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Attracting international migrant labor: Investment optimization to alleviate supply chain labor shortages

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ABSTRACT

The COVID-19 pandemic has disrupted supply chains globally with a major shortfall being that of labor shortages from production through distribution activities. In this paper, we construct a new supply chain network optimization model that includes both domestic labor and international migrant labor from multiple countries, with the latter made possible through investments in attracting labor subject to a budget constraint. We allow for different wage settings for domestic versus migrant labor and also have the flexibility of providing true information as to the wages of migrants or not. We derive variational inequality formulations of the model, along with qualitative properties, and present an algorithm that yields closed form expressions for the underlying problem variables at each iteration. The model is one of the very few variational inequality operations research models with nonlinear constraints. Three series of algorithmically solved numerical examples, motivated by a high value agricultural product — that of truffles, demonstrate the insights in terms of profits, prices, product path flows, and investments, with variations in the data including that of truthful and untruthful wages being used to attract migrant labor.

1. Introduction

The COVID-19 pandemic has demonstrated the importance of supply chains and their effective and efficient operation, with disruptions adversely affecting product prices and deliveries, as well as the prosperity of companies and the health and well-being of consumers [1]. The reasons for the disruptions have been multifaceted with shocks both on the demand side as well as on the supply side and challenges associated with transport [2–6]. One of the major characteristics of the pandemic has been that of labor shortages. Workers throughout the pandemic have been falling ill; some, sadly, have lost their lives, whereas others chose to switch jobs or to leave the labor force [7]. Furthermore, various countries imposed restrictions further impeding the flow of workers [8]. Even with vaccinations continuing, the challenges of ameliorating labor shortages throughout many supply chains remain [9]. Employers have had difficulties recruiting workers not only with advanced technical skills. but also those with low and middle level skills. Countries are increasingly looking towards immigration policy to mitigate the labor shortage crises (see [10,11] with the new variant Omicron adding to the complexities [12]. Attracting workers from other countries, who are known as international migrant laborers, may assist in alleviating labor shortages.

According to [13], noting the most recent estimate by the International Labour Organization, there are 164 million migrant laborers globally and, in many countries, they are a major proportion of the workforce, with contributions to economies both where they work and in terms of remittances to their countries of origins. Nevertheless, many migrant workers face inequality in terms of a wage gap (being paid less that the workers from the particular nation) among other discriminatory practices [14]. For example, migrant laborers in High Income Countries (HICs) earn, on the average, approximately 12.6% less than nationals, with the wages earned by migrants widening over the past 6 years or so in many HICs (see [13]). Furthermore, migrant laborers are among the most negatively affected workers by the economic recession due to the COVID-19 pandemic, in terms of job losses and a decrease in earnings for those who have been able to stay employed. This is happening despite the fact that the United Nations' Sustainable Development Goals (SDGs), in the framework of the UN agenda for 2030, have as their targets 8.5 and 8.8: having equal pay for work of equal value and protected labor rights for all workers, including migrant workers [13,15].

This paper aims to integrate and advance two streams of literature, which have received significant attention in the pandemic: that of

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incorporating labor into supply chain network modeling, analysis, and computations (see [16–20]) and problems of human migration, which have been exacerbated under COVID-19 (cf. [21–24]). Specifically, in this paper, we construct a supply chain network optimization model that captures the profit-maximizing behavior of a firm with respect to its supply chain network activities of production at multiple sites, the transport of the product to multiple storage sites, the storage at these facilities, and, finally, the ultimate distribution of the product to multiple points of demand. Associated with each of the supply chain network activities is a bound on domestic labor availability with possible investment in labor migration from other countries to attract workers. The migrants are responsive to the wages that they are told they will be paid for the respective supply chain network activities, as well as to the investments made. Such investments can include assistance with visas, additional training to bring the laborers to the same level of productivity as the domestic labor, assistance with relocation, enhanced marketing about various job openings, etc. see [9]. Associated with each supply chain link, hence, is a migrant attraction investment function that is distinct for each country and each link activity of production, transportation, storage, or distribution. This is quite reasonable since some migrants, for example, may be more or less adverse to moving across a greater distance from their present country of origin to work. We also include a budget constraint associated with the firm's total possible investment outlay.

Here, for the sake of flexibility, and, in order to evaluate what is happening in practice in some parts of the world, the model allows for wages paid to international migrants to be different (or the same) as the wages paid to the domestic workers. Distinct wages have been noted, for example, in the agricultural sector in the pandemic [25] as well as other sectors with migrants sometimes being paid much lower wages than nationals [26]. In addition, in the international labor migration attraction investment functions, the wages told to the prospective migrants can be the true ones or not. Hence, we allow for the investigation computationally of the effects of different wage scenarios (and wage information) and investments in attracting international migrant laborers on the profits of the firm, the product flows, the labor volumes of both the nationals and the international migrant workers, and the prices that the consumers pay for the product at different demand markets. In particular, in terms of the investigation of different wage scenarios, our model allows for the quantification of the impacts of:

1. paying international migrants the same wages for corresponding activities as the domestic workers are paid, under the scenario that the international migrant laborers are informed honestly of the wages that they will be paid;

2. paying international migrants less than the domestic workers are paid, but they are informed of this honestly before they migrate for work or

3. paying the international migrants less than the domestic workers but being dishonest as to what wages they will receive in order to attract them.

Of course, our supply chain optimization model is sufficiently flexible to allow for "honesty" in terms of the actual wages that the international migrants will be paid coming from certain countries and not telling others their true wages. However, the above three cases can be viewed as primary ones. Note that workers, migrants or domestic ones, should be paid the prevailing wages, and at least the minimum wage, if it exists, but in many cases they are not (cf. [27]). This unfairness, in terms of payment of proper wages to migrants, is unethical and can even be criminal if there are minimum wage regulations. Furthermore, it leads not only to wage gaps but can also exacerbate worker exploitation and human trafficking, which are not directly the focus of this paper, although there are contributions in this paper to this literature on the periphery (cf. [28–31], and the references therein).

2. Literature review and contributions in this paper

Models for investment optimization have been proposed, to-date, for supply chains with labor, but the focus was on optimization of labor productivity in the case of a single period model and a budget constraint, and accompanied by Lagrange analysis [20], as well as in the case of a multiperiod model in which investments could take place in each period (see [19]). Here, in contrast, the investments are in attracting international migrant labor in supply chains. We do not explicitly differentiate between skilled and lower skilled workers — the specific supply chain network activity and supply chain application will be clear as to the level of skill needed. For example, in the manufacturing of computer chips, skilled labor may be needed for production, whereas in the case of the agricultural sector lower skilled labor can be involved in the planting and harvesting of produce. The resulting constraints in the new model with the migrant labor attraction functions can be nonlinear, as opposed to linear, as in the preceding works. We use variational inequality theory as the methodological framework for the formulation, analysis, and solution of the model. There have been very few other research contributions in terms of variational inequality models with nonlinear constraints. Toyasaki et al. [32] introduced nonlinear constraints in a supply chain network equilibrium model focusing on sustainability in terms of end-of-life products. Nagurney et al. [33] proposed a supply chain game theory model with investments in cybersecurity and nonlinear budget constraints. Colajanni et al. [34], in turn, again, focused on cybersecurity investments in a game theory context, and with nonlinear constraints, but in the setting of a Generalized Nash Equilibrium, as opposed to a Nash Equilibrium [35,36], since the strategies in their supply chain network model influence not only the utility functions of the competing firms, but also their feasible sets. Furthermore, unlike the above-noted models with investments in labor productivity, the new model in this paper has wages that are fixed. Moreover, we allow for the investigation of the impacts of fair and equitable wages for migrant labor (or not). The use of variational inequality theory allows for further extensions of the framework in this paper to handle competition via game theory as in the above models with nonlinear constraints.

In addition, the contributions in this paper add to the growing literature on network models of human migration using the theory of variational inequalities (cf. [21,23,24,37–40]) but with significant differences in that here we use wages to be earned as a proxy for utility and also include supply chain network aspects, since we are focusing on labor migration. Such models are all mathematical models, as is the model in this paper. Prasad et al. [41], in turn, focus on the management of migrant supply networks, recognizing that they are critical issues both domestically and internationally. Applying a supply chain perspective, they propose an empirically based typology that can be utilized to assess various factors within a migrant network, notably: the pressure to migrate, network agility, and the total cost of migration. The focus is primarily on refugee flows, a topic also of concern, but using a modeling and computational framework, in [21].

The numerical examples that we solve are motivated by shortages of labor in the agricultural sector, an issue in many parts of the world in the COVID-19 pandemic (see [42,43]). Specifically, we consider the production and harvesting of truffles, which are now being grown in the United Kingdom. This agricultural product is one of the most expensive ones on the globe.

The novelty of the contributions in this paper are the following:

1. This is the first optimization model to integrate a supply chain network and investments in attracting international migrant labor.

2. The model can handle as many countries as are of interest in attracting migrant labor from.

3. The international migrant attraction functions can differ for each supply chain network activity and country from which the migrants originate from.

4. The model can handle different wages for national/domestic labor and for international migrant labor. The model can also evaluate impacts of truthful versus untruthful wage information provided to potential international migrants, as implemented in the attraction functions.

