Multicommodity Fresh Produce Trade Networks with Quality Deterioration Under Congestion and Transportation Capacities

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Abstract: This paper constructs a multicommodity spatial price equilibrium framework designed to allow for the quantification of the impacts of congestion and transportation capacities on trade flows and product prices while capturing the degradation of fresh produce quality through explicit quality deterioration formulas. We present the multicommodity fresh produce trade network model under equilibrium conditions both without and with minimum quality standards. Alternative variational inequality formulations of the governing equilibrium conditions are derived, with one of them exhibiting particularly favorable features for computational purposes. The numerical examples focus on the banana trade, since bananas are the most widely traded fresh produce commodity globally, with leading exporters being Ecuador and Costa Rica and major importers, the United States and the European Union. Through various scenarios, we simulate congestion and time delays in the Panama Canal and report on the impacts of ongoing disruptions, on shipment times, equilibrium shipment volumes, the supply and demand prices, quality levels at the supply markets and at the demand markets, and transportation costs. Our baseline numerical example results closely align with real-life data on export volumes of bananas, the associated supply and demand prices, and the transportation costs. The comprehensive numerical results reveal that reductions in transportation capacity and prolonged shipment times, as in the Panama Canal due to climate-related issues, lead to decreased banana shipment volumes, increased initial quality, but diminished final quality of bananas at demand markets. This exposes significant challenges for consumers, as well as for producers, as instances of transportation capacity disruptions and extended shipment times result in higher prices for lower-quality bananas.

Key words: trade, fresh produce, networks, quality deterioration, variational inequalities, bananas

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1. Introduction

Fresh produce, consisting of fruits and vegetables, is essential to healthy diets, and its availability and affordability are critical to food security as well as to global stability. According to the International Fresh Produce Association [43], global trade more than doubled between 2000 and 2018, with exports being behind expansion of the fruit and vegetable sector, with the expected fresh vegetable global revenue in 2022 at $691.20 billion US and that for fresh fruit at $622.80 billion US. However, the Food and Agriculture Organization [30] of the United Nations reports that, despite the global trade in fresh produce, the per capita consumption of vegetables and fruit does not meet the daily recommended level of 400 grams, and is, instead, in the range of 20-50% of the minimum recommended amount.

The Global Coalition for Fresh Produce [37] acknowledges that, although the fresh produce sector showed resilience during the COVID-19 pandemic, there remain numerous challenges for this sector, which have intensified over the past three years. Such challenges include: various cost increases, including that for fertilizer, transportation inefficiencies and delays, labor shortages, decreasing purchasing power of consumers, and various obstacles to international trade. In addition, it is essential to note the impacts of climate change on this important food sector ([97]) as well as the negative externalities associated with wars and strife, along with the related disasters. For example, Russia's blockade of maritime routes on the Black Sea, following its full-scale invasion of Ukraine on February 24, 2022, and its subsequent withdrawal from the Black Sea Grain Initiative after a year, have impacted prices and food security (cf. [62]) due to restrictions on the safe and secure export of agricultural products from Ukraine. Challenges remain in finding efficient alternative routes for exports, including that of food products, with congestion as well as capacity limitations playing roles (see [81] and [89]). Furthermore, as Nagurney [63] points out, labor strikes, such as the recent port strikes in 2024, significantly can hinder the flow of fresh produce, with bananas being particularly vulnerable due to their perishability and dependence on prompt transportation.

Also, as noted in Nagurney et al. [64], the Horn of Africa is having its worst drought recorded in modern history with a study by Cassidy [19] emphasizing that the drought has resulted in food insecurity for 21 million people in the region in countries such as Somalia, Kenya, and Ethiopia. And, a historical drought is resulting in major traffic jams and delays in the Panama Canal, which serves as a strategic gateway between the Pacific and Atlantic Oceans and is a major corridor for agricultural trade [58]. A special advisory was issued in late July with capacity restrictions leading to the number of vessels being reduced on a daily basis from 36 to 32. The resultant delays have increased the transportation costs. Furthermore, attacks by the Houthi rebels on commercial shipping on the Red Sea, which began in November 2023, are now escalating and are affecting transit through the Suez Canal, a major link in the global trade network, which handles 10% of global commerce (see [15]). The attacks are causing major carriers to reroute cargo, adding to both fuel costs and delivery times. The attacks have had repercussions on the trade of fresh produce from Europe to Asia [42]. The Ever Given incident that blocked the Suez Canal back in 2021 resulted in a container ship traffic jam that included cargo of apples from New Zealand (see [35]).

According to the International Fresh Produce Association [43], among the top ten fresh produce commodities in terms of global production are: bananas at 117 million tons, watermelon at 100 million tons, and apples at 87.2 million tons. Bananas are the top fruit product traded internationally. The OECD and COLEACP state in their report [78] that the five most important trade flows in fresh fruits are: the European Union to the European Union, Latin America to the European Union, Latin America to North America, Southeast Asia to Southeast Asia, and North America to North America.

Furthermore, as noted in Besik and Nagurney [9]; see, also Yu and Nagurney [101], trade in fresh produce can be complex since the quality of the fresh produce commodity deteriorates continuously over time, even under ideal conditions. Fresh produce, in terms of fruits and vegetables, is a food category with one of the highest food loss rates estimated by the Food and Agriculture Organization [29] to be at about 22% from post-harvest to distribution.

In this paper, we construct a novel multicommodity fresh produce trade network equilibrium model with quality deterioration, which captures the impacts of congestion on quality as the fresh produce commodities are transported from supply markets to demand markets. Our model adds to the literature in several significant ways. Our unique contribution resides in the application of a spatial price equilibrium framework to analyze the dynamic impacts of congestion over time. Additionally, our modeling framework incorporates minimum quality standards for fresh produce at demand markets and accounts for transportation capacities. The analysis is conducted through a numerical study focused on banana trade, with a specific emphasis on the trade between leading exporters Ecuador and Costa Rica, and major importers, namely the United States and the European Union through the Panama Canal. Our numerical results show that reductions in transportation capacity and increase in shipment times result in lower banana shipment volumes, increased initial quality, and diminished final quality at demand markets. This reveals unexpected challenges for consumers as they end up paying higher price for lower quality bananas. We also find that more stringent quality standards can benefit consumers and producers alike. Furthermore, findings reveal that certain non-lead banana export countries strategically capitalize on congestion to increase banana shipments and maintain high demand prices. We first provide an overview of related work and then delineate the contributions in this paper.

2. Related Work, Contributions, and Organization of the Paper

This paper focuses on the trade of fresh produce commodities whose quality deteriorates as they move through space and time, modeled using a spatial price equilibrium framework. Our work builds on and connects three core strands of literature: (i) spatial price equilibrium models in agricultural trade; (ii) supply chain network models for fresh produce that incorporate quality deterioration; and (iii) modeling efforts that introduce minimum quality standards and bounds on flows or initial product quality.

Although each of these research areas has advanced important insights, they tend to address the underlying challenges in isolation. Most spatial price equilibrium models applied to agriculture neglect perishability or rely on simplified representations. Quality deterioration models typically omit trade network features and route-specific shipment dynamics. Models with minimum quality standards often assume static quality or focus on a single product without deterioration effects.

This paper offers a unified modeling framework that advances the field in several important ways. We develop a multicommodity spatial trade network equilibrium model for fresh produce that: (1) explicitly captures quality deterioration using food science-based expressions, (2) accounts for congestion-induced shipment delays and their effect on product quality, and (3) incorporates minimum quality standards at demand markets, along with transportation and supply-side capacity constraints. To our knowledge, this is the first model to jointly address these factors within an agricultural spatial price equilibrium model formulation.

2.1 Agricultural Spatial Price Equilibrium Models

The seminal spatial price equilibrium models of Samuelson [84] and Takayama and Judge [91, 92], with variational inequality theory applied to extend them, for the first time, by Florian and Los [28], have been widely applied to various agricultural settings. Dafermos and Nagurney [23] conducted sensitivity analysis for general spatial price equilibrium problems using variational inequality theory whereas Nagurney and Aronson [64] were the first to construct a multiperiod spatial price equilibrium model with gains and losses to handle perishable agricultural products (and also financial ones). They utilized a generalized network approach and variational inequality theory. Yu and Nagurney [101], subsequently, but in the context of a supply chain network equilibrium model, used arc multipliers to assess food losses and waste with quality deterioration being associated with links and being known a priori. Nagurney and Besik [65] introduced flow-dependent arc multipliers for commodity trade in a spatial price equilibrium framework. Nolte [77] developed a spatial price equilibrium model for the global sugar market. Additionally, Grant, Hertel, and Rutherford [40] and Nagurney, Besik, and Dong [66] introduced spatial price equilibrium models with a primary focus on agriculture, particularly dairy products, and a range of trade mechanisms. The latter explored the variational inequality framework, while the former presented a mixed-complementarity framework within a partial equilibrium model. Van Campenhout, Pauw, and Minot [95] proposed a general equilibrium model to predict the impact of higher food prices on different subgroups of society in Uganda. Recently, Baptista, Unsal, and Spray [8] constructed a spatial general equilibrium model with households and multiple geographic regions to investigate the susceptibility of households to food insecurity resulting from climate-related shocks.

In contrast to prior literature, our paper introduces a multicommodity fresh produce trade spatial network equilibrium model. This model incorporates explicit quality deterioration formulae and places a primary emphasis on assessing the effects of congestion on the quality of fresh food. Consequently, our research distinguishes itself from earlier agricultural spatial price equilibrium models, marking a noteworthy and innovative contribution to the literature focusing on agricultural spatial price equilibrium models.

2.2 Fresh Produce Supply Chains and Quality Deterioration

The first study to integrate quality deterioration with food science formulae for fresh produce in competitive markets was done by Besik and Nagurney [9], who focused on farmers' markets. Nagurney, Besik, and Yu [67] then constructed extensions of that work for general supply chain networks. Capacities on initial product quality at production sites were included and consumers responded to the differentiated fresh produce commodities through prices that they were willing to pay, which depended both on commodity quantities and their average quality. Besik, Nagurney, and Dutta [10], subsequently, developed an integrated supply chain network model that consisted of two tiers with consideration of both fresh produce and minimally processed fresh produce by consumers.

Many authors have emphasized the importance of quality deterioration in fresh produce products as they traverse the supply chain ([1], [2], [3], [4], [12], [60], [90], [96]). Rong, Akkerman, and Grunow [83] developed a mixed-integer linear programming model that effectively integrated quality decay considerations into production and distribution planning for food products. In a similar vein, Lejarza and Baldea [56] proposed a mixed-integer programming model that captured multiple quality attributes associated with agricultural product quality degradation in supply chains. Ferrer et al. [27] presented an optimization model for scheduling wine grape harvesting, emphasizing operational costs and quality. Widodo et al. [99] investigated fresh produce supply chains by developing a mathematical model that encompasses the growth, harvesting, and deterioration of fresh produce products. Jonkman, Barbosa-Povoa, and Bloemhof [44] utilized mixed-integer linear programming to design a model for agro-food supply chains, incorporating quality constraints. Amorim, Costa, and Almada-Lobo [5] introduced demand functions that consider both product quality and price, allowing them to construct demand models for various products based on their age. Moreover, Ketzenberg, Bloemhof, and Gaukler [47] studied the utilization of time and temperature data for managing perishable goods within the framework of a retail operation that deals with products having uncertain lifespans, influenced by stochastic demand and potential lost sales. Chen and Chen [20] introduced a dynamic on-site direct-sales inventory model for perishable foods, while taking into account quality deterioration.

We also note the earlier works of Ghare and Schrader [36], Nahmias [75, 76], Silver, Pyke, and Peterson [87], and Coelho and Laporte [21] focusing on perishability in inventory management. Additionally, we note that numerous studies in the literature explore technology and data-driven approaches for agricultural and food systems, with further insights available in Papajorgji and Pardalos [79,80] and Kyrgiakos et al. [51].

In our paper, we also focus on the deterioration of fresh produce quality, employing explicit quality deterioration formulas. Our unique contribution lies in our utilization of a spatial price equilibrium framework to examine the effects of congestion over time as well as inclusion of minimum quality standards as well as transportation capacities. Our research holds significant relevance to real-world scenarios and introduces a new perspective to the existing body of literature on quality deterioration models within fresh produce supply chains.

2.3 Minimum Quality Standards and Bounds on Commodity Shipments and Initial Quality

Nagurney, Li, and Nagurney [72] introduced quality as a variable in spatial price equilibrium models, both static and dynamic ones, and utilized opportunity costs as we do here. However, that model had minimum quality standards imposed on the supply side, whereas, here, in contrast, the minimum quality standards are imposed on the commodities at the demand markets. Furthermore, our model is a multicommodity one plus quality of the commodities is not preserved over the transportation routes, but, instead, deteriorates according to physical conditions associated with food science associated with the specific fresh produce commodity. Nagurney and Li [70] developed a static and a dynamic supply chain network model with information asymmetry in product quality, in which the competing, profit-maximizing firms are aware of the quality of the product that they produce but consumers, at the demand markets, only know the average quality. They also showed how minimum quality standards can be incorporated. These models were also single product ones and assumed quality preservation from production sites to consumption sites. Both of the above papers utilized variational inequality theory for the static model development and projected dynamical systems theory (cf. [74]) for the construction of their dynamic counterparts. Additional models with quality as a variable is either a spatial price equilibrium setting or supply chain network one can be found in the book by Nagurney and Li [71].

Bounds on commodity shipments in spatial price equilibrium problems have been drawing increasing attention. Nagurney, Salarpour, and Dong [73], in their spatial price equilibrium model with trade measures relevant to the COVID-19 pandemic, included quotas, which corresponded to bounds on commodity flows from distinct supply markets to demand markets. Nagurney, Besik, and Dong [66], in turn, investigated tariff-rate quotas, which are two-tiered tariffs, in a multicommodity spatial price equilibrium framework. More recently, Birge et al. [11] studied spatial trade networks under capacities with a focus on energy applications. Nagurney et al. [69], in turn, building on the work in Nagurney et al. [68], which introduced exchange rates into spatial price equilibrium models using variational inequality theory, formulated a spatial price equilibrium model with exchange rates in which there are capacities at supply markets across all commodities as well as such capacities across transportation routes.

The exploration of bounds on initial quality in the existing literature has been limited. Nagurney, Besik, and Yu [67] constructed a competitive oligopolistic supply chain network model for fresh produce. In their modeling framework, food firms not only determine the quantity of fresh produce to produce but also make decisions regarding the initial quality of the produce. The authors additionally investigated fresh produce quality deterioration using explicit formulae based on time and temperature, with bounds on the initial quality. However, existing studies consider a constant time component in quality deterioration of agricultural products. In our paper, in contrast, we explore the intricacies of agricultural product transportation, where the time for shipment can depend on the volume of shipments. This perspective allows for a more nuanced quantification of fresh produce quality deterioration in trade networks.

2.4 Contributions in this Paper

The novelty of contributions in this paper include the following:

1. Quality deterioration of fresh produce is captured in a perfectly competitive spatial price equilibrium

model for the first time using kinetic formulae.

2. The final quality of the fresh produce includes the impacts of congestion on transportation time.

3. Consumers at the demand markets respond to both the quantity of the fresh produce as well as to its quality whereas the producers at the supply markets incur costs associated with quality in their supply price functions.

