Pharmaceutical Supply Chain Networks with Outsourcing Under Price and Quality Competition

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Abstract: In this paper, we present a pharmaceutical supply chain network model with outsourcing under price and quality competition, in both equilibrium and dynamic versions. We consider a pharmaceutical firm that is engaged in determining the optimal pharmaceutical flows associated with its supply chain network activities in the form of manufacturing and distribution. In addition to multimarket demand satisfaction, the pharmaceutical firm seeks to minimize its total cost, with the associated function also capturing the firm's weighted disrepute cost caused by possible quality issues associated with the contractors. Simultaneously, the contractors, who compete with one another in a noncooperative manner in prices a la Bertrand, and in quality, seek to secure manufacturing and distribution of the pharmaceutical product from the pharmaceutical firm. This game theory model allows for the determination of the optimal pharmaceutical product flows associated with the supply chain in-house and outsourcing network activities and provides the pharmaceutical firm with its optimal make-or-buy decisions and the optimal contractor-selections. We state the governing equilibrium conditions and derive the equivalent variational inequality formulation. We then propose dynamic adjustment processes for the evolution of the product flows, the quality levels, and the prices, along with stability analysis results. The algorithm yields a discretization of the continuous-time adjustment processes. We present convergence results and compute solutions to numerical examples to illustrate the generality and applicability of the framework.

Keywords: outsourcing, pharmaceutical products, healthcare, supply chains, supply chain networks, quality, competition, game theory, variational inequalities, dynamical systems

1. Introduction

Outsourcing is defined as the behavior of moving some of a firm's responsibilities and/or internal processes, such as product design or manufacturing, to a third party company (Chase, Jacobs, and Aquilano (2004)). Outsourcing of manufacturing/production has long been noted in operations and supply chain management in such industries as computer engineering and manufacturing, financial analysis, fast fashion apparel, and pharmaceuticals (cf. Austin, Hills, and Lim (2003), Nagurney and Yu (2011), and Hayes et al. (2005)). Evidence indicates that, in the case of the pharmaceutical industry, outsourcing results in the reduction of companies' overall costs (Garofolo and Garofolo (2010)), may increase operational efficiency and agility (Klopack (2000) and John (2006)), and allows such firms to focus on their core competitive advantages (Sink and Langley (1997)). In addition, outsourcing may possibly bring benefits from supportive government policies (Zhou (2007)).

Due to the above noted benefits there is, currently, a tremendous shift from in-house manufacturing towards outsourcing in US pharmaceutical companies. For example, Pfizer has increased the outsourcing of its drug manufacturing to 17% from less than 10% (Wheelwright and Downey (2008)). In addition, some pharmaceutical firms outsource to multiple contractors. For example, Bristol-Myers Squibb has had contracts with Celltrion Inc., a Korean company, and with Lonza Biologics, a Swiss company (Bristol-Myers Squibb (2005)). It is estimated that the US market for outsourced pharmaceutical manufacturing is expanding at the rate of 10% to 12% annually (Olson and Wu (2011)). Up to 40% of the drugs that Americans take are now imported, and more than 80% of the active ingredients for drugs sold in the United States are outsourced (Economy In Crisis (2010)). Moreover, pharmaceutical companies are increasingly farming out activities other than production, such as research and development activities (Higgins and Rodriguez (2006)).

However, the nation's growing reliance on sometimes uninspected foreign pharmaceutical suppliers has raised public and governmental awareness and concern, with outsourcing firms being faced with quality-related risks (Helm (2006), Liu and Nagurney (2011a), and Liu and Nagurney (2011b)). In 2006, Chinese-made cold medicine contained a toxic substance used in antifreeze, which can cause death (Bogdanich and Hooker (2007)). In 2008, fake heparin made by one Chinese manufacturer not only led to recalls of drugs in over ten European countries (Payne (2008)), but also caused the deaths of 81 American citizens (The New York Times (2011)). Given the growing volume of global outsourcing, quality issues in outsourced products must be of paramount concern, especially since adulterated pharmaceuticals, which are ingested, may negatively affect a patient's recovery or may even result in loss of life.

Although the need for regulations for the quality control of overseas pharmaceuticals and ingredients is urgent, due to the limited resources of the US Food and Drug Administration (FDA) that are available for global oversight, pharmaceutical firms, as well as citizens, cannot rely on the FDA to ensure the quality of outsourced pharmaceuticals (The New York Times (2011)). At the same time, pharmaceutical firms are under increasing pressure from consumers, the government, the medical insurance industry, and even the FDA, to reduce the prices of their pharmaceuticals (Enyinda, Briggs, and Bachkar (2009)). Thus, pharmaceutical companies must be prepared to adopt best practices aimed at safeguarding the quality of their supply chain networks. A more comprehensive supply chain network model that captures contractor selection, the minimization of the disrepute of the pharmaceutical firm, as well as the competition among contractors, is an imperative.

In this paper, we develop a pharmaceutical supply chain network model utilizing a game theory approach which takes into account the quality concerns in the context of global outsourcing. This model captures the behaviors of the pharmaceutical firm and its potential contractors with consideration of the transactions between them and the quality of the outsourced pharmaceutical product. The objective of each contractor is to maximize its total profit. The pharmaceutical firm seeks to minimize its total cost, which includes its weighted disrepute cost, which is influenced by the quality of the product produced by its contractors and the amount of product that is outsourced. The contractors compete with one another by determining the prices that they charge the pharmaceutical firm for manufacturing and delivering the product to the demand markets and the quality levels in order to maximize their profits.

We now provide a review of the relevant literature. We first note that only a small portion of the supply chain literature directly addresses and models the risk of quality and safety issues associated with outsourcing. Kaya (2011) considered an outsourcing model in which the supplier makes the quality decision and an in-house production model in which the manufacturer decides on the quality. Kaya and Özer (2009) employed vertical integration in their three-stage decision model to determine how the original equipment manufacturer's pricing strategy affects quality risk. The above two papers both used an effort function to capture the cost of quality in outsourcing. Balakrishnan, Mohan, and Seshadri (2008) considered quality risk in outsourcing as a recent supply chain phenomenon of outsourcing front-end business processes. In addition, Gray, Roth, and Tomlin (2008) proposed a metric of quality risk based on real data from the drug industry and provided empirical evidence that unobservability of quality risk causes the contract manufacturer to shirk on quality.

Obviously, in global manufacturing outsourcing, contractors do not only do production, but also are responsible for the sourcing of raw materials and conducting quality-related tasks such as inspections and incoming and outgoing quality control. They determine the quality of the materials that they purchase as well as the standard of their manufacturing activities. However, contract manufacturers may have less reason to be concerned with the quality (Amaral, Billington, and Tsay (2006)), since they may not have the same priorities as the original firms when making decisions about and trade-offs between cost and quality, which may lead them to expend less effort to ensure high quality.

Hence, quality should be quantified and incorporated into the make-or-buy, as well as the contractor-selection decision of pharmaceutical firms. If a contractor's poor quality product negatively affects the pharmaceutical firm's reputation, then outsourcing to that contractor will not be a wise choice.

Quality costs are defined as "costs incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved" (ASQC (1971) and BS (1990)). There are a variety of schemes by which quality costing can be implemented in organizations, as described in Juran and Gryna (1988) and Feigenbaum (1991). Based on the literature of quality cost, four categories of quality-related costs occur in the process of quality management, and are widely applied in organizations (see, e.g., Crosby (1979), Harrington (1987), Plunkett and Dale (1988), Juran and Gryna (1993), and Rapley, Prickett, and Elliot (1999)). They are prevention costs, appraisal costs, internal failure costs, and external failure costs. Quality cost is usually understood as the sum of the four categories of quality-related costs. Moreover, it is widely believed that the functions of the four quality-related costs are convex functions of the quality conformance level, which varies from a 0% defect-free level to a 100% defect-free level (see, e.g., Juran and Gryna (1988), Campanella (1990), Feigenhaum (1983), Porter and Rayner (1992), and Shank and Govindarajan (1994)).

The game theory supply chain network model developed in this paper is based on the following assumptions:

1. The pharmaceutical firm may contract the manufacturing and the delivery tasks to the contractors.

2. According to regulations (FDA (2002), U.S. Department of Health and Human Services, CDER, FDA, and CBER (2009), and the European Commission Health and Consumers Directorate (2010)) and the literature, before signing the contract, the pharmaceutical firm should have reviewed and evaluated the contractors' ability to perform the outsourcing tasks. Therefore, the production/distribution costs and the quality cost information of the contractors are assumed to be known by the firm.

3. In addition to paying the contractors, the pharmaceutical firm also pays the transaction cost and one category of the quality-related costs, the external failure costs. The transaction cost is the "cost of making each contract" (cf. Coase (1937) and also Aubert, Rivard, and Patry (1996)), which includes the costs of evaluating suppliers, negotiation costs, the monitoring and the enforcement of the contract in order to ensure the quality (Picot (1991), Franceschini (2003), Heshmati (2003), and Liu and Nagurney (2011a)). External failure costs are the compensation costs incurred when customers are unsatisfied with the quality of the products. The objective of the pharmaceutical firm is to minimize the total operational costs and transaction costs, along with the weighted individual disrepute.

4. For the in-house supply chain activities, it is assumed that the pharmaceutical firm can ensure a 100% perfect quality conformance level (see Schneiderman (1986) and Kaya (2011)).

