Supply Chain Network Design
Under
Profit Maximization and Oligopolistic Competition

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Abstract: In this paper, we model the supply chain network design problem with oligopolistic firms who are involved in the competitive production, storage, and distribution of a homogeneous product to multiple demand markets. The profit-maximizing firms select both the capacities associated with the various supply chain network activities as well as the product quantities. We formulate the governing Nash-Cournot equilibrium conditions as a variational inequality problem and identify several special cases of the model, notably, a generalization of a spatial oligopoly and a classical oligopoly problem to include design capacity variables. The proposed computational approach, which is based on projected dynamical systems, fully exploits the network structure of the problems and yields closed form solutions at each iteration. In order to illustrate the modeling framework and the algorithm, we also provide solutions to a spectrum of numerical supply chain network oligopoly design examples.

This paper makes a contribution to game theoretic modeling of competitive supply chain network design problems in an oligopolistic setting.

Keywords: profit-maximizing supply chains, supply chain design, network oligopolies, game theory, Nash equilibria, variational inequalities, projected dynamical systems

1. Introduction

Oligopolies are a fundamental economic organizational structure and capture numerous industries and their associated products and services ranging from airlines to particular retailers as well as high technology manufacturers, telecommunication companies, and specific energy providers. Hence, the formulation, analysis, and solution of oligopoly problems are all of wide theoretical and application-based interest in both economics and operations research (cf. Gabay and Moulin (1980), Murphy, Sherali, and Soyster (1982), Dafermos and Nagurney (1987), Tirole (1988), Flam and Ben-Israel (1990), Okuguchi and Szidarovsky (1990), Nagurney (1993, 2006a), Nagurney, Dupuis, and Zhang (1994), and the references therein). In addition, there has been growing interest in the modeling of oligopolies in a supply chain network context, derived, in part, from the recent, notable mergers and acquisitions of firms in such industries as: beverage, airline, financial services, and oil (cf. Nagurney (2009a), Nagurney and Qiang (2009), and the references therein), and the associated need to be able to identify and quantify potential synergies.

In particular, in this paper, we consider the modeling of the explicit supply chain network design problem in the context of oligopolies consisting of firms who compete in a Nash (1950, 1951)-Cournot (1838) framework. As noted in Brown et al. (2001), more firms now understand the potential benefits of controlling the supply chain as a whole. In this paper, we assume that the firms produce a homogeneous good and compete in a noncooperative manner. As in Nagurney (2009a,b), we depict each firm as a network of its economic activities of production, storage, and distribution to demand markets. Of course, the modeling of mergers and acquisitions in a supply chain network context (cf. Nagurney (2009a,b), Nagurney, Woolley, and Qiang (2009), and Nagurney and Woolley (2009)) may be viewed as a supply chain network design problem, broadly classified. However, in this paper, we make explicit the design variables associated with profit-maximizing, competing firms and their supply chains.

The supply chain network design model developed in this paper extends the aforementioned supply chain network models in several significant ways. First, here we explicitly consider oligopolistic behavior and associate demand market price functions with the demand markets, whereas Nagurney (2009b) assumed known demands and assumed that the link capacities were fixed and assigned a priori. In addition, we extend the (pre-merger) oligopolistic supply chain network model of Nagurney (2009a), substantively, by allowing for competition on the production side, in distribution and storage, and, finally, across the demand markets. Moreover, the capacities are now strategic decision variables. Finally, we also extend the original supply chain network design model of Nagurney (2009c), in which there is a single, cost-minimizing firm faced with known demands at the markets for its product and, hence, there is no competition. For flexibility, clarity, and continuity, the multifirm, multimarket, supply chain network design model developed in this paper is formulated as a variational inequality problem (cf. the book by Nagurney (1993), which also contains some of the history of the evolution of network models of firms and numerous network-based economic models).

Supply chain network equilibrium models, initiated by Nagurney, Dong, and Zhang (2002) have been developed, which focus on competition among decision-makers (such as manufacturers, distributors, and retailers) at a tier of the supply chain but cooperation between tiers. The relationships of such supply chain network equilibrium problems to transportation network equilibrium problems, which are characterized by user-optimizing behavior have also been established (cf. Nagurney (2006a,b)). Zhang, Dong, and Nagurney (2003) and Zhang (2006), on the other hand, modeled competition among supply chains in a supply chain economy context, but did not consider explicit firms. See the book by Nagurney (2006a) for a spectrum of supply chain network equilibrium models and applications.

The supply chain network design model developed in this paper, in contrast to the ones immediately above, is more detailed, since it is at the level of the firms. In addition, the model in this paper is the first to capture the design aspects of profit-maximizing oligopolistic firms, who compete in multiple functions of production, distribution, and storage, and who seek to determine not only the quantities of the product but also the capacities of the various manufacturing plants, distribution centers, etc. In addition, in this paper, we identify special cases of the proposed model. Furthermore, unlike much of the classical supply chain network literature (cf. Beamon (1998), Min and Zhou (2002), Handfield and Nichols Jr. (2002), and Meixell and Gargeya (2005) for surveys and Geunes and Pardalos (2003) for an annotated bibliography) in our framework we do not need discrete variables and we use continuous variables exclusively in our model formulations. In addition, we are not limited to linear costs; but, rather, the model can handle nonlinear costs, which can capture the reality of today's networks from transportation to telecommunication ones, which may be congested (cf. Nagurney and Qiang (2009)), and upon which supply chains depend for their functionality. The solution of our oligopolistic supply chain network design model yields the optimal supply chain network topology since those links with zero optimal capacities can, in effect, be eliminated.

This paper is organized as follows. In Section 2, we develop the supply chain network design model in which the firms seek to determine the capacities associated with manufacturing plants, distribution centers, and shipment links, as well as the production quantities so as to maximize their profits and to satisfy the associated demand at the multiple demand markets. We consider competition in production, distribution, and storage, in that we allow the underlying functions to depend on the flows not only of the particular firm but, in general, on the product flows of all the firms. In addition, we assume that the demand price of the product at any given demand market may, in general, depend upon the demand for the product at every market. Hence, we also assume the general situation that there is competition on the demand side. We provide the game theoretic formulation of the problem, state the governing Nash-Cournot equilibrium conditions, and provide alternative variational inequality formulations. In addition, we identify several oligopoly models that are special cases of the new model and that also enhance (since they include capacities as strategic decision variables) more classical models, be they spatial (see, e.g., Dafermos and Nagurney (1987)) or aspatial (cf. Cournot (1838) and Nagurney (1993) and the references therein).