4. Upper bounds are incorporated on the initial product quality achievable at the supply markets for the commodities as well as bounds on the shipment of each commodity on a transportation route.

5. Equilibrium conditions are presented for the fresh produce trade network model without minimum quality standards as well as for the one with minimum quality standards.

6. Alternative variational inequalities are presented, one of which has especially nice features for computational purposes.

7. Numerical examples based on the banana trade and impacts of the drought affecting the Panama Canal and ensuing congestion and transportation capacity reduction illustrate the modeling and algorithmic scheme.

2.5 Organization of the Paper

The paper is organized as follows. In Section 3, we first present some preliminaries associated with quality deterioration of fresh produce for easy reference. The multicommodity fresh produce trade network model with quality deterioration is then constructed with the version without minimum quality standards and then the one with such standards. The latter one also includes capacities on each commodity shipment on each route and an upper bound on the initial quality of each commodity produced at each supply market. The equilibrium conditions are detailed and a variational inequality formulation established. We also provide an alternative variational inequality formulation for the model with minimum quality standards. In Section 4, we outline an algorithm for computational purposes and show explicitly how, at each iteration, it resolves the variational inequality problem into subproblems that yield closed form expressions for the commodity shipment variables, the initial quality variables, and the Lagrange multipliers associated with the minimum quality variables. In addition, we provide some theoretical results. In Section 5, we illustrate the relevance and applicability of our modeling framework through a series of numerical examples, accompanied by sensitivity analysis, focused on trade of bananas through the Panama Canal. The paper concludes with Section 6 in which we summarize our results and present our conclusions.

3. The Multicommodity Fresh Produce Trade Network Model with Quality Deterioration

In this Section, we construct the multicommodity fresh produce trade network equilibrium model with quality deterioration. In Section 3.1, we present the model without minimum quality standards and then, in Section 3.2, we expand the equilibrium conditions to include minimum quality standards for the fresh produce at the demand markets as well as bounds on the initial quality at the supply markets for the commodities

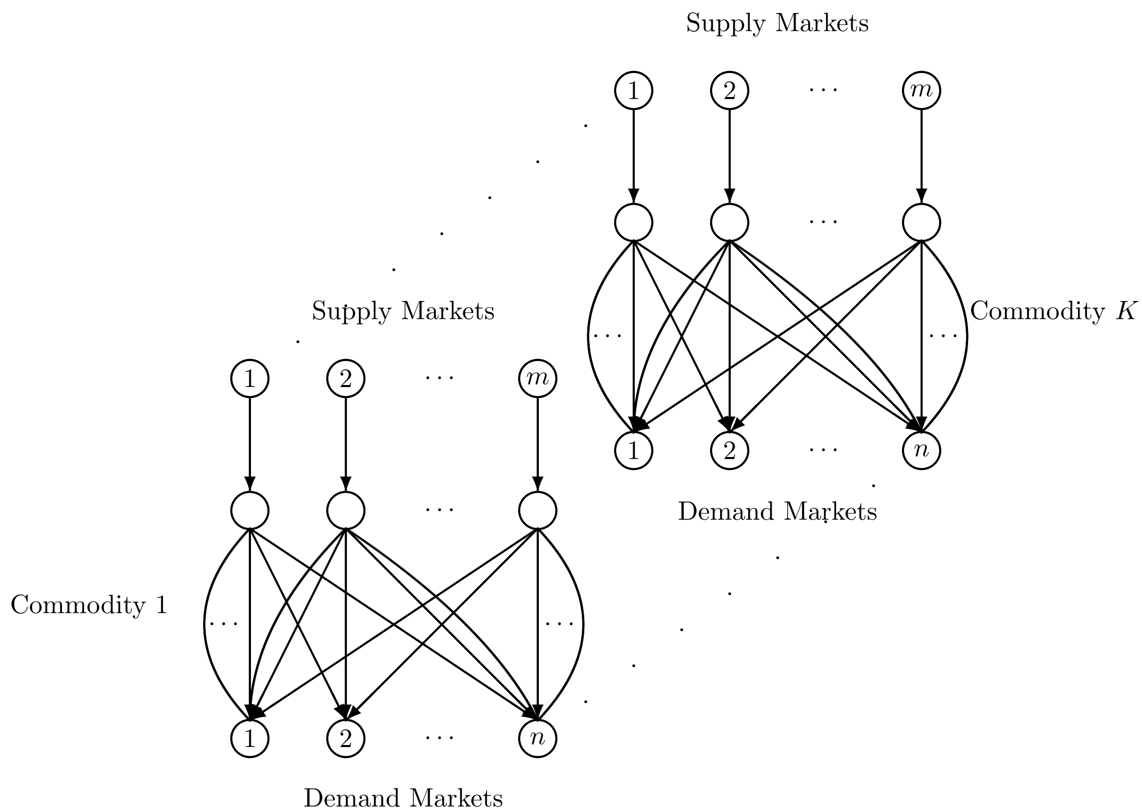


Figure 1: The Multicommodity Fresh Produce Trade Network

and bounds on the commodity flows on the transportation routes. There are m supply markets involved in the production of K fresh produce commodities, with a typical supply market denoted by i and a typical commodity by k. There are n demand markets in the trade network with a typical demand market denoted by j. Joining each pair of supply and demand markets are multiple routes with a typical route denoted by r. Without loss of generality, we assume that each pair of supply and demand markets is joined by P possible routes and emphasize that a route r joining a pair of supply and demand markets need not correspond to the same route r joining another pair of markets. The supply markets and the demand markets can be in the same or in different countries. The multicommodity network representation of the model as a multicopy network is given in Figure 1.

The basic notation for the models is given in Table 1. All vectors are assumed to be column vectors.

Since the focus of the modeling and analysis in this paper is fresh produce and its quality deterioration, we first provide some preliminaries.

Table 1: Notation for the Multicommodity Fresh Produce Trade Network Models

Notation	**Parameter Definition**
T^k_{ijr}	temperature for the shipment of commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$.
$\bar{Q}^k_{ijr}$	upper bound (capacity) on the volume of shipment of commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$.
$\bar{q}^{0k}_i$	upper bound on the initial quality of commodity k; $k=1,\ldots,K$ at supply market i; $i=1,\ldots,m$.
$\underline{q}^k_{ijr}$	minimum quality standard for commodity k; $k=1,\ldots,K$ produced at supply market i; $i=1,\ldots,m$ at demand market j; $j=1,\ldots,n$ transported on route r; $r=1,\ldots,P$.
Notation	**Variable Definition**
s^k_i	the supply of the commodity k; $k=1,\ldots,K$ at supply market i; $i=1,\ldots,m$. We group all the supplies into the vector $s\in R^{Km}_+$.
d^k_{ijr}	the demand for the commodity k; $k=1,\ldots,K$ at demand market j; $j=1,\ldots,n$ that was transported on route r; $r=1,\ldots,P$ from supply market i; $i=1,\ldots,m$. We group all the demands into the vector $d\in R^{KmnP}_+$.
Q^k_{ijr}	the shipment of the commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ on route r; $r=1,\ldots,P$. We group all the commodity shipments into the vector $Q\in R^{KmnP}_+$.
q^{0k}_i	the initial quality of commodity k; $k=1,\ldots,K$ produced at supply market i; $i=1,\ldots,m$. We group the initial commodity quality values into the vector $q^0\in R^{Km}_+$.
q^k_{ijr}	the quality of commodity k; $k=1,\ldots,K$ produced at i; $i=1,\ldots,m$ and arriving at j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$. We group these quality levels into the vector $q\in R^{KmnP}$.
t^k_{ijr}	the time for shipment of commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$. We group all such times into the vector $t\in R^{KmnP}_+$.
λ^k_{ijr}	the Lagrange multiplier associated with constraint (22a) for the commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$. We group all Lagrange multipliers into the vector $\lambda\in R^{KmnP}_+$.
γ^k_i	the Lagrange multiplier associated with constraint (22b) for the commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$. We group all these Lagrange multipliers into the vector $\gamma\in R^{Km}_+$.
μ^k_{ijr}	the Lagrange multiplier associated with constraint (22c) for the commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$. We group all such Lagrange multipliers into the vector $\mu\in R^{KmnP}_+$.
Notation	**Function Definition**
$\pi^k_i(s,q^0)$	the supply price function for commodity k; $k=1,\ldots,K$ at supply market i; $i=1,\ldots,m$. We group all these functions into the vector $\pi(s)\in R^{Km}$.
$OC^k_i(q^0)$	opportunity cost associated with commodity k; $k=1,\ldots,K$ and supply market i; $i=1,\ldots,m$. We group such functions into the vector $OC(q^0)\in R^{Km}$.
$\rho^k_{ijr}(d,q)$	the demand price function for commodity k; $k=1,\ldots,K$ produced at supply market i; $i=1,\ldots,m$ and shipped on route r; $r=1,\ldots,P$ at demand market j; $j=1,\ldots,n$. We group all the demand price functions into the vector $\rho(d,q)\in R^{KmnP}$.
$c^k_{ijr}(Q,t^k_{ijr},T^k_{ijr})$	the unit transportation cost associated with shipping the commodity k; $k=1,\ldots,K$ from supply market i; $i=1,\ldots,m$ to demand market j; $j=1,\ldots,n$ via route r; $r=1,\ldots,P$. We group all the unit transportation costs into the vector $c(Q,t,T)\in R^{KmnP}$, where T here is the vector of all the T^k_{ijr}.

Preliminaries

Here, we present some preliminaries on the concept of quality deterioration in the context of fresh produce commodities. It is important to note that defining the quality of fresh produce isn't a straightforward task due to its subjective nature, which can vary across different cultures and nations. According to Kader [45], fresh produce quality encompasses attributes such as color, appearance, flavor, texture, and nutritional value. Fresh produce commodities, being biological products, naturally experience a deterioration in quality over time, characterized by specific quality attributes ([85], [88]). In our paper, we establish a definition for the quality of fresh produce, similar to the approach taken by Besik and Nagurney [9], Nagurney, Besik, and Yu [67], and Besik, Nagurney, and Dutta [10]. We achieve this by employing kinetic functions that quantify the rate at which quality deteriorates over time for a relevant quality attribute.

Taoukis and Labuza [93] define the rate of quality deterioration as a function influenced by various factors, including the microenvironment, gas composition, relative humidity, and temperature. Labuza [52] captures the decay of a specific food attribute, denoted as q, over time t through the following differential equation:

$$\frac{\partial q}{\partial t} = -\kappa q^b = -Ae^{(-E_A/RT)}q^b. \tag{1}$$

In equation (1), κ represents the reaction rate, as defined by the Arrhenius formula, which is expressed as $-Ae^{(-E_A/RT)}$. Here, A stands for the pre-exponential constant, T denotes temperature, E_A represents activation energy, and R corresponds to the universal gas constant, as established by Arrhenius [7]. Additionally, b signifies the reaction order, which is a nonnegative integer belonging to the set $\mathrm{Z}^* = \{0\} \cup \mathrm{Z}^+$. In general, the deterioration of the fresh produce's quality attribute can be expressed in terms of its reaction order. Here, we consider fresh produce whose quality deteriorates with a reaction order $b = 0$. When the reaction order b is zero, indicated by $\frac{\partial q}{\partial t} = -\kappa$, the rate at which the food attribute quality, denoted as q, deteriorates over time t with an initial quality level of q^0, can be expressed using the function described in Tijskens and Polderdijk [94]:

$$q = q^0 - \kappa t. \tag{2}$$

Such quality functions are commonly known as zero-order quality decay functions. Broccoli [38], strawberries [55], watermelons [24], and bananas [100] are some examples of fresh produce items that exhibit a zero-order reaction for specific quality attributes.

We can rewrite the quality deterioration function in (2) according to our modeling framework. Given an initial quality (which is a variable in our framework) of q_i^{0k} of commodity k at supply market i, the commodity k's quality, when it reaches demand market j, and has been transported on route r, follows the formula:

$$q_{ijr}^k = q_i^{0k} - \kappa_{ijr}^k t_{ijr}^k, \tag{3}$$

where κ_{ijr}^k is the reaction rate for commodity k on route r and t_{ijr}^k is the time for commodity k on route r from i to j. The reaction rate is described on each link ijr by the Arrhenius formula, where T_{ijr}^k is the temperature associated with the link ijr, and commodity k, expressed as:

$$\kappa_{ijr}^k = Ae^{(-E_A/RT_{ijr}^k)}. \tag{4}$$

Note that different modes of transport associated with routes may have distinct reaction rates.

In this paper, we are interested in capturing congestion and associated delays, which can impact the quality of the fresh produce. Hence, we have the following time expressions:

$$t_{ijr}^k = g_{ijr}^k Q_{ijr}^k + h_{ijr}^k, \quad \forall k, i, j, r, \tag{5}$$

where $h_{ijr}^k > 0$, $\forall i, j, r, k$ and, typically, g_{ijr}^k is also greater than zero. Of course, if the time on a route is independent of the volume of the commodity flow, then we can set the $t_{ijr}^k = h_{ijr}^k, \quad \forall k, i, j, r$, with the g_{ijr}^k then equal to zero.

Note that, in view of (3) and (5), we have that the final quality, as perceived by consumers, of commodity k produced at i and transported to j via route k, q_{ijr}^k, satisfies the following expression:

$$q_{ijr}^k = q_i^{0k} - \kappa_{ijr}^k (g_{ijr}^k Q_{ijr}^k + h_{ijr}^k), \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, P. \tag{6}$$

According to (6), the time for transport on a link is an increasing function if the g coefficient is positive. This is quite reasonable since one expects that the greater the volume of the commodity and, hence, weight, the greater the time for loading and unloading and, also, a lengthier transport time.

The Conservation of Flow Equations

We now state the conservation of flow equations.

The quantity of commodity k produced at supply market i must be equal to the sum of the shipments of the commodity from i to all the demand markets; that is:

$$s_i^k = \sum_{j=1}^{n} \sum_{r=1}^{P} Q_{ijr}^k, \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{7}$$

Furthermore, the demand for each commodity k at each demand market j transported on route r must be equal to the shipment of that commodity to each demand market; that is

$$d_{ijr}^k = Q_{ijr}^k, \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, P. \tag{8}$$

Note that, here, the consumers at the demand markets care about the quality of the fresh produce commodities and, hence, they differentiate the commodity by supply market and by route. The models in this paper, hence, both without minimum quality standards and with such standards, are examples of product differentiation, in contrast to much of the earlier work on spatial price networks.

Also, all the commodity shipments must be nonnegative; that is:

$$Q^k_{ijr} \geq 0, \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n; r = 1, \ldots, P. \tag{9}$$

Re-Expression of the Supply Price and Demand Price Functions

We now redefine both the supply price functions and the demand price functions in terms of the commodity shipments, while retaining the respective quality levels (initial at the supply markets or that remaining upon arrival at the demand markets).

According to Table 1, the supply price of a commodity at a supply market can, in general, be a function not only of the supply of the commodity at the supply market (the amount produced) but also of the supplies of other commodities at the supply market as well as the supplies of the commodities at all other other supply markets, and their initial quality levels. Farmers must decide what commodities to produce and, clearly, there may be competition for resources such as fertilizers and even labor for their production activities.

