Multitiered Blood Supply Chain Network Competition: Linking Blood Service Organizations, Hospitals, and Payers

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Abstract

In this paper, we present a multitiered competitive supply chain network model for the blood banking industry, with a focus on the United States, that captures the economic interactions between three tiers of stakeholders; namely, the blood service organizations, the hospitals or medical centers, which transfuse blood to patients, and the payer groups that patients belong to. In addition, the supply chain framework for this life-saving product includes the competition among blood service organizations and their various supply chain activities. We model the behavior of each category of stakeholder and use the theory of variational inequalities to derive the equilibrium conditions for the entire supply chain. Illustrative examples are provided, along with qualitative properties, followed by an algorithm, accompanied by convergence results, that is used to solve simulated numerical examples. Results from these examples demonstrate that such a model can be effectively used to determine the prices and blood pathways from blood service organizations to hospitals to payers.

Keywords: game theory, blood supply chains, healthcare

1. Introduction

The blood banking industry world-wide is a capital-intensive industry in which, however, the major stakeholders such as the blood suppliers and hospitals / medical centers in many countries are nonprofit organizations. At every stage in the blood supply chain the blood suppliers (blood service organizations) incur high costs associated with the collection of whole blood, the processing of the collected blood and the segregation of the components, the testing for disease markers, storage of blood bags at the appropriate temperature, and, finally, distribution to hospitals and other medical facilities. With strict regulations enforced in the United States by the Food and Drug Administration and the introduction of new disease markers, such as the one for the Zika virus to ensure safety of the transfused blood, there is, in addition, a recognition of the importance of research and development, which also requires capital investment (Mulcahy et al. (2016)). The economic sustainability of the blood service organizations is hence critical to ensure safe and steady supply of the life-saving product, blood.

In the United States, the annual blood bank revenue is experiencing a decline, falling to 1.5 billion USD in 2014 from as high as 5 billion USD in 2008 (Wald (2014)) with the industry faced with, on the average, a decreasing demand and a rise in stiff competition (Nagurney (2017a)). This, in turn, has resulted in some significant developments in the blood banking industry such as mergers and consolidations (Toland (2014), Tracy (2015)). The growing trend of mergers is evident, for example, from the fact that the number of members in America's Blood Centers, the largest network of nonprofit community blood centers in North America, has dropped from 87 to 68 members due to 19 partnerships and mergers formed in the five years between 2010-2015 among their member blood banks (Masoumi, Yu, and Nagurney (2017)). In the United States, America's Blood Centers, which has in its network large organizations such as the New York Blood Center and OneBlood, supplies about 50 percent of the blood, while the American Red Cross supplies approximately 40 percent, with the remainder being collected by hospitals and medical centers (Nagurney (2017b)). In light of recent changes in the industry there needs to be an intensified focus on the economics of blood supply chain networks to identify alternative ways for blood banks to be cost-effective, and, hence, sustainable.

Hospitals and, similarly, medical centers, in turn, have to manage their blood inventory efficiently to minimize shortages and, at the same time, plan blood transfusions optimally to reduce overutilization or wastage of donated blood. Mergers and consolidations have been a characteristic of not only blood service organizations but also of hospitals, lately, and competition among hospitals has decreased (Kacik (2017), Gaynor, Mostashari, and

Ginsburg (2017)). Some policy makers are emphasizing the need for more competition among health care providers for better service to patients at lower prices (Hyde (2016)) and this has also been a recent topic of modeling research (see Nagurney and Li (2017)). On another spectrum, industry executives argue that they face sufficient competition from neighboring hospitals, new entrants, and alternative sources of health care. The cost of providing health care is continuing to increase in the United States and it is becoming more difficult to negotiate better reimbursement rates from hard-bargaining insurers (Dafny and Lee (2016)). Hospitals in the United States get reimbursed for their patients' procedures, including blood transfusions, by different types of payers such as private insurers and government programs such as Medicare.

The process of billing for blood products and receiving reimbursements for transfused units from different payer groups is a complicated process. It has been a topic of concern among industry professionals (America's Blood Centers (2017)) and policy makers. The 2018 proposed reimbursement rates for blood products by the Centers for Medicare and Medicaid Services showed reduction in prices for several products. These payment policies have been criticized by organizations, including the America's Blood Centers, who continue to push the Centers for Medicare and Medicaid Services for fairer payment strategies (America's Blood Centers (2017)). The Medicare reimbursement for blood in inpatient setting falls under MS-DRG (Medicare Severity Diagnosis Related Groups) which makes it difficult to separate the exact costs for the blood products (Toner et al. (2011)), while the outpatient reimbursement rates are determined using a cost-to-charge ratio methodology which uses data submitted by the hospitals and is susceptible to errors due to use of improper billing codes. For example, under the MS-DRG payment policy a hospital providing knee replacement to a patient gets reimbursed for the medical procedure as a whole and not separately for the amount of blood transfused as a part of the procedure. This policy often leads to underestimation of the costs incurred in procuring blood.

Inadequate reimbursements from the payers, in turn, affect the transactions between hospitals and blood service organizations who struggle to cover the rising cost of blood collection and testing, and, thereby, impact the economic stability of the entire blood supply chain (Mulcahy et al. (2016)). With the rise in competition among blood suppliers, the negotiation power lies with the hospitals who would agree to pay lower prices if they are unable to recover their own costs adequately. The blood banking industry, therefore, faces serious challenges due to a clear disconnect between the payments received and the actual cost of blood. However, the payment methods used by Medicare and other private insurance companies following their suit, at present, succeed in checking the overutilization of blood.

To solve the above mentioned issue Mulcahy et al. (2016) propose alternative payment methods that might be beneficial for all stakeholders in the industry. One of the alternatives is a cost-based reimbursement policy which would take into account the number of units of blood transfused and the acquisition cost of blood, in contrast to the current inpatient payment policy. The majority of blood transfusions, around 92-95 % occur in an inpatient setting (Mulcahy et al. (2016)). Hence, blood transfusions in inpatient setting account for a large portion of revenue generated from blood transfusion and needs to be studied extensively for possible policy reforms.

In this paper, we develop an equilibrium model for a multitiered competitive supply chain network for the blood banking industry, which takes into account both the logistical blood flows and the financial interactions between the different tiers of decision-maker/stakeholders. The contributions in this paper are manifold. Our model captures the decentralized nature of the blood supply chains in the United States. It is the first competitive supply chain network model in healthcare with multiple tiers, multiple paths, and multiple associated distinct types of stakeholders. The objective here is to determine the optimal flows along various paths from the blood service organizations to the hospitals, the amount of blood transfused by different hospitals to patients belonging to different payer groups to meet the demand, the price per unit that hospitals agree to pay to the different blood suppliers, and the reimbursements received by different hospitals from different payer groups. As mentioned earlier this paper not only looks at efficiently managing the product flow but also the economics of the supply chain which is critical. To the best of our knowledge, there does not exist any prior work which studies analytically the linkage between payer reimbursement and blood product pricing. We base our model to some extent on the cost-based alternative payment policy suggested in the report by Mulcahy et al. (2016) and provide a supply chain network framework that aims to solve the issue of the disconnect between costs incurred by blood service organizations, and hospitals, and the reimbursements received from payers. The results obtained from testing our model in simulated numerical examples provide significant insights to policy makers.

2. Literature Review

In this section, we highlight the existing literature on competitive supply chain networks and game theory that is relevant to the model in this paper, as well as the literature on blood supply chains, noting that, to-date, there has only been very limited research that integrates game theory and blood supply chains.

There exists a body of scientific literature that uses the concept of Nash equilibrium (cf. Nash (1950, 1951)) in decentralized supply chains, although, overall, the work is fairly recent (see, e.g., Nagurney, Dong, and Zhang (2002), Ha, Li, and Ng (2003), Bernstein and Federgruen (2005), Dong et al. (2005), Xiao and Yang (2008), Anderson and Bao (2010), Toyasaki, Daniele, and Wakolbinger (2014), and Saberi et al. (2018)), with the books by Nagurney (2006a) and Nagurney and Li (2016) providing extensive references. In particular, Nagurney, Dong, and Zhang (2002) developed an equilibrium model for a competitive supply chain network with separate tiers for multiple manufacturers, multiple retailers, and multiple demand markets. They formulated and solved the multitiered supply chain network equilibrium problem as a variational inequality problem to obtain the equilibrium product flows and prices. Dong et al. (2005) conceptualized the three tiers in their supply chain network to denote manufacturers who can use one of several shipment alternatives to send the products to the distributors who comprise the second tier, and, finally, to retailers who are faced with stochastic demand. Other papers dealing with competition among supply chain stakeholders and demand uncertainty include those by Tsiakis et al. (2001), Bernstein and Federgruen (2005), Xiao and Yang (2008), and Mahmoodi and Eshghi (2014).

Bernstein and Federgruen (2005) studied the equilibrium conditions in a two-echelon supply chain where a single supplier supplies materials to multiple competing retailers who face uncertain demand. The authors also explored the impacts of coordination between the two echelons through contracts. Mahmoodi and Eshghi (2014) considered price competition between two-tiered supply chains consisting of manufacturers and retailers. The authors proposed three different algorithms to obtain the equilibrium solutions in three possible industry structures and examined the effects of competition and demand uncertainty intensity on the solutions and supply chain profits in a numerical example. Farahani et al. (2014) provided a comprehensive literature review of competitive supply chain design models in which they classified the papers based on several major features of the models such as the number of tiers considered, the type of demand, the type of competition, etc.

While the majority of the papers discussed above deal with supply chain structures with two tiers, in reality, supply chain networks may be more complex and involve multiple network economic activities as well as several competing stakeholders. Nagurney (2010) proposed a supply chain competition model with activities such as manufacturing, storage, and distribution for profit-maximizing firms. Masoumi, Yu, and Nagurney (2012), in turn, constructed a supply chain network model for oligopolistic competition among pharmaceutical companies while taking into account the perishable nature of drugs, weheras Yu and Nagurney (2013) developed a competitive food supply chain network model, which also included perishability and price differentiation.

There also exists a wide range of literature on blood supply chains focusing on issues from donor motivation and blood collection to inventory planning policy to facility location. While some of the theoretical work on donor motivation focuses on altruism (Andreoni (1990), Evans and Ferguson (2014)), there are empirical studies (cf. Nguyen et al. (2008) and Schlumpf et al. (2008)) that address the various operational factors at the blood collection sites that affect donor retention and recruitment. Nagurney and Dutta (2018) used empirical findings to formulate a game theory model using variational inequality theory in which blood service organizations utilize their quality of service at the blood collection sites to compete with one another for donors.