In Section 3, we consider the solution of the supply chain network design model. We propose the Euler method, which is a special case of the general iterative scheme introduced by Dupuis and Nagurney (1993) for the determination of stationary points of projected dynamical systems; equivalently, solutions of variational inequality problems. We demonstrate that, in the context of the model, the Euler method resolves the supply chain network design problem into subproblems that can be solved, at each iteration, explicitly and in closed form. We provide the associated formulae. A variety of network economic equilibrium problems (and the associated tatonnement/adjustment processes) have been modeled and solved to-date as projected dynamical systems, including dynamic spatial price problems

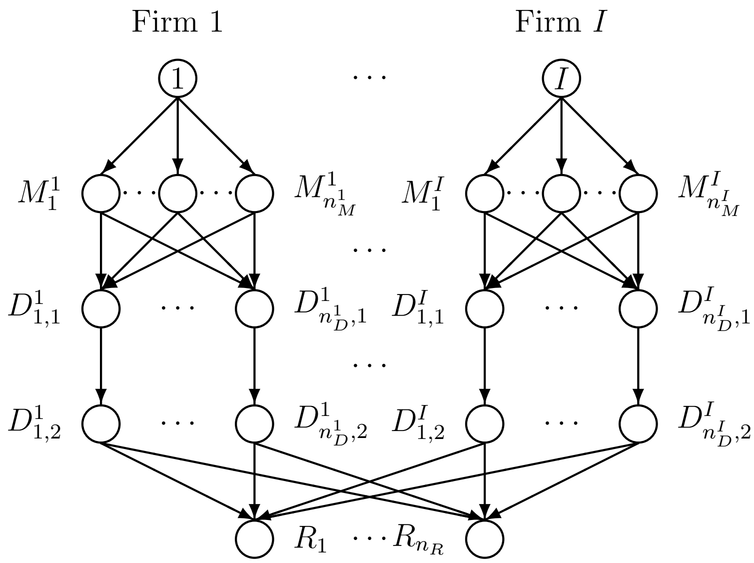

Figure 1: The Initial Supply Chain Network Topology of the Oligopoly

(see Nagurney, Takayama, and Zhang (1995) and Nagurney and Zhang (1996a)), dynamic network oligopolies (cf. Nagurney, Dupuis, and Zhang (1994)), and dynamic traffic network equilibrium problems (see, e.g., Zhang and Nagurney (1997) and Yang and Zhang (2009)).

In Section 4, we provide numerical examples. In Section 5, we summarize the results in this paper and give suggestions for future research.

2. The Supply Chain Network Design Model

In this Section, we develop the supply chain network design model with profit-maximizing firms and identify several special cases. We consider a finite number of I firms, with a typical firm denoted by i, who are involved in the production, storage, and distribution of a homogeneous product and who compete noncooperatively in an oligopolistic manner.

We assume that each firm is represented as an initial network of its economic activities (cf. Figure 1). Each firm seeks to determine its optimal link capacities and product quantities by using Figure 1 as a schematic. Each firm i; $i = 1, \ldots, I$, hence, is considering n^i_M manufacturing facilities/plants; n^i_D distribution centers, and serves the same n_R retail outlets/demand markets. Let L_i^0 denote the set of directed links representing the economic activities associated with firm i; $i = 1, \ldots, I$ and let $n_{L_i^0}$ denote the number of links in L_i^0 with n_{L^0} denoting the total number of links in the initial network L^0, where $L^0 \equiv \cup_{i=1,I} L_i^0$. Let $G^0 = [N^0, L^0]$ denote the graph consisting of the set of nodes N^0 and the set of links L^0 in Figure 1. Once the supply chain design problem is solved, those links with zero optimal capacities (and, hence, also zero product flows) can, in effect, be eliminated from the final supply chain network design topology.

The links from the top-tiered nodes i; $i = 1, \ldots, I$, representing the respective firm, in Figure 1 are connected to the manufacturing nodes of the respective firm i, which are denoted, respectively, by: $M_1^i, \ldots, M_{n_M^i}^i$, and these links represent the manufacturing links. The links from the manufacturing nodes, in turn, are connected to the distribution center nodes of each firm i; $i = 1, \ldots, I$, which are denoted by $D_{1,1}^i, \ldots, D_{n_D{}^i,1}^i$. These links correspond to the shipment links between the manufacturing plants and the distribution centers where the product is stored. The links joining nodes $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$ with nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = 1, \ldots, I$ correspond to the storage links. Finally, there are possible shipment links joining the nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = 1, \ldots, I$ with the demand market nodes: $R_1, \ldots, R_{n_R}$.

We emphasize that the network topology in Figure 1, which corresponds to G^0 is given here for descriptive purposes and, for definiteness. In fact, the model, described fully below, can handle any prospective supply chain network topology provided that there is a top-tiered node to represent each firm and bottom-tiered nodes to represent the demand markets with a sequence of directed links, corresponding to at least one path, joining each top-tiered node with each bottom-tiered node. The solution of the complete model will identify which links have positive capacities and, hence, should be retained in the final supply chain network design.

We assume that associated with each link (cf. Figure 1) of the network corresponding to each firm i; $i = 1, \ldots, I$ is a total cost associated with operating the link over the time horizon under consideration. We also assume that there is a total design cost associated with each link. We denote, without any loss in generality, the links by a, b, etc., the total operational cost on a link a by $\hat{c}_a$ and the total design cost by $\hat{\pi}_a$, for all links $a \in L^0$.

Let d_{R_k} denote the demand for the product at demand market R_k; $k = 1, \ldots, n_R$. Let x_p

denote the nonnegative flow of the product on path p joining (origin) node i; $i = 1, \ldots, I$ with a (destination) demand market node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^0_{R_k}} x_p = d_{R_k}, \quad k = 1, \ldots, n_R, \tag{1}$$

where $P^0_{R_k}$ denotes the set of paths connecting the (origin) nodes $i; i = 1, \ldots, I$ with (destination) demand market R_k. In particular, we have that $P^0_{R_k} = \cup_{i=1,\ldots,I} P^0_{R^i_k}$, where $P^0_{R^i_k}$ denotes the set of paths from origin node i to demand market k as in Figure 1.

According to (1), the demand at each demand market must be equal to the sum of the product flows from all firms to that demand market.

We assume that there is a demand price function associated with the product at each demand market. We denote the demand price at demand market R_k by ρ_{R_k} and we assume, as given, the demand price functions:

$$\rho_{R_k} = \rho_{R_k}(d), \quad k = 1, \ldots, n_R, \tag{2}$$

where d is the n_R-dimensional vector of demands at the demand markets. We assume that all vectors in this paper are column vectors. Note that we consider the general situation where the price for the product at a particular demand market may, in general, depend upon the demand for the product at the other demand markets. We assume that the demand price functions are continuous, continuously differentiable, and monotone decreasing. Note that the consumers at each demand market are indifferent as to which firm produced the homogeneous product.

In addition, we let f_a denote the flow of the product on link a. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^0} x_p \delta_{ap}, \quad \forall a \in L^0, \tag{3}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise. Here P^0 denotes the set of *all* paths in Figure 1, that is, $P^0 = \cup_{k=1,\ldots,n_R} P^0_{R_k}$. There are n_{P^0} paths in the network in Figure 1. We use P^0_i to denote the set of all paths from firm i to all the demand markets for $i = 1, \ldots, I$. There are $n_{P^0_i}$ paths from the firm i node to the demand markets.