The commodity supply price functions π^k_i, for all k, i, due to the conservation of flow equations (7), may be redefined as $\tilde{\pi}^k_i$, for all k, i, as follows:

$$\tilde{\pi}^k_i(Q, q^0) \equiv \pi^k_i(s, q^0), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{10}$$

Similarly, due to (8), plus (3), and (5) (or (6)), we can construct new commodity demand price functions $\tilde{\rho}^k_{ijr}$, for all i, k, j, r, such that:

$$\tilde{\rho}^k_{ijr}(Q, q^0) \equiv \rho^k_{ijr}(d, q), \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n. \tag{11}$$

Furthermore, in view of (6), can define the unit transportation cost functions $\tilde{c}^k_{ijr}$, for all i, k, j, r, as follows:

$$\tilde{c}^k_{ijr}(Q, T) \equiv c^k_{ijr}(Q, t^k_{ijr}, T^k_{ijr}), \quad k = 1, \ldots, K; i = 1, \ldots, m; j = 1, \ldots, n. \tag{12}$$

We emphasize, as noted in Table 1, the generality of the commodity unit transportation cost functions, where the unit transportation cost can depend on the vector of commodity shipments between all pairs of supply and demand markets, as well as on the temperature associated with the transportation route and the commodity. This level of generality allows one to further capture competition for transportation services among commodities as well as congestion, since the unit transportation cost functions of the routes are commodity flow-dependent.

We assume that the supply price, demand price, and the unit transportation cost functions are all continuous as are the opportunity cost functions (cf. Table 1).

3.1 The Multicommodity Trade Network Model Without Minimum Quality Standards and Bounds

The multicommodity fresh produce trade network equilibrium conditions are now stated. In Section 3.2, we expand the equilibrium conditions to include minimum quality standards at the demand markets as well as specific bounds.

Definition 1: The Multicommodity Fresh Produce Trade Network Equilibrium Conditions with Quality Deterioration But No Minimum Quality Standards and No Bounds

A multicommodity shipment and initial quality level pattern $(Q^*, q^{0*}) \in \mathcal{K}^1$*, where* $\mathcal{K}^1 \equiv \{(Q, q^0)|(Q, q^0) \in R_+^{KmnP+Km}\}$ *is a multicommodity fresh produce trade network equilibrium with quality deterioration if the following conditions hold: for all commodities* k*;* $k = 1, \dots, K$*; for all supply and demand market pairs:* (i, j)*;* $i = 1, \dots, m$*;* $j = 1, \dots, n$*, and for all routes* r*;* $r = 1, \dots, P$*:*

$$\tilde{\pi}_i^k(Q^*, q^{0*}) + \tilde{c}_{ijr}^k(Q^*, T) \begin{cases} = \tilde{\rho}_{ijr}^k(Q^*, q^{0*}), & \text{if } Q_{ijr}^{k*} > 0, \\ \geq \tilde{\rho}_{ijr}^k(Q^*, q^{0*}), & \text{if } Q_{ijr}^{k*} = 0, \end{cases} \tag{13}$$

and for all commodities k*;* $k = 1, \dots, K$*, and for all supply markets* i*;* $i = 1, \dots, m$*:*

$$OC_i^k(q^{0*}) \begin{cases} = \tilde{\pi}_i^k(Q^*, q^{0*}), & \text{if } q_i^{0k*} > 0, \\ \geq \pi_i^k(Q^*, q^{0*}), & \text{if } q_i^{0k*} = 0. \end{cases} \tag{14}$$

The multicommodity fresh produce trade network equilibrium conditions (13) state that, if there is a positive flow of a commodity on a route between a pair of supply and demand markets, then the supply price of the commodity at the supply market plus the unit transportation cost associated with transporting the commodity on the route is equal to the demand price of the commodity at the demand market. If the flow of a commodity is equal to zero on a route, then the demand market price of the commodity is less than or equal to the supply market price plus the unit transportation cost on the route for the commodity. In addition, the equilibrium conditions (14) state that, if the initial quality level of a commodity produced at a supply market is positive, then the opportunity cost associated with that commodity at the supply market is equal to its supply price; if, on the other hand, the initial quality is zero, then the opportunity cost is greater than or equal to the supply price at the supply market for that commodity. Equilibrium conditions (13) are extensions of the classical spatial price equilibrium conditions of Samuelson [84] and Takayama and Judge [91, 92] to include product quality (and deterioration) as well as product differentiation. Equilibrium conditions (14) that include opportunity cost functions are extensions of those introduced by Nagurney, Li, and Nagurney [72] to multiple commodities.

Theorem 1: Variational Inequality Formulation of the Multicommodity Fresh Produce Trade Network Equilibrium Conditions with Quality Deterioration But No Minimum Quality Standards or Bounds

A multicommodity shipment and initial quality level pattern $(Q^*, q^{0*}) \in \mathcal{K}^1$ *is a multicommodity fresh produce trade network equilibrium with quality deterioration but no minimum quality standards or bounds, according* *to Definition 1, if and only if it satisfies the variational inequality:*

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[\tilde{\pi}_i^k(Q^*,q^{0*})+\tilde{c}_{ijr}^k(Q^*,T)-\tilde{\rho}_{ijr}^k(Q^*,q^{0*})\right]\times(Q_{ijr}^k-Q_{ijr}^{k*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\left[OC_i^k(q^{0*})-\tilde{\pi}_i^k(Q^*,q^{0*})\right]\times(q_i^{0k}-q_i^{0k*})\geq 0,\quad \forall (Q,q^0)\in\mathcal{K}^1. \tag{15}$$

Proof: First, necessity is established; that is, we show that if $(Q^*,q^{0*})\in\mathcal{K}^1$ satisfies equilibrium conditions (13) and (14), then it also satisfies variational inequality (15). From the equilibrium conditions (13), for an equilibrium commodity shipment and initial quality level pattern, and for fixed k,l,i,j, we know that:

$$\left[\tilde{\pi}_i^k(Q^*,q^{0*})+\tilde{c}_{ijr}^k(Q^*,T)-\tilde{\rho}_{ijr}^k(Q^*,q^{0*})\right]\times(Q_{ijr}^k-Q_{ijr}^{k*})\geq 0,\quad \forall Q_{ijr}^k\geq 0, \tag{16}$$

because if $Q_{ijr}^{k*}>0$, then the left-hand side of (16) preceding the multiplication sign is zero, so (16) holds. Also, if $Q_{ijr}^{k*}=0$, then the left-hand side expression is nonnegative, and (16) holds, since Q_{ijr}^k is always greater than or equal to Q_{ijr}^{k*}. Since (16) is true for any k,i,j,r, summation of (16) over these indices yields:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[\tilde{\pi}_i^k(Q^*,q^{0*})+\tilde{c}_{ijr}^k(Q^*,T)-\tilde{\rho}_{ijr}^k(Q^*,q^{0*})\right]\times(Q_{ijr}^k-Q_{ijr}^{k*})\geq 0,\quad \forall Q\in R_+^{KmnP}. \tag{17}$$

In addition, from equilibrium conditions (14), it follows that, for a fixed k,i:

$$\left[OC_i^k(q^{0*})-\tilde{\pi}_i^k(Q^*,q^{0*})\right]\times(q_i^{0k}-q_i^{0k*})\geq 0,\quad \forall q_i^{0k}\geq 0. \tag{18}$$

Summing now (18) over all k,i gives us:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\left[OC_i^k(q^{0*})-\tilde{\pi}_i^k(Q^*,q^{0*})\right]\times(q_i^{0k}-q_i^{0k*})\geq 0,\quad \forall q\in R_+^{Km}. \tag{19}$$

Adding (17) and (19) results in variational inequality (15). Necessity has, thus, been established.

We now turn to establishing sufficiency. Setting $q_i^{0k}=q_i^{k*}$ for all k,i and $Q_{ijr}^k=Q_{ijr}^{k*}$ for all k,i,j,r except for $k=\tilde{k}$, $r=\tilde{r}$, $i=\tilde{\imath}$, and $j=\tilde{\jmath}$, and substituting the resultants into (15), reduces the variational inequality (15) to:

$$\left[\tilde{\pi}_{\tilde{\imath}}^{\tilde{k}}(Q^*,q^{0*})+\tilde{c}_{\tilde{\imath}\tilde{\jmath}\tilde{r}}^{\tilde{k}}(Q^*,T)-\tilde{\rho}_{\tilde{\imath}\tilde{\jmath}\tilde{r}}^{\tilde{k}}(Q^*,q^{0*})\right]\times(Q_{\tilde{\imath}\tilde{\jmath}\tilde{r}}^{\tilde{k}}-Q_{\tilde{\imath}\tilde{\jmath}\tilde{r}}^{\tilde{k}*})\geq 0,\quad \forall Q_{\tilde{\imath}\tilde{\jmath}\tilde{r}}^{\tilde{k}}\geq 0, \tag{20}$$

from which it follows that the multicommodity trade equilibrium conditions (13) hold.

Now, setting $Q_{ijr}^k=Q_{ijr}^{k*}$ for all k,i,j,r; and $q_i^{0k}=q_i^{0k*}$, except for $k=\tilde{k}$; $i=\tilde{\imath}$, and substituting the resultant values into (15), yields:

$$\left[OC_i^k(q^{0*})-\tilde{\pi}_{\tilde{\imath}}^{\tilde{k}}(Q,q^0)\right]\times(q_{\tilde{\imath}}^{0\tilde{k}}-q_{\tilde{\imath}}^{0\tilde{k}*})\geq 0,\quad \forall q_{\tilde{\imath}}^{0\tilde{k}}\geq 0, \tag{21}$$

from which it follows that the equilibrium conditions (14) must hold.

Sufficiency has been established. □

3.2 The Multicommodity Trade Network Model With Minimum Quality Standards and Upper Bounds

In this Subsection, we introduce minimum quality standards in the form of constraints and also upper bounds on the initial quality levels of the commodities and on the commodity shipments on routes. We then expand the equilibrium conditions (13) and (14), plus we add Lagrange multipliers to formulate an additional set of equilibrium conditions.

We are interested now in the satisfaction of the following constraints (cf. (3) and (6)):

$$q_i^{0k} - \kappa_{ijr}^k (g_{ijr}^k Q_{ijr}^k + h_{ijr}^k) \geq \underline{q}_{ijr}^k, \quad \forall k, i, j, r. \tag{22a}$$

Note that, according to (22a), there is a fixed minimum quality standard for each commodity being transported from each supply market to each demand market via a given route. One could also have a minimum quality standard independent of supply market and of route, as in the form of $\underline{q}_j^k$. Having $\underline{q}_{ijr}^k$ offers a greater degree of flexibility, however, and, hence, we focus on such minimum quality standards.

In addition, we have the following upper bound constraints:

$$q_i^{0k} \leq \bar{q}_i^{0k}, \quad \forall k, i, \tag{22b}$$

and

$$Q_{ijr}^k \leq \bar{Q}_{ijr}^k, \quad \forall k, i, j, r. \tag{22c}$$

From (22a) and (22b), we can infer that if $\bar{q}_i^{0k} - \kappa_{ijr}^k h_{ijr}^k < \underline{q}_{ijr}^k$, then route r is infeasible because of quality constraints for the commodity and pair of supply and demand markets and, hence, should be removed from the trade network.

We define the feasible set $\mathcal{K}^2 \equiv \{Q \in R_+^{KmnP} \, \text{and} \, q^0 \in R_+^{Km} : (22a) - (22c) \, \text{holds}\}$.

Variational Inequality Formulation of the Multicommodity Fresh Produce Trade Network Equilibrium Conditions with Quality Deterioration and with Quality Standards

We consider now the variational inequality problem: determine $(Q^*, q^{0*}) \in \mathcal{K}^2$ such that:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[\tilde{\pi}_i^k(Q^*, q^{0*}) + \tilde{c}_{ijr}^k(Q^*, T) - \tilde{\rho}_{ijr}^k(Q^*, q^{0*})\right] \times (Q_{ijr}^k - Q_{ijr}^{k*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\left[OC_i^k(q^{0*}) - \tilde{\pi}_i^k(Q^*, q^{0*})\right] \times (q_i^{0k} - q_i^{0k*}) \geq 0, \quad \forall (Q, q^0) \in \mathcal{K}^2. \tag{23}$$

Existence of a solution to variational inequality (23) is guaranteed sinc ethe feasible set $\mathcal{K}^2$ is compact (cf. [48]).

We now prove in the next theorem that a solution of variational inequality (23) satisfies an expanded set of equilibrium conditions to those stated in Definition 1.

Theorem 2: Expanded Multicommodity Trade Network Equilibrium Conditions Under Minimum Quality Standards and Bounds

Let λ be a vector in R_+^{KmnP}. Let also μ be a vector in R_+^{KmnP}, with γ being a vector in R_+^{Km}. The solution $(Q^*, q^{0*}) \in \mathcal{K}^2$ to variational inequality (23) is equivalent to the solution $(Q^*, q^{0*}, \lambda^*, \gamma^*, \mu^*) \in R_+^{KmnP+2Km+2KmnP}$ satisfying the following expanded equilibrium conditions: for all commodities k; $k = 1, \ldots, K$; for all supply and demand market pairs: (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and for all routes r; $r = 1, \ldots, P$:

$$\tilde{\pi}_i^k(Q^*, q^{0*}) + \tilde{c}_{ijr}^k(Q^*, T) + \lambda_{ijr}^{k*}\kappa_{ijr}^k g_{ijr}^k + \mu_{ijr}^{k*} \begin{cases} = \tilde{\rho}_{ijr}^k(Q^*, q^{0*}), & \text{if } Q_{ijr}^{k*} > 0, \\ \geq \tilde{\rho}_{ijr}^k(Q^*, q^{0*}), & \text{if } Q_{ijr}^{k*} = 0; \end{cases} \tag{24}$$

for all commodities k; $k = 1, \ldots, K$, and for all supply markets i; $i = 1, \ldots, m$:

$$OC_i^k(q^{0*}) - \sum_{j=1}^{n}\sum_{r=1}^{P} \lambda_{ijr}^{k*} + \gamma_i^{k*} \begin{cases} = \tilde{\pi}_i^k(Q^*, q^{0*}), & \text{if } q_i^{0k*} > 0, \\ \geq \pi_i^k(Q^*, q^{0*}), & \text{if } q_i^{0k*} = 0; \end{cases} \tag{25}$$

and for all commodities k; $k = 1, \ldots, K$, for all supply markets i; $i = 1, \ldots, m$, all demand markets j; $j = 1, \ldots, n$, and all routes r; $r = 1, \ldots, P$:

$$q_i^{0k*} - \kappa_{ijr}^k(g_{ijr}^k Q_{ijr}^{k*} + h_{ijr}^k) - \underline{q}_{ijr}^k \begin{cases} = 0, & \text{if } \lambda_{ijr}^{k*} > 0, \\ \geq 0, & \text{if } \lambda_{ijr}^{k*} = 0, \end{cases} \tag{26}$$

plus, for all commodities k; $k = 1, \ldots, K$ and all supply markets i; $i = 1, \ldots, m$:

$$\bar{q}_i^{0k} \begin{cases} = q_i^{0k*}, & \text{if } \gamma_i^{k*} > 0, \\ \geq q_i^{0k*}, & \text{if } \gamma_i^{k*} = 0, \end{cases} \tag{27}$$

and for all commodities k; $k = 1, \ldots, K$; for all supply and demand market pairs: (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and for all routes r; $r = 1, \ldots, P$:

$$\bar{Q}_{ijr}^k \begin{cases} = Q_{ijr}^{k*}, & \text{if } \mu_{ijr}^{k*} > 0, \\ \geq \bar{Q}_{ijr}^{k*}, & \text{if } \mu_{ijr}^{k*} = 0. \end{cases} \tag{28}$$

Proof: We introduce the function Φ:

$$\Phi(Q, q^0) = \sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\Big[\tilde{\pi}_i^k(Q^*, q^{0*}) + \tilde{c}_{ijr}^k(Q^*, T) - \tilde{\rho}_{ijr}^k(Q^*, q^{0*})\Big] \times Q_{ijr}^k$$

$$+ \sum_{k=1}^{K}\sum_{i=1}^{m}\Big[OC_i^k(q^{0*}) - \tilde{\pi}_i^k(Q^*, q^{0*})\Big] \times q_i^{0k}. \tag{29}$$

We know that (Q^*, q^{0*}) solves (23) if and only if (Q^*, q^{0*}) is a global minimum point of Φ in $\mathcal{K}^2$.

