Modeling of Supply Chain Risk Under Disruptions with Performance Measurement and Robustness Analysis

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Abstract: In this paper, we develop a new supply chain network model with multiple decision-makers associated at different tiers and with multiple transportation modes for shipment of the good between tiers. The model formulation captures supply-side risk as well as demand-side risk, along with uncertainty in transportation and other costs. The model also incorporates the individual attitudes towards disruption risks among the manufacturers and the retailers, with the demands for the product associated with the retailers being random. We present the behavior of the various decision-makers, derive the governing equilibrium conditions, and establish the finite-dimensional variational inequality formulation. We also propose a weighted supply chain performance and robustness measure based on our recently derived network performance/efficiency measure and provide supply chain examples for which the equilibrium solutions are determined along with the robustness analyses. This paper extends previous supply chain research by capturing supply-side disruption risks, transportation and other cost risks, and demand-side uncertainty within an integrated modeling and robustness analysis framework.

1. Introduction

Supply chain disruptions and the associated risk are major topics in theoretical and applied research, as well as in practice, since risk in the context of supply chains may be associated with the production/procurement processes, the transportation/shipment of the goods, and/or the demand markets. In fact, Craighead, Blackhurst, Rungtusanatham, and Handfield (2007) have argued that supply chain disruptions and the associated operational and financial risks are the most pressing issue faced by firms in today's competitive global environment. Notably, the focus of research has been on "demand-side" risk, which is related to fluctuations in the demand for products, as opposed to the "supply-side" risk, which deals with uncertain conditions that affect the production and transportation processes of the supply chain. For a discussion of the distinction between these two types of risk, see Snyder (2003).

For example, several recent major disruptions and the associated impacts on the business world have vividly demonstrated the need to address supply-side risk with a case in point being a fire in the Phillips Semiconductor plant in Albuquerque, New Mexico, causing its major customer, Ericsson, to lose $400 million in potential revenues. On the other hand, another major customer, Nokia, managed to arrange alternative supplies and, therefore, mitigated the impact of the disruption (cf. Latour (2001)). Another illustrative example concerns the impact of Hurricane Katrina, with the consequence that 10% - 15% of total U.S. gasoline production was halted, which not only raised the oil price in the U.S., but also overseas (see, e. g., Canadian Competition Bureau (2006)). Moreover, the world price of coffee rose 22% after Hurricane Mitch struck the Central American republics of Nicaragua, Guatemala, and Honduras, which also affected supply chains worldwide (Fairtrade Foundation (2002)). As summarized by Sheffi (2005) on page 74, one of the main characteristics of disruptions in supply networks is "the seemingly unrelated consequences and vulnerabilities stemming from global connectivity." Indeed, supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a holistic, system-wide approach to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers.

Indeed, rigorous modeling and analysis of supply chain networks, in the presence of possible disruptions is imperative since disruptions may have lasting major financial consequences. Hendricks and Singhal (2005) analyzed 800 instances of supply chain disruptions experienced by firms whose stocks are publicly traded. They found that the companies that suffered supply chain disruptions experienced share price returns 33 percent to 40 percent lower than the industry and the general market benchmarks. Furthermore, share price volatility was 13.5 percent higher in these companies in the year following a disruption than in the prior year. Based on their findings, it is evident that only well-prepared companies can effectively cope with supply chain disruptions. Wagner and Bode (2007), in turn, designed a survey to empirically study the responses from executives of firms in Germany regarding their opinions as to the factors that impact supply chain vulnerability. The authors found that demand-side risks are related to customer dependence while supply-side risks are associated with supplier dependence, single sourcing, and global sourcing.

The goal of supply chain risk management is to alleviate the consequences of disruptions and risks or, simply put, to increase the *robustness* of a supply chain. However, there are very few quantitative models for measuring supply chain robustness. For example, Bundschuh, Klajan, and Thurston (2003) discussed the design of a supply chain from both reliability and robustness perspectives. The authors built a mixed integer programming supply chain model with constraints for reliability and robustness. The robustness constraint was formulated in an implicit form: by requiring the suppliers' sourcing limit to exceed a certain level. In this way, the model builds redundancy into a supply chain. Snyder and Daskin (2005) examined supply chain disruptions in the context of facility location. The objective of their model was to select locations for warehouses and other facilities that minimize the transportation costs to customers and, at the same time, account for possible closures of facilities that would result in re-routing of the product. However, as commented in Snyder and Shen (2006), "Although these are multi-location models, they focus primarily on the local effects of disruptions." Santoso, Ahmed, Goetschalckx, and Shapiro (2005) applied a sample average approximation scheme to study the stochastic facility location problem by considering different disruption scenarios.

Tang (2006a) also discussed how to deploy certain strategies in order to enhance the robustness and the resiliency of supply chains. Kleindorfer and Saad (2005), in turn, provided an overview of strategies for mitigating supply chain disruption risks, which were exemplified by a case study in a chemical product supply chain. For a comprehensive review of supply chain risk management models to that date, please refer to Tang (2006b).

To-date, however, most supply disruption studies have focused on a local point of view, in the form of a single-supplier problem (see, e. g., Gupta (1996) and Parlar (1997)) or a two-supplier problem (see, e. g., Parlar and Perry (1996)). Very few papers have examined supply chain risk management in an environment with multiple decision-makers and in the case of uncertain demands (cf. Tomlin (2006)). We believe that it is imperative to study supply chain risk management from a holistic point of view and to capture the interactions among the multiple decision-makers in the various supply chain network tiers. Indeed, such a perspective has also been argued by Wu, Blackhurst, and Chidambaram (2006), who focused on inbound supply risk analysis. Towards that end, in this paper, we take an entirely different perspective, and we consider, for the first time, supply chain robustness in the context of multi-tiered supply chain networks with multiple decision-makers under equilibrium conditions. For a plethora of supply chain network equilibrium models and the associated underlying dynamics, see the book by Nagurney (2006a).