The blood collection process lays the foundation to the entire supply chain and given the associated problems it has been emphasized and incorporated in several papers in recent times. In Fortsch and Perera (2018) the authors proposed a policy for donor-arrival that is validated using real data to show how it can help deal with shortages and wastages, two of the most persistent challenges in blood supply chains. In Ayer et al. (2017), the authors worked with American Red Cross to develop a mathematical model for whole blood collection for cryo production that meets the demand while reducing the total collection cost. van Brummelen, de Kort and van Dijk (2015) studied the issue of long wait times for donors. They investigated an analytic waiting time computation method and presented a computation algorithm to approximate the total delay time approximation.

Since blood products are perishable and have very short shelf lives, with Red Blood Cells (RBCs) lasting up to 42 days, and platelets only 5 days, there has been emphasis on this aspect and, in turn, on inventory planning for blood products both at the blood bank level and at the hospital level (cf. Nahmias (1982) and Pierskalla (2004)). Sarhangian et al. (2017) studied the performance of threshold-based allocation policies for optimizing blood inventory taking into consideration the trade-off between age of the blood and availability. Although it has been controversial, extant literature shows no evidence to support that fresher blood has better clinical outcomes (Alexander et al. (2016)). Some of the other recent contributions in this area include those of Duan and Liao (2014), Gunpinar and Centeno (2015), and Dillon, Oliveira, and Abbasi (2017), Puranam et al. (2017). In addition to optimizing the inventory of RBCs, Duan and Liao (2014) tackle the issue of blood group compatibility and substitution, while Dillon, Oliveira, and Abbasi (2017) use stochastic programming to deal with the uncertainty in the demand for blood. Other recent papers that incorporate the stochastic nature of demand for blood products in their models include those by Nagurney, Masoumi, and Yu (2012), Fortsch and Khapalova (2016), Zahiri and Pishvaee (2017), and Najafi, Ahmadi and Zolfagharinia (2017), Ramezanian and Behboodi (2017). While Fortsch and Khapalova (2016) tested various forecasting techniques to better predict the demand for blood at the blood centers to reduce the uncertainty regarding the demand for blood, Najafi, Ahmadi and Zolfagharinia (2017) developed a bi-objective integer programming model for blood inventory management that provides solutions for handling issues of shortage and wastage by allowing transshipment between hospitals.

The various inherent challenges in the management of blood supply chains such as shortages, uncertainty in demand, dependence on voluntary donors, and perishability, along with emerging ones, such as competition, provide numerous opportunities and scope for research. A literature search, nevertheless, reveals that, in many cases, these issues are addressed in isolation. It is important to study the blood supply chain as a whole to understand and model the interplay between the different stakeholders. Among the integrated supply chain models reported in the literature review paper by Osorio, Brailsford, and Smith (2015), the most relevant to our work is that of Nagurney, Masoumi, and Yu (2012), where the authors developed a multicriteria optimization model taking into account all the major supply chain activities between a regional blood bank and its demand markets. Inspired by this work and the current competitive environment in the blood banking industry, Nagurney and Dutta (2017) developed an integrated network model to capture the competition among blood suppliers for donations as well as supply contracts with hospitals. Heidari-Fathian and Pasandideh (2017), in turn, developed a three-echelon optimization model including collection sites, donors, and demand points. They, further, differentiate between mobile and demountabl collection facilities in developing their location allocation and inventory planning model.

A major gap found in the extant literature is that the economic transactions included in the models are limited to cost of activities such as collection, testing, and inventory holding, and do not include the reimbursements or payments that the hospitals and blood service organizations receive to cover these costs. While the minimization of cost is an important aspect in blood supply chain optimization, there is a need for studies exploring the "two key economic relationships at the core of the U.S blood system: the relationship between hospitals and blood suppliers and the relationship between hospitals and health care payers" (Mulcahy et al. (2016)), specifically, in the context of competition.

In this paper, we have tried to bridge the gaps in extant literature on both blood supply chains and competitive supply chain networks with our multitiered model. We construct an integrated supply chain network model that includes all major stakeholders in the blood supply chain; namely, the blood banks or blood service organizations, the hospitals, and the patient payer groups. It is important to mention that our focus is on the blood industry in the United States and other countries may have more centralized blood systems, such as the United Kingdom, for example. However, the issue of efficient and effective management of blood is an universal issue. For an interesting article on the results of a survey of blood supply chain management in the Council of Europe (CoE) countries, and CoE observers (Australia, Canada, New Zealand, and the United States), see Follea (2013).

The rest of the paper is organized as follows. In Section 3, we present the multitiered blood supply chain network competition model, consisting of blood service organizations, hospitals, and the payers. The behavior of each class of decision-makers is described, and a unified variational inequality derived, whose solution yields equilibrium blood logistical flows and prices. Illustrative examples are presented for clarification and exposition purposes. We present numerical examples in Section 4, and summarize our results and present our conclusions in Section 5.

3. The Multitiered Blood Supply Chain Network Competition Model

The blood supply chain network (cf. Figure 1) consists of I competing blood service organizations (BSOs), with a typical BSO denoted by i, H_{n_H} hospitals, with a typical hospital denoted by j, and T_{n_T} payers, with a typical payer denoted by k. The BSOs are depicted by the top-most nodes in Figure 1 and the payers by the bottom nodes. Examples of patient payer types in the United States include: Medicare, Medicaid, other public health insurance programs, private health insurance such as UnitedHealthcare, the uninsured, etc. Each blood service organization i collects blood from n^i_C collection sites that include fixed and mobile sites. Once blood is collected by BSO i; $i = 1, \ldots, I$, it is sent to n^i_P component laboratories for testing and processing where whole blood is separated into components such as Red Blood Cells (RBCs), platelets, and plasma. In our model, we consider only RBCs since these are the most common blood products used for transfusion in surgeries, and, henceforth, whenever we use the term "blood" we imply RBCs.

Blood is shipped from the component laboratories of each BSO i to n^i_S storage facilities that constitute the fifth tier of the supply chain network. The next level of nodes represents the n^i_D; $i = 1, \ldots, I$, distribution centers. At times, the component laboratories, storage facilities, and distribution centers are not separate physical entities but exist within the blood centers. At the seventh tier blood reaches hospitals from multiple suppliers with whom they have contracts (Merola (2017)). The hospitals also compete with one another for patients. Each set of links between a pair of nodes denotes an activity along the supply chain such as, for example, the collection of whole blood from donors, shipment, testing and processing, storage, distribution, and, finally, transfusion. We now focus on the behavior

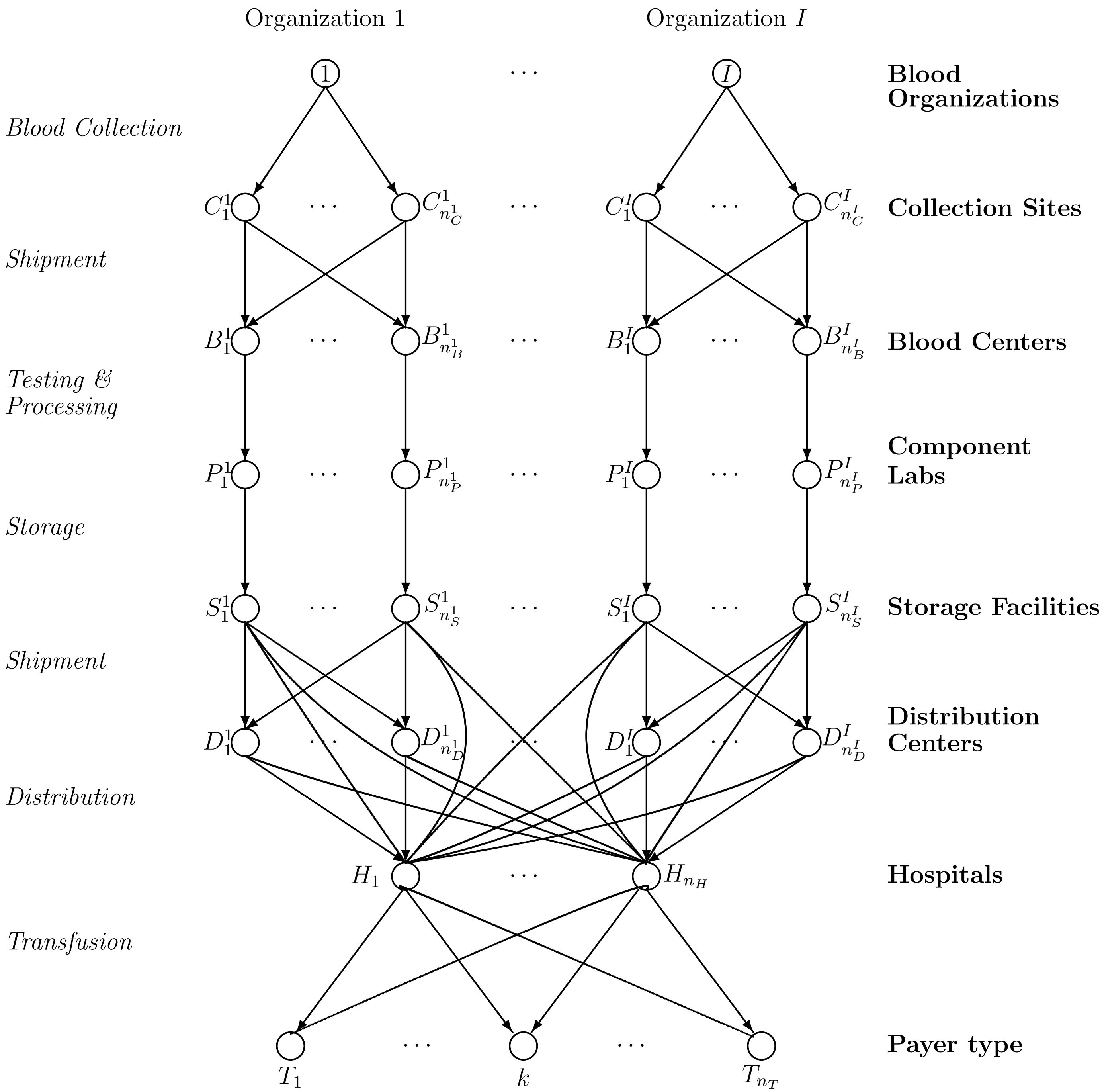

Figure 1: The Multitiered Blood Supply Chain Network Topology

of the decision-makers in the blood supply chain and their interplay. First, we discuss the behavior of the blood service organizations and then turn to the hospitals and, finally, the payers.

3.1 Behavior of the Blood Service Organizations and Their Optimality Conditions

A path is a sequence of links, which are directed, and originates at a top origin node representing a blood service organization and ending at a hospital node. N and L are defined as the sets of nodes and links, respectively, up to the seventh tier representing the hospitals with L^i denoting the set of links in BSO i's supply chain for $i = 1, \ldots, I$. Associated with each link a, $\forall a \in L$, is a total cost function $\hat{c}_a$ representing the cost for the activity.

