Dynamics of Quality as a Strategic Variable in Complex Food Supply Chain Network Competition: The Case of Fresh Produce

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Abstract:

In this paper, we construct a competitive food supply chain network model in which the profit-maximizing producers decide not only as to the volume of fresh produce produced and distributed using various supply chain network pathways, but they also decide, with the associated costs, on the initial quality of the fresh produce. Consumers, in turn, respond to the various producers' product outputs through the prices that they are willing to pay, given also the average quality associated with each producer or brand at the retail outlets. The quality of the fresh produce is captured through explicit formulae that incorporate time, temperature, and other link characteristics with links associated with processing, shipment, storage, etc. Capacities on links are also incorporated as well as upper bounds on the initial product quality of the firms at their production/harvesting sites. The governing concept of the competitive supply chain network model is that of Nash Equilibrium, for which alternative variational inequality formulations are derived, along with existence results. An algorithmic procedure, which can be interpreted as a discrete-time tatonnement process, is then described and applied to compute the equilibrium produce flow patterns and accompanying link Lagrange multipliers in a realistic case study, focusing on peaches, which includes disruptions.

Food, and especially fresh produce, in the form of fruits and vegetables, is essential to the well-being and health of consumers. In this paper we construct a supply chain network game theory model which captures competition among food firms, along with the quality associated with their fresh produce products

as they move along the pathways from production/harvesting, through storage and distribution, to the retail outlets. The food firms are profit-maximizers whereas the consumers reflect their preferences for the fresh produce by taking quality into consideration. Qualitative properties of the model are established and illustrative examples provide along with a case study on peaches, under both status quo and disruption scenarios. This is the first such general supply chain network model constructed, analyzed, and solved and has great relevance to this important industry.

Key words: food supply chains, quality deterioration, fresh produce, oligopolistic competition, game theory, networks

1. Introduction

Fresh produce, in the form of fruits and vegetables, is essential to human health and well-being. Given the consumer demand for fresh produce, year round, global supply chains have evolved in order to satisfy customers. With fresh produce criss-crossing the globe from producers to consumers, attention to quality is essential with time and distance playing critical roles, as well as environmental factors associated with the various supply chain network activities, including processing, transportation, storage, and distribution.

Food supply chains, as noted in Yu and Nagurney (2013), are distinct from other product supply chains. The basic difference between food supply chains and other supply chains, and this is especially characteristic of fresh produce, is the continuous and significant change in the quality of food products throughout the entire supply chain from the points of production/harvesting to points of demand/consumption (see Sloof, Tijskens, and Wilkinson (1996), Van der Vorst (2000), Lowe and Preckel (2004), Ahumada and Villalobos (2009), Blackburn and Scudder (2009), Akkerman, Farahani, Grunow (2010), Aiello, La Scalia, and Micale (2012), and Besik and Nagurney (2017)). Indeed, the quality of food products is decreasing with time, even with the use of advanced facilities and under the best processing, handling, storage, and shipment conditions (Sloof, Tijskens, and Wilkinson (1996) and Zhang, Habenicht, and Spieß (2003)). Propitiously, however, it has been discovered that the quality of fresh produce can be determined scientifically using chemical formulae, which include both time and temperature (cf. Labuza (1982), Taoukis and Labuza (1989), Tijskens and Polderdijk (1996), Rong, Akkerman, and Grunow (2011)). Hence, the fresh produce supply chain network topology can have a big impact on the quality of the fresh produce with "shorter" paths resulting in higher quality retention, provided also that environmental and handling factors are opportune. Moreover, the initial quality is also very important and food producers, such as farmers, have significant control over this important strategic variable at their production/harvesting sites. Clearly, there are great opportunities for enhanced decision-making in this realm that can be supported by appropriate models and methodological tools.

As for the literature on food supply chains, which is growing, given the great interest in this topic, we note that early contributions focused on perishability and, in particular, on inventory management (see Ghare and Schrader (1963), Nahmias (1982, 2011) and Silver, Pyke, and Peterson (1998) for reviews). More recently, some studies have proposed integrating more than a single supply chain network activity (see, e.g., Zhang, Habenicht, and Spieß (2003), Widodo et al. (2006), Ahumada and Villalobos (2011), and Kopanos, Puigjaner, and Georgiadis (2012)) and also have emphasized the need to bring greater realism to the underlying economics and competition (cf. Yu and Nagurney (2013)). Van der Vorst (2006) noted that it is essential to analyze food supply chains within the context of the full complexity of their network structure. Monteiro (2007), further, postulated that network economics (cf. Nagurney (1999)) provides a powerful framework in which the structure of the supply chain can be graphically captured and analyzed, and studied the traceability in food supply chains theoretically. Additional modeling and methodological contributions in the food supply chain and quality domain have been made by Blackburn and Scudder (2009) and by Rong, Akkerman, and Grunow (2011) with the paper by Besik and Nagurney (2017) formulating short fresh produce supply chains with the inclusion of the dynamics of quality, in the context of farmers' markets, while also capturing competition. For approaches to the quantification of quality in supply chain networks of manufactured products, including durable goods, we refer the interested reader to the book by Nagurney and Li (2016) and the references therein. For a recent book on perishable product supply chains with a variety of applications, see Nagurney et al. (2013).

In this paper, we construct a competitive supply chain network model for fresh produce under oligopolistic competition among the food firms, who are profit-maximizers. The firms have, as their strategic variables, not only the product flows on the pathways of their supply chain networks from the production/harvesting locations to the ultimate points of demand, but also the initial quality of the produce that they grow at their production locations. Associated with the various links representing the supply chain activities are total costs. The consumers at the retail outlets (demand points), differentiate the fresh produce from the distinct firms and reflect their preferences through the prices that they are willing to pay which depend on quantities of the produce as well as the average quality of the produce associated with the firm and retail outlet pair(s). Quality of the produce reaching a destination node depends on its initial quality and on the path that it took with each particular path consisting of specific links, with particular characteristics of physical features of time, temperature, etc., which are used to construct link multipliers, ultimately, yielding a path multiplier. The new model in this paper is related to the model of Yu and Nagurney (2013) but with significant differences:

1. The model herein explicitly captures quality of fresh produce using appropriate physical formulae, which depend on the particular type of fresh produce, whereas the model of Yu and Nagurney (2013) used arc multipliers to address perishability with wastage costs associated with discarding.

2. In this paper, quality is a strategic variable, whereas in Yu and Nagurney (2013) only path flows were strategic variables.

3. In addition, in the model in this paper, we include capacities in the links as well as on the initial quality of the fresh produce at the production/harvesting sites.

Besik and Nagurney (2017) modeled competition among farmers in farmers' markets, along with fresh produce quality deterioration along paths. However, in their paper the initial quality was assumed to be known and fixed a priori, whereas, in this paper, each food firm's initial product quality at each production site is a strategic variable. Furthermore, in the farmer's market supply chain network model there was only a single path from an origin node to a destination node. Here, in contrast, there are multiple paths. Moreover, the demand price functions in the new model depend on the average quality of a given firm/brand since there can be multiple paths.

The paper is organized as follows. In Section 2, we provide a few preliminaries, focusing on a review of fresh food quality deterioration and associated formulae. We then present the additional notation associated with quality and its deterioration for the general supply chain network model in this paper. In Section 3, we construct the model and derive alternative variational inequality formulations of the governing Nash (1950, 1951) equilibrium conditions. We also provide an existence result. In addition, we present several simple numerical examples for illustrative purposes.

In Section 4, an algorithmic scheme is outlined and its interpretation as a discrete-time tatonnement process given. It is then applied in Section 5 to compute solutions to numerical examples comprising a case study focusing on peaches from which managerial insights are drawn. A summary of our contributions as well as suggestions for future research are provided in the concluding Section 6.

2. A Few Preliminaries and Construction of Path Quality

In this Section, we first recall a few fundamental formulae associated with food quality and deterioration over time. We then generalize the quality path concept and formulae introduced in Besik and Nagurney (2017) to accommodate the possibility of multiple production/harvesting sites for each food firm, along with the inclusion of the initial quality as a strategic variable. We consider quality reaction orders of zero order and of the first order.

Fresh foods deteriorate since they are biological products, and, therefore, lose quality over time (Schouten et al. (2004), Singh et al. (2004)). As noted by Taoukis and Labuza (1989), the rate of quality deterioration can be represented as a function of the microenvironment, the gas composition, the relative humidity, and the temperature. Labuza (1984) demonstrated that the quality of a food attribute, Q, over time t, which can correspond, depending on the fruit or vegetable, to the color change, the moisture content, the amount of nutrition such as vitamin C, or the softening of the texture, can be formulated via the differential equation:

$$\frac{\partial Q}{\partial t} = -kQ^n = -Ae^{(-E/RT)}Q^n, \tag{1}$$

where in equation (1), k is the reaction rate and is defined by the Arrhenius formula, $Ae^{(-E/RT)}$, A is a pre-exponential constant, T is the temperature, E is the activation energy, and R is the universal gas constant (cf. Arrhenius (1889)). The parameter n in (1) is the reaction order. If the reaction order n is zero, that is, $\frac{\partial Q}{\partial t} = -k$, and the initial quality is denoted by Q_0, we can quantify the remaining quality Q_t at time t (Tijskens and Polderdijk (1996)) according to:

$$Q_t = Q_0 - kt. \tag{2}$$

Examples of fresh produce that follow a reaction order of zero include watermelons (see Dermesonlouoglou, Giannakourou, and Taoukis (2007)) and spinach (cf. Aamir et al. (2013)).

On the other hand, if the reaction order is 1, known as a *first order reaction*, the quality decay function is then given by the expression:

$$Q_t = Q_0e^{-kt}. \tag{3}$$

Popular fruits that follow first order kinetics include peaches (see Toralles et al. (2005)), apples (Tijskens (1979)), and strawberries (Castro et al. (2004)), as well as vegetables such as: peas, beans, and carrots (see Aamir et al. (2013)), avocados (Maftoonazad and Ramaswamy (2008)), and tomatoes (Krokida et al. (2003)).