Of course, in order to study supply chain robustness, an informative and effective performance measure is first required. Beamon (1998, 1999) reviewed the supply chain literature and suggested directions for research on supply chain performance measures, which should include criteria on efficient resource allocation, output maximization, and flexible adaptation to the environmental changes (see also, Lee and Whang (1999), Lambert and Pohlen (2001), and Lai, Ngai, and Cheng (2002)). We emphasize that different supply performance measures can be devised based on the specific nature of the problem. In any event, the discussion here is not meant to cover all the existing supply chain performance measures. Indeed, we are well aware that it is a daunting task to propose a supply chain performance measure that covers all aspects of supply chains. We believe that such a discussion will be an ongoing research topic for decades to follow. In this paper, we study supply chain robustness based on a novel network performance measure proposed by Qiang and Nagurney (2008), which captures the network flows, the costs, and the decision-makers' behavior under network equilibrium conditions.

In particular, the model developed in this paper extends the supply chain model of Nagurney, Dong, and Zhang (2002) with consideration of random demand (cf. Nagurney, Cruz, Dong, and Zhang (2005)). In order to study supply chain robustness, the new model contains the following novel features:

- We associate each process in a supply chain with random cost parameters to represent the impact of disruptions to the supply chain.
- We extend the aforementioned supply chain models to capture the attitude of the manufacturers and the retailers towards disruption risks.
- We propose a weighted performance measure to evaluate different supply chain disruptions.
- Different transportation modes are considered in the model (see also, e.g., Dong, Zhang, and Nagurney (2002) and Dong, Zhang, Yan, and Nagurney (2005)). In the multimodal transportation supply chain, alternative transportation modes can be used in the case of the failure of a transportation mode. Indeed, many authors have emphasized that redundancy needs to be considered in the design of supply chains in order to prevent supply chain disruptions. For example, Wilson (2007) used a system dynamic simulation to study the relationship between transportation disruptions and supply chain performance. The author found that the existence of transportation alternatives significantly improved supply chain performance in the case of transportation disruptions.

In this paper, we assume that the probability distributions of the disruption related cost parameters are known. This assumption is not unreasonable given today's advanced information technology and increasing awareness of the risks among managers. A great deal of disruption related information can be obtained from a careful examination and abstraction of the relevant data sources. Specifically, as indicated by Sheffi (2005) on page 55, "... as investigation boards and legal proceedings have revealed, in many cases relevant data are on the record but not funneled into a useful place or not analyzed to bring out the information in the data." Moreover, Holmgren (2007) also discussed ways to improve prediction of disruptions, using, for example: historical data analysis, mathematical modeling, and expert judgments. Furthermore, we assume that the random cost parameters are independent.

The organization of this paper is as follows. In Section 2, we present the model of a supply chain network faced with (possible) disruptions and in the case of random demands and multiple transportation modes. In Section 3, we provide a definition of a weighted supply chain performance measure with consideration of robustness. In Section 4, we present numerical examples in order to illustrate the model and concepts introduced in this paper. The paper concludes with Section 5, which summarizes the results obtained and provides suggestions for future research.

2. The Supply Chain Model with Disruption Risks and Random Demands

The topology of the supply chain network is depicted in Figure 1.

The supply chain model consists of m manufacturers, with a typical manufacturer denoted by i, n retailers with a typical retailer denoted by j, and o demand markets with a typical demand market denoted by k. Furthermore, we assume that there are g transportation modes from manufacturers to retailers, with a typical mode denoted by u and there are h transportation modes between retailers and demand markets, with a typical mode denoted by v. Typical transportation modes may include trucking, rail, air, sea, etc. By allowing multiple modes of transportation between successive tiers of the supply chain we also generalize the earlier models of Dong, Zhang, and Nagurney (2002) and Dong, Zhang, Yan, and Nagurney (2005).

Manufacturers are assumed to produce a homogeneous product, which can be purchased by retailers, who, in turn, make the product available to demand markets. Each process in the supply chain is associated with some random parameters that affect the cost functions. The relevant notation is summarized in Table 1.

Table 1: Notation for the Supply Chain Network Model

Notation	**Definition**
q	vector of the manufacturers' production outputs with components: $q_1, \ldots, q_m$
Q^1	mng-dimensional vector of product shipments between manufacturers and retailers via the transportation modes with component iju denoted by q_{ij}^u
Q^2	noh-dimensional vector of product shipments between retailers and demand markets via the transportation modes with component jkv denoted by q_{jk}^v
α	m-dimensional vector of nonnegative random parameters with α_i being the random parameter associated with the production cost of manufacturer i and the corresponding cumulative distribution function is given by $\mathcal{F}_i(\alpha_i)$
β	mng-dimensional vector of nonnegative random parameters with β_{ij}^u being the random parameter associated with the transportation cost of manufacturer i and retailer j via mode u and the corresponding cumulative distribution function is given by $\mathcal{F}_{ij}^u(\beta_{ij}^u)$
η	n-dimensional vector of nonnegative random parameters with η_j being the random parameter associated with the handling cost of retailer j and the corresponding cumulative distribution function is given by $\mathcal{F}_j(\eta_j)$
γ	n-dimensional vector of shadow prices associated with the retailers with component j denoted by γ_j
θ	m-dimensional vector of nonnegative weights with θ_i reflecting manufacturer i's attitude towards disruption risks
ϖ	n-dimensional vector of nonnegative weights with ϖ_j reflecting retailer j's attitude towards disruption risks
$f_i(q, \alpha_i) \equiv f_i(Q^1, \alpha_i)$	production cost of manufacturer i with random parameter α_i
$\hat{F}_i(q) \equiv \hat{F}_i(Q^1)$	expected production cost function of manufacturer i with marginal production cost with respect to q_{ij}^u denoted by $\frac{\partial \hat{F}_i(Q^1)}{\partial q_{ij}^u}$
$VF_i(Q^1)$	variance of the production cost of manufacturer i with marginal with respect to q_{ij}^u denoted by $\frac{\partial VF_i(Q^1)}{\partial q_{ij}^u}$