It is to be noted that the amount of blood collected is never equal to the amount of blood supplied to the hospitals. After collection at every stage of the supply chain, such as testing, processing, storage, a fraction of blood is wasted due to errors such as misfilled bags, incorrect labeling, etc., in addition to outdates due to the perishable nature of blood. To capture this loss along the supply chain, we utilize a generalized network approach with appropriate arc and path multipliers (see also, e.g., Masoumi, Yu, and Nagurney (2012) and Nagurney et al. (2013)) as defined in Table 1.

Table 1: Multiplier Notation for Blood Loss

Notation	**Definition**
α_a	The arc multiplier associated with link a, which represents the percentage of throughput on link a. $\alpha_a \in (0, 1]$; $a \in L$.
α_{ap}	The arc-path multiplier, which is the product of the multipliers of the links on path p that precede link a; $a \in L$ and $p \in P$; that is, $$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \emptyset, \end{cases}$$ where $\{a' < a\}_p$ denotes the set of the links preceding link a in path p and $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.
μ_p	The multiplier corresponding to the percentage of throughput on path p; that is, $\mu_p \equiv \prod_{a \in p} \alpha_a$; $p \in P$, where P is the set of all paths connecting the BSOs with the hospitals.

Let x_p denote the nonnegative flow of blood on a path p sent from a BSO to a hospital. Let the contracted amount of blood supplied by BSO i to hospital j be denoted by q_{ij}; $i = \ldots, I; j = H_1, \ldots, H_{n_H}$. We consider this to be the projected demand for a week. Let P^i_j denote the set of all paths joining BSO i with hospital j.

The conservation of flow equation that has to hold for each BSO i; $i = 1, \ldots, I$, at hospital

j; $j = H_1, \ldots, H_{n_H}$, is:

$$\sum_{p \in P_j^i} x_p \mu_p = q_{ij}, \tag{1}$$

that is, the sum of all the actual, after loss is factored in, path flows from a particular BSO to a particular hospital should be equal to their contracted supply amount. The total amount of blood supplied by a blood service organization i, q_i, can be written as $\sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} x_p \mu_p = q_i$.

Since the path flows must be nonnegative, we have that:

$$x_p \geq 0, \quad \forall p \in P. \tag{2}$$

Let f_a denote the flow of blood on link a. Then, the following conservation of flow equations must hold:

$$f_a = \sum_{p \in P} x_p \alpha_{ap}, \quad \forall a \in L. \tag{3}$$

According to equation (3), the initial blood product flow on link a is the sum of the product flows along paths that contain that link, taking into account possible losses in the preceding activities. We group all the flows corresponding to links in L into the vector $f \in R^{n_L}$ where n_L is the total number of elements in L. The total link cost on a link a is assumed to be, in general, a function of all the flows in the network. Therefore, we have that

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \tag{4}$$

The total cost on each link is assumed to be convex and continuously differentiable. The total cost incurred by a blood service organization will be the sum of all the total costs on links operated by the blood service organization. The price per unit charged by BSO i to hospital j is denoted by ρ_{ij}^{1*}. We discuss how the equilibrium prices are recovered, once the model is solved, later in this section. The revenue generated by each BSO is the product of the unit price and the amount of blood supplied.

As noted earlier, blood service organizations in the United States are predominantly nonprofits. Therefore, there is a utility associated with the service that they provide (cf. Nagurney, Alvarez Flores, and Soylu (2016) and Nagurney and Li (2017)). Let γ_{ij} correspond to a measurement of the satisfaction that blood service organization i derives from supplying blood to hospital j. The overall such "service" utility of blood service organization i associated with all the demand points is then given by $\sum_{j=H_1}^{H_{n_H}} \gamma_{ij} q_{ij}$. This service utility also represents altruism (cf. Nagurney, Alvarez Flores, and Soylu (2016)). In addition, each blood service organization i associates a weight ω_i with its service utility, which monetizes it. According to the function $\omega_i \sum_{j=H_1}^{H_{n_H}} \gamma_{ij} q_{ij}$, the greater the amount made available, the more patients that can benefit and, therefore, the greater the good that can be accomplished.

The utility function of blood service organization i; $i = 1, \ldots, I$, denoted by U_i, can be expressed as:

$$U_i = \sum_{j=H_1}^{H_{n_H}} \rho_{ij}^{1*} q_{ij} + \omega_i \sum_{j=H_1}^{H_{n_H}} \gamma_{ij} q_{ij} - \sum_{a \in L^i} \hat{c}_a(f), \tag{5a}$$

or, equivalently, in terms of path flows, through the use of equations (1) and (3).

$$\hat{U}_i(x) = \sum_{j=H_1}^{H_{n_H}} \rho_{ij}^{1*} \sum_{p \in P_j^i} x_p \mu_p + \omega_i \sum_{j=H_1}^{H_{n_H}} \gamma_{ij} \sum_{p \in P_j^i} x_p \mu_p - \sum_{a \in L^i} \tilde{\hat{c}}_a(x), \tag{5b}$$

with $\tilde{\hat{c}}_a(x) \equiv \hat{c}_a(f)$, $\forall a \in L$.

It is to be noted that the utility of each blood service organization is over a time horizon of a week.

The blood service organizations seek to maximize their utility, while competing for the quantity of blood supplied and also, given the generality of the total cost functions (4), through the other various activities, since they may, for example, compete for qualified staff, etc. Hence, each BSO has as its strategic variables, its path flows, with X_i denoting the vector of path flows corresponding to blood service organization i; $i = 1, \ldots, I$:

$$X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}, \tag{6}$$

where P^i denotes the set of all paths associated with BSO i and n_{P^i} denotes the number of paths from BSO i to the hospitals. X is the vector of all path flows, that is, $X \equiv \{\{X_i\} | i = 1, \ldots, I\}$. Further, we define the feasible set for blood service organization i as $K_i \equiv \{X_i | x_i \in R_+^{n_{P^i}}\}$. All vectors are column vectors.

The blood service organizations compete noncooperatively in an oligopolistic market framework in which each blood service organization selects its own optimal blood product flows to maximize its utility, given the optimal ones of its competitors. The governing equilibrium concept underlying the behavior of the blood service organizations is, therefore, that of Nash (1950, 1951) equilibrium. The optimality conditions for all the blood service organizations simultaneously can be expressed as the following variational inequality (cf. Gabay and Moulin (1980), Nagurney (1999)): determine $x^* \in K^1, K^1 \equiv \prod_{i=1}^{I} K_i$, such

that:
$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P^i_j}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}-\omega_i\gamma_{ij}\mu_p-\rho^{1*}_{ij}\mu_p\right]\times[x_p-x^*_p]\geq 0,\quad \forall x\in K^1, \tag{7}$$

where $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ for paths $p\in P^i_j$ is given by

$$\frac{\partial \hat{C}_p(x)}{\partial x_p}\equiv\sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\alpha_{ap}. \tag{8}$$

The optimality conditions as expressed by (7) provide a nice economic interpretation in that a blood service organization will supply blood to a hospital by a path p (flow on the path will be positive) if the "marginal total cost" on the path is exactly equal to the marginal utility associated with the weighted altruism of the pair (i,j) plus the marginal revenue associated with the path p, with yield loss accounted for.

3.2 Behavior of the Hospitals and Their Optimality Conditions

We now discuss the competition among the hospitals. Hospitals are the stakeholders in the blood supply chain network who are involved in transactions with both blood suppliers, the BSOs, and the patient payer groups.

Each hospital j decides to transfuse an amount q_{jk} of RBCs to patient group k. The total amount of blood transfused by hospital j; $j=H_1,\ldots,H_{n_H}$, cannot exceed the total amount it receives from its contracted suppliers. Therefore, the following condition must be satisfied

$$\sum_{k=T_1}^{T_{n_T}}q_{jk}\leq\sum_{i=1}^{I}q_{ij}, \tag{9a}$$

or, equivalently,

$$\sum_{k=T_1}^{T_{n_T}}q_{jk}\leq\sum_{i=1}^{I}\sum_{p\in P^i_j}x_p\mu_p. \tag{9b}$$

We denote the price charged by hospital j; $j=H_1,\ldots,H_{n_H}$, per unit of RBCs transfused as ρ^{2*}_j. Similar to blood service organizations, many hospitals are nonprofits and, hence, will have a weighted altruism factor in their utility function which is given as $\beta_j\sum_{k=T_1}^{T_{n_T}}\theta_{jk}q_{jk}$. In the case of a for profit hospital the weight β_j will simply be zero.

In addition to the cost of acquiring blood from the suppliers, hospitals incur a holding cost for maintaining a proper inventory of blood. This cost is denoted by h_j for hospital j;

$j = H_1, \ldots, H_{n_H}$, and is a function of $\sum_{k=T_1}^{T_{n_T}} q_{jk}$, the total amount of blood transfused at hospital j.

The optimization problem for hospital j; $j = H_1, \ldots, H_{n_H}$, then becomes

$$\text{Maximize} \quad \rho_j^{2*} \sum_{k=T_1}^{T_{n_T}} q_{jk} + \beta_j \sum_{k=T_1}^{T_{n_T}} \theta_{jk} q_{jk} - h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}) - \sum_{i=1}^{I} \rho_{ij}^{1*} \sum_{p \in P_j^i} x_p \mu_p, \tag{10}$$

subject to constraint (9b) and the nonnegativity constraints: $x_p \geq 0$, $\forall p \in P_j$, where P_j is the set of all paths terminating in j, and $q_{jk} \geq 0$ for all j and k.

We now obtain the optimality conditions of the hospitals, assuming that each hospital is faced with the above optimization problem, and that the hospitals compete in a noncooperative manner to maximize their utilities, given the actions of the other hospitals. It is to be noted that the hospitals seek to determine the optimal quantities to be supplied to the patient groups as well as the amount to be received from different suppliers. Assuming that the holding cost for each hospital is convex and continuous, the optimality conditions for all hospitals, simultaneously, coincide with the solution of the variational inequality: determine $(x^*, q^*, \eta^*) \in K^2$ satisfying

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\rho_{ij}^{1*} \mu_p - \eta_j^* \mu_p\right] \times [x_p - x_p^*] + \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[-\rho_j^{2*} - \beta_j \theta_{jk} + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^*)}{\partial q_{jk}} + \eta_j^*\right] \times [q_{jk} - q_{jk}^*]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \left[\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^* \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^*\right] \times [\eta_j - \eta_j^*] \geq 0, \quad \forall (x, q, \eta) \in K^2, \tag{11}$$

with feasible set K^2 defined as:

$$K^2 \equiv \{(x, q, \eta) | x \in R_+^{n_P}, q \in R_+^{n_H n_T}, \eta \in R_+^{n_H}\}. \tag{12}$$

Here η_j is the Lagrange multiplier associated with constraint (9) for hospital j, η is the n_H-dimensional vector of all the multipliers, and q denotes the $n_H n_T$-dimensional vector of blood flows between the hospitals and patient groups. For further background on such a derivation, see Nagurney, Dong, and Zhang (2002) and the references therein. Similar to (7), in the derivation of the variational inequality (11), we do not have the prices charged as variables. They become endogenous variables in the complete equilibrium model.