We now construct a generalization of each of the above quality deterioration functions in terms of a path concept, focusing on the supply chain network topology depicted in Figure 1. We let L^i denote the set of directed links in the supply chain network of food firm i; where $i = 1, \ldots, I$, which consists of a set of production links, L_1^i, and a set of post-harvest links, L_2^i, that is, $L^i \equiv L_1^i \cup L_2^i$. We let L denote the full set of links in the supply chain fresh produce network economy such that $L \equiv \cup_{i=1}^I L^i$. Besik and Nagurney (2017) introduced a slightly simpler path concept, in the case where the initial quality was fixed and was associated with each firm. Here, in contrast, we allow for a distinct initial quality q_{0a}^i associated with each top-tiered production link $a \in L_1^i$; $i = 1, \ldots, I$, since distinct production sites of a firm may have different associated quality of the produce that is harvested because of soil conditions, investment in irrigation, types of pesticides, and fertilizers used, etc.

We define a path p as a sequence of directed links joining an origin node firm i with a destination node k, corresponding to a retail outlet (demand market), where $k = R_1, \ldots, R_{n_R}$.

Furthermore, let β_b denote the quality decay incurred on link b, for $b \in L_2^i$, which is a factor that depends on the reaction order n, the reaction rate k_b, and the time t_b on link b, according to:

$$\beta_b \equiv \begin{cases} -k_b t_b, , & \text{if } n = 0, \, \forall b \in L_2^i, \, \forall i, \\ \\ e^{-k_b t_b}, & \text{if } n = 1, \, \forall b \in L_2^i, \, \forall i. \end{cases} \tag{4}$$

Here, k_b is the reaction constant related to the link b. Note that, according to (4) and the presentation above, we assume that there is no quality decay associated with the production/harvesting links. At the same time, there is an initial quality associated with these top-most links (cf. Figure 1) of the fresh produce, which will be strategic variables of the firms, along with the fresh produce path flows. Since each link on a path can have different associated temperature conditions, the differentiation over the temperature of the links is essential. Thus, the reaction rate is described in the following equation for each link b by the Arrhenius formula with the same parameters as in (1), except that the temperature is now denoted for each link b as T_b, where:

$$k_b = Ae^{(-E/RT_b)}, \quad \forall b \in L_2^i, \, \forall i. \tag{5}$$

Since we can have multiple paths from an origin node i to a destination node k, P_k^i denotes the set of all paths that have origin i and destination k. The quality q_p, over a path p, joining the origin node i, with a destination node k, with the incorporation of the quality deterioration of the fresh produce, is, hence:

$$q_p \equiv \begin{cases} q_{0a}^i + \sum\limits_{b \in p \cap L_2^i} \beta_b, & \text{if } \ n = 0, \, p \in P_k^i, \, \forall i, k, \\ \\ q_{0a}^i \prod\limits_{b \in p \cap L_2^i} \beta_b, & \text{if } \ n = 1, \, p \in P_k^i, \, \forall i, k, \end{cases} \tag{6}$$

where q_{0a}^i in (6) is the initial quality of the fresh produce on a top-most link a from an origin node i and in the path p under consideration.

3. The Competitive Fresh Produce Supply Chain Network Model with Quality and Associated Dynamics

In this Section, we present the competitive fresh produce supply chain network model, under oligopolistic competition, and with both path flows and initial quality as strategic variables. The fresh produce products are substitutable and are differentiated by food firm. Each multitiered supply chain network of firm i consists of n_M^i production/harvesting facilities:

$M^i_1, \ldots, M^i_{n^i_M}$; n^i_C processors: $C^i_{1,1}, \ldots, C^i_{n^i_C,1}$, and n^i_D distribution centers, $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$, and can serve the n_R retail outlets, denoted, respectively, by: $R_1, \ldots, R_{n_R}$. Different links connecting a pair of nodes correspond to distinct options. We assume that the time durations and temperatures of the links are fixed and known. Let $G = [N, L]$ denote the graph consisting of the set of nodes N and the set of links L in Figure 1. The supply chain network topology in Figure 1 can be adapted/modified according to the particular fresh produce product under investigation. The supply chain network topology is inspired by the one constructed in Yu and Nagurney (2013) but, here, the model is distinctly different with a focus on quality deterioration using explicit physical formulae associated with the various network economic links and associated activities of shipment, processing, storage, and distribution, as illustrated in Figure 1. Moreover, we include explicit capacities on the supply chain network links as well as an upper bound on the initial quality associated with the various production/harvesting sites.

We first present the variables and then the various functions associated with the supply chain network. The flow of the fresh produce product on path p joining an origin node i with a destination node k is denoted by x_p. For each path the following nonnegativity condition must hold:

$$x_p \geq 0, \quad \forall p \in P^i_k;\ i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{7}$$

Furthermore, q^i_{0a}, the initial quality of the fresh produce on the top-most links a of an origin node i, must be nonnegative, that is,

$$q^i_{0a} \geq 0, \quad \forall a \in L^i_1;\ i = 1, \ldots, I. \tag{8}$$

In addition, it is reasonable to assume that the quality is bounded from above by a maximum value; hence, we have that:

$$q^i_{0a} \leq \bar{q}^i_{0a}, \quad \forall a \in L^i_1;\ i = 1, \ldots, I, \tag{9}$$

where $\bar{q}^i_{0a}$ is the positive upper bound on the quality of the produce produced/harvested on link a of food firm i.

The conservation of flow equations that relate the link flows of each food firm i; $i = 1, \ldots, I$, to the path flows are given by:

$$f_l = \sum_{p \in P} x_p \delta_{lp}, \quad \forall l \in L^i;\ i = 1, \ldots, I, \tag{10}$$

where f_l denotes the flow on link l, $\delta_{lp} = 1$, if link l is contained in path p, and 0, otherwise, and P denotes the set of all paths. Therefore, since the supply chain networks of the firms

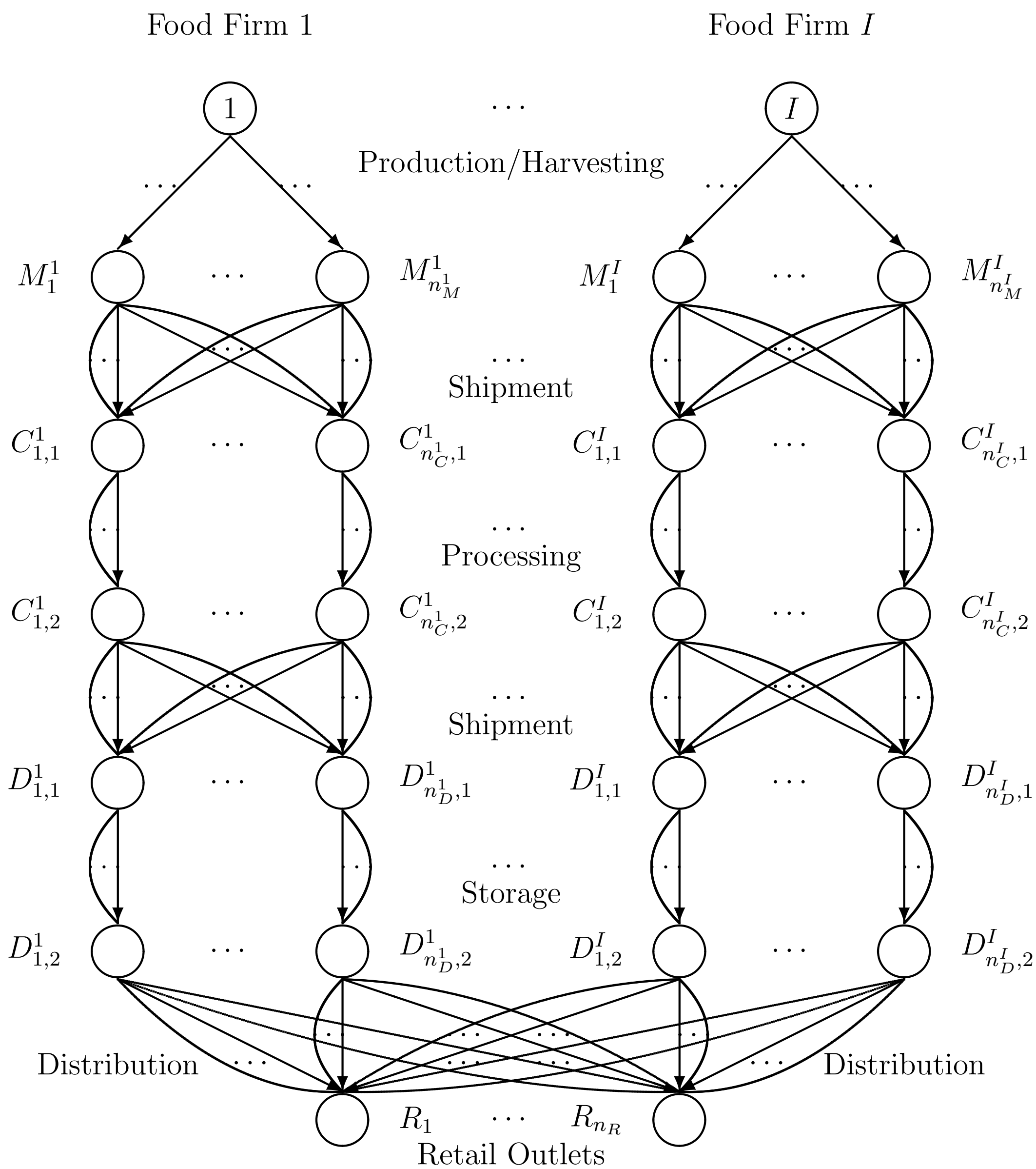

Figure 1: The Fresh Produce Supply Chain Network Topology with Quality Deterioration

do not share any common links, the flow of a firm's fresh produce product on a link is equal to the sum of that product's flows on paths that contain that link. We group the link flows into the vector $f \in R_+^{n_L}$, where n_L denotes the number of links in L. All vectors in this paper are column vectors.