5. The theoretical framework is that of variational inequalities and this work is one of the very few that includes nonlinear constraints in the model and these can arise due to the form that the international migrant attraction functions take.

6. The solution of a series of numerical examples, inspired by a high value agricultural product — that of truffles, having a variety of the above features, via the proposed algorithm with nice features for implementation, yields interesting insights.

The framework constructed here is inspired by issues that have become more notable in the pandemic such as limited national labor in many countries for various supply chain network activities of firms, which is handled by tightening the link labor bounds in our model, as need be; restrictions due to border controls and other regulations to decrease the chances of the spread of the coronavirus that causes COVID-19, which is handled by our model through the ability to reduce the number of countries from which migrant labor can be attracted, while, at the same time, allowing for laborers from other countries; and a reduced budget for attracting labor faced by a firm in the pandemic due to financial challenges. The latter is handled, as need be, by reducing the budget parameter in our model. Importantly, and as already emphasized, the model allows for the setting of the same or different wages for national laborers and international migrants, as well as the wage information that the international migrants are given. Such wage differentials have attracted the interest of policy makers in the pandemic. Of course, the supply chain network optimization model that is constructed in this paper with international migrant labor is also relevant to times of large labor resources, the free movement of workers across boarders, and rich financial budgets for attracting migrant labor, through the appropriate setting of various parameters in the model.

The paper is organized as follows. In Section 3, the investment supply chain network optimization model in attracting migrant laborers from multiple countries is developed and the optimality conditions are shown to satisfy a variational inequality. The model, under the above-described wage settings, can computationally reveal the impacts of distinct wage payment schemes and truthfulness in terms of the revelation of actual wages to be paid to international migrants. Some qualitative properties are also presented. An alternative variational inequality is then derived. The realization of the algorithm for the solution of the alternative variational inequality is given in Section 4. The algorithm results in closed form expressions for the variables (the product path flows, the investments, as well as several sets of Lagrange multipliers). Section 5 presents the solutions to three series of numerical examples, which are inspired by a valuable agricultural product - that of truffles, which are now being cultivated in the United Kingdom. Section 6 summarizes the results and presents our conclusions.

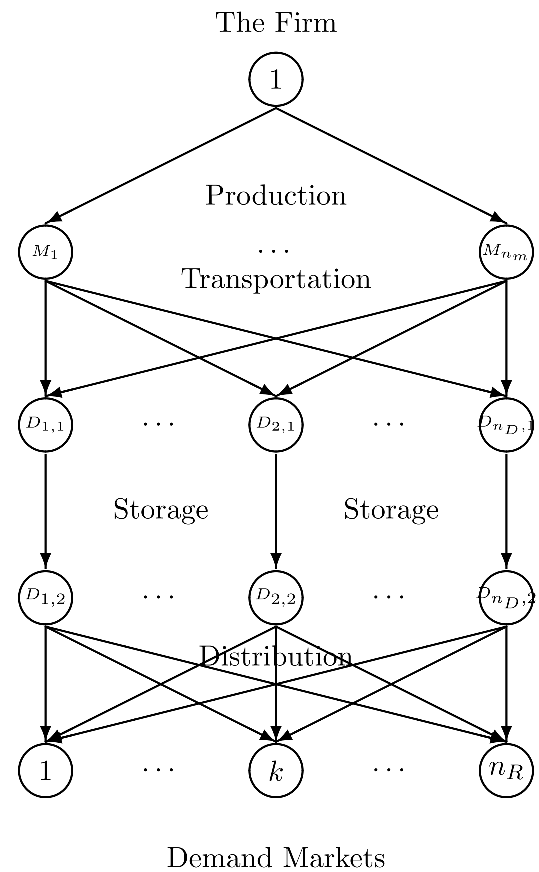

Fig. 1. The supply chain network topology for the model.

Table 1
Notation for the supply chain network model with international migrant labor — parameters.

Notation	Parameter definition
w_a^1	Hourly wage for a unit of labor on link a paid to domestic workers, $\forall a \in L$.
w_a^2	Hourly wage for a unit of labor on link a paid to international migrant workers, $\forall a \in L$.
$\tilde{w}_a^j$	Hourly wage for a unit of labor on link a that international migrant workers are told they will be paid, $\forall a \in L$, for all countries $j = 1, \dots, J$.
α_a	The link productivity on link a, $\forall a \in L$, which maps labor hours into product flow.
δ_{ap}	Indicator taking on the value 1 if link a is contained in path p and 0, otherwise.
$\bar{l}_a^1$	The maximum available domestic labor hours locally for work associated with link a, $a \in L$.
B	The amount of financing in the budget for investments in attracting migrant labor from different countries.

3. The supply chain network model with investments in attracting international migrant labor

We now construct the model. There are n_M possible production locations, n_D possible storage locations, and n_R demand markets. The supply chain network is represented by a graph $G = [N, L]$, where N is the set of nodes and L is the set of links. Links are denoted by a, b, etc. The supply chain network activities of production, transportation, storage, and distribution take place on the links. Please refer to Fig. 1 for the problem's supply chain network topology.

A path p is defined as a sequence of directed links from node 1 to a demand market node k; $k = 1, \dots, n_R$. There are n_L links in the supply chain and n_P paths. We denote the set of paths to demand market node k by P_k and the set of all paths by P. Different specific applications will differ in their supply chain network topologies by the number of relevant nodes and links and costs and demand price functions, etc. There are J countries that the labor can migrate from. We do not include the country the firm under consideration is located in this set of country indices.

The fundamental notation for the parameters is given in Table 1.

The notation for the variables is given in Table 2. All vectors are column vectors.

Note that, as mentioned in the Introduction, we, for purposes of generality and relevance, allow the wages for international migrant laborers to be the same (or different) from the wages of domestic laborers. In other words, the model allows for the flexibility of having

Table 2
Notation for the supply chain network model with international migrant labor — variables.

Notation	Variable definition
d_k	The demand for the product at demand market k; $k = 1, \ldots, n_R$. We group the demands into the vector $d \in R_+^{n_R}$.
x_p	The product flow on path p, $\forall p \in P$. The product path flows are grouped into the vector $x \in R_+^{n_P}$
f_a	The product flow on link a, $\forall a \in L$. The product link flows are grouped into the vector $f \in R_+^{n_L}$.
l_a^1	The hours of domestic labor available for link a supply chain activity, $\forall a \in L$.
l_a^2	The hours of international migrant labor available for link a supply chain activity, $\forall a \in L$.
v_a^j	The investment in attracting migrant labor from country j; $j = 1, \ldots, J$ for link a, $\forall a \in L$. The investments in attracting labor are grouped into the vector $v \in R_+^{J n_L}$.
η	The nonnegative Lagrange multiplier associated with the budget constraint.
λ_a	The nonnegative Lagrange multiplier associated with the bound on domestic labor hours on link a. We group the Lagrange multipliers into the vector $\lambda \in R_+^{n_L}$.
δ_a^1	The nonnegative Lagrange multiplier associated with the constraint guaranteeing that l_a^1 is nonnegative on link a. We group all such Lagrange multipliers into the vector $\delta^1 \in R_+^{n_L}$.
δ_a^2	The nonnegative Lagrange multiplier associated with the constraint guaranteeing that l_a^2 is nonnegative on link a. We group all such Lagrange multipliers into the vector $\delta^2 \in R_+^{n_L}$.

Table 3
Notation for the supply chain network model with international migrant labor — functions.

Notation	Function definition
$\hat{c}_a(f)$	The total operational cost function associated with link a, not including the labor cost, $\forall a \in L$.
$\rho_k(d)$	The demand price function for the product at demand market k; $k = 1, \ldots, n_R$.
$g_a^j(\tilde{w}_a^j, v_a^j)$	The investment function associated with link a in attracting migrant workers from country j to work on link a of the firm's supply chain network, $\forall a \in L$ and $j = 1, \ldots, J$.

$w_a^1 = w_a^2$, for some $a \in L$, whereas for other links $b \in L$, we may have: $w_b^1 \neq w_b^2$. Also, the wage used in the attraction function in attracting international migrant workers from a specific country j for a supply chain activity a, $\tilde{w}_a^j$, can be the actual wage that will be paid (or not). We investigate the implications of different wage settings in the numerical examples, for which the data and computed solutions are reported in Section 5.

In Table 3, the notation for the functions is provided.

We first present the constraints and then we construct the objective function of the firm.

Conservation of Flow Equations

The sum of the product path flows to each demand market must be equal to the demand at the demand market:

$$\sum_{p \in P_k} x_p = d_k, \quad k = 1, \ldots, n_R, \tag{1}$$

with all the path flows being nonnegative:

$$x_p \geq 0, \quad \forall p \in P. \tag{2}$$

In addition, the amount of product flow on each link must be equal to the sum of product flows on paths that contain that link:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L. \tag{3}$$

Labor Constraints

Furthermore, we assume linear production functions here, as done in prior work (cf. [16–18]), but we extend them to differentiate between domestic labor and international migrant labor. Hence, we have that

$$f_a = \alpha_a (l_a^1 + l_a^2), \quad \forall a \in L. \tag{4}$$

According to (4), the amount of product flow on a supply chain link is equal to the hours of labor availability on the link, which represents a supply chain network activity, times the productivity factor of labor associated with the link. Here we assume that, through investments, the productivity of migrant labor, for a given activity, will be the same as that of the domestic workers where the firm's supply chain network activity is located.