Of course, we also have that the path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^0. \tag{4}$$

Let $u_a, a \in L^0$, denote the design capacity of link a, where we must have that

$$f_a \leq u_a, \quad \forall a \in L^0, \tag{5}$$

or, in view of (3)

$$\sum_{p \in P^0} x_p \delta_{ap} \leq u_a, \quad \forall a \in L^0. \tag{6}$$

In other words, the product flow on each link is bounded by the capacity (which is a strategic decision variable) on the link.

The total operational cost on a link, be it a manufacturing/production link, a shipment/distribution link, or a storage link is assumed, in general, to be a function of the flows of the product on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L^0, \tag{7}$$

where f is the vector of all the link flows.

In addition, we assume that the total design cost associated with each link is a function of the design capacity on the link, that is,

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^0. \tag{8}$$

Let X_i denote the vector of strategy variables associated with firm i; $i = 1, \ldots, I$, where X_i is the vector of path flows associated with firm i, and the vector of link capacities, that is, $X_i \equiv \{\{\{x_p\}|p \in P_i^0\}; \{\{u_a\}|a \in L_i^0\}\} \in R_+^{n_{P_i^0}+n_{L_i^0}}$. X is then the vector of all the firms' strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}\}$.

The profit function U_i of firm i; $i = 1, \ldots, I$, is the difference between the firm's revenue and its total costs, that is,

$$U_i = \sum_{k=1}^{n_R} \rho_{R_k}(d) \sum_{p \in P^0_{R^i_k}} x_p - \sum_{a \in L_i^0} \hat{c}_a(f) - \sum_{a \in L_i^0} \hat{\pi}_a(u_a). \tag{9}$$

In view of (1) – (9), we may write:

$$U = U(X), \tag{10}$$

where U is the I-dimensional vector of the firms' profits.

We now consider the oligopolistic market mechanism in which the I firms select their supply chain network link capacities and product path flows in a noncooperative manner, each one trying to maximize its own profit. We seek to determine a path flow and capacity pattern X for which the I firms will be in a state of equilibrium as defined below.

Definition 1: Supply Chain Network Design Cournot-Nash Equilibrium

A path flow and design capacity pattern $X^* \in \mathcal{K}^0 = \prod_{i=1}^I \mathcal{K}_i^0$ *is said to constitute a supply chain network design Cournot-Nash equilibrium if for each firm* i*;* $i = 1, \ldots, I$*:*

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in \mathcal{K}_i^0, \tag{11}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ *and* $\mathcal{K}_i^0 \equiv \{X_i | X_i \in R_+^{n_{P_i^0}+n_{L_i^0}}$*, and (6) is satisfied*$\}$.

Note that, according to (11), a supply chain network design Cournot-Nash equilibrium has been established if no firm can increase its profits unilaterally.

The variational inequality formulation of the Cournot-Nash (Cournot (1838), Nash (1950, 1951)) supply chain network design problem satisfying Definition 1 is given in the following theorem.

Theorem 1

Assume that for each firm i*;* $i = 1, \ldots, I$*, the profit function* $U_i(X)$ *is concave with respect to the variables in* X_i*, and is continuously differentiable. Then* $X^* \in \mathcal{K}^0$ *is a supply chain network design Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} U_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in \mathcal{K}^0, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ *denotes the inner product in the corresponding Euclidean space and* $\nabla_{X_i} U_i(X)$ *denotes the gradient of* $U_i(X)$ *with respect to* X_i*. The solution of variational inequality (12)* *is equivalent to the solution of variational inequality: determine* $(x^*, u^*, \lambda^*) \in \mathcal{K}^1$ *satisfying:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P^0_{R^i_k}}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^0}\lambda^*_a\delta_{ap}-\rho_{R_k}(x^*)-\sum_{l=1}^{n_R}\frac{\partial \rho_{R_l}(x^*)}{\partial d_{R_k}}\sum_{p\in P^0_{R^i_k}}x^*_p\right]\times[x_p-x^*_p]$$

$$+\sum_{a\in L^0}\left[\frac{\partial \hat{\pi}_a(u^*_a)}{\partial u_a}-\lambda^*_a\right]\times[u_a-u^*_a]$$

$$+\sum_{a\in L^0}\left[u^*_a-\sum_{p\in P^0}x^*_p\delta_{ap}\right]\times[\lambda_a-\lambda^*_a]\geq 0,\quad \forall (x,u,\lambda)\in\mathcal{K}^1, \tag{13}$$

where $\mathcal{K}^1 \equiv \{(x,u,\lambda)|x\in R^{n_{P^0}}_+; u\in R^{n_{L^0}}_+; \lambda\in R^{n_{L^0}}_+\}$ *and* $\frac{\partial \hat{C}_p(x)}{\partial x_p}\equiv \sum_{b\in L^0_i}\sum_{a\in L^0_i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}$ *for paths* $p\in P^0_i$*;* $i=1,\ldots,I$*.*

Proof: Variational inequality (12) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). Here we have also utilized the fact that the demand price functions (2) can be reexpressed in light of (1) directly as a function of path flows as can the total operational cost functions (7). Variational inequality (13) follows, in turn, from Bertsekas and Tsitsiklis (1989) with notice that λ^* corresponds to the vector of optimal Lagrange multipliers associated with constraints (6).

Variational inequality (13) can be put into standard form as below: determine $X^*\in\mathcal{K}$ such that

$$\langle F(X^*)^T, X-X^*\rangle \geq 0,\quad \forall X\in\mathcal{K}, \tag{14}$$

where $\mathcal{K}$ is closed and convex and $F(X)$ is a continuous function from $\mathcal{K}$ to R^n. Indeed, we can define $\mathcal{K}\equiv\mathcal{K}^1$ and let $F(X)$ be the vector with $n_{P^0}+2n_{L^0}$ components given by the specific terms preceding the first multiplication sign in (13), the second in (13), and the third in (13).

It is interesting to relate this supply chain network design oligopoly model to the spatial oligopoly model proposed by Dafermos and Nagurney (1987), which is done in the following corollary.

Corollary 1

Assume that there are I firms in the supply chain network design oligopoly model and that each firm is considering a single manufacturing plant and a single distribution center. Assume also that the distribution costs from each manufacturing plant to the distribution center and the storage costs are all equal to zero. Then the resulting model is a generalization of the spatial oligopoly model of Dafermos and Nagurney (1987) with the inclusion of design capacities as strategic variables and whose underlying initial network structure is given in Figure 2.

Proof: Follows from Dafermos and Nagurney (1987) and Nagurney (1993).

