**A General Multitiered Supply Chain Network Model of Quality Competition with Suppliers**

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**Abstract**

In this paper, we develop a general multitiered supply chain network equilibrium model consisting of competing suppliers and competing firms who purchase components for the assembly of their final branded products and, if capacity permits, and it enhances profits, produce their own components. The competitive behavior of each tier of decision-makers is described along with their strategic variables, which include quality of the components and, in the case of the firms, the quality of the assembly process itself. The governing equilibrium conditions of the supply chain network are formulated as a variational inequality and qualitative properties are presented. The algorithm, accompanied with convergence results, is then applied to numerical supply chain network examples, along with sensitivity analysis in which the impacts of capacity disruptions and complete supplier elimination are investigated.

**Keywords:** supply chains, networks, suppliers, quality competition, game theory, variational inequalities

## 1. Introduction

Quality has been recognized as "the single most important force leading to the economic growth of companies in international markets" (Feigenbaum (1982) page 22), and, in the long run, as the most important factor affecting a business unit's performance and competitiveness, relative to the quality levels of its competitors (Buzzell and Gale (1987)). Many firms, however, with pressure to lower their costs, continue to subcontract manufacturing to lower cost countries globally, with their supply chain networks being more exposed to quality failures (Marucheck et al. (2011)) and with the policing and monitoring of product safety also extremely difficult (Tang (2008)).

In recent years, there have been numerous examples of finished product failures due to the poor quality of a suppliers' components. For example, the toy manufacturer, Mattel, in 2007, recalled 19 million toy cars because of suppliers' lead paint and poorly designed magnets, which could harm children if ingested (Story and Barboza (2007)). In 2013, four Japanese car-makers, along with BMW, recalled 3.6 million vehicles because the airbags supplied by Takata Corp., the world's second-largest supplier of airbags, were at risk of rupturing and injuring passengers (Kubota and Klayman (2013)). The recalls are still ongoing and have expanded to other companies as well (Tabuchi and Jensen (2014)). Most recently, the defective ignition switches in General Motors (GM) vehicles, which were produced by Delphi Automotive in Mexico, have been linked to 13 deaths, due to the fact that the switches could suddenly shut off engines with no warning (Stout, Ivory, and Wald (2014) and Bomey (2014)). In addition, serious quality shortcomings and failures associated with suppliers have also occurred in finished products such as aircraft (Drew (2014)), pharmaceuticals (Rao (2014)), and also food (Strom (2013) and McDonald (2014)). In 2009, over 400 peanut butter products were recalled after 8 people died and more than 500 people, half of them children, were sickened by salmonella poisoning, the source of which was a peanut butter plant in Georgia (Harris (2009)).

Product quality is an important feature that enables firms to maintain and even to improve their competitive advantage and reputation. However, numerous finished products are made of raw materials as well as components and it is usually the case that the components and materials are produced and supplied not by the firms that process them into products but by suppliers in globalized supply chain networks. For example, Sara Lee bread, an everyday item, is made with flour from the US, vitamin supplements from China, gluten from Australia, honey from Vietnam and India, and other ingredients from Switzerland, South America, and Russia (Bailey (2007)), let alone automobiles and aircrafts which are made up of thousands of different components.

Therefore, the quality of a finished/final product depends not only on the quality of the firm that produces and delivers it, but also on the quality of the components provided by the firm's suppliers (Robinson and Malhotra (2005) and Foster (2008)). It is the suppliers that determine the quality of the materials that they purchase as well as the standards of their manufacturing activities. However, suppliers may have less reason to be concerned with quality (Nagurney, Li, and Nagurney (2013) and Amaral, Billington, and Tsay (2006)). In Mattel's case, some of the suppliers were careless, others flouted rules, and others simply avoided obeying the rules (Tang (2008)). With non-conforming components, it may be challenging and very difficult for firms to produce high quality finished products even if they utilize the most superior production and transportation delivery techniques.

Furthermore, since suppliers may be located both on-shore and off-shore, supply chain networks of firms may be more vulnerable to disruptions around the globe than ever before. Photos of Honda automobiles under 15 feet of water were some of the most appalling images of the impacts of the Thailand floods of 2011. Asian manufacturing plants affected by the catastrophe were unable to supply components for cars, electronics, and many other products (Kageyama (2011)). In the same year, the triple disaster in Fukushima affected far more than the manufacturing industry in Japan. A General Motors plant in Louisiana had to shut down due to a shortage of Japanese-made components after the disaster took place (Lohr (2011)). Under such disruptions, suppliers may not even be capable of performing their production tasks, let alone guaranteeing the quality of the components.

In addition, the number of suppliers that a firm may be dealing with can be vast. For example, according to Seetharaman (2013), Ford, the second largest US car manufacturer, had 1,260 suppliers at the end of 2012 with Ford purchasing approximately 80 percent of its parts from its largest 100 suppliers. Due to increased demand, many of the suppliers, according to the article were running "flat out" with the consequence that there were quality issues. In the case of Boeing, according to Denning (2013), complex products such as aircraft involve a necessary degree of outsourcing, since the firm lacks the necessary expertise in some areas, such as, for example, engines and avionics. Nevertheless, as noted therein, Boeing significantly increased the amount of outsourcing for the 787 Dreamliner airplane over earlier planes to about 70 percent, whereas for the 737 and 747 airplanes it had been at around 35-50 percent. Problems with lithium-ion batteries produced in Japan grounded several flights and resulted in widespread media coverage and concern for safety of the planes because of that specific component (see Parker (2014)).

Recently, there has been increasing attention from researchers to supplier-manufacturer supply chain networks with quality in both operations research and economics. However, in the literature, most models are based on a single firm - single supplier supply chain network (cf. Reyniers and Tapiero (1995), Tagaras and Lee (1996), Starbird (1997), Baiman, Fischer, and Rajan (2000), Lim (2001), Hwang, Radhakrishnan, and Su (2006), Zhu, Zhang, and Tsung (2007), Chao, Iravani, and Savaskan (2009), Hsieh and Liu (2010), and Xie et al. (2011)), which, given the reality of many finished product supply chains, as noted above, may be limiting in terms of both scope and practice. Specifically, although focused, simpler models, may yield closed form analytical solutions, more general frameworks, that are computationally tractable, are also needed, given the size and complexity of real-world global supply chains.

In addition to the above mentioned literature, the following papers considered more than one firm and/or supplier in their models. Economides (1999) studied a dual-monopolist problem with two components, and concluded that vertical integration could guarantee higher quality. El Ouardighi and Kim (2010) formulated a dynamic game in which a supplier collaborated with two firms on design quality improvements. Pennerstorfer and Weiss (2012) studied a wine supply chain network with multiple suppliers and firms, and each firm made identical decisions on quality. Furthermore, in the models developed by Hong and Hayya (1992), Rosenthal, Zydiak, and Chaudhry (1995), Jayaraman, Srivastava, and Benton (1999), Ghodsypour and O'Brien (2001), and Rezaei and Davoodi (2008), multiple firms and/or suppliers were involved with quality being considered as input parameters, and the decisions on quality were not provided.

Different from the above models, the main contributions of this paper are: 1. We formulate the supply chain network problem with multiple nonidentical competing firms and their potential suppliers who also compete in quality. 2. The model is general and not limited to a small number of firms, suppliers, or components or limited to specific functional forms in terms of costs or demand price functions. 3. The solution of the proposed game theory model provides equilibrium decisions on the in-house and contracted component production and quality levels, component prices, product quantities, and the quality preservation/decay levels of the assembly processes simultaneously. Decisions on the prices and quality levels of the final products are determined through information provided via the demand price functions and the quality aggregation functions. 4. Based on this model, the value of each supplier to each firm can be identified, as illustrated in the analysis in Section 5. This information is crucial in facilitating strategy design and development in supplier management especially in response to supplier disruptions. 5. Along with the general multitiered supply chain network model, we also provide a general computational procedure with explicit formulae at each iteration. 6. The qualitative properties of the solution to the proposed model, in terms of existence and uniqueness, and the convergence criteria of the computational procedure are presented.

In several of the authors' previous product quality supply chain papers, the quality competition among firms (Nagurney and Li (2014a, b)) and that among their potential contractors who produce final products for the firms (Nagurney, Li, and Nagurney (2013) and Nagurney and Li (2015)) were modeled. The equilibrium product quantity, quality, and price decisions were presented. Nevertheless, the focus of this paper is on modeling the behavior of suppliers, who provide components to the firms, and in determining the equilibrium decisions on in-house and contracted component quality. In addition, decisions on supplier selections are also given.

Specifically, in this paper, the potential suppliers may either provide distinct components to the firms, or provide the same component, in which case, they compete noncooperatively with one another in terms of quality and prices. The firms, in turn, are responsible for assembling the products under their brand names using the components needed and transporting the products to multiple demand markets. They also have the option of producing their own components, if necessary. The firms compete in product quantities, the quality preservation levels of their assembly processes, the contracted component quantities produced by the suppliers, and in in-house component quantities and quality levels. Each of the firms aims to maximize profits. The quality of an end product is determined by the qualities and quality levels of its components, produced both by the firms and the suppliers, the importance of the quality of each component to that of the end product, and the quality preservation level of its assembly process. Consumers at the demand markets respond to both the prices and the quality of the end products.

As in our previous quality competition papers (cf. Nagurney and Li (2014a, b, 2015), Nagurney, Li, and Nagurney (2013, 2014), and Nagurney et al. (2013a, 2014)), we define quality as "the degree to which a specific product conforms to a specification," which is how well the product is conforming (Shewhart (1931), Juran (1951), Levitt (1972), Gilmore (1974), Crosby (1979), and Deming (1986)). When the quality is at a 0% conformance level, it means that the product does not conform to the specification at all, and when it is 100%, the product conforms perfectly. This conformance-to-specification definition makes quality relatively straightforward to quantify, which is essential for firms and researchers who are eager to measure it, manage it, model it, compare it across time, and to also make associated decisions (Shewhart (1931)). In addition, with notice that consumers' needs and desires for a product are actually governed by specific requirements or standards on design and production and these can be correctly translated to a specification by, for example, engineers (Oliver (1981)), the conformance-to-specification definition is quite general. In addition to consumers' needs, the specification can also include both international and domestic standards (Yip (1989)), and, in order to gain marketing advantages in the competition with other firms, the competitors' specifications.

The reasons that we adopt this definition rather than defining quality as a) a binary variable representing whether a product is defective or not, or b) a value of defect rate, are as the following. First, it is very difficult to define and identify defectives in reality. If defectives can always be well-defined and successfully screened out before they reach consumers, quality failure incidents caused by defectives, as we listed and discussed above, would never occur. Secondly, consumers' demand in terms of quality is not simply based on whether a product is defective or not. It also matters to consumers as to how well the product serves their needs and expectations, since consumers are willing to pay more for products that can satisfy their needs in a better way, that is, are of better quality. Therefore, the conformance-to-specification definition is a more general expression of quality than a) and b). In addition, given the different characteristics of and functions that competing products can perform, quality should not only be limited to whether a product is defective or not, but must capture the degree that can take on different values for different products.

In this paper, since we are dealing with supply chain networks in which finished products are assembled from multiple components, we also need to characterize and quantify quality of the finished product. Hence, in our new framework, we provide a formula to quantify it based on the quality of the individual components. We assume in the model that each component's quality ranges from a lower bound of 0, which can represent the 0% conformance level or the value of the associated minimum quality standard, to an imposed upper bound, which, depending upon the application, can represent perfect quality, if it is achievable by the manufacturer/producer. Quality levels with lower and upper bounds can also be found in Akerlof (1970) ($q \in [0, 2]$), Leland (1979) ($q \in [0, 1]$), Chan and Leland (1982) ($q \in [q_0, q_H]$), Lederer and Rhee (1995) ($q \in [0, 1]$), Acharyya (2005) ($q \in [q_0, \bar{q}]$), Chambers, Kouvelis, and Semple (2006) ($q \in [0, q_{max}]$), and in Nagurney, Li, and Nagurney (2013) and Nagurney and Li (2015) ($q \in [0, q^U]$). Reyniers and Tapiero (1995), Tagaras and Lee (1996), Baiman, Fischer, and Rajan (2000), Hwang, Radhakrishnan, and Su (2006), Hsieh and Liu (2010), and Lu, Ng, and Tao (2012) modelled quality as probabilities, which are between 0 and 1.

This paper is organized as follows. In Section 2, we develop the multitiered supply chain network model with competing suppliers and competing firms. We describe their strategic variables and their competitive behavior and derive the variational inequality formulations for each tier followed by a unified variational inequality. In Section 3, we present qualitative properties of the equilibrium pattern, in particular, existence and uniqueness results. In Section 4, we outline the algorithm, along with conditions for convergence, which is then applied in Section 5 to compute solutions to numerical supply chain network examples accompanied by sensitivity analysis. We also discuss the results in order to provide managerial insights. We summarize our results and present our conclusions in Section 6.

## 2. A Multitiered Supply Chain Network Game Theory Model with Suppliers and Quality Competition

In this section, we develop a multitiered supply chain network game theory model with suppliers and firms that procure components from the suppliers for their products, which are differentiated by brand. We consider a supply chain network consisting of $I$ firms, with a typical firm denoted by $i$, $n_S$ suppliers, with a typical supplier denoted by $j$, and a total of $n_R$ demand markets, with a typical demand market denoted by $k$.

The firms compete noncooperatively, and each firm corresponds to an individual brand representing the product that it produces. We assume that product $i$, which is the product produced by firm $i$, requires $n_{l^i}$ different components, and the total number of different components required by the $I$ products is $n_l$. We allow for the situation that each supplier may be able to produce a variety of components for each firm.

The $I$ firms are involved in the processes of assembling the products using the components needed, transporting the products to the demand markets, and, possibly, producing the components of the products. The suppliers, in turn, are involved in the processes of producing and delivering the components of the products to the firms. Both in-house and contracted component production activities are captured in the model. Firms' and suppliers' production capacities/abilities are also considered.

The network topology of the problem is depicted in Figure 1. The first two sets of links from the top are links corresponding to distinct supplier components. The links from the top-tiered nodes $j; j = 1, \ldots, n_S$, representing the suppliers, are connected to the associated manufacturing nodes, denoted by nodes $1, \ldots, n_l$. These links represent the manufacturing activities of the suppliers. The next set of links that emanates from $1, \ldots, n_l$ to the firms, denoted by nodes $1, \ldots, I$, reflects the transportation of the components to the associated firms. In addition, the links that connect nodes $1^i, \ldots, n^i_{l^i}$, which are firm $i$'s component manufacturing nodes, and firm $i$ are the manufacturing links of firm $i$ for producing its components.

