
ρ_{11}	700.25	CW_{11}	560,973.35	U_1	1,180,812.05
ρ_{21}	726.76	CW_{21}	338,916.81	U_2	724,196.08

Table 4: Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.1

It is seen from Table 3 and Table 4 that Cargill's soybeans have an associated higher demand, a lower price, a higher associated consumer welfare, and Cargill enjoys a higher profit than does ADM, but with a lower average quality of its soybeans at Demand Market 1. For both firms, the production site in the United States produces the greatest amount of soybeans with also the highest quality as compared to the overseas sites. The reported demand prices of soybeans are close to the actual soybean price per ton, which was between 550-600 dollars in China in 2018 (Gu and Mason (2018)).

Example 5.2: Example 5.1 with a Strict Quota and Sensitivity Analysis

This example considers the same differentiated product supply chain network problem as in Example 5.1, but with the imposition of a strict quota of $\bar{Q} = 1200$ by the Chinese government on imports from US production sites, that is, the production site P_1^1 of Cargill and the production site P_1^2 of ADM. We report the equilibrium soybean flows, equilibrium quality levels, equilibrium demands, and average quality results in Table 5 and Table 6.

Equilibrium Flows	**Results**	**Equilibrium Quality**	**Results**
Q_{111}^*	528.96	q_{11}^*	72.13
Q_{121}^*	697.99	q_{12}^*	87.25
Q_{131}^*	692.63	q_{13}^*	86.58
Q_{211}^*	671.04	q_{21}^*	100.00
Q_{221}^*	708.75	q_{22}^*	100.00

Table 5: Equilibrium Soybean Flows and Equilibrium Quality Levels for Example 5.2

Demand	**Results**	**Average Quality**	**Results**
d_{11}^*	1,919.58	$\hat{q}_{11}$	82.84
d_{21}^*	1,379.79	$\hat{q}_{21}$	100.00

Table 6: Equilibrium Demands and Average Quality for Example 5.2

Notice that the equilibrium soybean flows, Q_{111}^* and Q_{211}^*, decrease from their values in Example 5.1, due to the strict quota imposed on the soybeans from the United States. Meanwhile, the soybean exports from other countries, Q_{121}^*, Q_{131}^*, and Q_{221}^*, increase. This results from the fact that firms now produce more in countries that are not restricted by the quota, that is, in Brazil and Argentina.

Furthermore, the equilibrium Lagrange multiplier, which is the equivalent tariff, is:

$$\lambda^* = 29.91.$$

Note that the Lagrange multiplier is positive since the volume of equilibrium soybean flows to China from the US production sites is equal to the imposed quota $\bar{Q} = 1200$.

When the above strict quota is imposed on United States' imports into China, the quality level associated with Cargill's production site located in the United States, q_{11}^*, decreases from its value in Example 5.1. Interestingly, the quality level of ADM's soybeans produced in the United States, q_{21}^*, does not show a change. However, q_{12}^* and q_{13}^*, denoting the quality of Cargill's soybeans produced in Brazil and Argentina, increase. Similarly, the quality of ADM's soybeans produced in Brazil, q_{22}^*, increases to its upper bound. These results indicate that, when a strict quota is imposed, the quality levels of the products produced under the quota are affected negatively or stay the same, whereas the quality of the products that are produced at sites not subject to the quota increases. Producers may wish to sustain prices as high as feasible and that can be accomplished through higher product quality.

Next, we report the equilibrium incurred demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, and the consumer welfare and the profits achieved by Cargill and ADM, in dollars, in Table 7.

Demand Price	**Results**	**Consumer Welfare**	**Results**	**Profits**	**Results**
ρ_{11}	706.16	CW_{11}	552,718.33	U_1	1,181,876.72
ρ_{21}	741.20	CW_{21}	333,167.37	U_2	728,637.06

Table 7: Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.2

Notice that, at the demand market, which is in China, the prices of soybeans increase as compared to their values in Example 5.1. The consumer welfare associated with both firms' soybeans at the demand market decreases. Hence, consumers in China are negatively impacted by the strict quota.

Interestingly, due to increases in prices, the profits of both firms increase, as compared to Example 5.1 without the quota. The imposed quota does create an advantage for firms in this example. The two firms, by having soybean production sites in countries not under the strict quota, are able to expand their production at those sites. Hence, in a sense, they are more resilient to the imposition of a quota than they might be otherwise. In a later example, Example 5.4, we consider the same problem as in Example 5.2 but with Cargill's production site in the United States shut down.

Sensitivity Analysis: Impacts of a Quality Coefficient Change in a Cost Functions

We now test the robustness of the managerial insights achieved in Example 5.2. Specifically, we test whether the following four insights obtained from the solution of Example 5.2 also hold when varying the quality coefficient in a production cost function, $\hat{f}_{12}$, not under a strict quota.

Recall that, in Example 5.2, with the imposition of a strict quota, $\bar{Q} = 1200$, by China on imports from the United States, the following results were observed:

1. The soybean flows from the United States production sites, Q^*_{111} and Q^*_{211}, decreased. Meanwhile, the soybean exports from other countries, Q^*_{121}, Q^*_{131}, and Q^*_{221}, increased.

2. The quality levels of the soybeans produced in the United States, q^*_{11} and q^*_{21}, decreased or stayed the same. However, q^*_{12}, q^*_{13}, and q^*_{22}, denoting the quality of the soybeans produced elsewhere, increased.

3. At the demand market, the prices of soybeans increased.

4. The consumer welfare associated with both firms' soybeans decreased, but both firms made higher profits.

In Tables 8 and 9 we provide sensitivity analysis results for Example 5.2 by varying the coefficient of q^2_{12} in $\hat{f}_{12}$, with and without the strict quota. The values of the coefficient are: 0.2, 0.4, 0.6, 0.8, and 1, respectively. "Coeff." in Tables 8 and 9 denotes "Coefficient."