5. The contractors, who produce for the same pharmaceutical firm, compete a la Bertrand (cf. Bertrand (1838)) by determining their optimal prices and quality levels in order to maximize their profits.

We develop both a static version of the model (at the equilibrium state) and also a dynamic one using the theory of projected dynamical systems (cf. Nagurney and Zhang (1996)). In our earlier research on pharmaceutical supply chain networks (cf. Masoumi, Yu, and Nagurney (2012)), we focused on Cournot competition (cf. Cournot (1838)), where firms selected their optimal product flows under perishability of the pharmaceutical product, and we did not consider outsourcing. Moreover in that work, the underlying dynamics were not identified. This paper adds to the growing literature on supply chain network equilibrium problems introduced by Nagurney, Dong, and Zhang (2002) (see also Nagurney et al. (2002), Dong, Zhang, and Nagurney (2004), Dong et al. (2005), and Cruz (2008)), with the inclusion of contractor behavior, Bertrand competition in prices, as well as quality competition, and dynamics.

This paper is organized as follows. In Section 2, we describe the decision-making behavior of the pharmaceutical firm and the competing contractors. We then develop the game theory model, state the equilibrium conditions, and derive the equivalent variational inequality formulation. We assume that the demand for the pharmaceutical product is known at the various demand markets since the pharmaceutical firm can be expected to have good in-house forecasting abilities. Hence, we focus on cost minimization associated with the pharmaceutical firm but profit maximization for the contractors who compete on prices and quality.

In Section 3, we provide a dynamic version of the model through a description of the underlying adjustment processes associated with the product flows, the quality levels, and the contractor prices. We show that the projected dynamical system has stationary points that coincide with the solutions for relevant corresponding the variational inequality problem. We also provide stability analysis results.

In Section 4, we describe the algorithm, which yields closed form expressions, at each iteration, for the contractor prices and the quality levels, with the product flows being solved exactly using an equilibration algorithm. The algorithm provides a discrete-time version of the continuous-time adjustment processes given in Section 3. We illustrate the concepts through small examples, and include sensitivity analysis results. We then apply the algorithm in Section 5 to demonstrate the modeling and computational framework on larger examples. We explore the case of a disruption in the pharmaceutical supply chain network and two cases focused on opportunity costs. We summarize our results and give our conclusions in Section 6.

2. The Pharmaceutical Supply Chain Network Model with Outsourcing and Price and Quality Competition

In this section, we develop the pharmaceutical supply chain network model with outsourcing and with price and quality competition among the contractors. We assume that a pharmaceutical firm is involved in the processes of in-house manufacturing and distribution of a pharmaceutical product, and may also contract its manufacturing and distribution activities to contractors, who may be located overseas. We seek to determine the optimal product flows of the firm to its demand markets, along with the prices the contractors charge the firm for production and distribution, and the quality levels of their products.

For clarity and definiteness, we consider the network topology of the pharmaceutical firm depicted in Figure 1. In the supply chain network, there are n_M manufacturing facilities or plants that the pharmaceutical firm owns and n_R demand markets, such as hospitals, clinics, drugstores, etc. Some of the links from the top-tiered node 0, representing the pharmaceutical firm, are connected to its manufacturing facility/plant nodes, which are denoted, respectively, by: $M_1, \dots, M_{n_M}$ and these, in turn, are connected to the demand nodes: $R_1, \dots, R_{n_R}$.

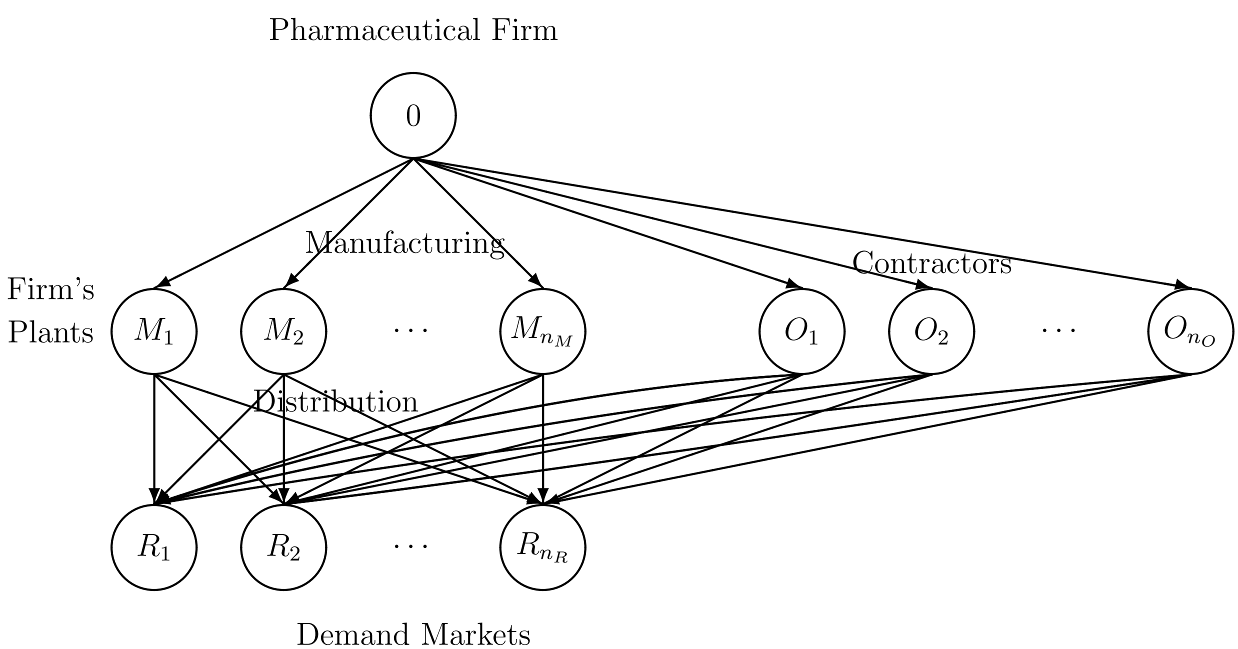

Figure 1: The Pharmaceutical Supply Chain Network Topology with Outsourcing

As also depicted in Figure 1, we capture the outsourcing of the pharmaceutical product in terms of its production and delivery. There are n_O contractors available to the pharmaceutical firm. The firm may potentially contract to any of these contractors who then also distribute the outsourced product that they manufacture to the n_R demand markets. The first set of outsourcing links directly link the top-most node 0 to the n_O contractor nodes, $O_1, \ldots, O_{n_O}$, which correspond to their respective manufacturing activities, and the next set of outsourcing links emanate from the contractor nodes to the demand markets and reflect the delivery of the outsourced pharmaceutical product to the demand markets.

Let $\mathcal{G} = [\mathcal{N}, \mathcal{L}]$ denote the graph consisting of nodes $[\mathcal{N}]$ and directed links $[\mathcal{L}]$ as in Figure 1. The top set of links consists of the manufacturing links, whether in-house or outsourced (contracted), whereas the next set of links consists of the distribution links. For simplicity, we let $n = n_M + n_O$ denote the number of manufacturing plants, whether in-house or belonging to the contractors. The notation for the model is given in Table 1. The vectors are assumed to be column vectors. The optimal/equilibrium solution is denoted by "*".

Table 1: Notation for the Game Theoretic Supply Chain Network Model with Outsourcing

Notation	**Definition**
Q_{jk}	the nonnegative amount of pharmaceutical product produced at manufacturing plant j and delivered to demand market k. We group the $\{Q_{jk}\}$ elements into the vector $Q \in R_+^{nn_R}$.
d_k	the demand for the product at demand market k, assumed known and fixed.
q_j	the nonnegative quality level of the pharmaceutical product produced by contractor j. We group the $\{q_j\}$ elements into the vector $q \in R_+^{n_O}$.
π_{jk}	the price charged by contractor j for producing and delivering a unit of the product to k. We group the $\{\pi_{jk}\}$ elements for contractor j into the vector $\pi_j \in R_+^{n_R}$ and then group all such vectors for all the contractors into the vector $\pi \in R^{n_O n_R}$.
$f_j(\sum_{k=1}^{n_R} Q_{jk})$	the total production cost at manufacturing plant j; $j = 1, \ldots, n_M$ owned by the pharmaceutical firm.
$q\prime$	the average quality level.
$tc_j(\sum_{k=1}^{n_R} Q_{n_M+j,k})$	the total transaction cost associated with the firm transacting with contractor j; $j = 1, \ldots, n_O$.
$\hat{c}_{jk}(Q_{jk})$	the total transportation cost associated with delivering the product manufactured at j to k; $j = 1, \ldots, n_M$; $k = 1, \ldots, n_R$.
$\hat{sc}_{jk}(Q, q)$	the total cost of contractor j; $j = 1, \ldots, n_O$, to produce and distribute the product to demand market k; $k = 1, \ldots, n_R$.
$\hat{qc}_j(q)$	quality cost faced by contractor j; $j = 1, \ldots, n_O$.
$oc_{jk}(\pi_{jk})$	the opportunity cost associated with pricing the product by contractor j; $j = 1, \ldots, n_O$ and delivering it to k; $k = 1, \ldots, n_R$.
$dc(q\prime)$	the cost of disrepute, which corresponds to the external failure quality cost.