The rest of the links in Figure 1 are links corresponding to the finished products. The link con-

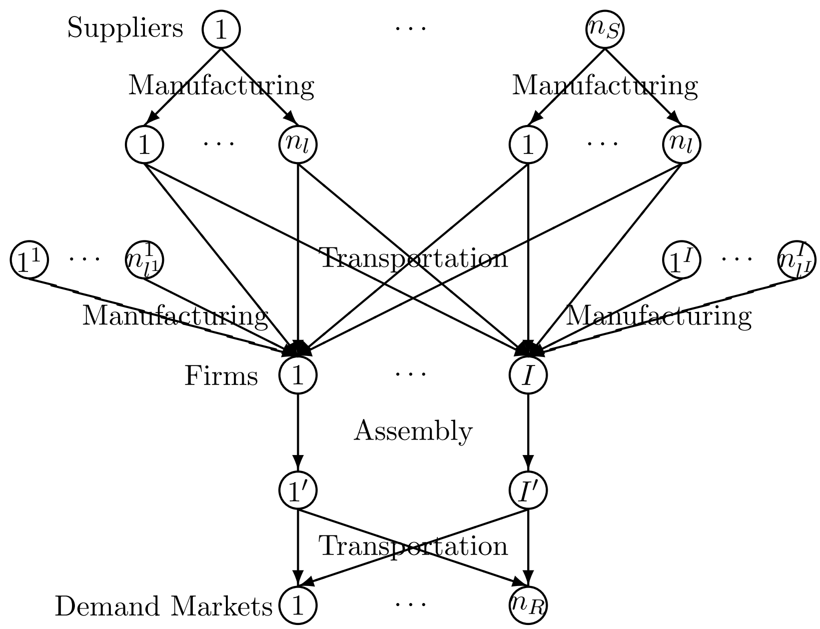


Figure 1: The Multitiered Supply Chain Network Topology

necting firm $i$ and node $i'$, which is the assembly node of firm $i$, represents the activity of assembling firm $i$'s product using the components needed, which may be produced by firm $i$, the suppliers, or both. Finally, the links joining nodes $1', \ldots, I'$ with demand market nodes $1, \ldots, n_R$ correspond to the transportation of the products to the demand markets.

In this paper, we seek to determine the optimal component production quantities and quality levels, both by the firms and by the suppliers, the optimal product shipments from the firms to the demand markets, the optimal quality preservation levels of the assembly processes of the firms, and the prices that the suppliers charge the firms for producing and delivering the components. The firms compete noncooperatively under the Cournot-Nash equilibrium concept in product shipments, in-house and contracted component production quantities, in-house component quality levels, and the quality preservation levels of the assembly processes. The suppliers, in turn, compete in Bertrand fashion in the prices that they charge the firms and the quality levels of the components produced by them. We assume that, through the transaction activities, there is no information asymmetry between firms and suppliers, and firms and suppliers are able to make estimations on each others' and their competitors' cost information.

The model in this paper aims at providing the final equilibrium decisions for firms and suppliers. Since there is no information asymmetry among firms and suppliers and estimations can be made, we assume that all firms and suppliers make their decisions simultaneously. Moreover, each firm and each supplier make quality and quantity/price decisions at the same stage. In Hotelling (1929), Shaked and Sutton (1982), Motta (1993), Aoki and Prusa (1997), Lehmann-Grube (1997), and Banker, Khosla, and Sinha (1998), firms first decided on product quality, and, at the second stage, product quantity/price was determined. Nevertheless, in Leland (1977), Dixit (1979), Gal-Or (1983), Porteus (1986), Cheng (1991), Lederer and Rhee (1995), Starbird (1997), Zhu, Zhang, and Tsung (2007), Xu (2009), Shi, Liu, and Petruzzi (2013), and Ouardighi and Kogan (2013), and in Pennerstorfer and Weiss (2012)'s model for the wine industry, decision-makers determined quality and quantity/price in one stage. Brekkle, Siciliani, and Straume (2010) modeled both one-stage and two-stage scenarios. The above two-stage models reflect the presumption that quantity and price decisions entail more flexibility than firms' quality positioning. However, this is not always true. For example, for critical needs products and products with a steady production rate and demand, such as vaccines, medicines, food, and important agricultural products, the quantity and price decisions can be as flexible as that for quality.

The notation for the variables and parameters in the model is given in Table 1. The functions in the model are given in Table 2. The vectors are assumed to be column vectors. The optimal/equilibrium solution is denoted by "*".

The following conservation of flow equation must hold:

$$Q_{ik} = d_{ik}, \quad i = 1, \dots, I; k = 1, \dots, n_R. \tag{1}$$

Hence, the quantity of a firm's brand-name product consumed at a demand market is equal to the amount shipped from the firm to that demand market. In addition, the shipment volumes must be nonnegative, that is:

$$Q_{ik} \geq 0, \quad i = 1, \dots, I; k = 1, \dots, n_R. \tag{2}$$

In addition, we quantify the quality levels of the components as values between 0 and the perfect quality, that is:

$$\bar{q}_{il} \geq q^S_{jil} \geq 0, \quad j = 1, \dots, n_S; i = 1, \dots, I; l = 1, \dots, n_{l^i}, \tag{3}$$

$$\bar{q}_{il} \geq q^F_{il} \geq 0, \quad i = 1, \dots, I; l = 1, \dots, n_{l^i}, \tag{4}$$

where $\bar{q}_{il}$ is the value representing the prefect quality level associated with firm $i$'s component $l$; $i = 1, \dots, I; l = 1, \dots, n_{l^i}$, and 0 can represent the 0% conformance level or the value for the associated minimum quality standard.

The average quality level of product $i$'s component $l$ is determined by all the quantities and quality levels of that component, produced both by firm $i$ and by the suppliers, that is:

$$q_{il} = \frac{q^F_{il} Q^F_{il} + \sum_{j=1}^{n_S} Q^S_{jil} q^S_{jil}}{Q^F_{il} + \sum_{j=1}^{n_S} Q^S_{jil}}, \quad i = 1, \dots, I; l = 1, \dots, n_{l^i}. \tag{5}$$

In Chao, Iravani, and Savaskan (2009), the quality failure rate of a finished product was modeled as a weighted summation of those of its components. However, Economides (1999) considered that the quality of the composite good is the minimum quality of the quality levels of its components. Combining the above opinions, Pennerstorfer and Weiss (2012) presented three forms of quality aggregation that the quality of the final product can be modeled as the weighted summation, the minimum, or the maximum of the quality of suppliers.

Table 1: Notation for the Variables and Parameters in the Multitiered Supply Chain Network Game Theory Model with Suppliers and Quality

| Notation | Definition |
|---|---|
| $Q^S_{jil}$ | the nonnegative amount of firm $i$'s component $l$ produced by supplier $j$; $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$. For firm $i$, we group its $\{Q^S_{jil}\}$ elements into the vector $Q^S_i \in R^{n_S n_{l^i}}_+$. All the $\{Q^S_{jil}\}$ elements are grouped into the vector $Q^S \in R^{n_S \sum_{i=1}^I n_{l^i}}_+$. |
| $q^S_{jil}$ | the quality of firm $i$'s component $l$ produced by supplier $j$. For supplier $j$, we group its $\{q^S_{jil}\}$ elements into the vector $q^S_j \in R^{\sum_{i=1}^I n_{l^i}}_+$, and group all such vectors into the vector $q^S \in R^{n_S \sum_{i=1}^I n_{l^i}}_+$. |
| $CAP^S_{jil}$ | the capacity of supplier $j$ for producing firm $i$'s component $l$. |
| $\pi_{jil}$ | the price charged by supplier $j$ for producing one unit of firm $i$'s component $l$. For supplier $j$, we group its $\{\pi_{jil}\}$ elements into the vector $\pi_j \in R^{\sum_{i=1}^I n_{l^i}}_+$, and group all such vectors into the vector $\pi \in R^{n_S \sum_{i=1}^I n_{l^i}}_+$. |
| $Q^F_{il}$ | the nonnegative amount of firm $i$'s component $l$ produced by firm $i$ itself. For firm $i$, we group its $\{Q^F_{il}\}$ elements into the vector $Q^F_i \in R^{n_{l^i}}_+$, and group all such vectors into the vector $Q^F \in R^{\sum_{i=1}^I n_{l^i}}_+$. |
| $q^F_{il}$ | the quality of firm $i$'s component $l$ produced by firm $i$ itself. For firm $i$, we group its $\{q^F_{il}\}$ elements into the vector $q^F_i \in R^{n_{l^i}}_+$, and group all such vectors into the vector $q^F \in R^{\sum_{i=1}^I n_{l^i}}_+$. |
| $CAP^F_{il}$ | the capacity of firm $i$ for producing its component $l$. |
| $q_{il}$ | the average quality of firm $i$'s component $l$, produced both by the firm and by the suppliers. |
| $Q_{ik}$ | the nonnegative shipment of firm $i$'s product from firm $i$ to demand market $k$; $k = 1, \ldots, n_R$. For firm $i$, we group its $\{Q_{ik}\}$ elements into the vector $Q_i \in R^{n_R}_+$, and group all such vectors into the vector $Q \in R^{In_R}_+$. |
| $\alpha^F_i$ | the quality preservation level of the assembly process of firm $i$. We group all $\{\alpha^F_i\}$ elements into the vector $\alpha^F \in R^I_+$. |
| $q_i$ | the quality associated with firm $i$'s product. We group all $\{q_i\}$ elements into the vector $q \in R^I_+$. |
| $d_{ik}$ | the demand for firm $i$'s product at demand market $k$. We group all $\{d_{ik}\}$ elements into the vector $d \in R^{In_R}_+$. |
| $\theta_{il}$ | the amount of component $l$ needed by firm $i$ to produce one unit product $i$. |
| $\omega_{il}$ | the ratio of the importance of the quality of firm $i$'s component $l$ in one unit product $i$ to the quality associated with one unit product $i$ (i.e., $q_i$). |

We all know that a product is composed of or is a mixture of different components/materials with distinct functions, and they synergize in contributing to the performance of the final product. This synergy mechanism should be captured and considered in the quality expression of the final product. In addition, different components are of different importance to the quality of the final product. For example, for an automobile, the quality of the spare tire is not as critical as that of the ignition switch, the engine, or the airbags, so it is far from the truth to consider that the quality of a car can

Table 2: Functions for the Multitiered Supply Chain Network Game Theory Model with Suppliers and Quality

| Function | Definition |
|---|---|
| $f^F_{il}(Q^F, q^F)$ | firm $i$'s production cost for producing its component $l$; $i = 1, \ldots, I$; $l = 1, \ldots, n_l$. |
| $f_i(Q, \alpha^F)$ | firm $i$'s cost for assembling its product using the components needed. |
| $tc^F_{ik}(Q, q)$ | firm $i$'s transportation cost for shipping its product to demand market $k$; $k = 1, \ldots, n_R$. |
| $c_{ijl}(Q^S)$ | the transaction cost paid by firm $i$ for transacting with supplier $j$; $j = 1, \ldots, n_S$, for its component $l$. |
| $f^S_{jl}(Q^S, q^S)$ | supplier $j$'s production cost for producing component $l$. |
| $tc^S_{jil}(Q^S, q^S)$ | supplier $j$'s transportation cost for shipping firm $i$'s component $l$. |
| $oc_{jil}(\pi)$ | the opportunity cost of supplier $j$ associated with pricing firm $i$'s component $l$ at $\pi_{jil}$ for producing and transporting it. |
| $\rho_{ik}(d, q)$ | the demand price for firm $i$'s product at demand market $k$. |

be represented by the quality of the spare tire only because the spare tire is of extremely low or high quality. Therefore, the weighted summation expression is most general among the three expressions of product quality in Pennerstorfer and Weiss (2012). Moreover, one should not neglect the fact that, the quality of a product is also affected by its assembly processes, in which the components should be fitted together properly in order to achieve particular functions of the product.

Thus, in this paper, we model the quality of a finished product $i$ as a function determined by the average quality levels of its components, the importance of the quality of the components to the quality of the product, and the quality preservation level of the assembly process of firm $i$. It is expressed as:

$$q_i = \alpha^F_i (\sum_{l=1}^{n_{l_i}} \omega_{il} q_{il}), \quad i = 1, \ldots, I; l = 1. \tag{6}$$

Note that $\alpha^F_i$ captures the percentage of the quality preservation of product $i$ in the assembly process of the firm and lies between 0 and 1, that is:

$$0 \leq \alpha^F_i \leq 1, \quad i = 1, \ldots, I, \tag{7}$$

where 0 can also be used to represent the value for the quality preservation standard of the assembly process.

The decay of quality can hence be captured by $1-\alpha^F_i$. In Nagurney and Masoumi (2012), Masoumi, Yu, and Nagurney (2012), Nagurney and Nagurney (2012), Yu and Nagurney (2013), and in Nagurney et al. (2013b), arc multipliers that are similar to $\alpha^F_i$ are used to model the perishability of particular products, such as pharmaceuticals, human blood, medical nuclear products, and fresh produce, in terms of the percentages of flows that reach the successor nodes in supply chain networks.

We also assume that the importance of the quality levels of all components of product $i$ sums up to 1, that is:

$$\sum_{l=1}^{n_{li}} \omega_{il} = 1, \quad i = 1, \ldots, I. \tag{8}$$

In view of (1), (5), and (6), we can redefine the transportation cost functions of the firms $tc_{ik}^F(Q, q)$ and the demand price functions $\rho_{ik}(d, q)$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$, in quantities and quality levels of the components, both by the firms and by the suppliers, the quantities of the products, and the quality preservation levels of the assembly processes, that is:

$$\hat{tc}_{ik}^F = \hat{tc}_{ik}^F(Q, Q^F, Q^S, q^F, q^S, \alpha^F) = tc_{ik}^F(Q, q), \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{9}$$

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(Q, Q^F, Q^S, q^F, q^S, \alpha^F) = \rho_{ik}(d, q), \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{10}$$

The generality of the expressions in (9) and (10) allows for modeling and application flexibility. The demand price functions (10) are, typically, assumed to be monotonically decreasing in product quantities but increasing in terms of product quality levels.

As noted in Table 2, the assembly cost functions, the production cost functions, the transportation cost functions, the transaction cost functions, and the demand price functions are general functions in vectors of quantities and/or quality levels, since one supplier's or one firm's decisions will affect their competitors' costs and, hence, their decisions as well. In this way, the interaction among firms and that among suppliers in their competition for resources and technologies are captured.

Furthermore, the cost functions measure not only the monetary costs in the corresponding processes, but also other important factors, such as the time spent in conforming the processes and the costs of ensuring and assuring quality in these processes (e.g., scrap costs, screening costs, rework costs, and the investments for quality engineering and training). The compensation costs incurred when customers are dissatisfied with the quality of the products, such as warranty charges and the complaint adjustment cost, are also included, which can be utilized to measure the disrepute costs of the firms. The costs related to quality are all convex functions of quality conformance levels (see, e.g., Feigenbaum (1983), Juran and Gryna (1988), Campanella (1990), Porter and Rayner (1992), and Shank and Govindarajan (1994)). In addition, since, as argued by Bender et al. (1985), one of the most important factors that must be considered in selecting suppliers is their cost, we assume that the cost functions of the suppliers are known by the firms.