Notation	Definition
$c_{ij}^u(q_{ij}^u, \beta_{ij}^u)$	transaction cost between manufacturer i and retailer j via transportation mode u with the random parameter β_{ij}^u
$\hat{C}_{ij}^u(q_{ij}^u)$	expected transaction cost between manufacturer i and retailer j via transportation mode u with marginal transaction cost denoted by $\frac{\partial \hat{C}_{ij}^u(q_{ij}^u)}{\partial q_{ij}^u}$
$VC_{ij}^u(q_{ij}^u)$	variance of the transaction cost between manufacturer i and retailer j via transportation mode u with marginal denoted by $\frac{\partial VC_{ij}^u(q_{ij}^u)}{\partial q_{ij}^u}$
$c_j(Q^1, Q^2, \eta_j)$	handling cost of retailer j with random parameter η_j
$\hat{C}_j^1(Q^1, Q^2)$	expected handling cost of retailer j with marginal handling cost with respect to q_{ij}^u denoted by $\frac{\partial \hat{C}_j^1(Q^1,Q^2)}{\partial q_{ij}^u}$ and the marginal handling cost with respect to q_{jk}^v denoted by $\frac{\partial \hat{C}_j^1(Q^1,Q^2)}{\partial q_{jk}^v}$
$VC_j^1(Q^1, Q^2)$	variance of the handling cost of retailer j with marginal with respect to q_{ij}^u denoted by $\frac{\partial VC_j^1(Q^1,Q^2)}{\partial q_{ij}^u}$ and the marginal with respect to q_{jk}^v denoted by $\frac{\partial VC_j^1(Q^1,Q^2)}{\partial q_{jk}^v}$
$c_{jk}^v(Q^2)$	unit transaction cost between retailer j and demand market k via transportation mode v
$d_k(\rho_3)$	random demand at demand market k with expected value $\hat{d}_k(\rho_3)$
ρ_3	vector of prices of the product at the demand markets with ρ_{3k} denoting the demand price at demand market k

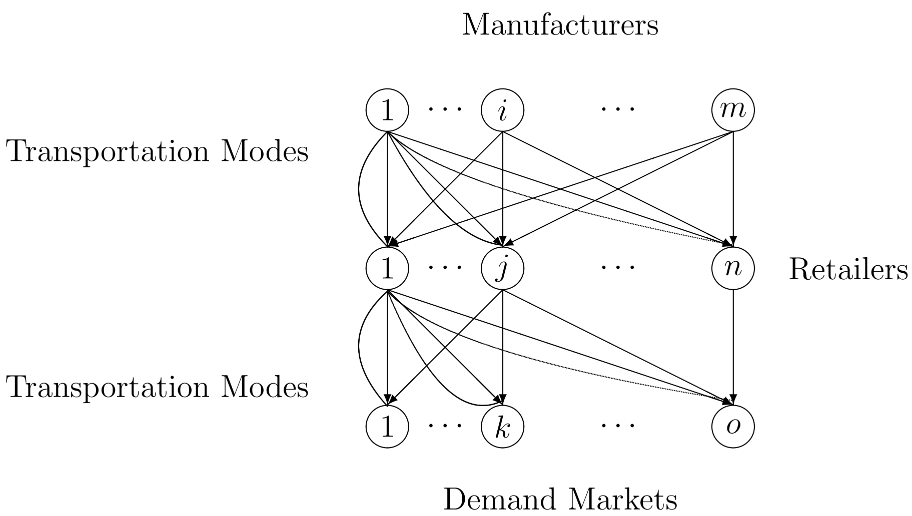

Figure 1: The Multitiered Network Structure of the Supply Chain

2.1 The Behavior of the Manufacturers

We assume a homogeneous product economy meaning that all manufacturers produce the same product which is then shipped to the retailers, who, in turn, sell the product to the demand markets.

Since the total amount of the product shipped from a manufacturer via different transportation modes has to be equal to the amount of the production of each manufacturer, we have the following relationship between the production of manufacturer i and the shipments to the retailers:

$$q_i = \sum_{j=1}^{n} \sum_{u=1}^{g} q_{ij}^u, \; i = 1, \ldots, m. \tag{1}$$

We assume that disruptions will affect the production processes of manufacturers, the impact of which is reflected in the production cost functions. For each manufacturer i, there is a random parameter α_i that reflects the impact of disruption to his production cost function. The expected production cost function is given by:

$$\hat{F}_i(Q^1) \equiv \int_{\alpha_i} f_i(Q^1, \alpha_i) d\mathcal{F}_i(\alpha_i), \quad i = 1, \ldots, m. \tag{2}$$

We further denote the variance of the above production cost function as $VF_i(Q^1)$ where $i = 1, \ldots, m$.