Coeff. of q^2_{12} in $\hat{f}_{12}$	**Quota**	Q^*_{111}	Q^*_{121}	Q^*_{131}	Q^*_{211}	Q^*_{221}	q^*_{11}	q^*_{12}	q^*_{13}	q^*_{21}	q^*_{22}
0.2	with	525.90	716.00	679.90	674.10	705.56	71.71	100.00	84.99	100.00	100.00
0.2	without	743.97	621.85	570.98	778.96	611.51	100	93.28	71.37	100.00	93.06
0.4	with	528.96	697.99	692.63	671.04	708.75	72.13	87.25	86.58	100.00	100.00
0.4	without	756.70	591.26	585.9	779.32	612.32	100.00	73.91	73.24	100.00	93.18
0.6	with	532.48	677.26	707.29	667.52	712.42	72.61	72.56	88.41	100.00	100.00
0.6	without	765.06	571.19	595.68	779.57	612.85	100.00	61.2	74.46	100.00	93.26
0.8	with	534.99	662.5	717.72	665.01	715.03	72.95	62.11	89.72	100.00	100.00
0.8	without	770.96	557.01	602.59	779.74	613.22	100.00	52.22	75.32	100.00	93.32
1.0	with	536.86	651.45	725.53	663.14	716.99	73.21	54.29	90.69	100.00	100.00
1.0	without	775.36	546.46	607.74	779.87	613.5	100.00	45.54	75.97	10.000	93.36

Table 8: Equilibrium Soybean Flows and Equilibrium Quality Levels with a Varying Quality Coefficient in $\hat{f}_{12}$ with and without the Strict Quota

Coeff. of q^2_{12} in $\hat{f}_{12}$	**Quota**	ρ_{11}	ρ_{21}	CW_{11}	CW_{21}	U_1	U_2
0.2	with	708.39	740.58	553996.44	333104.16	1183676.53	727808.73
0.2	without	703.45	726.19	562679.86	338344.67	1182645.7	722969.50
0.4	with	706.16	741.20	552718.33	333167.37	1181876.72	728637.06
0.4	without	700.25	726.76	560973.35	338916.81	1180812.05	724196.08
0.6	with	703.80	741.91	551248.08	333240.18	1180660.12	729587.16
0.6	without	698.39	727.14	559855.12	339292.45	1179609.38	725001.41
0.8	with	702.27	742.42	550202.67	333292.02	1179796.75	730260.94
0.8	without	697.20	727.41	559065.70	339557.98	1178759.8	725570.68
1.0	with	701.20	742.80	549421.21	333330.80	1179152.28	730763.64
1.0	without	696.36	727.61	558478.65	339755.64	1178127.73	725994.42

Table 9: Equilibrium Demand Prices, Consumer Welfare, and Profits with a Varying Quality Coefficient in $\hat{f}_{12}$ with and without the Strict Quota

As can be observed from Tables 8 and 9, points 1, 2, 3, and 4 above hold even with a varying quality coefficient in $\hat{f}_{12}$. The generality of the model and computational procedure allows for such flexibility. Of course, sensitivity analyses can be conducted by varying other coefficients in the other production cost functions, as well as in the transportation cost functions, both individually and jointly.

Sensitivity Analysis: Impacts of Changes in the Strict Quota

We now provide a sensitivity analysis examining the effects of the value of the strict quota on the equilibrium product flows, the quality levels, and the average quality levels, demands, demand prices, profits, consumer welfare, and the Lagrange multiplier (i.e., the equivalent tariff) at equilibrium. The strict quota, $\bar{Q}$, varies from 1600 to 1200, 800, 400, and 0 tons. The results are shown in Figures 4 and 5.

As revealed in Figures 4.a and 4.b, as the strict quota imposed by China on the US production sites becomes tighter, the soybean flows from the United States, Q^*_{111} and Q^*_{211}, decrease to 0, whereas Q^*_{121}, Q^*_{131}, and Q^*_{221}, the soybean flows from production sites in countries not under the quota, increase.

Furthermore, as can be seen in Figures 4.c and 4.d, the quality levels at Cargill's and ADM's non-US sites, q^*_{12}, q^*_{13}, and q^*_{22}, increase. This result further indicates that tighter (lower) quotas may lead to an increase in both the product flows and the quality levels at the production sites where the quota is not imposed.

However, the product quality at Cargill's production site in the United States, q^*_{11}, decreases as the associated soybean flow declines when the strict quota tightens (cf. Figure 4.c). There's no value of it when $\bar{Q}$ becomes 400 or 0 due to no associated flows of soybeans; that is, the same holds with no value of q^*_{21} when $\bar{Q}$ is 0. Interestingly, q^*_{21}, the quality of ADM's soybeans at its United States site, remains the same (at the maximum value) as $\bar{Q}$ decreases from 1600 to 400 (cf. Figure 4.d). This is due to the high coefficient of the average quality, $\hat{q}_{21}$, in the demand price function of ADM, $\rho_{21}(d, \hat{q})$, which indicates a high correlation of $\hat{q}_{21}$ with respect to ρ_{21}, as compared to the other coefficients in the demand price functions. Thus, q^*_{21} needs to be at the maximum value (the upper bound) in order to maintain high prices. To illustrate this point, in Table 10, we report the values of q^*_{21}, for $\bar{Q} = 1200$ and 800, as the coefficient of $\hat{q}_{21}$ in $\rho_{21}(d, \hat{q})$ ranges from 1, to 1.20, 1.50, 1.80, and to 2.00. The coefficient of $\hat{q}_{11}$ in $\rho_{21}(d, \hat{q})$ remains constant at 0.7.