Please note that, since, as mentioned, defectives are hard to define and identify in reality, in the context of this paper, firms and suppliers may produce defectives. However, they are not motivated to do so. As reflected in the demand price functions, consumers pay more for products with higher quality, and, hence, good quality products and components will lead to more revenue. Thus, essentially, the component quality, the quality preservation/decay level, and hence product quality are highly dependent on the costs related to quality, the prices of the components, and consumers sensitivity to product quality.

Furthermore, in this paper, we assume that transportation activities affect quality in terms of quality preservation, and, thus, quality does not deteriorate during transportation, but, as mentioned, may deteriorate in the assembly processes.

This model is also capable of handling the case of outsourcing by setting each $n_{l^i}$; $i = 1, \ldots, I$, to 1. In such a case, the contractors do the outsourced jobs of producing products and transporting them to the firms, and the firms do the packaging and labeling for their products and may also produce in-house. In addition, when the number of firms and the number of the suppliers are one, this model is able to capture the case of a single firm - single supplier supply chain where the firm procures from one exclusive supplier, without competition, as in the models in related literature (cf. Section 1 and Example 1).

### 2.1 The Behavior of the Firms and Their Optimality Conditions

Given the prices $\pi_i^*$ of the components that the suppliers charge firm $i$, and the quality $q^{S^*}$ of the components produced by the suppliers, the objective of firm $i$; $i = 1, \ldots, I$, is to maximize its utility/profit $U_i^F$. It is the difference between its total revenue and its total cost. The total cost includes the assembly cost, the production costs, the transportation costs, the transaction costs, and the payments to the suppliers.

Hence, firm $i$ seeks to

$$\text{Maximize}_{Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F} \quad U_i^F = \sum_{k=1}^{n_R} \hat{\rho}_{ik}(Q, Q^F, Q^S, q^F, q^{S^*}, \alpha^F) d_{ik} - f_i(Q, \alpha^F) - \sum_{l=1}^{n_{l^i}} f_{il}^F(Q^F, q^F)$$

$$- \sum_{k=1}^{n_R} \hat{tc}_{ik}^F(Q, Q^F, Q^S, q^F, q^{S^*}, \alpha^F) - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} c_{ijl}(Q^S) - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} \pi_{jil}^* Q_{jil}^S \tag{11}$$

subject to:

$$\sum_{k=1}^{n_R} Q_{ik}\theta_{il} \le \sum_{j=1}^{n_S} Q_{jil}^S + Q_{il}^F, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{12}$$

$$CAP_{jil}^S \ge Q_{jil}^S \ge 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{13}$$

$$CAP_{il}^F \ge Q_{il}^F \ge 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{14}$$

and (1), (2), (4), and (7).

We assume that all the cost functions and demand price functions in (11) are continuous and twice continuously differentiable. The cost functions are convex in quantities and/or quality levels and have bounded second-order partial derivatives. The demand price functions have bounded first-order and second-order partial derivatives. Constraint (12) captures the material requirements in the assembly process. Constraints (13) and (14) indicate that the component production quantities should be nonnegative and limited by the associated capacities, which can capture the abilities of producing. If a supplier or a firm is not capable of producing a certain component, the associated capacity should be 0.

The firms compete in the sense of Nash (1950, 1951). The strategic variables for each firm $i$ are the product shipments to the demand markets, the in-house component production quantities, the contracted component production quantities, which are produced by the suppliers, the quality levels of the in-house produced components, and the quality preservation level of its assembly process.

We define the feasible set $\overline{K}_i^F$ as $\overline{K}_i^F \equiv \{(Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F) | (1), (2), (4), (7), \text{and } (12) - (14)$ are satisfied$\}$. All $\overline{K}_i^F$; $i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\overline{\mathcal{K}}^F \equiv \Pi_{i=1}^I \overline{K}_i^F$.

**Definition 1: A Cournot-Nash Equilibrium**

*A product shipment, in-house component production, contracted component production, in-house component quality, and assembly quality preservation pattern* $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}) \in \overline{\mathcal{K}}^F$ *is said to constitute a Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$,*

$$U_i^F(Q_i^*, \hat{Q}_i^*, Q_i^{F^*}, \hat{Q}_i^{F^*}, Q_i^{S^*}, \hat{Q}_i^{S^*}, q_i^{F^*}, \hat{q}_i^{F^*}, \alpha_i^{F^*}, \hat{\alpha}_i^{F^*}, \pi_i^*, q^{S^*}) \geq$$
$$U_i^F(Q_i, \hat{Q}_i^*, Q_i^F, \hat{Q}_i^{F^*}, Q_i^S, \hat{Q}_i^{S^*}, q_i^F, \hat{q}_i^{F^*}, \alpha_i^F, \hat{\alpha}_i^{F^*}, \pi_i^*, q^{S^*}),$$
$$\forall (Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F) \in \overline{K}_i^F, \tag{15}$$

*where*

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$
$$\hat{Q}_i^{F^*} \equiv (Q_1^{F^*}, \ldots, Q_{i-1}^{F^*}, Q_{i+1}^{F^*}, \ldots, Q_I^{F^*}),$$
$$\hat{Q}_i^{S^*} \equiv (Q_1^{S^*}, \ldots, Q_{i-1}^{S^*}, Q_{i+1}^{S^*}, \ldots, Q_I^{S^*}),$$
$$\hat{q}_i^{F^*} \equiv (q_1^{F^*}, \ldots, q_{i-1}^{F^*}, q_{i+1}^{F^*}, \ldots, q_I^{F^*}),$$

*and*

$$\hat{\alpha}_i^{F^*} \equiv (\alpha_1^{F^*}, \ldots, \alpha_{i-1}^{F^*}, \alpha_{i+1}^{F^*}, \ldots, \alpha_I^{F^*}).$$

According to (15), a Cournot-Nash equilibrium is established if no firm can unilaterally improve upon its profit by selecting an alternative vector of product shipments, in-house component production quantities, contracted component production quantities, in-house component quality levels, and the quality preservation level of its assembly process.

We now derive the variational inequality formulation of the Cournot-Nash equilibrium (see Cournot (1838), Nash (1950, 1951), and Gabay and Moulin (1980)) in the following theorem.

**Theorem 1: Variational Inequality Formulation for Firms' Problems**

*Assume that, for each firm i; $i = 1, \ldots, I$, the utility function $U_i^F(Q, Q^F, Q^S, q^F, \alpha^F, \pi_i^*, q^{S^*})$ is concave with respect to its variables in $Q_i$, $Q_i^F$, $Q_i^S$, $q_i^F$, and $\alpha_i^F$, and is continuous and twice continuously differentiable. Then $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}) \in \overline{\mathcal{K}}^F$ is a Counot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)$$
$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{jil}^S}\times(Q_{jil}^S-Q_{jil}^{S^*})$$

$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial q_{il}^F}\times(q_{il}^F-q_{il}^{F^*})$$

$$-\sum_{i=1}^{I}\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial \alpha_i^F}\times(\alpha_i^F-\alpha_i^{F^*})\geq 0,\quad \forall(Q,Q^F,Q^S,q^F,\alpha^F)\in\overline{\mathcal{K}}^F, \tag{16}$$

*with notice that: for $i=1,\ldots,I$; $k=1,\ldots,n_R$:*

$$-\frac{\partial U_i^F}{\partial Q_{ik}}=\Bigg[\frac{\partial f_i(Q,\alpha^F)}{\partial Q_{ik}}+\sum_{h=1}^{n_R}\frac{\partial \hat{tc}_{ih}^F(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial Q_{ik}}-\sum_{h=1}^{n_R}\frac{\partial\hat{\rho}_{ih}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial Q_{ik}}\times d_{ih}$$
$$-\hat{\rho}_{ik}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)\Bigg],$$

*for $i=1,\ldots,I$; $l=1,\ldots,n_{l^i}$:*

$$-\frac{\partial U_i^F}{\partial Q_{il}^F}=\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f_{im}^F(Q^F,q^F)}{\partial Q_{il}^F}+\sum_{h=1}^{n_R}\frac{\partial \hat{tc}_{ih}^F(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial Q_{il}^F}-\sum_{h=1}^{n_R}\frac{\partial\hat{\rho}_{ih}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial Q_{il}^F}\times d_{ih}\right],$$

*for $j=1,\ldots,n_S$; $i=1,\ldots,I$; $l=1,\ldots,n_{l^i}$:*

$$-\frac{\partial U_i^F}{\partial Q_{jil}^S}=\left[\pi_{jil}^*+\sum_{h=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{ihm}(Q^S)}{\partial Q_{jil}^S}+\sum_{h=1}^{n_R}\frac{\partial \hat{tc}_{ih}^F(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial Q_{jil}^S}-\sum_{h=1}^{n_R}\frac{\partial\hat{\rho}_{ih}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial Q_{jil}^S}\times d_{ih}\right],$$

*for $i=1,\ldots,I$; $l=1,\ldots,n_{l^i}$:*

$$-\frac{\partial U_i^F}{\partial q_{il}^F}=\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f_{im}^F(Q^F,q^F)}{\partial q_{il}^F}+\sum_{h=1}^{n_R}\frac{\partial \hat{tc}_{ih}^F(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial q_{il}^F}-\sum_{h=1}^{n_R}\frac{\partial\hat{\rho}_{ih}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial q_{il}^F}\times d_{ih}\right],$$

*for $i=1,\ldots,I$:*

$$-\frac{\partial U_i^F}{\partial \alpha_i^F}=\left[\frac{\partial f_i(Q^F,\alpha^F)}{\partial \alpha_i^F}+\sum_{h=1}^{n_R}\frac{\partial \hat{tc}_{ih}^F(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial \alpha_i^F}-\sum_{h=1}^{n_R}\frac{\partial\hat{\rho}_{ih}(Q,Q^F,Q^S,q^F,q^{S^*},\alpha^F)}{\partial \alpha_i^F}\times d_{ih}\right],$$

*or, equivalently, in view of (1) and (12), $(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\lambda^*)\in\mathcal{K}^F$ is a vector of the equilibrium product shipment, in-house component production, contracted component production, in-house component quality, and assembly quality preservation pattern and Lagrange multipliers if and only if it satisfies the variational inequality*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{ik}}+\sum_{l=1}^{n_{l^i}}\lambda_{il}^*\theta_{il}\right]\times(Q_{ik}-Q_{ik}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{il}^F}-\lambda_{il}^*\right]\times(Q_{il}^F-Q_{il}^{F^*})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{jil}^S}-\lambda_{il}^*\right]\times(Q_{jil}^S-Q_{jil}^{S^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial q_{il}^F}\right]\times(q_{il}^F-q_{il}^{F^*})$$

$$+\sum_{i=1}^{I}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial \alpha_i^F}\right]\times(\alpha_i^F-\alpha_i^{F^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[\sum_{j=1}^{n_S}Q_{jil}^{S^*}+Q_{il}^{F^*}-\sum_{k=1}^{n_R}Q_{ik}^{*}\theta_{il}\right]\times(\lambda_{il}-\lambda_{il}^{*})\geq 0,\quad \forall(Q,Q^F,Q^S,q^F,\alpha^F,\lambda)\in\mathcal{K}^F, \tag{17}$$

*where $\mathcal{K}^F \equiv \Pi_{i=1}^{I}K_i^F$ and $K_i^F \equiv \{(Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F, \lambda_i)|\lambda_i \geq 0$ with (2), (4), (7), (13), and (14) satisfied$\}$. $\lambda_i$ is the $n_{li}$-dimensional vector with component $l$ being the element $\lambda_{il}$ corresponding to the Lagrange multiplier associated with the $(i,l)$-th constraint (12). Both the above-defined feasible sets are convex.*

**Proof**: Please see the Appendix.

For additional background on the variational inequality problem, please refer to the book by Nagurney (1999).

### 2.2 The Behavior of the Suppliers and Their Optimality Conditions

Opportunity costs of the suppliers are considered in this model. As in Nagurney, Li, and Nagurney (2013) and Nagurney and Li (2015), we capture each opportunity cost with a general function that depends on the entire vector of prices, since the opportunity costs of a supplier may also be affected by the prices charged by the other suppliers.

Given the $Q^{S^*}$ determined by the firms, the objective of supplier $j$; $j = 1,\ldots,n_S$, is to maximize its total profit, denoted by $U_j^S$. Its revenue is obtained from the payments of the firms, while its costs are the costs of production and delivery and the opportunity costs. The strategic variables of a supplier are the prices that it charges the firms and the quality levels of the components that it produces.

The decision-making problem for supplier $j$ is as the following:

$$\text{Maximize}_{\pi_j,q_j^S}\quad U_j^S=\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\pi_{jil}Q_{jil}^{S^*}-\sum_{l=1}^{n_l}f_{jl}^S(Q^{S^*},q^S)-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}tc_{jil}^S(Q^{S^*},q^S)-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}oc_{jil}(\pi) \tag{18}$$

subject to:

$$\pi_{jil}\geq 0,\quad j=1,\ldots,n_S;i=1,\ldots,I;l=1,\ldots,n_{l^i}, \tag{19}$$

and (3).

We assume that the cost functions of each supplier are continuous, twice continuously differentiable, and convex, and have bounded second-order partial derivatives.

The suppliers compete in a noncooperative in the sense of Nash (1950, 1951), with each one trying to maximize its own profit. We define the feasible sets $K_j^S \equiv \{(\pi_j, q_j^S)|\pi_j \in R_+^{\sum_{i=1}^{I}n_{li}}$ and $q_j^S$ satisfies (3) for $j\}$, $\mathcal{K}^S \equiv \Pi_{j=1}^{n_S}K_j^S$, and $\overline{\mathcal{K}} \equiv \overline{\mathcal{K}}^F \times \mathcal{K}^S$. All the above-defined feasible sets are convex.

**Definition 2: A Bertrand-Nash Equilibrium**

*A price and contracted component quality pattern $(\pi^*, q^{S^*}) \in \mathcal{K}^S$ is said to constitute a Bertrand-Nash equilibrium if for each supplier $j$; $j = 1,\ldots,n_S$,*

$$U_j^S(Q^{S^*},\pi_j^*,\hat{\pi}_j^*,q_j^{S^*},\hat{q}_j^{S^*})\geq U_j^S(Q^{S^*},\pi_j,\hat{\pi}_j^*,q_j^S,\hat{q}_j^{S^*}),\quad\forall(\pi_j,q_j^S)\in K_j^S, \tag{20}$$

*where*

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_S}^*)$$

*and*

$$\hat{q}_j^{S^*} \equiv (q_1^{S^*}, \ldots, q_{j-1}^{S^*}, q_{j+1}^{S^*}, \ldots, q_{n_S}^{S^*}).$$

According to (20), a Bertrand-Nash equilibrium is established if no supplier can unilaterally improve upon its profit by selecting an alternative vector of prices that it charges the firms and the quality levels of the components that it produces.