We now discuss the economic interpretation of the hospitals' optimality conditions and the justification of the h_j functions. From the first term in (11) we can infer that if there is a positive flow of RBCs between a blood service organization and a hospital, then η_j^* is precisely equal to the hospital's payment to the supplier, ρ_{ij}^{1*}. From the second term of (11) we see that if q_{jk}^* is positive, that is, if patients from payer group k get transfusions from hospital j, then the unit price charged by hospital j, ρ_j^{2*}, plus its marginal service utility, $\beta_j\theta_{jk}$, is exactly equal to its marginal cost of holding inventory plus its unit cost of procuring blood (since $\eta_j^* = \rho_{ij}^{1*}$). Further, from the third term in (11) we can infer that if η_j^* is positive, then the amount of blood received by hospital j is exactly equal to the amount of blood transfused at hospital j. Hence, we can say that the inventory holding cost of a hospital is a function of the total amount of blood transfused at a hospital.

3.3 Behavior of the Payer Groups and Equilibrium Conditions

As mentioned earlier, the payers are conceptualized as patients belonging to different payer groups. Since most of the surgeries requiring blood transfusion such as knee replacements, cardiovascular surgery, organ transplants, etc., are planned ahead of time, we assume that the demand for blood at each payer node depends on the reimbursement that the payers (insurers) are willing to give. The type of insurance is also known before blood transfusion takes place. Since for a patient getting treatment at a hospital, the blood required for transfusion will be provided by that particular hospital, we expect that the demand at each hospital from each payer group may be different. Demand at patient payer group k for transfusions at hospital j is denoted by d_{jk}. The amount that payer type k is willing to reimburse to hospital j is given as ρ_{jk}^3. Thus, we have that

$$d_{jk} = d_{jk}(\rho^3), \quad \forall j, \forall k, \tag{13}$$

where ρ^3 is the $n_H n_T$-dimensional vector of payer prices. The demand is assumed to be monotonically decreasing in the reimbursement for the hospital, but increasing in the reimbursements for other hospitals. However, it is to be noted that if the demand is quite price inelastic then the coefficients should be set accordingly.

In reality, healthcare payments received by different hospitals from the same payer might vary significantly (Luhby (2013)). The reimbursement or payer price can depend on several factors such as the payer mix, whether it is a teaching or non-teaching hospital, the hospital's location, its healthcare network, etc. For example, Medicare pays a higher rate to teaching hospitals while private insurance companies negotiate better rates for hospitals in their network.

The payers take into account not only the price charged by the hospitals in determining which hospital to choose, but also the transaction cost. Let c_{jk} denote the transaction cost between hospital j and payer group k. We assume that the transaction cost is continuous, positive, and of the general form

$$c_{jk} = c_{jk}(q), \quad \forall j, \forall k. \tag{14}$$

Following Nagurney, Dong, and Zhang (2002) (see also Nagurney (2006b)), we have that the equilibrium conditions are: For all hospitals j; $j = H_1, \ldots, H_{n_H}$, and payers $k = T_1, \ldots, T_{n_T}$:

$$\rho_j^{2*} + c_{jk}(q^*) \begin{cases} = \rho_{jk}^{3*} & \text{if} \quad q_{jk}^* > 0, \\ \geq \rho_{jk}^{3*} & \text{if} \quad q_{jk}^* = 0. \end{cases} \tag{15}$$

and

$$d_{jk}(\rho^{3*}) \begin{cases} = q_{jk}^* & \text{if} \quad \rho_{jk}^{3*} > 0, \\ \leq q_{jk}^* & \text{if} \quad \rho_{jk}^{3*} = 0. \end{cases} \tag{16}$$

Conditions (15) imply that, in equilibrium, if q_{jk}^* is positive, that is, there are patients at the hospital k that get blood transfusions from hospital j, then the price charged by the hospital plus the transaction cost does not exceed the price that the payer is willing to reimburse. Conditions (16) state that, if the equilibrium price that a particular payer group is willing to pay for the blood product from a particular hospital is positive, then the quantity of blood obtained from a hospital is precisely equal to the demand of blood for that payer group. These conditions correspond to the well-known spatial price equilibrium conditions but applied to an entirely novel context of multitiered blood supply chain networks (cf. Takayama and Judge (1971), Nagurney (1999), and the references therein).

In equilibrium, conditions (15) and (16) will have to hold for all k, and can, in turn, be expressed as the variational inequality problem (see, e.g., Nagurney (1999)): determine $(q^*, \rho^{3*}) \in K^3$, such that

$$\sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [\rho_j^{2*} + c_{jk}(q^*) - \rho_{jk}^{3*}] \times [q_{jk} - q_{jk}^*] + \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [q_{jk}^* - d_{jk}(\rho^{3*})] \times [\rho_{jk}^3 - \rho_{jk}^{3*}] \geq 0,$$

$$\forall (q, \rho^3) \in K^3, \tag{17}$$

where the feasible set $K^3 \equiv \{(q, \rho^3) \in R_+^{2n_H n_T}\}$.

3.4 The Equilibrium Conditions of the Blood Supply Chain

In equilibrium, the amount of blood supplied by the blood service organizations must be equal to the amount of blood received by the hospitals. In addition, the amount of blood transfused by the hospitals must be equal to the amount needed by the patients. Furthermore, the equilibrium quantities and price pattern in the blood supply chain must satisfy the sum of the inequalities (7), (11), and (17), to formalize the agreements between the tiers. Hence, although there is competition across a tier of decision-makers, whether BSOs or hospitals, there is cooperation between tiers and the prices assist in this. We now state this explicitly in the following definition.

Definition 1: Multitiered Blood Supply Chain Network Equilibrium

The equilibrium state of the supply chain is one where the blood product (RBC) flows between the three distinct tiers of decision makers coincide and the blood flows and prices satisfy the sum of the optimality conditions (7), (11), and (17).

We now establish the following:

Theorem 1: Variational Inequality Formulation of the Multitiered Blood Supply Chain Network Equilibrium

The equilibrium conditions governing the multitiered blood supply chain network are equivalent to the solution of the variational inequality problem given by: determine $(x^*, q^*, \eta^*, \rho^{3*}) \in K^4$ *satisfying:*

$$\sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \eta_j^* \mu_p \right] \times [x_p - x_p^*]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [c_{jk}(q^*) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^*)}{\partial q_{jk}} + \eta_j^* - \beta_j \theta_{jk} - \rho_{jk}^{3*}] \times [q_{jk} - q_{jk}^*]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \left[\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^* \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^* \right] \times [\eta_j - \eta_j^*] + \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [q_{jk}^* - d_{jk}(\rho^{3*})] \times [\rho_{jk}^3 - \rho_{jk}^{3*}] \geq 0,$$

$$\forall (x, q, \eta, \rho^3) \in K^4, \tag{18}$$

where $K^4 \equiv \{(x, q, \eta, \rho^3) \in R_+^{n_P + 2n_H n_T + n_H}\}$.

Proof: We first establish necessity, that the equilibrium conditions imply variational inequality (18). Observe that, indeed, the summation of (7), (11), and (17), yields variational inequality (18), after algebraic simplification.

For sufficiency we now establish the converse, that is, that a solution to variational inequality (18) satisfies the sum of inequalities (7), (11), and (17), and is, therefore, an equilibrium according to Definition 1. To inequality (18) we add the term $-\rho_{ij}^{1*} \mu_p + \rho_{ij}^{1*} \mu_p$ to the term in the first set of brackets preceding the multiplication sign and we add the term $-\rho_j^{2*} + \rho_j^{2*}$ to the term in brackets preceding the second multiplication sign. Such "terms" do not change the value of the inequality since their value is equal to zero, with the resulting inequality of the form

$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P_j^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}-\omega_i\gamma_{ij}\mu_p-\eta_j^*\mu_p-\rho_{ij}^{1*}\mu_p+\rho_{ij}^{1*}\mu_p\right]\times[x_p-x_p^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[c_{jk}(q^*)+\frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^*)}{\partial q_{jk}}+\eta_j^*-\beta_j\theta_{jk}-\rho_{jk}^{3*}-\rho_j^{2*}+\rho_j^{2*}]\times[q_{jk}-q_{jk}^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\left[\sum_{i=1}^{I}\sum_{p\in P_j^i}x_p^*\mu_p-\sum_{k=T_1}^{T_{n_T}}q_{jk}^*\right]\times[\eta_j-\eta_j^*]+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[q_{jk}^*-d_{jk}(\rho^{3*})]\times[\rho_{jk}^3-\rho_{jk}^{3*}]\geq 0,$$

$$\forall (x,q,\eta,\rho^3)\in K^4, \tag{19}$$

which, in turn, can be rewritten as

$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P_j^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}-\omega_i\gamma_{ij}\mu_p-\rho_{ij}^{1*}\mu_p\right]\times[x_p-x_p^*]+\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\left[\rho_{ij}^{1*}\mu_p-\eta_j^*\mu_p\right]\times[x_p-x_p^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}\left[-\rho_j^{2*}-\beta_j\theta_{jk}+\frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^*)}{\partial q_{jk}}+\eta_j^*\right]\times[q_{jk}-q_{jk}^*]+\sum_{j=H_1}^{H_{n_H}}\left[\sum_{i=1}^{I}\sum_{p\in P_j^i}x_p^*\mu_p-\sum_{k=T_1}^{T_{n_T}}q_{jk}^*\right]\times[\eta_j-\eta_j^*]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[\rho_j^{2*}+c_{jk}(q^*)-\rho_{jk}^{3*}]\times[q_{jk}-q_{jk}^*]+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}[q_{jk}^*-d_{jk}(\rho^{3*})]\times[\rho_{jk}^3-\rho_{jk}^{3*}]\geq 0,$$

$$\forall (x,q,\eta,\rho^3)\in K^4. \tag{20}$$

But inequality (20) is equivalent to the price and product flow pattern satisfying the sum of (7), (11), and (17). The proof is complete. □

We can rewrite the variational inequality (18) in standard variational inequality form (see Nagurney (1999)), that is: determine $Y^*\in\mathcal{K}\subset R^N$, such that

$$\langle F(Y^*),Y-Y^*\rangle\geq 0,\quad \forall Y\in\mathcal{K}, \tag{21}$$

where $Y\equiv(x,q,\eta,\rho^3)$, $F(Y)\equiv(F_p,F_{jk}^1,F_j,F_{jk}^2)_{p\in P_j^i}; i=1,\ldots,I; j=H_1,\ldots,H_{n_H}; k=T_1,\ldots,T_{n_T}$, and the specific components of F are given by the functional terms preceding the multiplication signs in (18), and $\mathcal{K}\equiv K^4$. The term $\langle\cdot,\cdot\rangle$ denotes the inner product in N-dimensional Euclidean space, where N here is $n_P+2n_Hn_T+n_H$.

