Networks in Economics and Finance in *Networks* and Beyond: A Half Century Retrospective

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Abstract:

This paper presents a panoramic view of research on economic and financial networks in the journal *Networks*, since its inception half a century ago. This paper focuses on both the breadth and depth of the journal articles, and within the context of earlier contributions, as well as more recent related ones in other scientific publications. From network optimization to game theory and a plethora of equilibrium concepts, along with novel dynamical systems frameworks, the journal has led the way in advancing economic and financial network models, algorithms, and applications. Moreover, *Networks* has helped to attract researchers in a variety of disciplines to the science of networks and the formulation and solution of associated problems drawn from the real world.

Key words: economic networks; financial networks; spatial price equilibrium; financial equilibrium; game theory; variational inequalities; projected dynamical systems

1. Introduction

Networks, from transportation and logistical ones, to communications and energy, have provided the foundation and connectivity for the flow of people, and the exchange of goods, information, and services across space and time. Intimately related to such physical networks, in which the identification of nodes, links, and associated flows with physical entities is well-understood, are economic and financial networks. The importance of all such network systems to the functioning of our societies and economies, coupled with the need to understand their interrelationships, has spurred numerous advances in methodologies for their modeling, analysis, and solution, under different behavioral concepts associated with usage and management.

The origins of network theory can be traced back to the 1700s, to the classical paper of Euler [64], the earliest paper on *graph* theory. By a graph in this context is meant, mathematically, a means of abstractly representing a system by its representation in terms of vertices (or nodes) and edges (arcs or, equivalently, links) joining pairs of vertices. Euler sought to determine whether it was possible to walk around Königsberg (later renamed Kaliningrad) by crossing the seven bridges over the River Pregel exactly once. The problem was depicted as a graph with the vertices representing land masses and the edges – bridges.

Interestingly, one of the first network models was for a financial system. Specifically, Quesnay [188], in his *Tableau Economique*, conceptualized the circular flow of financial funds in an economy as a network. His fundamental idea has been utilized in the construction of financial flow of funds accounts, which provide a statistical description of the flows of money and credit in an economy (see [37]). This work also inspired the first paper on financial networks in *Networks*, by Nagurney and Hughes [144]. The network model, with an accompanying decomposition algorithm, can be applied to calculate reconciled values of outstanding financial instruments, tangible assets, and net worth. The reconciled dataset can then be utilized as a base line for an empirical general equilibrium model and for macromonetary policy analysis.

Cournot [42], in his classical work in economics, was inspired by competition in a spring water product duopoly. His model, which considered two spatially separated markets in which the cost associated with transporting the product was included, implicitly assumed a network. Pigou [184], subsequently, studied a transportation network with two routes and observed that the decision-making behavior of the users leads to different flow patterns. The network consisted of a directed graph, that is, with the links represented by arrows, as well as the resulting flows on the links. Kohl [101] and, later, Knight [100] also considered congestion in their transportation network models as had Pigou. Such early contributions already recognized the relevance of economic activity on infrastructure, concomitant with the users' behavior.

Network theory has evolved over many years into a powerful, dynamic formalism for analyzing and solving numerous complex problems. The science of networks continues to impact a plethora of disciplines, from operations research and the management sciences (OR/MS) to economics and finance, with new applications regularly being discovered. It is also bringing different scientific disciplines closer together such as, for example, physics and computer science closer to OR/MS.

This paper is focused on the journal *Networks*, as a significant publication outlet for contributions to networks in economics and finance, since its inception half a century ago. Here, we also discuss related publications, which further place into context contributions in *Networks* and more deeply accentuate their impact. The journey of *Networks* has been scientifically exciting and very rich intellectually. Congratulations to the journal, its editors, and authors on this landmark half century anniversary!

2. More on the Origins

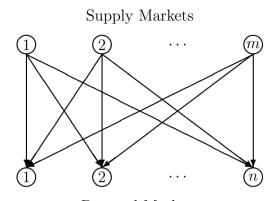
Before presenting a detailed analysis of *Networks*' role in advancing the science of networks in economics and finance, it is essential to further expand upon some of the foundations/origins.

Due to the prevalence of transportation networks, along with their scope and scale, it is not surprising that such networks attracted interest from economists, and, later, by operations researchers. Indeed, it is important to note that much of the initial work on transportation network modeling was conducted by economists. For example, after the first graph theory book (cf. König [102]) was published, the economists Kantorovich [94], Hitchcock [90], and Koopmans [103] considered the network flow problem associated with this classical minimum cost transportation problem. They provided insights into the special network structure of such problems, which enabled network-based algorithms. Both Kantorovich and Koopmans were recipients of the Nobel Prize in Economic Sciences. Interestingly, as noted in Nagurney [134], the study of network flows precedes that of optimization techniques, in general, with seminal work done by Dantzig [55] in linear programming with the simplex method, and later adapted for the classical transportation problem [56].

2.1 Networks in Economics

The need to formulate and compute solutions to network-based equilibrium problems, in which there are multiple interacting entities, in both spatial economics and in transportation, gave rise to the development of appropriate rigorous methodologies. Enke [63], writing in the journal *Econometrica*, proposed an analogue computer consisting of electronic circuits as a means of solving spatial price economic equilibrium problems. Such problems, unlike oligopolies, which are examples of imperfect competition, represent perfect competition, in which goods are produced, consumed, and traded, in the presence of transportation costs. The Nobel laureate in Economic Sciences Paul Samuelson [195] revisited the spatial price equilibrium problem. He recognized and utilized the network structure, which was bipartite (the same structure as in the classical transportation problems), that is, consisting of two sets of nodes (cf. Figure 1). Enke's use of analog computational machines was soon superseded by digital computers, with accompanying algorithms, based on mathematical programming, with quadratic programming being utilized, under linearity (and separability) assumptions on the underlying economic functions.