The domestic worker labor hours available for each link activity cannot exceed the bound on domestic labor hours of availability; that is:

$$l_a^1 \leq \bar{l}_a^1, \quad \forall a \in L. \tag{5a}$$

Furthermore, we must have that the domestic labor hours available are nonnegative:

$$l_a^1 \geq 0, \quad \forall a \in L. \tag{5b}$$

Also, the amount of international migrant labor hours available for a link a is:

$$l_a^2 = \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^j), \quad \forall a \in L, \tag{6a}$$

with nonnegativity of international migrant labor hours also holding:

$$l_a^2 \geq 0, \quad \forall a \in L. \tag{6b}$$

Constraints on Investments

The investments by the firm in attracting international migrant labor must be nonnegative:

$$v_a^j \geq 0, \quad j = 1, \ldots, J; \forall a \in L. \tag{7}$$

Finally, the firm's budget constraint in terms of attracting migrants is:

$$\sum_{j=1}^{J} \sum_{a \in L} v_a^j \leq B. \tag{8}$$

Here, for simplicity, we assume that all the links $a \in L$ of the firm's supply chain network are located in the same country. If they are not, the above model could still be applicable/relevant, but one would have to make sure that the summations over the countries in the constraints above would exclude the country that the link a is in.

The Optimization Problem

The optimization problem faced by the firm in optimizing its supply chain network can now be stated. Specifically, the firm seeks to maximize its objective function, denoted by U, which represents its profits, subject to the constraints (1)–(8):

$$\text{Maximize} \quad U = \sum_{k=1}^{n_R} \rho_k(d) d_k - \sum_{a \in L} \hat{c}_a(f) - \sum_{j=1}^{J} \sum_{a \in L} v_a^j - \sum_{a \in L} (w_a^1 l_a^1 + w_a^2 l_a^2). \tag{9}$$

The first term after the equal sign in (9), $\sum_{k=1}^{n_R} \rho_k(d) d_k$, is the revenue; the second term, $\sum_{a \in L} \hat{c}_a(f)$, represents the total operational costs, whereas the third term, $\sum_{j=1}^{J} \sum_{a \in L} v_a^j$, is the total investments. The last term, $\sum_{a \in L} (w_a^1 l_a^1 + w_a^2 l_a^2)$, in (9) is the payout to labor for wages.

We assume that the demand price functions are monotone decreasing and that each $\rho_k(d) d_k$ is concave for each k, and that the total operational link cost functions are convex with both the demand price functions and the operational cost functions being continuously differentiable. Furthermore, we assume that the investment functions $g_a^j(\tilde{w}_a^j, v_a^j)$ are all concave, which is reasonable, since we can expect diminishing returns to such investments and that they are bounded. These are also assumed to be continuously differentiable.

Making use of the various expressions comprising the feasible set for the above problem enables one to transform the objective function in (9) to be in path flow and investment amount variables only. We proceed as follows. In lieu of (1), we can define demand price functions $\tilde{\rho}_k(x) \equiv \rho_k(d)$, for $k = 1, \dots, n_R$, and, in lieu of (3), we can define link operational total cost functions $\tilde{c}_a(x) \equiv \hat{c}_a(f)$, for $a \in L$. Furthermore, using (4) and (6a), we note that

$$l_a^1 = \frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^j), \quad \forall a \in L. \tag{10}$$

Then, the firm's supply chain network optimization problem (9) can be re-expressed as:

$$\text{Maximize} \quad \tilde{U}(x,v) = \sum_{k=1}^{n_R} \tilde{\rho}_k(x) \sum_{p \in P_k} x_p - \sum_{a \in L} \tilde{c}_a(x) - \sum_{j=1}^J \sum_{a \in L} v_a^j - \sum_{a \in L} w_a^1 \left(\frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^j)\right) - \sum_{a \in L} w_a^2 \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^j); \tag{11a}$$

equivalently:

$$\text{Maximize} \quad \tilde{U}(x,v) = \sum_{k=1}^{n_R} \tilde{\rho}_k(x) \sum_{p \in P_k} x_p - \sum_{a \in L} \tilde{c}_a(x) - \sum_{j=1}^J \sum_{a \in L} v_a^j - \sum_{a \in L} w_a^1 \left(\frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a}\right) + \sum_{a \in L} (w_a^1 - w_a^2) \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^j), \tag{11b}$$

subject to:

$$\frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^J g_a^j(\tilde{w}_a, v_a^j) \le \bar{l}_a^1, \quad \forall a \in L, \tag{12a}$$

$$\frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^J g_a^j(\tilde{w}_a, v_a^j) \ge 0, \quad \forall a \in L, \tag{12b}$$

$$\sum_{j=1}^J g_a^j(\tilde{w}_a, v_a^j) \ge 0, \quad \forall a \in L, \tag{13}$$

$$\sum_{j=1}^J \sum_{a \in L} v_a^j \le B, \tag{14}$$

$$x_p \ge 0, \quad \forall p \in P, \tag{15}$$

$$v_a^j \ge 0, \quad j = 1, \dots, j; \forall a \in L. \tag{16}$$

We define the feasible set $K^1 \equiv \{(x,v) \in R_+^{n_P + Jn_L}$ and satisfying $(12a,b)-(14)\}$. Here we assume that for each link $a \in L$, $w_a^1 \ge w_a^2$, which is reasonable since one can expect the firm to pay domestic workers at least the amount that it is paying to international migrant laborers. And, if it pays migrants the same wage that it pays the domestic workers on each link, then the fifth term after the equal sign in the objective function $(11b)$ is equal to 0. The objective function in $(11b)$, under our assumptions, is concave and the underlying functions are continuously differentiable. The underlying feasible set K^1 is convex, since constraints (14), (15), and (16) are linear and the investment functions are assumed to be concave, so minus each investment function is convex. Hence, it follows from the classical theory of variational inequalities (see [44] and [45]) that the optimal solution $(x^*, v^*) \in K^1$ satisfies the variational inequality problem:

$$-\sum_{p \in P} \frac{\partial \tilde{U}(x^*, v^*)}{\partial x_p} \times (x_p - x_p^*) - \sum_{j=1}^J \sum_{a \in L} \frac{\partial \tilde{U}(x^*, v^*)}{\partial v_a^j} \times (v_a^j - v_a^{j*}) \ge 0, \ \forall (x,v) \in K^1. \tag{17}$$

By expanding out the partial derivatives of the utility functions in (17), we obtain the equivalent variational inequality: determine $(x^*, v^*) \in K^1$, such that

$$\sum_{k=1}^{n_R} \sum_{p \in P_k} \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L} \frac{w_a^1}{\alpha_a} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^*\right] \times \left[x_p - x_p^*\right] + \sum_{j=1}^J \sum_{a \in L} \left[-(w_a^1 - w_a^2) \frac{\partial g_a^j(\tilde{w}_a^j, v_a^{j*})}{\partial v_a^j} + 1\right] \times \left[v_a^j - v_a^{j*}\right] \ge 0, \quad \forall (x,v) \in K^1, \tag{18}$$

where

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L} \sum_{b \in L} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}, \ \ \forall p \in P, \ \text{and} \ \frac{\partial \tilde{\rho}_l(x)}{\partial x_p} \equiv \frac{\partial \rho_l(d)}{\partial d_k}, \forall p \in P_k, \forall k. \tag{19}$$

A solution $(x^*, v^*) \in K^1$ to both variational inequalities (17) and (18) exists since the feasible set K^1 is compact and the underlying functions, under our imposed assumptions, are continuous.