The variational inequality formulation of the Bertrand-Nash equilibrium according to Definition 2 (see Bertrand (1883), Nash (1950, 1951), Gabay and Moulin (1980), Nagurney (2006)) is given in the following theorem.

**Theorem 2: Variational Inequality Formulation for Suppliers' Problems**

*Assume that, for each supplier $j$; $j = 1, \ldots, n_S$, the profit function $U_j^S(Q^{S^*}, \pi, q^S)$ is concave with respect to the variables in $\pi_j$ and $q_j^S$, and is continuous and twice continuously differentiable. Then $(\pi^*, q^{S^*}) \in \mathcal{K}^S$ is a Bertrand-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:*

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) - \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial q_{jil}^S} \times (q_{jil}^S - q_{jil}^{S^*}) \geq 0,$$

$$\forall (\pi, q^S) \in \mathcal{K}^S, \tag{21}$$

*with notice that: for $j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$:*

$$-\frac{\partial U_j^S}{\partial \pi_{jil}} = \sum_{g=1}^{I}\sum_{m=1}^{n_{l^i}} \frac{\partial oc_{jgm}(\pi)}{\partial \pi_{jil}} - Q_{jil}^{S^*},$$

*for $j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$:*

$$-\frac{\partial U_j^S}{\partial q_{jil}^S} = \sum_{m=1}^{n_l} \frac{\partial f_{jm}^S(Q^{S^*}, q^S)}{\partial q_{jil}^S} + \sum_{g=1}^{I}\sum_{m=1}^{n_{l^i}} \frac{\partial tc_{jgm}^S(Q^{S^*}, q^S)}{\partial q_{jil}^S}.$$

### 2.3 The Equilibrium Conditions for the Multitiered Supply Chain Network with Suppliers and Quality Competition

In equilibrium, the optimality conditions for all firms and the optimality conditions for all suppliers must hold simultaneously, according to the definition below.

**Definition 3: Multitiered Supply Chain Network Equilibrium with Suppliers and Quality Competition**

*The equilibrium state of the multitiered supply chain network with suppliers is one where both variational inequalities (16), or, equivalently, (17), and (21) hold simultaneously.*

**Theorem 3: Variational Inequality Formulation for the Multitiered Supply Chain Network Equilibrium with Suppliers and Quality Competition**

*The equilibrium conditions governing the multitiered supply chain network model with suppliers and quality competition are equivalent to the solution of the variational inequality problem: determine* $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*}) \in \overline{\mathcal{K}}$*, such that:*

$$
\begin{aligned}
&-\sum_{i=1}^{I}\sum_{k=1}^{n_R}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)\\
&-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})\\
&-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*})\\
&-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial q_{il}^F} \times (q_{il}^F - q_{il}^{F^*})\\
&-\sum_{i=1}^{I}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial \alpha_i^F} \times (\alpha_i^F - \alpha_i^{F^*})\\
&-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*)\\
&-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial q_{jil}^S} \times (q_{jil}^S - q_{jil}^{S^*}) \geq 0, \quad \forall (Q, Q^S, Q^F, q^F, \alpha^F, \pi, q^S) \in \overline{\mathcal{K}},
\end{aligned}
\tag{22}
$$

*or, equivalently: determine* $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \lambda^*, \pi^*, q^{S^*}) \in \mathcal{K}$*, such that:*

$$
\begin{aligned}
&\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{ik}} + \sum_{l=1}^{n_{li}}\lambda_{il}^*\theta_{il}\right] \times (Q_{ik} - Q_{ik}^*)\\
&+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{il}^F} - \lambda_{il}^*\right] \times (Q_{il}^F - Q_{il}^{F^*})\\
&+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{jil}^S} - \lambda_{il}^*\right] \times (Q_{jil}^S - Q_{jil}^{S^*})\\
&+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial q_{il}^F}\right] \times (q_{il}^F - q_{il}^{F^*})\\
&+\sum_{i=1}^{I}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial \alpha_i^F}\right] \times (\alpha_i^F - \alpha_i^{F^*})\\
&+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[\sum_{j=1}^{n_S}Q_{jil}^{S^*} + Q_{il}^{F^*} - \sum_{k=1}^{n_R}Q_{ik}^*\theta_{il}\right] \times (\lambda_{il} - \lambda_{il}^*)\\
&+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}}\right] \times (\pi_{jil} - \pi_{jil}^*)
\end{aligned}
$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_j^S(Q^{S^*},\pi^*,q^{S^*})}{\partial q_{jil}^S}\right]\times(q_{jil}^S-q_{jil}^{S^*})\geq 0,\quad \forall(Q,Q^F,Q^S,q^F,\alpha^F,\lambda,\pi,q^S)\in\mathcal{K}, \tag{23}$$

*where* $\mathcal{K}\equiv\mathcal{K}^F\times\mathcal{K}^S$.

**Proof:** Summation of variational inequalities (16) (or (17)) and (21) yields variational inequality (22) (or (23)). A solution to variational inequality (22) (or (23)) satisfies the sum of (16) (or (17)) and (21) and, hence, is an equilibrium according to Definition 3.□

We now put variational inequality (23) into standard form (cf. Nagurney (1999)): determine $X^*\in\mathcal{K}$ where $X$ is a vector in $R^N$, $F(X)$ is a continuous function such that $F(X): X\mapsto\mathcal{K}\subset R^N$, and

$$\langle F(X^*), X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}, \tag{24}$$

where $\langle\cdot,\cdot\rangle$ is the inner product in the $N$-dimensional Euclidean space, $N=In_R+3\sum_{i=1}^{I}n_{l^i}+3n_S\sum_{i=1}^{I}n_{l^i}+I$, and $\mathcal{K}$ is closed and convex. We define the vector $X\equiv(Q,Q^F,Q^S,q^F,\alpha^F,\lambda,\pi,q^S)$ and the vector $F(X)\equiv(F^1(X),F^2(X),F^3(X),F^4(X),F^5(X),F^6(X),F^7(X),F^8(X))$, such that:

$$F^1(X)=\left[-\frac{\partial U_i^F(Q,Q^F,Q^S,q^F,\alpha^F,\pi_i,q^S)}{\partial Q_{ik}}+\sum_{l=1}^{n_{l^i}}\lambda_{il}\theta_{il};i=1,\ldots,I;k=1,\ldots,n_R\right],$$

$$F^2(X)=\left[-\frac{\partial U_i^F(Q,Q^F,Q^S,q^F,\alpha^F,\pi_i,q^S)}{\partial Q_{il}^F}-\lambda_{il};i=1,\ldots,I;l=1,\ldots,n_{l^i}\right],$$

$$F^3(X)=\left[-\frac{\partial U_i^F(Q,Q^F,Q^S,q^F,\alpha^F,\pi_i,q^S)}{\partial Q_{jil}^S}-\lambda_{il};j=1,\ldots,n_S;i=1,\ldots,I;l=1,\ldots,n_{l^i}\right],$$

$$F^4(X)=\left[-\frac{\partial U_i^F(Q,Q^F,Q^S,q^F,\alpha^F,\pi_i,q^S)}{\partial q_{il}^F};i=1,\ldots,I;l=1,\ldots,n_{l^i}\right],$$

$$F^5(X)=\left[-\frac{\partial U_i^F(Q,Q^F,Q^S,q^F,\alpha^F,\pi_i,q^S)}{\partial \alpha_i^F};i=1,\ldots,I\right],$$

$$F^6(X)=\left[\sum_{j=1}^{n_S}Q_{jil}^S+Q_{il}^F-\sum_{k=1}^{n_R}Q_{ik}\theta_{il};i=1,\ldots,I;l=1,\ldots,n_{l^i}\right],$$

$$F^7(X)=\left[-\frac{\partial U_j^S(Q^S,\pi,q^S)}{\partial \pi_{jil}};j=1,\ldots,n_S;i=1,\ldots,I;l=1,\ldots,n_{l^i}\right],$$

$$F^8(X)=\left[-\frac{\partial U_j^S(Q^S,\pi,q^S)}{\partial q_{jil}^S};j=1,\ldots,n_S;i=1,\ldots,I;l=1,\ldots,n_{l^i}\right]. \tag{25}$$

Hence, (23) can be put into standard form (24).

Similarly, we also put variational inequality (22) into standard form: determine $Y^*\in\overline{\mathcal{K}}$ where $Y$ is a vector in $R^M$, $G(Y)$ is a continuous function such that $G(Y): Y\mapsto\overline{\mathcal{K}}\subset R^M$, and

$$\langle G(Y^*), Y-Y^*\rangle\geq 0,\quad \forall Y\in\overline{\mathcal{K}}, \tag{26}$$

where $M = In_R + 2\sum_{i=1}^{I} n_{l^i} + 3n_S \sum_{i=1}^{I} n_{l^i} + I$, and $\overline{\mathcal{K}}$ is closed and convex. We define $Y \equiv (Q, Q^F, Q^S, q^F, \alpha^F, \pi, q^S)$, $G(Y) \equiv (-\frac{\partial U_i^F}{\partial Q_{ik}}, -\frac{\partial U_i^F}{\partial Q_{il}^F}, -\frac{\partial U_i^F}{\partial Q_{jil}^S}, -\frac{\partial U_i^F}{\partial q_{il}^F}, -\frac{\partial U_i^F}{\partial \alpha_i^F}, -\frac{\partial U_j^S}{\partial \pi_{jil}}, -\frac{\partial U_j^S}{\partial q_{jil}^S}); j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$. Hence, (22) can be put into standard form (26).

The equilibrium solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ to (26) and the $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ in the equilibrium solution to (24) are equivalent for this multitiered supply chain network problem. In addition to $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$, the equilibrium solution to (24) also contains the equilibrium Lagrange multipliers $(\lambda^*)$.

## 3. Qualitative Properties

In this Section, we present some qualitative properties of the solution to variational inequality (24) and (26), equivalently, (23) and (22). In particular, we provide the existence result and the uniqueness result. We also investigate the properties of the function $F$ given by (25) that enters variational inequality (24) and the function $G$ that enters variational inequality (26).

In a multitiered supply chain network with suppliers, it is reasonable to expect that the price charged by each supplier $j$ for producing one unit of firm $i$'s component $l$, $\pi_{jil}$, is bounded by a sufficiently large value, since, in practice, each supplier cannot charge unbounded prices to the firms. Therefore, the following assumption is not unreasonable:

**Assumption 1**

*Suppose that in our multitiered supply chain network model with suppliers and quality competition, there exist a sufficiently large* $\Pi$*, such that,*

$$\pi_{jil} \leq \Pi, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{27}$$

With this assumption, we have the following existence result.

**Theorem 4: Existence**

*With Assumption 1 satisfied, there exists at least one solution to variational inequality (24) and (26), equivalently, (23) and (22).*

**Proof**: Please see the Appendix.

**Theorem 5: Monotonicity**

*Under the assumptions in Theorems 1 and 2, the* $F(X)$ *that enters variational inequality (24), is monotone, that is,*

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, \tag{28}$$

*and the* $G(Y)$ *that enters variational inequality (27) is also monotone,*

$$\langle G(Y') - G(Y''), Y' - Y'' \rangle \geq 0, \quad \forall Y', Y'' \in \overline{\mathcal{K}}. \tag{29}$$

**Proof**: Please see the Appendix.

**Definition 4: Strict Monotonicity**

*The function $G(Y)$ in variational inequality (26) is strictly monotone on $\bar{\mathcal{K}}$ if*

$$\langle G(Y') - G(Y''), Y' - Y'' \rangle > 0, \quad \forall Y', Y'' \in \bar{\mathcal{K}}, Y' \neq Y''. \tag{30}$$

The monotonicity of the function $G$ is closely related to the positive-definiteness of its Jacobian $\nabla G$ (cf. Nagurney (1999)). Particularly, if $\nabla G$ is positive-definite, $G$ is strictly monotone.

**Theorem 6: Uniqueness**

*Assume that the strict monotonicity condition (30) is satisfied. Then, if variational inequality (26) admits a solution, $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$, that is the only solution.*

**Proof**: Under the strict monotonicity assumption given by (30), the proof follows the standard variational inequality theory (cf. Kinderlehrer and Stampacchia (1980)). □

**Theorem 7: Lipschitz Continuity**

*The function that enters the variational inequality problem (24) is Lipschitz continuous, that is,*

$$\| F(X') - F(X'') \| \leq L \| X' - X'' \|, \quad \forall X', X'' \in \mathcal{K}, \text{ where } L > 0. \tag{31}$$

**Proof**: Since we have assumed that all the cost functions have bounded second-order partial derivatives, and the demand price functions have bounded first-order and second-order partial derivatives, the result is direct by applying a mid-value theorem from calculus to the $F(X)$ that enters variational inequality (24). □

## 4. The Algorithm

We employ the modified projection method (see Korpelevich (1997) and Nagurney (1999)) for the computation of the solution for the multitiered supply chain network game theory model with suppliers and quality competition. It has been effectively used in large-scale supply chain network equilibrium problems (cf. Liu and Nagurney (2009)). The statement of the modified projection method is as follows, where $\mathcal{T}$ denotes an iteration counter:

### The Modified Projection Method

**Step 0: Initialization**

Start with $X^0 \in \mathcal{K}$ (cf. (24)). Set $\mathcal{T} := 1$ and select $a$, such that $0 < a \leq \frac{1}{L}$, where $L$ is the Lipschitz continuity constant (cf. 31) for $F(X)$ (cf. (24))

**Step 1: Construction and Computation**

Compute $\overline{X}^{\mathcal{T}-1}$ by solving the variational inequality subproblem:

$$\langle \overline{X}^{\mathcal{T}-1} + (aF(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}), X - \overline{X}^{\mathcal{T}-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{32}$$

**Step 2: Adaptation**

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + (aF(\overline{X}^{\mathcal{T}-1}) - X^{\mathcal{T}-1}), X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{33}$$

**Step 3: Convergence Verification**

If $\mid X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1} \mid \leq \epsilon$, for all $l$, with $\epsilon > 0$, a prespecified tolerance, then stop; else set $\mathcal{T} := \mathcal{T} + 1$, and go to step 1.

Steps (32) and (33) yield closed form expressions for the variables.

**Theorem 8: Convergence**

*If Assumption 1 is satisfied, and the function $F(X)$ is monotone and Lipschitz continuous, then the modified projection method described above converges to the solution of variational inequality (24).*

**Proof**: According to Korpelevich (1977) and Nagurney (1999), the modified projection method converges to the solution of the variational inequality problem of the form (24), provided that the function $F$ that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 4, monotonicity follows from Theorem 5, and Lipschitz continuity, in turn, follows from Theorem 7. $\square$

## 5. Numerical Examples and Sensitivity Analysis

In this Section, we applied the modified projection method, as described in Section 4, to several numerical examples accompanied by extensive sensitivity analysis. The modified projected method was implemented in Matlab on a Lenovo Z580. We set $a = 0.003$ in the algorithm with the convergence tolerance $\epsilon = 10^{-4}$. The product and component quantities were initialized to 30 and the prices, quality levels, quality preservation levels, and the Lagrange multipliers to 0.