In addition, since the link flows must satisfy capacity constraints, we have that:

$$f_l \leq u_l, \quad \forall l \in L, \tag{11}$$

where u_l denotes the positive upper bound on link l.

Also, observe that, in view of the conservation of flow equations (10), we can rewrite (11) in terms of path flows as:

$$\sum_{p \in P} x_p \delta_{lp} \leq u_l, \quad \forall l \in L. \tag{12}$$

In general, consumers, at the retail outlets, respond not only to the quantities available of the product but also to their average quality, where the average quality of the product at retail outlet k, associated with the fresh produce product of firm i, and denoted by $\hat{q}_{ik}$, is given by the expression:

$$\hat{q}_{ik} = \frac{\sum_{p \in P_k^i} q_p x_p}{\sum_{p \in P_k^i} x_p}, \quad i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}, \tag{13}$$

where recall that q_p is fresh produce product specific with its value computed according to (6). We group the average product quality of all firms into the vector $\hat{q} \in R_+^{I \times n_R}$. We exclude all food firm / retail outlet pairs that do not conduct business with one another so that the denominator in (13) is never equal to zero.

The demand for food firm i's fresh food product at retail outlet k is denoted by d_{ik} and is equal to the sum of all the fresh produce flows on paths joining (i, k), so that:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{14}$$

We group the demands for the fresh food products of all firms at all retail outlets into the vector $d \in R_+^{I \times n_R}$.

We now present the underlying functions in the competitive supply chain network model with quality.

We denote the demand price of food firm i's product at retail outlet k by ρ_{ik} and assume that

$$\rho_{ik} = \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{15}$$

Note that the price of food firm i's product at a particular retail outlet may depend not only on the demands for and the average quality of its product, but also on the demands for and the average quality of the other substitutable food products at all the retail outlets. These demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

The cost of production/harvesting at firm i's production site a depends, in general, on the initial quality q_{0a}^i, and the product flow on the production/harvesting link, that is,

$$\hat{z}_a = \hat{z}_a(f_a, q_{0a}^i), \quad \forall a \in L_1^i;\ i = 1, \ldots, I. \tag{16}$$

Furthermore, we define the operational cost functions associated with the remaining links in the supply chain network as:

$$\hat{c}_b = \hat{c}_b(f), \quad \forall b \in L_2^i;\ i = 1, \dots, I. \tag{17}$$

The total operational cost on each such link is assumed to be convex and continuously differentiable and the same holds for each production/harvesting cost function.

Let X_i denote the vector of path flows associated with firm i; $i = 1, \dots, I$, where $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$, $P^i \equiv \cup_{k=R_1,\dots,R_{n_R}} P_k^i$, and n_{P^i} denotes the number of paths from firm origin node i to the retail outlets. Then, X is the vector of all the food firms' path flow strategies, that is, $X \equiv \{\{X_i\}|i = 1, \dots, I\}$. Similarly, the vector of initial quality levels, associated with firm i; $i = 1, \dots, I$, is denoted by q_0^i, where $q_0^i \equiv \{\{q_{0a}^i\}|a \in L_1^i\}\} \in R_+^{n_{L_1^i}}$, where $n_{L_1^i}$ is the number of top-most links in firm i's supply chain network. Finally, q_0 is the vector of all initial quality levels of the food firms; that is, $q_0 \equiv \{\{q_0^i\}|i = 1, \dots, I\}$.

The utility of food firm i is its profit, which is the difference between its revenue and its total costs, where the total costs are composed of the total costs on the production/harvesting links and the total operational costs on the post-harvest links in its supply chain network. Hence, the utility of firm i; $i = 1, \dots, I$, denoted by U_i, is expressed as:

$$U_i = \sum_{k=R_1}^{R_{n_R}} \rho_{ik}(d, \hat{q}) d_{ik} - (\sum_{a \in L_1^i} \hat{z}_a(f_a, q_{0a}^i) + \sum_{b \in L_2^i} \hat{c}_b(f)). \tag{18}$$

In view of (6), (13), and (14), we can rewrite (15) as:

$$\hat{\rho}_{ik}(x, q_0) \equiv \rho_{ik}(d, \hat{q}), \quad i = 1, \dots, I;\ k = R_1, \dots, R_{n_R}. \tag{19}$$

In lieu of the constraints (10) and (12), and the functional expressions (16), (17), and (19), we can define $\hat{U}_i(X, q_0) \equiv U_i$ for all firms i; $i = 1, \dots, I$, with the I-dimensional vector $\hat{U}$ being the vector of the profits of all the firms:

$$\hat{U} = \hat{U}(X, q_0). \tag{20}$$

In the competitive oligopolistic market framework, each firm selects its product path flows as well as its initial quality levels at its production sites in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below:

Definition 1: Supply Chain Network Nash Equilibrium with Fresh Produce Quality

A fresh produce path flow pattern and initial quality level $(X^*, q_0^*) \in K = \prod_{i=1}^I K_i$ *constitutes a supply chain network Nash Equilibrium with fresh produce quality if for each food firm* i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, X_{-i}^*, q_0^{i*}, q_0^{-i*}) \geq \hat{U}_i(X_i, X_{-i}^*, q_0^i, q_0^{-i*}), \quad \forall (X_i, q_0^i) \in K_i, \tag{21}$$

where $X_{-i}^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$, $q_0^{-i*} \equiv (q_0^{1*}, \ldots, q_0^{i-1*}, q_0^{i+1*}, \ldots, q_0^{I*})$ *and* $K_i \equiv \{(X_i, q_0^i) | X_i \in R_+^{n_{P^i}}, q_0^i \in R_+^{n_{L_1^i}},$ (9) *and* (12) *hold for* $l \in L^i\}$.

In other words, an equilibrium is established if no food firm can unilaterally improve upon its profit by altering its product flows and initial quality at production sites in its supply chain network, given the product flows and initial quality decisions of the other firms.

Next, we derive alternative variational inequality formulations of the Nash Equilibrium for the fresh produce supply chain network under oligopolistic competition satisfying Definition 1, in terms of path flows and initial quality levels (see Nash (1950, 1951)).

The λ_a; $a \in L_1$ and γ_l; $l \in L$ are the Lagrange multipliers associated with constraints (9) and (12) (or (11)), respectively. We group these Lagrange multipliers into the n_{L_1}-dimensional vector λ and the n_L-dimensional vector γ, respectively.

Theorem 1: Variational Inequality Formulation of the Governing Equilibrium Conditions

Assume that, for each food firm i; $i = 1, \ldots, I$, *the profit function* $\hat{U}_i(X, q_0)$ *is concave with respect to the variables* X_i *and* q_0^i, *and is continuously differentiable. Then* $(X^*, q_0^*) \in K$ *is a supply chain network Nash Equilibrium with fresh produce quality according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^I \langle \nabla_{X_i} \hat{U}_i(X^*, q_0^*), X_i - X_i^* \rangle - \sum_{i=1}^I \langle \nabla_{q_0^i} \hat{U}_i(X^*, q_0^*), q_0^i - q_0^{i*} \rangle \geq 0, \quad \forall (X, q_0) \in K, \tag{22}$$

where $\langle \cdot, \cdot \rangle$ *denotes the inner product in the corresponding Euclidean space. Furthermore,* $\nabla_{X_i} \hat{U}_i(X, q_0)$ *denotes the gradient of* $\hat{U}_i(X, q_0)$ *with respect to* X_i *and* $\nabla_{q_0^i} \hat{U}_i(X, q_0)$ *denotes the gradient of* $\hat{U}_i(X, q_0)$ *with respect to* q_0^i. *The solution of variational inequality (22), in turn, is equivalent to the solution of the variational inequality: determine* $(x^*, q_0^*, \lambda^*, \gamma^*) \in K^1$

satisfying:

$$\sum_{i=1}^{I}\sum_{k=R_1}^{R_{n_R}}\sum_{p\in P_k^i}\left[\frac{\partial \hat{Z}^i(x^*,q_0^{i*})}{\partial x_p}+\frac{\partial \hat{C}^i(x^*)}{\partial x_p}+\sum_{l\in L^i}\gamma_l^*\delta_{lp}-\hat{\rho}_{ik}(x^*,q_0^*)-\sum_{j=R_1}^{R_{n_R}}\frac{\partial \hat{\rho}_{ij}(x^*,q_0^*)}{\partial x_p}\sum_{r\in P_j^i}x_r^*\right]$$

$$\times[x_p-x_p^*]+\sum_{i=1}^{I}\sum_{a\in L_1^i}\left[\frac{\partial \hat{Z}^i(x^*,q_0^{i*})}{\partial q_{0a}^i}+\lambda_a^*-\sum_{j=R_1}^{R_{n_R}}\frac{\partial \hat{\rho}_{ij}(x^*,q_0^*)}{\partial q_{0a}^i}\sum_{r\in P_j^i}x_r^*\right]\times[q_{0a}^i-q_{0a}^{i*}]$$

$$+\sum_{i=1}^{I}\sum_{a\in L_1^i}[\bar{q}_{0a}^i-q_{0a}^{i*}]\times[\lambda_a-\lambda_a^*]+\sum_{i=1}^{I}\sum_{l\in L^i}\left[u_l-\sum_{r\in P}x_r^*\delta_{lr}\right]\times[\gamma_l-\gamma_l^*]\geq 0,\quad \forall(x,q_0,\lambda,\gamma)\in K^1, \tag{23}$$

where $K^1\equiv\{(x,q_0,\lambda,\gamma)|x\in R_+^{n_P},\ q_0\in R_+^{n_{L_1}},\ \lambda\in R_+^{n_{L_1}},\ \gamma\in R_+^{n_L}\}$ *and for each path* p*;* $p\in P_k^i$*;* $i=1,\ldots,I$*;* $k=R_1,\ldots,R_{n_R}$*:*

$$\frac{\partial \hat{Z}^i(x,q_0^i)}{\partial x_p}\equiv\sum_{a\in L_1^i}\frac{\partial \hat{z}_a(f_a,q_{0a}^i)}{\partial f_a}\delta_{ap}, \tag{24a}$$

$$\frac{\partial \hat{C}^i(x)}{\partial x_p}\equiv\sum_{b\in L_2^i}\sum_{l\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_l}\delta_{lp}, \tag{24b}$$

$$\frac{\partial \hat{\rho}_{ij}(x,q_0)}{\partial x_p}\equiv\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}}+\frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}}\left(\frac{q_p}{\sum_{r\in P_k^i}x_r}-\frac{\sum_{r\in P_k^i}q_rx_r}{(\sum_{r\in P_k^i}x_r)^2}\right). \tag{24c}$$