We now provide an alternative variational inequality, equivalent to the one in (18), which we will utilize for computational purposes. The alternative variational inequality below follows from analogous results in [40] and [46]. According to Table 2, we associate a nonnegative Lagrange multiplier λ_a with each link labor bound constraint $(12a)$, for each $a \in L$, and the nonnegative Lagrange multiplier η with the budget constraint (14). Then, the equivalent variational inequality to the one in (18) is: determine $(x^*, \lambda^*, v^*, \eta^*, \delta^{1*}, \delta^{2*}) \in K^2$, where $K^2 \equiv \{(x, \lambda, v, \eta, \delta^1, \delta^2) | (x, \lambda, v, \eta, \delta^1, \delta^2) \in R_+^{n_P + n_L + Jn_L + 1 + 2n_L}\}$, such that

$$\begin{aligned}
&\sum_{k=1}^{n_R} \sum_{p \in P_k} \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L} \frac{w_a^1}{\alpha_a} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* + \sum_{a \in L} \frac{\lambda_a^*}{\alpha_a} \delta_{ap} - \frac{\delta_a^{1*}}{\alpha_a} \delta_{ap}\right] \times \left[x_p - x_p^*\right] \\
&+ \sum_{a \in L} \left[\bar{l}_a^1 - \frac{\sum_{p \in P} x_p^* \delta_{ap}}{\alpha_a} + \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^{j*})\right] \times \left[\lambda_a - \lambda_a^*\right] \\
&+ \sum_{j=1}^J \sum_{a \in L} \left[1 + \eta^* - (w_a^1 - w_a^2 + \lambda_a^* - \delta_a^{1*} + \delta_a^{2*}) \frac{\partial g_a^j(\tilde{w}_a^j, v_a^{j*})}{\partial v_a^j}\right] \times \left[v_a^j - v_a^{j*}\right] \\
&+ \left[B - \sum_{j=1}^J \sum_{a \in L} v_a^{j*}\right] \times \left[\eta - \eta^*\right] \\
&+ \sum_{a \in L} \left[\frac{\sum_{p \in P} x_p^* \delta_{ap}}{\alpha_a} - \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^{j*})\right] \times \left[\delta_a^1 - \delta_a^{1*}\right] \\
&+ \sum_{a \in L} \left[\sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^{j*})\right] \times \left[\delta_a^2 - \delta_a^{2*}\right] \ge 0, \quad \forall (x, \lambda, v, \eta, \delta^1, \delta^2) \in K^2.
\end{aligned} \tag{20}$$

We now put variational inequality (20) into standard form (cf. [45]): determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \ge 0, \quad \forall X \in \mathcal{K}, \tag{21}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ is the inner product in $\mathcal{N}$-dimensional Euclidean space.

We set $\mathcal{K} \equiv R_+^{n_P + n_L + Jn_L + 1 + 2n_L}$, $X \equiv (x, \lambda, v, \eta, \delta^1, \delta^2)$, and $\mathcal{N} = n_P + n_L + Jn_L + 1 + 2n_L$. We define the vector $F \equiv (F_1, F_2, F_3, F_4, F_5, F_6)$, with components of F_1 consisting of the elements: $\left[\frac{\partial \tilde{C}_p(x)}{\partial x_p} + \sum_{a \in L} \frac{w_a^1}{\alpha_a} \delta_{ap} - \tilde{\rho}_k(x) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x)}{\partial x_p} \sum_{q \in P_l} x_q + \sum_{a \in L} \frac{\lambda_a}{\alpha_a} \delta_{ap} - \frac{\delta_a^1}{\alpha_a} \delta_{ap}\right]$, $\forall p \in P$; the components of F_2 consisting of elements: $\left[\bar{l}_a^1 - \frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} + \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^j)\right]$, $\forall a \in L$; the components of F_3 being: $\left[1 + \eta - (w_a^1 - w_a^2 + \lambda_a - \delta_a^1 + \delta_a^2) \frac{\partial g_a^j(\tilde{w}_a^j, v_a^j)}{\partial v_a^j}\right]$, $\forall a \in L$, and the component of F_4 being: $\left[B - \sum_{j=1}^J \sum_{a \in L} v_a^j\right]$. Finally, the components of F_5 are the elements: $\frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^j)$, for $a \in L$, whereas the components of F_6 are: $\sum_{j=1}^J g_a^j(\tilde{w}_a^j, v_a^{j*})$, for $a \in L$. With such definitions, variational inequality (20) clearly coincides with variational inequality (21).

4. The algorithm

The algorithm that we implement and apply in the next Section to compute solutions to numerical examples, with a goal of gaining insights into impacts of wage settings and investments in attracting international migrant labor, is the modified projection method of [47]. For easy reference and completeness, we now recall the algorithm, and also spell out the form that it takes in the solution of our model as governed by variational inequality (20). Notably, each iteration of the algorithm yields closed form expressions for each of the variables (the product path flows, the Lagrange multipliers associated with the domestic labor bounds, the link investments in attracting migrant labor from different countries, and the Lagrange multiplier associated with the investment budget constraint) at each of the two steps.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $\tau := 1$, where τ is the iteration counter, and let β be a scalar such that $0 < \beta \leq \frac{1}{\omega}$, where ω is the Lipschitz constant (cf. (25) below).

Step 1: Computation

Compute $\bar{X}^\tau$ satisfying the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{22}$$

Step 2: Adaptation

Compute X^τ satisfying the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{23}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then terminate the algorithm; else, set $\tau := \tau + 1$ and go to Step 1.

The modified projection method is guaranteed to converge if $F(X)$ in (20) is monotone and Lipschitz continuous and there is a solution. The function $F(X)$ is monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{24}$$

$F(X)$ is Lipschitz continuous if there is a constant $\omega > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \omega \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{25}$$

We now report the explicit form that Step 1 (cf. (22)) takes for the determination of each of the variables. The analogues for Step 2, as in (23), readily follow.

Explicit Formulae at Iteration τ for the Product Path Flows in Step 1

The closed form expression for each path flow in Step 1 in the solution of variational inequality (21); equivalently, (20), is, for each $p \in P_k$, $\forall k$:

$$\bar{x}_p^\tau = \max\{0, x_p^{\tau-1} + \beta(\tilde{\rho}_k(x^{\tau-1}) + \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^{\tau-1})}{\partial x_p} \sum_{q \in P_l^i} x_q^{\tau-1} - \frac{\partial \tilde{C}_p(x^{\tau-1})}{\partial x_p}$$
$$- \sum_{a \in L} (\frac{w_a^1 + \lambda_a^{\tau-1} - \delta_a^{1(\tau-1)}}{\alpha_a}) \delta_{ap})\}. \tag{26}$$

Explicit Formulae at Iteration τ for the Link Domestic Labor Bound Lagrange Multipliers in Step 1

The closed form expression for each Lagrange multiplier for each link $a \in L$ in Step 1 at an iteration τ is:

$$\bar{\lambda}_a^\tau = \max\{0, \lambda_a^{\tau-1} + \beta(-\bar{l}_a^1 + \frac{\sum_{p \in P} x_p^{\tau-1} \delta_{ap}}{\alpha_a} - \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^{j\tau-1}))\}. \tag{27}$$

Explicit Formulae at Iteration τ for Investments in Attracting International Migrant Labor in Step 1

The closed form expression for the investment in attracting migrant labor from different countries in Step 1 is, for each country j; $j = 1, \ldots, J$, and for each link $a \in L$:

$$\bar{v}_a^{j\tau} = \max\{0, v_a^{\tau-1} + \beta(-1 - \eta^{\tau-1} + (w_a^1 - w_a^2 + \lambda_a^{\tau-1} - \delta_a^{1(\tau-1)}$$
$$+ \delta_a^{2(\tau-1)}) \frac{\partial g_a^j(\tilde{w}_a^j, v_a^{j\tau-1})}{\partial v_a^j})\}. \tag{28}$$

Explicit Formula at Iteration τ for the Lagrange Multiplier Associated with the Budget Constraint in Step 1

The closed form expression for the Lagrange multiplier associated with the budget constraint in Step 1 at an iteration τ is:

$$\bar{\eta}^\tau = \max\{0, \eta^{\tau-1} + \beta(\sum_{j=1}^{J} \sum_{a \in L} v_a^{j\tau-1} - B)\}. \tag{29}$$

Explicit Formulae at Iteration τ for the Lagrange Multipliers Associated with Domestic Labor Nonnegativity Constraints in Step 1

The closed form expression for the Lagrange multiplier associated with the domestic labor nonnegativity constraint in Step 1 is, for each link $a \in L$:

$$\bar{\delta}_a^{1\tau} = \max\{0, \delta_a^{1(\tau-1)} + \beta(-\frac{\sum_{p \in P} x_p^{\tau-1} \delta_{ap}}{\alpha_a} + \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^{j\tau-1}))\}. \tag{30}$$

Explicit Formulae at Iteration τ for the Lagrange Multipliers Associated with International Migrant Labor Nonnegativity Constraints in Step 1

The closed form expression for the Lagrange multiplier associated with the international migrant labor nonnegativity constraint in Step 1 is, for each link $a \in L$:

$$\bar{\delta}_a^{2\tau} = \max\{0, \delta_a^{2(\tau-1)} + \beta(-\sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^{j\tau-1}))\}. \tag{31}$$

The analogues of expressions (26) through (31) for Step 2 (cf. (23)) of the modified projection method follow in a straightforward manner.

5. Numerical examples

The modified projection method was coded in FORTRAN and a Linux system at the University of Massachusetts Amherst used for the computation of solutions to the numerical examples satisfying variational inequality (20). The demand for each demand market was initialized at 40.00 and equally distributed among the paths connecting each demand market from the origin node 1 (the Firm). The investments were initialized to 0.00 as were all the Lagrange multipliers. The algorithm was considered to have converged if the absolute difference of the path flows differed by no more than 10^{-7} and the same for investments and the Lagrange multipliers.