**Example 1**

Consider the supply chain network topology given in Figure 2 in which Firm 1 serves Demand Market 1 and procures the components of its product from Supplier 1. The firm also has the option of producing the components needed by itself. The product of Firm 1 requires only one component $1^1$. 2 units of $1^1$ are needed for producing one unit of Firm 1's product. Thus,

$$\theta_{11} = 2.$$

Component $1^1$ corresponds to node 1 in the second tier and node $1^1$ in the third tier in Figure 2 below.

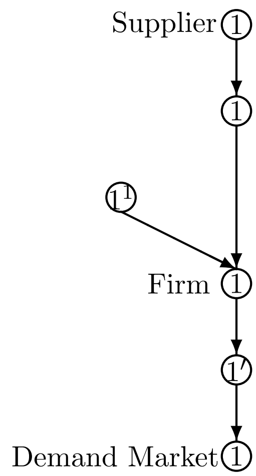


Figure 2: Supply Chain Network Topology for Example 1

The data are as follows.

The capacity of the supplier is:

$$CAP^S_{111} = 120.$$

The firm's capacity for producing its component is:

$$CAP^F_{11} = 80.$$

The value that represents the perfect component quality is:

$$\bar{q}_{11} = 75.$$

The supplier's production cost is:

$$f^S_{11}(Q^S_{111}, q^S_{111}) = 5Q^S_{111} + 0.8(q^S_{111} - 62.5)^2.$$

The supplier's transportation cost is:

$$tc^S_{111}(Q^S_{111}, q^S_{111}) = 0.5Q^S_{111} + 0.2(q^S_{111} - 125)^2 + 0.3Q^S_{111}q^S_{111},$$

and its opportunity cost is:

$$oc_{111}(\pi_{111}) = 0.7(\pi_{111} - 100)^2.$$

The firm's assembly cost is:

$$f_1(Q_{11}, \alpha^F_1) = 0.75Q^2_{11} + 200\alpha^{F^2}_1 + 200\alpha^F_1 + 25Q_{11}\alpha^F_1.$$

The firm's production cost for producing its component is:

$$f^F_{11}(Q^F_{11}, q^F_{11}) = 2.5Q^{F^2}_{11} + 0.5(q^F_{11} - 60)^2 + 0.1Q^F_{11}q^F_{11},$$

and its transaction cost is:

$$c_{111}(Q_{111}^S) = 0.5Q_{111}^{S^2} + Q_{111}^S + 100.$$

The firm's transportation cost for shipping its product to the demand market is:

$$tc_{11}^F(Q_{11}, q_1) = 0.5Q_{11}^2 + 0.02q_1^2 + 0.1Q_{11}q_1,$$

and the demand price function at Demand Market 1 is:

$$\rho_{11}(d_{11}, q_1) = -d_{11} + 0.7q_1 + 1000,$$

where $q_1 = \alpha_1^F \omega_{11} \frac{Q_{11}^F q_{11}^F + Q_{111}^S q_{111}^S}{Q_{11}^F + Q_{111}^S}$ and $\omega_{11} = 1$.

The equilibrium solution that we obtain using the modified projection method is:

$$Q_{11}^* = 89.26, \quad Q_{11}^{F^*} = 60.16, \quad Q_{111}^{S^*} = 118.38, \quad q_{11}^{F^*} = 71.17,$$

$$q_{111}^{S^*} = 57.25, \quad \pi_{11}^* = 184.53, \quad \alpha_1^{F^*} = 1.00, \quad \lambda_{11}^* = 305.25.$$

with the induced demand, demand price, and product quality being

$$d_{11} = 89.26, \quad \rho_{11} = 954.10, \quad q_1 = 61.94.$$

The profit of the firm is 33,331.69, and the profit of the supplier is 13,218.67.

For this example, the eigenvalues of the symmetric part of the Jacobian matrix of $G(Y^*)$ (cf. (27)) are 0.0016, 0.0101, 0.0140, 0.0169, 0.0439, 0.0503, 5.5468, which are all positive. Therefore, $\nabla G(Y^*)$ is positive-definite, and $G(Y^*)$ is locally strictly monotone at $Y^*$.

**Sensitivity Analysis**

In Example 1, the capacities of the firm and the supplier do not constrain the production of the components, since, at the equilibrium, the component quantities are lower than the associated capacities. However, in some cases, due to disruptions to capacities, such as disasters and strikes, firms and suppliers may not always be able to operate under desired capacities. In this sensitivity analysis, we investigate the impacts of the capacities that constrain the production of the components on the quantities, prices, quality levels, and the profits of the firm and the supplier.

First, we maintain the capacity of the firm at 80, and vary the capacity of the supplier from 0 to 20, 40, 60, 80, 100, and 120. The results of equilibrium quantities, quality levels, prices, and profits are shown in Figures 3 and 4.

As indicated in Figure 3.b, when the capacity of the supplier is 0, the firm has to produce the components for its product by itself, at full capacity, which is 80. This production pressure limits the firm's ability to produce with high quality, which causes a low in-house component quality (cf. Figure 3.d). Based on the data in this example, purchasing components from the supplier is always cheaper than producing them in-house. Therefore, as the capacity of the supplier increases, the firm buys more components from the supplier and tends to be more dependent on the supplier in component production.

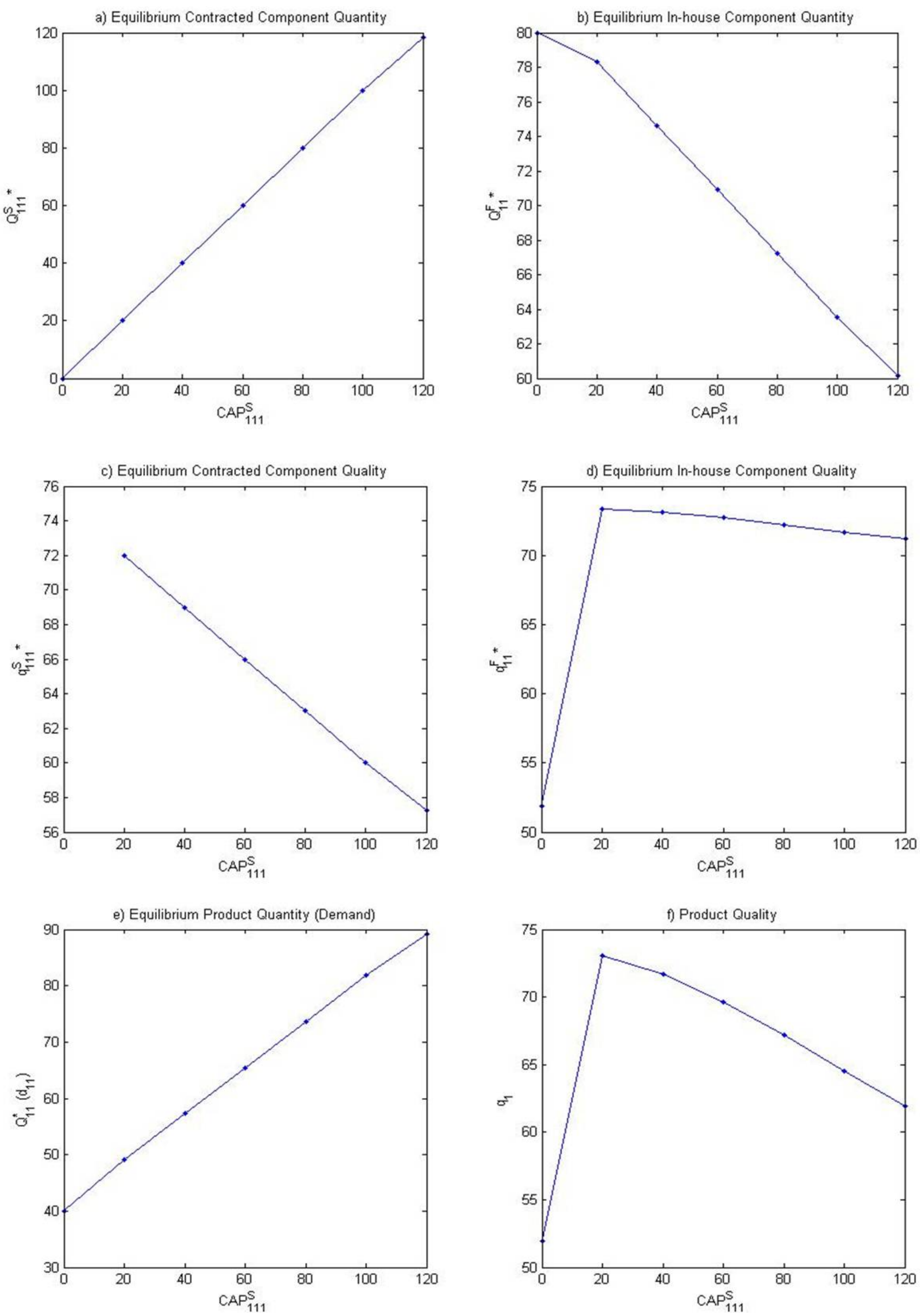


Figure 3: Equilibrium Component Quantities, Equilibrium Component Quality Levels, Equilibrium Product Quantity (Demand), and Product Quality as the Capacity of the Supplier Varies

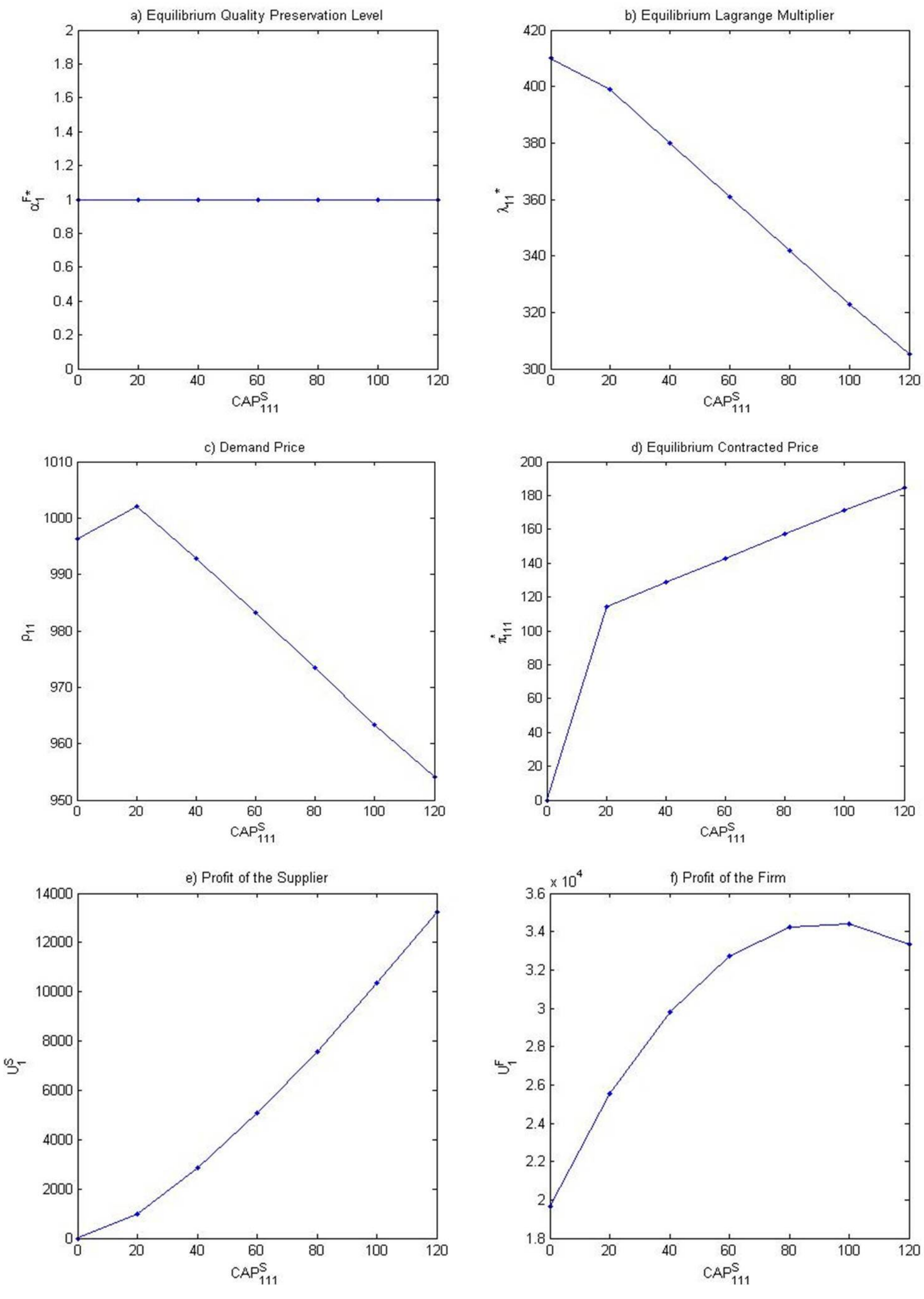


Figure 4: Equilibrium Quality Preservation Level, Equilibrium Lagrange Multiplier, Demand Price, Equilibrium Contracted Price, the Supplier's Profit, and the Firm's Profit as the Capacity of the Supplier Varies

Thus, the contracted component quantity increases (cf. Figure 3.a), and the in-house component quantity decreases (cf. Figure 3.b). In addition, with more components provided by the supplier, the firm is now able to assemble more products for profit maximization, which leads to an increase in demand (cf. Figure 3.e) and in profit (cf. Figure 4.f).

Since there is no competition on the supplier's side, as the firm becomes more dependent on the supplier, it charges more to the firm to maximize its profit (cf. Figure 4.d). For the same reason, the supplier's incentive to improve quality decreases, which leads to a reduction in contracted quality (cf. Figure 3.c). After the capacity of the supplier achieves a certain value (e.g., 100), as the capacity of the supplier increases, the contracted quantity and price keep increasing. This results in an extremely high payment to the supplier and a large transaction cost, and hence a decline in the profit of the firm (cf. Figure 4.f). The profit of the supplier always increases as its capacity expands (cf. Figure 4.e).

Moreover, when the supplier's capacity is 20, the in-house component quality achieves a higher value than before (cf. Figure 3.d), because the firm is able to pay for quality improvement for more profit at this point. However, it decreases ever after, since, given the high payment to the supplier and the high transaction cost, the firm is unable to produce a higher quality anymore. This also explains the trend of the product quality (cf. Figure 3.f) and that of the demand price (cf. Figure 4.c). The highest product quality and the highest demand price are achieved when the supplier's capacity is 20, after which they decrease.