The variables in the variational inequality problem are: the blood product (RBC) flows from the blood service organizations to the hospitals, x, the quantities of blood transfused by the hospitals to the patient groups, q, the prices associated with transfusing and storing blood by the hospitals, η, and the demand market prices or reimbursement rates, ρ^3.

We now discuss how to recover the blood service organizations' equilibrium prices, ρ^{1*}_{ij}, for all i,j, and the hospitals' equilibrium prices, ρ^{2*}_j, for all j, from the solution of the variational inequality (18). In the previous discussion in Section 3.2 we mention that if there is positive flow of blood products between a blood service organization and hospital, then η^*_j is precisely equal to the hospital's payment to the supplier, ρ^{1*}_{ij}. η^*_j is obtained from the solution of the inequality (18). On the other hand, prices charged by the hospitals, ρ^{2*}_js, can be obtained by finding a $q^*_{jk} > 0$, and then from (15) setting

$$\rho^{2*}_j = \rho^{3*}_{jk} - c_{jk}(q^*),$$

where ρ^{3*}_{jk} is obtained from the solution of variational inequality (18).

We now establish the result that, in equilibrium, the sum of the amounts of blood supplied to each hospital is equal to the sum of the amounts of blood transfused at that hospital. This implies that each hospital, assuming utility maximization, purchases from the blood service organizations equal the amount of blood that is actually transfused to the patients. We make use of variational inequality (18) to establish the above-mentioned result. From the third term in (18) we can see that if $\eta^*_j > 0$, then we have $\sum_{i=1}^{I}\sum_{p\in P^i_j} x^*_p\mu_p = \sum_{k=T_1}^{T_{n_T}} q^*_{jk}$. In other words, the "market clears" for hospital j. Let us now consider the case where $\eta^*_j = 0$. We see from (7) that, if $x^*_p > 0$, then we have that

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i\gamma_{ij}\mu_p + \rho^{1*}_{ij}\mu_p,$$

and, if $x^*_p = 0$, then we have that

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} \geq \omega_i\gamma_{ij}\mu_p + \rho^{1*}_{ij}\mu_p.$$

Hence, from the first term in inequality (18), we can say that, if $\eta^*_j = 0$, then

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i\gamma_{ij}\mu_p \geq 0,$$

which implies that $x^*_p = 0, p \in P^i_j, \quad \forall i,j$. It follows then from the third term of (18) that $\sum_{k=T_1}^{T_{n_T}} q^*_{jk} = 0$, and, hence, the market clears also in this case since the flow into a hospital is equal to the flow out and equal to zero. We have, thus, established the following:

Corollary 1

The market for the blood product clears for each hospital at the multitiered blood supply chain network equilibrium.

3.5. Illustrative Examples

In this section we present two examples to illustrate some of the above mentioned concepts. The blood supply chain topology is depicted in Figure 2.

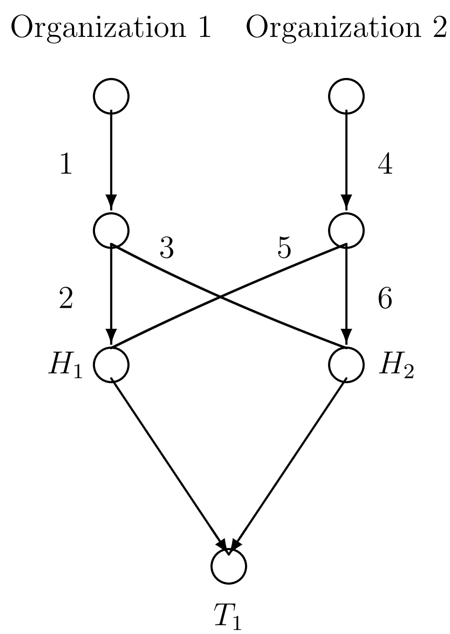

Figure 2: The Blood Supply Chain Network Topology for the Illustrative Examples

Example 1

There are two blood service organizations supplying blood to two hospitals. The hospitals in turn treat patients who belong to the group Payer type 1. From each BSO there are two paths reaching each hospital. For simplicity, each path consists of two links. The paths are defined as follows: $p_1 = (1, 2)$, $p_2 = (1, 3)$, $p_3 = (4, 5)$, and $p_4 = (4, 6)$. The time horizon is assumed to be a week.

The total link cost functions are:

$$\hat{c}_1(f_1) = f_1^2 + 1.5f_1, \quad \hat{c}_2(f_2) = f_2^2 + 2f_2, \quad \hat{c}_3(f_3) = f_3^2 + 2.5f_3,$$

$$\hat{c}_4(f_4) = f_4^2 + 2f_4, \quad \hat{c}_5(f_5) = f_5^2 + 2f_5, \quad \hat{c}_6(f_6) = f_6^2 + 2.5f_6.$$

In this example we assume that no amount of blood is lost in the supply chain; hence, the arc-path multipliers, μ_ps are all equal to 1, $\forall p$. Again, for ease of calculation, the parameters associated with the altruism components of the utility functions are all zero, i.e, $\omega_1 = \omega_2 = \beta_{H_1} = \beta_{H_2} = 0$.

The holding cost functions for the two hospitals are:

$$h_{H_1}(q_{H_1T_1}) = 1.5 \times q_{H_1T_1}, \quad h_{H_2}(q_{H_2T_1}) = 1.5 \times q_{H_2T_1}.$$

The transaction cost functions between the hospitals and payers are:

$$c_{H_1T_1}(q_{H_1T_1}) = q_{H_1T_1} + 100, \quad c_{H_2T_1}(q_{H_2T_1}) = q_{H_2T_1} + 100.$$

The demand price functions are:

$$d_{H_1T_1} = -0.005\rho^3_{H_1T_1} + 0.002\rho^3_{H_2T_1} + 100, \quad d_{H_2T_1} = -0.005\rho^3_{H_2T_1} + 0.002\rho^3_{H_1T_1} + 100.$$

Using inequality (18) we obtain ten linear equations as follows:

$$4x^*_{p_1} + 2x^*_{p_2} + 3.5 - \eta^*_{H_1} = 0,$$

$$4x^*_{p_2} + 2x^*_{p_1} + 4 - \eta^*_{H_2} = 0,$$

$$4x^*_{p_3} + 2x^*_{p_4} + 4 - \eta^*_{H_1} = 0,$$

$$4x^*_{p_4} + 2x^*_{p_3} + 4.5 - \eta^*_{H_2} = 0,$$

$$q^*_{H_1T_1} + \eta^*_{H_1} - \rho^{3*}_{H_1T_1} + 101.5 = 0,$$

$$q^*_{H_2T_1} + \eta^*_{H_2} - \rho^{3*}_{H_2T_1} + 101.5 = 0,$$

$$x^*_{p_1} + x^*_{p_3} - q^*_{H_1T_1} = 0,$$

$$x^*_{p_2} + x^*_{p_4} - q^*_{H_2T_1} = 0,$$

$$q^*_{H_1T_1} + 0.005\rho^{3*}_{H_1T_1} - 0.002\rho^{3*}_{H_2T_1} - 100 = 0,$$

$$q^*_{H_2T_1} + 0.005\rho^{3*}_{H_2T_1} - 0.002\rho^{3*}_{H_1T_1} - 100 = 0.$$

The equilibrium blood path flows from the blood service organizations to the hospitals obtained by solving the above equations are: $x^*_{p_1} = x^*_{p_2} = 49.29, x^*_{p_3} = x^*_{p_4} = 49.21$.

In the absence of the altruism factors, we have that the η^*_js are precisely equal to the prices charged by the blood service organizations, ρ^{1*}_{ij}s. The equilibrium prices charged by the BSOs are: $\eta^*_{H_1} = 299.25, \eta^*_{H_2} = 299.75$, respectively. This means that Hospital 1 agrees to pay \$299.25 per unit of blood and Hospital 2 agrees to pay \$299.75 per unit of blood. The price per unit charged by the hospitals are: $\rho^{2*}_{H_1} = 300.75$ and $\rho^{2*}_{H_2} = 301.25$. This makes sense and is fair since Hospital 2 pays a higher price for acquiring the blood, the price charged is also slightly higher than that of Hospital 1.

At equilibrium, the quantities of blood transfused at each hospital are: $q^*_{H_1T_1} = q^*_{H_2T_1} = 98.50$. Lastly, the reimbursements that Payer type 1 is willing to pay to Hospital 1 and Hospital 2 are: $\rho^{3*}_{H_1T_1} = 499.25$ and $\rho^{3*}_{H_2T_1} = 499.75$, respectively.

Example 2

In this example, the data remain as in Example 1, except that we modify the arc multipliers so that not all are equal to 1:

$$\alpha_1 = 1, \quad \alpha_2 = 0.95, \quad \alpha_3 = 1, \quad \alpha_4 = 1, \quad \alpha_5 = 1, \quad \alpha_6 = 0.98.$$

Hence, the path multipliers are:

$$\mu_{p_1} = 1 \times 0.95 = 0.95, \quad \mu_{p_2} = 1, \quad \mu_{p_3} = 1, \quad \mu_{p_4} = 1 \times 0.98 = 0.98.$$

Again, using inequality (18), we obtain the following set of equations:

$$3.9x^*_{p_1} + 2x^*_{p_2} + 3.5 - 0.95\eta^*_{H_1} = 0,$$

$$4x^*_{p_2} + 2x^*_{p_1} + 4 - \eta^*_{H_2} = 0,$$

$$4x^*_{p_3} + 2x^*_{p_4} + 4 - \eta^*_{H_1} = 0,$$

$$3.96x^*_{p_4} + 2x^*_{p_3} + 4.5 - 0.98\eta^*_{H_2} = 0,$$

$$q^*_{H_1T_1} + \eta^*_{H_1} - \rho^{3*}_{H_1T_1} + 101.5 = 0,$$

$$q^*_{H_2T_1} + \eta^*_{H_2} - \rho^{3*}_{H_2T_1} + 101.5 = 0,$$

$$0.95x^*_{p_1} + x^*_{p_3} - q^*_{H_1T_1} = 0,$$

$$x^*_{p_2} + 0.98x^*_{p_4} - q^*_{H_2T_1} = 0,$$

$$q^*_{11} + 0.005\rho^{3*}_{H_1T_1} - 0.002\rho^{3*}_{H_2T_1} - 100 = 0,$$

$$q^*_{H_2T_1} + 0.005\rho^{3*}_{H_2T_1} - 0.002\rho^{3*}_{H_1T_1} - 100 = 0.$$

The equilibrium path flows are now: $x^*_{p_1} = 48.35, x^*_{p_2} = 51.36, x^*_{p_3} = 52.53, x^*_{p_4} = 48.10$.