The numerical examples are inspired by recent issues surrounding agricultural supply chains in the UK. The UK has been pummeled with shortfalls in labor due to COVID-19 as well as Brexit. We are particularly interested in fresh produce, which requires minimal processing, and, hence, the production links correspond also to harvesting and some basic packaging. Even prior to the pandemic and Brexit, many domestic laborers in the UK shied away from farm work (see [48]) with only 1% of pickers and packers being British citizens in 2019, with the percentage rising to 11% in the summer of 2020 due to a British campaign. Obtaining sufficient truckers to transport produce and other perishables has also been a challenge in the UK and this has drawn attention in the news with the government getting involved and issuing special visas to both farmers and truckers [49]. Fresh fruit and vegetables, as well as milk, have gone to waste because of canceled or delayed deliveries (see [50]). According to [51], in 2019, an estimated 18% of workers in the UK were migrant workers, with migrants being over-represented in transport and storage at 28%. It is clear that international migrant labor is essential to supply chains, including agricultural ones, in the United Kingdom as well as beyond — in Australia (see [52]), the United States (see [53]), and the European Union ([54]).

The numerical examples are quite broad with the data being motivated by a very interesting agricultural product, now being grown in the UK — truffles [55]. Truffles are a high value agricultural product, and are considered a delicacy, with challenges associated with production and harvesting [56]. As noted in [57], in the Fall of 2021, due to a shortage, white truffle prices were about $4,500 a pound, whereas, in 2019, white truffle prices were in the range $1,100 to $1,200 a pound. Different types of truffles command different prices (see Truffle.[58]) but their costliness and desirability are well-known. They are considered among the most expensive foods on the planet [56]. Although southern Europe is a primary growth area for this agricultural delicacy, with changes in climate plus advances in science, the UK is also becoming a location for the farming of truffles. The cost and price data, as well as the wages and the profits, in the numerical examples are in British pounds. The unit for the truffle product flows is a pound of weight. According to [59], vegetable pickers in the UK, due to shortages of labor are being paid 30 pounds an hour to pick the produce.

The numerical examples are organized into three series.

Series 1 Numerical Examples

In this series of examples, we explore the impacts of different wage settings, both truthful and untruthful ones.

Example 1: Baseline Example: Migrant Workers Earn the Same Wage as Domestic Laborers and Migrants Are Told Their Truthful Wages in the Attraction Functions

Example 1 has the supply chain network topology depicted in Fig. 2. The agricultural firm has two locations requiring production/harvesting of fresh produce, which consists of truffles, which then must be transported to a single distribution center from which the produce is distributed to three demand markets.

The total operational link cost functions are:

$$\hat{c}_a(f) = 2.5f_a^2, \quad \hat{c}_b(f) = 2.5f_b^2, \quad \hat{c}_c(f) = .5f_c^2, \quad \hat{c}_d(f) = .5f_d^2,$$

$$\hat{c}_e(f) = f_e^2 + 2f_e, \quad \hat{c}_f(f) = .5f_f^2, \quad \hat{c}_g(f) = .5f_g^2, \quad \hat{c}_h(f) = .5f_h^2.$$

The demand price functions are:

$$\rho_1(d) = -5d_1 + 800, \quad \rho_2(d) = -5d_2 + 850, \quad \rho_3(d) = -5d_3 + 900.$$

The α link parameters are:

$$\alpha_a = .55, \quad \alpha_b = .50, \quad \alpha_c = .35, \quad \alpha_d = .35,$$
$$\alpha_e = .60, \quad \alpha_f = .38, \quad \alpha_g = .36, \quad \alpha_h = .40.$$

We assume that the supply chain firm considers a single country to obtain international migrants from, but the specific country can differ from supply chain network activity to activity. Hence, we can suppress the superscript j in the international migrant attraction functions and in the investments. The international migrant attraction functions are of the form: $g_a(\tilde{w}_a, v_a) = \tilde{w}_a v_a - \gamma_a v_a^2$, for all links $a \in L$. These functions are concave. The γ parameters in these functions are:

$$\gamma_a = .2, \quad \gamma_b = .2, \quad \gamma_c = .4, \quad \gamma_d = .4,$$
$$\gamma_e = .3 \quad \gamma_f = .4, \quad \gamma_g = .4, \quad \gamma_h = .4.$$

The wages in Example 1 are:

$$w_a^1 = w_a^2 = \tilde{w}_a = 30, \quad w_b^1 = w_b^2 = \tilde{w}_b = 20,$$

$$w_c^1 = w_c^2 = \tilde{w}_c = 18, \quad w_d^1 = w_d^2 = \tilde{w}_d = 18,$$

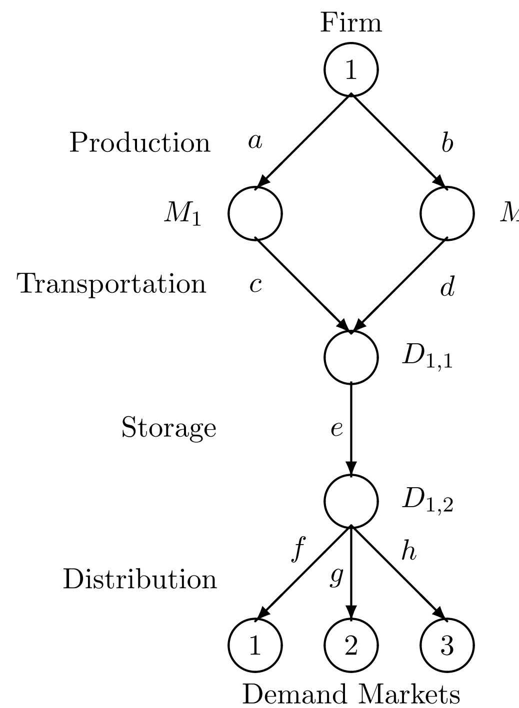

Fig. 2. The supply chain network topology for the numerical examples.

$$w_e^1 = w_e^2 = \tilde{w}_e = 17, \quad w_f^1 = w_f^2 = \tilde{w}_f = 19,$$

$$w_g^1 = w_g^2 = \tilde{w}_g = 19, \quad w_h^1 = w_h^2 = \tilde{w}_h = 19.$$

Hence, in Example 1, we assume that the international migrants are paid the same hourly wage for each supply chain network activity as are the domestic workers and that the international migrants are informed of the truthful wage when contacted about migrating for work.

The bounds on domestic labor are: $\bar{l}_a^1 = 100$, for all links in the supply chain from a through h and the budget $B = 1,000$.

The paths (see Fig. 2) are defined as follows: path $p_1 = (a, c, e, f)$, path $p_2 = (b, d, e, f)$, path $p_3 = (a, c, e, g)$, path $p_4 = (b, d, e, g)$, path $p_5 = (a, c, e, h)$, and path $p_6 = (b, d, e, h)$.

The modified projection method computes the following optimal product path flow pattern:

$$x_{p_1}^* = 9.60, \quad x_{p_2}^* = 10.41, \quad x_{p_3}^* = 11.74,$$
$$x_{p_4}^* = 12.55, \quad x_{p_5}^* = 17.51, \quad x_{p_6}^* = 18.32.$$

The equilibrium link flows and labor values are reported in Table 4, whereas the optimal investments and Lagrange multipliers λ_a^*, for all $a \in L$, are reported in Table 5.

The domestic labor on supply chain links of c, d, and e is at the capacity of 100, with international migrant labor hours being positive on those links. Hence, the associated Lagrange multipliers λs on those links are positive.

We note that all other Lagrange multipliers, that is, the elements of δ_a^{1*} and δ_a^{2*} for all $a \in L$, and η^* are equal to 0.00. The total investment outlay is: 3.69.

The demand price at the first demand market is 699.99 and at the second demand market the price is: 728.52, with the demand price at the third demand market being: 756.66. The corresponding respective demands are: 20.00, 24.30, 35.84.

The firm earns a profit of: 41,453.10.

Example 2: Domestic Workers Earn a Higher Wage at Production Sites than Migrants and Are Told Their Truthful Wages

Table 4
Optimal link flows and domestic and international migrant labor values for examples 1, 2, 3.

Notation	Optimal value		
	Example 1	Example 2	Example 3
f_a^*	38.86	38.85	38.85
f_b^*	41.28	41.27	41.27
f_c^*	38.86	38.85	38.85
f_d^*	41.28	41.27	41.27
f_e^*	80.13	80.12	80.13
f_f^*	20.00	20.00	20.00
f_g^*	24.30	24.29	24.29
f_h^*	35.84	35.83	35.83
l_a^{1*}	70.65	0.00	0.00
l_b^{1*}	82.56	0.00	0.00
l_c^{1*}	100.00	100.00	100.00
l_d^{1*}	100.00	100.00	100.00
l_e^{1*}	100.00	100.00	100.00
l_f^{1*}	52.64	52.63	52.63
l_g^{1*}	67.49	67.48	67.48
l_h^{1*}	89.59	89.58	89.58
l_a^{2*}	0.00	70.64	70.64
l_b^{2*}	0.00	82.54	82.55
l_c^{2*}	11.01	11.01	11.01
l_d^{2*}	17.94	17.91	17.92
l_e^{2*}	33.56	33.54	33.54
l_f^{2*}	0.00	0.00	0.00
l_g^{2*}	0.00	0.00	0.00
l_h^{2*}	0.00	0.00	0.00

Table 5
Optimal link international migrant attraction investments and domestic labor bound Lagrange multipliers for examples 1, 2, and 3.