Therefore, in the case of this example, the supplier would want to prevent disruptions to its own capacity in order to maintain a good profit. However, such disruptions may be beneficial for the firm's profit and the quality of the product at the demand market. Hence, it may be wise for the firm to contract with competing suppliers who have capacities that are not so high to harm the profit of the firm.

As already noted, when the capacity of the supplier is 0, the quantity of the in-house produced component is bounded by the capacity of the firm, which is 80. This happens because the firm can actually produce more to improve its profit with higher capacity. When the capacity of the firm is 80.78 or higher, the in-house component production does not have to operate at full capacity.

We then maintain the capacity of the supplier at 120, and vary the capacity of the firm from 0 to 20, 40, 60, and 80. The results are reported in Figures 5 and 6.

Most of the trends in Figures 5 and 6 follow a similar logic as that for Figures 3 and 4. However, as revealed in Figures 6.e and f, as the capacity of the firm increases, the profit of the supplier decreases, but that of the firm increases. Now, with higher capacity, the firm is more capable of producing more to satisfy the greater demand by itself, which weakens its dependence on the supplier and leads to a decline in the supplier's profit. Therefore, disruptions to the firm's capacity would benefit the profit of the supplier, but jeopardize the profit of the firm and the quality of the product at the demand market. Thus, the supplier would want to produce for firms who have low capacities and are, hence, more dependent on suppliers in component production.

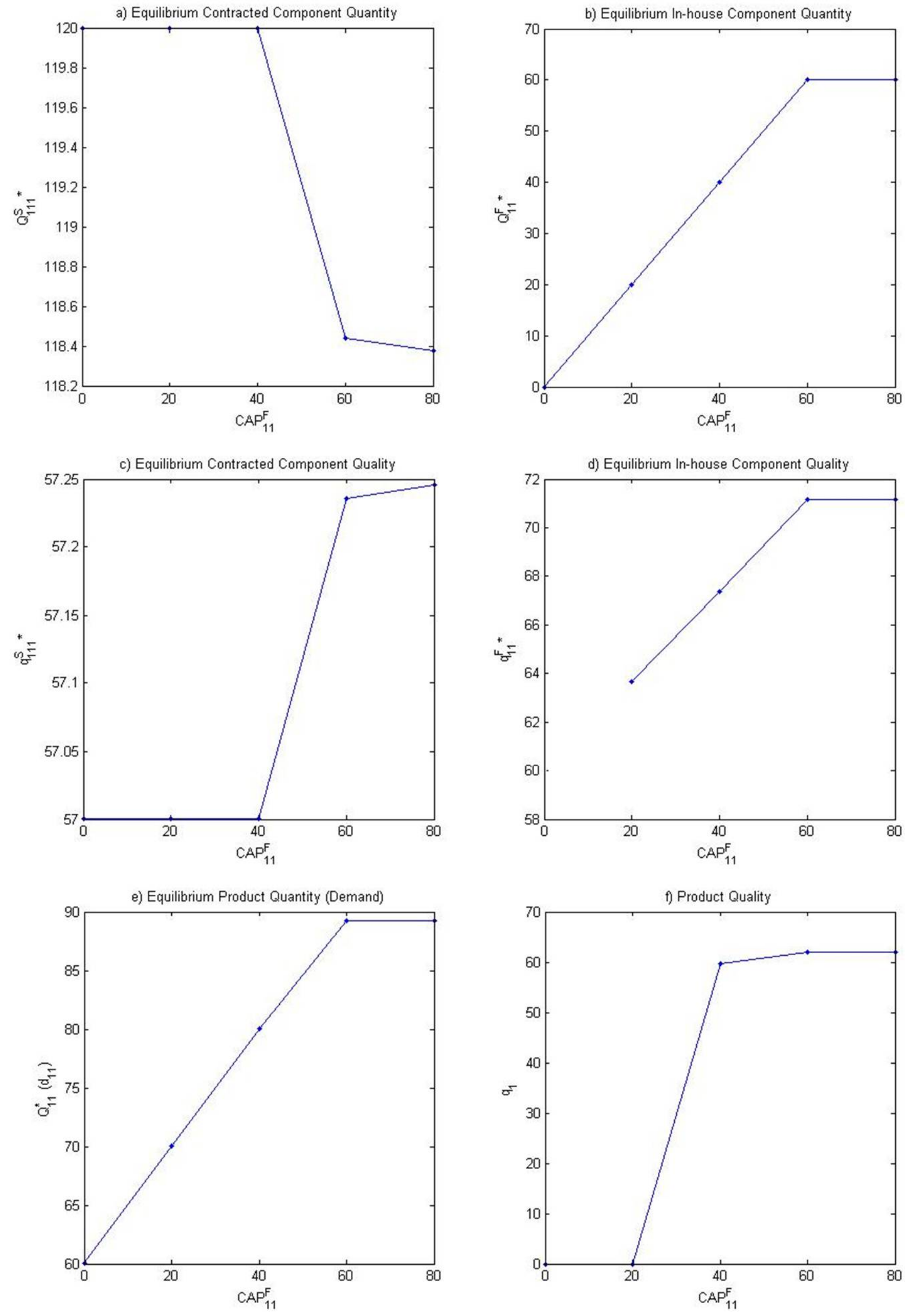


Figure 5: Equilibrium Component Quantities, Equilibrium Component Quality Levels, Equilibrium Product Quantity (Demand), and Product Quality as the Capacity of the Firm Varies

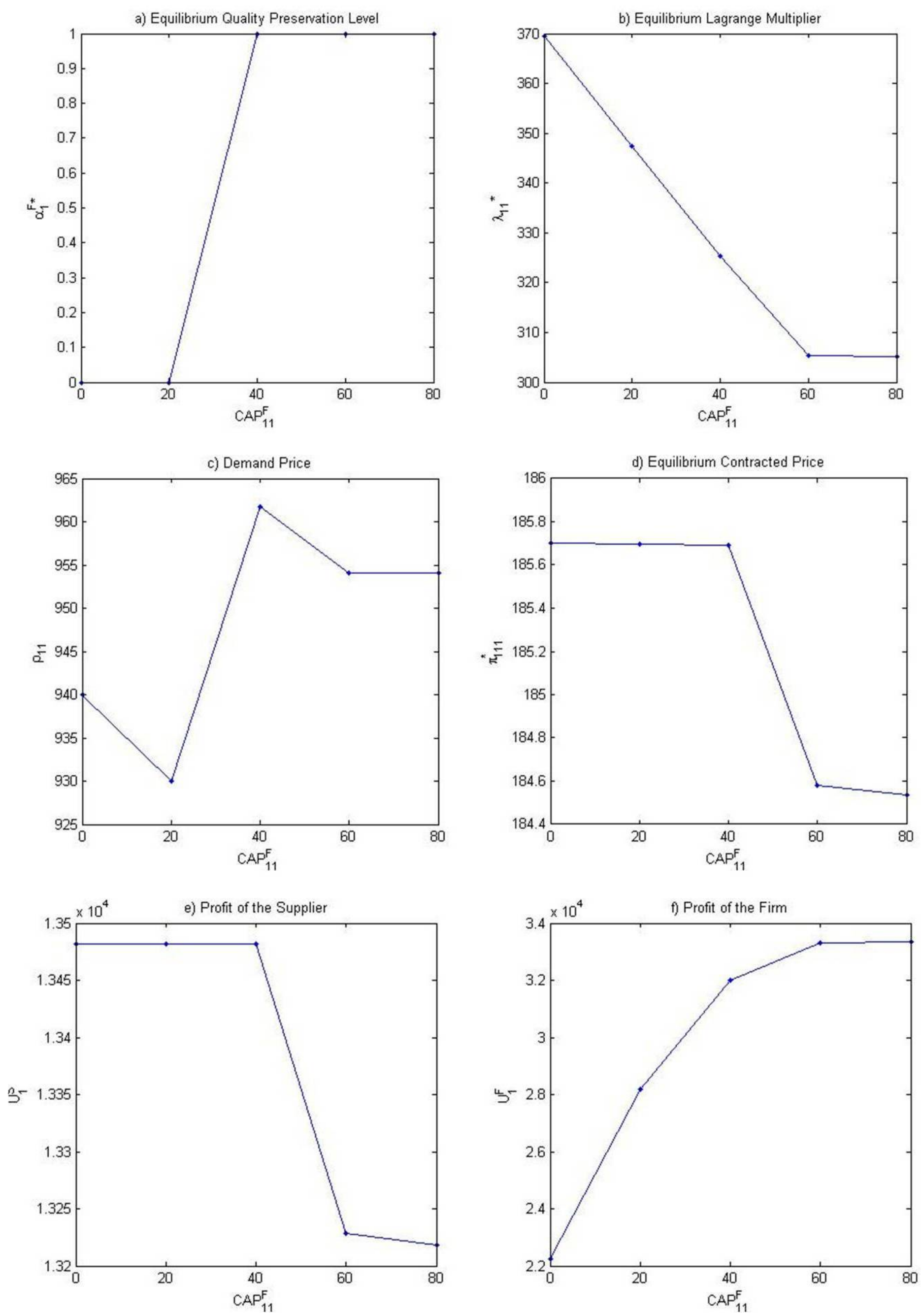


Figure 6: Equilibrium Quality Preservation Level, Equilibrium Lagrange Multiplier, Demand Price, Equilibrium Contracted Price, the Supplier's Profit, and the Firm's Profit as the Capacity of the Firm Varies

As shown in Figure 5.a, when the capacity of the firm is 0, 20, and 40, the quantity of contracted component production is bounded by the capacity of the supplier. Actually, when the capacity of the supplier is no less than 141.71, 133.99, and 126.20, respectively, the supplier does not need to operate at full capacity.

**Investing in Capacity Changing**

This sensitivity analysis further sheds light on the investments in capacity changing for the supplier and for the firm. If the investment is higher than the associated profit improvement, it is not wise for the supplier or the firm to invest in themselves' or each other's capacity changing. Tables 3 and 4 below show the maximum acceptable investments for capacity changing for this sensitivity analysis. The first number in each cell is the maximum acceptable investment for the supplier, and the second is that for the firm. In the italic cells, the two numbers are with different signs.

In Tables 3 and 4, for the cells in which both numbers are negative, it is not wise for the firm or the supplier to change the capacities at all, because their profits would decrease with the associated capacity change. For the italic cells that are with two opposite-sign numbers, the one with the negative number should prevent the other from investing in the associated capacity change, or, it should ask the other for a compensation which will prevent its profit from being compromised. This situation may occur only in 4 cases when the supplier's capacity varies (cf. Table 3). However, in Table 4, it happens very often when the firm's capacity varies, which is consistent with the results in the above sensitivity analysis. For the numbers that are 0, the associated profits will not be affected by the corresponding capacity changes.

In addition, if there is a capacity changing offer that costs more than the summation of the two numbers in the associated cell, it is not worthwhile for the supplier or the firm to accept the offer, since more profit cannot be obtained by doing so. If the offer costs less, the two parties should consider investing in the associated capacity change, and, if possible, negotiate on the separation of the payment between themselves.

Table 3: Maximum Acceptable Investments ($\times 10^3$) for Capacity Changing when the Capacity of the Firm Maintains 80 but that of the Supplier Varies

| From \ To | $CAP^S_{111}$=0 | 20 | 40 | 60 | 80 | 100 | 120 |
|---|---|---|---|---|---|---|---|
| $CAP^S_{111}$=0 | – | 0.97, 5.89 | 2.86, 10.17 | 5.08, 13.09 | 7.57, 14.62 | 10.37, 14.77 | 13.22, 13.69 |
| 20 | -0.97, -5.89 | – | 1.90, 4.28 | 4.09, 7.20 | 6.60, 8.73 | 9.40, 8.88 | 12.25, 7.80 |
| 40 | -2.86, -10.17 | -1.90, -4.28 | – | 2.20, 2.92 | 4.70, 4.45 | 7.51, 4.60 | 10.36, 3.52 |
| 60 | -5.06, -13.09 | -4.09, -7.20 | -2.20, -2.92 | – | 2.50, 1.53 | 5.31, 1.68 | 8.16, 0.60 |
| 80 | -7.57, -14.62 | -6.60, -8.73 | -4.70, -4.45 | -2.50, -1.53 | – | 2.81, 0.15 | *5.66, -0.93* |
| 100 | -10.37, -14.77 | -9.40, -8.88 | -7.51, -4.60 | -5.31, -1.68 | -2.81, -0.15 | – | *2.85, -1.08* |
| 120 | -13.22, -13.69 | -12.25, -7.80 | -10.36, -3.52 | -8.16, -0.60 | *-5.65, 0.93* | *-2.85, 1.08* | – |

**Example 2**

In Example 2, there are 2 firms competing with each other with differentiated but substitutable products in Demand Market 1. The firms can procure the components for producing their products

Table 4: Maximum Acceptable Investments ($\times 10^3$) for Capacity Changing when the Capacity of the Supplier Maintains 120 but that of the Firm Varies

| From \ To | $CAP^F_{11}$=0 | 20 | 40 | 60 | 80 |
|---|---|---|---|---|---|
| $CAP^F_{11}$=0 | – | 0.00, 5.94 | 0.00, 9.77 | *-0.25, 11.10* | *-0.26, 11.10* |
| 20 | 0.00, -5.94 | – | 0.00, 3.83 | *-0.25, 5.16* | *-0.26, 5.16* |
| 40 | 0.00, -9.77 | 0.00, -3.83 | – | *-0.25, 1.33* | *-0.26, 1.33* |
| 60 | *0.25, -11.10* | *0.25, -5.16* | *0.25, -1.33* | – | *-0.01, 0.004* |
| 80 | *0.26, -11.10* | *0.26, -5.16* | *0.26, -1.33* | *0.01, -0.004* | – |

from Suppliers 1 and 2 who also compete noncooperatively, and they can also produce the components needed by themselves.

Two components are required by the product of Firm 1, components $1^1$ and $2^1$. 1 unit of $1^1$ and 2 units of $2^1$ are required for producing 1 unit of Firm 1's product. In order to produce 1 unit Firm 2's product, 2 units of $1^2$ and 1 unit of $1^2$ are needed. Therefore,

$$\theta_{11} = 1, \quad \theta_{12} = 2, \quad \theta_{21} = 2, \quad \theta_{22} = 1.$$

The ratio of the importance of the quality of the components to the quality of one unit product is:

$$\omega_{11} = 0.2, \quad \omega_{12} = 0.8, \quad \omega_{21} = 0.4, \quad \omega_{22} = 0.6.$$

The network topology of Example 2 is as in Figure 7. Components $1^1$ and $2^1$ are the same component, which correspond to nodes 1's in the second tier of the figure. Components $2^1$ and $2^2$ are the same component, and they correspond to nodes 2's in the second tier.