The equilibrium prices charged by the BSOs are: $\eta^*_{H_1} = 310.29, \eta^*_{H_2} = 307.63$, respectively. The price per unit charged by Hospital 1 and Hospital 2 are $\rho^{2*}_{H_1} = 311.75$ and $\rho^{2*}_{H_2} = 307.63$, respectively.

At equilibrium, the quantities of blood transfused at each hospital are: $q^*_{H_1T_1} = 98.46, q^*_{H_2T_1} = 98.49$. The reimbursements that Payer type 1 is willing to pay to Hospital 1 and Hospital 2 are: $\rho^{3*}_{H_1T_1} = 510.26$, and $\rho^{3*}_{H_2T_1} = 506.12$, respectively.

If we compare the results in the two examples, we can see that even under the consideration that a fraction of the collected blood perishes or is wasted along the supply chain, the amount of blood transfused at each hospital remains almost same. However, while in Example 1 all the paths had similar flows, in case of Example 2, the paths without loss have higher flows than those with loss. Due to wastage of some amount of collected blood the cost is likely to increase, and hence, the equilibrium prices obtained in Example 2 are higher. The payer agrees to pay higher rates in Example 2 which enables the hospitals to pay higher prices to the blood suppliers to cover the cost of the wasted blood and still meet the demand. Hence, the blood supply chain functions efficiently even in the face of loss of blood products.

4. Numerical Examples

In this Section, we present further numerical examples, based on a more elaborate network, that we solve using the modified projection method of Korpelevich (1977). For the detailed statement of the algorithm and the convergence conditions please refer to The Algorithm section of the Appendix.

In our network structure shown in Figure 3 there are two blood service organizations, one a smaller regional blood bank, and the other a larger one. Both of these blood service organizations supply blood to two hospitals which treat patients belonging to three payer groups: two private ones. Given that hospitals and blood centers are facing shortages, it is not unreasonable to assume that each hospital has more than one supplier to mitigate the risk of running out of blood as observed in the Northeastern part of US. We now provide the data for this problem.

Baseline Example

In Lagerquist et al. (2017), the authors provide an analysis of the cost of transfusing one unit of RBC in a Canadian hospital. Based on their data, the per unit cost of inventory and storage at the hospital was obtained as 30.80 CAD.

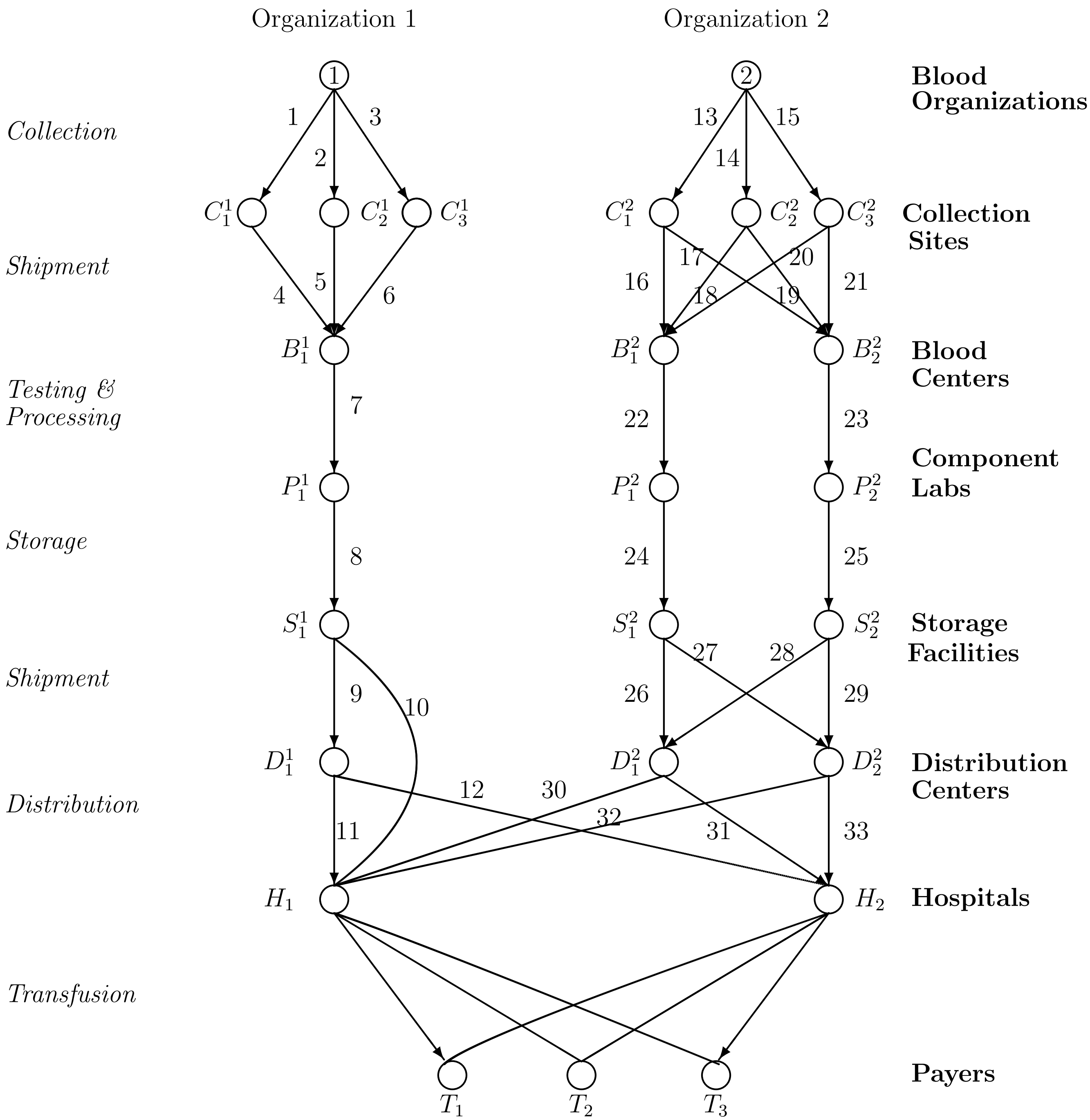

Figure 3: The Supply Chain Network Topology for Numerical Examples

Using this information and converting it to USD the inventory holding costs for the two hospitals are constructed as:

$$h_{H_1}(\sum_{k=T_1}^{T_3} q_{H_1k}) = 23.6 \times (q_{H_1T_1} + q_{H_1T_2} + q_{H_1T_3}),$$

and

$$h_{H_2}(\sum_{k=T_1}^{T_3} q_{H_2k}) = 24 \times (q_{H_2T_1} + q_{H_2T_2} + q_{H_2T_3}).$$

Below, we present the transaction cost functions for this problem. It is to be noted that this cost might include various costs that are not directly associated with the procurement of blood, and maintaining its inventory such as cost of cross-matching, transfusion, and administrative costs for billing, etc. Linear cost functions are used which are given as follows:

$$c_{H_1T_1}(q_{H_1T_1}) = 0.5q_{H_1T_1} + 10, \quad c_{H_1T_2}(q_{H_1T_2}) = 0.5q_{H_1T_2} + 9, \quad c_{H_1T_3}(q_{H_1T_3}) = 0.5q_{H_1T_3} + 8,$$

$$c_{H_2T_1}(q_{H_2T_1}) = 0.5q_{H_2T_1} + 10, \quad c_{H_2T_2}(q_{H_2T_2}) = 0.5q_{H_2T_2} + 10, \quad c_{H_2T_3}(q_{H_2T_3}) = 0.5q_{H_2T_3} + 8.$$

While it is mentioned in the introduction that reimbursements received by a hospital from different payers might vary, the transaction costs might also vary depending on the type of payer. According to Ho and Lee (2017) average cost per patient for a hospital varies from one payer to another due to long-term relationships with particular insurance companies or due to " complementarities in information systems with some insurers."

Assuming that the overall base weekly demand for RBCs across all payer types at each hospital is 250 units the demand price functions are constructed as follows:

$$d_{H_1T_1} = -0.007\rho^3_{H_1T_1} + 0.001\rho^3_{H_2T_1} + 100, \quad d_{H_2T_1} = -0.005\rho^3_{H_2T_1} + 0.003\rho^3_{H_1T_1} + 100,$$

$$d_{H_1T_2} = -0.007\rho^3_{H_1T_2} + 0.001\rho^3_{H_2T_2} + 50, \quad d_{H_2T_2} = -0.005\rho^3_{H_2T_2} + 0.003\rho^3_{H_1T_2} + 50,$$

$$d_{H_1T_3} = -0.007\rho^3_{H_1T_3} + 0.001\rho^3_{H_3T_1} + 100, \quad d_{H_2T_3} = -0.005\rho^3_{H_2T_3} + 0.003\rho^3_{H_1T_3} + 100.$$

The weights associated with the altruism components of the blood service organizations' objective function are $\omega_1 = \omega_2 = 1$. The coefficients of the altruism function are assumed to be $\gamma_{1H_1} = 1, \gamma_{1H_2} = 1, \gamma_{2H_1} = 1, \gamma_{2H_2} = 1$. The hospitals also have an altruism component in their objective functions and the associated weights are assumed to be $\beta_{H_1} = \beta_{H_2} = 1$, while the coefficients are $\theta_{H_1T_1} = 1, \theta_{H_1T_2} = 1, \theta_{H_1T_3} = 2, \theta_{H_2T_1} = 1, \theta_{H_2T_2} = 1, \theta_{H_2T_3} = 2$. Both hospitals associate greater service utility in treating patients belonging to government payer program.

We implemented the modified projection method in FORTRAN and a Linux system at the University of Massachusetts Amherst was used for the computations. The algorithm was initialized by setting all the variables equal to 0.00 and with ψ set to .05. The algorithm was considered to have converged when the absolute value of the difference of each successive iteration was less than or equal to 10^{-4}. The equilibrium conditions held with an excellent accuracy.

In Table 2 we provide the total link cost functions, the arc multipliers associated with each link as well as the computed equilibrium link flows. The total link cost functions capture the fact that the two most expensive operations for the blood services organizations are collection of blood from donors, and testing and processing of the collected units.