In spatial price economic equilibrium problems, unlike classical transportation problems, the supplies and the demands are variables, rather than fixed quantities. The seminal works of Samuelson [195] and Takayama and Judge [206, 207] were, later, extended by many researchers, motivated, in part, by the wide-range of applications, including commodity markets in agriculture [92] and energy [106] to even finance [121]. Authors, including Asmuth, Eaves, and Peterson [10], Pang and Lee [181], Florian and Los [70], Harker [86, 87], Dafermos and Nagurney [52], Nagurney and Kim [148], Nagurney [128], and the references therein, advanced spatial price economic equilibrium modeling to incorporate, among other features, multiple commodities, and asymmetric supply price and demand functions, as well as other



Demand Markets Figure 1: A Bipartite Network with Directed Links

extensions, such as the consideration of general underlying transportation networks (see also [50]). Such general, realistic spatial price network models were made possible by advances in complementarity theory (cf. [1] and the references therein), as well as variational inequality theory (cf. [47, 88, 97, 126]). Variational inequality theory allowed for the formulation and solution of equilibrium problems for which no optimization reformulation of the governing equilibrium conditions was available.

Interestingly, almost parallel to the initial work in spatial economics, with the goal of reformulating the underlying equilibrium conditions of the spatial price economic equilibrium problem as an associated optimization problem (under appropriate assumptions), was the research on the network modeling of congested urban transportation systems. Unlike the original spatial price equilibrium models, however, congested urban transportation networks may assume many different topologies. Wardrop [216] proposed two fundamental principles of travel behavior, later termed *user-optimization* and *system-optimization* by Dafermos and Sparrow [53]. The pioneering book *Studies in the Economics of Transportation* by the economists Beckmann, McGuire, and Winsten [18] detailed, for the first time, a rigorous mathematical formulation of the conditions described by Wardrop's first principle that allowed for the ultimate solution of the traffic network equilibrium problem in the context of certain link cost functions that are increasing functions of the flows on the links. The authors established that the optimality conditions in the form of Kuhn-Tucker [105] conditions of an appropriately constructed optimization problem coincided with Wardrop's first principle. According to the first principle, no traveler on the transportation network, acting unilaterally, will have any incentive to alter his route (assuming rational cost (time)-minimizing behavior) since his travel cost (travel time) is minimal. Independently, Charnes and Cooper [29, 30] in articles on traffic network equilibrium with fixed origin-destination demands, noted the relation to Nash [173] equilibrium, also emphasized by Dafermos and Sparrow [53]. Under system-optimization, in contrast, there is a central controller that routes the traffic from origins to destination nodes so that the total cost in the network is minimized. Earlier, Prager [185] had sketched out a formulation related to Wardrop's principles, recognizing that the travel cost/time on a link may depend upon the flows on other links [22].

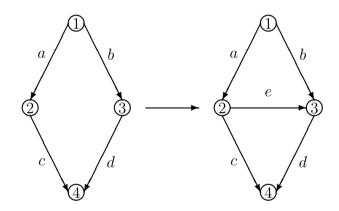


Figure 2: The Braess Network Topologies Before and After the Addition of a New Link e

Dafermos and Sparrow [53] proposed algorithms for the determination of the resulting flows on both user-optimized and system-optimized networks that were based on path flows. Fascinatingly, the renowned Braess [24] paradox paper, which "rediscovered" Wardrop's [216] two principles of travel behavior, which he, at that time, was unaware of (see [25, 139]), illustrated that the addition of a link may result in travel time increasing, under "useroptimizing" behavior for all travelers! Please refer to Figure 2 for the Braess networks before and after the addition of link e, which results in a new path for travelers between the origin node 1 and the destination node 4. Note that the Braess paradox can only occur in user-optimized networks and not in system-optimized ones. Moreover, it is as relevant today to another decentralized network par excellence - the Internet (see [104] and [153]) as it is to transportation networks.

2.2 Networks in Finance

As noted earlier, the first network model in finance was a model of a financial system. Copeland [41], two centuries later, recognized the conceptualization of the interrelationships among financial funds as a network and asked the question, "Does money flow like water or electricity?" In addition, he provided a "wiring diagram for the main money circuit." Hence, early on, there was interest in finding commonalities among different network systems.

With advances in optimization and associated mathematical programming, beginning with linear and nonlinear programming, financial network modeling first focused on financial optimization. The seminal work of the Nobel laureate Harry Markowitz [115, 116] in portfolio optimization established a new era in financial economics, with relevance to this day. Markowitz's model was based on mean-variance portfolio selection, with the average and the variability of portfolio returns determined in terms of the mean and covariance of the corresponding investments. As noted in Nagurney [134], although many financial optimization problems, including Markowitz's, had an underlying network structure, and the advantages of network programming were becoming increasingly evident [30], it was only some time later that financial network optimization models were developed. In fact, the structure of Markowitz's portfolio optimization pair as in Figure 3, with flows on the links corresponding to investments and the demand being the size of the financial assets to be invested. Early models were those of Charnes and Miller [33] and Charnes and Cooper [31].