Coefficient of $\hat{q}_{21}$ in $\rho_{21}(d, \hat{q})$	q^*_{21} under $\bar{Q} = 1200$	q^*_{21} under $\bar{Q} = 800$
1.00	10.00	10.00
1.20	10.00	10.00
1.50	59.82	47.84
1.80	100.00	100.00
2.00	100.00	100.00

Table 10: Values of q^*_{21} with Varying Coefficients

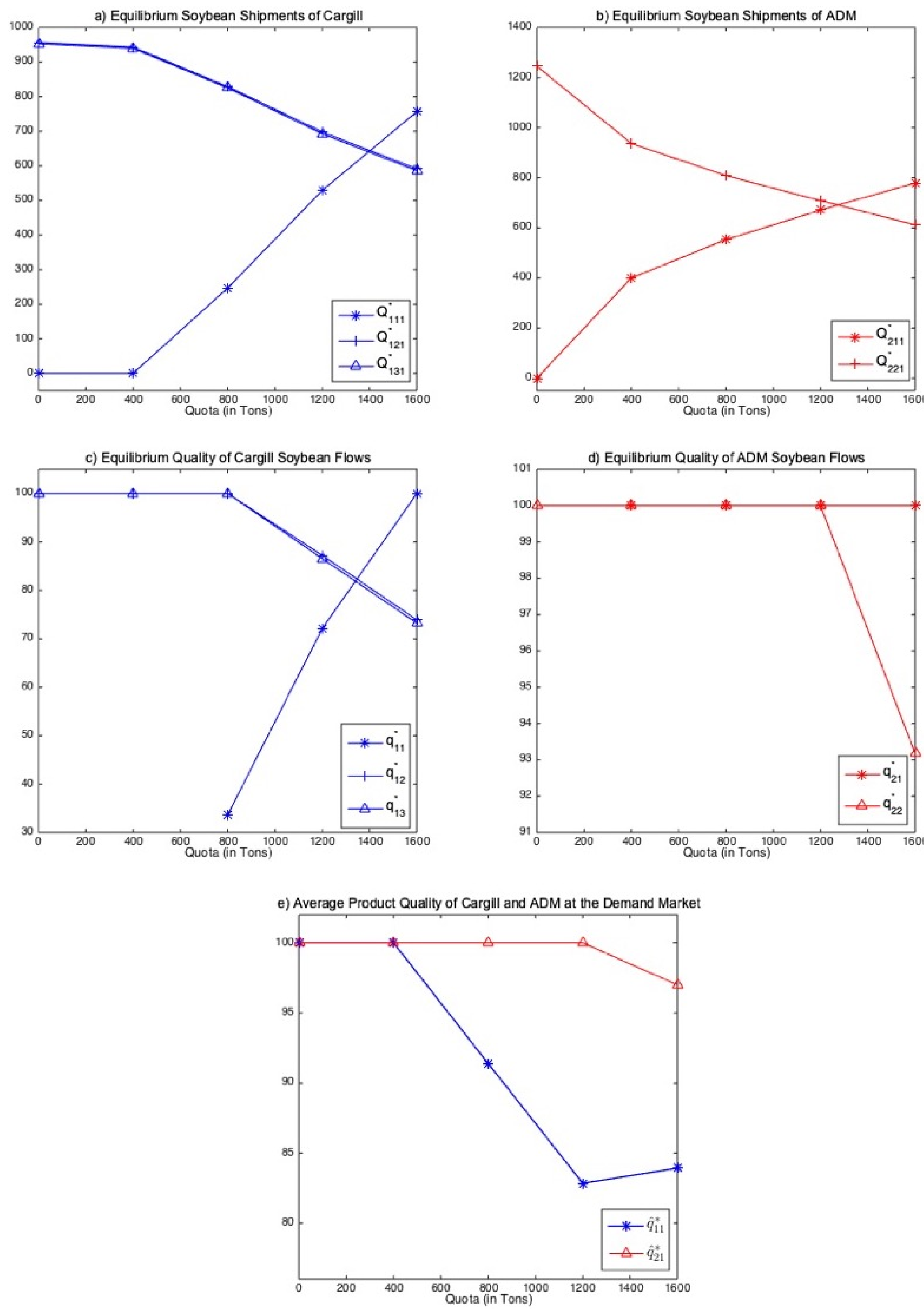

Figure 4: Equilibrium Soybean Flows, Equilibrium Quality Levels, and Average Quality Levels as Quota Decreases for Example 5.2