In addition to the computed equilibrium link flow values in Table 2, which are obtained from the equilibrium path flows, the other computed equilibrium values of the variables are:

$$\eta^*_{H_1} = 184.92, \quad \eta^*_{H_2} = 194.67,$$

$$q^*_{H_1T_1} = 98.46, \quad q^*_{H_1T_2} = 48.61, \quad q^*_{H_1T_3} = 98.55,$$

$$q^*_{H_2T_1} = 99.43, \quad q^*_{H_2T_2} = 49.53, \quad q^*_{H_2T_3} = 99.41.$$

and

$$\rho^{3*}_{H_1T_1} = 257.76, \quad \rho^{3*}_{H_1T_2} = 231.83, \rho^{3*}_{H_1T_3} = 245.80$$

$$\rho^{3*}_{H_2T_1} = 268.39, \quad \rho^{3*}_{H_2T_2} = 233.43, \quad \rho^{3*}_{H_2T_3} = 266.37.$$

Using the procedure described in Section 3, the equilibrium prices of the BSOs and those of the hospitals are recovered as follows: $\rho^{1*}_{1H_1} = \rho^{1*}_{2H_1} = 184.92$ and $\rho^{1*}_{1H_2} = \rho^{1*}_{2H_2} = 194.67$. Also, $\rho^{2*}_{H_1} = 198.52$ and $\rho^{2*}_{H_2} = 208.67$..

Also, for completeness, we report the incurred demands at the equilibrium prices at the different payers:

$$d_{H_1T_1} = 98.46, \quad d_{H_1T_2} = 48.61, \quad d_{H_1T_3} = 98.55,$$

$$d_{H_2T_1} = 99.43, \quad d_{H_2T_2} = 49.53, \quad d_{H_2T_3} = 99.41.$$

Finally, the incurred utilities of the blood service organizations and the hospitals at the equilibrium pattern are presented. The utility of BSO 1 is: 25,187.59 and that of BSO 2: 47,806.95. The utility of Hospital H_1 is: 985.40 and that of Hospital H_2: 495.25.

We now consider three variants of the baseline example, in which the weights associated with altruism are modified. Specific changes made to the basline example are reported below.

Table 2: Definition of Links, Associated activity, Arc Multipliers, Total Operational Link Cost Functions, and Equilibrium Link Solution

Link a	From Node	To Node	Activity	α_a	$\hat{c}_a(f)$	f_a^*
1	1	C_1^1	Collection	1.00	$0.45_1^2 + 0.6f_1$	49.96
2	1	C_2^1	Collection	1.00	$0.35f_2^2 + 0.5f_2$	57.35
3	1	C_3^1	Collection	1.00	$0.32f_3^2 + 0.6f_3$	64.24
4	C_1^1	B_1^1	Shipment	1.00	$0.09f_4^2 + 0.36f_4$	49.96
5	C_2^1	B_1^1	Shipment	1.00	$0.12f_5^2 + 0.5f_5$	57.35
6	C_3^1	B_1^1	Shipment	1.00	$0.1f_6^2 + 0.35f_6$	64.24
7	B_1^1	P_1^1	Testing/ Processing	0.98	$0.5f_7^2 + 0.86f_7$	171.55
8	P_1^1	S_1^1	Storage	1.00	$0.12f_8^2 + 0.5f_8$	168.12
9	S_1^1	D_1^1	Shipment	1.00	$0.09f_9^2 + 0.5f_9$	64.36
10	S_1^1	H_1	Distribution	1.00	$0.05f_{10}^2 + 0.68f_{10}$	103.76
11	D_1^1	H_1	Distribution	1.00	$0.04f_{11}^2 + 0.8f_{11}$	0.00
12	D_1^1	H_2	Distribution	1.00	$0.06f_{12}^2 + 0.8f_{12}$	64.36
13	2	C_1^2	Collection	1.00	$0.3f_{13}^2 + 0.8f_{13}$	104.72
14	2	C_2^2	Collection	1.00	$0.25f_{14}^2 + 0.65f_{14}$	138.95
15	2	C_3^2	Collection	1.00	$0.32f_{15}^2 + 0.6f_{15}$	97.36
16	C_1^2	B_1^2	Shipment	1.00	$0.1f_{16}^2 + 0.28f_{16}$	82.57
17	C_1^2	B_2^2	Shipment	1.00	$0.15f_{17}^2 + 0.3f_{17}$	22.15
18	C_2^2	B_1^2	Shipment	1.00	$0.15f_{18}^2 + 0.35f_{18}$	33.17
19	C_2^2	B_2^2	Shipment	1.00	$0.12f_{19}^2 + 0.45f_{19}$	105.78
20	C_3^2	B_1^2	Shipment	1.00	$0.16f_{20}^2 + 0.5f_{20}$	53.18
21	C_3^2	B_2^2	Shipment	1.00	$0.08f_{21}^2 + 0.6f_{21}$	44.17
22	B_1^2	P_1^2	Testing/Processing	0.98	$0.4f_{22}^2 + 0.65f_{22}$	168.93
23	B_2^2	P_2^2	Testing/Processing	0.97	$0.45f_{23}^2 + 0.8f_{23}$	172.11
24	P_1^2	S_1^2	Storage	0.96	$0.02f_{24}^2 + 0.05f_{24}$	165.55
25	P_2^2	S_2^2	Storage	1.00	$0.04f_{25}^2 + 0.07f_{25}$	166.94
26	S_1^2	D_1^2	Shipment	1.00	$0.2f_{26}^2 + 0.4f_{26}$	80.61
27	S_1^2	D_2^2	Shipment	1.00	$0.18f_{27}^2 + 0.6f_{27}$	78.32
28	S_2^2	D_1^2	Shipment	1.00	$0.12f_{28}^2 + 0.45f_{28}$	99.96
29	S_2^2	D_2^2	Shipment	1.00	$0.15f_{29}^2 + 0.5f_{29}$	66.98
30	D_1^2	H_1	Distribution	1.00	$0.08f_{30}^2 + 0.5f_{30}$	75.12
31	D_1^2	H_2	Distribution	1.00	$0.1f_{31}^2 + 0.6f_{31}$	105.45
32	D_2^2	H_1	Distribution	1.00	$0.12f_{32}^2 + 0.35f_{32}$	66.75
33	D_2^2	H_2	Distribution	1.00	$0.16f_{33}^2 + 0.4f_{33}$	78.55

The remainder of the data remains as in the baseline example. The computed equilibrium flows are reported in Table 3 for all the variants.

Variant 1

In Variant 1 the weights of the BSOs are all set to zero, that is, $\omega_1 = \omega_2 = 0$.

Besides the reported link equilibrium values in Table 3, the modified projection method also yielded the following equilibrium values for the other variables:

$$\eta^*_{H_1} = 283.54, \quad \eta^*_{H_2} = 293.33,$$

$$q^*_{H_1T_1} = 97.87, \quad q^*_{H_1T_2} = 48.02, \quad q^*_{H_1T_3} = 97.96,$$

$$q^*_{H_2T_1} = 99.23, \quad q^*_{H_2T_2} = 49.33, \quad q^*_{H_2T_3} = 99.21,$$

and

$$\rho^{3*}_{H_1T_1} = 356.08, \quad \rho^{3*}_{H_1T_2} = 330.15, \quad \rho^{3*}_{H_1T_3} = 344.12,$$

$$\rho^{3*}_{H_2T_1} = 366.95, \quad \rho^{3*}_{H_2T_2} = 332.00, \quad \rho^{3*}_{H_2T_3} = 364.93.$$

For completeness we also report the recovered prices at the top tier and the middle tier. Specifically, we have: $\rho^{1*}_{1H_1} = \rho^{1*}_{2H_1} = 283.54$ and $\rho^{1*}_{1H_2} = \rho^{1*}_{2H_2} = 293.33$. In addition, we have that: $\rho^{2*}_{H_1} = 297.14$ and $\rho^{2*}_{H_2} = 307.33$.

Also, for completeness, we would like to mention that the incurred demands at the equilibrium prices at the different payers are exactly equal to the corresponding q^*_{jk} value, which conforms well with the equilibrium conditions.

The utility of BSO 1 is now: 24,952.65 and that of BSO 2 is: 47,365.72, whereas the utility of Hospital H_1 is: 979.48 and that of Hospital H_2 is: 493.29.

Variant 2

In Variant 2, only the weights associated with the hospitals are set to zero, that is, we have $\beta_{H_1} = \beta_{H_2} = 0$ with all the rest of the data as in the baseline example. Please refer to Table 3 for the computed equilibrium link flows.

As can be seen from Table 3 all the equilibrium link flows are higher than the corresponding values for Variant 1.

The other computed equilibrium values of the variables are:

$$\eta^*_{H_1} = 184.74, \quad \eta^*_{H_2} = 194.49,$$

Table 3: Links and Link Equilibrium Solution for Variant Examples

Link a	From Node	To Node	Variant 1 f_a^*	Variant 2 f_a^*	Variant 3 f_a^*
1	1	C_1^1	49.71	49.94	49.68
2	1	C_2^1	57.08	57.32	57.04
3	1	C_3^1	63.93	64.20	63.89
4	C_1^1	B_1^1	49.71	49.92	49.68
5	C_2^1	B_1^1	57.08	57.32	57.04
6	C_3^1	B_1^1	63.93	64.20	63.89
7	B_1^1	P_1^1	170.72	171.44	170.61
8	P_1^1	S_1^1	167.31	168.01	167.20
9	S_1^1	D_1^1	64.26	64.34	64.25
10	S_1^1	H_1	103.05	103.67	102.95
11	D_1^1	H_1	0.00	0.00	0.00
12	D_1^1	H_2	64.26	64.34	64.25
13	2	C_1^2	104.22	104.66	104.16
14	2	C_2^2	138.29	138.27	138.21
15	2	C_3^2	96.89	97.30	96.83
16	C_1^2	B_1^2	82.18	82.52	82.13
17	C_1^2	B_2^2	22.04	22.13	22.03
18	C_2^2	B_1^2	33.01	33.15	32.99
19	C_2^2	B_2^2	105.28	105.72	105.21
20	C_3^2	B_1^2	52.93	53.15	52.90
21	C_3^2	B_2^2	43.96	44.15	43.94
22	B_1^2	P_1^2	168.12	168.82	168.02
23	B_2^2	P_2^2	171.29	172.00	171.18
24	P_1^2	S_1^2	164.76	165.45	164.66
25	P_2^2	S_2^2	166.15	166.84	166.04
26	S_1^2	D_1^2	80.23	80.56	80.18
27	S_1^2	D_2^2	77.94	78.27	77.89
28	S_2^2	D_1^2	99.49	99.90	99.42
29	S_2^2	D_2^2	66.66	66.94	66.62
30	D_1^2	H_1	74.53	75.04	74.45
31	D_1^2	H_2	105.18	105.42	105.15
32	D_2^2	H_1	66.28	66.68	66.22
33	D_2^2	H_2	78.33	78.52	78.30

$$q^*_{H_1T_1} = 98.41, \quad q^*_{H_1T_2} = 48.56, \quad q^*_{H_1T_3} = 98.42,$$

$$q^*_{H_2T_1} = 99.41, \quad q^*_{H_2T_2} = 49.46, \quad q^*_{H_2T_3} = 99.42,$$

and

$$\rho^{3*}_{H_1T_1} = 267.54, \quad \rho^{3*}_{H_1T_2} = 241.62, \quad \rho^{3*}_{H_1T_3} = 265.55,$$

$$\rho^{3*}_{H_2T_1} = 278.20, \quad \rho^{3*}_{H_2T_2} = 253.33, \quad \rho^{3*}_{H_2T_3} = 276.20.$$

As for the equilibrium prices at the top and middle tiers, these are now: $\rho^{1*}_{1H_1} = \rho^{1*}_{2H_1} = 184.74$ and $\rho^{1*}_{1H_2} = \rho^{1*}_{2H_2} = 194.49$. In addition, we have: $\rho^{2*}_{H_1} = 278.20$ and $\rho^{2*}_{H_2} = 218.49$

As in the above examples, the q^*_{jk} value coincides with the incurred equilibrium demand $d_{jk}(\rho^{3*}), \forall j, k$.