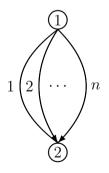


Figure 3: Network Structure of the Classical Portfolio Optimization Problem

The 1970s saw the greater utility of networks for financial applications, with, among the first financial network optimization models in the literature being a series of currency translating models. For example, Rutenberg [193] noted that the translation among different currencies could be performed through the use of arc multipliers. Rutenberg's network model was multiperiod with linear costs on the links (a characteristic common to the earlier financial networks models). The nodes of such generalized networks represented a particular currency in a specific period and the flow on the arcs the amount of cash moving from one period and/or currency to another with Christofides, Hewins, and Salkin [35] and Shapiro and Rutenberg [197], among others, proposing related financial network models. Golden, Liberatore, and Lieberman [82] constructed a generalized network flow model that allows for gains and losses on link flows to account for cash value changes. The authors also conducted sensitivity analysis on the borrowing interest rate, the bounds on the link flows, and the forecast horizon. Interestingly, arc multipliers and generalized networks were subsequently also applied in the context of perishable product spatial price equilibrium problems [138] and supply chain networks from food [224] to blood [152].

As overviewed in Nagurney [133], where additional references can be found, Barr [15] and Srinivasan [204] utilized networks to construct cash management problems, with Crum [43] introducing a generalized linear network model for the cash management of a multinational firm. A series of related cash management problems were modeled as network problems in subsequent years by, among others, Crum and Nye [45] and Crum, Klingman, and Tavis [44] as linear network flow problems in which the cost on an arc was a linear function of the flow. In many financial network optimization problems, nevertheless, the objective function must be nonlinear due to the modeling of the risk function and, therefore, often, such financial problems are more appropriately modeled as nonlinear, rather than linear, network flow problems. Mulvey [122] not only identified that the Markowitz [115, 116] mean-variance minimization problem was, in fact, a network optimization problem with a nonlinear objective function, but he also presented a collection of nonlinear financial network models that were based on previous cash flow and portfolio models in which the original authors (see, e.g., [192] and [202]) had not identified and, consequently, had not exploited the underlying network structure. Additional financial network optimization models and associated references can be found in the paper by Mulvey [122], in the book by Nagurney and Siokos [162], and in the volume edited by Nagurney [129].

In addition to the early literature on financial networks, with a focus on optimization, there was a growing literature on financial networks for financial equilibrium. As noted in Nagurney [133], Thore [208] proposed networks for the study of systems of linked portfolios, with his work recognizing the contributions of Charnes and Cooper [31], who had showed that systems of linked accounts could be represented as a network. In such a financial network, the nodes correspond to balance sheets and the links to the credit and debit entries. Thore [208] considered credit networks, with the goal of constructing a tool for studying money and credit streams in an economy, based on the behavior of banks and other financial institutions, and utilized linear programming. Thore [209] then extended the basic financial network model to incorporate holdings of financial reserves under uncertainty. Fei [65] had earlier proposed a graph theoretic approach to the credit system. Storoy, Thore, and Boyer [205], subsequently, constructed a network of the interconnection of capital markets, in which the utility function of a sector was no longer limited to being a linear function. The authors illustrated how decomposition theory of mathematical programming could enable the computation of the equilibrium. Thore [210], in his book, which appears to be the first book on financial networks, further investigated network models of linked portfolios, financial intermediation, and decentralization/decomposition theory. However, the computational techniques at that time were not sufficiently well-developed to handle such problems in practice.

Interestingly, Thore [211], exploiting the ideas of Samuelson [195] and Takayama and Judge [207] for spatial price equilibrium problems, as discussed above, later proposed an international financial network for the Euro dollar market, conceptualizing it as a logistical system. In that paper, as in Thore's preceding papers on financial networks, the microbehavioral unit was an individual bank, savings and loan, or other financial intermediary with the portfolio options described in some optimizing framework, with the portfolios linked together into a network with a separate portfolio visualized as a node and assets and liabilities as directed links. In such financial systems, equilibrium was central, along with the role of prices in the equilibrating mechanism. The Arrow-Debreu economic model (cf. [9, 57]) greatly influenced the rigorous approaches to both economic and financial equilibrium, along with the price determination. In addition, the importance of the inclusion of incorporating dynamics into the modeling of such financial network systems was also being stressed [212]. Arrow, Debreu, and Kydland were all awarded the Nobel Prize in Economic Sciences.

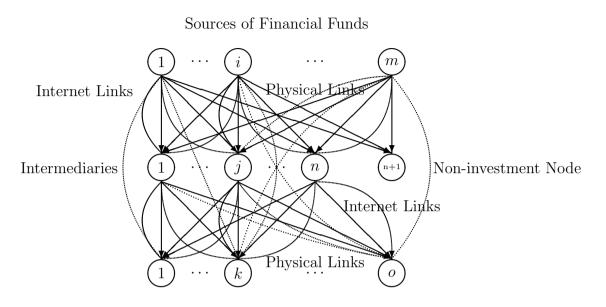
2.3 Some Additional Methodological Advances

Subsequent notable methodological contributions were made by Smith [201] and Dafermos [47], who identified Smith's formulation of traffic network equilibrium conditions as a variational inequality (VI) problem. This fundamental discovery allowed for the modeling of asymmetric interactions associated with the link travel costs (resulting in no equivalent optimization reformulation of the equilibrium conditions) and established the methodology of variational inequalities as a primary tool for both the qualitative analysis and the solution of such and other related equilibrium problems, many of which had a network structure. For additional background, see the book by Nagurney [128]. Today, VI theory is as relevant to the Internet, which is characterized by decentralized decision-making (cf. [22, 191, 194]), as it is to transportation networks [198], as well as to electric power generation and distribution networks [223], supply chain networks (see [131] and [149]), and financial networks with and without intermediation (see [113, 145, 146, 162] and the references therein). For a depiction of a financial network with intermediation and electronic transactions, see Figure 4.