We construct the Lagrangian function

$$L(Q, q^0, \lambda) = \Phi + \sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P} \lambda_{ijr}^k \left[-q_i^{0k} + \kappa_{ijr}^k(g_{ijr}^k Q_{ijr}^k + h_{ijr}^k) + \underline{q}_{ijr}^k\right]$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\gamma_i^k\left[q_i^{0k}-\bar{q}_i^{0k}\right]+\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\mu_{ijr}^k\left[Q_{ijr}^k-\bar{Q}_{ijr}^k\right] \tag{30}$$

where λ_{ijr}^k is the Lagrange multiplier associated with constraint (22a) for k,i,j,r; γ_i^k is the Lagrange multiplier associated with constraint (22b), for k,i, and μ_{ijr}^k is the Lagrange multiplier associated with constraint (22c) for k,i,j,r. The KKT conditions are then:

$$\frac{\partial L(Q^*,q^{0*},\lambda^*)}{\partial Q_{ijr}^k}\geq 0,\quad Q_{ijr}^{k*}\geq 0,\frac{\partial L(Q^*,q^{0*},\lambda^*)}{\partial Q_{ijr}^k}Q_{ijr}^{k*}=0,\quad \forall k,i,j,r, \tag{31}$$

$$\frac{\partial L(Q^*,q^{0*},\lambda^*)}{\partial q_i^{0k}}\geq 0,\quad q_i^{0k*}\geq 0,\frac{\partial L(Q^*,q^{0*},\lambda^*)}{\partial q_i^{0k}}q_i^{0k*}=0,\quad \forall k,i, \tag{32}$$

$$\lambda_{ijr}^{k*}\left[-q_i^{0k*}+\kappa_{ijr}^k(g_{ijr}^kQ_{ijr}^{k*}+h_{ijr}^k)+\underline{q}_{ijr}^k\right]=0,\quad \lambda_{ijr}^{k*}\geq 0,\left[-q_i^{0k*}+\kappa_{ijr}^k(g_{ijr}^kQ_{ijr}^{k*}+h_{ijr}^k)+\underline{q}_{ijr}^k\right]\leq 0,\forall k,i,j,r. \tag{33}$$

$$\gamma_i^{k*}\left[q_i^{0k}-\bar{q}_i^{0k}\right]=0,\quad \gamma_i^{k*}\geq 0,\left[q_i^{k*}-\bar{q}_i^k\right]\leq 0,\quad \forall k,i, \tag{34}$$

$$\mu_{ijr}^{k*}\left[Q_{ijr}^{k*}-\bar{Q}_{ijr}^k\right]=0,\quad \mu_{ijr}^{k*}\geq 0,\left[Q_{ijr}^{k*}-\bar{Q}_{ijr}^k\right]\leq 0,\quad \forall k,i,j,r. \tag{35}$$

Since Φ is linear as are the constraints in (22a)-(22c), the KKT conditions are both necessary and sufficient for (Q^*,q^{0*}) to be a minimum point.

It is easy to check that conditions (33) are equivalent to (26), that conditions (34) are equivalent to (27), and that conditions (35) are equivalent to (28).

Also, since

$$\frac{\partial L(Q^*,q^{0*},\lambda^*)}{Q_{ijr}^k}=\tilde{\pi}_i^k(Q^*,q^{0*})+\tilde{c}_{ijr}^k(Q^*,T)+\lambda_{ijr}^{k*}\kappa_{ijr}^kg_{ijr}^k-\tilde{\rho}_{ijr}^k(Q^*,q^{0*})+\mu_{ijr}^{k*} \tag{36}$$

we can rewrite (31) as: for all commodities k; $k=1,\ldots,K$; for all supply and demand market pairs: (i,j); $i=1,\ldots,m$; $j=1,\ldots,n$, and for all routes r; $r=1,\ldots,P$:

$$\tilde{\pi}_i^k(Q^*,q^{0*})+\tilde{c}_{ijr}^k(Q^*,T)+\lambda_{ijr}^{k*}\kappa_{ijr}^kg_{ijr}^k+\mu_{ijr}^{k*}\begin{cases}=\tilde{\rho}_{ijr}^k(Q^*,q^{0*}), & \text{if } Q_{ijr}^{k*}>0,\\ \geq\tilde{\rho}_{ijr}^k(Q^*,q^{0*}), & \text{if } Q_{ijr}^{k*}=0,\end{cases} \tag{37}$$

which corresponds to (24).

In addition, since

$$\frac{\partial L(Q^*,q^{0*},\lambda^*)}{\partial q_i^{0k}}=\left[OC_i^k(q^{0*})-\tilde{\pi}_i^k(Q^*,q^{0*})-\sum_{j=1}^{n}\sum_{r=1}^{P}\lambda_{ijr}^{k*}+\gamma_i^{k*}\right], \tag{38}$$

we can rewrite (32) as: for all commodities k; $k=1,\ldots,K$, and for all supply markets i; $i=1,\ldots,m$:

$$OC_i^k(q^{0*})-\sum_{j=1}^{n}\sum_{r=1}^{P}\lambda_{ijr}^{k*}+\gamma_i^{k*}\begin{cases}=\tilde{\pi}_i^k(Q^*,q^{0*}), & \text{if } q_i^{0k*}>0,\\ \geq\pi_i^k(Q^*,q^{0*}), & \text{if } q_i^{0k*}=0,\end{cases} \tag{39}$$

which coincides with (25).

The conclusion follows. □

We now provide an alternative variational inequality to (15) with Lagrange multipliers, which we will utilize for computational purposes.

Theorem 3: Alternative Variational Inequality Formulation of the Multicommodity Fresh Produce Trade Network Equilibrium Conditions with Quality Deterioration and with Quality Standards

A multicommodity, initial quality level, and Lagrange multiplier pattern $(Q^*, q^{0*}, \lambda^*, \gamma^*, \mu^*) \in \mathcal{K}^3$, where $\mathcal{K}^3 \equiv \{(Q, q^0, \lambda, \gamma, \mu) | (Q, q^0, \lambda, \gamma, \mu) \in R_+^{KmnP+2Km+2KmnP}\}$ satisfies equilibrium conditions (24), (25), and (26) if and only if it satisfies the variational inequality problem:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\Big[\tilde{\pi}_i^k(Q^*, q^{0*}) + \tilde{c}_{ijr}^k(Q^*, T) + \lambda_{ijr}^{k*}\kappa_{ijr}^k g_{ijr}^k + \mu_{ijr}^{k*} - \tilde{\rho}_{ijr}^k(Q^*, q^{0*})\Big] \times (Q_{ijr}^k - Q_{ijr}^{k*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\Big[OC_i^k(q^{0*}) - \sum_{j=1}^{n}\sum_{r=1}^{P}\lambda_{ijr}^{k*} + \gamma_i^{k*} - \tilde{\pi}_i^k(Q^*, q^{0*})\Big] \times (q_i^{0k} - q_i^{0k*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}(q_i^{0k*} - \kappa_{ijr}^k(g_{ijr}^k Q_{ijr}^{k*} + h_{ijr}^k) - \underline{q}_{ijr}^k) \times (\lambda_{ijr}^k - \lambda_{ijr}^{k*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\left[\bar{q}_i^{0k} - q_i^{0k*}\right] \times (\gamma_i^k - \gamma_i^{k*}) + \sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[\bar{Q}_{ijr}^k - Q_{ijr}^{k*}\right] \times (\mu_{ijr}^k - \mu_{ijr}^{k*}) \geq 0, \quad \forall (Q, q^0, \lambda, \gamma, \mu) \in \mathcal{K}^3. \tag{40}$$

Proof: Follows using similar arguments as those in the proof of Theorem 2.

We now put variational inequality (40) into standard form VI$(F, \mathcal{K})$ (cf. [61]). In particular, we are interested in determining a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{41}$$

with F being a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, where $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

We define $X \equiv (Q, q^0, \lambda, \gamma, \mu)$, $\mathcal{K} \equiv \mathcal{K}^3$, and $\mathcal{N} \equiv KmnP + 2Km + 2KmnP$ and

$$F(X) \equiv (F_1(X), F_2(X), F_3(X), F_4(X), F_5(X))$$

where $F_1(X)$ consists of the elements: $\big[\tilde{\pi}_i^k(Q, q^0) + \tilde{c}_{ijr}^k(Q, T) + \lambda_{ijr}^k\kappa_{ijr}^k g_{ijr}^k + \mu_{ijr}^k - \tilde{\rho}_{ijr}^k(Q, q^0)\big], \forall k, i, j, r$; the components of $F_2(X)$ are: $\big[OC_i^k(q^0) - \sum_{j=1}^n\sum_{r=1}^P \lambda_{ijr}^k + \gamma_i^k - \tilde{\pi}_i^k(Q, q^0)\big], \forall k, i$, and $F_3(X)$ is comprised of the elements: $\left[q_i^{0k} - \kappa_{ijr}^k(g_{ijr}^k Q_{ijr}^k + h_{ijr}^k) - \underline{q}_{ijr}^k\right]$, $\forall k, i, j, r$ with $F_4(X)$ consisting of the elements: $\bar{q}_i^{0k} - q_i^k$, $\forall k, i$, and $F_5(X)$ consisting of the elements: $\bar{Q}_{ijr}^k - Q_{ijr}^k$, $\forall k, i, j, r$.

Variational inequality (40) has now been put into standard form (41).

4. The Computational Procedure

The computational procedure that we implement and apply in Section 5 to compute solutions to a series of numerical examples is the modified projection method of Korpelevich [50]. For easy reference, it statement is now recalled. An iteration is denoted by τ.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{42}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{43}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The convergence of this modified projection method is guaranteed if the function $F(X)$ that enters the variational inequality problem (41) is monotone and Lipschitz continuous.

Recall that the function $F(X)$ is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{44}$$

Also, $F(X)$ is Lipschitz continuous, if there exists a Lipschitz constant, $\eta > 0$, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{45}$$

Note that the feasible set $\mathcal{K}^3$ underlying the multicommodity fresh produce trade network equilibrium model with quality deterioration and minimum quality standards and bounds satisfying variational inequality (40) is that of the nonnegative orthant. Hence, the solution of each of the subproblems in (42) and (43) in the multicommodity flow, initial uality levels, and the Lagrange multiplier variables can be readily obtained via closed-form expressions, as made explicit below.

Explicit Formulae at Iteration τ for the Multicommodity Fresh Produce Flows in Step 1

The closed-form expressions for the multicommodity fresh produce flows in the solution of variational inequality (40) are:

$$\bar{Q}_{ijr}^{k\tau} = \max\{0, Q_{ijr}^{k\tau-1} + \beta(\tilde{\rho}_{ijr}^{k}(Q^{\tau-1}, q^{0\tau-1}) - (\tilde{\pi}_i^k(Q^{\tau-1}, q^{0\tau-1}) + \tilde{c}_{ijr}^k(Q^{\tau-1}, T) + \lambda_{ijr}^{k\tau-1}\kappa_{ijr}^k g_{ijr}^k + \mu_{ijr}^{k\tau-1}))\},$$
$$\forall k, i, j, r. \tag{46}$$

Explicit Formulae at Iteration τ for the Initial Quality Levels in Step 1

The closed-form expressions for the initial quality levels in variational inequality (40) are:

$$\bar{q}_i^{0k\tau} = \max\{0, q_i^{0k\tau-1} + \beta(-OC_i^k(q^{0k\tau-1}) + \sum_{j=1}^{n}\sum_{r=1}^{P}\lambda_{ijr}^{k\tau-1} - \gamma_i^{k\tau-1} + \tilde{\pi}_i^k(Q^{\tau-1}, q^{0\tau-1}))\}, \quad \forall k, i. \tag{47}$$

Explicit Formulae at Iteration τ for the Lagrange Multipliers in Step 1

The closed-form expressions for the Lagrange multipliers in variational inequality (40) are:

$$\bar{\lambda}_{ijr}^{k\tau} = \max\{0, \lambda_{ijr}^{k\tau-1} + \beta(-q_i^{0k\tau-1} + \kappa_{ijr}^k(g_{ijr}^k Q^{l\tau-1} + h_{ijr}^k) + \underline{q}_{ijr}^k)\}, \quad \forall k, i, j, r. \tag{48a}$$

$$\bar{\gamma}_i^{k\tau} = \max\{0, \gamma_i^{k\tau-1} + \beta(q_i^{0k\tau-1} - \bar{q}_i^{0k})\}, \quad \forall k, i. \tag{48b}$$

$$\bar{\mu}_{ijr}^{k\tau} = \max\{0, \mu_{ijr}^{k\tau-1} + \beta(Q_{ijr}^{k\tau-1} - \bar{Q}_{ijr}^k)\}, \quad \forall k, i, j, r. \tag{48c}$$

The explicit formulae for the variables in (43) in Step 2 readily follow.

We now proceed to investigate monotonicity of the function $F(X)$ for our model as in variational inequality (40).

For our model,

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle$$
$$= \sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{P}\left[(\tilde{\pi}_i^k(Q^1, q^{01}) + \tilde{c}_{ijr}^k(Q^1, T) - \tilde{\rho}_{ijr}^k(Q^1, q^{01})) - (\tilde{\pi}_i^k(Q^2, q^{02}) + \tilde{c}_{ijr}^k(Q^2, T) - \tilde{\rho}_{ijr}^k(Q^2, q^{02}))\right]$$
$$\times \left[Q_{ijr}^1 - Q_{ijr}^2\right]$$
$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\left[(OC_i^k(q^{01}) - \tilde{\pi}_i^k(Q^1, q^{01})) - (OC_i^k(q^{02}) - \tilde{\pi}_i^k(Q^2, q^{02}))\right] \times \left[q_i^{0k1} - q_1^{0k2}\right]. \tag{49}$$

It follows then that $F(X)$ is monotone in X, if $\tilde{F}^1$ with components: $\tilde{\pi}_i^k(Q, q^0) + \tilde{c}_{ijr}^k(Q, T) - \tilde{\rho}_{ijr}^k(Q, q^0)$, $\forall k, i, j, r$ is monotone in Q and if $\tilde{F}^2$ with components: $OC_i^k(q^0) - \tilde{\pi}_i^k(Q, q^0)$, $\forall k, i$ is monotone in q.