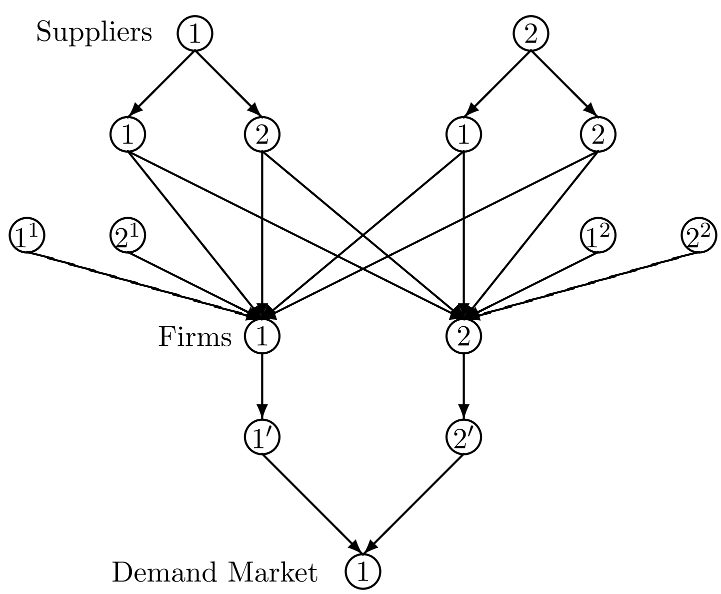


Figure 7: Supply Chain Network Topology for Example 2

The other data are as follows:

The capacities of the suppliers are:

$$CAP^S_{111} = 80, \quad CAP^S_{112} = 100, \quad CAP^S_{121} = 100, \quad CAP^S_{122} = 60,$$

$$CAP^S_{211} = 60, \quad CAP^S_{212} = 100, \quad CAP^S_{221} = 100, \quad CAP^S_{222} = 50.$$

The firms' capacities for in-house component production are:

$$CAP^F_{11} = 30, \quad CAP^F_{12} = 30, \quad CAP^F_{21} = 30, \quad CAP^F_{22} = 30.$$

The values representing the perfect component quality are:

$$\bar{q}_{11} = 60, \quad \bar{q}_{12} = 75, \quad \bar{q}_{21} = 60, \quad \bar{q}_{22} = 75.$$

The suppliers' production costs are:

$$f^S_{11}(Q^S_{111}, Q^S_{121}, q^S_{111}, q^S_{121}, q^S_{211}, q^S_{221}) = 0.4(Q^S_{111} + Q^S_{121}) + 1.5(q^S_{111} - 50)^2 + 1.5(q^S_{121} - 50)^2 + q^S_{211} + q^S_{221},$$
$$f^S_{12}(Q^S_{112}, Q^S_{122}, q^S_{112}, q^S_{122}, q^S_{212}, q^S_{222}) = 0.4(Q^S_{112} + Q^S_{122}) + 2(q^S_{112} - 45)^2 + 2(q^S_{122} - 45)^2 + q^S_{212} + q^S_{222},$$
$$f^S_{21}(Q^S_{211}, Q^S_{221}, q^S_{211}, q^S_{221}, q^S_{111}, q^S_{121}) = Q^S_{211} + Q^S_{221} + 2(q^S_{211} - 31.25)^2 + 2(q^S_{221} - 31.25)^2 + q^S_{111} + q^S_{121},$$
$$f^S_{12}(Q^S_{212}, Q^S_{222}, q^S_{212}, q^S_{222}, q^S_{112}, q^S_{122}) = Q^S_{212} + Q^S_{222} + (q^S_{212} - 85)^2 + (q^S_{222} - 85)^2 + q^S_{112} + q^S_{122}.$$

Their transportation costs are:

$$tc^S_{111}(Q^S_{111}, q^S_{111}) = 0.2Q^S_{111} + 1.2(q^S_{111} - 41.67)^2, \quad tc^S_{112}(Q^S_{112}, q^S_{112}) = 0.1Q^S_{112} + 1.2(q^S_{112} - 37.5)^2,$$
$$tc^S_{121}(Q^S_{121}, q^S_{121}) = 0.2Q^S_{121} + 1.4(q^S_{121} - 39.29)^2, \quad tc^S_{122}(Q^S_{122}, q^S_{122}) = 0.1Q^S_{122} + 1.1(q^S_{122} - 36.36)^2,$$
$$tc^S_{211}(Q^S_{211}, q^S_{211}) = 0.3Q^S_{211} + 1.3(q^S_{211} - 30.77)^2, \quad tc^S_{212}(Q^S_{212}, q^S_{212}) = 0.4Q^S_{212} + 1.7(q^S_{212} - 32.35)^2,$$
$$tc^S_{221}(Q^S_{221}, q^S_{221}) = 0.2Q^S_{221} + 1.3(q^S_{221} - 30.77)^2, \quad tc^S_{222}(Q^S_{222}, q^S_{222}) = 0.1Q^S_{222} + 1.5(q^S_{222} - 30)^2.$$

The opportunity costs of the suppliers are:

$$oc_{111}(\pi_{111}, \pi_{211}) = 5(\pi_{111} - 80)^2 + 0.5\pi_{211}, \quad oc_{112}(\pi_{112}, \pi_{212}) = 9(\pi_{112} - 80)^2 + \pi_{212},$$

$$oc_{121}(\pi_{121}, \pi_{221}) = 5(\pi_{121} - 100)^2 + \pi_{221}, \quad oc_{122}(\pi_{122}, \pi_{222}) = 7.5(\pi_{122} - 50)^2 + 0.1\pi_{222},$$
$$oc_{211}(\pi_{211}, \pi_{111}) = 5(\pi_{211} - 50)^2 + 2\pi_{111}, \quad oc_{212}(\pi_{212}, \pi_{112}) = 8(\pi_{212} - 70)^2 + 0.5\pi_{112},$$
$$oc_{221}(\pi_{221}, \pi_{121}) = 9(\pi_{221} - 60)^2 + \pi_{121}, \quad oc_{222}(\pi_{222}, \pi_{122}) = 8(\pi_{222} - 60)^2 + 0.5\pi_{122}.$$

The firms' assembly costs are:

$$f_1(Q_{11}, \alpha^F_1) = 3Q^2_{11} + 0.5Q_{11}\alpha^F_1 + 100\alpha^{F^2}_1 + 50\alpha^F_1,$$

$$f_2(Q_{21}, \alpha^F_2) = 2.75Q^2_{21} + 0.6Q_{21}\alpha^F_2 + 100\alpha^{F^2}_2 + 50\alpha^F_2.$$

Their production costs for producing components are:

$$f^F_{11}(Q^F_{11}, q^F_{11}) = Q^{F^2}_{11} + 0.0001Q^F_{11}q^F_{11} + 1.1(q^F_{11} - 36.36)^2,$$

$$f_{12}^F(Q_{12}^F, q_{12}^F) = 1.25Q_{12}^{F^2} + 0.0001Q_{12}^F q_{12}^F + 1.2(q_{12}^F - 41.67)^2,$$
$$f_{21}^F(Q_{21}^F, q_{21}^F) = Q_{21}^{F^2} + 0.0001Q_{21}^F q_{21}^F + 1.5(q_{21}^F - 33.33)^2,$$
$$f_{22}^F(Q_{22}^F, q_{22}^F) = 0.75Q_{22}^{F^2} + 0.0001Q_{22}^F q_{22}^F + 1.25(q_{22}^F - 36)^2.$$

The transaction costs are:

$$c_{111}(Q_{111}^S) = 0.5Q_{111}^{S^2} + Q_{111}^S + 100, \quad c_{112}(Q_{112}^S) = 0.5Q_{112}^{S^2} + 0.5Q_{112}^S + 150,$$
$$c_{121}(Q_{211}^S) = 0.75Q_{211}^{S^2} + 0.75Q_{211}^S + 150, \quad c_{122}(Q_{212}^S) = Q_{212}^{S^2} + Q_{212}^S + 100,$$
$$c_{211}(Q_{121}^S) = 0.75Q_{121}^{S^2} + Q_{121}^S + 150, \quad c_{212}(Q_{122}^S) = 0.5Q_{122}^{S^2} + 0.75Q_{122}^S + 100,$$
$$c_{221}(Q_{221}^S) = 0.8Q_{221}^{S^2} + 0.25Q_{221}^S + 100, \quad c_{222}(Q_{222}^S) = 0.5Q_{222}^{S^2} + Q_{222}^S + 175.$$

The firms' transportation costs are:

$$tc_{11}^F(Q_{11}, q_1) = 3Q_{11}^2 + 0.3Q_{11}q_1 + 0.25q_1, \quad tc_{21}^F(Q_{21}, q_2) = 3Q_{21}^2 + 0.3Q_{21}q_2 + 0.1q_2,$$

and the demand price functions are:

$$\rho_{11}(d_{11}, d_{21}, q_1, q_2) = -3d_{11} - 1.3d_{21} + q_1 + 0.74q_2 + 2200,$$
$$\rho_{21}(d_{21}, d_{11}, q_2, q_1) = -3.5d_{21} - 1.4d_{11} + 1.1q_2 + 0.9q_1 + 1800,$$

where $q_1 = \alpha_1^F(\omega_{11}\frac{Q_{11}^F q_{11}^F + Q_{111}^S q_{111}^S + Q_{211}^S q_{211}^S}{Q_{11}^F + Q_{111}^S + Q_{211}^S} + \omega_{12}\frac{Q_{12}^F q_{12}^F + Q_{112}^S q_{112}^S + Q_{212}^S q_{212}^S}{Q_{12}^F + Q_{112}^S + Q_{212}^S})$ and $q_2 = \alpha_2^F(\omega_{21}\frac{Q_{21}^F q_{21}^F + Q_{121}^S q_{121}^S + Q_{221}^S q_{221}^S}{Q_{21}^F + Q_{121}^S + Q_{221}^S} + \omega_{22}\frac{Q_{22}^F q_{22}^F + Q_{122}^S q_{122}^S + Q_{222}^S q_{222}^S}{Q_{22}^F + Q_{122}^S + Q_{222}^S})$.

The modified projection method converges to the following equilibrium solution:

$$Q_{11}^* = 93.56, \quad Q_{21}^* = 71.34,$$
$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$
$$Q_{111}^{S^*} = 27.37, \quad Q_{112}^{S^*} = 100.00, \quad Q_{121}^{S^*} = 45.44, \quad Q_{122}^{S^*} = 23.35,$$
$$Q_{211}^{S^*} = 36.19, \quad Q_{212}^{S^*} = 57.12, \quad Q_{221}^{S^*} = 67.24, \quad Q_{222}^{S^*} = 17.99,$$
$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.15, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.71,$$
$$q_{111}^{S^*} = 46.30, \quad q_{112}^{S^*} = 42.19, \quad q_{121}^{S^*} = 44.83, \quad q_{122}^{S^*} = 41.94,$$
$$q_{211}^{S^*} = 31.06, \quad q_{212}^{S^*} = 51.85, \quad q_{221}^{S^*} = 31.06, \quad q_{222}^{S^*} = 52.00,$$
$$\pi_{111}^* = 82.74, \quad \pi_{112}^* = 85.56, \quad \pi_{121}^* = 104.54, \quad \pi_{122}^* = 51.56,$$
$$\pi_{211}^* = 53.62, \quad \pi_{212}^* = 73.57, \quad \pi_{221}^* = 63.74, \quad \pi_{222}^* = 61.12,$$
$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$
$$\lambda_{11}^* = 109.83, \quad \lambda_{12}^* = 187.06, \quad \lambda_{21}^* = 172.34, \quad \lambda_{22}^* = 76.58,$$

and the induced demands, demand prices, and product quality levels are:

$$d_{11} = 93.56, \quad d_{21} = 71.34, \quad \rho_{11} = 1,901.07, \quad \rho_{21} = 1,504.22,$$
$$q_1 = 44.06, \quad q_2 = 41.13.$$

The firms' profits are 94,610.69 and 57,787.69, respectively, and those of the suppliers are 15,671.13 and 6923.20.

The eigenvalues of the symmetric part of the Jacobian matrix of $G(Y)$ (cf. (27)) are 0.0089, 0.0098, 0.0100, 0.0102, 0.0107, 0.0135, 0.0147, 0.0151, 0.0158, 0.0164, 0.0198, 0.0198, 0.0201, 0.0224, 0.0254, 0.0298, 0.0409, 0.0492, 0.0540, 0.0564, 0.0578, 0.0605, 0.0650, 0.0660, 0.1000, 0.1000, 0.1000, 0.1063, 0.1500, 0.1600, 0.1600, 0.1600, 0.1800, 0.1800, 2.0280, 2.1399, which are all positive. Thus, $G(Y^*)$ is locally strictly monotone at $Y^*$.

**Supplier Disruption Analysis and the Values of Suppliers**

As mentioned in the Introduction, the manufacturing plants of suppliers may be located in different geographical locations around the globe, which increases the vulnerability of the supply chain networks of the firms to the disruptions that happen to the suppliers, such as those caused by natural disasters. In this analysis, we model and analyze the impacts of the disruptions to Suppliers 1 and 2 on the profits of the firms and the demands, prices, and quality levels of the products.

We also evaluate the values of the two suppliers and which one of them is more important to the firms. With the values of the suppliers and the importance level of them to the firms, the firms can make more specific and targeted efforts in their supplier management strategies and in the contingency plans in handling the disruptions to their suppliers.

First, we present the following disruption. The data are as in Example 2, except that Supplier 1 is no longer available for the firms to contract with or to produce or transport the components needed.

The equilibrium solution achieved by the modified projection method is:

$$Q_{11}^* = 65.00, \quad Q_{21}^* = 65.00,$$

$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$

$$Q_{111}^{S^*} = 0.00, \quad Q_{112}^{S^*} = 0.00, \quad Q_{121}^{S^*} = 0.00, \quad Q_{122}^{S^*} = 0.00,$$

$$Q_{211}^{S^*} = 35.00, \quad Q_{212}^{S^*} = 100.00, \quad Q_{221}^{S^*} = 100.00, \quad Q_{222}^{S^*} = 35.00,$$

$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.16, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.75,$$

$$q_{211}^{S^*} = 31.06, \quad q_{212}^{S^*} = 51.85, \quad q_{221}^{S^*} = 31.06, \quad q_{222}^{S^*} = 52.00,$$

$$\pi_{211}^* = 53.50, \quad \pi_{212}^* = 76.25, \quad \pi_{221}^* = 65.56, \quad \pi_{222}^* = 62.19,$$

$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$

$$\lambda_{11}^* = 107.53, \quad \lambda_{12}^* = 448.93, \quad \lambda_{21}^* = 242.02, \quad \lambda_{22}^* = 95.98,$$

and the induced demands, demand prices, and product quality levels are:

$$d_{11} = 65.00, \quad d_{21} = 65.00 \quad \rho_{11} = 1,998.07, \quad \rho_{21} = 1,569.17,$$

$$q_1 = 47.12, \quad q_2 = 41.14.$$

The firms' profits are 80,574.83 and 57,406.47, respectively, and Supplier 2's profit is 13,635.49.