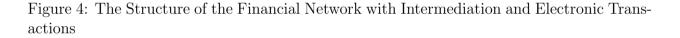
Indeed, many complex systems in which decision-makers/agents compete for scarce resources on a network, be it a physical one, as in the case of congested urban transportation systems, or a more abstract one, as in economic and financial problems, can be formulated and studied as network equilibrium problems. Applications of network equilibrium problems are common in many disciplines, including economics, finance, and in engineering, operations research, and management science (cf. [69, 128, 131]). See the classical book on networks by Ford and Fulkerson [71] and the book by Ahuja, Magnanti, and Orlin [2] for a reference to network flows with a focus on linear, rather than nonlinear, problems and many interesting applications.

3. The First Decade of Networks

With the work of Beckmann, McGuire, and Winsten [18] and Dafermos and Sparrow [53] setting the stage for the rigorous modeling of network problems under different behavioral principles, the journal *Networks* provided an excellent outlet for further advances. Rosenthal [190], in his remarkable paper in *Networks*, proposed a discrete flow approach, rather



Demand Markets - Uses of Funds



than a continuous one, to network equilibrium problems, formulated as an *n*-person noncooperative game with pure strategies. He demonstrated that pure-strategy Nash equilibria exist and that any solution to an integer-variable analogue of the usual network equilibrium model is such a Nash equilibrium. Gazis [77] then provided an excellent critical survey of transportation networks in *Networks*. Golden [81] constructed a nonlinear multicommodity network optimization model with quadratic cost functions and applied it to a specific large scale network in the form of a port planning problem with consideration of foreign ports.

Golden and Magnanti [83] produced a bibliography in *Networks* for deterministic network optimization, with an emphasis on algorithms, along with the accompanying theory. Assad [11], in his comprehensive survey on multicommodity network flows in *Networks*, emphasized that equilibrium problems, including those in transportation, had renewed substantial interest in applying mathematical programming. He discussed solution techniques and applications, recognizing that such network modelling arises naturally whenever commodities, vehicles, or even messages needed to be shipped/transported.

Already, in the first decade of its existence, it was apparent that *Networks* was the "go-to" journal for innovative, substantive articles, including those with an economic bent.

4. Papers of the 1980s and the Impact

With the publication of the paper by Dafermos [47], which unveiled the theory of variational inequalities as a powerful framework for the formulation and solution of equilibrium problems, and, in particular, network equilibrium problems, the momentum of important contributions to the science of networks accelerated and increased.

Dafermos [48], in her *Networks* paper, proposed a general multimodal transportation network equilibrium problem with elastic demand. The model dramatically extended the types of transportation network equilibrium problems that could be rigorously formulated, analyzed qualitatively, and solved, as well as applied in practice. The theoretical framework was that of VI theory. The proposed algorithm resolved the asymmetric problem into a series of single-modal problems, amenable to solution via quadratic programming. This very general model not only was an advance in transportation but also enabled the establishment of connections to other models/problems. For example, using an elastic traffic network equilibrium model and variational inequality theory, Dafermos and Nagurney [51] demonstrated an isomorphism between spatial price and traffic network equilibrium problems, which was further elaborated upon by Dafermos [49] in the context of multicommodity / multimodal networks. Hence, many spatial price equilibrium problems could be transformed into and solved as transportation network equilibrium problems. It is important to mention that the model of Dafermos [48] had elastic demands as had the original transportation network equilibrium model of Beckmann, McGuire, and Winsten (BMW) [18]. Subsequently, Nagurney and Liu [150], see also [131, 132, 151], were able to answer the fundamental question in the BMW book regarding unsolved problems, on page 106, "The unsolved problems concern the application of this model to particular cases. In particular, the problem of generation and distribution of electric energy in a network comes to mind." Specifically, using the work of Dafermos [48] and also an extension of finite-dimensional variational inequalities to evolutionary ones (cf. [54]), they showed that electric power generation and distribution problems

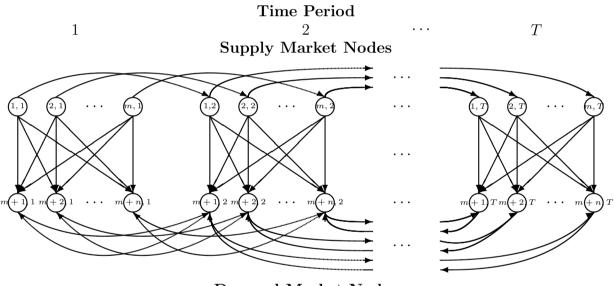
could be reformulated and solved as transportation network equilibrium problems. Liu and Nagurney [113] then, addressing the question of Copeland [41], noted earlier, as to how does money flow, established a transformation of financial network equilibrium problems with intermediation to transportation equilibrium problems. Hence, theoretical results had shown that both electricity and money flows like transportation flows.