For each a*;* $a\in L_1^i$*;* $i=1,\ldots,I$*,*

$$\frac{\partial \hat{Z}^i(x,q_0^i)}{\partial q_{0a}^i}\equiv\frac{\partial \hat{z}_a(f_a,q_{0a}^i)}{\partial q_{0a}^i}, \tag{24d}$$

$$\frac{\partial \hat{\rho}_{ij}(x,q_0)}{\partial q_{0a}^i}\equiv\sum_{h=R_1}^{R_{n_R}}\sum_{s\in P_h^i}\frac{x_s}{\sum_{r\in P_h^i}x_r}\frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}}\frac{\partial q_s}{\partial q_{0a}^i}. \tag{24e}$$

In particular, if link a *is not included in path* s*,* $\frac{\partial q_s}{\partial q_{0a}^i}=0$*; if link* a *is included in path* s*, following (6), we have:*

$$\frac{\partial q_s}{\partial q_{0a}^i}=\begin{cases}1, & \text{if } n=0,\\ \prod_{b\in s\cap L_2^i}\beta_b, & \text{if } n=1.\end{cases} \tag{24f}$$

Proof: (22) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). Under the imposed assumptions, (22) holds if and only if (see, e.g., Bertsekas and

Tsitsiklis (1989)) the following holds:

$$\sum_{i=1}^{I}\sum_{k=R_1}^{R_{n_R}}\sum_{p\in P_k^i}\left[-\frac{\partial \hat{U}_i}{\partial x_p}+\sum_{l\in L^i}\gamma_l^*\delta_{lp}\right]\times[x_p-x_p^*]+\sum_{i=1}^{I}\sum_{a\in L_1^i}\left[-\frac{\partial \hat{U}_i}{\partial q_{0a}^i}+\lambda_a^*\right]\times[q_{0a}^i-q_{0a}^{i*}]$$

$$+\sum_{i=1}^{I}\sum_{a\in L_1^i}[\bar{q}_{0a}^i-q_{0a}^{i*}]\times[\lambda_a-\lambda_a^*]+\sum_{i=1}^{I}\sum_{l\in L^i}\left[u_l-\sum_{r\in P}x_r^*\delta_{lr}\right]\times[\gamma_l-\gamma_l^*]\geq 0,\quad \forall (x,q_0,\lambda,\gamma)\in K^1. \tag{25}$$

For each path p; $p\in P_k^i$, we have that

$$\begin{aligned}
\frac{\partial \hat{U}_i}{\partial x_p}=&\frac{\partial[\sum_{j=R_1}^{R_{n_R}}\rho_{ij}(d,\hat{q})d_{ij}-(\sum_{e\in L_1^i}\hat{z}_e(f_e,q_{0e}^i)+\sum_{b\in L_2^i}\hat{c}_b(f))]}{\partial x_p}\\
=&\frac{\partial[\sum_{j=R_1}^{R_{n_R}}\rho_{ij}(d,\hat{q})d_{ij}]}{\partial x_p}-\frac{\partial[\sum_{e\in L_1^i}\hat{z}_e(f_e,q_{0e}^i)]}{\partial x_p}-\frac{\partial[\sum_{b\in L_2^i}\hat{c}_b(f)]}{\partial x_p}\\
=&\sum_{j=R_1}^{R_{n_R}}\left[\frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial d_{ik}}\frac{\partial d_{ik}}{\partial x_p}+\frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial \hat{q}_{ik}}\frac{\partial \hat{q}_{ik}}{\partial x_p}\right]-\sum_{a\in L_1^i}\sum_{e\in L_1^i}\frac{\partial \hat{z}_e(f_e,q_{0e}^i)}{\partial f_a}\frac{\partial f_a}{\partial x_p}\\
&-\sum_{b\in L_2^i}\sum_{l\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_l}\frac{\partial f_l}{\partial x_p}\\
=&\rho_{ik}(d,\hat{q})+\sum_{j=R_1}^{R_{n_R}}\left[\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}}d_{ij}+\frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}}d_{ij}\left(\frac{\partial \hat{q}_{ik}}{\partial[\sum_{r\in P_k^i}q_rx_r]}\frac{\partial[\sum_{r\in P_k^i}q_rx_r]}{\partial x_p}\right.\right.\\
&\left.\left.+\frac{\partial \hat{q}_{ik}}{\partial[\sum_{r\in P_k^i}x_r]}\frac{\partial[\sum_{r\in P_k^i}x_r]}{\partial x_p}\right)\right]-\sum_{a\in L_1^i}\frac{\partial \hat{z}_a(f_a,q_{0a}^i)}{\partial f_a}\delta_{ap}-\sum_{b\in L_2^i}\sum_{l\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_l}\delta_{lp}\\
=&\rho_{ik}(d,\hat{q})+\sum_{j=R_1}^{R_{n_R}}\left[\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}}+\frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}}\left(\frac{q_p}{\sum_{r\in P_k^i}x_r}-\frac{\sum_{r\in P_k^i}q_rx_r}{(\sum_{r\in P_k^i}x_r)^2}\right)\right]d_{ij}\\
&-\sum_{a\in L_1^i}\frac{\partial \hat{z}_a(f_a,q_{0a}^i)}{\partial f_a}\delta_{ap}-\sum_{b\in L_2^i}\sum_{l\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_l}\delta_{lp},
\end{aligned}\tag{26}$$

and for each link a; $a\in L_1^i$, we know that

$$\begin{aligned}
\frac{\partial \hat{U}_i}{\partial q_{0a}^i}=&\frac{\partial[\sum_{j=R_1}^{R_{n_R}}\rho_{ij}(d,\hat{q})d_{ij}-(\sum_{e\in L_1^i}\hat{z}_e(f_e,q_{0e}^i)+\sum_{b\in L_2^i}\hat{c}_b(f))]}{\partial q_{0a}^i}\\
=&\frac{\partial[\sum_{j=R_1}^{R_{n_R}}\rho_{ij}(d,\hat{q})d_{ij}]}{\partial q_{0a}^i}-\frac{\partial[\sum_{e\in L_1^i}\hat{z}_e(f_e,q_{0e}^i)]}{\partial q_{0a}^i}-\frac{\partial[\sum_{b\in L_2^i}\hat{c}_b(f)]}{\partial q_{0a}^i}\\
=&\sum_{h=R_1}^{R_{n_R}}\sum_{j=R_1}^{R_{n_R}}\frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial \hat{q}_{ih}}\frac{\partial \hat{q}_{ih}}{\partial q_{0a}^i}-\frac{\partial \hat{z}_a(f_a,q_{0a}^i)}{\partial q_{0a}^i}
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} d_{ij} \frac{\partial \hat{q}_{ih}}{\partial [\sum_{r\in P_h^i} q_r x_r]} \frac{\partial [\sum_{r\in P_h^i} q_r x_r]}{\partial q_{0a}^i} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i} \\
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} d_{ij} \frac{1}{\sum_{r\in P_h^i} x_r} \sum_{s\in P_h^i} \frac{\partial [\sum_{r\in P_h^i} q_r x_r]}{\partial q_s} \frac{\partial q_s}{\partial q_{0a}^i} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i} \\
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \sum_{s\in P_h^i} \frac{x_s}{\sum_{r\in P_h^i} x_r} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} \frac{\partial q_s}{\partial q_{0a}^i} d_{ij} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}. \qquad (27)
\end{aligned}
$$

By making use of (14), (19), and the definitions in (24(a))-(24(e)), variational inequality (23) is immediate.

In addition, it is obvious that $\frac{\partial q_s}{\partial q_{0a}^i} = 0$ if link a is not included in path s. If link a is included in path s, $\frac{\partial q_s}{\partial q_{0a}^i}$ in (24f) follows directly from (6). $\square$

For further background on variational inequalities, see the books by Kinderlehrer and Stampacchia (1980) and Nagurney (1999).

Existence of a solution $(X^*, q_0^*) \in K$ to variational inequality (22) follows from the classical theory of variational inequalities since the feasible set K is compact, that is, closed and bounded, and the components of the gradients are continuous under our imposed assumptions.

3.1 Simple Illustrative Examples

In this Subsection, we provide illustrative numerical examples. There are two food firms, Food Firm 1 and Food Firm 2, competing in a duopolistic manner to sell their fresh produce products, which are substitutable. There is a single retail outlet R_1. The supply chain network topology is given in Figure 2.