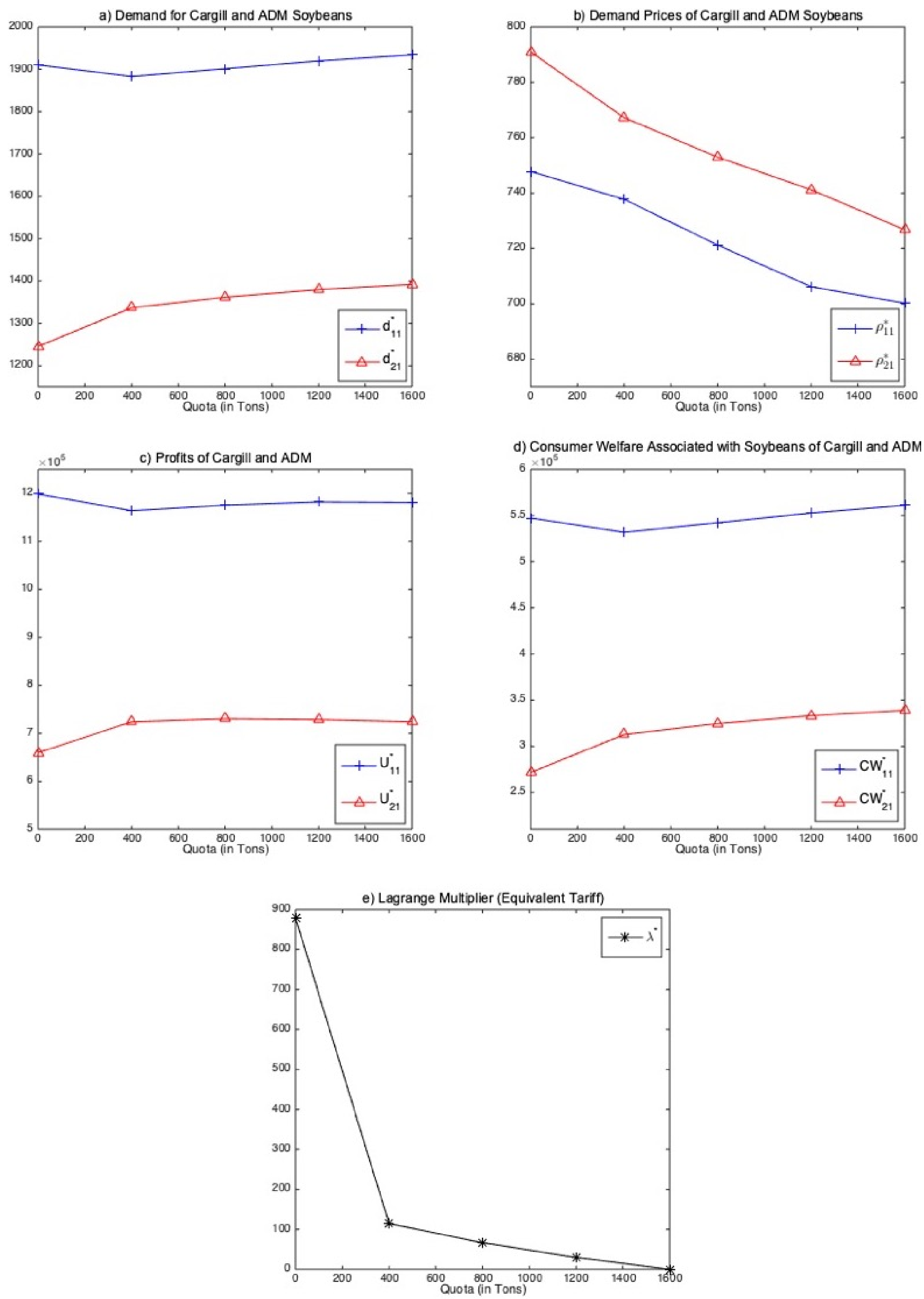

Figure 5: Equilibrium Demands, Equilibrium Demand Prices, Profits, Consumer Welfare, and Lagrange Multipliers at Equilibrium as Quota Decreases for Example 5.2

The results in Table 10 reveal that q_{21}^* increases to its maximum value (upper bound) as the coefficient of $\hat{q}_{21}$ in $\rho_{21}(d,\hat{q})$ becomes higher; nevertheless, at the same coefficient, q_{21}^* decreases or stays the same as $\bar{Q}$ tightens. Hence, we can draw the conclusion that a tighter strict quota may negatively impact the quality of the product at the production sites where the quota is imposed. This result adds new insights on the effects of quotas on product quality to the literature.

Moreover, as shown in Figure 4.e, as the strict quota becomes tighter, the average quality at the demand market, $\hat{q}_{21}$ increases; $\hat{q}_{11}$ drops first and then improves due to the enhancement in quality in Brazil and Argentina.

Figures 5.a, 5.c, and 5.d show similar patterns. It is worth noting that, when the strict quota is 0, the demand for Cargill's soybeans is around 1,910 tons due to more exports from Brazil and Argentina (cf. Figure 5.a). This value is even higher than its demand when the quota is looser. Similar results are obtained for profits and consumer welfare. The profit of Cargill, U_1, with the strict quota being 0, is approximately 1.16 million dollars more than the profit when the quota is 400 tons (cf. Figure 5.c).

Last, but not least, as expected, the demand prices increase as the strict quota becomes tighter (cf. Figure 5.b). CW_{11} and CW_{21}, the consumer welfare, achieve their highest values when the quota is the greatest (cf. Figure 5.d). Finally, as the quota tightens, the Lagrange multiplier, i.e., the equivalent tariff (cf. Section 3.3), increases, reflecting the more restrictive trade policy interventions.

Next, we provide another detailed sensitivity analysis by changing the value of the tariff and report the associated results.

Example 5.3: Example 5.1 with Tariffs on Soybeans from the United States and Sensitivity Analysis

In Example 5.3, we investigate numerically the impacts of tariffs imposed by China on soybeans from the United States. We consider the same supply chain topology and the same data as in Example 5.1. The tariff is $\tau^* = 10.00$ dollars on imports of soybeans from the United States. We report the computed results for the equilibrium soybean flows, in tons, the equilibrium equilibrium quality levels, the equilibrium demands, and the average quality in Table 11 and Table 12.

Notice that the equilibrium demands for soybeans of both Cargill and ADM are lower than their values in Example 5.1. Similar to the discussion of the strict quota in Example 5.2, tariffs create a negative impact on the quality levels of the products. Observe that the

Equilibrium Flows	**Results**	**Equilibrium Quality**	**Results**
Q^*_{111}	685.38	q^*_{11}	93.46
Q^*_{121}	624.46	q^*_{12}	78.06
Q^*_{131}	619.09	q^*_{13}	77.39
Q^*_{211}	735.79	q^*_{21}	100.00
Q^*_{221}	653.62	q^*_{22}	99.46

Table 11: Equilibrium Soybean Flows and Equilibrium Quality Levels for Example 5.3

Demand	**Results**	**Average Quality**	**Results**
d^*_{11}	1,928.93	$\hat{q}_{11}$	83.32
d^*_{21}	1,389.42	$\hat{q}_{21}$	99.75

Table 12: Equilibrium Demands and Average Quality for Example 5.3

quality level q^*_{11} decreases from its value in Example 5.1, when a tariff is imposed for the soybeans produced at P^1_1 and at P^2_1.

The incurred equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, the consumer welfare associated with the soybeans of the two firms at the demand market, and the profits achieved by Cargill and ADM, in dollars, are reported in Table 13.

Demand Price	**Results**	**Consumer Welfare**	**Results**	**Profits**	**Results**
ρ_{11}	701.76	CW_{11}	558,116.66	U_1	1,174,437.42
ρ_{21}	734.52	CW_{21}	337,834.91	U_2	718,677.19

Table 13: Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.3

Observe, from Table 13, that, after the imposition of the tariff, the equilibrium soybean demand prices associated with Cargill and ADM at Demand Market 1 are higher than their values in Example 5.1. Moreover, the consumer welfare associated with both firms decreases from that obtained in Example 5.1. This means that the introduction of tariffs creates a negative impact on Chinese consumers. As consumers at Demand Market 1 suffer from tariffs, the firms, Cargill and ADM, are faced with a profit decrease as compared to the profits enjoyed in Example 5.1, in which there were no trade policy instruments in the form of quotas or tariffs imposed.