2.1 The Behavior of the Pharmaceutical Firm

Since quality is a major issue in the pharmaceutical industry and, hence, it is the industry selected to focus on in our model, we first discuss how we capture quality. We assume that in-house activities can ensure a 100% perfect quality conformance level. The quality conformance level of contractor j is denoted by q_j, which varies from a 0% defect-free level to a 100% defect-free level, such that

$$0 \leq q_j \leq q^U, \quad j = 1, \ldots, n_O, \tag{1}$$

where q^U is the value representing perfect quality achieved by the pharmaceutical firm in its in-house manufacturing.

The quality level associated with the product of the pharmaceutical firm is, hence, an average quality level that is determined by the quality levels decided upon by the contractors and the outsourced product amounts. Thus, the average quality level for the pharmaceutical firm's product, both in-house and outsourced, can be expressed as

$$q\prime = \frac{\sum_{j=n_M+1}^{n} \sum_{k=1}^{n_R} Q_{jk} q_{j-n_M} + (\sum_{j=1}^{n_M} \sum_{k=1}^{n_R} Q_{jk}) q^U}{\sum_{k=1}^{n_R} d_k}. \tag{2}$$

The pharmaceutical firm selects the product flows Q, whereas the contractors, who compete with one another, select their respective quality level q_j and price vector π_j for contractor $j = 1, \ldots, n_O$.

Since we are presenting a game theory model, we use terms from game theory.

The objective of the pharmaceutical firm is to maximize its utility (cf. (3) below), represented by minus its total costs that include the production costs, the transportation costs, the payments to the contractors, the total transaction costs, along with the weighted cost of disrepute, with the nonnegative term ω denoting the weight that the firm imposes on the disrepute cost function. The firm's utility function is denoted by U_0 and, hence, the firm seeks to

$$\text{Maximize}_Q \quad U_0(Q, q^*, \pi^*) = -\sum_{j=1}^{n_M} f_j(\sum_{k=1}^{n_R} Q_{jk}) - \sum_{j=1}^{n_M} \sum_{k=1}^{n_R} \hat{c}_{jk}(Q_{jk}) - \sum_{j=1}^{n_O} \sum_{k=1}^{n_R} \pi^*_{jk} Q_{n_M+j,k}$$

$$-\sum_{j=1}^{n_O} tc_j(\sum_{k=1}^{n_R} Q_{n_M+j,k}) - \omega dc(q\prime). \tag{3}$$

subject to:

$$\sum_{j=1}^{n} Q_{jk} = d_k, \quad k = 1, \ldots, n_R, \tag{4}$$

$$Q_{jk} \geq 0, \quad j = 1, \ldots, n; k = 1, \ldots, n_R, \tag{5}$$

with $q\prime$ in (3) as in (2).

Note that (3) is equivalent to minimizing the total costs. Also, according to (4) the demand at each demand market must be satisfied. This is important since the firm is dealing with pharmaceutical products. We assume that all the cost functions in (3) are continuous, continuously differentiable, and convex. We define the feasible set K^0 as follows: $K^0 \equiv \{Q | Q \in R_+^{nn_R} \text{ with (4) satisfied}\}$. K^0 is closed and convex. The following theorem is immediate.

Theorem 1

The optimality conditions for the pharmaceutical firm, faced with (3) and subject to (4) and (5), with $q\prime$ as in (2) embedded into $dc(q\prime)$, and under the above imposed assumptions, coincide with the solution of the following variational inequality (cf. Nagurney (1999) and Bazaraa, Sherali, and Shetty (1993)): determine $Q^* \in K^0$

$$-\sum_{h=1}^{n} \sum_{l=1}^{n_R} \frac{\partial U_0(Q^*, q^*, \pi^*)}{\partial Q_{hl}} \times (Q_{hl} - Q_{hl}^*) \geq 0, \quad \forall Q \in K^0, \tag{6}$$

with notice that: for $h = 1, \ldots, n_M$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_0}{\partial Q_{hl}} = \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial Q_{hl}} \right]$$

$$= \left[\frac{\partial f_h(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \frac{\partial \hat{c}_{hl}(Q_{hl})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial q\prime} \frac{q^U}{\sum_{k=1}^{n_R} d_k} \right],$$

and for $h = n_M + 1, \ldots, n$; $l = 1, \ldots, n_R$:

$$-\frac{\partial U_0}{\partial Q_{hl}} = \left[\pi_{h-n_M,l}^* + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial Q_{hl}} \right]$$

$$= \left[\pi_{h-n_M,l}^* + \frac{\partial tc_{h-n_M}(\sum_{k=1}^{n_R} Q_{hk})}{\partial Q_{hl}} + \omega \frac{\partial dc(q\prime)}{\partial q\prime} \frac{q_h^*}{\sum_{k=1}^{n_R} d_k} \right].$$

2.2 The Behavior of the Contractors and Their Optimality Conditions

The objective of the contractors is profit maximization. Their revenues are obtained from the purchasing activities of the pharmaceutical firm, while their costs are the costs of production and distribution, the quality cost, and the opportunity cost. Opportunity cost is defined as "the loss of potential gain from other alternatives when one alternative is chosen"

(New Oxford American Dictionary, 2010). In this model, the contractors' opportunity costs are functions of the prices charged, since, if the values are too low, they may not recover all of their costs, whereas if they are too high, then the firm may select another contractor. In our model, these are the only costs that depend on the prices that the contractors charge the pharmaceutical firm and, hence, there is no double counting. We note that the concept of opportunity cost (cf. Mankiw (2011)) is very relevant to both economics and operations research. It has been emphasized in pharmaceutical firm competition by Grabowski and Vernon (1990), Palmer and Raftery (1999), and Cockburn (2004). Gan and Litvinov (2003) also constructed opportunity cost functions that are functions of prices as we consider here (see Table 1) but in an energy application.

Interestingly, Leland (1979), inspired by the work of the Nobel laureate Akerlof (1970) on quality, noted that drugs (pharmaceuticals) must satisfy federal safety standards and, in his model, introduced opportunity costs that are functions of quality levels.

We emphasize that general opportunity cost functions include both explicit and implicit costs (Mankiw (2011)) with the explicit opportunity costs requiring monetary payment, and including possible anticipated regulatory costs, wage expenses, and the opportunity cost of capital (see Porteus (1986)), etc. Implicit opportunity costs are those that do not require payment, but to the decision-maker, still need to be monetized, for the purposes of decision-making, and can include the time and effort put in (see Payne, Bettman, and Luce (1996)), and the profit that the decision-maker could have earned, if he had made other choices (Sandoval-Chavez and Beruvides (1998)).

Please note that, as presented in Table 1, $\hat{sc}_{jk}(Q,q)$, which is contractor j's cost function associated with producing and delivering the pharmaceutical firm's product to demand market k, only captures the cost of production and delivery. It depends on both the quantities and the quality levels. However, $\hat{qc}_j(q)$ is the cost associated with quality management, and reflects the "cost incurred in ensuring and assuring quality as well as the loss incurred when quality is not achieved," and is over and above the cost of production and delivery activities. As indicated in the Introduction, $\hat{qc}_j(q)$ is a convex function in the quality levels. Thus, $\hat{sc}_{jk}(Q,q)$ and $\hat{qc}_j(q)$ are two entirely different costs, and they do not overlap.

Each contractor has, as its strategic variables, its quality level, and the prices that it charges the pharmaceutical firm for production and distribution to the demand markets. We denote the utility of each contractor j by U_j, with $j = 1,\dots,n_O$, and note that it represents the profit. Hence, each contractor j; $j = 1, \ldots, n_O$ seeks to:

$$\text{Maximize}_{q_j,\pi_j} \quad U_j(Q^*, q, \pi) = \sum_{k=1}^{n_R} \pi_{jk} Q^*_{n_M+j,k} - \sum_{k=1}^{n_R} \hat{sc}_{jk}(Q^*, q) - \hat{qc}_j(q) - \sum_{k=1}^{n_R} oc_{jk}(\pi_{jk}) \quad (7)$$

subject to:

$$\pi_{jk} \geq 0, \quad k = 1, \ldots, n_R, \quad (8)$$

and (1) for each j.

We assume that the cost functions in each contractor's utility function are continuous, continuously differentiable, and convex. Moreover, we assume that the contractors compete in a noncooperative in the sense of Nash (1950, 1951), with each one trying to maximize its own profits.

We define the feasible sets $K^j \equiv \{(q_j, \pi_j) | \pi_j \text{ satisfies (8) and } q_j \text{ satisfies (1) for } j\}$. We also define the feasible set $\mathcal{K}^1 \equiv \prod_{j=1}^{n_O} K^j$ and $\mathcal{K} \equiv K^0 \times \mathcal{K}^1$. We observe that all the above-defined feasible sets are closed and convex.

Definition 1: A Nash-Bertrand Equilibrium with Price and Quality Competition

A quality level and price pattern $(q^*, \pi^*) \in \mathcal{K}^1$ *is said to constitute a Bertrand-Nash equilibrium if for each contractor* j; $j = 1, \ldots, n_O$

$$U_j(Q^*, q_j^*, \hat{q_j^*}, \pi_j^*, \hat{\pi_j^*}) \geq U_j(Q^*, q_j, \hat{q_j^*}, \pi_j, \hat{\pi_j^*}), \quad \forall (q_j, \pi_j) \in K^j, \quad (9)$$

where

$$\hat{q_j^*} \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_{n_O}^*), \quad (10)$$

$$\hat{\pi_j^*} \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_O}^*). \quad (11)$$

According to (9), a Nash-Bertrand equilibrium is established if no contractor can unilaterally improve upon its profits by selecting an alternative vector of quality levels and prices charged to the pharmaceutical firm.