Notation	Optimal value		
	Example 1	Example 2	Example 3
v_a^*	0.00	2.39	1.78
v_b^*	0.00	4.31	2.80
v_c^*	0.62	0.62	0.62
v_d^*	1.02	1.02	1.02
v_e^*	2.05	2.05	2.05
v_f^*	0.00	0.00	0.00
v_g^*	0.00	0.00	0.00
v_h^*	0.00	0.00	0.00
λ_a^*	0.00	0.00	0.00
λ_b^*	0.00	0.00	0.00
λ_c^*	0.06	0.06	0.06
λ_d^*	0.06	0.06	0.06
λ_e^*	0.06	0.06	0.06
λ_f^*	0.00	0.00	0.00
λ_g^*	0.00	0.00	0.00
λ_h^*	0.00	0.00	0.00

Example 2 has the same data as that in Example 1, except that now the domestic workers earn a higher wage at the two production sites with $w_a^1 = 40$ and $w_b^1 = 30$.

The modified projection method yields the following optimal product path flow pattern:

$$x_{p_1}^* = 9.60, \quad x_{p_2}^* = 10.40, \quad x_{p_3}^* = 11.74,$$
$$x_{p_4}^* = 12.55, \quad x_{p_5}^* = 17.51, \quad x_{p_6}^* = 18.32.$$

The optimal link flows and labor values are reported in Table 4 and the optimal investments and Lagrange multipliers λ_a^*, for all $a \in L$, are reported in Table 5.

In Example 2, we see that the firm now invests in attracting international migrants also on links a and b, the production/picking sites for truffles of the firm. It, hence, obtains international migrant laborers for links a through e, whereas in Example 1, it had international migrants working only on links c, d, and e. Since domestic workers are now more expensive, the firm hires exclusively migrant laborers on the production links a and b. The total investment outlay increases from 3.69 to 10.39. We now have δ_a^{1*} and δ_b^{1*} being positive at values of 9.97 and 9.95, respectively. All other δ^* values, including the δ^{2*}s as well as η^* are equal to 0.00.

The demand price at the first demand market is now 700.00. The demand price at the second demand market is: 728.54, with the demand price at the third demand market being: 756.67, with the corresponding respective demands of: 20.00, 24.29, and 35.83.

The firm earns a profit of: 41,444.64. Observe that the profit has now decreased, as compared to that in Example 1.

Example 3: Domestic Workers Earn a Higher Wage at Production Sites than Migrants but Migrants Are Told Untruthfully That They Will Be Paid the Same Wage as the Domestic Workers

In Example 3, we investigate the impact of the firm being untruthful. Specifically, the firm now tells the international migrant laborers that it will pay them the same (higher) wage at each production site that it is paying its domestic laborers, but it actually will pay them less. The data, hence, are exactly as in Example 2, but now we have that $\tilde{w}_a = 40$ and $\tilde{w}_b = 30$.

The modified projection method computes the product path flow pattern:

$$x_{p_1}^* = 9.60, \quad x_{p_2}^* = 10.40, \quad x_{p_3}^* = 11.74,$$
$$x_{p_4}^* = 12.55, \quad x_{p_5}^* = 17.51, \quad x_{p_6}^* = 18.32.$$

Please refer to Table 4 for the computed link flows and labor values and to Table 5 for the computed investments and Lagrange multipliers λ_a^*, for all $a \in L$. The $\delta_a^{2*} = 0.00$ for all links in the supply chain network and $\eta^* = 0.00$, as in the preceding examples.

As in Example 2, the firm uses exclusively international migrant laborers at its production/picking sites on links a and b with the Lagrange multipliers $\delta_a^{1*} = \delta_b^{1*} = 9.97$. The total investment outlay now decreases to 8.27, since not as much is needed as in Example 2 due to the untruthfulness about the wages that migrants will be paid for work at the production sites for truffles. Migrant laborers, again, work on links a through e.

The demand price at the first demand market remains at 700.00. The demand price at the second demand market is: 728.53, with the demand price at the third demand market being: 756.67, with the corresponding respective demands of: 20.00, 24.29, and 35.83. The demand prices and the demands are essentially unchanged for their values in Example 2.

The firm earns a profit of: 41,447.33. Observe that the profit now increases suggesting that, without oversight, “cheating can pay”.

The product path flows are essentially the same in Examples 1, 2, and 3 in this series due to the demand price functions, in part, with the reallocation of labor from domestic ones to migrants.

Series 2 Numerical Examples

In this series of examples, we investigate the impacts of increases in the price that consumers are willing to pay for the truffles at the demand markets. Specifically, we conduct sensitivity analysis associated with the intercept in the demand price functions.

For each of the examples in this series, we report the computed link flows and the domestic and international migrant labor values in Table 6 and the migrant attraction function investments and domestic labor Lagrange multipliers in Table 7.

Example 4: Example 1 Data With All the Demand Price Function Intercepts Doubled

The data in Examples 4 through 7 are identical to the data in Example 1 but with the intercept for each demand function increased. In Example 4, we consider a doubling of the intercept so that the demand price functions are now:

The demand price functions are now:

$$\rho_1(d) = -5d_1 + 1600, \quad \rho_2(d) = -5d_2 + 1700, \quad \rho_3(d) = -5d_3 + 1800.$$

The modified projection method yields the following product path flow pattern:

$$x^*_{p_1} = 22.90, \quad x^*_{p_2} = 23.70, \quad x^*_{p_3} = 27.32,$$
$$x^*_{p_4} = 28.12, \quad x^*_{p_5} = 39.33, \quad x^*_{p_6} = 40.13.$$

The demand price at the first demand market is: 1,366.98 and at the second demand market the price is: 1,422.80, with the demand price at the third demand market being: 1,482.17. The respective demands at the demand markets are: 46.60, 55.44, and 79.46.

The consumers are now willing to pay a higher price for the truffles at each demand market, the demand increases at each market, and the firm now enjoys a profit of: 171,789.94. This profit is more than four times that in Example 1.

The domestic labor on each supply chain link is at its bound of 100, with the associated Lagrange multipliers now all positive. In addition, there is a positive investment on each link for attracting migrant labor and all values of migrant labor hours are positive on each link. The total investment outlay is: 57.96. With the demand being more than twice that in Example 1 at each demand market, all the domestic labor is hired and migrant labor is also hired across all the supply chain network links from production through distribution.

Example 5: Example 1 Data With All the Demand Price Function Intercepts Tripled

In Example 5, the intercept of each demand price function is triple that of the corresponding intercept in Example 1. Hence, demand price functions are now:

$$\rho_1(d) = -5d_1 + 2400, \quad \rho_2(d) = -5d_2 + 2550, \quad \rho_3(d) = -5d_3 + 2700.$$

The modified projection method computes the product path flow pattern:

$$x^*_{p_1} = 24.05, \quad x^*_{p_2} = 24.84, \quad x^*_{p_3} = 30.74,$$
$$x^*_{p_4} = 31.53, \quad x^*_{p_5} = 46.28, \quad x^*_{p_6} = 47.07.$$

The demand price at the first demand market is now: 2,155.57. The price at the second demand market is: 2,238.66, with the demand price at the third demand market being: 2,326.58. The corresponding demands are: 48.89, 62.27, and 93.35.

The firm now enjoys a profit of: 349,417.22, which is more than double the profit in Example 4. Clearly, doing marketing to enhance consumers' value of truffles can yield economic benefits.

The domestic labor on each supply chain link is, as in Example 4, at its bound of 100, with the associated Lagrange multipliers again all positive. In addition, there is a positive investment on each link for attracting migrant labor and all values of migrant labor hours are positive on each link. The total investment outlay is now 86.15, which exceeds that in Example 4 by more than 25%. The Lagrange multipliers: δ^{1*}_a and δ^{2*}_a are equal to 0.00 for all links $a \in L$, and η^* remains at 0.00.

Example 6: Example 1 Data With All the Demand Price Function Intercepts Quadrupled

The demand price functions in Example 6 have each demand price function intercept in Example 1 quadrupled so that the functions are:

$$\rho_1(d) = -5d_1 + 3200, \quad \rho_2(d) = -5d_2 + 3400, \quad \rho_3(d) = -5d_3 + 3600.$$

The algorithm computes the product path flow pattern:

$$x^*_{p_1} = 21.62, \quad x^*_{p_2} = 22.41, \quad x^*_{p_3} = 30.58,$$
$$x^*_{p_4} = 31.37, \quad x^*_{p_5} = 48.87, \quad x^*_{p_6} = 49.66.$$

The demand price at the first demand market is now: 3,090.21. The price at the second demand market is: 2,979.84. The demand price at the third demand market is: 3,205.88. The corresponding respective demands are: 44.03, 61.96, and 98.53.

The firm's profit is: 525,725.19, which is more than 50% higher than the profit in Example 5.

Table 6
Optimal link flows and domestic and international migrant labor values for examples 4 through 7.