Haurie and Marcotte [89], in their *Networks* paper, utilized variational inequality theory, contributing to the literature in a significant way. They introduced a noncooperative game on a congested transportation network and proved that the asymptotic behavior of the Nash-Cournot equilibrium yields (under suitable assumptions) a flow vector corresponding to a Wardrop equilibrium. Economists had investigated that Cournot oligopolies with an infinite number of firms led to perfect competition (cf. [75, 118, 178]) but not on a network. As mentioned earlier, Dafermos and Sparrow [53] had established a relationship between traffic network equilibria and Nash equilibria as had Rosenthal [190] in his *Networks* paper and Devarajan [59], as well. However, Haurie and Marcotte [89] did not require integrability of the user link cost functions. Recall that Rosenthal [190] had considered a discrete version of the traffic network equilibrium problem with specific link cost functions.

Dafermos and Nagurney [52], subsequently, constructed a general oligopoly model with spatially separated markets on a bipartite network and proved that it generates a general spatial price equilibrium model as an extreme, limiting case. The analysis made use of the fact that the governing equilibrium conditions of both the spatial oligopoly and the spatial price economic equilibrium problem could be formulated as variational inequalities. The oligopoly was governed by a Nash [172, 173] equilibrium. Gabay and Moulin [74] had shown that oligopolistic market equilibria under Nash equilibrium, under suitable assumptions could be formulated as variational inequality problems.

The 1980s also revealed a renewed focus on algorithm development for solving equilibrium problems, as evidenced by a series of publications in *Networks* for traffic network equilibrium problems, including ones that could be reformulated as optimization problems (cf. [58, 68, 99]), as well as spatial price equilibrium problems [182]. Importantly, the introduction of stochastic elements to traffic assignment models was initiated by Sheffi and Powell [199] in their paper in *Networks*.

New models of spatial price equilibrium problems, along with algorithms, were published in the regional science literature. Some notable examples include the already noted paper of Florian and Los [70], and papers by Friesz, Harker, and Tobin [72], Nagurney [124, 125], and Nagurney and Aronson [138] in *Networks*. The latter work introduced a general, dynamic spatial price equilibrium model with gains and losses, of relevance to a multiplicity of perishable products [166]. It allowed for inventorying at the supply markets and at the demand markets, as well as backordering at the demand markets. For a graphical depiction of the model, see Figure 5. In addition, the model has inspired research on multitiered, multiperiod supply chain network equilibrium problems [114].



Demand Market Nodes

Figure 5: Network Structure of the Multiperiod Spatial Price Equilibrium Model with Gains and Losses

Today, the study of the modeling, analysis, and solution of a variety of game theory problems on networks, including routing games, has crossed disciplines, and has helped in building bridges, linking, for example, economics, OR/MS, and computer science (see, e.g., [177, 191]).

5. The 1990s with a Surge in Publications on Economic and Financial Networks

With the development of new network models, algorithms, and more powerful computers as well as the growing influence of the Internet, the 1990s saw an explosion of research interest in economic and financial networks. In 1991, Nagurney and Zhao [171], inspired by the relevance of policies for market equilibrium problems in practice, and, building on earlier work on spatial price equilibrium, in their paper in Networks, constructed a market disequilibrium model with price controls in the form of price floors and ceilings. Thev identified the problem's network structure, and proposed a network decomposition scheme for computation of the product flows and excess supplies and demands. Qiu [187], in turn, conducted qualitative analysis for an oligopolistic market equilibrium problem defined on a bipartite network. Wie [220], also writing in *Networks*, in a paper in 1993, introduced a differential game model of Nash equilibrium on a congested traffic network over a simple network. Wie and Tobin [221], subsequently, considered the problem of a dynamic Nash equilibrium traffic assignment with schedule delays on congested networks and formulated it as an N-person nonzero-sum differential game in which each player represents an origindestination pair.

In 1993, Dupuis and Nagurney [61] (see also the book by Nagurney and Zhang [167]) introduced a new dynamical system termed a *projected dynamical system*, with accompanying theory, analysis, algorithms, and dynamic models. They established that the sets of solutions to a projected dynamical system, which has a discontinuous right-hand-side, and, hence, is nonclassical, coincides with the sets of solutions to an appropriately defined finite-dimensional variational inequality problem. This contribution provided a natural underlying dynamics to a multiplicity of equilibrium problems, including network equilibrium problems, which had previously been studied primarily in the steady-state. The framework also yielded continuous time and discrete time tatonnement processes, with the latter corresponding to algorithms. In a 1995 paper in *Networks*, Nagurney, Takayama, and Zhang [164] proposed a projected dynamical systems model for spatial price network equilibrium and implemented the algorithm on a massively parallel architecture.

To-date, the theory of projected dynamical systems has been used to model and analyze, including from a stability standpoint: dynamic spatial price problems [168], dynamic traffic network problems (cf. [169, 170]), dynamic oligopolies [226], dynamic financial network problems [60], and even a spectrum of supply chain network problems, in which there are multiple tiers of decision-makers, each with his own objective function and constraints (cf. [131] and the references therein).

Projected dynamical systems have also found application and success in population games and evolutionary dynamics in economics [196] and even fascinatingly, in neuroscience [80]. Their relationships, in the infinite-dimensional context, to evolutionary variational inequalities were established by Cojocaru, Daniele, and Nagurney [38, 39].