Next, we present the variational inequality formulation of the Bertrand-Nash equilibrium according to Definition 1 (see Bertrand (1883), Nash (1950, 1951), Gabay and Moulin (1980), and Nagurney (2006)).

Theorem 2

Assume that, for each contractor j; $j = 1, \ldots, n_O$, *the profit function* $U_j(Q, q, \pi)$ *is concave with respect to the variables* $\{\pi_{j1}, \ldots, \pi_{jn_R}\}$ *and* q_j, *and is continuous and continuously differentiable. Then* $(q^*, \pi^*) \in \mathcal{K}^1$ *is a Bertrand-Nash equilibrium according to Definition 1 if* *and only if it satisfies the variational inequality:*

$$-\sum_{j=1}^{n_O}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial q_j}\times(q_j-q_j^*)-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial \pi_{jk}}\times(\pi_{jk}-\pi_{jk}^*)\geq 0,\quad \forall (q,\pi)\in\mathcal{K}^1. \tag{12}$$

with notice that: for $j=1,\ldots,n_O$*:*

$$-\frac{\partial U_j}{\partial q_j}=\sum_{k=1}^{n_R}\frac{\partial \hat{sc}_{jk}(Q^*,q)}{\partial q_j}+\frac{\partial \hat{qc}_j(q)}{\partial q_j}, \tag{13}$$

and for $j=1,\ldots,n_O; k=1,\ldots,n_R$*:*

$$-\frac{\partial U_j}{\partial \pi_{jk}}=\frac{\partial oc_{jk}(\pi_{jk})}{\partial \pi_{jk}}-Q^*_{n_M+j,k}. \tag{14}$$

2.3 The Equilibrium Conditions for the Supply Chain Network with Outsourcing and with Price and Quality Competition

In equilibrium, the optimality conditions for all contractors and the optimality conditions for the pharmaceutical firm must hold simultaneously, according to the definition below.

Definition 2: Supply Chain Network Equilibrium with Outsourcing and with Price and Quality Competition

The equilibrium state of the pharmaceutical supply chain network with outsourcing is one where both variational inequalities (6) and (12) hold simultaneously.

The following theorem is then immediate.

Theorem 3

The equilibrium conditions governing the pharmaceutical supply chain network model with outsourcing are equivalent to the solution of the variational inequality problem: determine $(Q^*,q^*,\pi^*)\in\mathcal{K}$*, such that:*

$$-\sum_{h=1}^{n}\sum_{l=1}^{n_R}\frac{\partial U_0(Q^*,q^*,\pi^*)}{\partial Q_{hl}}\times(Q_{hl}-Q_{hl}^*)-\sum_{j=1}^{n_O}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial q_j}\times(q_j-q_j^*)$$

$$-\sum_{j=1}^{n_O}\sum_{k=1}^{n_R}\frac{\partial U_j(Q^*,q^*,\pi^*)}{\partial \pi_{jk}}\times(\pi_{jk}-\pi_{jk}^*)\geq 0,\quad \forall (Q,q,\pi)\in\mathcal{K}. \tag{15}$$

We now put variational inequality (15) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in the N-dimensional Euclidean space, and $\mathcal{K}$ is closed and convex. We define the vector $X \equiv (Q, q, \pi)$. Also, here $N = nn_R + n_O + n_O n_R$. Note that (16) may be rewritten as:

$$\sum_{i=1}^{N} F_i(X^*) \times (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}. \tag{17}$$

The components of F are then as follows. The first nn_R components of F are given by: $-\frac{\partial U_0(Q,q,\pi)}{\partial Q_{hl}}$ for $h = 1, \ldots, n$; $l = 1, \ldots, n_R$; the next n_O components of F are given by: $-\frac{\partial U_j(Q,q,\pi)}{\partial q_j}$ for $j = 1, \ldots, n_O$, and the subsequent $n_O n_R$ components of F are given by: $-\frac{\partial U_j(Q,q,\pi)}{\partial \pi_{jk}}$ with $j = 1, \ldots, n_O$; $k = 1, \ldots, n_R$. Hence, (15) can be put into standard form (16).

3. The Underlying Dynamics

We now describe the underlying dynamics until the equilibrium satisfying variational inequality (15) is achieved. We identify dynamic adjustment processes for the evolution of the firm's pharmaceutical product flows, both in-house and outsourced, and the contractors' quality levels and prices charged to the pharmaceutical firm. In Section 4, we provide a discrete-time version of the continuous time adjustment processes in the form of an algorithm.

Observe that, for a current vector of product flows, quality levels, and prices at time t, $X(t) = (Q(t), q(t), \pi(t))$, $-F_i(X(t)) = \frac{\partial U_0(Q(t),q(t),\pi(t))}{\partial Q_{hl}}$, for $i = 1, \dots, nn_R$ and $h = 1, \dots, n$; $l = 1, \dots, n_R$, and is given by minus the value of the expressions following (6), is the marginal utility of the pharmaceutical firm with respect to the product flow produced at h and distributed to demand market l. Similarly, $-F_i(X(t)) = \frac{\partial U_j(Q(t),q(t),\pi(t))}{\partial q_j}$, given by minus the value in (13), is contractor j's marginal utility (profit) with respect to its quality level associated with the pharmaceutical product that it produced and distributed to the demand markets, with $j = 1, \dots, n_O$ and $i = nn_R+1, \dots, nn_R+n_O$. Finally, $-F_i(X(t)) = \frac{\partial U_j(Q(t),q(t),\pi(t))}{\partial \pi_{jk}}$, given by minus the value in (14), is contractor j's marginal utility (profit) with respect to its price charged for delivering the product that it produced to k, with $j = 1, \dots, n_O$; $k = 1, \dots, n_R$, and $i = nn_R + n_O + 1, \dots, n$.

We emphasize that it is imperative that the constraints be satisfied, consisting of the demand constraints for the pharmaceutical firm, and the nonnegativity assumption on the product flows, as well as the nonnegativity assumption on the contractors' quality levels and the prices that they charge.

Specifically, we propose the following pertinent ordinary differential equation (ODE) for the adjustment processes of the product flows, quality levels, and contractor prices, in vector form, as:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \tag{18}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{19}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P(X) = \text{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{20}$$

and where $\| \cdot \| = \langle x, x \rangle$ and $-F(X) = \nabla U(Q, q, \pi)$, where $\nabla U(Q, q, \pi)$ is the vector of marginal utilities as described above.

We now further interpret ODE (18) in the context of the supply chain network game theory model with price and quality competition among the contractors. Observe that ODE (18) guarantees that the product flows, quality levels, and the contractor prices are always nonnegative and that the demand constraints (4) are also satisfied. If one were to consider, instead, the ordinary differential equation: $\dot{X} = -F(X)$, or, equivalently, $\dot{X} = \nabla U(X)$, such an ODE would not ensure that $X(t) \geq 0$, for all $t \geq 0$, unless additional restrictive assumptions were to be imposed. Moreover, it would not be guaranteed that the demand at the demand markets would be satisfied and that the upper bound on the quality levels would be met.

Recall now the definition of $F(X)$ for the model, which captures the behavior of all the decision-makers in the supply chain network with outsourcing and competition in an integrated manner. The *projected dynamical system* (18) states that the rate of change of the product flows, quality levels, and contractor prices is greatest when the pharmaceutical firm's and contractors' marginal utilities are greatest. If the marginal utility of a contractor with respect to its quality level is positive, then the contractor will increase its quality level; if it is negative, then it will decrease the quality. Note that the quality levels will also never be outside their upper bound. A similar adjustment behavior holds for the contractors in terms of the prices that they charge, but without an upper bound on the prices. This type of behavior is rational from an economic standpoint. Therefore, ODE (18) corresponds to reasonable continuous adjustment processes for the supply chain network game theory model with outsourcing and quality and price competition.

Although the use of the projection on the right-hand side of ODE (18) guarantees that the underlying variables are always nonnegative, and that the other constraints are satisfied, it also raises the question of existence of a solution to ODE (18), since this ODE is nonstandard due to its discontinuous right-hand side. Dupuis and Nagurney (1993) constructed the fundamental theory with regards to existence and uniqueness of projected dynamical systems as defined by (18). We cite the following theorem from that paper. See also the book by Nagurney and Zhang (1996). For a plethora of dynamic, multitiered supply chain network models under Cournot competition, see the book by Nagurney (2006).

Theorem 4

X^* solves the variational inequality problem (15), equivalently, (16), if and only if it is a stationary point of the ODE (18), that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{21}$$

This theorem provides the necessary and sufficient condition for a pattern $X^* = (Q^*, q^*, \pi^*)$ to be an equilibrium, according to Definition 2, in that $X^* = (Q^*, q^*, \pi^*)$ is a stationary point of the adjustment processes defined by ODE (18), that is, X^* is the point at which $\dot{X} = 0$.