Without Supplier 1 and with the firms' limited in-house production capacities, there is no competition on the suppliers' side and the firms have to depend more on the supplier in component production. Therefore, as shown by the results, 3 out of the 4 contracted component quantities produced by Supplier 2 increase. Supplier 2 charges the firms more than before, and its profit improves. Without Supplier 1, the firms are not able to provide as many products as before, and hence, the demands at the demand market decrease. The quality of the products of Firms 1 and 2 increase, and the prices at the demand market increase.

Under this disruption, the profit of Firm 1 decreases by 14.84%, and that of Firm 2 decreases by 0.66%. Therefore, from this perspective, Supplier 1 is more important to Firm 1 than to Firm 2. The value of Supplier 1 to Firm 1 is 14,035.86, and that to Firm 2 is 381.22, which are measured by the associated profit declines.

We then present the disruption in which Supplier 2 is no longer available to the firms. The other data are the same as in Example 2.

The modified projection method converges to the following equilibrium solution:

$$Q_{11}^* = 65.00, \quad Q_{21}^* = 63.79,$$

$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$

$$Q_{111}^{S^*} = 35.00, \quad Q_{112}^{S^*} = 100.00, \quad Q_{121}^{S^*} = 97.58, \quad Q_{122}^{S^*} = 33.79,$$

$$Q_{211}^{S^*} = 0.00, \quad Q_{212}^{S^*} = 0.00, \quad Q_{221}^{S^*} = 0.00, \quad Q_{222}^{S^*} = 0.00,$$

$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.16, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.75,$$

$$q_{111}^{S^*} = 46.30, \quad q_{112}^{S^*} = 42.19, \quad q_{121}^{S^*} = 44.83, \quad q_{122}^{S^*} = 41.94,$$

$$\pi_{111}^* = 83.50, \quad \pi_{112}^* = 85.56, \quad \pi_{121}^* = 109.76, \quad \pi_{122}^* = 52.25,$$

$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$

$$\lambda_{11}^* = 119.17, \quad \lambda_{12}^* = 442.79, \quad \lambda_{21}^* = 256.75, \quad \lambda_{22}^* = 86.75.$$

The induced demands, demand prices, and product quality levels are:

$$d_{11} = 65.00, \quad d_{21} = 63.79, \quad \rho_{11} = 1,996.05, \quad \rho_{21} = 1,570.59,$$

$$q_1 = 42.82, \quad q_2 = 42.11.$$

The firms' profits are 83,895.42 and 53,610.96, respectively, and Supplier 1's profit is 22,729.18.

The impacts of the disruption to Supplier 2 follow similar logic as those brought about by the disruption to Supplier 1. The contracted component quantities by Supplier 1 increase, and its profit increases. The demands at the demand market decrease. Firm 1's product quality decreases, and Firm 2's increases. The prices at the demand market increase.

Without Supplier 2, Firm 1's profit declines by 11.33%, and that of Firm 2 reduces by 7.23%. Thus, Supplier 2 is more important to Firm 1 than to Firm 2 under this disruption. The value of Supplier 2 to Firm 1 is 10,715.27, and that to Firm 2 is 4,176.73.

In addition, according to the above results, Supplier 1 is more important than Supplier 2 to Firm 1, and to Firm 2, Supplier 2 is more important.

For completeness, the disruption in which neither of the supplier is no longer available to the firms is also considered. The other data are the same as in Example 2.

The equilibrium solution obtained using the modified projection method is:

$$Q_{11}^* = 15.00, \quad Q_{21}^* = 15.00,$$

$$Q_{11}^{F^*} = 15.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$

$$Q_{111}^{S^*} = 0.00, \quad Q_{112}^{S^*} = 0.00, \quad Q_{121}^{S^*} = 0.00, \quad Q_{122}^{S^*} = 0.00,$$

$$Q_{211}^{S^*} = 0.00, \quad Q_{212}^{S^*} = 0.00, \quad Q_{221}^{S^*} = 0.00, \quad Q_{222}^{S^*} = 0.00,$$

$$q_{11}^{F^*} = 37.29, \quad q_{12}^{F^*} = 45.08, \quad q_{21}^{F^*} = 35.71., \quad q_{22}^{F^*} = 37.90,$$

$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$

$$\lambda_{11}^* = 30.46, \quad \lambda_{12}^* = 967.28, \quad \lambda_{21}^* = 772.88, \quad \lambda_{22}^* = 22.63.$$

The induced demands, demand prices, and the product quality levels are:

$$d_{11} = 15.00, \quad d_{21} = 15.00, \quad \rho_{11} = 2,206.42, \quad \rho_{21} = 1,806.40,$$

$$q_1 = 43.52, \quad q_2 = 37.02.$$

The firms' profits are 30,016.91 and 24,391.32, respectively.

Compared to Example 2, without the suppliers, the demands at the demand market decrease, the firms' product quality levels decrease, and the prices at the demand market increase. Firm 1's profit deceases by 68.27%, Firm 2's reduces by 57.79%. The value of the suppliers to Firm 1 is 64,593.78, and that to Firm 2 is 33,396.37.

## 6. Summary and Conclusions

In this paper, we develop a general multitiered supply chain network equilibrium model with a focus on quality in which suppliers compete to produce components that are utilized by competing firms as they assemble final products that are differentiated by brands. The firms can also produce components in-house, depending on their capacities. We model the competitive behavior of the two tiers of decision-makers as they identify their optimal strategies in terms of quantity and quality with the assembling firms also identifying their assembly quality preservation levels. The suppliers charge the firms prices for the components that they supply.

The novelty of our framework lies in its generality and its computability. Rather than focus, as some of the literature does, on one supplier-one manufacturer studies, here we do not limit the number of components needed for the finished product, the number of suppliers, the number of firms, nor the number of demand markets. Moreover, we provide a framework for tracking the quality of the product from the component level, through the assembly process into the final product, and ultimate distribution to the demand markets.

We derive the unified variational inequality formulation of the governing equilibrium conditions, provide qualitative properties of the equilibrium solution pattern, in terms of existence and uniqueness results, and propose an algorithm along with conditions for convergence. Our framework is illustrated with numerical examples, accompanied by sensitivity analysis that explores such critical issues as the impacts of capacity disruptions and the potential investments in capacity enhancements. We also conduct sensitivity analysis to reveal the impacts of specific supplier unavailability along with their values as reflected in the profits of the firms and in the quality of the finished products. With knowledge of the values of the suppliers to the firms, the firms can make more specific, targeted efforts in their supplier management strategies and in their contingency plans in the case of supplier disruptions.

Future research may include the solution of larger numerical examples as well as the computational exploration of enhancements to the algorithmic scheme (see, for example, Fu and He (2010) and He, Yuan, and Zhang (2004)). .

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**Appendix**

**Proof of Theorem 1:**

For a given firm $i$, under the imposed assumptions, (16) holds if and only if (see Bertsekas and Tsitsiklis (1989) page 287) the following holds:

$$\sum_{k=1}^{n_R}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{ik}}+\sum_{l=1}^{n_{li}}\lambda_{il}^*\theta_{il}\right]\times(Q_{ik}-Q_{ik}^*)$$
$$+\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{il}^F}-\lambda_{il}^*\right]\times(Q_{il}^F-Q_{il}^{F^*})$$
$$+\sum_{j=1}^{n_S}\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial Q_{jil}^S}-\lambda_{il}^*\right]\times(Q_{jil}^S-Q_{jil}^{S^*})$$
$$+\sum_{l=1}^{n_{li}}\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial q_{il}^F}\right]\times(q_{il}^F-q_{il}^{F^*})$$
$$+\left[-\frac{\partial U_i^F(Q^*,Q^{F^*},Q^{S^*},q^{F^*},\alpha^{F^*},\pi_i^*,q^{S^*})}{\partial \alpha_i^F}\right]\times(\alpha_i^F-\alpha_i^{F^*})$$
$$+\sum_{l=1}^{n_{li}}\left[\sum_{j=1}^{n_S}Q_{jil}^{S^*}+Q_{il}^{F^*}-\sum_{k=1}^{n_R}Q_{ik}^*\theta_{il}\right]\times(\lambda_{il}-\lambda_{il}^*)\geq 0,\quad \forall(Q_i,Q_i^F,Q_i^S,q_i^F,\alpha_i^F,\lambda_i)\in K_i^F. \qquad (A1)$$

Variational inequality (A1) holds for each firm $i$; $i=1,\ldots,I$, and, hence, the summation of (A1) yields variational inequality (17).□

**Proof of Theorem 4:**

We first prove that there exists at least one solution to variational inequality (26) (cf. (22)). Note that, all the component quality levels, $q_{jil}^S$ and $q_{il}^F$, and quality preservation levels $\alpha_i$ are bounded due to constraints (3), (4), and (7). Because of constraint (12), all the product quantities $Q_{ik}$ are also bounded, since the components quantities are nonnegative and capacitated (cf. (13) and (14)). Therefore, with Assumption 1, the feasible set of variational inequality (26) is bounded. Since the cost functions and the demand price functions are continuously differentiable, and the feasible set is convex and compact, the existence of a solution to (26) (cf. (22)) is guaranteed (cf. Kinderlehrer and Stampacchia (1980) and Theorem 1.5 in Nagurney (1999)). Because (26) and (24) (cf. (23)) are equivalent (Nagurney and Dhanda (2000)), the existence of (24) is also guaranteed. □

**Proof of Theorem 5:**

Let $Y'=(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})$, $Y''=(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})$, $X'=(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\lambda',\pi',q^{S'})$, and $X''=(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\lambda'',\pi'',q^{S''})$. Then the left-hand-side of (29) can be expended to:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial Q_{ik}})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial Q_{ik}})\right]\times(Q'_{ik}-Q''_{ik})$$
$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial Q_{il}^F})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial Q_{il}^F})\right]\times(Q_{il}^{F'}-Q_{il}^{F''})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial Q_{jil}^S})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial Q_{jil}^S})\right]\times(Q_{jil}^{S'}-Q_{jil}^{S''})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial q_{il}^F})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial q_{il}^F})\right]\times(q_{il}^{F'}-q_{il}^{F''})$$

$$+\sum_{i=1}^{I}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial \alpha_i^F})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial \alpha_i^F})\right]\times(\alpha_i^{F'}-\alpha_i^{F''})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_j^S(Q^{S'},\pi',q^{S'})}{\partial \pi_{jil}})-(-\frac{\partial U_j^S(Q^{S''},\pi'',q^{S''})}{\partial \pi_{jil}})\right]\times(\pi'_{jil}-\pi''_{jil})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_j^S(Q^{S'},\pi',q^{S'})}{\partial q_{jil}^S})-(-\frac{\partial U_j^S(Q^{S''},\pi'',q'')}{\partial q_{jil}^S})\right]\times(q_{jil}^{S'}-q_{jil}^{S''})$$

$$+\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\sum_{l=1}^{n_{li}}\lambda'_{il}\theta_{il}-\sum_{l=1}^{n_{li}}\lambda'_{il}\theta_{il}\right]\times(Q'_{ik}-Q''_{ik})+\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\sum_{l=1}^{n_{li}}\lambda''_{il}\theta_{il}-\sum_{l=1}^{n_{li}}\lambda''_{il}\theta_{il}\right]\times(Q'_{ik}-Q''_{ik})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}[\lambda'_{il}-\lambda'_{il}]\times(Q_{il}^{F'}-Q_{il}^{F''})+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}[\lambda''_{il}-\lambda''_{il}]\times(Q_{il}^{F'}-Q_{il}^{F''})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}[\lambda'_{il}-\lambda'_{il}]\times(Q_{jil}^{S'}-Q_{jil}^{S''})+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}[\lambda''_{il}-\lambda''_{il}]\times(Q_{jil}^{S'}-Q_{jil}^{S''}). \tag{A2}$$

where $\lambda'_{il},\lambda''_{il}\geq 0, i=1,\ldots,I, l=1,\ldots,n_{li}$.

After combining terms, (A2) reduces to

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial Q_{ik}}+\sum_{l=1}^{n_{li}}\lambda'_{il}\theta_{il})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial Q_{ik}}+\sum_{l=1}^{n_{li}}\lambda''_{il}\theta_{il})\right]$$
$$\times(Q'_{ik}-Q''_{ik})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial Q_{il}^F}-\lambda'_{il})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial Q_{il}^F}-\lambda''_{il})\right]\times(Q_{il}^{F'}-Q_{il}^{F''})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial Q_{jil}^S}-\lambda'_{il})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial Q_{jil}^S}-\lambda''_{il})\right]\times(Q_{jil}^{S'}-Q_{jil}^{S''})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial q_{il}^F})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial q_{il}^F})\right]\times(q_{il}^{F'}-q_{il}^{F''})$$

$$+\sum_{i=1}^{I}\left[(-\frac{\partial U_i^F(Q',Q^{S'},Q^{F'},q^{F'},\alpha^{F'},\pi',q^{S'})}{\partial \alpha_i^F})-(-\frac{\partial U_i^F(Q'',Q^{S''},Q^{F''},q^{F''},\alpha^{F''},\pi'',q^{S''})}{\partial \alpha_i^F})\right]\times(\alpha_i^{F'}-\alpha_i^{F''})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_j^S(Q^{S'},\pi',q^{S'})}{\partial \pi_{jil}})-(-\frac{\partial U_j^S(Q^{S''},\pi'',q^{S''})}{\partial \pi_{jil}})\right]\times(\pi'_{jil}-\pi''_{jil})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(-\frac{\partial U_j^S(Q^{S'},\pi',q^{S'})}{\partial q_{jil}^S})-(-\frac{\partial U_j^S(Q^{S''},\pi'',q'')}{\partial q_{jil}^S})\right]\times(q_{jil}^{S'}-q_{jil}^{S''})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{li}}\left[(\sum_{j=1}^{n_S}Q_{jil}^{S'}+Q_{il}^{F'}-\sum_{k=1}^{n_R}Q'_{ik}\theta_{il})-(\sum_{j=1}^{n_S}Q_{jil}^{S''}+Q_{il}^{F''}-\sum_{k=1}^{n_R}Q''_{ik}\theta_{il})\right]\times(\lambda'_{il}-\lambda''_{il}). \tag{A3}$$

The expression in (A2), equivalently, (A3), is greater than or equal to zero, since we have assumed in Theorems 1 and 2 that the profit functions are concave with respect to associated variables. (A2) is derived from the left-hand-side of (29), so (29) holds true, and, hence, the $G(Y)$ that enters variational inequality (26) is monotone. Because (A3) is also the left-hand-side of (28), the $F(X)$ that enters variational inequality (24) is also monotone. □