This decade, filled with many novel, creative contributions to the science of networks, also benefited from the recognition that advances in both economics and finance could be made through computational methods. For example, the first book in the series: Advances in Computational Economics, was the book, Network Economics: A Variational Inequality Approach, by Nagurney, and published in 1993 [126], with the second edition, in 1999 [128]. The Society of Computational Economics was founded in 1995.

The paper, noted earlier, by Nagurney and Hughes [144] was the first paper in *Networks* on financial networks. The authors emphasized the need for the development of empirical general equilibrium models that would capture the economic behavior of sectors in determining financial and capital flows. Underlying such models was a balanced set of financial accounts with the origins of flow of funds accounting dating to Copeland [41]. The network approach of Nagurney and Hughes [144] could be applied to create such a balanced set of accounts. The classical, renowned in economics, Walrasian price equilibrium problem [214], a general economic equilibrium problem, was identified by Zhao and Nagurney [228] to be isomorphic to a network equilibrium problem, which they then exploited for computational purposes. The special network structure was identical to that of the portfolio optimization problem as depicted in Figure 3.

Nagurney and Dong [140], writing in *Networks*, proposed a financial equilibrium network model with transaction costs for an economy with multiple sectors, each of which seeks to determine the optimal composition of its portfolio in terms of assets and liabilities. The variational inequality formulation of the equilibrium conditions was derived and an algorithm that exploited the network structure was applied to determine the equilibrium asset, liability, and price pattern in a series of numerical examples. Nagurney [127] had earlier developed a financial equilibrium model with general utility functions, generalizing the quadratic ones of Nagurney, Dong, and Hughes [142], who were the first to propose variational inequality theory for general financial equilibrium problems.

Nagurney and Siokos [163], in their paper in *Networks*, introduced a dynamic multi-sector, multi-instrument financial network model with futures, utilizing the theory of projected dynamical systems for its formulation, analysis, and solution. They also demonstrated that their model could be reformulated as an optimization problem.

6. The New Millennium, Networks Cross More Disciplines, and the Financial Collapse - The 2000-2009 Decade

With the advent of the new millennium, it was appropriate and reflective, to have the 30th anniversary paper by the journal's first Editor-in-Chief Frisch [73] on the early days of *Networks* published in the journal. On page 6 of the article, he wrote: "When we started *Networks*, we did not anticipate the explosion of the field of networks." This statement resonates and is as true today as it was at the beginnings of the journal.

Nagurney [130] reported in *Networks* on recent developments in Network Economics based on papers presented at the 2002 Computing in Economics and Finance Conference, which took place in Aix en Provence, France. At the conference, speakers presented their research on computational economics and finance topics with two sessions specifically devoted to Network Economics. As emphasized in the article, the role of networks in economics and finance was gaining prominence for several reasons: (1) the emergence of industries that are network-based, from transportation and logistics companies, to telecommunication, energy, and power companies; (2) the recognition of the interdependence between/among many network systems, such as telecommunications with finance, telecommunications with transportation in the form of electronic commerce, for example, and telecommunications with a variety of energy transmission mechanisms; (3) the recognition that new relationships between economic agents in terms of competition and cooperation are giving rise to new supply chains as well as new financial networks; (4) the relevance of networks in terms of infrastructure and the pricing of their usage as well as the management of risk and uncertainty surrounding networks; and (5) interest in the dynamics surrounding networks and their evolution over space and time. At the conference there were several papers on supply chains and electronic commerce as well as on electronic financial transactions.

Geunes and Pardalos [78], also writing in *Networks*, presented an elegant annotated bibliography on network optimization in supply chain management and financial engineering, emphasizing their real-world relevance, including in the context of large-scale problems. They also identified them as still emerging fields. In terms of finance, their emphasis was on contributions that assume a single decision maker (e.g., an individual or organization), who seeks to maximize future wealth by allocating resources to investments and/or assets. However, they noted that, in addition to such "microlevel" or "single-investor" problems, a substantial body of literature addresses equilibrium in financial networks with multiple sectors and financial intermediaries. They highlighted as representative of such research being that of Nagurney [127], Nagurney and Siokos [163], and Nagurney and Dong [140], which characterizes equilibrium funds flows among and between various financial sources, financial intermediaries, and demand markets, using dynamical systems analysis and variational inequalities. Clearly, networks were becoming more prominent, more visible, and even more useful in applications essential to modern economies and societies.

With the growth of the Internet (cf. Clark [36] for an excellent volume on its history and possible future directions) and innovations both in supply chains, with precursors in spatial price equilibrium problems [136], and in financial networks, along with growing interactions among network systems (see [146] and the book [131]), it was clear that there was a new momentum with exciting opportunities. The volume on innovations in financial and economic networks edited by Nagurney [130] contains refereed contributions by scholars from around the globe including work by Boginski, Butenko, and Pardalos [20] for the (massive) stock market graph, focusing on the United States. The authors establish, for the first time in the field of finance, the power law model, a construct from the network science literature, introduced by physicists, which we return to later. Several papers (see [84] and [123]) focus on stochastic network approaches for financial optimization problems. Also, with the interest in modeling the behavior of various decision-makers in supply chains, the concept of *supply chain network equilibrium* was introduced by Nagurney, Dong, and Zhang [143] (cf. Figure 6 for a topological depiction) and, subsequently, its transformation into a transportation

network equilibrium problem also identified [132].