As noted earlier, we assume that each manufacturer has g types of transportation modes available to ship the product to the retailers, the cost of which is also subject to disruption impacts. The expected transportation cost function is given by:

$$\hat{C}^u_{ij}(q^u_{ij}) \equiv \int_{\beta^u_{ij}} c^u_{ij}(q^u_{ij}, \beta^u_{ij}) d\mathcal{F}^u_{ij}(\beta^u_{ij}), \; i = 1, \ldots, m; \; j = 1, \ldots, n; \; u = 1, \ldots, g. \tag{3}$$

We further denote the variance of the above transportation cost function as $VC^u_{ij}(Q^1)$ where $i = 1, \ldots, m$; $j = 1, \ldots, n$; $u = 1, \ldots, g$.

It is well-known in economics that variance may be used to measure risk (see, e.g., Silberberg and Suen (2000) with Tomlin (2006) using such an approach to study risks in applications to supply chains). Therefore, we assign a nonnegative weight θ_i to the variance of the cost functions for each manufacturer to reflect his attitude towards disruption risks. The larger the weight is, the larger the penalty a manufacturer imposes on the risk, and, therefore, the more risk-averse the manufacturer is.

Let ρ^{u*}_{1ij} denote the price charged for the product by manufacturer i to retailer j when the product is shipped via transportation mode u. Hence, manufacturers can price according to their locations as well as according to the transportation modes utilized. Each manufacturer faces two objectives: to maximize his expected profit and to minimize the disruption risks adjusted by his risk attitude. Therefore, the objective function for manufacturer i; $i = 1, \ldots, m$ can be expressed as follows:

$$\text{Maximize} \quad \sum_{j=1}^{n} \sum_{u=1}^{g} \rho^{u*}_{1ij} q^u_{ij} - \hat{F}_i(Q^1) - \sum_{j=1}^{n} \sum_{u=1}^{g} \hat{C}^u_{ij}(q^u_{ij}) - \theta_i \left[VF_i(Q^1) + \sum_{j=1}^{n} \sum_{u=1}^{g} VC^u_{ij}(q^u_{ij}) \right] \tag{4}$$

subject to:

$$q^u_{ij} \geq 0, \; \text{for all } i, j, \; \text{and } u.$$

The first term in (4) represents the revenue. The second term is the expected disruption related production cost. The third term is the expected disruption related transportation cost. The fourth term is the cost of disruption risks adjusted by each manufacturer's attitude.

We assume that, for each manufacturer, the production cost function and the transaction cost function without disruptions are continuously differentiable and convex. It is easy to verify that $\hat{F}_i(Q^1)$, $VF_i(Q^1)$, $\hat{C}^u_{ij}(q^u_{ij})$, and $VC^u_{ij}(q^u_{ij})$ are also continuously differentiable and convex. Furthermore, we assume that manufacturers compete in a noncooperative fashion in the sense of Nash (1950, 1951). Hence, the optimality conditions for all manufacturers simultaneously (cf. Bazaraa, Sherali, and Shetty (1993) and Nagurney (1999)) can be expressed as the following variational inequality: determine $Q^{1*} \in R^{mng}_+$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}\left[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q^u_{ij}} + \frac{\partial \hat{C}^u_{ij}(q^{u*}_{ij})}{\partial q^u_{ij}} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q^{u*}_{ij}} + \frac{\partial VC^u_{ij}(q^{u*}_{ij})}{\partial q^{u*}_{ij}}) - \rho^{u*}_{1ij}\right]$$
$$\times[q^u_{ij} - q^{u*}_{ij}] \geq 0, \quad \forall Q^1 \in R^{mng}_+. \tag{5}$$

2.2 The Behavior of the Retailers

The retailers, in turn, are involved in transactions both with the manufacturers and the demand markets since they must obtain the product to deliver to the consumers at the demand markets.

Let ρ^{v*}_{2jk} denote the price charged for the product by retailer j to demand market k when the product is shipped via transportation mode v. Hence, retailers can price according to their locations as well as according to the transportation modes utilized. This price is determined endogenously in the model along with the prices associated with the manufacturers, that is, the ρ^{u*}_{1ij}, for all i, j and u. We assume that certain disruptions will affect the retailers' handling processes (e. g., the storage and display processes). An additional random risk/disruption related random parameter η_j is associated with the handling cost of retailer j. Recall that we also assume that there are h types of transportation modes available to each retailer for shipping the product to the demand markets. The expected handling cost is given by:

$$\hat{C}_j^1(Q^1, Q^2) \equiv \int_{\eta_j} c_j(Q^1, Q^2, \eta_j) d\mathcal{F}_j(\eta_j), \; j = 1, \ldots, n. \tag{6}$$

We further denote the variance of the above handling cost function as $VC_j^1(Q^1, Q^2)$ where $j = 1, \ldots, n$.

Furthermore, similar to the case for the manufacturers, we associate a nonnegative weight ϖ_j to the variance of each retailer's handling cost according to his attitude towards risk. Each retailer faces two objectives: to maximize his expected profit and to minimize the disruption risks adjusted by his risk attitude. Therefore, the objective function for retailer j; $j = 1, \ldots, n$ can be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{o} \sum_{v=1}^{h} \rho_{2jk}^{v*} q_{jk}^v - \hat{C}_j^1(Q^1, Q^2) - \sum_{i=1}^{m} \sum_{u=1}^{g} \rho_{1ij}^{u*} q_{ij}^u - \varpi_j VC_j^1(Q^1, Q^2) \tag{7}$$

subject to:

$$\sum_{k=1}^{o} \sum_{v=1}^{h} q_{jk}^v \leq \sum_{i=1}^{m} \sum_{u=1}^{g} q_{ij}^u \tag{8}$$

and the nonnegativity constraints: $q_{ij}^u \geq 0$ for all i, j, and u; $q_{jk}^v \geq 0$ for all j, k, and v.