Hence, the above supply chain network design oligopoly model, as a spcial case, provides us with an extension to the spatial oligopoly model of Dafermos and Nagurney (1987). It is also interesting to note that in the new supply chain network design oligopoly model there is competition on the supply and distribution sides as well as on the demand side, since the cost functions associated with production, with distribution, and with storage may, in general, depend upon not only the flows of the particular firm but, rather, on the flows of all the firms. In Dafermos and Nagurney (1987) and in Nagurney (1993) the network structure of the spatial oligopoly problem is depicted as a bipartite graph with the manufacturing assumed to take place at the manufacturing/production nodes. In the supply chain network design formalism here, we associate the production with links, and the outputs are, hence, the link flows, and, thus, Figure 2 makes this explicit. Dafermos and Nagurney (1987) also establish the relationships between spatial Cournot oligopolies and perfectly competitive spatial price equilibrium problems (cf. Dafermos and Nagurney (1987)). Nagurney, Dupuis, and Zhang (1994), in turn, developed a dynamic version of the model of Dafermos and Nagurney (1987) using projected dynamical systems theory (see also Nagurney and Zhang (1996b)). Of course, those models, unlike the ones proposed in this paper, did not have the capacity design variables in their formulations. Our framework allows for such a generalization and, as we will see in the following sections, without much additional cost in terms of computations and problem solution.

It is also interesting to relate the supply chain network oligopoly model to the classi-

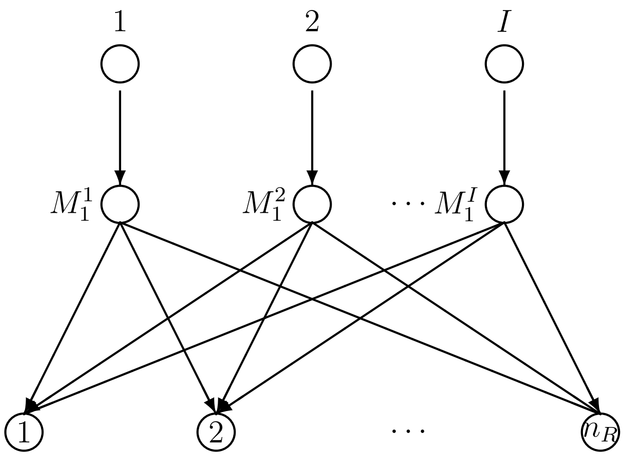

Figure 2: The Initial Network Topology of the Spatial Oligopoly Design Problem

cal Cournot (1838) oligopoly model in quantity variables, which has been formulated as a variational inequality problem by Gabay and Moulin (1980) and has been studied by both economists and operations researchers (cf. Murphy, Sherali, and Soyster (1982), Flam and Ben-Israel (1990), Nagurney (1993), and the references therein). Indeed, we have the following corollary, the proof of which is immediate.

Corollary 2

Assume that there is a single manufacturing plant associated with each firm in the above supply chain network design model and a single distribution center. Assume also that there is a single demand market. Assume that the manufacturing cost of each firm depends only upon its own output. Then, if the storage and distribution cost functions are all identically equal to zero the above design model collapses to an extension of the classical oligopoly model in quantity variables and with capacity design variables. Furthermore, if $I = 2$, one then obtains a generalization of the classical duopoly model.

The initial network structure for the classical oligopoly problem (see also Nagurney (1993, 2009a)) is depicted in Figure 3. The classical model is an aspatial model since there are no explicit transportation/transaction costs between the firms/manufacturers and the demand

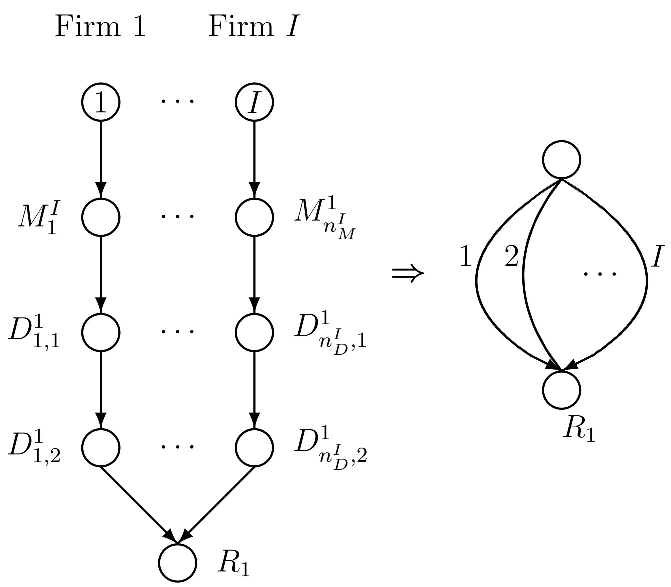

Figure 3: The Initial Network Topology of the "Classical" Oligopoly Design Problem

market.

Existence results for both spatial and aspatial oligopoly problems can be found in Nagurney and Zhang (1996b) and the references therein. Typically, either strong monotonicity or coercivity conditions are imposed on the relevant functions in order to guarantee existence of a Cournot-Nash equilibrium in such applications.

We emphasize that with the variational inequality formulation of the competitive supply chain network design problem, as given above, we can now:

1. consider problems in which there are nonlinearities as well as asymmetries in the underlying functions, for which an optimization formulation would no longer suffice and

2. obtain further insights into competitive oligopolistic equilibrium problems that characterize a variety of industries with the generalization of the inclusion of explicit design variables.

Moreover, as is well-known, intuition in the case of equilibrium problems, as opposed to optimization problems, may be easily confounded (as in the Braess paradox; see Braess (1968) and Braess, Nagurney, and Wakolbinger (2005)). Hence, an oligopolistic supply chain network design framework, which allows for the computation of solutions, can allow firms to investigate their optimal strategies in the presence of competition.

3. The Algorithm

In this Section, we recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of supply chain network design problems governed by variational inequality (13) yields subproblems that can be solved explicitly and in closed form.

Specifically, recall that at an iteration τ of the Euler method (see also Nagurney and Zhang (1996b)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{15}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous (see also (14)).

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996b), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996b) and the references therein.

Explicit Formulae for the Euler Method Applied to the Supply Chain Network Design Variational Inequality (13)

The elegance of this procedure for the computation of solutions to the supply chain network design problem modeled in Section 2 can be seen in the following explicit formulae. Indeed, (15) for the supply chain design network oligopoly problem governed by variational inequality problem (13) yields the following closed form expressions for the product path flows, the design capacities, and the Lagrange multipliers, respectively:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\rho_{R_k}(x^\tau) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^\tau)}{\partial d_{R_k}} \sum_{p \in P^0_{R^i_k}} x_p^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L^0} \lambda_a^\tau \delta_{ap})\}, \; \forall i, \forall k, \forall p \in P^0_{R^i_k}; \tag{17}$$

$$u_a^{\tau+1} = \max\{0, u_a^\tau + a_\tau(\lambda_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L^0; \tag{18}$$

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + a_\tau(\sum_{p \in P^0} x_p^\tau \delta_{ap} - u_a^\tau)\}, \quad \forall a \in L^0. \tag{19}$$

In the next Section, we solve supply chain network oligopoly design problems using this algorithmic scheme.