3.1 Stability Under Monotonicity

We now turn to the questions as to whether and under what conditions does the adjustment processes defined by ODE (21) approach an equilibrium? We first note that Lipschitz continuity of $F(X)$ (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) guarantees the existence of a unique solution to (22) below, where we have that $X^0(t)$ satisfies ODE (18) with product flow, quality level, and price pattern (Q^0, q^0, π^0). In other words, $X^0(t)$ solves the initial value problem (IVP)

$$\dot{X} \;=\; \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) \;=\; X^0, \tag{22}$$

with $X^0(0) = X^0$. For convenience, we sometimes write $X^0 \cdot t$ for $X^0(t)$.

We present the following definitions of stability for these adjustment processes, which are adaptations of those introduced in Zhang and Nagurney (1995). Hereafter, we use $B(X, r)$ to denote the open ball with radius r and center X.

Definition 3

An equilibrium product flow, quality level, and price pattern X^* is stable, if for any $\epsilon > 0$, there exists a $\delta > 0$, such that for all initial $X \in B(X^*, \delta)$ and all $t \geq 0$

$$X(t) \in B(X^*, \epsilon). \tag{23}$$

The equilibrium point X^* is unstable, if it is not stable.

Definition 4

An equilibrium product flow, quality level, and price pattern X^* is asymptotically stable, if it is stable and there exists a $\delta > 0$ such that for all initial product flows, quality levels, and prices $X \in B(X^*, \delta)$

$$\lim_{t \to \infty} X(t) \longrightarrow X^*. \tag{24}$$

Definition 5

An equilibrium product flow, quality level, and price pattern X^* *is globally exponentially stable, if there exist constants* $b > 0$ *and* $\mu > 0$ *such that*

$$\|X^0(t) - X^*\| \leq b\|X^0 - X^*\|e^{-\mu t}, \quad \forall t \geq 0, \ \forall X^0 \in \mathcal{K}. \tag{25}$$

Definition 6

An equilibrium product flow, quality level, and price pattern X^* *is a global monotone attractor, if the Euclidean distance* $\|X(t) - X^*\|$ *is nonincreasing in* t *for all* $X \in \mathcal{K}$*.*

Definition 7

An equilibrium X^* *is a strictly global monotone attractor, if* $\|X(t) - X^*\|$ *is monotonically decreasing to zero in* t *for all* $X \in \mathcal{K}$*.*

We now tackle the stability of the adjustment processes under various monotonicity conditions.

Recall (cf. Nagurney (1999)) that $F(X)$ is *monotone* if

$$\langle F(X) - F(X^*), X - X^* \rangle \geq 0, \quad \forall X, X^* \in \mathcal{K}. \tag{26}$$

$F(X)$ is *strictly monotone* if

$$\langle F(X) - F(X^*), X - X^* \rangle > 0, \quad \forall X, X^* \in \mathcal{K}, \ X \neq X^*. \tag{27}$$

$F(X)$ is *strongly monotone* X^*, if there is an $\eta > 0$, such that

$$\langle F(X) - F(X^*), X - X^* \rangle \geq \eta \|X - X^*\|^2, \quad \forall X, X^* \in \mathcal{K}. \tag{28}$$

The monotonicity of a function F is closely related to the

positive-definiteness of its Jacobian ∇F (cf. Nagurney (1999)). Specifically, if ∇F is positive-semidefinite, then F is monotone; if ∇F is positive-definite, then F is strictly monotone; and, if ∇F is strongly positive-definite, in the sense that the symmetric part of $\nabla F, (\nabla F^T + \nabla F)/2$, has only positive eigenvalues, then F is strongly monotone.

In the context of our supply chain network game theory model with outsourcing, where $F(X)$ is the vector of negative marginal utilities as follows (17), we note that if the utility functions are twice differentiable and the Jacobian of the negative marginal utility functions (or, equivalently, the negative of the Hessian matrix of the utility functions) for the integrated model is positive-definite, then the corresponding $F(X)$ is strictly monotone.

We now present an existence and uniqueness result, the proof of which follows from the basic theory of variational inequalities (cf. Nagurney (1999)).

Theorem 5

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (16); equivalently, to variational inequality (15).

The following theorem summarizes the stability properties of the utility gradient processes, under various monotonicity conditions on the marginal utilities.

Theorem 6

(i). *If $F(X)$ is monotone, then every supply chain network equilibrium, as defined in Definition 2, provided its existence, is a global monotone attractor for the utility gradient processes.*

(ii). *If $F(X)$ is strictly monotone, then there exists at most one supply chain network equilibrium. Furthermore, given existence, the unique equilibrium is a strictly global monotone attractor for the utility gradient processes.*

(iii). *If $F(X)$ is strongly monotone, then the unique supply chain network equilibrium, which is guaranteed to exist, is also globally exponentially stable for the utility gradient processes.*

Proof: The stability assertions follow from Theorems 3.5, 3.6, and 3.7 in Nagurney and Zhang (1996), respectively. The uniqueness in (ii) is a classical variational inequality result, whereas existence and uniqueness as in (iii) follows from Theorem 5. □

4. The Algorithm

As mentioned in Section 3, the projected dynamical system yields continuous-time adjustment processes. However, for computational purposes, a discrete-time algorithm, which serves as an approximation to the continuous-time trajectories is needed.

We now recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, iteration τ of the Euler method (see also Nagurney and Zhang (1996)) is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{29}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (16).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \rightarrow 0$, as $\tau \rightarrow \infty$. Specific conditions for convergence of this scheme as well as various applications to the solutions of other game theory models can be found in Nagurney, Dupuis, and Zhang (1994), Nagurney, Takayama, and Zhang (1995), Cruz (2008), Nagurney (2010), and Nagurney and Li (2012).

We emphasize that this is the first time that this algorithm is being adapted and applied for the solution of supply chain network game theory problems under Nash-Bertrand competition and with outsourcing.

Note that, at each iteration τ, $X^{\tau+1}$ in (29) is actually the solution to the strictly convexquadratic programming problem given by:

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X\rangle - \langle X^{\tau} - a_{\tau}F(X^{\tau}), X\rangle. \tag{30}$$

As for solving (30) in order to obtain the values of the product flows at each iteration τ, we can apply the exact equilibration algorithm, originated by Dafermos and Sparrow (1969) and applied to many different applications of networks with special structure (cf. Nagurney (1999) and Nagurney and Zhang (1996)). See also Nagurney and Zhang (1997) for an application to fixed demand traffic network equilibrium problems.

Furthermore, in light of the nice structure of the underlying feasible set $\mathcal{K}$, we can obtain the values for the quality variables explicitly according to the following closed form expressions for contractor j; $j = 1, \ldots, n_O$:

$$q_j^{\tau+1} = \min\{q^U, \max\{0, q_j^\tau + a_\tau(-\sum_{k=1}^{n_R} \frac{\partial \hat{sc}_{jk}(Q^\tau, q^\tau)}{\partial q_j} - \frac{\partial \hat{qc}_j(q^\tau)}{\partial q_j})\}\}. \tag{31}$$

Also, we have the following explicit formulae for the contractor prices: for $j = 1, \ldots, n_O$; $k = 1, \ldots, n_R$:

$$\pi_{jk}^{\tau+1} = \max\{0, \pi_{jk}^\tau + a_\tau(-\frac{\partial oc_{jk}(\pi_{jk}^\tau)}{\partial \pi_{jk}} + Q_{n_M+j,k}^\tau)\}. \tag{32}$$

We now provide the convergence result. The proof is direct from Theorem 5.8 in Nagurney and Zhang (1996).

Theorem 7

In the supply chain network model with outsourcing, let $F(X) = -\nabla U(Q, q, \pi)$ be strongly monotone. Also, assume that F is uniformly Lipschitz continuous. Then there exists a unique equilibrium product flow, quality level, and price pattern $(Q^*, q^*, \pi^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by (29) above, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (Q^*, q^*, π^*).

Note that convergence also holds if $F(X)$ is monotone (cf. Theorem 8.6 in Nagurney and Zhang (1996)) provided that the price iterates are bounded. We know that the product flow iterates as well as the quality level iterates will be bounded due to the constraints. Clearly, in practice, contractors cannot charge unbounded prices for production and delivery. Hence, we can also expect the existence of a solution, given the continuity of the functions that make up $F(X)$, under less restrictive conditions that that of strong monotonicity.

4.1 An Illustrative Example, a Variant, and Sensitivity Analysis

We now provide a small example to clarify ideas, along with a variant, and also conduct a sensitivity analysis exercise. The supply chain network consists of the pharmaceutical firm, a single contractor, and a single demand market, as depicted in Figure 2.

The data are as follows. The firm's production cost function is:

$$f_1(Q_{11}) = Q_{11}^2 + Q_{11}$$

and its total transportation cost function is:

$$\hat{c}_{11}(Q_{11}) = .5Q_{11}^2 + Q_{11}.$$

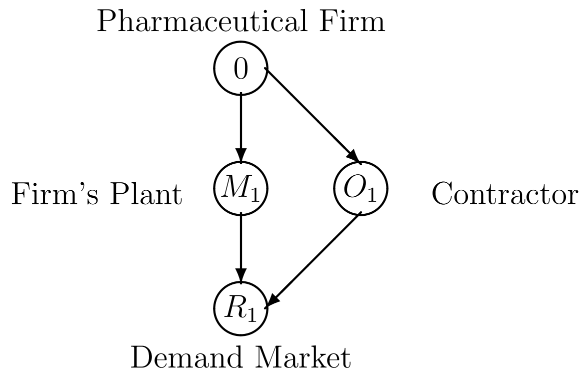

Figure 2: Supply Chain Network for an Illustrative Numerical Example

The firm's transaction cost function associated with the contractor is given by:

$$tc_1(Q_{21}) = .05Q_{21}^2 + Q_{21}.$$

The demand for the pharmaceutical product at demand market R_1 is 1,000 and q^U is 100 and the weight ω is 1.