Objective function (7) expresses that the difference between the revenues minus the expected handling cost, the payout to the manufacturers and the weighted disruption risk is to be maximized. Constraint (8) states that retailers cannot purchase more product from a retailer than is available in stock.

As noted in Table 1, γ_j is the Lagrange multiplier associated with constraint (8) for retailer j. Furthermore, we assume that, for each retailer, the handling cost without disruptions is continuously differentiable and convex. It is easy to verify that $\hat{C}_j^1(Q^1, Q^2)$ and $VC_j^1(Q^1, Q^2)$ are also continuously differentiable and convex. We assume that retailers compete with one another in a noncooperative manner, seeking to determine their optimal shipments from the manufacturers and to the demand markets. The optimality conditions for all retailers simultaneously coincide with the solution of the following variational inequality: determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R_+^{mng+noh+n}$

satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}\left[\frac{\partial \hat{C}_j^1(Q^{1*},Q^{2*})}{\partial q_{ij}^u}+\rho_{1ij}^{u*}+\varpi_j\frac{\partial VC_j^1(Q^{1*},Q^{2*})}{\partial q_{ij}^u}-\gamma_j^*\right]\times\left[q_{ij}^u-q_{ij}^{u*}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}\left[-\rho_{2jk}^{v*}+\gamma_j^*+\frac{\partial \hat{C}_j^1(Q^{1*},Q^{2*})}{\partial q_{jk}^v}+\varpi_j\frac{\partial VC_j^1(Q^{1*},Q^{2*})}{\partial q_{jk}^v}\right]\times\left[q_{jk}^v-q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g}q_{ij}^{u*}-\sum_{k=1}^{o}\sum_{v=1}^{h}q_{jk}^{v*}\right]\times\left[\gamma_j-\gamma_j^*\right]\geq 0,\quad \forall (Q^1,Q^2,\gamma)\in R_+^{mng+noh+n}. \quad (9)$$

2.3 The Market Equilibrium Conditions

We now turn to a discussion of the market equilibrium conditions. Subsequently, we construct the equilibrium condition for the entire supply chain network.

The equilibrium conditions associated with the product shipments that take place between the retailers and the consumers are the *stochastic economic equilibrium* conditions, which, mathematically, take on the following form: for any retailer with associated demand market k; $k=1,\dots,o$:

$$\hat{d}_k(\rho_3^*)\begin{cases}\leq \sum_{j=1}^{o}\sum_{v=1}^{h}q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^*=0,\\ = \sum_{j=1}^{o}\sum_{v=1}^{h}q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^*>0,\end{cases} \quad (10a)$$

$$\rho_{2jk}^{v*}+c_{jk}^v(Q^{2*})\begin{cases}\geq \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*}=0,\\ = \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*}>0.\end{cases} \quad (10b)$$

Conditions ($10a$) state that, if the expected demand price at demand market k is positive, then the quantities purchased by consumers at the demand market from the retailers in the aggregate is equal to the demand at demand market k. Conditions ($10b$) state, in turn, that in equilibrium, if the consumers at demand market k purchase the product from retailer j via transportation mode v, then the price charged by the retailer for the product plus the unit transaction cost is equal to the price that the consumers are willing to pay for the product. If the price plus the unit transaction cost exceeds the price the consumers are willing to pay at the demand market then there will be no transaction between the retailer and demand market via that transportation mode.

Equilibrium conditions ($10a$) and ($10b$) are equivalent to the following variational inequality problem, after summing over all demand markets: determine $(Q^{2*}, \rho_3^*) \in R_+^{noh+o}$ satisfying:

$$\sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*]$$

$$+\sum_{k=1}^{o}\sum_{j=1}^{n}\sum_{v=1}^{h}(\rho_{2jk}^{v*} + c_{jk}^{v}(Q^{2*}) - \rho_{3k}^*) \times [q_{jk}^{v} - q_{jk}^{v*}] \geq 0, \ \forall \rho_3 \in R_+^o, \forall Q^2 \in R_+^{noh}, \quad (11)$$

where ρ_3 is the o-dimensional vector with components: $\rho_{31}, \ldots, \rho_{3o}$ and Q^2 is the noh-dimensional vector.

Remark: In this paper, we are interested in the cases where the expected demands are positive, that is, $\hat{d}_k(\rho_3) > 0$, $\forall \rho_3 \in R_+^o$ for $k = 1, \ldots, o$. Furthermore, we assume that the unit transaction costs: $c_{jk}^v(Q^2) > 0$, $\forall j, k$, $\forall Q^2 \neq 0$.

Under the above assumptions, we have that $\rho_{3k}^* > 0$ and $\hat{d}_k(\rho_3^*) = \sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h q_{jk}^{v*}$, $\forall k$. This can be shown by contradiction. If there exists a $\bar{k}$ where $\rho_{3\bar{k}}^* = 0$, then according to (10a) we have that $\sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h q_{jk}^{v*} \geq \hat{d}_{\bar{k}}(\rho_3^*) > 0$. Hence, there exists at least a $(j,\bar{k})$ pair such that $q_{j\bar{k}}^{v*} > 0$, which means that $c_{j\bar{k}}^v(Q^{2*}) > 0$ by assumption. From conditions (10b), we have that $\rho_{2j\bar{k}}^{v*} + c_{j\bar{k}}^v(Q^{2*}) = \rho_{3\bar{k}} > 0$, which leads to a contradiction.