Notation	Optimal value			
	Example 4	Example 5	Example 6	Example 7
f^*_a	89.54	101.07	101.07	101.08
f^*_b	91.96	103.44	103.45	103.45
f^*_c	89.54	101.07	101.07	101.08
f^*_d	91.96	103.44	103.45	103.45
f^*_e	181.50	204.51	204.52	204.54
f^*_f	46.60	48.89	44.03	39.18
f^*_g	55.44	62.27	61.96	61.65
f^*_h	79.46	93.35	98.53	103.71
l^{1*}_a	100.00	100.00	100.00	100.00
l^{1*}_b	100.00	100.00	100.00	100.00
l^{1*}_c	100.00	100.00	100.00	100.00
l^{1*}_d	100.00	100.00	100.00	100.00
l^{1*}_e	100.00	100.00	100.00	100.00
l^{1*}_f	100.00	100.00	100.00	100.00
l^{1*}_g	100.00	100.00	100.00	100.00
l^{1*}_h	100.00	100.00	100.00	100.00
l^{2*}_a	62.81	83.76	83.77	83.78
l^{2*}_b	83.91	106.88	106.89	106.91
l^{2*}_c	155.84	188.76	188.78	188.80
l^{2*}_d	162.73	195.55	195.56	195.59
l^{2*}_e	202.50	240.83	240.83	240.83
l^{2*}_f	22.64	28.65	15.87	3.10
l^{2*}_g	54.00	72.97	72.10	71.25
l^{2*}_h	98.64	133.39	146.32	159.27

Table 7
Optimal link international migrant attraction investments and domestic labor bound Lagrange multipliers for examples 4 through 7.

Notation	Optimal value			
	Example 4	Example 5	Example 6	Example 7
v^*_a	2.12	2.85	2.85	2.85
v^*_b	4.39	5.67	5.67	5.67
v^*_c	11.70	16.64	16.64	16.65
v^*_d	12.53	18.33	18.33	18.34
v^*_e	17.03	28.33	28.33	28.33
v^*_f	1.22	1.56	0.85	0.16
v^*_g	3.04	4.21	4.16	4.10
v^*_h	5.93	8.56	9.67	10.87
λ^*_a	0.03	0.03	0.03	0.03
λ^*_b	0.05	0.06	0.06	0.06
λ^*_c	0.12	0.21	0.21	0.21
λ^*_d	0.13	0.30	0.30	0.30
λ^*_e	0.15	395.82	907.84	1419.82
λ^*_f	0.06	0.06	0.05	0.05
λ^*_g	0.06	0.06	0.06	0.06
λ^*_h	0.07	0.08	0.10	0.10

The domestic labor on each supply chain link is, as in Examples 4 and 5, at its bound of 100, with the associated Lagrange multipliers all positive. There is, again, a positive investment on each link for attracting migrant labor and all values of migrant labor hours are positive on each link. The total investment outlay is now 86.50, which is almost identical to the investment total in Example 5. The Lagrange multipliers: δ^{1*}_a and δ^{2*}_a are equal to 0.00 for all links $a \in L$, and η^* is also 0.00. We note that in all the numerical examples, to this point, and the following ones, the variational inequality (20) holds with excellent accuracy. Furthermore, the prices obtained are reasonable, since truffles are such an exclusive, high value agricultural product.

Example 7: Example 1 Data With All the Demand Price Function Intercepts Quintupled

And, finally, in Example 7, each demand price function intercept is 5 times the value found in Example 1. Hence, we have that:

$$\rho_1(d) = -5d_1 + 4000, \quad \rho_2(d) = -5d_2 + 4250, \quad \rho_3(d) = -5d_3 + 4500.$$

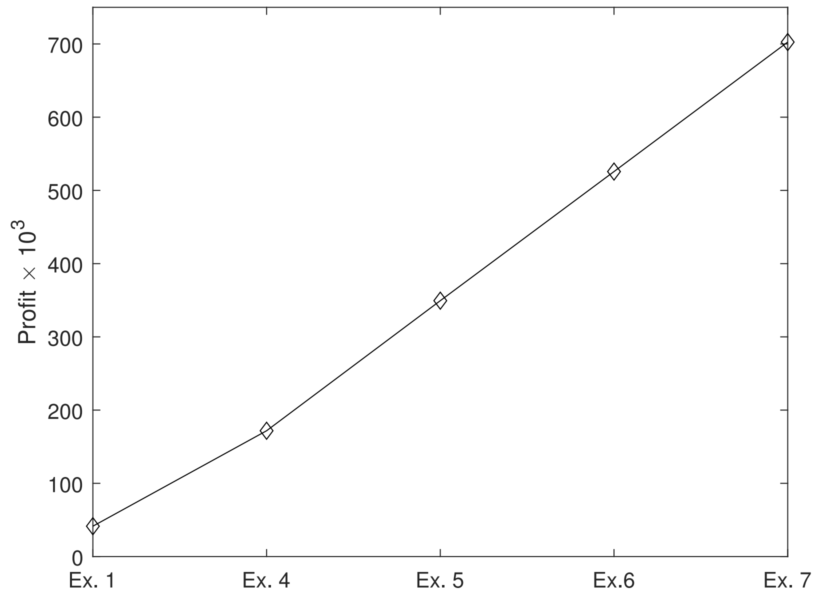

Fig. 3. Effect on profit of the firm when the demand price function intercepts are doubled, tripled, and so on, with Example 1 being the baseline.

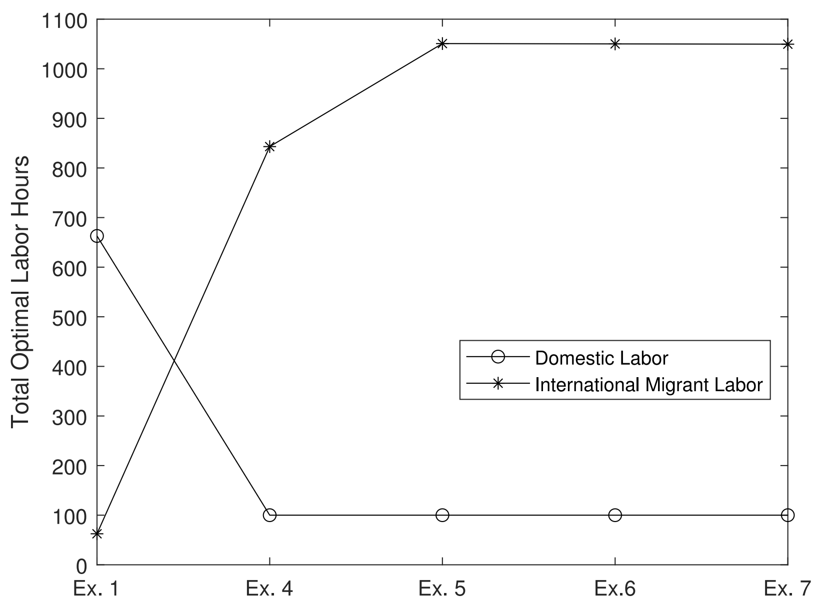

Fig. 4. Effect on optimal total labor hours of domestic labor and of international migrant labor in the supply chain network when the demand price function intercepts are doubled, tripled, and so on, with example 1 being the baseline.

The computed optimal product path flow pattern is:

$$x^*_{p_1} = 19.19, \quad x^*_{p_2} = 19.99, \quad x^*_{p_3} = 30.43,$$
$$x^*_{p_4} = 31.22, \quad x^*_{p_5} = 51.46, \quad x^*_{p_6} = 52.25.$$

The demand price at the first demand market is: 3,804.11. The price at the second demand market is: 3,941.75 and that at the third demand market is: 4,085.17. The respective demands are: 39.18, 61.65, and 103.71.

The firm's profit is now: 702,571.38, a sizeable increase from that in Example 6, with the total demand close to that in Example 6. The total investment outlay is almost identical to that in Example 6 and is at 86.97.

The domestic labor on each supply chain link is, as in Examples 4, 5, and 6, at its bound of 100, with the associated Lagrange multipliers all positive. There is, again, a positive investment on each link for attracting migrant labor and all values of migrant labor hours are positive on each link. The Lagrange multipliers: δ^{1*}_a and δ^{2*}_a are equal to 0.00 for all links $a \in L$, and η^* is also 0.00.

In Fig. 3, the profits are displayed for Example 1, which serves as the baseline, and for Examples 4, 5, 6, and 7. It is clear from Fig. 3 that a firm can greatly benefit by having consumers recognize the value of its product and be willing to pay a higher price for it, even in the pandemic. Recall that, in Example 4, the demand price function intercepts were doubled for all the demand markets, as compared to their values in Example 1, tripled in Example 5, and, so on, until they were quintupled in Example 7.

In Fig. 4, we display the total labor hours at optimality for Example 1 and Examples 4 through 7, in the supply chain network of the firm for both the domestic laborers and the international migrant laborers. We see that, clearly, with consumers being willing to pay a higher price for the truffles, and the bounds of domestic labor being at a value of 100 for the links, international migrant labor is essential. Further, we see that, with Example 5 onwards, the total amount of international migrant labor remains essentially that same, at about 1,050 h, and, from Table 7, we see that the investments are essentially the same for links a through e in Examples 5 through 7, signifying a kind of stabilization.