The cost of production/harvesting depends on the initial quality and on the product flow on the production/harvesting links. In Figure 2, there are two production/harvesting links that belong, respectively, to set L_1^1 and set L_1^2. The cost of production/harvesting is higher for Food Firm 1 since it uses better machinery and has invested more into the necessary chemicals to maintain the soil quality, with the top-tiered link cost functions being:

$$\hat{z}_1(f_1, q_{01}^1) = f_1^2 + 8f_1 + 3q_{01}^1, \quad \hat{z}_7(f_7, q_{07}^2) = f_7^2 + 3q_{07}^2.$$

There are ten additional links, belonging to the sets L_2^1 and L_2^2, in the supply chain

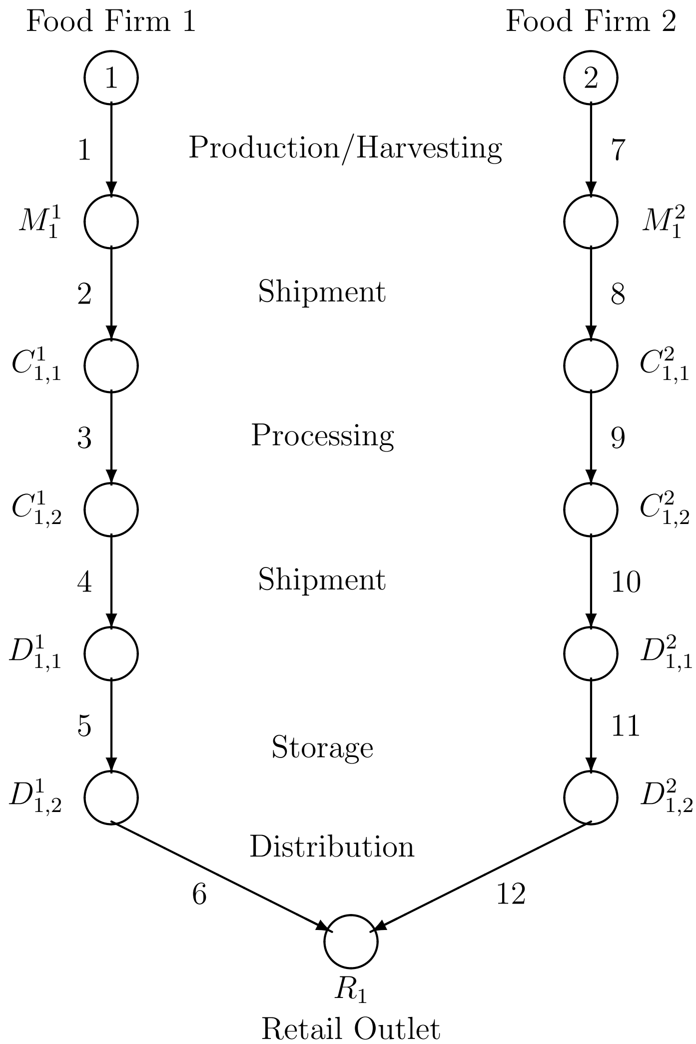

Figure 2: Supply Chain Network Topology for the Illustrative Examples

network and their total link cost functions are:

$$\hat{c}_2(f_2) = 5f_2^2 + 10f_2,\ \hat{c}_3(f_3) = 2f_3^2,\ \hat{c}_4(f_4) = 2f_4^2 + f_4,\ \hat{c}_5(f_5) = 3f_5^2,\ \hat{c}_6(f_6) = f_6^2 + f_6,$$

$$\hat{c}_8(f_8) = f_8^2 + f_8,\ \hat{c}_9(f_9) = 3f_9^2 + f_9,\ \hat{c}_{10}(f_{10}) = 2f_{10}^2,\ \hat{c}_{11}(f_{11}) = 6f_{11}^2 + f_{11},\ \hat{c}_{12}(f_{12}) = 6f_{12}^2 + f_{12}.$$

The total link cost functions are constructed according to the assumptions made for Food Firm 1, Food Firm 2, and Retail Outlet R_1.

Table 1 displays the quality decay β_b incurred on the links, when the reaction order $n = 0$ and when $n = 1$.

The reaction rate and quality decay rate on each link are calculated according to (5) and (6), where the universal gas constant and activation energy are taken as 8.314 $Jmol^{-1}K^{-1}$ and 150 $kJmol^{-1}$, respectively.

Table 1: Parameters for the Calculation of Quality Decay for the Illustrative Examples

Link b	**Hours**	**Temperature (Celsius)**	β_b ($n=0$)	β_b ($n=1$)
2	48	22	-0.1784	0.8366
3	10	22	-0.0372	0.9635
4	12	10	-0.0167	0.9835
5	10	22	-0.0372	0.9635
6	10	22	-0.0372	0.9635
8	4	22	-0.0149	0.9852
9	2	22	-0.0074	0.9926
10	2	5	-0.0004	0.9995
11	8	22	-0.0297	0.9707
12	2	22	-0.0149	0.9852

The shipment time is longer for Food Firm 1 than for Food Firm 2 because of their respective distances to their processing facilities, which can be seen from the time difference between link 2 and link 8.

There are two paths, p_1 and p_2, in this supply chain network, defined as: $p_1 = (1, 2, 3, 4, 5, 6)$ and $p_2 = (7, 8, 9, 10, 11, 12)$. Since there exists one path for each food firm, we know, because of conservation of flow, that: $d^*_{11} = x^*_{p_1}$ and $d^*_{21} = x^*_{p_2}$.

We now consider different quality decay functions and provide specific details in the examples below.

Example 1a: Linear Quality Decay (Zero Order Kinetics)

As mentioned in Section 2, in a zero order quality decay function, the reaction order $n = 0$, and the quality q_p over a path p can be determined by the appropriate formula in (6), for $n = 0$. The initial quality variables are q^1_{01} and q^2_{07}.

The demand price functions are:

$$\hat{\rho}_{11}(x, q_0) \equiv \rho_{11}(d, \hat{q}) = -2x_{p_1} - x_{p_2} + \frac{q_{p_1} x_{p_1}}{x_{p_1}} + 100$$

and

$$\hat{\rho}_{21}(x, q_0) \equiv \rho_{21}(d, \hat{q}) = -3x_{p_2} - x_{p_1} + \frac{q_{p_2} x_{p_2}}{x_{p_2}} + 90,$$

with the path quality q_p for the two paths constructed according to (6), for $n = 0$ for $i = 1, 2$, given by:

$$q_{p_1} = q^1_{01} + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 = q^1_{01} - 0.3066,$$

$$q_{p_2} = q_{07}^2 + \beta_8 + \beta_9 + \beta_{10} + \beta_{11} + \beta_{12} = q_{07}^2 - 0.0674.$$

We set the upper bounds (capacities) on the links to 200. Since the capacities on the links are high, we know that (cf. variational inequality (23)) the equilibrium Lagrange multipliers γ^* associated with the links will be equal to 0. Similarly, for simplicity, and exposition purposes, we set the initial quality bounds to 100, so that the corresponding equilibrium Lagrange multipliers λ^*, based on the data, will also be 0.

Given the data, it is also reasonable to expect that $x_{p_1}^* > 0$, $x_{p_2}^* > 0$, $q_{01}^{1*} > 0$, and $q_{07}^{2*} > 0$. Hence, in order to obtain the equilibrium path flows and the equilibrium initial quality levels that satisfy variational inequality (23), the following expressions must be equal to 0:

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial x_{p_1}} + \frac{\partial \hat{C}^1(x^*)}{\partial x_{p_1}} - \hat{\rho}_{11}(x^*, q_0^*) - \frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial x_{p_1}} x_{p_1}^* = 0, \tag{28}$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial q_{01}^1} - \frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1}^* = 0, \tag{29}$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial x_{p_2}} + \frac{\partial \hat{C}^2(x^*)}{\partial x_{p_2}} - \hat{\rho}_{21}(x^*, q_0^*) - \frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial x_{p_2}} x_{p_2}^* = 0, \tag{30}$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial q_{07}^2} - \frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2}^* = 0. \tag{31}$$

Utilizing the functions for this example and $(24a) - (24f)$, we can construct:

$$\frac{\partial \hat{C}^1(x^*)}{\partial x_{p_1}} = 10x_{p_1}^* + 10 + 4x_{p_1}^* + 4x_{p_1}^* + 1 + 6x_{p_1}^* + 2x_{p_1}^* + 1 = 26x_{p_1}^* + 12,$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial x_{p_1}} = 2x_{p_1}^* + 8,$$

$$-\hat{\rho}_{11}(x^*, q_0^*) = 2x_{p_1}^* + x_{p_2}^* - \frac{(q_{01}^{1*} - 0.3066)x_{p_1}^*}{x_{p_1}^*} - 100 = 2x_{p_1}^* + x_{p_2}^* - q_{01}^{1*} - 99.6934,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial x_{p_1}} x_{p_1}^* = 2x_{p_1}^*,$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial q_{01}^1} = 3,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1}^* = -x_{p_1}^*.$$

Analogous expressions for path p_2 are:

$$\frac{\partial \hat{C}^2(x^*)}{\partial x_{p_2}} = 2x_{p_2}^* + 1 + 6x_{p_2}^* + 1 + 4x_{p_2}^* + 12x_{p_2}^* + 1 + 12x_{p_2}^* + 1 = 36x_{p_2}^* + 4,$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial x_{p_2}} = 2x_{p_2}^*,$$

$$-\hat{\rho}_{21}(x^*, q_0^*) = 3x_{p_2}^* + x_{p_1}^* - \frac{(q_{07}^{2*} - 0.0674)x_{p_2}^*}{x_{p_2}^*} - 90 = 3x_{p_2}^* + x_{p_1}^* - q_{07}^{2*} - 89.9326,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial x_{p_2}} x_{p_2}^* = 3x_{p_2}^*,$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial q_{07}^2} = 3,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2} = -x_{p_2}^*.$$

Grouping the terms above corresponding to each equation (28) – (31) we obtain the following system of equations:

$$32x_{p_1}^* + x_{p_2}^* - q_{01}^{1*} = 79.6934,$$

$$3 - x_{p_1}^* = 0,$$

$$x_{p_1}^* + 44x_{p_2}^* - q_{07}^{2*} = 85.9326,$$

$$3 - x_{p_2}^* = 0,$$

with solution:

$$x_{p_1}^* = 3, \; x_{p_2}^* = 3, \quad q_{01}^{1*} = 19.3066, \; q_{07}^{2*} = 49.0674.$$

Hence, the path quality levels are: $q_{p_1} = 19$ and $q_{p_2} = 49$, the demand prices are: $\rho_{11} = 110$ and $\rho_{21} = 127$, with Food Firm 1 enjoying a profit (in dollars) of $\hat{U}_1(X^*, q_0^*) = 87.0000$ and Food Firm 2 a profit of $\hat{U}_2(X^*, q_0^*) = 51.0000$.

Observe that Food Firm 1 has a higher profit than Food Firm 2, although its fresh produce is of lower quality, both at its production site and at the retail outlet.

Next, we present an example with exponential quality decay.