4. Numerical Examples

In this Section, we present numerical supply chain network design oligopoly examples of increasing complexity. We implemented the Euler method, as discussed in Section 3. The algorithm code were implemented in FORTRAN and the system used for the computations was a Unix system at the University of Massachusetts Amherst. The convergence tolerance was: $|X^{\tau+1} - X^\tau| \leq 10^{-6}$ for all the examples. The sequence $\{a_\tau\}$ used (cf. (15)) was: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots, \}$. We initialized the Euler method as follows. We set the demand for the product at 100 for each demand market and equally distributed the demand among all the paths. The initial link design capacities as well as the initial Lagrange multipliers were all set to zero.

Example 1

Example 1 consisted of four firms with the prospective supply chain network topology as in Figure 4. Each firm was considering a single manufacturing plant, a single distribution center, and a single demand market.

For simplicity, we let all the total operational cost functions on the links be equal and given by:

$$\hat{c}_a(f) = 2f_a^2 + f_a, \quad \forall a \in L_i^0; i = 1, 2, 3, 4. \tag{20}$$

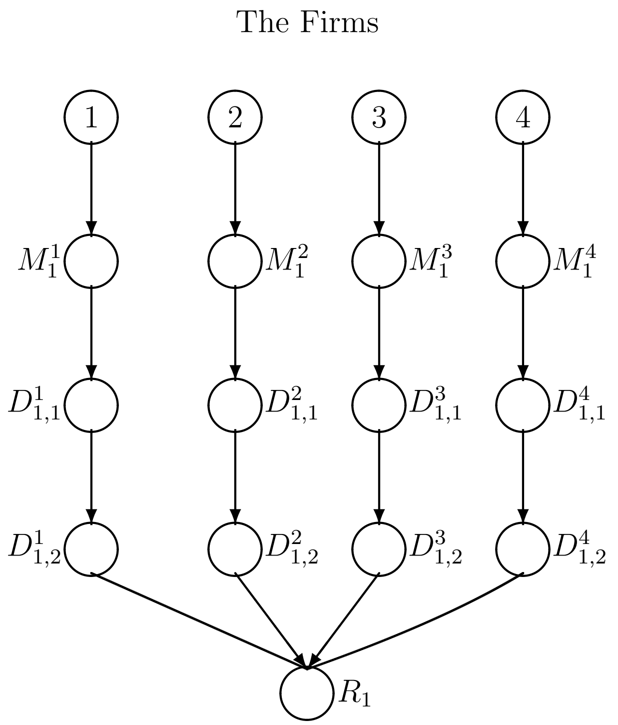

Figure 4: The Initial Network Topology for Examples 1 and 2

The total design cost functions on all the links were:

$$\hat{\pi}_a(u_a) = 5u_a^2 + u_a, \quad \forall a \in L_i^0; i = 1, 2, 3, 4. \tag{21}$$

The demand price function at the single demand market was:

$$\rho_{R_1}(d) = -d_{R_1} + 200. \tag{22}$$

We denoted the paths by p_1, p_2, p_3, and p_4 corresponding to firm 1 through firm 4, respectively, with each path originating in its top-most firm node and ending in the demand market node (cf. Figure 4).

The Euler method converged to the equilibrium solution:

$$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = x_{p_4}^* = 3.15,$$

$$u_a^* = 3.15, \forall a \in L^0, \quad f_a^* = 3.15, \forall a \in L^0,$$

$$\lambda_a^* = 32.46, \forall a \in L^0.$$

Hence, the supply chain network design, since all links had positive capacities (as well as positive flows), had the topology as in Figure 4.

The demand was 12.60 and the demand market price was $\rho_{R_1} = 187.40$. Each firm earned a profit of 287.88.

Example 2

Example 2 was constructed from Example 1 and had the same data except that the demand price at the demand market was greatly reduced to:

$$\rho_{R_1}(d) = -d_{R_1} + 5. \tag{23}$$

The Euler method converged to the solution with all path flows, link flows, and design capacities equal to 0.00 with the Lagrange multipliers all equal to 1.05. Hence, not one of the firms enters into this market and none of these firms produces the product. Therefore, the final supply chain network design is essentially the null set.

Example 3

Example 3 had the initial supply chain network topology as in Figure 5, where we also define the links. Specifically, there were two firms considering two manufacturing plants each, two distribution centers each, and considering serving a single demand market.

The total cost data, along with the computed link flows, capacities, and Lagrange multipliers, are reported in Table 1. This example represents the following scenario. Let the first firm be located in the US, for example, where there are higher manufacturing costs and also higher costs associated with both constructing the manufacturing plants and the distribution centers; see: $\hat{c}_1$, $\hat{c}_5$, and $\hat{\pi}_1$, $\hat{\pi}_5$. The second firm, on the other hand, is located outside the US where there are lower manufacturing costs and storage costs as well as associated design costs for the facilities; see $\hat{c}_{11}$, $\hat{c}_{15}$, and $\hat{\pi}_{11}$, $\hat{\pi}_{15}$. However, the demand market is in the US