Sensitivity Analysis: Impact of Changes in Tariffs

In this sensitivity analysis, the equilibrium soybean flows, the equilibrium quality levels, the average quality levels, the equilibrium demands, prices, consumer welfare, and profits are computed for a range of tariffs: $\tau^* = 20.00$, $\tau^* = 29.90$ and $\tau^* = 40.00$. According to Rapoza (2019), in 2019, China imposed an 8 dollar tariff per bushel of soybeans. In this sensitivity analysis, we test our model with higher tariffs. The results are reported in Tables 14, 15, and 16.

τ^*	Q^*_{111}	Q^*_{121}	Q^*_{131}	Q^*_{211}	Q^*_{221}
20.00	606.79	661.38	656.02	702.79	681.92
29.90	528.96	697.99	692.63	671.04	708.75
40.00	449.69	735.28	729.91	638.70	736.07

Table 14: Equilibrium Soybean Flows for Example 5.5 under Various Tariffs

τ^*	q^*_{11}	q^*_{12}	q^*_{13}	q^*_{21}	q^*_{22}	$\hat{q}_{11}$	$\hat{q}_{21}$
20.00	82.74	82.67	82.00	100.00	100.00	82.47	100.00
29.90	72.13	87.25	86.58	100.00	100.00	82.84	100.00
40.00	61.32	91.10	91.24	100.00	100.00	84.74	100.00

Table 15: Equilibrium Quality Levels and Average Quality Levels for Example 5.3 under Various Tariffs

Similar conclusions to those obtained for Example 5.2 can be drawn from the results in Tables 14, 15, and 16, further supporting the equivalence between strict quotas and tariffs derived in Section 3.3.

τ^*	d^*_{11}	d^*_{21}	ρ_{11}	ρ_{21}	CW_{11}	CW_{21}	U_1	U_2
20.00	1,924.19	1,384.71	703.53	738.09	555,378.50	335,548.02	1,169,791.64	713,460.32
29.90	1,919.58	1,379.79	706.16	741.20	552,718.33	333,167.37	1,166,059.22	708,571.37
40.00	1,914.88	1,374.78	709.71	744.36	550,015.49	330,751.37	1,163,040.11	703,900.27

Table 16: Equilibrium Demands, Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.3 under Various Tariffs

For example, the results in Table 14 show that, when the tariff increases, the equilibrium flows from the production sites under the tariff decrease. This result is the same as in Example 5.2 with the strict quota. Moreover, the equilibrium quality levels for the soybeans produced in the US decrease, as the tariff increases. Additionally, the results in Table 16 reveal a decrease in equilibrium demands as the tariff increases.

Similar to the discussion for Example 5.2, the demand prices of the Cargill and ADM soybeans increase and the associated consumer welfare decreases, when a higher tariff is imposed. Furthermore, notice that, when the tariff $\tau^* = 29.90$, the computed equilibrium solution is equal to that reported in Example 5.2. Hence, we have numerically illustrated/verified the equivalence between strict quotas and tariffs, where $\tau^* = \lambda^* > 0$, further supporting the theoretical results obtained in Section 3.3.

Example 5.4: Example 5.2 with Cargill's Production Site in the United States Shut Down

In Example 5.4, the same differentiated product supply chain network problem for soybeans as in Example 5.2 is considered, but with Cargill's soybean production site in the United States, P_1^1, shut down. Thus, the corresponding node and associated links are removed from the supply chain network in Figure 3, yielding the supply chain network in Figure 6. The strict quota of 1200 is now imposed only on ADM's soybeans from the United States produced at P_1^2.

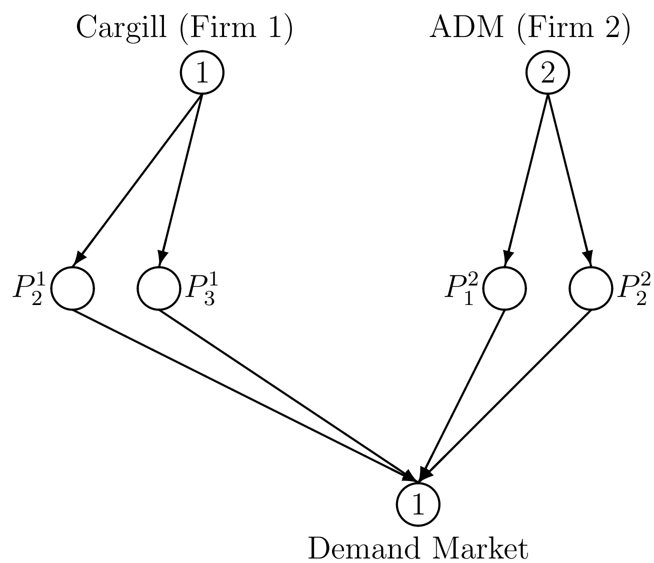

Figure 6: The Supply Chain Network Topology for Example 5.4

We report the equilibrium soybean flows, in tons, the equilibrium quality levels, the equilibrium demands for soybeans at Demand Market 1, and the average quality levels in Table 17 and Table 18.

Furthermore, the equilibrium Lagrange multiplier, which is the equivalent tariff, becomes:

$$\lambda^* = 0.00.$$

Due to the shutdown of Cargill's production site in the United States, its soybean flows from the other sites, that are located in Brazil and Argentina, increase. Meanwhile, with less

Equilibrium Flows	Results	Equilibrium Quality	Results
Q^*_{111}	-	q^*_{11}	-
Q^*_{121}	930.73	q^*_{12}	100.00
Q^*_{131}	926.80	q^*_{13}	100.00
Q^*_{211}	788.93	q^*_{21}	100.00
Q^*_{221}	633.23	q^*_{22}	99.36

Table 17: Equilibrium Soybean Flows and Equilibrium Quality Levels for Example 5.4

Demand	Results	Average Quality	Results
d^*_{11}	1,857.53	$\hat{q}_{11}$	100.00
d^*_{21}	1,422.16	$\hat{q}_{21}$	98.38

Table 18: Equilibrium Demands and Average Quality for Example 5.4

competition from Cargill, ADM delivers more soybeans to China from the cheaper site, P^2_1, in the United States, and fewer soybeans from the more expensive site, P^2_2, in Brazil. Since the strict quota is only imposed on ADM in this example, there is more opportunity for ADM to increase its production in the US. Furthermore, ADM's soybean production in the US site does not meet the strict quota, resulting in a zero equilibrium Lagrange multiplier.