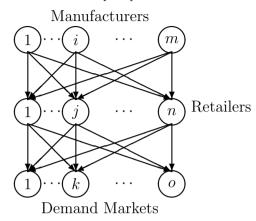


Figure 6: The Network Structure of the Supply Chain at Equilibrium

In addition, given the growing interconnections between/among different network systems because of the Internet, there were models being constructed that integrated social and financial networks [165], as well as social networks and supply chains [46]. Many of the models took a "supernetwork" perspective as promulgated by Nagurney and Dong [140]. For a supernetwork representation of the Cruz, Nagurney, and Wakolbinger [46] model, see Figure 7.

At the cusp of the new millennium, "network science" was becoming a term that was receiving growing attention with a (2006) report by the National Research Council (NRC) [174] noting that this "new" research field was focusing on an interdisciplinary perspective for complex network systems. As emphasized in the overview by Alderson [4], operations research and its fundamental and wide ranging contributions to networks were essentially ignored except for an introductory chapter in the anthology by Newman, Barabási, and Watts [176] that cited Ahuja, Magnanti, and Orlin [2] and Nagurney [126] as "exemplars." Some of the research questions of interest to physicists, sociologists, etc., that helped in promoting network science as a discipline, included: the identification of whether there was any network structure in various complex systems; were there any underlying universal laws, with power laws, based on physics (and the use of statistical mechanics), gaining specific prominence (see [3]), and how to quantify vulnerabilities and fragilities in complex networks (see [4]). Early survey papers further disseminated research results with such foci [3, 13, 14, 175, 217].

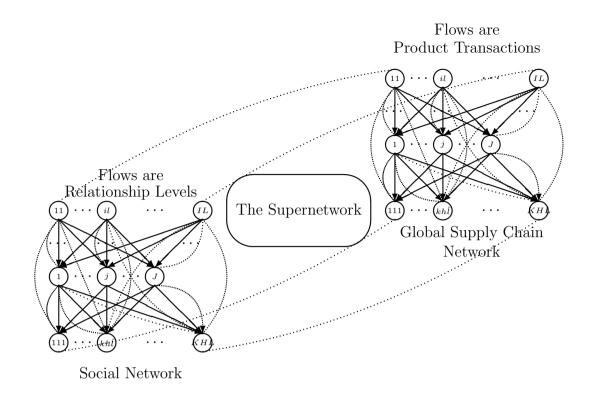


Figure 7: The Multilevel Supernetwork Structure of the Integrated Global Supply Chain Network / Social Network System

Issues of scale-free properties [13] as well as small world networks [218] were intensively studied in different systems. In addition, much of the existing work emphasized descriptive approaches to network dynamics, specifically, the dynamics related to network formation and growth, and also the dynamical behavior taking place on these networks (but not including optimizing behavior of individual decision-makers that was prominent in the OR literature with many highlights noted above).

In the new millennium, as mentioned above, there was growing interest in assessing network vulnerabilities and fragilities. This was, due, in part, to prominent disasters, from natural ones, such as hurricanes, earthquakes, and floods, to technological ones, including electric power failures, Internet disruptions, due to cyberattacks, etc. (cf. [160] and the references therein). There was also increasing interest in assessing default by firms. For example, Eisenberg and Noe [62] considered default by firms that are part of a single clearing mechanism and recognized that one of the most characteristic aspects of the present financial environment is the network of interconnections among firms. They provided conditions for the existence and uniqueness of a clearing vector for a complex financial system. Moreover, the collapse of the financial system in the United States in 2008 and 2009 had global ramifications, further generating interest in conceptualizing financial systems as networks, for purposes of identifying not only connectivity but also the propagation of potential failures and the determination of the importance of nodes and links (and their ranking). For example, the more recent theories of scale-free and small world networks in complex network research were helping in the understanding of the vulnerability of some real-world networks (see [7, 34, 91]). However, much of that literature was concerned with the topological characteristics of networks [27] even though Barabási [12] was emphasizing the need to move beyond structure and topology to include dynamics on the links.

In a series of papers, including several published in the physics literature, since it was imperative to share contributions with researchers in this discipline, given their intense interest in networks, Nagurney and Qiang [154, 155, 156, 186] proposed a network performance measure, inspired by transportation network equilibrium models, both fixed and elastic demand ones, as a basis for evaluating critical network infrastructure. The performance measure was well-defined, even in the case of disconnected networks, captured the behavior of users in decentralized networks, and also could be applied to dynamic networks [158]. As demonstrated in Qiang and Nagurney [186], the new network performance measure provided more reasonable results in terms of node and link importance identification and ranking than the measure of Latora and Marchiori [107, 108, 109, 110]; notably, in the case of the Braess paradox network it was shown that the addition of a new link results in a decrease in network performance.

In 2008 and 2009, the world reeled from the effects of the financial credit crisis, with major banks and lending institutions closing, including Lehman Brothers; others merging, and the financial services landscape forever altered [160]. Given the importance of financial networks to the global economy, researchers, especially physicists, were attracted to their study (see [26, 96, 179]), with notice that operations researchers Boginski, Butenko, and Pardalos [20] also utilized complex network constructs and analysis for the stock market graph. Complex network measures such as node degree centrality and node importance were garnering importance in the broader economics and financial literature [5, 6, 21, 93]. Although there was a literature starting on financial contagion (see, e.g., [117]) there was

only a minimal literature that considered financial network vulnerability. Nagurney and Qiang [159] proposed a financial network performance measure with critical node and link identification utilizing the financial model with electronic transactions developed by Liu and Nagurney [113], which was an extension of the earlier financial network equilibrium models of Nagurney and Ke [145, 146] published in financial journals. That work demonstrated the flexibility of the original Nagurney and Qiang [154, 157] performance measure, which had been inspired, in part, by the work in general transportation network equilibrium modeling of Dafermos [48] in *Networks*.