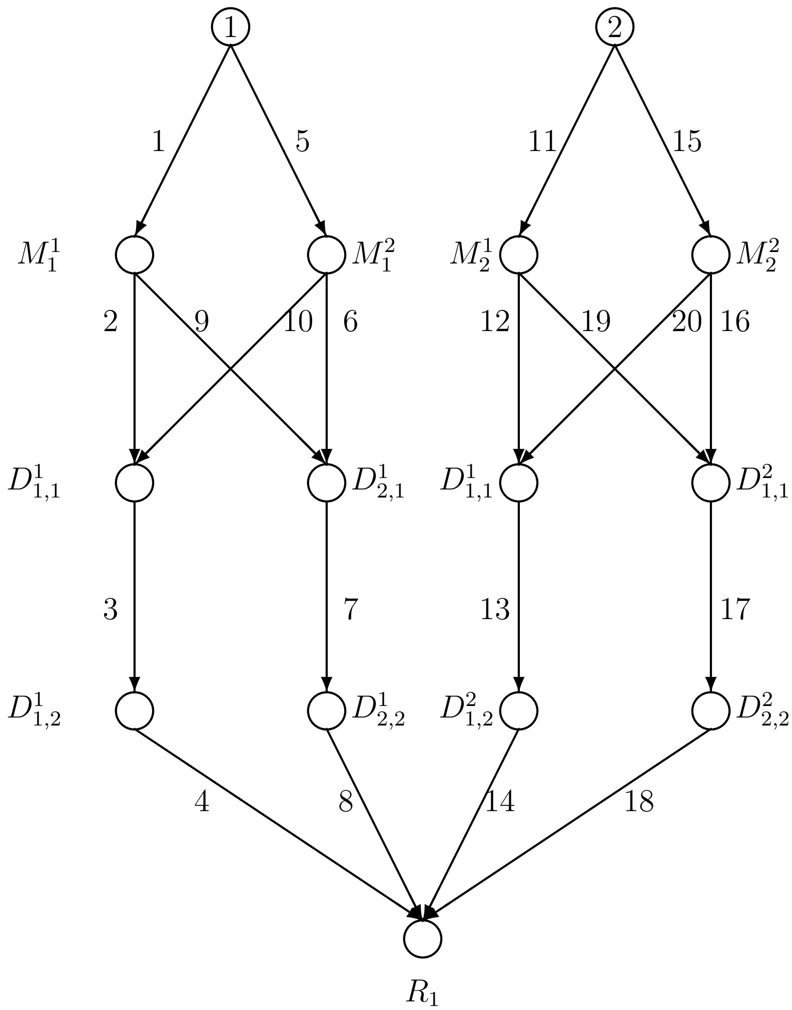

Figure 5: The Initial Supply Chain Network Topology for Examples 3 and 4

Table 1: Total Cost Functions and Solution for Example 3

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$2f_1^2 + 2f_1$	$5u_1^2 + u_1$	7.28	7.28	74.05
2	$f_2^2 + 2f_2$	$.5u_2^2 + u_2$	3.98	3.98	4.98
3	$2.5f_3^2 + f_3$	$5u_3^2 + 2u_3$	7.54	7.54	76.23
4	$f_4^2 + f_4$	$.5u_4^2 + u_4$	7.54	7.54	8.54
5	$3f_5^2 + 2f_5$	$6u_5^2 + u_5$	5.73	5.73	70.05
6	$.5f_6^2 + f_6$	$.5u_6^2 + u_6$	2.17	2.17	3.17
7	$1.5f_7^2 + f_7$	$10u_7^2 + u_7$	5.47	5.47	110.07
8	$f_8^2 + f_8$	$.5u_8^2 + u_8$	5.47	5.47	6.48
9	$.5f_9^2 + f_9$	$.5u_9^2 + u_9$	3.30	3.30	4.30
10	$f_{10}^2 + f_{10}$	$.5u_{10}^2 + u_{10}$	3.56	3.56	4.56
11	$.5f_{11}^2 + f_{11}$	$4u_{11}^2 + u_{11}$	7.15	7.15	58.23
12	$f_{12}^2 + f_{12}$	$.5u_{12}^2 + u_{12}$	2.98	2.98	3.98
13	$.5f_{13}^2 + f_{13}$	$2.5u_{13}^2 + u_{13}$	7.14	7.14	36.72
14	$4f_{14}^2 + f_{14}$	$5u_{14}^2 + 5u_{14}$	7.14	7.14	76.19
15	$f_{15}^2 + f_{15}$	$3u_{15}^2 + u_{15}$	8.12	8.12	49.76
16	$f_{16}^2 + f_{16}$	$.5u_{16}^2 + u_{16}$	3.96	3.96	4.96
17	$.5f_{17}^2 + f_{17}$	$2.5u_{17}^2 + u_{17}$	8.13	8.13	41.66
18	$3.5f_{18}^2 + f_{18}$	$4u_{18}^2 + 2u_{18}$	8.13	8.13	66.89
19	$f_{19}^2 + f_{19}$	$.5u_{19}^2 + u_{19}$	4.17	4.17	5.17
20	$.5f_{20}^2 + f_{20}$	$.5u_{20}^2 + u_{20}$	4.16	4.16	5.16

and, hence, the second firm faces higher transportation costs to deliver the product to the demand market, as can be seen from the function data in Table 1; see $\hat{c}_{14}$ and $\hat{c}_{18}$ versus $\hat{c}_4$ and $\hat{c}_8$.

The demand price function was:

$$\rho_{R_1}(d) = -d_{R_1} + 300. \tag{24}$$

For completeness, we also provide the computed equilibrium path flows. There were four paths for each firm and we label the paths as follows (please refer to Figure 5): for firm 1:

$$p_1 = (1, 2, 3, 4), \quad p_2 = (1, 9, 7, 8), \quad p_3 = (5, 6, 7, 8), \quad p_4 = (5, 10, 3, 4),$$

for firm 2:

$$p_5 = (11, 12, 13, 14), \quad p_6 = (11, 19, 17, 18), \quad p_7 = (15, 16, 17, 18), \quad p_8 = (15, 20, 13, 14).$$

The computed equilibrium path flow pattern was:

$$x^*_{p_1} = 3.98, \quad x^*_{p_2} = 3.30, \quad x^*_{p_3} = 2.17, \quad x^*_{p_4} = 3.56,$$

$$x^*_{p_5} = 2.98, \quad x^*_{p_6} = 4.17, \quad x^*_{p_7} = 3.96, \quad x^*_{p_8} = 4.16.$$

Example 4

Example 4 also had the initial supply chain network topology as in Figure 5. Example 4 had the same data as Example 3 except we reduced the capacity design costs associated with the shipment links from the second firm to the demand market as in Table 2; see $\hat{\pi}_{14}$ and $\hat{\pi}_{18}$.

The total cost data, along with the computed link flows, capacities, and Lagrange multipliers, are reported in Table 2.

The demand price was: 267.47 and the total profit earned by both firms was: 4,493.29.

For completeness, we also provide the computed equilibrium path flows, which were:

$$x^*_{p_1} = 3.92, \quad x^*_{p_2} = 3.25, \quad x^*_{p_3} = 2.13, \quad x^*_{p_4} = 3.51,$$

$$x^*_{p_5} = 4.04, \quad x^*_{p_6} = 5.19, \quad x^*_{p_7} = 4.89, \quad x^*_{p_8} = 5.60.$$

One can see that the second firm manufactured a greater volume of the product than the first firm and provided more of the product to the consumers at the demand market. Hence, in this example the firm that had lower overseas costs was more competitive once the shipment costs were further reduced.

Note that the final supply chain network topology under the optimal design for Example 4 remained as in Figure 5.