5. Numerical Study on the Banana Trade

In this numerical study section, we focus on the banana trade. As mentioned in the Introduction, bananas are immensely popular fresh produce items worldwide ([49], [86]). Over the period from 2000 to 2017, global banana production displayed a compounded annual growth rate of 3.2 percent, reaching an all-time high of 114 million tons in 2017, according to the Food and Agriculture Organization [31]. Bananas are cultivated in over 130 nations around the globe [25]. According to the Food and Agriculture Organization [32], the estimated export volume of bananas was 19.1 million tons in 2022, with Ecuador being the largest exporter, followed by the Philippines, Costa Rica, and Guatemala. Between 2015 and 2017, Ecuador accounted for a total share of 29.1 percent; Costa Rica had 10.9 percent, and Guatemala had 10.4 percent of the export shares [25]. On the demand side, the European Union stands out as the largest importer, accounting for an annual average of 32 percent of the total global imports between 2010 and 2016, with the United States following at 25 percent [31].

In this numerical study, we analyze the banana commodity trade flows between supply markets in Ecuador and Costa Rica, and demand markets in the United States and the European Union. The data for our analysis are sourced from the FAOSTAT [26] database. We now document the data sources that we use to construct the functions and the parameters for the numerical examples. According to data available online, Ecuador's total banana export volume in 2021 reached 6.81 million tons. Within these exports, 0.68 million tons (10 percent of the total) were shipped to the United States, and 1.86 million tons of bananas were sent to European Union countries, constituting approximately 27 percent of Ecuador's total banana exports in 2021. It is noteworthy to highlight that, in 2021, the second-largest importer of bananas from Ecuador was the Russian Federation, with 0.15 million tons (22 percent of the total). Additionally, Turkiye stands out as one of the major importers of Ecuadorian bananas, acquiring 0.44 million tons (6.5 percent of the total) in 2021.

As for Costa Rica, the total banana exports in 2021 amounted to 2.3 million tons. Among these, 0.79 million tons (equivalent to 35 percent of total exports) of bananas from Costa Rica were destined for the United States, and 1.01 million tons (equivalent to 44 percent of total exports) were exported to European Union countries from Costa Rica in 2021. It is crucial to acknowledge that our analysis is limited by the absence of FAOSTAT [26] data for 2022 and beyond. The Food and Agriculture Organization [32] reports that Ecuador and Costa Rica encountered a reduction in their banana exports attributed to production shortages stemming from a lack of essential fertilizers imported from the Russian Federation and Ukraine. Consequently, in 2022, Ecuador witnessed a notable 12 percent decline in its total banana export volume, at 5.8 million tons. Similarly, Costa Rica experienced a 9.2 percent decrease in export volume of bananas, with the total falling to 2.1 million tons of exported bananas.

Moreover, concerning the prices in the supply markets for bananas, those originating from Ecuador were reported at 321 dollars per ton, while bananas from Costa Rica were reported at 464.3 dollars per ton for 2021 [26]. The average import price of bananas in the United States was at 515 dollars in the United States in 2023 [41]. In the European Union, the average import price of bananas was at 645 euros (696 dollars) in

2022 [32]. It is important to clarify that our reference to "dollars" in this context pertains specifically to U.S. dollars.

The transportation cost of bananas is reported to be between 15 and 20 percent of the average demand market price of bananas ([6], [98]). Certain shipping companies offer shipment price quotes between Ecuador and the European Union at an average of 2,962.5 euros per container (equivalent to 145 dollars per ton) [13]. Similarly, the average cost per container between Ecuador and the U.S. is reported at 2,945 dollars (approximately 133 dollars per ton) [13]. The shipment price quotes between Costa Rica to the European Union and to the U.S. are at an average of 4,250 euros per container (208 dollars per ton), and 2,980 dollars per container (135 dollars per ton) [14]. We observe that the price quotes from the online reference consider the origin as the United States or Europe and the destination as Ecuador or Costa Rica. However, we assume that the price quotes for freight services remain the same when the origin and destinations are swapped. There are additional references citing the cost of shipping a container from Latin America to a port in Rotterdam, the Netherlands, at 5,000 euros per container (245 dollars per ton) (cf. Liang [59] for more references). Additionally, in 2022, due to supply chain shortages and congestion at destination ports, certain ocean freight shipping rates for bananas surged to as high as 12,000 dollars per container [39]. Furthermore, when determining the per-ton cost of shipping, we assume that a 40ft container has a capacity of 22 tons of bananas [54], and that the exchange rate is considered to be 1.08 dollars per euro [33].

When considering the trade networks for bananas, which are primarily produced in Latin American nations, with transportation to the demand markets in the European Union, and the United States, one cannot overlook the significance of the Panama Canal. While the primary goal in the building of the Panama Canal was not to specifically accommodate the banana trade, the Panama Canal impacted both the efficiency and the economic viability of the banana sector by facilitating the movement of this fresh fruit between Latin America and the United States, to start [17]. According to the Canal de Panama [16] report on the principal commodities shipped through the Panama Canal, bananas ranked number one among the refrigerated foods category for the fiscal years 2020 to 2022.

Establishing a direct link with the central theme of this paper, which revolves around the ramifications of congestion and the imposition of minimum quality standards on the trade of fresh produce, the Food and Agriculture Organization [32] highlights a significant decline in banana export volume. This decline can be attributed to the stringent minimum quality standards in place, colder temperatures during production, disruptions to transportation networks, and the adverse effects of Russia's war on Ukraine. Consequently, the focus on banana trade in this Section aligns seamlessly with the scope of our modeling and algorithmic framework and the objectives of this paper.

The modified projection method for the solution of the variational inequality (40) for the multicommodity fresh produce trade network equilibrium model is implemented in Python with $\beta = 0.05$, with the convergence tolerance being 10^{-6} ; that is, the modified projection method is deemed to have converged if the absolute value of the difference of each successive variable (commodity shipment, initial quality level, and Lagrange

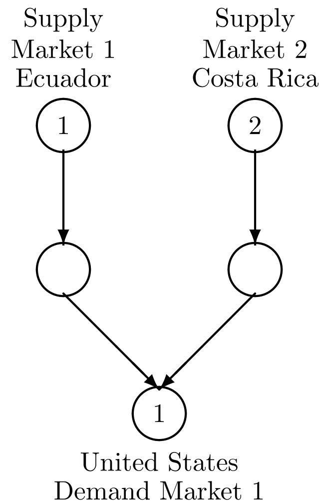


Figure 2: The Trade Network Topology for Examples 1 Through 3

multiplier) iterate differs by no more than this value.

In the next Subsections, we present numerical examples to illustrate our modeling framework and to showcase our results.

5.1 Example 1: Baseline Scenario

Example 1 serves as a baseline example. The trade network topology of Example 1 is provided in Figure 2. It is also the trade network topology for Examples 2 and 3. In Example 1, there are two supply markets: Supply Market 1 in Ecuador and Supply Market 2 in Costa Rica. There is a single demand market, Demand Market 1, in the United States. The commodity, bananas, is transported to the demand market via the route using the Panama Canal. Hence, we have: $i = \{1, 2\}$, $j = \{1\}$, $r = \{1\}$, and $k = \{1\}$. The banana shipments are in metric tons.

In this example, we construct the supply and demand market price functions, along with the functions for the unit transportation costs, and the opportunity costs to accurately mirror real-world prices and costs reported earlier. Following the calibration of prices and costs to align with the real-world setting, we proceed to present various scenarios in the following Subsections, building upon the foundation established in Example 1.

First, we establish the parameters for the banana quality deterioration function. Yan, Sousa-Gallagher, and Oliveira [100] examined various quality indicators related to banana quality deterioration, such as changes in color and moisture content. Through their experiments, they identified the most appropriate quality deterioration function for these attributes. In our research, we focus specifically on the color change aspect to evaluate banana quality decline, utilizing a zero-order deterioration model. The activation energy and pre-exponential constant used in our model are based on the data provided in their study for different quality factors. Since determining these parameters independently is beyond the scope of this paper, we adopt the values directly from their findings. The universal gas constant is known to be $R = 8.314 Jmol^{-1}K^{-1}$.

The pre-exponential factor is taken as $A = 0.007hour^{-1}$, and the activation energy is $E_A = 32.39kJmol^{-1}$.

The suggested practice is to store bananas within the temperature range of 13.3°C to 14.4°C once they are separated from the trees [18]. Hence, we have the parameters: $T^1_{111} = 14°C = 287.15°K$, $T^1_{211} = 14°C = 287.15°K$ and we calculate the reaction rate according to (4), which yields: $\kappa^1_{111} = 0.007hour^{-1}$, and $\kappa^1_{211} = 0.007hour^{-1}$. The minimum quality standards are set to: $\underline{q}^1_{11} = 60$, and $\underline{q}^1_{21} = 60$. Here, we represent quality levels as being between 0 and 100; hence, the upper bounds on the initial quality of bananas at Supply Markets 1 and 2 are: $\overline{q}^1_{11} = 100$ and $\overline{q}^1_{21} = 100$. Furthermore, the upper bounds on the volumes of shipment of bananas are: $\overline{Q}^1_{111} = 1,000,000$ and $\overline{Q}^1_{211} = 1,000,000$.

The variables are: $Q^1_{111}, Q^1_{211}, q^{01}_1, q^{01}_2, \lambda^1_{111}, \lambda^1_{211},, \gamma^1_1, \gamma^1_2, \mu^1_{111}, \mu^1_{211}$. The expressions for the final quality according to (3) are: $q^1_{111} = q^{01}_1 - \kappa^1_{111}t^1_{111} = q^{01}_1 - 0.007t^1_{111}$, $q^1_{211} = q^{01}_2 - \kappa^1_{211}t^1_{211} = q^{01}_2 - 0.007t^1_{211}$.

On average, the Panama Canal transit can be completed in approximately 8-10 hours [46]. Hence, we set the additional parameters in (5) to: $g^1_{111} = g^1_{211} = 0.001$ hours per ton and $h^1_{111} = h^1_{211} = 10$ hours. Consequently, equations in (5) for this example, in hours, are: $t^1_{111} = g^1_{111}Q^1_{111} + h^1_{111} = 0.001Q^1_{111} + 10$, and $t^1_{211} = g^1_{211}Q^1_{211} + h^1_{211} = 0.001Q^1_{211} + 10$.

For Example 1, we have that: $s^1_1 = Q^1_{111}$ and $s^1_2 = Q^1_{211}$, and the commodity supply price functions for Supply Market 1 in Ecuador and Supply Market 2 in Costa Rica, in dollars per ton, are:

$$\tilde{\pi}^1_1(Q, q^0) = 0.00025Q^1_{111} + 0.0001Q^1_{211} + 0.2q^{01}_1 + 0.1q^{01}_2 + 100,$$

$$\tilde{\pi}^1_2(Q, q^0) = 0.0003Q^1_{211} + 0.00015Q^1_{111} + 0.2q^{01}_2 + 0.1q^{01}_1 + 100.$$

In the supply price functions, the coefficients associated with the shipment volumes, the initial quality levels, and the fixed terms are denoted in dollars, dollars per ton, and dollars per ton, respectively. Specifically, the coefficient multiplied by the initial quality expression establishes the corresponding dollars per ton value for a specific level of initial quality in bananas.

The opportunity cost functions associated with the bananas and Supply Markets 1 and 2 in Ecuador and Costa Rica, respectively, are: $OC^1_1(q^0) = 4.66q^{01}_1$ dollars per ton and $OC^1_2(q^0) = 5.78q^{01}_2$ dollars per ton. In the opportunity cost functions, the product of the coefficient and the initial quality expression determines the cost in dollars per ton for the level of initial quality in bananas.

Since $d^1_{111} = Q^1_{111}$ and $d^1_{211} = Q^1_{211}$, we can express the demand price functions, in dollars per ton, as:

$$\tilde{\rho}^1_{111}(Q, q^0) = -0.00012Q^1_{111} - 0.0001Q^1_{211} + 1.72(q^{01}_1 - \kappa^1_{111}t^1_{111}) + 0.66(q^{01}_2 - \kappa^1_{211}t^1_{211}) + 500,$$

$$\tilde{\rho}^1_{211}(Q, q^0) = -0.00015Q^1_{211} - 0.0001Q^1_{111} + 1.32(q^{01}_2 - \kappa^1_{211}t^1_{211}) + 1.29(q^{01}_1 - \kappa^1_{111}t^1_{111}) + 600.$$

Both demand functions can be rewritten as:

$$\tilde{\rho}^1_{111}(Q, q^0) = -0.000134Q^1_{111} - 0.000104Q^1_{211} + 1.72q^{01}_1 + 0.66q^{01}_2 + 499.83,$$

$$\tilde{\rho}^1_{211}(Q, q^0) = -0.000159Q^1_{211} - 0.000109Q^1_{111} + 1.29q^{01}_2 + 1.32q^{01}_1 + 599.89.$$

The demand price functions for bananas sourced from Ecuador and Costa Rica are configured to have a similar structure. Nevertheless, the volume of bananas shipped from Ecuador to the United States exerts a more substantial impact on the demand price functions. Analogous to the supply price functions, the coefficients pertaining to shipment volumes, initial quality levels, and fixed terms are delineated in dollars, dollars per ton, and dollars per ton, respectively.

Finally, the unit transportation cost functions associated with shipping bananas from the two supply markets via the Panama Cana, in dollars per ton, are:

$$\tilde{c}^1_{111}(Q,T) = 0.000212Q^1_{111}, \quad \tilde{c}^1_{211}(Q,T) = 0.000184Q^1_{211}.$$

We make the assumption that, due to Ecuador's geographical location further from the Panama Canal, the corresponding unit transportation cost is assigned a slightly higher coefficient. The cost functions include the unit transportation cost of a shipment of bananas from Ecuador and Costa Rica to the United States, typically through a temperature-controlled container. Hence, the unit cost of keeping the bananas at 14°C while shipping them from Ecuador and Costa Rica is included in the unit transportation cost function.

Table 2 displays the equilibrium solutions for Example 1 (and also for Examples 2 and 3). The equilibrium shipment volumes are expressed in tons.

Table 2: Computed Equilibrium Values of Shipment Volumes, Initial Quality Levels, and Lagrange Multipliers for Examples 1 Through 3

Variable	**Example 1**	**Example 2**	**Example 3**
Q^{1*}_{111}	681,427.10	692,355.58	52,142.90
Q^{1*}_{211}	790,480.01	796,103.65	52,142.91
q^{01*}_1	80.13	84.91	100.00
q^{01*}_2	80.17	80.85	100.00
λ^{1*}_{111}	0.00	17.95	674,320.29
λ^{1*}_{211}	0.00	0.00	829,293.74
γ^{1*}_1	0.00	0.00	674,002.54
γ^{1*}_2	0.00	0.00	828,869.20
μ^{1*}_{111}	0.00	0.00	0.00
μ^{1*}_{211}	0.00	0.00	0.00

Observe that the equilibrium volumes of banana shipments in Example 1 correspond closely to the reported actual export volumes of bananas from Ecuador and Costa Rica to the United States, as detailed at the outset of Section 5, based on the FAOSTAT [26] database. Hence, according to Table 2, in Example 1, as well as in reality, Ecuador exports a lower volume of bananas to the United States compared to Costa Rica. Furthermore, the initial quality levels at equilibrium are nearly identical for bananas from Ecuador and Costa Rica in Example 1 as both countries are located in similar climates.