2.4 The Equilibrium Conditions of the Supply Chain

In equilibrium, we must have that the optimality conditions for all manufacturers, as expressed by (4), the optimality conditions for all retailers, as expressed by (9), and as well as the equilibrium conditions for all the demand markets, as expressed by (11), must hold simultaneously (see also Nagurney, Cruz, Dong, and Zhang (2005)). Hence, the product shipments of the manufacturers with the retailers must be equal to the product shipments that retailers accept from the manufacturers. We now formally state the equilibrium conditions for the entire supply chain network as follows:

Definition 1: Supply Chain Network Equilibrium with Uncertainty and Random Demands

The equilibrium state of the supply chain network with disruption risks and random demands is one where the flows of the product between the tiers of the decision-makers coincide and the flows and prices satisfy the sum of conditions (4), (9), and (11).

The summation of inequalities (4), (9), and (11), after algebraic simplification, yields the following result (see also Nagurney (1999, 2006a)).

Theorem 1: Variational Inequality Formulation

A product shipment and price pattern $(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in R_+^{mng+noh+n+o}$ *is an equilibrium pattern of the supply chain model according to Definition 1, if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u})$$

$$+\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*] \times [q_{ij}^u - q_{ij}^{u*}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{g}[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\gamma_j^* + c_{jk}^v(Q^{2*}) - \rho_{3k}^*] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right] \times \left[\gamma_j - \gamma_j^*\right] + \sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}. \tag{12}$$

For easy reference in the subsequent sections, variational inequality problem (12) can be rewritten in standard variational inequality form (cf. Nagurney (1999)) as follows: determine $X^* \in \mathcal{K}$:

$$\langle F(X^*), X - X^* \rangle \geq 0, \ \ \forall X \in \mathcal{K} \equiv R_+^{mng+noh+n+o}, \tag{13}$$

where $X \equiv (Q^1, Q^2, \gamma, \rho_3)$, $F(X) \equiv (F_{iju}, F_{jkv}, F_j, F_k)_{i=1,\cdots,m;j=1,\cdots,n;k=1,\cdots,o;u=1,\cdots,g;v=1,\cdots,h}$, and the specific components of F are given by the functional terms preceding the multiplication signs in (12). The term $< \cdot, \cdot >$ denotes the inner product in N-dimensional Euclidean space.

Note that the equilibrium values of the variables in the model (which can be determined from the solution of either variational inequality (12) or (13)) are: the equilibrium product shipments between manufacturers and the retailers given by Q^{1*}, and the equilibrium product shipments transacted between the retailers and the demand markets given by Q^{2*}, as well as the equilibrium prices: ρ_3^* and γ^*. We now discuss how to recover the prices ρ_1^* associated with the top tier of nodes of the supply chain network and the prices ρ_2^* associated with the middle tier.

First, note that, from (5), we have that if $q_{ij}^{u*} > 0$, then price $\rho_{1ij}^{u*} = \frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^{u*}} + \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^{u*}})$. One the other hand, from (9), it follows that, if $q_{jk}^{v*} > 0$, the price $\rho_{2j}^* = \gamma_j^* + \frac{\partial \hat{C}_j^1(Q^{1*},Q^{2*})}{\partial q_{jk}^v} + \varpi_j \frac{\partial VC_j^1(Q^{1*},Q^{2*})}{\partial q_{jk}^v}$. These expressions can be utilized to obtain all such prices for all modes and decision-makers.

3. A Weighted Supply Chain Performance Measure

In this Section, we first propose a supply chain network performance measure. Then, we provide the definition of supply chain network robustness, and follow with the definition for a weighted supply chain performance measure.

3.1 A Supply Chain Network Performance Measure

Recently, Qiang and Nagurney (2008) (see also Nagurney and Qiang (2007a, b, c)) proposed a network performance measure, which captures flows, costs, and behavior under network equilibrium conditions. Based on the measure in the above paper(s), we propose the following definition of a supply chain network performance measure.

Definition 2: The Supply Chain Network Performance Measure

The supply chain network performance measure, $\mathcal{E}$, for a given supply chain, and expected demands: $\hat{d}_k$; $k = 1, 2, \ldots, o$, is defined as follows:

$$\mathcal{E} \equiv \frac{\sum_{k=1}^{o} \frac{\hat{d}_k}{\rho_{3k}}}{o}, \tag{14}$$

where o is the number of demand markets in the supply chain network, and $\hat{d}_k$ and ρ_{3k} denote, respectively, the expected equilibrium demand and the equilibrium price at demand market k.

Note that the equilibrium price is equal to the unit production and transaction costs plus the weighted marginal risks for producing and transacting one unit from the manufacturers to the demand markets (see also Nagurney (2006b)). According to the above performance measure, a supply chain network performs well in network equilibrium if, on the average, and across all demand markets, a large demand can be satisfied at a low price. Therefore, in this paper, we apply the above performance measure to assess the robustness of particular supply chain networks. From the discussion in Section 2.3, we have that $\rho_{3k} > 0$, $\forall k$. Therefore, the above definition is well-defined.

Furthermore, since each individual may have different opinions as to the risks, we need a "basis" to compare supply chain performance under different risk attitudes and to understand how risk attitudes affect the performance of a supply chain. Hence, we define $\mathcal{E}^0$ as the supply chain performance measure where the $\hat{d}_k$ and the ρ_{3k}; $k = 1, \ldots, o$, are obtained by assuming that the weights that reflect the manufacturers and the retailers' attitudes towards the disruption risks are zero. This definition excludes individuals' subjective differences in a supply chain and, with this definition, we are ready to study supply chain network robustness.

3.2 Supply Chain Robustness Measurement

Robustness has a broad meaning and is often couched in different settings. Generally speaking, robustness means that the system performs well when exposed to uncertain future conditions and perturbations (cf. Bundschuh, Klabjan, and Thurston (2003), Snyder (2003), and Holmgren (2007)).