The contractor's total cost of production and distribution function is:

$$\hat{s}c_{11}(Q_{21}, q_1) = Q_{21}q_1.$$

Its total quality cost function is given by:

$$\hat{q}c_1(q_1) = 10(q_1 - 100)^2.$$

The contractor's opportunity cost function is:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2.$$

The pharmaceutical firm's cost of disrepute function is:

$$dc(q\prime) = 100 - q\prime$$

where $q\prime$ (cf. (1)) is given by: $\frac{Q_{21}q_1 + Q_{11}100}{1000}$.

We set the convergence tolerance to 10^{-3} so that the algorithm was deemed to have converged when the absolute value of the difference between each product flow, each quality level, and each price was less than or equal to 10^{-3}. So, in effect, a stationary point had been achieved. The Euler method was initialized with $Q^0_{11} = Q^0_{21} = 500$, $q^0_1 = 1$, and $\pi^0_{11} = 0$. The sequence $\{a_\tau\}$ was set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

The Euler method converged in 87 iterations and yielded the following product flow, quality level, and price pattern:

$$Q^*_{11} = 270.50, \quad Q^*_{21} = 729.50, \quad q^*_1 = 63.52, \quad \pi^*_{11} = 739.50.$$

The total cost incurred by the pharmaceutical firm is 677,128.65 with the contractor earning a profit of 213,786.67. The value of $q\prime$ is 73.39.

The Jacobian matrix of $F(X) = -\nabla U(Q, q, \pi)$, for this example, denoted by $J(Q_{11}, Q_{21}, q_1, \pi_{11})$, is

$$J(Q_{11}, Q_{21}, q_1, \pi_{11}) = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & .1 & -.001 & 1 \\ 0 & 1 & 20 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

This Jacobian matrix is positive-definite, and, hence, -$\nabla U(Q, q, \pi)$ is strongly monotone (see also Nagurney (1999)). Thus, both the existence and the uniqueness of the solution to variational inequality (15) with respect to this example are guaranteed. Moreover, the equilibrium solution, reported above, is globally exponentially stable.

We then constructed a variant of this example. The transportation cost function was reduced by a factor of 10 so that it is now:

$$\hat{c}_{11}(Q_{11}) = .05Q^2_{11} + .1Q_{11}$$

with the remainder of the data as in the original example above. This could capture the situation of the firm moving its production facility closer to the demand market.

The Euler method again required 87 iterations for convergence and yielded the following equilibrium solution:

$$Q^*_{11} = 346.86, \quad Q^*_{21} = 653.14, \quad q^*_1 = 67.34, \quad \pi^*_{11} = 663.15.$$

The pharmaceutical firm's total costs are now 581,840.07 and the contractor's profits are 165,230.62. The value of $q\prime$ is now 78.67. The average quality increased, with the quantity of the product produced by the firm having increased. Also, the price charged by the contractor decreased but the quality level of its product increased.

For both the examples, the underlying constraints are satisfied, consisting of the demand constraint, the nonnegativity constraints, as well as the upper bound on the contractor's quality level. In addition, the variational inequality for this problem is satisfied.

It is easy to verify that the Jacobian of F for the variant is positive-definite with the only change in the Jacobian matrix above being that the 3 is replaced by 2.1.

We then proceeded to conduct a sensitivity analysis exercise. We returned to the original example and increased the demand for the pharmaceutical product at R_1 in increments of 1,000. The results of the computations are reported in Figures 3, 4, and 5 for the equilibrium product flows, the quality levels and the average quality $q\prime$, and, finally, the equilibrium prices.

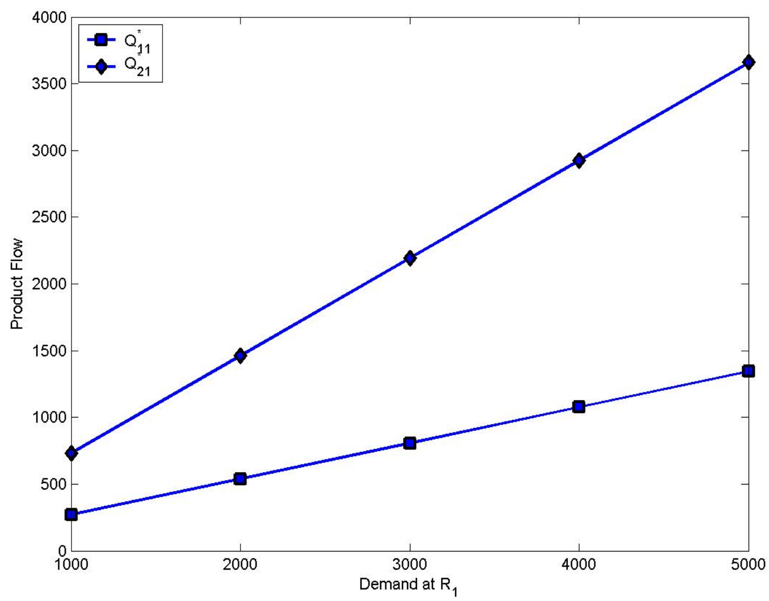

Figure 3: Equilibrium Product Flows as the Demand Increases for the Illustrative Example

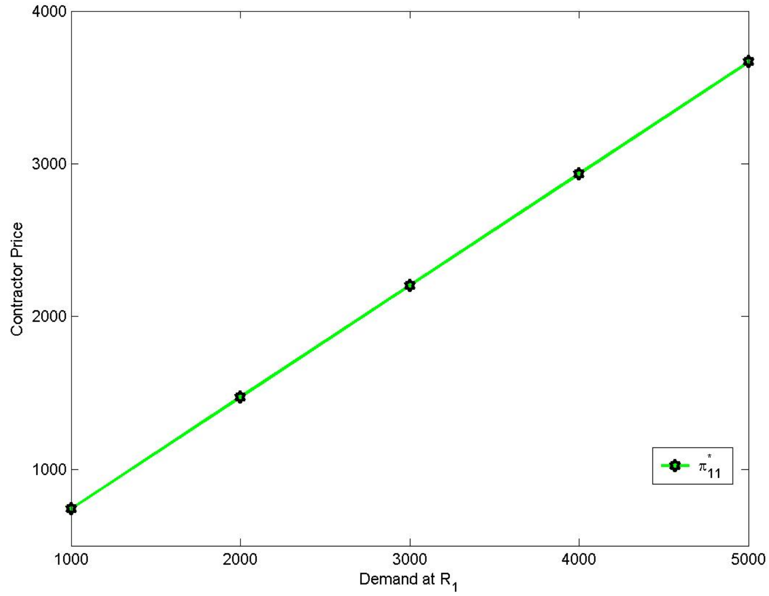

Figure 4: Equilibrium Contractor Prices as the Demand Increases for the Illustrative Example

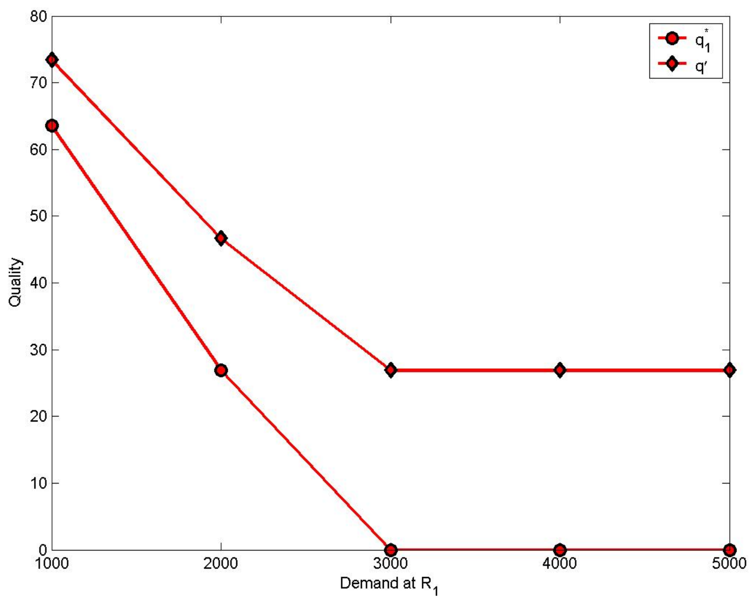

Figure 5: Equilibrium Contractor Quality Level and the Average Quality as the Demand Increases for the Illustrative Example

It is interesting to observe that, when the demand increased past a certain point, the contractor's equilibrium quality level decreased to zero and stayed at that level. Such unexpected insights may be obtained through a modeling and computational framework that we have developed. Of course, the results in this subsection are based on constructed examples. One may, of course, conduct other sensitivity analysis exercises and also utilize different underlying cost functions in order to tailor the general framework to specific pharmaceutical firms' needs and situations.