We provide results for the computed equilibrium incurred demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, the consumer welfare at the demand market, and the profits achieved by Cargill and ADM, in dollars, in Table 19.

Demand Price	Results	Consumer Welfare	Results	Profits	Results
ρ_{11}	728.31	CW_{11}	517,560.59	U_1	1,131,885.86
ρ_{21}	741.75	CW_{21}	353,945.28	U_2	756,415.15

Table 19: Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.4

As compared to the results for Example 5.2, the quality levels of soybeans produced by Cargill, including the average quality, increase to the maximum level, after its production site in the United States is shut down. ADM's quality levels of its soybeans, in turn, decrease or remain the same, with its average quality decreased. Furthermore, the consumer welfare associated with Cargill's soybeans decreases due to less demand and a higher price, but the consumer welfare associated with ADM's soybeans improves. It is also worth noting that the shut down of Cargill's production site in the United States harms its profit, whereas ADM now achieves a higher profit than before.

Example 5.5: Example 5.1 with a New Demand Market in the United States and Additional Data

In this example, we add a new demand market, Demand Market 2, which is located in the United States, to the supply chain network in Example 5.1, as shown in Figure 7. The domestic consumption of soybeans in the United States is also worth analyzing, even with the United States being one of the top soybean exporters in the world. In Figure 7, we do not include the transportation links from Brazil and Argentina to the United States, since clearly such trade would not be cost-efficient, with the US being an exporter. The new demand market, hence, induces two additional paths and path flows, denoted by Q_{112} and Q_{212}.

The total soybean production outputs at Cargill's and ADM's production sites (cf. Figure 7) must satisfy the following expressions:

$$s_{11} = Q_{111} + Q_{112},\ s_{12} = Q_{121},\ s_{13} = Q_{131},\ s_{21} = Q_{211} + Q_{212},\ s_{22} = Q_{221}.$$

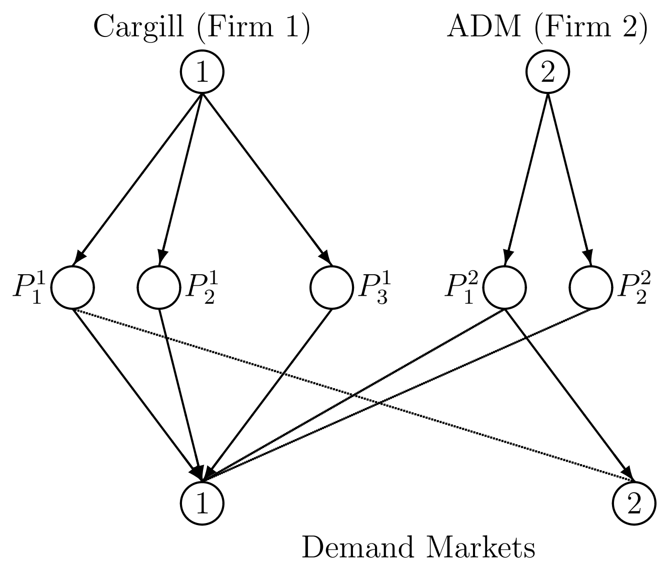

Figure 7: The Supply Chain Network Topology for Example 5.5

Furthermore, the updated production cost functions of Cargill and ADM are:

$$f_{11}(s_{11}, q_{11}) = 0.04s_{11}^2 + 0.35s_{11} + 0.4s_{11}q_{11} + 0.6q_{11}^2,$$

$$f_{12}(s_{12}, q_{12}) = 0.05s_{12}^2 + 0.35s_{12} + 0.4s_{12}q_{12} + 0.4q_{12}^2,$$

$$f_{13}(s_{13}, q_{13}) = 0.05s_{13}^2 + 0.8s_{13} + 0.4s_{13}q_{13} + 0.4q_{13}^2,$$

$$f_{21}(s_{21}, q_{21}) = 0.06s_{21}^2 + 0.5s_{21} + 1.2s_{21}q_{21} + q_{21}^2,$$

$$f_{22}(s_{22}, q_{22}) = 0.07s_{22}^2 + 0.3s_{22} + 1.3s_{22}q_{22} + 1.5q_{22}^2.$$

There are two additional total transportation cost functions associated with Cargill and ADM shipping their soybeans to Demand Market 2, given by:

$$\hat{c}_{112}(Q_{112}, q_{11}) = 0.002Q_{112}^2+0.02Q_{112}+0.8q_{11}^2, \qquad \hat{c}_{212}(Q_{212}, q_{21}) = 0.002Q_{212}^2+0.04Q_{212}+q_{21}^2.$$

The additional demand price functions for the soybeans of Cargill and ADM at Demand Market 2 are:

$$\rho_{12}(d, \hat{q}) = 1100 - (0.25d_{12} + 0.2d_{22}) + 0.9\hat{q}_{12},$$

$$\rho_{22}(d, \hat{q}) = 1400 - (0.3d_{22} + 0.25d_{12}) + 1.4\hat{q}_{22},$$

with the additional average quality $\hat{q}_{12}$ and $\hat{q}_{22}$ at Demand Market 2 given by:

$$\hat{q}_{12} = \frac{Q_{112}q_{11}}{Q_{112}} = q_{11}, \quad \hat{q}_{22} = \frac{Q_{212}q_{21}}{Q_{212}} = q_{21}.$$

The computed equilibrium soybean flows, in tons, equilibrium quality levels, equilibrium demands, and average quality are given in Tables 20 and 21.