Example 1b: Exponential Quality Decay (First Order Kinetics)

Example 1b is constructed from Example 1a and has the same data except that the product now has an exponential quality decay with a reaction order $n = 1$. The quality levels of the paths are constructed using the appropriate expression in (6) and the β_b values in Table 1, for $n = 1$ for $i = 1, 2$, yielding:

$$q_{p_1} = q_{01}^1 \times \beta_2 \times \beta_3 \times \beta_4 \times \beta_5 \times \beta_6 = (q_{01}^1)(0.7359),$$

$$q_{p_2} = q_{07}^2 \times \beta_8 \times \beta_9 \times \beta_{10} \times \beta_{11} \times \beta_{12} = (q_{07}^2)(0.9418).$$

We now proceed to solve the equations (28) – (31) for this example, with the following terms for paths p_1 and p_2 presented for completeness and convenience:

$$-\hat{\rho}_{11}(x^*, q_0^*) = 2x_{p_1}^* + x_{p_2}^* - \frac{(q_{01}^{1*})(0.7359)(x_{p_1}^*)}{x_{p_1}^*} - 100 = 2x_{p_1}^* + x_{p_2}^* - (q_{01}^{1*})(0.7359) - 100,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1} = -0.7359 x_{p_1}^*,$$

$$-\hat{\rho}_{21}(x^*, q_0^*) = 3x_{p_2}^* + x_{p_1}^* - \frac{(q_{07}^{2*})(0.9418)(x_{p_2}^*)}{x_{p_2}^*} - 90 = 3x_{p_2}^* + x_{p_1}^* - (q_{07}^{2*})(0.9418) - 90,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2} = -0.9418 x_{p_2}^*.$$

Grouping the terms above corresponding to each equation, we obtain the following system of equations:

$$32x_{p_1}^* + x_{p_2}^* - 0.7359 q_{01}^{1*} = 80,$$

$$3 - 0.7359 x_{p_1}^* = 0,$$

$$x_{p_1}^* + 44x_{p_2}^* - 0.9418 q_{07}^{2*} = 86,$$

$$3 - 0.9418 x_{p_2}^* = 0.$$

Straightforward calculations yield the following equilibrium path flows and equilibrium initial quality levels:

$$x_{p_1}^* = 4.0766, \; x_{p_2}^* = 3.1854, \quad q_{01}^{1*} = 72.8857, \; q_{07}^{2*} = 61.8329.$$

The path quality levels are, hence, $q_{p_1} = 53.6366$ and $q_{p_2} = 58.2342$.

Notice that, even though the initial quality of Food Firm 1's fresh produce is higher, Food Firm 2 sells its fresh produce with a higher quality at the retail outlet.

By substituting the equilibrium path flows and the equilibrium initial qualities into the demand price functions, we obtain the following equilibrium demand prices at the retail outlet for Food Firm 1 and Food Firm 2, respectively: $\rho_{11} = 142.30$ and $\rho_{21} = 134.60$. The price of Food Firm 2's fresh produce is higher than that of Food Firm 1's. Furthermore, the profits of the food firms are calculated, in dollars, as $\hat{U}_1(X^*, q_0^*) = 47.2497$ and $\hat{U}_2(X^*, q_0^*) = 37.7258$. Notice that, when the quality decay becomes exponential, the profits of the food firms decrease significantly. It would be reasonable to expect that food firms invest more to keep the initial quality high, hence, the end quality would not get too low with a faster quality degradation. This causes the total cost to become higher, therefore the the profits to decrease.

4. The Algorithm

The algorithm that we apply to compute the solution to variational inequality (23) represented in standard variational inequality form as: determine $Y^* \in \mathcal{K}$ where Y is a vector in R^N, $F(Y)$ is a continuous function such that $F(Y) : X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(Y^*), Y - Y^* \rangle \geq 0, \qquad \forall Y \in \mathcal{K}, \tag{32}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space, is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, at an iteration $\tau+1$ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$Y^{\tau+1} = P_{\mathcal{K}}(Y^\tau - \alpha_\tau F(Y^\tau)), \tag{33}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$, F is the function that enters the variational inequality problem, and $\{a_\tau\}$ is a sequence constructed as below. For variational inequality (23) the feasible set $\mathcal{K} = K^1$ and $N = n_P + 2n_{L_1} + n_L$.

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} \alpha_\tau = \infty$, $\alpha_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. Conditions for convergence of this algorithm as well as solutions to a spectrum of applications can be found in Nagurney (2006), Nagurney et al. (2013), Toyasaki, Daniele, and Wakolbinger (2014), and Nagurney and Li (2016), and the references therein.

4.1 Explicit Formulae for the Euler Method Applied to the Model

A nice feature of the Euler method applied to the competitive supply chain network model with quality is that the variables can be determined at each iteration $\tau + 1$ using closed form expressions, because of the simplicity of the feasible set. We now provide the explicit formulae.

The closed form expressions for the fresh produce path flows at iteration $\tau + 1$ are as follows. For each path $p \in P_k^i$, $\forall i, k$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + \alpha_\tau(\hat{\rho}_{ik}(x^\tau, q_0^\tau) + \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial x_p} \sum_{r \in P_j^i} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial x_p} - \frac{\partial \hat{C}^i(x^\tau)}{\partial x_p} - \sum_{l \in L^i} \gamma_l^\tau \delta_{lp})\}. \tag{34}$$

For each initial quality level $a \in L_1^i$, $\forall i$, in turn, compute:

$$q_{0a}^{i\tau+1} = \max\{0, q_{0a}^{i\tau} + \alpha_\tau(\sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial q_{0a}^i} \sum_{r \in P_j^i} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial q_{0a}^i} - \lambda_a^\tau)\}. \quad (35)$$

The Lagrange multiplier for each top-most link $a \in L_1^i$; $i = 1, \ldots, I$, associated with the initial quality bounds is computed as:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + \alpha_\tau(q_{0a}^{i\tau} - \bar{q}_{0a}^i)\}. \quad (36)$$

Finally, the Lagrange multiplier for each link $l \in L^i$; $i = 1, \ldots, I$, associated with the link capacities is computed according to:

$$\gamma_l^{\tau+1} = \max\{0, \gamma_l^\tau + \alpha_\tau(\sum_{r \in P} x_r^\tau \delta_{lr} - u_l)\}. \quad (37)$$

We note that expressions (34) through (37) may be interpreted as a discrete-time adjustment or tatonnement process with the food firms updating at each discrete point in time their fresh produce path flows, their initial quality levels as well as the Lagrange multipliers associated with the initial quality levels at the production sites, and the Lagrange multipliers associated with the link capacities, until an equilibrium is achieved. Observe that these computations can all be done simultaneously and, hence, in parallel. Moreover, at each iteration, only the iterates from the preceding iteration (point in time) are needed for these computations.

Next, we present larger numerical examples comprising a case study focusing on peaches.

5. A Case Study of Peaches

In this Section, we focus on the peach market in the United States, specifically in Western Massachusetts. Peaches [*Prunus persica (L.)* Batsch] are very vital for fresh produce markets in the United States and also all over the world. It is noted that, in 2015, the United States peach production was 825,415 tons in volume, and 606 million dollars in worth (USDA NASS (2016), Zhao et al. (2017)). For our case study, we selected two orchards from Western Massachusetts: Apex Orchards and Cold Spring Orchard, located, respectively, in Shelburne, MA and Belchertown, MA. The supply chain network topology for the peach case study is illustrated in Figure 3. We assume that the Apex Orchards farm has two production sites, two processors, and two distribution centers. After production/harvesting, Apex Orchards can ship its peaches to processors $C_{1,1}^1$ or $C_{2,1}^1$, with shipment depicted via links 4, 5, 6, and

7 in Figure 3. Similarly, after processing, Apex Orchards can transfer its peaches to $D^1_{1,1}$ or $D^1_{2,1}$, as shown in Figure 3 via links 12, 13, 14, and 15. Cold Spring Orchard is smaller in size and, therefore, it only has only a single production site, one processor, and a single distribution center. Both of the orchards sell their peaches to two retailers, Whole Foods, located in Hadley, MA, and Formaggio Kitchen, located in Cambridge, MA. The mode of transportation for both of the orchards is trucks.