The equilibrium supply prices, demand prices, opportunity costs, and unit transportation costs are reported in Table 3 and are denoted in dollars per ton.

The final quality values of bananas shipped from Ecuador and Costa Rica to the United States via the Panama Canal are calculated using (3) as: $q^1_{111} = 75.29$ and $q^1_{211} = 74.57$, with the time for the transportation of the bananas being (cf. (5)), respectively: $t^1_{111} = 691.42$ hours and $t^1_{211} = 800.48$ hours. While freight shipment times from Ecuador and Costa Rica to the United States can vary, some shipping services report durations of up to 40 days (BR Logistics [13,14]). Furthermore, Li [57] reports comparable shipment durations to those presented in our study.

The quality of bananas shipped to the United States remains comparable for shipments from Ecuador and from Costa Rica. The higher transportation time for bananas from Costa Rica to the Uited States than from Ecuador is a direct outcome of the high shipment volumes destined for the United States from Costa Rica, since transportation times are inherently tied to the quantity of shipments.

Table 3: Function Values at the Equilibrium for Examples 1 Through 3

Function	**Example 1**	**Example 2**	**Example 3**
$\tilde{\pi}^1_1$	373.45	377.76	148.25
$\tilde{\pi}^1_2$	463.40	467.34	153.46
$\tilde{\rho}^1_{111}$	517.91	524.54	631.32
$\tilde{\rho}^1_{211}$	608.85	613.83	743.56
OC^1_1	373.45	395.71	466.00
OC^1_2	463.40	467.34	578.00
$\tilde{c}^1_{111}$	144.46	146.77	11.05
$\tilde{c}^1_{211}$	145.44	146.48	9.59

Table 3 indicates that the equilibrium supply price of bananas in Costa Rica surpasses that in Ecuador in Example 1. The demand price for bananas originating from Ecuador is lower than that for bananas from Costa Rica in Example 1. Additionally, it is worth mentioning that the equilibrium demand and supply prices for Example 1 in Table 3 are close to the actual prices observed in practice, as previously mentioned in Section 5. The transportation cost for bananas from Ecuador to the United States is slightly lower than the transportation cost from Costa Rica to the United States in Example 1. This can be attributed to the higher volume of banana shipments from Costa Rica to the United States, despite Ecuador being geographically located slightly farther from the Panama Canal than Costa Rica. We also note that the transportation costs computed in Example 1 closely align with shipping quotes available through online resources, such as those from BR Logistics [13, 14]. The equilibrium conditions (24) hold with excellent accuracy. The equilibrium opportunity costs for both supply markets are equal to their corresponding supply prices, in accordance with the equilibrium conditions in (25). Furthermore, equilibrium conditions (26) - (28) also hold with all the Lagrange multipliers being equilibrium to 0.00 in Example 1.

5.2 Example 2: More Stringent Minimum Quality Standards for Bananas from Ecuador

In Example 2, we increase the minimum quality standards for the bananas shipped from Ecuador to $\underline{q}^1_{11} = 80$. The remaining data are as in Example 1. We present the equilibrium solutions in Table 2 and the corresponding function values in Table 3.

The equilibrium shipment volume of bananas from Ecuador increases from the value in Example 1 as the minimum quality standard for bananas at Demand Market 1 in the United States increases. Similarly, the equilibrium shipment volume of bananas from Costa Rica increases from the value in Example 1. Examining the initial quality level, we observe an increase in the initial quality level of bananas in Ecuador as compared to its value in Example 1. This change complements the higher minimum quality standard in the United States for bananas shipped from Ecuador. Interestingly, the initial quality of bananas in Costa Rica also increases, albeit slightly, when the initial quality of bananas from Ecuador increases. This is due to competition.

The final quality levels of bananas from Ecuador and Costa Rica transported to the United States via the Panama Canal are, respectively: $q^1_{111} = 80.00$ and $q^1_{211} = 75.21$ with the time for the transportation of bananas being, respectively: $t^1_{111} = 702.35$ hours and $t^1_{211} = 806.10$ hours. We observe that the quality of bananas shipped via the Panama Canal from Ecuador to the United States reaches its minimum level of 80, leading to a positive Lagrange multiplier associated with equation (22a) for Supply Market 1 in Ecuador. Consequently, the equilibrium condition (26) holds. Furthermore, the quality of bananas from Costa Rica transported via the Panama Canal to the United States is higher than its value in Example 1.

The equilibrium supply price of bananas in Ecuador increases from its value in Example 1 due to a rise in the minimum quality standard for bananas in the United States. Similarly, we note a comparable increase in the equilibrium supply price of bananas in Costa Rica shipped to the United States, albeit without an associated increase in the minimum quality standard for Costa Rica.

When examining the equilibrium demand prices, a similar trend emerges. The equilibrium demand prices for bananas produced in both Ecuador and in Costa Rica and transported via the Panama Canal increase from their values in Example 1. This can be attributed to the higher initial quality at both supply markets. The costs of transportation and the opportunity costs also exhibit an increase from their values in Example 1. In Example 2, we observe a slightly higher cost of transportation of bananas from Ecuador to the demand market in the United States than that from Costa Rica. In Example 2, it is easy to verify that, in addition to the already noted equilibrium condition (26) holding, equilibrium conditions (24), (25), (27), and (28) are also satisfied.

5.3 Example 3: Congestion in the Panama Canal and Transportation Capacity Reduction

Example 3 is crafted to showcase the effects of climate change, and the possible droughts, such as the recent one that has affected the Panama Canal, which has led to a substantial congestion issue resulting in delays of shipments [82]. In Example 3, we update the time for transportation of bananas to the United States via the Panama Canal from Ecuador and Costa Rica to: $t^1_{111} = g^1_{111}Q^1_{111} + h^1_{111} = 0.1Q^1_{111} + 500$ and $t^1_{211} = g^1_{211}Q^1_{211} + h^1_{211} = 0.1Q^1_{211} + 500$. We also reduce the transportation capacity to $\frac{1}{10}$ of each of the respective values in Example 1; hence, $\overline{Q}^1_{111} = 100,000$ and $\overline{Q}^1_{211} = 100,000$. The remaining data are as in Example 1. We display the equilibrium solutions for Example 3 in Table 2 and the associated function values at the equilibrium in Table 3.

Compared to the equilibrium banana shipments in Example 1, we observe a significant decrease in equilibrium banana shipment volumes from both Ecuador and Costa Rica to the United States because of drought and congestion-related delays as well as limited shipments in the Panama Canal in Example 3. According to Table 2 the equilibrium initial quality levels reach their upper bound of 100 for bananas from Ecuador and Costa Rica in Example 3. Furthermore, the final quality of bananas shipped from Ecuador and Costa Rica to the United States via the Panama Canal are computed as: $q^1_{111} = 60.00$ and $q^1_{211} = 60.00$. The quality of bananas produced at both supply markets arriving in the United States reach their minimum standards. This implies that, in the presence of decreased transportation capacity and increased congestion through the Panama Canal, the initial quality of bananas reaches its maximum in order to meet the quality standards set by the demand market in the United States.

The time for transportation of the bananas is, respectively, is: $t^1_{111} = 5,714.29$ hours and $t^1_{211} = 5,714.30$ hours. We observe that, for Example 3, in which we capture the impacts of drought-related congestion, the time for transportation increases drastically. Consequently, we observe that the qualitylevels of bananas transported to the demand market decrease from their values in Example 1. The Lagrange multipliers associated with equations (22a) and (22b) are positive, as to be expected, since the quality levels of bananas produced at both supply markets, arriving in the United States via the Panama Canal, are at their lower bounds and, hence, equilibrium condition (26) holds, and, since the initial quality levels are at their upper bounds, with equilibrium condition (27) also being satisfied.

Observe, from Table 3, that, while the equilibrium supply prices for bananas in Ecuador and in Costa Rica exhibit a significant decline from their values in Example 1, the equilibrium demand prices for bananas produced in Ecuador and Costa Rica, and transported via the Panama Canal to the United States, increase significantly compared to their values in Example 1. The drop in supply market prices can be attributed to the decrease in shipment volumes, leading to a reduction in both supply market prices. However, due to the decline in shipment volumes resulting from capacity and congestion issues, the demand market prices increase compared to their values in Example 1. It is interesting to note that, due to congestion and reduced transportation capacity, consumers in demand markets pay a higher price for lower-quality bananas compared to Examples 1 and 2.

Hence, we can see, from Example 3, that a decreased transportation capacity and subsequent congestion in the Panama Canal have an impact on both the quality and the volume of banana commodity shipments, coupled with declines in the supply market prices and increases in the demand market prices. This is a quite significant result, reflecting the impacts of climate change in the form of drought-related delays and congestion on fresh produce commodity trade networks, of relevance to practice. Furthermore, both producers (the banana farmers) and the consumers are negatively impacted in terms of prices as well as the volumes of trade in bananas.

5.4 Example 4: Additional Demand Market

In this example, we expand upon the network topology presented in Example 1 by introducing a new demand market, Demand Market 2, located in the European Union (EU). The network topology is shown

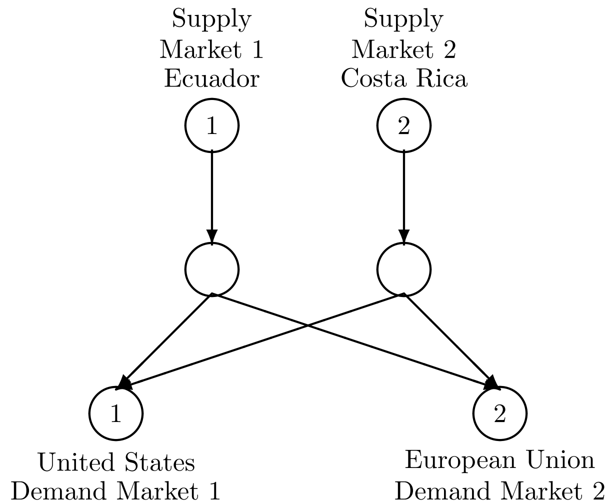


Figure 3: The Trade Network Topology for Example 4

in Figure 3. Hence, we define the following index sets: $i = \{1, 2\}$, $j = \{1, 2\}$, $r = \{1\}$, and $k = \{1\}$. In addition to the variables presented in Examples 1 through 3, we introduce new variables: Q^1_{121}, Q^1_{221}, $\lambda^1_{121}, \lambda^1_{221}$, μ^1_{121}, μ^1_{221}.

We set the new temperature parameters as follows: $T^1_{121} = 14°C = 287.15°K$, $T^1_{221} = 14°C = 287.15°K$ and $\kappa^1_{121} = 0.007hour^{-1}$, and $\kappa^1_{221} = 0.007hour^{-1}$, keeping the other temperature parameters as in Example 1. These are needed for calculating the final quality of bananas at the demand markets. Hence, the expressions for the final quality of bananas from Ecuador and Costa Rica at the new demand market in the European Union, according to (3), are: $q^1_{121} = q^{01}_1 - \kappa^1_{121}t^1_{121} = q^{01}_1 - 0.007t^1_{121}$, $q^1_{221} = q^{01}_2 - \kappa^1_{221}t^1_{221} = q^{01}_2 - 0.007t^1_{211}$. Furthermore, the transportation capacities on banana shipments from Ecuador and Costa Rica to the European Union are set to: $\overline{Q}^1_{121} = 2,000,000$ and $\overline{Q}^1_{221} = 2,000,000$.

Since we now introduce a new demand market, additional parameters in equation (5) are needed and they are: $g^1_{121} = 0.0005$ hours per ton, $g^1_{221} = 0.0008$ hours per ton, and $h^1_{121} = h^1_{221} = 10$ hours. Consequently, the shipment times on the additional links are: $t^1_{121} = g^1_{121}Q^1_{121} + h^1_{121} = 0.0005Q^1_{111} + 10$ and $t^1_{221} = g^1_{221}Q^1_{221} + h^1_{211} = 0.0008Q^1_{211} + 10$.

Also, since there are additional variables in Example 4, representing shipments to the European Union, we now have: $s^1_1 = Q^1_{111} + Q^1_{121}$ and $s^1_2 = Q^1_{211} + Q^1_{221}$. Hence, we expand the commodity supply price functions for Supply Market 1 in Ecuador and Supply Market 2 in Costa Rica, in dollars per ton, according to:

$$\tilde{\pi}^1_1(Q, q^0) = 0.00006(Q^1_{111} + Q^1_{121}) + 0.000054(Q^1_{211} + Q^1_{221}) + 0.2q^{01}_1 + 0.1q^{01}_2 + 100,$$

$$\tilde{\pi}^1_2(Q, q^0) = 0.00009(Q^1_{211} + Q^1_{221}) + 0.00007(Q^1_{111} + Q^1_{121}) + 0.2q^{01}_2 + 0.1q^{01}_1 + 100.$$

Given the presence of an additional demand market in this example, we now have that: $d^1_{121} = Q^1_{121}$ and $d^1_{221} = Q^1_{221}$. The corresponding demand price functions, in dollars per ton, for the second demand market,

Table 4: Computed Equilibrium Values of Shipment Volumes, Initial Quality Levels, and Lagrange Multipliers for Example 4

Variable	**Example 4**
Q^{1*}_{111}	681,959.81
Q^{1*}_{211}	789,489.61
Q^{1*}_{121}	1,847,141.62
Q^{1*}_{221}	1,017,761.05
q^{01*}_{1}	80.12
q^{01*}_{2}	80.21
λ^{1*}_{111}	0.00
λ^{1*}_{211}	0.00
λ^{1*}_{121}	0.00
λ^{1*}_{221}	0.00
γ^{1*}_{1}	0.00
γ^{1*}_{2}	0.00
μ^{1*}_{111}	0.00
μ^{1*}_{211}	0.00
μ^{1*}_{121}	0.00
μ^{1*}_{221}	0.00

are:

$$\tilde{\rho}^1_{121}(Q,q^0) = -0.0002Q^1_{121} - 0.00015Q^1_{221} + 5(q^{01}_1 - \kappa^1_{121}t^1_{121}) + 2(q^{01}_2 - \kappa^1_{221}t^1_{221}) + 700,$$

$$\tilde{\rho}^1_{221}(Q,q^0) = -0.00018Q^1_{221} - 0.0002Q^1_{121} + 5(q^{01}_2 - \kappa^1_{221}t^1_{221}) + 3(q^{01}_1 - \kappa^1_{121}t^1_{121}) + 700.$$

Finally, the additional unit transportation cost functions associated with shipping bananas from the two supply markets via the Panama Canal, to the European Union, in dollars per ton, are:

$$\tilde{c}^1_{121}(Q,T) = 0.000174Q^1_{121}, \quad \tilde{c}^1_{221}(Q,T) = 0.000287Q^1_{221}.$$

Note that, in Example 4, we expanded the supply price functions and we added new demand market price functions and unit transportation costs to reflect the real-world prices and costs reported at the beginning of Section 5. The remaining data used in Example 4 are the same as the data in Example 1.

In Table 4, we display the equilibrium solutions for Example 4. The equilibrium shipment volumes are expressed in tons.