Table 2: Total Cost Functions and Solution for Example 4

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$2f_1^2 + 2f_1$	$5u_1^2 + u_1$	7.17	7.17	72.87
2	$f_2^2 + 2f_2$	$.5u_2^2 + u_2$	3.92	3.92	4.92
3	$2.5f_3^2 + f_3$	$5u_3^2 + 2u_3$	7.42	7.42	75.01
4	$f_4^2 + f_4$	$.5u_4^2 + u_4$	7.42	7.42	8.42
5	$3f_5^2 + 2f_5$	$6u_5^2 + u_5$	5.64	5.64	68.93
6	$.5f_6^2 + f_6$	$.5u_6^2 + u_6$	2.13	2.13	3.14
7	$1.5f_7^2 + f_7$	$10u_7^2 + u_7$	5.39	5.39	108.30
8	$f_8^2 + f_8$	$.5u_8^2 + u_8$	5.39	5.39	6.39
9	$.5f_9^2 + f_9$	$.5u_9^2 + u_9$	3.25	3.25	4.25
10	$f_{10}^2 + f_{10}$	$.5u_{10}^2 + u_{10}$	3.51	3.51	4.50
11	$.5f_{11}^2 + f_{11}$	$4u_{11}^2 + u_{11}$	9.23	9.23	74.74
12	$f_{12}^2 + f_{12}$	$.5u_{12}^2 + u_{12}$	4.04	4.04	5.04
13	$.5f_{13}^2 + f_{13}$	$2.5u_{13}^2 + u_{13}$	9.64	9.64	49.24
14	$4f_{14}^2 + f_{14}$	$.5u_{14}^2 + u_{14}$	9.64	9.64	10.64
15	$f_{15}^2 + f_{15}$	$3u_{15}^2 + u_{15}$	10.49	10.49	63.90
16	$f_{16}^2 + f_{16}$	$.5u_{16}^2 + u_{16}$	4.89	4.89	5.89
17	$.5f_{17}^2 + f_{17}$	$2.5u_{17}^2 + u_{17}$	10.08	10.08	51.44
18	$3.5f_{18}^2 + f_{18}$	$.5u_{18}^2 + u_{18}$	10.08	10.08	11.08
19	$f_{19}^2 + f_{19}$	$.5u_{19}^2 + u_{19}$	5.19	5.19	6.19
20	$.5f_{20}^2 + f_{20}$	$.5u_{20}^2 + u_{20}$	5.60	5.60	6.60

Example 5

Example 5 had the same data as Example 4 but now we added a second demand market with the initial supply chain network topology being as depicted in Figure 6. The links are labeled on that figure. We assumed that the second demand market was located in the US with the shipment costs set to reflect this scenario. The demand price function for the first demand market remained as in Examples 3 and 4. The demand price function for the new demand market was:

$$\rho_{R_2}(d) = -2d_{R_2} + 500. \tag{25}$$

The remainder of the input data and the computed solution are given in Table 3.

For completeness, we also provide the computed path flows. We retained the numbering and the definitions of the first eight paths for the first demand market as in Examples 3 and 4 (but associated now with Figure 6). The new paths for the first firm to the second demand market are defined as:

$$p_9 = (1, 2, 3, 21), \quad p_{10} = (1, 9, 7, 22), \quad p_{11} = (5, 6, 7, 22), \quad p_{12} = (5, 10, 3, 21),$$

and for the second firm to the second demand market as:

$$p_{13} = (11, 12, 13, 23), \quad p_{14} = (11, 19, 17, 24), \quad p_{15} = (15, 16, 17, 24), \quad p_{16} = (15, 20, 13, 23).$$

The computed path flow solution follows:

$$x^*_{p_1} = 0.00, \quad x^*_{p_2} = 0.00, \quad x^*_{p_3} = 0.00, \quad x^*_{p_4} = 0.00,$$

$$x^*_{p_5} = 0.49, \quad x^*_{p_6} = 0.72, \quad x^*_{p_7} = 0.79, \quad x^*_{p_8} = 1.84.$$

$$x^*_{p_9} = 5.75, \quad x^*_{p_{10}} = 4.54, \quad x^*_{p_{11}} = 3.04, \quad x^*_{p_{12}} = 5.08,$$

$$x^*_{p_{13}} = 6.49, \quad x^*_{p_{14}} = 8.15, \quad x^*_{p_{15}} = 7.55, \quad x^*_{p_{16}} = 7.84.$$

It is very interesting to note that the first firm no longer provides any of the product to the first demand market and, in fact, all its associated path flows to the first demand market are now equal to zero as are the link flows on links 4 and 8. In addition, the associated design capacities on links 4 and 8 are also equal to zero. Thus, for Example 5, the optimal supply chain network design is as given in Figure 7.

Table 3: Total Cost Functions and Solution for Example 5

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$2f_1^2 + 2f_1$	$5u_1^2 + u_1$	10.29	10.29	104.12
2	$f_2^2 + 2f_2$	$.5u_2^2 + u_2$	5.75	5.75	6.75
3	$2.5f_3^2 + f_3$	$5u_3^2 + 2u_3$	10.83	10.83	109.09
4	$f_4^2 + f_4$	$.5u_4^2 + u_4$	0.00	0.00	0.04
5	$3f_5^2 + 2f_5$	$6u_5^2 + u_5$	8.12	8.12	98.65
6	$.5f_6^2 + f_6$	$.5u_6^2 + u_6$	3.04	3.04	4.04
7	$1.5f_7^2 + f_7$	$10u_7^2 + u_7$	7.58	7.58	151.81
8	$f_8^2 + f_8$	$.5u_8^2 + u_8$	0.00	0.00	0.56
9	$.5f_9^2 + f_9$	$.5u_9^2 + u_9$	4.54	4.54	5.54
10	$f_{10}^2 + f_{10}$	$.5u_{10}^2 + u_{10}$	5.08	5.08	6.08
11	$.5f_{11}^2 + f_{11}$	$4u_{11}^2 + u_{11}$	15.84	15.84	127.61
12	$f_{12}^2 + f_{12}$	$.5u_{12}^2 + u_{12}$	6.98	6.98	7.98
13	$.5f_{13}^2 + f_{13}$	$2.5u_{13}^2 + u_{13}$	16.66	16.66	84.34
14	$4f_{14}^2 + f_{14}$	$.5u_{14}^2 + u_{14}$	2.33	2.33	3.33
15	$f_{15}^2 + f_{15}$	$3u_{15}^2 + u_{15}$	18.02	18.02	109.00
16	$f_{16}^2 + f_{16}$	$.5u_{16}^2 + u_{16}$	8.34	8.34	9.34
17	$.5f_{17}^2 + f_{17}$	$2.5u_{17}^2 + u_{17}$	17.20	17.20	87.04
18	$3.5f_{18}^2 + f_{18}$	$.5u_{18}^2 + u_{18}$	1.51	1.51	2.51
19	$f_{19}^2 + f_{19}$	$.5u_{19}^2 + u_{19}$	8.86	8.86	9.86
20	$.5f_{20}^2 + f_{20}$	$.5u_{20}^2 + u_{20}$	9.68	9.68	10.68
21	$.5f_{21}^2 + f_{21}$	$2.5u_{21}^2 + f_{21}$	10.83	10.83	23.66
22	$f_{22}^2 + f_{22}$	$.5u_{22}^2 + f_{22}$	7.58	7.58	17.16
23	$2f_{23}^2 + f_{23}$	$.5u_{23}^2 + f_{23}$	14.33	14.33	15.33
24	$1.5f_{24}^2 + f_{22}$	$.5u_{24}^2 + f_{24}$	15.70	15.70	16.69