5. Additional Numerical Examples

In this section, we applied the Euler method to compute solutions to examples that are larger than those in the preceding section. We report all of the input data as well as the output. The Euler method was initialized as in the illustrative example, except that the initial product flows were equally distributed among the available options for each demand market. We used the same convergence tolerance as previously.

Example 1 and Sensitivity Analysis

Example 1 consists of the topology given in Figure 6. There are two manufacturing plants owned by the pharmaceutical firm and two possible contractors. The firm must satisfy the demands for its pharmaceutical product at the two demand markets. The demand for the pharmaceutical product at demand market R_1 is 1,000 and it is 500 at demand market R_2. q^U is 100 and the weight ω is 1.

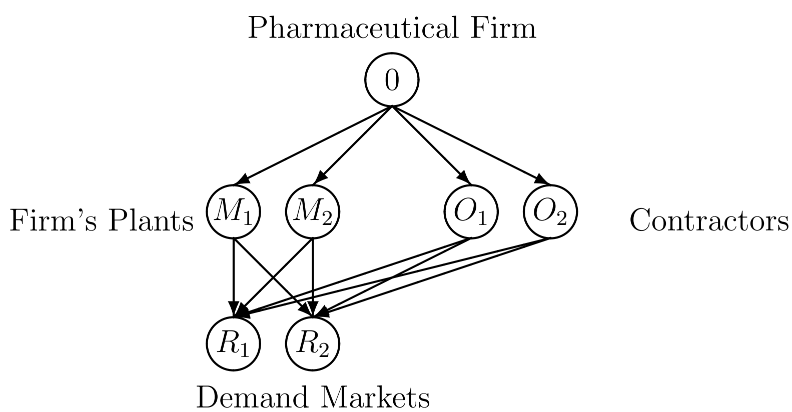

Figure 6: Supply Chain Network Topology for Example 1

The production cost functions at the plants are:

$$f_1(\sum_{k=1}^{2} Q_{1k}) = (Q_{11}+Q_{12})^2 + 2(Q_{11}+Q_{12}), \quad f_2(\sum_{k=1}^{2} Q_{2k}) = 1.5(Q_{21}+Q_{22})^2 + 2(Q_{21}+Q_{22}).$$

The total transportation cost functions are:

$$\hat{c}_{11}(Q_{11}) = 1.5Q_{11}^2 + 10Q_{11}, \quad \hat{c}_{12}(Q_{12}) = 1Q_{12}^2 + 25Q_{12},$$

$$\hat{c}_{21}(Q_{21}) = 1Q_{21}^2 + 5Q_{21}, \quad \hat{c}_{22}(Q_{22}) = 2.5Q_{22}^2 + 40Q_{22}.$$

The transaction cost functions are:

$$tc_1(Q_{31}+Q_{32}) = .5(Q_{31}+Q_{32})^2 + .1(Q_{31}+Q_{32}), \quad tc_2(Q_{41}+Q_{42}) = .25(Q_{41}+Q_{42})^2 + .2(Q_{41}+Q_{42}).$$

The contractors' total cost of production and distribution functions are:

$$\hat{sc}_{11}(Q_{31}, q_1) = Q_{31}q_1, \quad \hat{sc}_{12}(Q_{32}, q_1) = Q_{32}q_1,$$

$$\hat{sc}_{21}(Q_{41}, q_2) = 2Q_{41}q_2, \quad \hat{sc}_{22}(Q_{42}, q_2) = 2Q_{42}q_2.$$

Their total quality cost functions are given by:

$$\hat{qc}_1(q_1) = 5(q_1 - 100)^2, \quad \hat{qc}_2(q_2) = 10(q_1 - 100)^2.$$

The contractors' opportunity cost functions are:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 10)^2, \quad oc_{12}(\pi_{12}) = (\pi_{12} - 10)^2,$$

$$oc_{21}(\pi_{21}) = (\pi_{21} - 5)^2, \quad oc_{22}(\pi_{22}) = .5(\pi_{22} - 20)^2.$$

The pharmaceutical firm's cost of disrepute function is:

$$dc(q\prime) = 100 - q\prime$$

where $q\prime$ (cf. (1)) is given by: $\frac{Q_{31}q_1+Q_{32}q_1+Q_{41}q_2+Q_{42}q_2+Q_{11}100+Q_{12}100+Q_{21}100+Q_{22}100}{1500}$.

The Euler method converged in 153 iterations and yielded the following equilibrium solution. The computed product flows are:

$$Q_{11}^* = 95.77, \quad Q_{12}^* = 85.51, \quad Q_{21}^* = 118.82, \quad Q_{22}^* = 20.27,$$

$$Q_{31}^* = 213.59, \quad Q_{32}^* = 224.59, \quad Q_{41}^* = 571.83, \quad Q_{42}^* = 169.63.$$

The computed quality levels of the contractors are:

$$q_1^* = 56.18, \quad q_2^* = 25.85,$$

and the computed prices are:

$$\pi_{11}^* = 223.57, \quad \pi_{12}^* = 122.30, \quad \pi_{21}^* = 290.92, \quad \pi_{22}^* = 189.61.$$

The total cost of the pharmaceutical firm is 610,643.26 and the profits of the contractors are: 5,733.83 and 9,294.44.

The value of $q\prime$ is 50.55.

In order to investigate the stability of the computed equilibrium for Example 1 (and a similar analysis holds for the subsequent examples), we constructed the Jacobian matrix as follows. The Jacobian matrix of $F(X) = -\nabla U(Q, q, \pi)$, for this example, denoted by $J(Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, Q_{32}, Q_{41}, Q_{42}, q_1, q_2, \pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$, is

$$J(Q_{11}, Q_{12}, Q_{21}, Q_{22}, Q_{31}, Q_{32}, Q_{41}, Q_{42}, q_1, q_2, \pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$$
$$= \begin{pmatrix} 5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67\times10^{-4} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -6.67\times10^{-4} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67\times10^{-4} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & -6.67\times10^{-4} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

This Jacobian matrix is positive-semidefinite. Hence, according to Theorem 6 we know that, since $F(X)$ is monotone, every supply chain network equilibrium, as defined in Definition 2, is a global monotone attractor for the utility gradient process.

Interestingly, in this example, the pharmaceutical firm pays relatively higher prices for the products with a lower quality level from contractor O_2. This happens because the firm's demand is fixed in each demand market, and, therefore, there is no pressure for quality improvement from the demand side (as would be the case if te demands were elastic (cf. Nagurney and Li (2012)). As reflected in the transaction costs with contractors O_1 and O_2, the pharmaceutical firm is willing to pay higher prices to O_2, despite a lower quality level.

We then conducted sensitivity analysis. In particular, we investigated the effect of increases in the demands at both demand markets R_1 and R_2. The results for the new equilibrium product flows are depicted in Figure 7, with the results for the equilibrium quality levels and the equilibrium prices in Figure 8 and Figure 9, respectively. The contractors consistently provide the majority of the product to the demand markets.