Equilibrium Flows	**Results**	**Equilibrium Quality**	**Results**
Q_{111}^*	87.50	q_{11}^*	100.00
Q_{121}^*	913.30	q_{12}^*	100.00
Q_{131}^*	909.37	q_{13}^*	100.00
Q_{211}^*	150.11	q_{21}^*	77.79
Q_{221}^*	1,126.34	q_{22}^*	100.00
Q_{112}^*	1,469.53	-	-
Q_{212}^*	1,422.13	-	-

Table 20: Equilibrium Soybean Flows and Equilibrium Quality Levels for Example 5.5

Demand	**Results**	**Average Quality**	**Results**
d_{11}^*	1,910.17	$\hat{q}_{11}$	100.00
d_{21}^*	1,276.44	$\hat{q}_{21}$	97.39
d_{12}^*	1,469.53	$\hat{q}_{12}$	100.00
d_{22}^*	1,422.13	$\hat{q}_{22}$	77.79

Table 21: Equilibrium Demands and Average Quality for Example 5.5

Observe that the equilibrium soybean flow from Cargill's United States production site to Demand Market 1, Q_{111}^*, decreases greatly from its value reported in Example 5.1. Also, now the majority of the soybeans produced at P_1^1 are shipped to Demand Market 2 in the United

States, since the transportation cost of sending soybeans domestically is cheaper. Moreover, the equilibrium soybean flows of Cargill's production sites in Brazil and Argentina increase from their values in Example 5.1 with the new demand market. A similar pattern is observed for ADM's soybean flows.

The equilibrium demands d_{11}^* and d_{21}^* decrease from their values in Example 5.1. Notice that the equilibrium demand for ADM's soybeans at Demand Market 2 in the United States is higher than that at Demand Market 1 in China. The equilibrium quality levels q_{12}^*, q_{13}^*, and q_{22}^* have higher values than in Example 5.1. Moreover, the average quality of Cargill's and ADM's soybeans at the demand markets is at the respective upper bound.

In Table 22, we provide the equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1 and Demand Market 2, in dollars, the consumer welfare at each demand market, and the achieved profits of Cargill and ADM.

Demand Price	**Results**	**Consumer Welfare**	**Results**	**Profits**	**Results**
ρ_{11}	741.66	CW_{11}	547,310.17	U_1	1,809,217.78
ρ_{21}	774.97	CW_{21}	285,128.51	U_2	1,405,249.62
ρ_{12}	538.19	CW_{12}	269,938.75	-	-
ρ_{22}	714.89	CW_{22}	303,368.87	-	-

Table 22: Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.5

With the introduction of Demand Market 2, the demand prices of the Cargill and ADM soybeans at Demand Market 1 increase from their values in Example 5.1. The consumer welfare associated with both firms' soybeans at Demand Market 1 decreases due to higher prices and lower demands. Both firms achieve a higher profit with the addition of a new demand market. ADM, in particular, enjoys a significant profit increase from that in Example 5.1.

Example 5.6: Example 5.5 with a Strict Quota

In Example 5.6, we consider the same supply chain topology and the data as in Example 5.5, but we assume that China now imposes a strict quota of $\bar{Q} = 100$ on its imports from the United States. Similar to the previous sections, we report the computed equilibrium soybean flows, in tons, the equilibrium quality levels, the equilibrium demands, and the average quality in Tables 23 and 24.

As expected, the equilibrium demands d_{11}^* and d_{21}^* decrease from their values in Example 5.5 with the introduction of the quota, whereas d_{12}^* and d_{22}^* increase. Notice that the equilibrium quality level of soybeans of ADM, produced in the United States, decreases from its

Equilibrium Flows	Results	Equilibrium Quality	Results
Q_{111}^*	19.30	q_{11}^*	100.00
Q_{121}^*	945.79	q_{12}^*	100.00
Q_{131}^*	941.86	q_{13}^*	100.00
Q_{211}^*	80.70	q_{21}^*	67.35
Q_{221}^*	1,182.64	q_{22}^*	100.00
Q_{112}^*	1,476.77	-	-
Q_{212}^*	1,428.25	-	-

Table 23: Equilibrium Soybean Flows and Equilibrium Quality Levels for Example 5.6

Demand	Results	Average Quality	Results
d_{11}^*	1,906.95	$\hat{q}_{11}$	100.00
d_{21}^*	1,263.34	$\hat{q}_{21}$	97.91
d_{12}^*	1,476.77	$\hat{q}_{12}$	100.00
d_{22}^*	1,428.25	$\hat{q}_{22}$	67.35

Table 24: Equilibrium Demands and Average Quality for Example 5.6

value in Example 5.5, whereas the remaining equilibrium quality levels are the same as in Example 5.5. Interestingly, the average quality of ADM's soybeans at Demand Market 1 in China increases from its value in Example 5.5. In contrast, the average quality of ADM's soybeans at Demand Market 2 in the United States decreases.

Moreover, the equilibrium Lagrange multiplier or the equivalent tariff is:

$$\lambda^* = 12.15$$

since the volume of equilibrium soybean flows from the US production sites is at the imposed strict quota.

The equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1 and Demand Market 2, in dollars, the consumer welfare at each demand market, and the achieved profits of Cargill and ADM are reported in Table 25.

Demand Price	Results	Consumer Welfare	Results	Profits	Results
ρ_{11}	745.25	CW_{11}	545,470.00	U_1	1,812,002.21
ρ_{21}	781.57	CW_{21}	279,305.28	U_2	1,403,470.22
ρ_{12}	535.16	CW_{12}	272,607.69	-	-
ρ_{22}	696.62	CW_{22}	305,984.29	-	-

Table 25: Equilibrium Demand Prices, Consumer Welfare, and Profits for Example 5.5

Note that the consumer welfare associated with consumers in China, represented by

Demand Market 1, which has imposed the quota on production sites in the US, associated with Cargill's soybeans, CW_{11}, and with ADM's soybeans, CW_{21}, declines. This is in contrast to the values in Example 5.5, signifying a negative impact on the consumers. However, the consumer welfare of the consumers in the US (Demand Market 2), increases.