The equilibrium volumes of banana shipments in Example 4 closely match the reported actual export volumes of bananas from Ecuador and Costa Rica to the United States and the European Union, as reported at the beginning of Section 5, with reference to the FAOSTAT [26] database. In reality, as well as in this example, Ecuador ends up exporting a higher volume of bananas to the European Union than to the United States as does Costa Rica. The initial quality levels of the bananas at the equilibrium are at similar levels for Ecuador and Costa Rica in this example. Note that the equilibrium banana shipment values in Example 4 are similar to those for Example 1 for exports from Ecuador and Costa Rica to the United States. These shipment times are also consistent with those reported in [13], [14], and [34].

Observe that shipment times from Latin America to the European Union are longer than those to the United States, which is to be expected, given the respective distances involved. The final quality levels of bananas shipped from Ecuador and Costa Rica to the United States and the European Union via the Panama Canal are: $q^1_{111} = 75.28$, $q^1_{211} = 74.63$, $q^1_{121} = 73.59$, and $q^1_{211} = 74.46$. The final quality of bananas is slightly lower in the European Union than in the United States. This difference can be attributed to the longer distances and shipping times from Latin America to the European Union. In Example 4, all the Lagrange multipliers are equal to 0.00 at the equilibrium.

The equilibrium supply market prices, demand market prices, opportunity costs, and unit transportation costs are reported in Table 5 and are denoted in dollars per ton.

Table 5: Function Values at the Equilibrium for Example 4

Function	**Example 4**
$\tilde{\pi}^1_1$	373.38
$\tilde{\pi}^1_2$	463.75
$\tilde{\rho}^1_{111}$	517.96
$\tilde{\rho}^1_{211}$	609.01
$\tilde{\rho}^1_{121}$	694.78
$\tilde{\rho}^1_{221}$	755.31
OC^1_1	373.38
OC^1_2	463.75
$\tilde{c}^1_{111}$	144.57
$\tilde{c}^1_{211}$	145.26
$\tilde{c}^1_{121}$	321.40
$\tilde{c}^1_{221}$	292.09

Observe, from Table 5, that the equilibrium supply prices closely resemble the values found in both Example 1 and the real-world under the updated supply price functions for this example. Additionally, the demand prices for bananas exported from Ecuador and Costa Rica to the United States exhibit comparatively lower values than those for bananas shipped to the European Union. Notably, the equilibrium demand prices for Ecuadorian bananas in the European Union closely mirror the actual import prices of bananas in that region, as detailed in the Food and Agriculture Organization [32] report. We observe similar equilibrium opportunity cost values in this example as in Example 1. The transportation costs associated with shipping bananas from both Ecuador and Costa Rica to the European Union are higher than those to the United States, primarily due to the longer distances involved. It is essential to underscore that these values align with the shipping quotes provided earlier in real-life scenarios, particularly in light of escalating ocean freight costs attributable to pandemic-induced constraints, shortages, and disruptions to transportation.

Table 6: Equilibrium Solutions for Shipment Volumes, Initial Quality Levels, and Lagrange Multipliers for Sensitivity Analysis on Transportation Capacity

Variable	Example 4	Scenario 1: Capacity 20% Down	Scenario 2: Capacity 40% Down
Q_{111}^{1*}	681,959.81	694,355.71	600,000.00
Q_{211}^{1*}	789,489.61	796,302.04	600,000.00
Q_{121}^{1*}	1,847,141.62	1,600,000.00	1,200,000.00
Q_{221}^{1*}	1,017,761.05	1,108,438.63	1,200,000.00
q_1^{01*}	80.12	78.12	70.51
q_2^{01*}	80.21	78.82	70.78
μ_{111}^{1*}	0.00	0.00	71.50
μ_{211}^{1*}	0.00	0.00	103.06
μ_{121}^{1*}	0.00	78.63	201.65
μ_{221}^{1*}	0.00	0.00	15.03

5.5 Sensitivity Analysis

In this Subsection, we conduct a sensitivity analysis to examine the equilibrium solutions and function values under different scenarios, with a specific focus on the impacts of transportation capacity reductions and increases in shipment times.

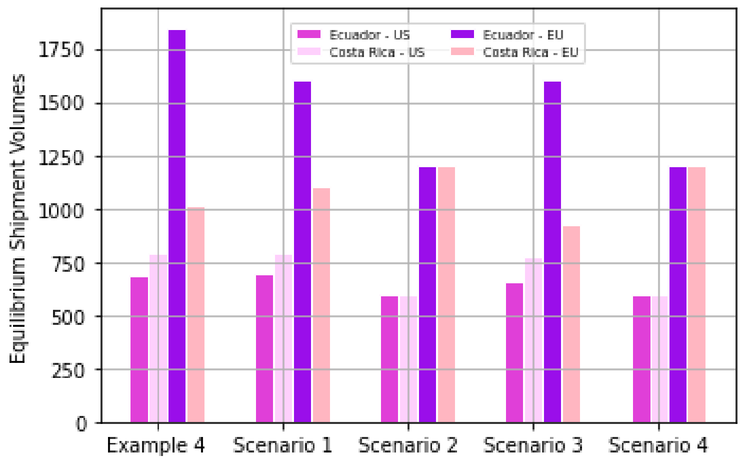
5.5.1 Sensitivity Analysis on Transportation Capacity

In this Subsection, we conduct a sensitivity analysis for two scenarios: Scenario 1 involves a 20% transportation capacity reduction, while Scenario 2 explores a 40% capacity reduction, as compared to the capacities in Example 4. Hence, we have that: $\overline{Q}_{111}^1 = \overline{Q}_{211}^1 = 800,000$ tons and $\overline{Q}_{121}^1 = \overline{Q}_{221}^1 = 1,600,000$ tons in Scenario 1 and $\overline{Q}_{111}^1 = \overline{Q}_{211}^1 = 600,000$ tons and $\overline{Q}_{121}^1 = \overline{Q}_{221}^1 = 1,200,000$ tons in Scenario 2. Recent reports indicate that the Panama Canal's capacity is operating at 40% of its maximum capacity [53]. The remaining data are as in Example 4. We report the equilibrium solutions and the function values in Tables 6 and 7, respectively. Please also note that some of the Lagrange multipliers are omitted in Table 6 since they were at 0.00. Figures 4 and 5 graphically display the equilibrium shipments and demand prices for all sensitivity analysis scenarios, respectively.

Observe, from Table 6 and Figure 4, that, as the capacity on the transportation routes decreases by 20% in Scenario 1, the equilibrium banana shipments from Ecuador and from Costa Rica to the United States slightly increase, while the equilibrium banana shipments from Ecuador to the European Union decrease, as compared to their values in Example 4. In fact, in Scenario 1, the equilibrium banana shipments reach their upper bound of $1,600,000.00$ under a 20% capacity reduction scenario for banana shipments from Ecuador to the European Union and, hence, the equilibrium Lagrange multiplier, μ_{121}^{1*}, is greater than zero. In Scenario 1, we obtain a slight increase in equilibrium banana shipments from Costa Rica to the European Union as compared to the corresponding value in Example 4.

In Scenario 1, the initial quality levels of bananas decrease from their values in Example 4. The final quality levels of bananas in Scenario 1 at the demand markets are: $q_{111}^1 = 73.18$, $q_{211}^1 = 73.18$, $q_{121}^1 = 72.44$,

Figure 4: Equilibrium Solutions for Shipment Volumes in Thousands of Tons under Different Scenarios



and $q^1_{211} = 72.54$. The final quality of bananas at the demand markets is lower than their levels in Example 4. This indicates that the capacity reductions may adversely affect the quality of bananas, suggesting that drought-related capacity decreases in the Panama Canal can result in a lower quality of bananas for consumers.

When we decrease the transportation capacities in Scenario 2 by 40%, as compared to the capacities in Example 4, the equilibrium banana shipments from Ecuador and from Costa Rica to the United States decrease from their values in Example 4 and Scenario 1, and reach their upper bounds. Additionally, the equilibrium banana shipments from Ecuador to the European Union also decrease from their values in Example 4 and in Scenario 1, while the equilibrium banana shipments from Costa Rica to the European Union increase compared to their values in Example 4 and in Scenario 1. The increase in equilibrium banana trade between Costa Rica and the European Union can be explained by Costa Rica taking advantage of the lack of competition from Ecuador; thus, capitalizing on the capacity reductions. However, we acknowledge that this situation may change if we further decrease the capacity on shipments. In Scenario 2 (cf. Table 6), all the equilibrium banana shipment values reach their upper bounds, with the Lagrange multipliers associated with the constraints being greater than zero.

The equilibrium initial quality levels of bananas reach their lowest levels when the capacity reductions are the greatest, as in Scenario 2, in comparison with the results in Example 4 and in Scenario 1. The final quality levels of bananas are: $q^1_{111} = 65.74$, $q^1_{211} = 66.51$, $q^1_{121} = 65.74$, and $q^1_{211} = 63.99$. These results underscore our finding that the quality of bananas received by consumers deteriorates as the shipments through the Panama Canal become more restricted.

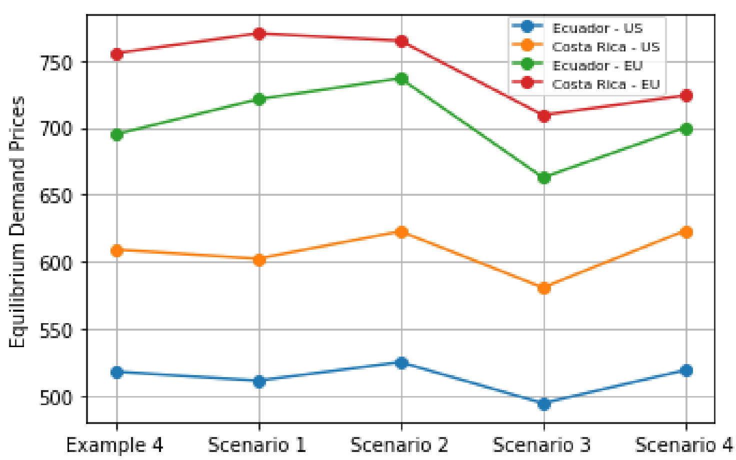
Furthermore, observe, from Table 7 and Figure 5, that, in Scenario 1, the equilibrium demand prices in the United States exhibit a decrease from their values in Example 4 for bananas exported from Ecuador and from Costa Rica. These decreases can be attributed to the increases in equilibrium shipment volumes from Ecuador and Costa Rica to the United States in Scenario 1. However, when examining the equilibrium

Table 7: Function Values at the Equilibrium for Sensitivity Analysis Examples on Transportation Capacity

Function	**Example 4**	**Scenario 1: Capacity 20% Down**	**Scenario 2: Capacity 40% Down**
$\tilde{\pi}_1^1$	373.38	364.02	326.28
$\tilde{\pi}_2^1$	463.75	455.61	409.15
$\tilde{\rho}_{111}^1$	517.96	511.22	524.98
$\tilde{\rho}_{211}^1$	609.01	602.13	622.61
$\tilde{\rho}_{121}^1$	694.78	721.06	736.73
$\tilde{\rho}_{221}^1$	755.31	770.18	764.73
OC_1^1	373.38	364.02	326.28
OC_2^1	463.75	455.61	409.15
$\tilde{c}_{111}^1$	144.57	147.20	127.20
$\tilde{c}_{211}^1$	145.26	146.52	110.39
$\tilde{c}_{121}^1$	321.40	278.40	208.79
$\tilde{c}_{221}^1$	292.09	318.12	344.40

demand prices for bananas in the European Union from both Ecuador and Costa Rica, an increase is observed in Scenario 1. The decrease in equilibrium banana shipments from Ecuador to the European Union leads to a rise in demand prices. Under Scenario 1 with a transportation capacity reduction, consumers in the European Union end up paying more for lower quality bananas in comparison to Example 4.

Figure 5: Demand Prices in Dollars at the Equilibrium under Different Scenarios



When examining equilibrium banana demand prices in Scenario 2, we observe higher demand prices for Ecuadorian and Costa Rican bananas in the United States than the values in Example 4 and Scenario 1. The increases in equilibrium demand prices in the United States for bananas from Ecuador and from Costa Rica result from lower equilibrium banana shipments from these countries to the demand markets. Analyzing the equilibrium demand price of bananas from Ecuador in the European Union in Scenario 2, we observe that the value is higher than the values exhibited in Example 4 and Scenario 1 due to lower levels of equilibrium shipments even though the final quality levels are lower than the values in Example 4 and Scenario 4. The equilibrium Costa Rican banana price in the European Union in Scenario 2 is higher than in Example 4, despite the equilibrium shipments from Costa Rica to the United States being higher than in Example 4. This discrepancy arises because the demand prices of Costa Rican bananas depend not only on the volume of shipments from Costa Rica to the European Union but also on volume of shipments from Ecuador to the European Union. This result reflects the competition dynamics, illustrating Costa Rica's advantage in capitalizing on the banana market even in the presence of capacity reductions.

In Scenarios 1 and 2, reductions in transportation capacity lead to decreases in equilibrium supply prices and opportunity costs for bananas from Ecuador and Costa Rica. In Scenario 1, equilibrium transportation costs are higher than in Example 4 for Ecuadorian and Costa Rican banana shipments to the United States. However, further capacity reductions in Scenario 2 result in decreased transportation costs. Additionally, the equilibrium transportation costs decrease in Scenarios 1 and 2 for banana shipments from Ecuador to the European Union under reduced capacities. In terms of the equilibrium banana shipments between Costa Rica and the European Union, transportation costs increase as the equilibrium shipments increase in Scenarios 1 and 2.

In summary, our main findings from our sensitivity analysis on transportation capacity reductions indicate that, when there is a capacity reduction, equilibrium shipments are adversely affected. However, some export countries may be able to take advantage of the capacity reduction. Additionally, banana demand prices increase as conditions in the Panama Canal worsen, leading to a deterioration in the final quality of bananas. This means that consumers at the demand markets pay higher prices for lower-quality bananas. Export countries experience a drop in supply prices as capacity reductions affect shipment levels and, of course, production volumes. These results reinforce the negative impacts of transportation capacity reduction on banana farmers as well as the consumers of bananas.

5.5.2 Sensitivity Analysis on Shipment Time Increases and Transportation Capacity Reductions

We now explore the impacts of shipment times in addition to the capacity reductions previously examined, aiming to investigate the effects of congestion further. We increase the parameters associated with transportation times as follows: $g^1_{111} = g^1_{121} = 0.005$ hours per ton, $g^1_{211} = 0.0025$ hours per ton, $g^1_{211} = 0.004$ hours per ton, and $h^1_{111} = h^1_{121} = h^1_{211} = h^1_{221} = 100$ hours. We introduce two additional scenarios: In Scenario 3, we explore the consequences of a 20% transportation capacity reduction coupled with transportation time increases through the adjustment of parameters. In Scenario 4, we conduct an analysis on the impacts of a 40% transportation capacity reductions and shipment time increases. The remaining data are as in Example 4. We report the equilibrium solutions and the corresponding function values in Tables 8 and 9, respectively. The values of the equilibrium shipments and the demand prices are displayed in Figures 4 and 5, respectively.