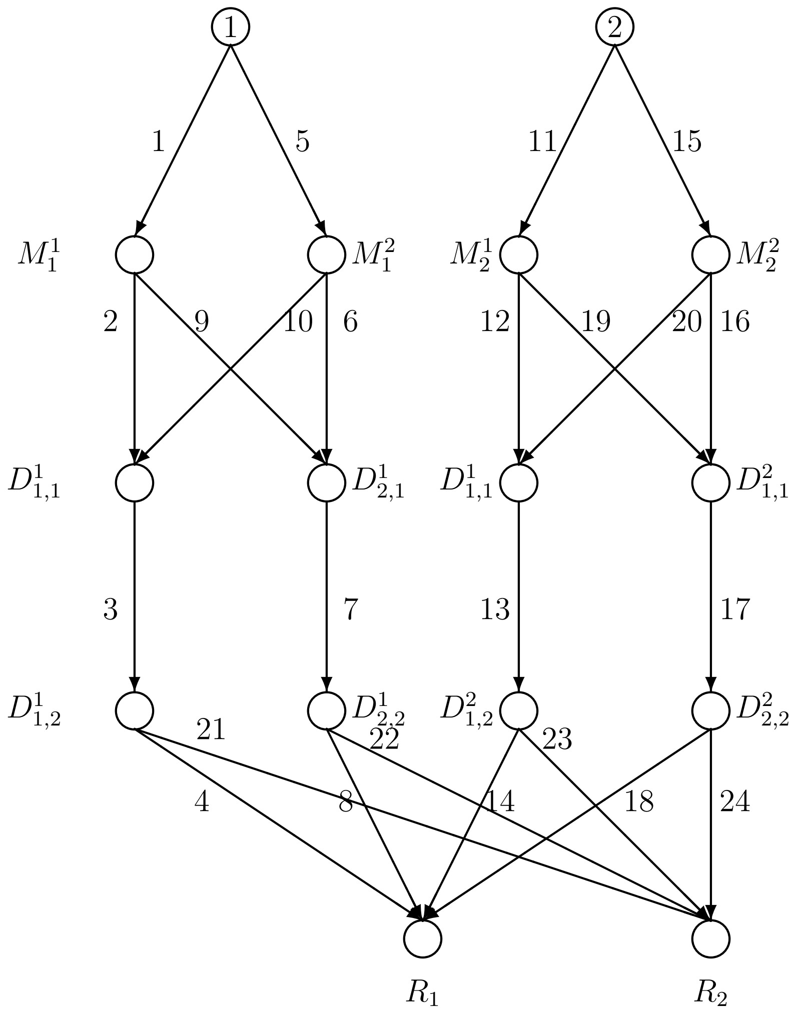

Figure 6: The Initial Supply Chain Network Topology for Example 5

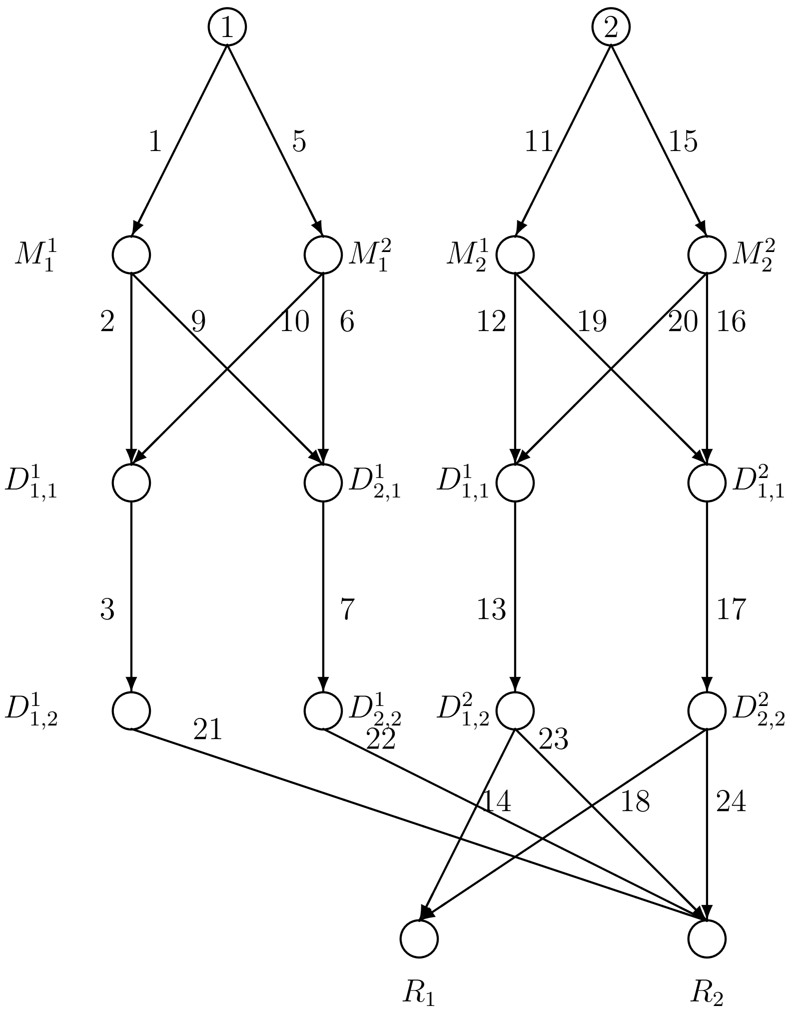

Figure 7: The Optimal Supply Chain Network Design for Example 5

5. Summary and Conclusions

In this paper, we developed a multimarket supply chain network design model in an oligopolistic setting. The firms select not only their optimal product flows but also the capacities associated with the various supply chain activities of production/manufacturing, storage, and distribution/shipment. We formulated the supply chain network design problem as a variational inequality problem and then proposed an algorithm, which fully exploits the underlying structure of these network problems, and yields closed form expressions at each iterative step.

The network formalism proposed here, which captures competition on the production, distribution, as well as demand market dimensions, enables the investigation of economic issues surrounding supply chain network design. In addition, it allows for the identification (and generalization) of special cases of oligopolistic market equilibrium problems, spatial, as well as aspatial, through the underlying network structure, that have appeared in the literature. Furthermore, the network structure allows one to visualize graphically the proposed supply chain network topology and the final optimal/equilibrium design. Importantly, this paper illustrates the power of computational methodologies to explore issues regarding competing firms and network design. Moreover, we demonstrate that such problems can be formulated and solved without using discrete variables but, rather only continuous variables.

The research in this paper can be extended in several directions. One can construct multiproduct versions of the oligopolistic supply chain network design model developed here, and one can also consider more explicitly international/global issues with the incorporation of exchange rates and risk. It would also be worthwhile to formulate supply chain network redesign oligopolistic models. Obviously, further computational experimentation as well as theoretical developments and empirical applications would also be of value, but we leave such research questions/problems for the future.

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