The introduction of the strict quota to Example 5.5 creates a profit increase and a profit decrease for Cargill and ADM, respectively. Having multiple production sites in Brazil and Argentina that are not under the strict quota saves Cargill from a profit drop. On the other hand, ADM is faced with a profit decrease due to having only single production site over which a strict quota is not imposed, in Brazil.

5.1 Managerial Insights

As noted in Table 1 of Choi (2019), delineating insights and managerial implications is imperative. From the above numerical examples, and accompanying sensitivity analysis, a spectrum of managerial insights is revealed.

Specifically, from the consumer's perspective, the results consistently and unanimously show that consumer welfare declines for consumers in the country imposing a strict quota or tariff on an imported product. Hence, a government may wish to loosen a quota (equivalently, reduce a tariff) so as not to adversely affect its own consumers.

Producing firms, as also critical stakeholders in competitive supply chain networks, should expand their demand markets within their own countries. This allows for a basic, but, effective, redesign of the supply chain network under a tariff or quota and results in higher profits for the firms. Also, firms should expand the number of production sites to countries not under a tariff or quota to maintain or improve upon their profits if some of their production sites are in countries subject to such trade policy instruments.

Finally, the examples numerically support our theoretical finding that a tariff has the equivalent impact on product flows and product quality as a strict quota, provided that the tariff is set to the Lagrange multiplier associated with the strict quota constraint and the constraint is tight. Hence, governments have the flexibility of imposing either a tariff or a quota to obtain equivalent trade flows and product quality levels. The imposition of a tariff may be more advisable/favored by a government, since it requires less "policing" and also yields financial rewards.

6. Summary and Conclusions and Suggestions for Future Research

Supply chain networks provide the pathways for the production and distribution of products to consumers across the globe and serve as the critical infrastructure for world trade. Governments, in their desire to influence world trade, utilize trade policies, in the form of quotas or tariffs. The recent dynamism associated with the imposition of such policies worldwide merits closer attention from modeling and computational perspectives. Although such topics have a long history in the economics literature, they have been less explored in operations research / operations management, especially from a supply chain perspective. Furthermore, the identification of the impacts on product quality and how consumers are affected by such trade policy instruments, within the context of realistic supply chain networks, is relevant for producers, consumers, and policy makers alike.

In this paper, we add to the supply chain network, game theory, and trade policy literature by constructing an oligopolistic supply chain network equilibrium model with differentiated products in which firms have multiple production sites and multiple possible demand markets. The firms compete in product quantities and product quality, subject to minimum quality standards, along with upper bounds on quality. The model is then extended to include trade policy instruments in the form of a strict quota or a tariff. We identify the governing equilibrium conditions, noting that the strict quota model is characterized by a Generalized Nash Equilibrium, rather than a Nash Equilibrium, as is the case of the other two models. We demonstrate that, nevertheless, the underlying equilibrium conditions for all three models can be formulated and analyzed as an appropriate variational inequality problem, for which an effective computational scheme is also provided, which yields closed form expressions for the variables in each of the two steps of the procedure.

We establish theoretically, and also support the results numerically, that the equilibrium Lagrange multiplier associated with the strict quota constraint in the strict quota model, if assigned as a tariff, when the strict quota is tight, yields the same equilibrium product flows and product quality levels for the tariff model as obtained for the quota model. This equivalence in a competitive supply chain network allows decision-makers the option of using either a strict quota or a tariff to obtain identical results. We also construct consumer welfare measures for the models.

In summary, our theoretical contributions in this paper are the following:

1. The construction of the first general (not limited to a specific number of firms or demand markets or functional forms) oligopolistic supply chain network equilibrium models with strategic variables of quantities and product quality that incorporate multiple trade policy instruments;

2. The establishment of the equivalence of a tariff with that of a strict quota, provided that

the strict quota constraint is tight, and the Lagrange multiplier associated with it is the set tariff. This provides decision-makers and policy makers, including governments, with the flexibility of imposing either a tariff or a strict quota in practice.

3. The construction of a formula for the determination of the consumer welfare, under a tariff or quota, in competitive supply chain networks.

4. The rigorous formulation of all the models as variational inequality problems, either with a tariff or a strict quota, and with minimum and maximum quality bounds. In the case of the model with a strict quota, the governing equilibrium conditions are those of a Generalized Nash Equilibrium, because of the shared/common constraint. There are very few GNE supply chain models in the literature to-date.

5. A proposed algorithm, with nice features for computations, for the new models, accompanied by convergence conditions.

Illustrative examples are provided, along with numerical examples inspired by an important agricultural product - that of soybeans. The numerical examples, accompanied by sensitivity analysis, reveal that a government, in imposing trade policy instruments in the form of strict quotas or tariffs, may decrease the welfare of its own consumers. The computational framework includes the quantitative measurement of the impacts of a production site disruption as well as the addition of a demand market.

In summary, the practical insights from our framework, are the following:

1. Governments should be cautious in imposing trade policy instruments in the form of tariffs or quotas on products in competitive, that is, oligopolistic, supply chain networks, since the consumer welfare of consumers in their own country can decrease as a result.

2. Governments, by imposing a tariff or quota, may help firms in their country garner enhanced profits but at the expense of consumers.

3. Producers should expand the geographic dispersion of their production sites to reduce the impact of imposed tariffs or quotas.

4. Producers should actively expand their demand markets in countries not under trade policy instrument regimes, since doing so can lead to higher profits.

In this paper, the focus was on products in which there were not multiple different tiers of suppliers in the supply chain networks. The investigation of tariffs and quotas in more complex, multitiered supply chain networks, in which there is assembly, etc., clearly merits study. In addition, it would be very worthwhile to model a government that is interested in enforcing a trade policy that maximizes total consumer welfare in its own country. Also, it would be very interesting to consider the redesign of supply chain networks in the presence of trade policy instruments such as quotas or tariffs. We leave such work for future research.

Acknowledgments

We sincerely thank the Editor and three anonymous referees for their many valuable suggestions and comments on earlier versions of this paper.

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