Therefore, we propose the following rationale to assess the robustness of a supply chain: assume that all the random parameters take on a given threshold probability value; say, for example, 95%. Moreover, assume that all the cumulative distribution functions for random parameters have inverse functions. Hence, we have that: $\alpha_i = \mathcal{F}_i^{-1}(.95)$, for $i = 1, \ldots, m$; $\beta_{ij}^u = \mathcal{F}_{ij}^{u^{-1}}(.95)$, for $i = 1, \ldots, m$; $j = 1, \ldots, n$, and so on. With the disruption related parameters given, we can calculate the supply chain performance measure according to the definition given by (14). Let $\mathcal{E}_w$ denote the supply chain performance measure with random parameters fixed at a certain level as described above. For example, when $w = 0.95$, $\mathcal{E}_w$ is the supply chain performance with all the random risk parameters fixed at the value of a 95% probability level. Then, the supply chain network robustness measure, $\mathcal{R}$, is given by the following:

$$\mathcal{R} = \mathcal{E}^0 - \mathcal{E}_w, \tag{15}$$

where $\mathcal{E}^0$ gauges the supply chain performance based on the model introduced in Section 2, but with weights related to risks being zero.

$\mathcal{E}^0$ examines the "base" supply chain performance while $\mathcal{E}_w$ assesses the supply chain performance measure at some prespecified uncertainty level. If their difference is small, a supply chain maintains its functionality well and we consider the supply chain to be robust at the threshold disruption level. Hence, the lower the value of $\mathcal{R}$, the more robust a supply chain is.

Notably, the above robustness definition has implications for network resilience as well. *Resilience* is a general and conceptual term, which is hard to quantify. McCarthy (2007) defined resilience "... as the ability of a system to recover from adversity, either back to its original state or an adjusted state based on new requirements, ...". For a comprehensive discussion of resilience, please refer to the Critical Infrastructure Protection Program (2007). Because our supply chain measure is based on the network equilibrium model, a network that is qualified as being robust according to our measure is also resilient provided that its performance after experiencing the disruption(s) is close to the "original value." Interestingly, this idea is in agreement with Hansson and Helgesson (2003), who proposed that robustness can be treated as a special case of resilience.

3.3 A Weighted Supply Chain Performance Measure

Note that different supply chains may have different requirements regarding the performance and robustness concepts introduced in the previous sections. For example, in the case of a supply chain of a toy product one may focus on how to satisfy demand in the most cost efficient way and not care too much about supply chain robustness. A medical/healthcare supply chain, on the other hand, may have a requirement that the supply chain be highly robust when faced with uncertain conditions. Hence, in order to be able to examine and to evaluate the different application-based supply chains from both perspectives, we now define a weighted supply chain performance measure as follows:

$$\hat{\mathcal{E}} = (1-\epsilon)\mathcal{E}^0 + \epsilon(-\mathcal{R}), \tag{16}$$

where $\epsilon \in [0, 1]$ is the weight that is placed on the supply chain robustness.

When ϵ is equal to 1, the performance of a supply chain hinges only on the robustness measure, which may be the case for a medical/healthcare supply chain, noted above. In contrast, when ϵ is equal to 0, the performance of the supply chain depends solely on how well it can satisfy demands at low prices. The supply chain of a toy product in the above discussion falls into this category.

4. Examples

The supply chain network topology for the numerical examples is depicted in Figure 2 below. There are assumed to be two manufacturers, two retailers, and two demand markets. There are two modes of transportation available between each manufacturer and retailer pair and between each retailer and demand market pair. These examples are solved by the modified projection method of Korpelevich (1977); see also, e.g., Nagurney (2006a).

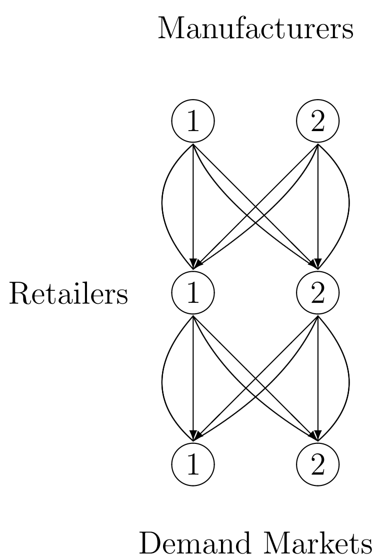

Figure 2: The Supply Chain Network for the Numerical Examples

Example 1

In the first example, for illustration purposes, we assumed that all the random parameters followed uniform distributions. The relevant parameters are as follows:

$\alpha_i \sim [0,2]$ for $i=1,2$; $\beta_{ij}^u \sim [0,1]$ for $i=1,2$; $j=1,2$; $u=1,2$; $\eta_j \sim [0,3]$ for $j=1,2$.

We further assumed that the demand functions followed a uniform distribution given by $[200-2\rho_{3k}, 600-2\rho_{3k}]$, for $k=1,2$. Hence, the expected demand functions are:

$$\hat{d}_k(\rho_3) = 400 - 2\rho_{3k}, \quad \text{for } k=1,2.$$

The production cost functions for the manufacturers are given by:

$$f_1(Q^1,\alpha_1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2\alpha_1(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u),$$

$$f_2(Q^1,\alpha_2) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2\alpha_2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u).$$

The expected production cost functions for the manufacturers are given by:

$$\hat{F}_1(Q^1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u),$$

$$\hat{F}_2(Q^1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u).$$

The variances of the production cost functions for the manufacturers are given by:

$$VF_1(Q^1) = \frac{4}{3}(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2,$$

$$VF_2(Q^1) = \frac{4}{3}(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$c_{ij}^1(q_{ij}^1,\beta_{ij}^1) = .5(q_{ij}^1)^2 + 3.5\beta_{ij}^1 q_{ij}^1, \text{ for } i=1,2; j=1,2,$$

$$c_{ij}^2(q_{ij}^2,\beta_{ij}^2) = (q_{ij}^2)^2 + 5.5\beta_{ij}^2 q_{ij}^2, \text{ for } i=1,2; j=1,2.$$