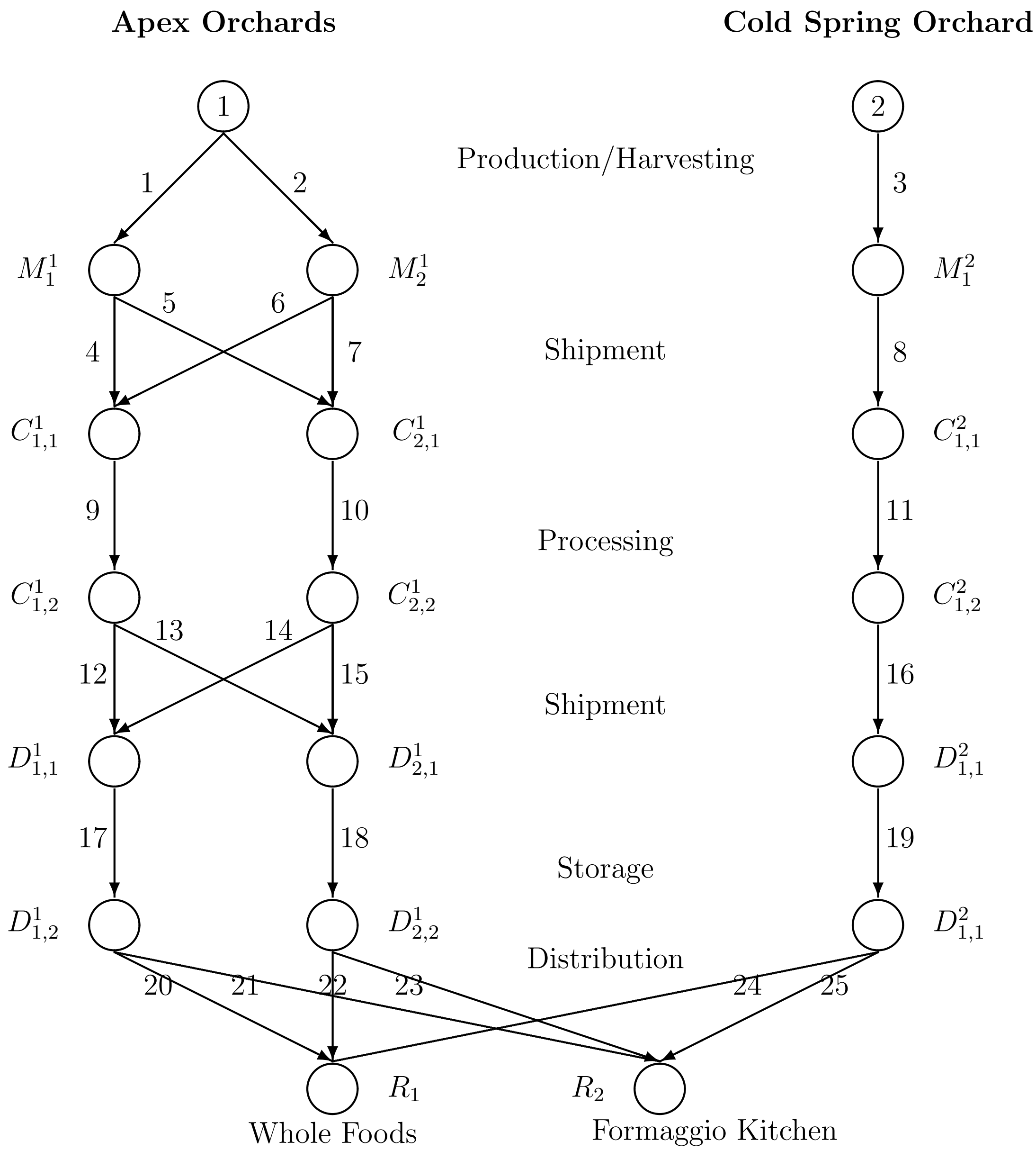

Figure 3: Supply Chain Network Topology of the Case Study Focusing on Peaches

According to Toralles et. al (2005), the color change attribute of peaches, in the form of browning, follows a first-order, that is, an exponential decay function. In Table 2, following

(4), we calculate the link quality degradation, β_b, when the reaction order is $n = 1$, for the supply chain network topology in Figure 3. The universal gas constant and activation energy are taken as 8.314 $Jmol^{-1}K^{-1}$ and 147.9 $kJmol^{-1}$, respectively (Toralles et al. (2005)). The harvesting season for peaches is usually mid-July to mid -September in Massachusetts, and the temperature reported in Table 3, for the processing, shipment, and distribution links, is the average temperature of these months. It is assumed that, since the Apex Orchards farm is located at a higher altitude, that the average temperature of Apex's operations is lower.

We assume that the shipment and distribution operations are made at an average temperature, since the orchards do not have the necessary technology in their trucks to keep the temperature at sufficient levels to deter quality degradation. Furthermore, according to Crisosto and Valero (2008) the ideal storage temperature of peaches is between -1 C$^\circ$ and 1 C$^\circ$. In this case study, the Apex Orchards owner is assumed to have the technology to keep the storage temperature at 1 C$^\circ$ and Cold Spring Orchard is assumed to decrease the storage temperature only to 18 C$^\circ$. The product flows and capacities are in pecks and we emphasize that a peck of peaches is approximately 12 pounds of peaches.

In Tables 3 and 4, we report the total production / harvesting cost functions, the upper bounds on the initial quality, the total operational cost functions, and the link flow capacities. We constructed the cost functions, in Table 3 and 4, through the data, gathered from Sumner and Murdock (2017), in which the authors made a sample cost analysis. We also utilize from Dris and Jain (2007) to construct the total storage link cost functions. The time horizon, under consideration, is that of a week.

The Euler method (cf. (34) – (37)) is implemented in FORTRAN and a Linux system at the University of Massachusetts used for the computations. The sequence $a_\tau = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-7}, that is, the Euler method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. The algorithm was initialized with each path flow set equal to 1, each Lagrange multiplier set equal to 0, and each initial quality level set to 50.

Example 1 - Baseline

We assume that the consumers at the retailers are discerning about the quality of the peaches that they are buying. The demand price functions are constructed for Apex Orchards and Cold Spring Orchard based on information from the orchards themselves and also the retailers, along with observed prices of peaches at the retail level. In the case study, both of the

Table 2: Parameters for the Calculation of Quality Decay for the Peach Case Study

Link b	**Hours**	**Temperature (Celsius)**	β_b $(n = 1)$
4	1	23	0.9961
5	2	23	0.9922
6	2	23	0.9922
7	1	23	0.9961
8	2	27	0.9913
9	3	18	0.9906
10	3	18	0.9906
11	4	25	0.9836
12	1	23	0.9961
13	2	23	0.9922
14	2	23	0.9922
15	1	23	0.9961
16	3	27	0.9870
17	48	1	1.0000
18	72	1	1.0000
19	96	18	0.7397
20	2	27	0.9913
21	4	27	0.9827
22	1	27	0.9956
23	4	27	0.9827
24	0.5	27	0.9978
25	4	27	0.9827

Table 3: Total Production / Harvesting Cost Functions, Link Capacities, and Upper Bounds on Initial Quality

Link a	$\hat{z}_a(f_a, q^i_{0a})$	u_a	$\bar{q}^i_{0a}$
1	$.002f_1^2 + f_1 + 0.7q^1_{01} + .01(q^1_{01})^2$	200	98
2	$.002f_2^2 + f_2 + 0.7q^1_{02} + .01(q^1_{02})^2$	200	95
3	$.002f_3^2 + f_3 + 0.5q^2_{03} + .001(q^1_{03})^2$	150	90

orchards sell their peaches to two retailers, Whole Foods, located in Hadley, MA, and Formaggio Kitchen, located in Cambridge, MA. It is known that both retailers sell high quality food products, with Formaggio Kitchen selling peaches at a higher price due to its emphasis on quality. Therefore, in the demand price functions, the coefficients of the average quality levels, representing the sensitivity to the food quality, are higher for Formaggio Kitchen than those for Whole Foods. Furthermore, through conversations at the retailers, we concluded

Table 4: Total Operational Link Cost Functions and Link Capacities

Link b	$\hat{c}_b(f)$	u_b
4	$.001f_4^2 + .7f_4$	150
5	$.002f_5^2 + .7f_5$	150
6	$.001_6^2 + .5f_6$	120
7	$.002f_7^2 + .5f_7$	120
8	$.002f_8^2 + .9f_8$	100
9	$.0025f_9^2 + 1.2f_9$	200
10	$.0025f_{10}^2 + 1.2_{10}$	200
11	$.0026f_{11}^2 + 1.5f_{11}$	150
12	$.001f_{12}^2 + .6f_{12}$	150
13	$.002f_{13}^2 + .6f_{13}$	150
14	$.001f_{14}^2 + .6f_{14}$	150
15	$.002f_{15}^2 + .6f_{15}$	150
16	$.002f_{16}^2 + .6f_{16}$	120
17	$.003f_{17}^2 + .5f_{17}$	150
18	$.0037f_{18}^2 + .9f_{18}$	150
19	$.002f_{19}^2 + .7f_{19}$	120
20	$.002f_{20}^2 + .6f_{20}$	150
21	$.003f_{21}^2 + .7f_{21}$	120
22	$.002f_{22}^2 + .6f_{22}$	150
23	$.003f_{23}^2 + .7f_{23}$	100
24	$.002f_{24}^2 + .6f_{24}$	100
25	$.003f_{25}^2 + .7f_{25}$	100

that Apex Orchards sell their peaches at a higher price. Therefore, the demand price functions for Apex Orchards include higher constant terms than those in Cold Spring Orchard's demand price functions. The corresponding demand price functions for the peaches of Apex Orchards and of Cold Spring Orchard, at the retailers R_1 and R_2, are as follows:

Apex Orchards: $\rho_{11} = -.02d_{11} - .01d_{21} + 0.008\hat{q}_{11} + 20, \quad \rho_{12} = -.02d_{12} - .01d_{22} + 0.01\hat{q}_{12} + 22;$

Cold Spring Orchard: $\rho_{21} = -.02d_{21} - .015d_{11} + 0.008\hat{q}_{21} + 18, \quad \rho_{22} = -.02d_{22} - .015d_{12} + 0.01\hat{q}_{22} + 19.$

In Tables 5 and 6, we report the equilibrium solution. We report the equilibrium link flows rather than the equilibrium path flows for compactness.

Table 5: Example 1 Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	133.43	97.54	0.00	0.00
2	166.57	95.00	0.00	0.05
3	100.00	65.61	0.00	0.00

Table 6: Example 1 Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	69.31	0.00
5	64.12	0.00
6	90.33	0.00
7	76.24	0.00
8	100.00	6.53
9	159.64	0.00
10	140.36	0.00
11	100.00	0.00
12	81.96	0.00
13	77.68	0.00
14	68.04	0.00
15	72.32	0.00
16	100.00	0.00
17	150.00	6.95
18	150.00	6.19
19	100.00	0.00
20	64.84	0.00
21	85.16	0.00
22	65.10	0.00
23	84.90	0.00
24	47.80	0.00
25	52.20	0.00

The demand price of the peaches, evaluated at the computed equilibrium solution, for each orchard, in dollars, per peck, is as follows:

Apex Orchards:

$$\rho_{11} = 17.67, \quad \rho_{12} = 19.00,$$

Cold Spring Orchard:

$$\rho_{21} = 15.47, \quad \rho_{22} = 15.86,$$

with the computed equilibrium demands being: $d^*_{11} = 129.95$, $d^*_{12} = 170.05$, $d^*_{21} = 47.80$, $d^*_{22} = 52.20$.

These prices (recall that they are per peck) are very reasonable.

The average quality of the peaches of the orchards at the retailers, at the equilibrium, is:

Apex Orchards:

$$\hat{q}_{11} = 93.40, \quad \hat{q}_{12} = 92.56,$$

Cold Spring Orchard:

$$\hat{q}_{21} = 46.60, \quad \hat{q}_{22} = 45.90.$$

The profits of the orchards, in dollars, at the equilibrium solution, are:

$$U_1 = 3,302.01, \quad U_2 = 787.65.$$

Recall that the time period in question is that of a week. Notice that the Apex Orchards farm enjoys a higher profit by selling its peaches at higher prices and at a higher average quality.