7. The Last Decade - From 2010

With the impact of the financial collapse on the global economy, financial networks, as a framework for modeling not only optimization problems but, notably, financial systems, were being adapted by the financial community. Much of the interest was in quantifying systemic risk. Billio et al. [19] proposed econometric measures of connectedness based on principal-components analysis and Granger-causality networks, and applied them to the monthly returns of hedge funds, banks, broker/dealers, and insurance companies. The authors found that all four sectors had become highly interrelated over the previous decade, affecting the level of systemic risk in the finance and insurance industries via a complex and time-varying network of relationships. Gai and Kapadia [76], building on the work of Allen and Gale [6], developed an analytical model of contagion in financial networks and suggested that financial systems exhibit a robust-yet-fragile tendency: while the probability of contagion may be low, the effects can be widespread when problems occur.

The following year, Nagurney [135] edited a special issue of the journal *Computational Management Science* devoted to financial networks. In the special issue, a wide range of papers, both theoretical and empirical, contributed to the fascination and relevance of this application area of networks. Topics included in the special issue, among others, were: advances in the empirical market graph for the US [200], a study of the stock market as a complex network [17]; the use of financial networks for the study of contagion (see [85, 203]), dynamic network formation game theory modeling of borrowers and sellers [67], as well as a multitiered financial network model [95], and a framework to address impacts of corporate financial networks on supply chain networks [112]. There was continuing interest

in identifying appropriate measures to assess the performance and vulnerability of financial networks, coupled with systemic risk [16]. Rogers and Veraat [189] extended the work of [62] by introducing costs of default if loans have to be called in by a failing bank, in which case, they noted that, in general, many different clearing vectors can arise. The authors then identified cases in which consortia of banks may have the means and incentives to rescue failing banks. Petrone and Latora [183], in turn, proposed a dynamic model that combines credit risk techniques with a contagion mechanism on the network of exposures among banks and illustrated how the model can be applied on the network of the European Global Systemically Important Banks.

And, in the journal *Networks*, Boyles and Waller [23] utilized ideas from portfolio optimization to study a minimum cost flow problem in which the arc costs are uncertain, and the decision maker wishes to minimize both the expected flow cost and the variance of this cost. They presented two optimality conditions, one based on cycle marginal costs, and the other – based on concepts of network equilibrium. In addition to providing algorithms, they also quantified the value of information. Also writing in *Networks*, Matsypura and Timkovsky [119] developed a heuristic network flow algorithm for an extension of the problem of margining option portfolios in practice and demonstrated a high efficiency of the proposed algorithm in a computational study.

Also, various themes, with economic underpinnings and origins in previous decades, continued to appear in *Networks*, notably, the price of anarchy and a variety of network routing games (cf. [40, 66, 98, 120, 219]), further cementing the great influence of the originators of the concepts of user-optimization and system-optimization (see also [28, 180]). Vulnerability issues in networks with financial underpinnings continued to be explored by researchers in *Networks* in this decade in a spectrum of critical node detection and related problems (see, e.g., [8, 79, 213, 215]). In addition, more complex supply chain network models were also becoming the setting for the assessment of network performance and the identification of supplier importance as well as that of the components of suppliers to firms as well as full supply chains [111].

Indeed, it was evident that, in the last decade there was sustained, continuing interest in many network systems, and their interrelationships, with the dominance of the Internet in economic transactions, communications, entertainment, and even education, making it increasingly exposed and vulnerable to cyberattacks. Nagurney [137], citing the paper of Nagurney and Aronson [138] in *Networks*, developed a network economic model for cybercrime with a focus on financial services in which the goods (such as hacked/stolen credit cards) were viewed as being perishable, due to their decrease in value over time. The model was a spatial price economic equilibrium one and added to the literature on cybercrime and cybersecurity, with a network economics focus, which also enabled insights for policymakers. This work further led to the development of game theory models for cybersecurity investment vs. competition and assessment of network vulnerability (see [161]). In fact, as noted by Clark [36], the developers of the Internet did not consider issues of cybersecurity nor any underlying economics, with new visions for an Internet, now being conceptualized to overcome some of the shortcomings [222].

8. Concluding Comments and Thoughts

This article has provided a panoramic view and discussion of the contributions of many researchers to the study of economic and financial networks in *Networks* since its establishment a half a century ago. The research has been placed in the context of the earliest relevant publications, some dating back centuries, in order to provide a proper historical scientific perspective, along with the accentuation of highlights of more recent research in other outlets. The goal was to demonstrate the broad reach of the power of networks and the impact in abstracting complex phenomena in the real world. The relevance, generality, and flexibility of network methodologies, from models to qualitative analysis, algorithms and computations, as well as applications, continue to build bridges to different scientific disciplines. New synergies are being made possible, as well as insights, through the recognition of research on economic and financial networks in *Networks* by not only operations researchers and management scientists, but also by the finance community, by economists (including regional scientists), and even by physicists, computer scientists, and, of course, engineers.

It is the expectation that the next half century will see further dramatic interest in economic and financial networks, and the science of networks, with the journal continuing to be a prominent outlet for highly original, creative research on these as well as many other related topics.

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