The expected transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$\hat{C}^1_{ij}(q^1_{ij}) = .5(q^1_{ij})^2 + 1.75q^1_{ij}, \text{ for } i = 1, 2; j = 1, 2,$$

$$\hat{C}^2_{ij}(q^2_{ij}) = .5(q^2_{ij})^2 + 2.75q^2_{ij}, \text{ for } i = 1, 2; j = 1, 2.$$

The variances of the transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$VC^1_{ij}(q^1_{ij}) = 1.0208(q^1_{ij})^2, \text{ for } i = 1, 2; j = 1, 2,$$

$$VC^2_{ij}(q^2_{ij}) = 2.5208(q^2_{ij})^2, \text{ for } i = 1, 2; j = 1, 2.$$

The handling costs of the retailers, in turn, are given by:

$$c_j(Q^1, Q^2, \eta_j) = .5(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij})^2 + \eta_j(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij}), \text{ for } j = 1, 2.$$

The expected handling costs of the retailers are given by:

$$\hat{C}^1_j(Q^1, Q^2) = .5(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij})^2 + 1.5(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij}), \text{ for } j = 1, 2.$$

The variance of the handling costs of the retailers are given by:

$$VC_j(Q^1, Q^2) = \frac{3}{4}(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij})^2, \text{ for } j = 1, 2.$$

The unit transaction costs from the retailers to the demand markets are given by:

$$c^1_{jk}(Q^2) = .3q^1_{jk}, \quad \text{for } j = 1, 2; \ k = 1, 2,$$

$$c^2_{jk}(Q^2) = .6q^2_{jk}, \quad \text{for } j = 1, 2; \ k = 1, 2.$$

We assumed that the manufacturers and the retailers placed zero weights on the disruption risks as discussed in Section 3.1 to compute $\mathcal{E}^0$.

In the equilibrium, under the expected costs and demands, we have that the equilibrium shipments between manufacturers and retailers are: $q_{ij}^{1*} = 8.5022$, for $i = 1,2$; $j = 1,2$; $q_{ij}^{2*} = 3.7511$, for $i = 1,2$; $j = 1,2$; whereas the equilibrium shipments between the retailers and the demand markets are: $q_{jk}^{1*} = 8.1767$, for $j = 1,2$; $k = 1,2$; $q_{jk}^{2*} = 4.0767$, for $j = 1,2$; $k = 1,2$. Finally, the equilibrium prices are: $\rho_{31}^* = \rho_{32}^* = 187.7466$ and the expected equilibrium demands are: $\hat{d}_1 = \hat{d}_2 = 24.5068$. The supply chain performance measure is equal to $\mathcal{E}^0 = 0.1305$. Now, assume that $w = .95$; that is, all the random cost parameters are fixed at a 95% probability level. The resulting supply chain performance measure is computed as $\mathcal{E}_w$=0.1270. If we let $\epsilon = .5$ (cf. (16)), which means that we place equal emphasis on performance and robustness of the supply chain, the weighted supply chain performance measure is $\hat{\mathcal{E}} = 0.0635$.

Example 2

For the same network structure and cost and demand functions, we now assume that the relevant parameters are changed as follows: $\alpha_i \sim [0,4]$ for $i = 1,2$; $\beta_{ij}^u \sim [0,2]$ for $i = 1,2$; $j = 1,2$; $u = 1,2$; $\eta_j \sim [0,6]$ for $j = 1,2$.

In the equilibrium, under the expected costs and demands, we have that the equilibrium shipments between manufacturers and retailers are now: $q_{ij}^{1*} = 8.6008$, for $i = 1,2$; $j = 1,2$; $q_{ij}^{2*} = 3.3004$, for $i = 1,2$; $j = 1,2$; whereas the equilibrium shipments between the retailers and the demand markets are: $q_{jk}^{1*} = 7.9385$, for $j = 1,2$; $k = 1,2$; $q_{jk}^{2*} = 3.9652$, for $j = 1,2$; $k = 1,2$. Finally, the equilibrium prices are: $\rho_{31}^* = \rho_{32}^* = 188.0963$ and the expected equilibrium demands are: $\hat{d}_1 = \hat{d}_2 = 23.8074$. The supply chain performance measure is equal to $\mathcal{E}^0 = 0.1266$. Similar to the above example, let us assume that $w = .95$; that is, all the random cost parameters are fixed at a 95% probability level. The resulting supply chain performance measure is now: $\mathcal{E}_w$=0.1194. If we let $\epsilon = .5$, the weighted supply chain performance measure is $\hat{\mathcal{E}} = 0.0597$.

Observe that first example leads to a better measure of performance since the uncertain parameters do not have as great of an impact as in the second one for the cost functions under the given threshold level.

5. Summary and Conclusions

In this paper, we developed a novel supply chain network model to study the demand-side as well as the supply-side risks, with the demand being random and the supply-side risks modeled as uncertain parameters in the underlying cost functions. This supply chain model generalizes several existing models by including multiple transportation modes from the manufacturers to the retailers, and from the retailers to the demand markets. We also proposed a weighted supply chain performance and robustness measure based on our recently derived network performance/efficiency measure and illustrated the supply chain network model through numerical examples for which the equilibrium prices and product shipments were computed and robustness analyses conducted. For future research, we plan on constructing further comprehensive metrics in order to evaluate supply chain network performance and to also apply the results in this paper to empirically-based supply chain networks in different industries.

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