Observe from Table 5 that the equilibrium initial quality at Apex Orchards' production site corresponding to link 1 is at its upper bound and, hence, the corresponding Lagrange multiplier λ^*_1 is positive. In addition, note that the flows on both links 17 and 18 corresponding to Apex Orchards' storage facilities are at their upper bounds, and, therefore, the associated link Lagrange multipliers γ^*_{17} and γ^*_{18} are positive. Finally, the flow on link 8 associated with shipment of the peaches from Cold Spring Orchard's production site is at its capacity and, therefore, the corresponding Lagrange multiplier γ^*_8 is also positive. The orchards may wish to invest in enhancing their capacity with Apex Orchards focusing on the storage facilities and Cold Spring Orchard on its freight shipment capacity.

Indeed, when we raised u_8 to 150, while keeping all the other data as above, the profit of Cold Spring orchard increased to 921.74 whereas that of Apex Orchards (because of the competition) decreased to 3,272.11.

On the other hand, when we raised both u_{17} and u_{18} to 200 and kept all the other data as in Example 1 above, then the profit enjoyed by Apex Orchards increased to 3,884.80 and that of Cold Spring Orchard decreased to 696.87.

Finally, we had both orchards make investments in enhancing capacity so that $u_8 = 150$ and u_{17} and u_{18} both equal to 200 with the remainder of the data as in Example 1. The profit garnered by Apex Orchards was now 3,844.89 and that of Cold Spring: 815.37. Both firms gain as compared to the profit values in Example 1. Interestingly, the demand prices were now lower but the average quality higher with $\rho_{11} = 16.54$, $\rho_{12} = 17.94$, $\rho_{21} = 14.64$, and $\rho_{22} = 15.10$, and $\hat{q}_{11} = 93.79$, $\hat{q}_{12} = 92.92$, $\hat{q}_{21} = 63.93$, and $\hat{q}_{22} = 67.96$. Hence, by investing in supply chain infrastructure both producers and consumers gain.

Example 2 - Disruption Scenario 1

Example 2 is constructed from Example 1. We now consider a disruption scenario in which a natural disaster has significantly affected the capacity of the orchard production sites of both orchards. Such an incident occurred in 2016 in the Northeast of the United States when extreme weather in terms of cold temperatures "decimated" the peach crop (cf. Tuohy (2016)).

Example 2 has the same data as Example 1 except for the following changes to capture the impacts of the natural disaster. We now have the following capacities on the production/harvesting links: $u_1 = 100$, $u_2 = 150$, and $u_3 = 80$.

The computed equilibrium solution is reported in Tables 7 and 8.

Table 7: Example 2 Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	100.00	75.54	8.17	0.00
2	150.00	75.54	8.18	0.00
3	80.00	11.02	7.78	0.00

The demand price of the peaches, evaluated at the computed equilibrium solution, for each orchard, in dollars, per peck, is as follows:

Apex Orchards:

$$\rho_{11} = 18.12, \quad \rho_{12} = 19.40,$$

Cold Spring Orchard:

$$\rho_{21} = 15.74, \quad \rho_{22} = 16.05,$$

with the computed equilibrium demands being: $d_{11}^* = 104.67$, $d_{12}^* = 145.33$, $d_{21}^* = 37.67$, $d_{22}^* = 42.33$.

Table 8: Example 2 Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	50.28	0.00
5	49.72	0.00
6	82.76	0.00
7	67.24	0.00
8	80.00	0.00
9	133.04	0.00
10	116.96	0.00
11	80.00	0.00
12	80.78	0.00
13	52.25	0.00
14	69.22	0.00
15	47.75	0.00
16	80.00	0.00
17	150.00	0.17
18	100.00	0.00
19	80.00	0.00
20	67.2	0.00
21	82.79	0.00
22	37.46	0.00
23	62.54	0.00
24	37.67	0.00
25	42.33	0.00

The average quality of the peaches of the orchards at the retailers, at the equilibrium, is:

Apex Orchards:

$$\hat{q}_{11} = 73.42, \quad \hat{q}_{12} = 72.68,$$

Cold Spring Orchard:

$$\hat{q}_{21} = 7.83, \quad \hat{q}_{22} = 7.71.$$

The profits of the orchards, in dollars, at the equilibrium solution, are:

$$U_1 = 2,984.07, \quad U_2 = 675.72.$$

Observe from the above equilibrium solution that all the production sites are now at their capacities and, hence, the corresponding link Lagrange multipliers are all positive. Also, observe that the average quality of each orchard's peaches has decreased at each retailer, as compared to the results for Example 1. The demand prices have increased but more for the peaches of Apex Orchards than those from Cold Spring Orchard. As expected, the profit is reduced for both orchards because of the limitations on how many pecks of peaches they can produce and harvest due to the disruption caused by the natural disaster.

Example 3 - Disruption Scenario 2

Example 3 is also constructed from the baseline Example 1 but now we illustrate how another type of supply chain disruption can be analyzed within our model. In particular, we consider a disruption that affects transportation in that the links 5 and 6 associated with the supply chain network of Apex Orchards (cf. Figure 3) are no longer available. This can occur and has occurred in western Massachusetts as a result of flooding. In order to handle this situation, we keep the data as in Example 1 but the upper bounds on these links are now set to zero so that: $u_5 = 0$ and $u_6 = 0$.

The computed new equilibrium solution is reported in Tables 9 and 10.

Table 9: Example 3 Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	150.00	84.50	0.00	0.00
2	120.00	84.50	0.00	0.00
3	100.00	65.59	0.00	0.00

Table 10: Example 3 Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	150.00	6.94
5	0.00	78.33
6	0.00	79.92
7	120.00	7.27
8	100.00	6.75
9	150.00	0.00
10	120.00	0.00
11	100.00	0.00
12	85.36	0.00
13	64.64	0.00
14	64.64	0.00
15	55.36	0.00
16	100.00	0.00
17	150.00	0.40
18	120.00	6.19
19	100.00	0.00
20	66.22	0.00
21	83.78	0.00
22	48.52	0.00
23	71.48	0.00
24	47.86	0.00
25	52.14	0.00

The demand price of the peaches, evaluated at the computed equilibrium solution, for each orchard, in dollars, per peck, is:

Apex Orchards:

$$\rho_{11} = 17.89, \quad \rho_{12} = 19.19,$$

Cold Spring Orchard:

$$\rho_{21} = 15.69, \quad \rho_{22} = 16.09,$$

with the computed equilibrium demands being: $d_{11}^* = 114.74$, $d_{12}^* = 155.26$, $d_{21}^* = 47.86$, $d_{22}^* = 52.14$.

The average quality of the peaches of the orchards at the retailers, at the equilibrium, is:

Apex Orchards:

$$\hat{q}_{11} = 82.32, \quad \hat{q}_{12} = 81.46,$$

Cold Spring Orchard:

$$\hat{q}_{21} = 46.59, \quad \hat{q}_{22} = 45.88.$$

The profits of the orchards, in dollars, at the equilibrium solution, are:

$$U_1 = 3,074.72, \quad U_2 = 811.35.$$

Apex Orchards farm experiences a loss in profits, whereas its competitor, Cold Spring Orchards, garners a higher profit, as compared to the baseline Example 1. Both orchards raise their prices and the average quality of their produce drops although much more significantly for Apex Orchards, which has suffered a supply chain disruption in terms of transportation/shipment possibilities.

We then addressed the following questions: What would be the impact on profits if only link 5 was restored to its original capacity of 150 (and link 6 remained unavailable)? What would be the impact on profits if only link 6 was restored to its original capacity of 120 (and link 5 remained unavailable)? We found the following: The profit of Apex Orchards was 3,272.78 with link 5 restored only and that of Cold Spring Orchard was: 787.64. On the other hand, if link 6 was restored only, then Apex Orchards garnered 3,283.32 in profit and Cold Spring Orchard 787.67 in profit. Hence, given the choice, Apex Orchards should advocate for restoration of link 6 versus link 5 if only one link restoration is feasible.

6. Summary and Conclusions and Suggestions for Future Research

In this paper, we constructed a general framework for the modeling, analysis, and computation of solutions to competitive fresh produce supply chain networks in which food firm owners seek to maximize their profits while determining both the initial quality of the fresh produce with associated costs as well as the fresh produce flows along pathways of their supply chain network through the various activities of harvesting, processing, storage, and distribution. In our framework, we utilize explicit formulae associated with quality deterioration on the supply chain network links which are a function of physical characteristics, including temperature and time. The prices at the retailers are a function not only of the demand for the produce but also of the average quality level of the produce at the retailers.

The governing Nash Equilibrium conditions are stated and alternative variational inequality formulations provided, along with existence results. An algorithmic scheme is outlined, which can be interpreted as a discrete-time adjustment process, and which yields closed form expressions at each iteration for the product path flows, the initial quality levels, as well as the Lagrange multipliers associated with the link capacities and the initial quality upper bounds. Stylistic examples are provided to illustrate the framework and a case study on peaches, consisting of numerical examples under status quo and disruption scenarios, is then presented, along with the computed equilibrium patterns.

The modeling and computational framework proposed in this paper can be extended in multiple, interesting directions. For example, we may introduce additional tiers of decision-makers, such as wholesalers, and capture the impacts of their decision-making on both the final quality levels as well as the prices. Ideas from supply chain network equilibria as in the work of Nagurney, Dong, and Zhang (2002) could then be applied (see also Nagurney (2006) and additional references therein). In addition, food firms might share links, in which case the concept of a Generalized Nash Equilibrium would come into play as investigated in Nagurney, Yu, and Besik (2017), but without the consideration of quality aspects. Furthermore, it would be very interesting to include such policy interventions as minimum quality standards, which would lead to nonlinear constraints, as well as tariffs, and to also include exchange rates for global fresh produce supply chain networks. We leave such research questions for the future.

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