Observe, from Table 8 and Figure 4, that, in Scenario 3, all equilibrium banana exports decrease from their values in Example 4 due to a combination of reduced transportation capacity and increased shipment time. Similar to Scenario 1, the equilibrium volume of banana shipments from Ecuador to the European

Table 8: Equilibrium Solutions for Shipment Volumes, Initial Quality Levels, and Lagrange Multipliers for Sensitivity Analysis on Shipment Times and Transportation Capacity

Variable	**Example 4**	**Scenario 3: Capacity 20% Down & Time Increase**	**Scenario 4: Capacity 40% & Time Increase**
Q^{1*}_{111}	681,959.81	661,399.98	600,000.00
Q^{1*}_{211}	789,489.61	773,839.53	600,000.00
Q^{1*}_{121}	1,847,141.62	1,600,000.00.	1,200,000.00
Q^{1*}_{221}	1,017,761.05	929,356.43	1,200,000.00
q^{01*}_{1}	80.12	88.70	81.70
q^{01*}_{2}	80.21	87.78	94.29
λ^{1*}_{111}	0.00	0.00	24.86
λ^{1*}_{211}	0.00	69.38	0.00
λ^{1*}_{121}	0.00	59.17	24.89
λ^{1*}_{221}	0.00	0.00	130.02
μ^{1*}_{111}	0.00	0.00	60.85
μ^{1*}_{211}	0.00	0.00	97.80
μ^{1*}_{121}	0.00	30.04	160.18
μ^{1*}_{221}	0.00	0.00	27.20

Union reaches its capacity, with $\mu^{1*}_{121} > 0$ in Scenario 3. Note that, in Scenario 3, the volume of equilibrium shipments between Costa Rica and the European Union decreases is lower than in Example 4, whereas in Scenario 2, previously reported in Table 6, such a decrease was not observed. The reduction in transportation capacity, combined with an increase in shipping time in Scenario 3, magnifies the impacts of congestion, leading to a decrease in the volume of equilibrium banana shipments.

The equilibrium initial quality levels in Scenario 3 are higher than those observed in Example 4. Furthermore, the shipment times in Scenario 3 are: $t^1_{111} = 3,406.99$ hours, $t^1_{211} = 3,969.19$ hours, $t^1_{121} = 4,100.00$ hours, and $t^1_{221} = 3,817.42$ hours. It is evident that the shipment times exhibit an increase from their values in Example 4, as expected. Although the equilibrium initial quality levels exhibit an increase, the final equilibrium quality levels of bananas in Scenario 3 are: $q^1_{111} = 64.85$, $q^1_{211} = 60.00$, $q^1_{121} = 60.00$, and $q^1_{211} = 61.06$. It is noteworthy that, despite higher initial quality levels compared to those in Example 4, the final equilibrium quality levels of bananas at the demand markets are lower than their values in Example 4. In fact, the equilibrium final quality of bananas from Costa Rica to the United States and from Ecuador to the European Union reaches its lower bound of 60.00. Consequently, the equilibrium Lagrange multipliers associated with the lower bound constraint in (22a) are greater than zero. Therefore, given the transportation capacity reductions and the shipping time increases, the initial quality of bananas must increase to be sold at an acceptable quality. This may exert more pressure on banana farmers in the long run to enhance the initial quality of their bananas.

Furthermore, in Scenario 4, all equilibrium shipments, as presented in Table 8 and Figure 5, reach their upper bounds, with the only exception being an increase in Q^{1*}_{221}, representing banana shipments between Costa Rica and the European Union. This aligns closely with our findings for Scenario 3. Despite increased shipping times, Costa Rica manages to boost its shipment levels, capitalizing on Ecuador's inability to ship bananas due to congestion issues in the Panama Canal.

In Scenario 4, the equilibrium initial quality levels surpass those observed in Example 4 for Ecuador and for Costa Rica. However, the equilibrium initial quality level of bananas from Ecuador are at a lower level than their value in Scenario 3, whereas the equilibrium initial quality of bananas from Costa Rica is higher than its equilibrium value under Scenario 3. Additionally, the shipment times in Scenario 3 are: $t^1_{111} = 3,100.0$ hours, $t^1_{211} = 3,099.99$ hours, $t^1_{121} = 3,099.99$ hours, and $t^1_{221} = 4,900.00$ hours. The shipment times exhibit an increase from their values in Example 4 but are also slightly lower than the values reported for Scenario 3, attributable to a decrease in equilibrium shipment values. The final equilibrium quality levels of bananas at the demand markets are: $q^1_{111} = 60.00$, $q^1_{211} = 72.60$, $q^1_{121} = 60.00$, and $q^1_{211} = 60.00$, with all shipments meeting or exceeding their minimum quality standards. This implies that even though the equilibrium initial quality is at a relatively high value, the final quality of bananas is at its lower bound for most shipments, except those destined for the United States from Costa Rica. This result is intriguing, particularly for Costa Rican bananas sold in the United States. For these bananas, an overall increase in the equilibrium final quality is observed. This is because the equilibrium initial quality of bananas from Costa Rica needed to be high to comply with the acceptable quality standards in the European Union. Due to the shorter distances between the United States and Costa Rica in comparison to the European Union, bananas ultimately attain a higher final equilibrium quality in the United States, despite starting at the same initial quality level.

Table 9: Function Values at the Equilibrium for Sensitivity Analysis Examples on Shipment Times and Transportation Capacity

Function	**Example 4**	**Scenario 3: Capacity 20% Down & Time Increase**	**Scenario 4: Capacity 40% & Time Increase**
$\tilde{\pi}^1_1$	373.38	354.17	330.97
$\tilde{\pi}^1_2$	463.75	438.01	415.03
$\tilde{\rho}^1_{111}$	517.96	494.39	519.11
$\tilde{\rho}^1_{211}$	609.01	580.64	623.23
$\tilde{\rho}^1_{121}$	694.78	662.72	700.00
$\tilde{\rho}^1_{221}$	755.31	709.31	724.00
OC^1_1	373.38	413.34	380.72
OC^1_2	463.75	507.39	545.05
$\tilde{c}^1_{111}$	144.57	140.22	127.20
$\tilde{c}^1_{211}$	145.26	142.39	110.39
$\tilde{c}^1_{121}$	321.40	278.40	208.79
$\tilde{c}^1_{221}$	292.09	266.72	344.40

Observe from Figure 5 and Table 9 that, in Scenario 3, the equilibrium banana demand prices in the United States and in the European Union exhibit a decrease from their values in Example 4 for exports from Ecuador and from Costa Rica. These values are even lower than those reported for Scenario 2. This is due to the worsened quality levels and increased transportation shipments. The equilibrium supply prices are calculated to be lower than those obtained for Example 4. However, we observe an increase in the equilibrium opportunity costs in Scenario 3 compared to their values in Example 4. The equilibrium transportation costs are lower than those reported for Example 4 due to a decrease in equilibrium shipments. The equilibrium supply prices are lower than their values in Example 4 in Scenario 3 for Ecuador and Costa Rica even though the equilibrium initial quality levels are higher than those reported in Example 4. This is because of the lower equilibrium shipment volumes in Scenario 3 when compared to Scenario 2.

In Scenario 4, with further reductions in transportation capacity and increased shipment times, all equilibrium banana prices in the United States increase from their values in Example 4 and in Scenario 3. The equilibrium price of Ecuadorian bananas sold in the European Union is higher than its value in Example 4 and Scenario 3. The price of Costa Rican bananas sold in the European Union is higher than its value under Scenario 3 at equilibrium but lower than its reported equilibrium demand price value in Example 4. This lower price of Costa Rican bananas in the European Union can be explained by the increased levels of shipments for Scenario 4. The opportunity cost associated with bananas from Ecuador is lower in Scenario 4 than its value in Example 4 but higher than its value in Scenario 3. Furthermore, the opportunity cost associated with bananas from Costa Rica is higher in Scenario 4 than its value in Example 4 and Scenario 3. In Scenario 4, the equilibrium transportation costs between Ecuador and the United States and the European Union are lower than their values in Scenario 3 and Example 4. Additionally, the equilibrium transportation cost between Costa Rica and the United States is lower than its values in Example 4 and Scenario 3, whereas the equilibrium transportation cost between Costa Rica and the European Union is higher than its values reported in Example 4 and Scenario 3 due to increased level of shipments.

To summarize, our findings suggest that, when the shipment time increases, there is an overall increase in the equilibrium initial quality levels, but also a decrease in the final equilibrium quality levels. However, as observed in the case of Costa Rica in Scenario 4, some countries may take advantage of the improved initial quality levels if the distances between the supply and demand markets are shorter. The equilibrium demand prices of bananas are observed to be lower as the shipment times increase and the transportation capacity is kept at 20%, attributable to lower levels of quality of bananas sold at demand markets. However, if the transportation capacity is further reduced (by 40 %), while keeping the shipment time increases at the same levels, as in Scenario 4, we observe that the equilibrium banana prices increase at the demand markets overall. This amplifies our findings from the previous section, as this time consumers are willing to pay a higher price even for lower-quality bananas in the presence of increased shipment times, especially when the transportation capacity is reduced significantly. These results highlight the importance of the congestion issues occurring in the Panama Canal as well as the appeal of bananas to consumers at the demand markets.

5.6 Managerial and Policy Insights

The results of our numerical study provide several important insights for practitioners and policymakers managing global trade in fresh produce, particularly in the context of perishability, quality standards, transportation constraints, and sustainability.

Strategic Value of Minimum Quality Standards. As demonstrated in Example 2, increasing minimum quality standards at the demand markets not only elevates the final quality of fresh produce but also incentivizes higher initial quality production at the supply end. However, this also increases opportunity costs for producers. Policymakers should consider providing support to producers, such as quality-enhancing technologies or subsidies, to avoid marginalizing smaller suppliers when enforcing stricter quality standards.

Prioritizing Investment in Infrastructure. Example 3 illustrates that congestion in key trade routes, such as the Panama Canal, significantly degrades quality and reduces shipment volumes. These disruptions lead to price increases for lower-quality commodities, harming both producers and consumers. Investments in alternate trade corridors or expanded canal capacity could alleviate these issues. Logistics planners should also consider adaptive routing and real-time quality tracking to respond dynamically to delays.

Role of Transportation Capacity Management. Our sensitivity analyses reveal that reduced transportation capacity results in decreased supply prices and quality, but increased consumer prices. Exporting countries with flexible routing or diversified access to ports (e.g., Costa Rica in Scenario 4) can partially offset these challenges. Trade alliances and regional agreements could help to redistribute shipment loads more effectively across supply chains to prevent bottlenecks.

Temporal Risk Mitigation. Longer shipment times, as shown in the sensitivity analysis, exacerbate quality loss even when initial quality is high. This underscores the need for tighter integration of food science with logistics, such as pre-cooling technologies, enhanced packaging, and predictive deterioration modeling, to ensure quality preservation under uncertain transit times.

Implications for Food Waste. Our model highlights that if shipments arrive below minimum acceptable quality levels, they may be rejected or discarded, directly contributing to food waste. This is especially evident in scenarios with high congestion or reduced transportation capacity. Policymakers and supply chain managers should focus on preventive measures, such as enforcing cold chain standards, improving transparency in transit conditions, and establishing secondary markets for downgraded but still consumable produce to reduce postharvest food losses.

Demand Market Implications. From the demand side, quality-sensitive consumers (e.g., in the European Union) are shown to pay premium prices despite lower quality under constrained scenarios. Demand forecasting models that incorporate elasticity with respect to quality may aid retailers and importers in price optimization and procurement decisions during disruptions.

In summary, our results highlight the interconnectedness of production quality, logistics constraints, and market access in the fresh produce trade. The model provides a quantitative foundation for evaluating trade-offs and designing interventions that promote both resilience and sustainability in perishable supply chains.

6. Conclusions

In this paper, we develop novel multicommodity spatial price equilibrium models (without and with bounds) to analyze the evolving effects of congestion over time on trade of fresh produce while incorporating the degradation of its quality through explicit quality deterioration formulas. The bounds include minimum quality standards for fresh produce at the demand markets and transportation capacities on trade routes. The methodology utilized in the formulation and qualitative analysis of the models is the theory of variational inequalities. We demonstrate the applicability of our modeling framework via a numerical study on banana trade, focused on the top exporters Ecuador and Costa Rica as well as the top importers, the United States and the European Union. In addition, the Panama Canal, now undergoing severe capacity limitations due to a drought, is included as a component in the transportation routes. Our theoretical and computational framework, accompanied by explicit numerical examples, for which all the input and output data are reported, is of significant relevance to real-world scenarios. Furthermore, it provides a unique perspective within the existing body of literature on quality deterioration models for fresh produce supply chains. We build and test various scenarios computationally to capture congestion in the Panama Canal through our variational inequality model and report the impacts of such disruptions on equilibrium shipment volumes and times, supply and demand prices, quality levels (initial at the supply markets and final ones at the demand markets), and transportation costs. We note that our baseline example and accompanying numerical results align very closely with the real-life export volumes of bananas, demand and supply prices, as well as transportation costs.

Our numerical examples, along with the sensitivity analysis results, indicate that a reduction in transportation capacity and an increase in shipment times, such as the one observed in the Panama Canal due to drought and climate change-related issues, result in decreased export volumes, higher initial quality levels, but lower final quality of bananas at demand markets. In one of the scenarios, with both the highest transportation capacity reduction (40%) and increased shipment times, the lowest overall banana quality levels occur for consumers at the demand markets. In this scenario, the equilibrium demand prices are also found to be higher than in most of the other tested scenarios. This implies that, in scenarios involving transportation capacity disruptions and increased shipment times, leading to congestion, consumers at demand markets may end up paying more for the lowest quality bananas, while overall production and shipments also decrease.

Additionally, our findings suggest that more stringent quality standards lead to an increase in the final quality levels of bananas at the demand markets and a rise in the demand prices of bananas. In one of our examples, we only impose higher quality standards on Ecuadorian bananas and not on those from Costa Rica. However, due to competition, we observe an overall increase in the final quality of Costa Rican bananas in the demand market, even without the imposition of higher quality standards. In this scenario, we also observe an increase in equilibrium banana shipments, and in the demand and supply prices. This suggests that higher minimum quality standards for fresh produce at the demand markets can prove advantageous for both consumers and producers. Consumers, recognizing the enhanced quality, may willingly pay for higher-quality bananas. Despite the higher supply price, this commitment to quality results in increased export volumes, ultimately, benefiting consumers and producers alike.

Furthermore, our findings indicate that certain export countries, not holding the lead exporter position, can leverage congestion to boost their banana shipments while maintaining demand prices at a level that enables them to capitalize on the congestion and the reduction in prevalence of product of a main competitor. This outcome holds true for bananas from Costa Rica, where Ecuador serves as the lead exporter to the European Union. Upon incorporating the impacts of congestion, we observe that Costa Rica seizes the opportunity to increase both its banana shipments and the demand prices. Interestingly, we also observe an improvement in the quality level of bananas from Costa Rica in the United States, attributed to the higher initial quality levels.

Possible additional research directions can include the construction of fresh produce trade models with quality deterioration on general networks and the inclusion of various disruption scenarios to trade with associated probabilities. Another potential avenue for future research is to extend the model to incorporate a broader range of destination markets, along with additional explanatory variables, such as geographical proximity, common language, and shared borders, using data from the CEPII Gravity database [22]. This would enhance the understanding of trade flow determinants and shed light on regional disparities in access to high-quality fresh produce.

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