Series 3 Numerical Examples

In the third, final series of numerical examples, we first, again, explore the impacts of being untruthful in terms of wages in recruiting international migrant laborers and then we investigate the impact of a tighter budget on investments. For each of the examples in this series, we report the computed link flows and the domestic and international migrant labor values in Table 8 and the migrant attraction function investments and domestic labor Lagrange multipliers in Table 9.

Example 8

The data in Example 8 are as in Example 7 except that, now, we consider untruthfulness in informing migrant laborers prior to hiring about their wages. In fact, we set the $\tilde{w}$s to values even higher than those that were being paid to domestic and migrant workers, who were paid, the same in Example 7. Specifically, the wage settings are now:

$$w_a^1 = w_a^2 = 30, \tilde{w}_a = 50, \quad w_b^1 = w_b^2 = 20, \tilde{w}_b = 40,$$

$$w_c^1 = w_c^2 = 18, \tilde{w}_c = 38, \quad w_d^1 = w_d^2 = 18, \tilde{w}_d = 38,$$

$$w_e^1 = w_e^2 = 17, \tilde{w}_e = 37, \quad w_f^1 = w_f^2 = 19, \tilde{w}_f = 39,$$

$$w_g^1 = w_g^2 = 19, \tilde{w}_g = 39, \quad w_h^1 = w_h^2 = 19, \tilde{w}_h = 29.$$

The computed product path flow pattern is:

$$x_{p_1}^* = 62.84, \quad x_{p_2}^* = 63.65, \quad x_{p_3}^* = 74.08,$$
$$x_{p_4}^* = 74.88, \quad x_{p_5}^* = 104.80, \quad x_{p_6}^* = 105.61.$$

The demand price at the first demand market is: 3,367.57. The price at the second demand market is: 3,505.20 and that at the third demand market is: 3,658.35. The demand market prices are all lower than in Example 7. The demands are now: 126.49, 148.96, and 210.41. These are all higher than their respective values in Example 7.

The firm's profit is now: 1,085,374.25, a big increase from the profit in Example 7, which was 702,571.38. The total investment outlay now increases to: 116.15. We now see a huge increase in the hiring of international migrant laborers, which are all attracted to the work, however, under false pretenses in the form of higher wages than either the domestic laborers or the migrant laborers are being paid!

The domestic labor on each supply chain link is, as in Examples 4, 5, and 6, at its bound of 100, with the associated Lagrange multipliers all positive. There is, again, a positive investment on each link for attracting migrant labor and all values of migrant labor hours are positive on each link and these are much higher than those in Example 7. The Lagrange multipliers: δ_a^{1*} and δ_a^{2*} are equal to 0.00 for all links $a \in L$, and η^* is also 0.00.

Example 9 — Same Data as Example 8 with Budget Decrease

Example 9 has the identical data to the data in Example 8 but with a decrease in the budget B from 1000 to 100.

The new computed product path flow pattern is:

$$x_{p_1}^* = 58.22, \quad x_{p_2}^* = 58.09, \quad x_{p_3}^* = 69.06,$$
$$x_{p_4}^* = 68.88, \quad x_{p_5}^* = 94.19, \quad x_{p_6}^* = 94.01.$$

The product path flows all decrease in comparison with their values in Example 8.

The demand price at the first demand market is: 3,418.44. The price at the second demand market is: 3,560.32 and that at the third demand market is: 3,747.19. The respective demands are now: 116.31, 137.94, and 188.20. These are all lower than their respective values in Example 8.

The firm's profit is now: 1,069,553.88, a decrease from the profit in Example 8, which is to be expected. The total investment outlay is now at the bound of 100, with η^* being positive and at a value of 1068.04.

The domestic labor on each supply chain link is, as in Example 8, at its bound of 100, with the associated Lagrange multipliers all positive. Notably, the value of the Lagrange multipliers, the λ^*s, is quite high now, demonstrating that the firm should try to make more domestic labor interested and willing to work in the production and harvesting of truffles. There is, again, a positive investment on each link for attracting migrant labor and all values of migrant labor hours are positive on each link and these are much higher than those in Example 7. The Lagrange multipliers: δ_a^{1*} and δ_a^{2*} are equal to 0.00 for all links $a \in L$, and η^* is also 0.00.

Obviously, this is another example of "cheating" in that international migrants are attracted to work through being told that they will be paid higher wages than what they will actually be paid. This example, cheating or not, demonstrates also the importance of having a sufficient budget in order to be able to attract the needed international migrant labor. Furthermore, through the use of Lagrange multipliers, we can see the value of increasing the availability of domestic labor in this endeavor, as well as the budget for investing in attracting international migrant labor. The above examples are stylized, but do provide insights and, importantly, demonstrate both the breadth of the model and the effectiveness of the computational procedure.

We also emphasize that the numerical examples, focusing on a high value agricultural product, with complete input and output data reported, illustrate both the effectiveness of the computational procedure, as well as the information that the solution of the model reveals in terms of the optimal product path flows, the optimal national and international labor values needed, the optimal Lagrange multipliers associated with labor, budget, and other constraints, as well as the profit earned by the firm under different scenarios. The model can be applied to distinct products not only in agriculture but also even to high technology through appropriate parameterization. Future research may include the expansion to include additional tiers of suppliers as well as additional competitors using game theory.

Table 8
Optimal link flows and domestic and international migrant labor values for examples 8 and 9.

Notation	Optimal value	
	Example 8	Example 9
f_a^*	241.72	221.48
f_b^*	244.14	220.97
f_c^*	241.72	221.48
f_d^*	244.14	220.97
f_e^*	485.86	442.45
f_f^*	126.49	116.31
f_g^*	148.96	137.94
f_h^*	210.41	188.20
l_a^{1*}	100.00	100.00
l_b^{1*}	100.00	100.00
l_c^{1*}	100.00	100.00
l_d^{1*}	100.00	100.00
l_e^{1*}	100.00	100.00
l_f^{1*}	100.00	100.00
l_g^{1*}	100.00	100.00
l_h^{1*}	100.00	100.00
l_a^{2*}	339.49	302.68
l_b^{2*}	388.28	341.95
l_c^{2*}	590.63	532.79
l_d^{2*}	597.54	531.35
l_e^{2*}	709.77	637.42
l_f^{2*}	232.86	206.09
l_g^{2*}	313.78	283.16
l_h^{2*}	426.03	370.50

Table 9
Optimal link international migrant attraction investments and domestic labor bound Lagrange multipliers for Examples 8 and 9.

Notation	Optimal value	
	Example 8	Example 9
v_a^*	6.98	6.21
v_b^*	10.23	8.95
v_c^*	19.58	17.10
v_d^*	19.89	17.04
v_e^*	23.76	20.70
v_f^*	6.39	5.61
v_g^*	8.85	7.90
v_h^*	20.47	16.56
λ_a^*	0.02	22.50
λ_b^*	0.03	29.35
λ_c^*	0.04	43.95
λ_d^*	0.05	43.87
λ_e^*	0.04	43.49
λ_f^*	0.03	30.97
λ_g^*	0.03	32.71
λ_h^*	0.08	67.86

6. Summary and conclusions

The COVID-19 pandemic is transforming our societies and economies and is also demonstrating the criticality of labor resources to supply chains. Many countries have been grappling with shortages of workers from the agricultural and manufacturing sectors to various services including healthcare. Attracting international migrant labor may help to assuage shortages in domestic labor.

In this paper, we aim to establish the foundation for the integration of labor in supply chains coupled with international human migration, with the latter focused on labor. We provide an advance to the existing literature through the synthesis of a supply chain network optimization model with labor that includes both domestic as well as migrant labor with the latter requiring investments subject to a budget constraint. The model has the flexibility to handle wages that are the same or different for domestic and migrant laborers and also the use of the truthful wage in the migration attraction functions that will be paid the migrant laborers for their work or not.

The theoretical framework that we utilize for the formulation, analysis, and solution of three series of numerical examples is the theory of variational inequalities. The alternative variational inequality that we construct is in path flow and investment variables plus several sets of Lagrange multipliers, including those associated with the upper bounds on domestic labor on the various supply chain network links and the budget constraint. The alternative variational inequality allows for the implementation of an elegant computational method with closed form expressions for the underlying variables at each iteration. To-date, there are very few variational inequality models for operations research problems with nonlinear constraints. Hence, our work also adds to that literature.

We provide three series of numerical examples, with complete input and out data with the examples being motivated by a fairly rare, high priced, high value agricultural product — truffles, which are now even being grown in the United Kingdom. The numerical examples reveal the benefits of having Lagrange multiplier information, and show that generating additional interest in an agricultural product in terms of prices that consumers are willing to pay, can yield sizeable increases in profit. The examples also show the impacts if a firm is untruthful in reporting wages that international migrants will actually be paid in attracting them, which suggests that policy makers need to be aware of such "cheating" since it can lead to higher profits.

In addition to the possible extensions noted at the end of Section 5, the model in this paper can be extended to include, for example, competition among different firms for international migrant labor and also to allow for different productivity factors associated with different levels of skilled migrants. In addition, it would be interesting and valuable to include a tier of brokers, along with their behavior, who work to procure migrant laborers for firm' supply chain networks.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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This paper is dedicated to freedom-loving people everywhere.

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