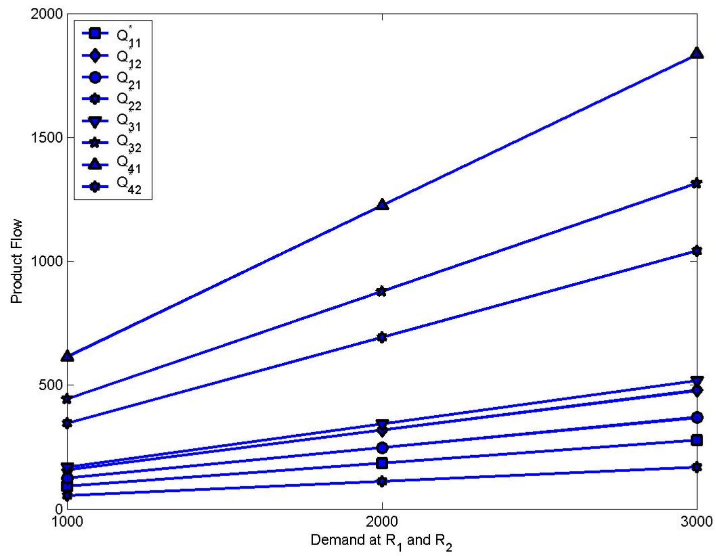

Figure 7: Equilibrium Product Flows as the Demand Increases for Example 1

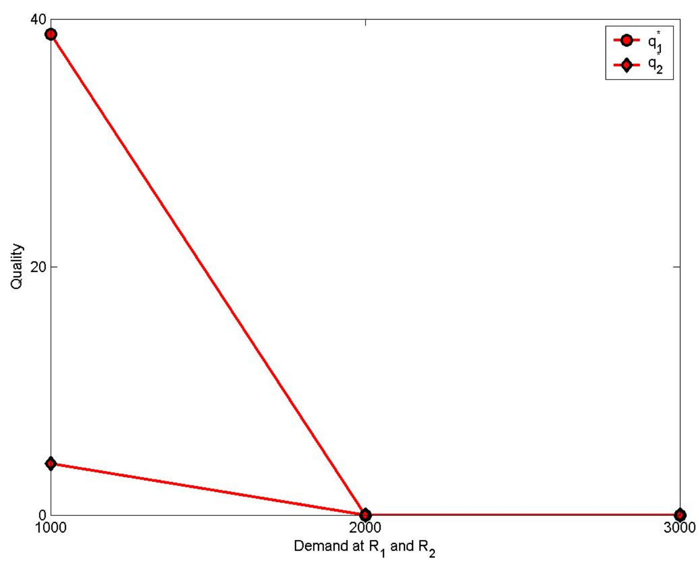

Figure 8: Equilibrium Quality Levels as the Demand Increases for Example 1

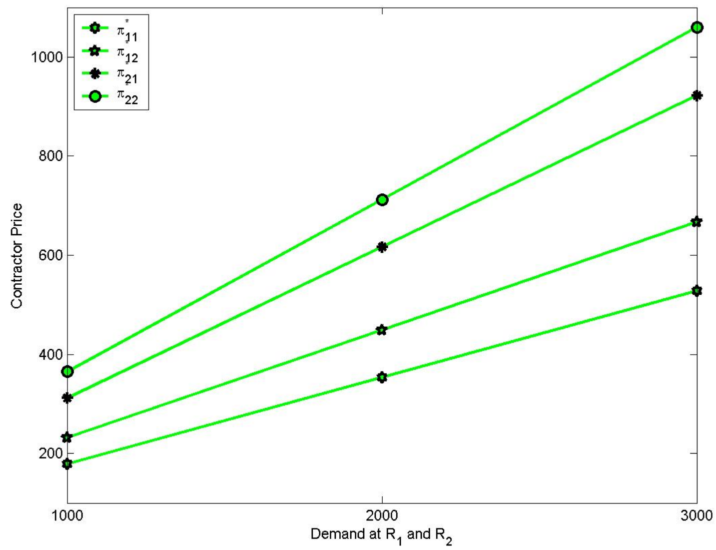

Figure 9: Equilibrium Contractor Prices as the Demand Increases for Example 1

Observe from Figure 8 that as the demand increases the quality levels for both contractors drops to zero.

Example 2

We then considered a disruption to the original supply chain in Example 1. No business is immune from supply chain disruptions and, as noted by Purtell (2010), pharmaceuticals are especially vulnerable since they are high-value, highly regulated products. Moreover, pharmaceutical disruptions may not only increase costs but may also create health hazards and expose the pharmaceutical companies to damage to their brands and reputations (see also Nagurney, Yu, and Qiang (2011), Masoumi, Yu, and Nagurney (2012), and Qiang and Nagurney (2012)).

Specifically, we considered the following disruption. The data are as in Example 1 but contractor O_2 is not able to provide any production and distribution services. This could arise due to a natural disaster, adulteration in its production process, and/or an inability to procure an ingredient. Hence, the topology of the disrupted supply chain network is as in Figure 10.

The Euler method converged in 73 iterations and yielded the following new equilibrium

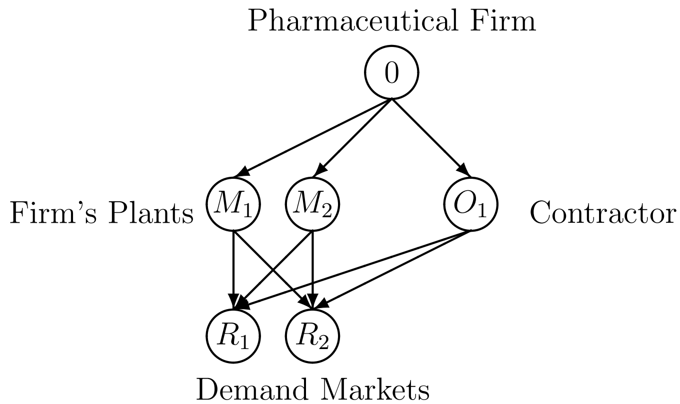

Figure 10: Supply Chain Network Topology for Example 2

solution. The computed product flows are:

$$Q^*_{11} = 218.06, \quad Q^*_{12} = 141.79, \quad Q^*_{21} = 260.20, \quad Q^*_{22} = 25.96, \quad Q^*_{31} = 521.74, \quad Q^*_{32} = 332.25.$$

The computed quality level of the remaining contractor is:

$$q^*_1 = 14.60,$$

and the computed prices are:

$$\pi^*_{11} = 531.74, \quad \pi^*_{12} = 176.12.$$

With the decrease in competition among the contractors, since there is now only one, rather than two, as in Example 1, the quality level of contractor O_1 dropped, but the prices that it charged increased. The new average quality level is $q\prime$=51.38. The total cost of the pharmaceutical firm is now 1,123,226.62 whereas the profit of the first contractor is now 123,460.67, a sizable increase relative to that with competition as in Example 1.

Example 3

The data for Example 3 are the same as for Example 1, except that the opportunity cost functions oc_{11}, oc_{12}, oc_{21}, oc_{22} are all equal to 0.00.

The Euler method converged in 76 iterations and yielded the following equilibrium solution. The computed product flows are:

$$Q^*_{11} = 451.17, \quad Q^*_{12} = 396.50, \quad Q^*_{21} = 548.73, \quad Q^*_{22} = 103.39,$$

$$Q^*_{31} = 0.00, \quad Q^*_{32} = 0.00, \quad Q^*_{41} = 0.00, \quad Q^*_{42} = 0.00.$$

The computed quality levels of the contractors are:

$$q_1^* = 100.00, \quad q_2^* = 100.00,$$

and the computed prices are:

$$\pi_{11}^* = 3,060.70, \quad \pi_{12}^* = 2,515.08, \quad \pi_{21}^* = 3,060.56, \quad \pi_{22}^* = 2,515.16.$$

The total cost of the pharmaceutical firm is 2,171,693.16 and the profits of the contractors are 0.00 and 0.00. The value of $q\prime$ is 100.00.

Because the opportunity costs are all zero, in order to improve the total profit, the contractors will charge the pharmaceutical firm very large prices. Thus, the original firm would rather produce by itself than outsource to the contractors.

One can see, from this example, that, in addition to the total revenue term, each contractor must consider an outsourcing price related term, such as the opportunity cost, in its objective function. Without considering such a function, the outsourcing quantities will all be zero (cf. (14)), and, hence, a contractor would not secure any contracts from the pharmaceutical firm.

Example 4

The data for Example 4 are the same as for Example 1, except the opportunity cost functions and the demand. The demand in R_1 is now 700, and the demand in R_2 is 100.

The contractors' opportunity cost functions now become:

$$oc_{11}(\pi_{11}) = .5(\pi_{11} - 2)^2 - 15265.29, \quad oc_{12}(\pi_{12}) = (\pi_{12} - 1)^2 - 513.93,$$

$$oc_{21}(\pi_{21}) = (\pi_{21} - 1)^2 - 35751.25, \quad oc_{22}(\pi_{22}) = .5(\pi_{22} - 2)^2 - 613.20.$$

The Euler method converged in 14 iterations and yielded the following equilibrium solution. The computed product flows are:

$$Q_{11}^* = 69.12, \quad Q_{12}^* = 19.65, \quad Q_{21}^* = 77.99, \quad Q_{22}^* = 0.00,$$

$$Q_{31}^* = 174.72, \quad Q_{32}^* = 45.34, \quad Q_{41}^* = 378.17, \quad Q_{42}^* = 35.00.$$

The computed quality levels of the contractors are:

$$q_1^* = 77.99, \quad q_2^* = 58.68,$$

and the computed prices are:

$$\pi_{11}^* = 176.73, \quad \pi_{12}^* = 23.67, \quad \pi_{21}^* = 190.08, \quad \pi_{22}^* = 37.02.$$

The total cost of the pharmaceutical firm is 204,701.28 and the profits of the contractors are: 12,366.75 and 7,614.84. The incurred opportunity costs at the equilibrium prices are all zero. Thus, in this example, the equilibrium prices that the contractors charge the firm are such that they are able to adequately recover their costs, and secure contracts.

The value of $q\prime$ is 72.61.

6. Summary and Conclusions

The pharmaceutical industry is faced with numerous challenges driven by economic pressures, coupled with the need to produce and deliver medicinal drugs, which may not only assist in the health of consumers but may also be life-saving. Increasingly pharmaceutical firms have turned to outsourcing the production and delivery of their pharmaceutical products.

In this paper, we developed a supply chain network game theory model, in both equilibrium and dynamic versions, to capture contractor selection, based on the competition among the contractors in the prices that they charge as well as the quality levels of the pharmaceutical products that they produce. We introduced a disrepute cost associated with the average quality at the demand markets. We assumed that the pharmaceutical firm is cost-minimizing whereas the contractors are profit-maximizing.

We utilized variational inequality theory for the formulation of the governing Nash-Bertrand equilibrium conditions and then revealed interesting dynamics associated with the evolution of the firm's product flows, and the contractors' prices and quality levels. We provided stability analysis results as well as an algorithm that can be interpreted as a discretization of the continuous-time adjustment processes. We illustrated the methodological framework through a series of numerical examples for which we reported the complete input and output data for transparency purposes. Our numerical studies included sensitivity analysis results as well as a disruption to the supply chain network in that a contractor is no longer available for production and distribution. We also discussed the scenario in which the opportunity costs on the contractors' side are identically equal to zero and the scenario in which the opportunity costs at the equilibrium are all zero.

This paper is a contribution to the literature on outsourcing with a focus on quality with an emphasis on the pharmaceutical industry which is a prime example of an industry where the quality of a product is paramount. It also is an interesting application of game theory and associated methodologies. The ideas in this paper may be adapted, with appropriate modifications, to other industries.

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