The utility of BSO 1 is now: 25,156.20 and that of BSO 2: 47,747.99, whereas the utility of Hospital H_1 is: -0.02 and that of Hospital H_2 is: 0.01.

This result is quite interesting. First, note that the entire supply chain of each BSO is captured in the model. As for the hospitals, the focus is on its blood supply operations, but each hospital engages in numerous other activities. The utilities of both hospitals without the altruism component of the objective functions are essentially zero, which implies economic sustainability on the part of the blood operations. It is important to emphasize that the hospitals are nonprofits and, were they for profit organizations, then their respective objective functions would be modified from those in (5.10). Moreover, it is to be noted that in the United States, blood transfusion costs account for 1% of a hospital's budget, typically, which is considerd to be high (Hemez (2016)).

Variant 3

In Variant 3, all the weights associated with altruism for all the BSOs and all the hospitals were identically equal to zero, with the rest of the data as in the baseline example. This would correspond, in effect, to the stakeholders in terms of the BSOs and the hospitals being non altruistic and operating, more or less, in a profit-like manner. Please refer to Table 3 for the computed equilibrium link flow pattern. Observe that, of the three Variant examples, the equilibrium link flows are the lowest for Variant 3. Also, note that the highest equilibrium link flows occur in the baseline example. The computed equilibrium values for the other variables are:

$$\eta^*_{H_1} = 283.36, \quad \eta^*_{H_2} = 293.15,$$

$$q^*_{H_1T_1} = 97.82, \quad q^*_{H_1T_2} = 47.94, \quad q^*_{H_1T_3} = 97.83,$$

$$q^*_{H_2T_1} = 99.21, \quad q^*_{H_2T_2} = 49.26, \quad q^*_{H_2T_3} = 99.22,$$

and

$$\rho^{3*}_{H_1T_1} = 365.87, \quad \rho^{3*}_{H_1T_2} = 339.94, \quad \rho^{3*}_{H_1T_3} = 363.87,$$

$$\rho^{3*}_{H_2T_1} = 376.76, \quad \rho^{3*}_{H_2T_2} = 351.78, \quad \rho^{3*}_{H_2T_3} = 374.76.$$

The recovered equilibrium prices charged by the hospitals to the BSOs are: $\rho^{1*}_{1H_1} = \rho^{1*}_{2H_1} = 283.36$ and $\rho^{1*}_{1H_2} = \rho^{1*}_{2H_2} = 293.15$. The recovered prices at the hospitals are: $\rho^{2*}_{H_1} = 306.96$ and $\rho^{2*}_{H_2} = 317.15$. The utility of BSO 1 is now: 24,921.40 and that of BSO 2: 47,307.50, whereas the utility of Hospital H_1 is now: -.12 and that of Hospital H_2: .06. Again, the utilities of both hospitals are essentially zero in this variant, which represents that none of the stakeholders assign a positive value to the altruism component in their respective objective functions.

Additional Discussion of the Numerical Results

As can be seen from the numerical results in the examples the equilibrium prices increase down the tiers, which is very reasonable economic behavior. Furthermore, as can be seen from Variant 3, the equilibrium prices charged to the various payers are the highest of all the examples. There are few studies that report price per pint of blood charged in various parts of the country. In Toner at al. (2011), the authors report that the average cost of acquisition of one unit of RBCs in the West is 228.31 with a standard deviation of 42. According to Ellingson et al. (2017), the interquartile range for the price paid by hospitals in the United States for a unit of leukocyte-reduced RBCs in 2015 was from 197 to 228, while that of non-leukocyte-reduced RBCs was 185 to 205.

We emphasize that, although the network for these examples is stylized, they, nevertheless, illustrate important features of this unique supply chain in which the product cannot be produced but must be donated, and it then undergoes multiple activities of testing, processing, and distribution to hospitals, with subsequent dissemination to needy patients for the medical procedures. Moreover, the model captures, in a novel way, that payments for blood services can depend on the method of payment and reimbursement to hospitals.

5. Summary and Conclusion

In this paper, we developed a mathematical model that integrates the behaviors of three major stakeholders in the blood supply chain: blood service organizations, hospitals and medical centers, and patient payer groups. The model captures the current competitive landscape of the blood banking industry in the United States, and explores a cost-based pricing scheme for blood products that is aimed at bridging the disconnect between actual costs of acquiring a unit of red blood cells for transfusion and the payments received by the hospitals and the blood suppliers. The model optimizes the flow of blood through the paths joining the blood service organizations with the hospitals, the amount of blood transfused to each patient payer group at each hospital, and determines the equilibrium prices charged by the blood service organizations, and the reimbursements received by the hospitals. To the best of our knowledge, this is the first supply chain network model to include the complex economic interplays between the different tiers of decision-makers in the blood supply chain. We also quantify and incorporate the nonprofit or altruistic nature of blood centers and hospitals through a service utility component in their utility functions.

The theory of variational inequalities was utilized to formulate the equilibrium conditions for each stakeholder, and, subsequently, the entire integrated supply chain. Examples are presented for illustrative purposes. Further, the modified projection method is applied to solve numerical examples consisting of complex network structures.

The equilibrium prices obtained from the examples reveal how the prices increase as the blood service organizations and hospitals act less altruistically. Under every scenario that is examined the prices obtained closely resemble those in the actual world. The results also show that the equilibrium prices increase in progression down the tiers, which ensures the economic stability of the blood supply chain. In terms of policy implications the results show the benefit of having a pricing scheme for blood products based on the volume of blood transfused and the actual costs of all the supply chain operations, and how the reimbursements to hospitals vary by payer type.

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Appendix

Qualitative Properties

We provide some qualitative properties of the solution to the variational inequality (18). We first present the existence results. Since the feasible set underlying the variational inequality problem (18), K^4, is not compact it is not possible to derive existence of a solution from the sole assumption of continuity of the function $F(Y)$ (cf. Kinderlehrer and Stampacchia (1980)). However, we can impose a rather weak condition to ensure the existence of a solution pattern. Let

$$K_b \equiv \{(x, q, \eta, \rho^3) | 0 \leq x \leq b_1; 0 \leq q \leq b_2; 0 \leq \eta \leq b_3; 0 \leq \rho^3 \leq b_4\}, \qquad (A.1)$$

where $b = (b_1, b_2, b_3, b_4)$ and $x \leq b_1; q \leq b_2; \eta \leq b_3; \rho^3 \leq b_4$ means that $x_p \leq b_1; q_{jk} \leq b_2; \eta_j \leq b_3; \rho^3_{jk} \leq b_4$ for all $p \in P^i_j, \forall i, j, k$. Then K_b is a bounded, closed convex subset of $R_+^{n_P+2n_Hn_T+n_H}$. Thus, the following variational inequality:

$$\langle F(Y^b), Y - Y^b \rangle \geq 0, \quad \forall Y^b \in K_b, \qquad (A.2)$$

admits at least one solution $Y^b \in K_b$, from the standard theory of variational inequalities, since K_b is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980)(see also Theorem 1.5 in Nagurney (1999)), we have:

Lemma A.1

Variational inequality (21) admits a solution if and only if there exists a $b > 0$ such that variational inequality (23) admits a solution in K_b with

$$x^b < b_1, \quad q^b < b_2, \quad \eta^b < b_3, \quad \rho^{3b} < b_4. \qquad (A.3)$$

Under the conditions in Theorem 2 below it is possible to construct the upper bounds b_1, b_2, b_3, and b_4 large enough so that the restricted variational inequality (A.2) will satisfy the boundedness condition (A.3) and, thus, existence of a solution to the original variational inequality problem according to Lemma 1 will hold.

Theorem A.1: Existence of a Solution

Suppose that there exist positive constants M, N, and R with $R > 0$ such that:

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} - \omega_i \gamma_{ij} \mu_p \geq M, \quad \forall x \quad \textit{with} \quad x_p \geq N, p \in P^i_j, \forall i, j, \qquad (A.4)$$

$$c_{jk}(q) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk})}{\partial q_{jk}} \geq M, \quad \forall q \quad \textit{with} \quad q_{jk} \geq N, \forall j, k,$$

$$d_{jk}(\rho^3) \leq N, \quad \forall \rho^3 \quad with \quad \rho^3_{jk} > R, \forall j, k. \tag{A.5}$$

Then variational inequality (21); equivalently, variational inequality (18), admits at least one solution.

Proof: Follows from Lemma 1. See also the proof of existence for Proposition 1 in Nagurney and Zhao (1993) and the existence proof in Nagurney, Dong, and Zhang (2003). □

We argue that, from an economics perspective, assumptions (A.4) and (A.5) are reasonable, since when the flow of RBCs on a path between a blood service organization and a hospital pair is large, we can expect the "marginal" cost on the path minus the marginal service utility associated with the weighted altruism to exceed a positive lower bound. Similarly, when the amount of blood transfused by a hospital to a patient group is positive, the transaction cost between the pair and marginal cost of holding the blood in inventory by the hospital will exceed a lower bound. Lastly, in the case where the demand market price is very high, the demand for the product can be expected to be low (even if slightly).

Lemma A.2: Monotonicity

Assume that the link total cost functions and the inventory holding cost functions are convex, the transaction cost functions are monotone increasing, and the demand functions are monotone decreasing functions. Then the vector function F that enters the variational inequality (21) is monotone, that is,

$$\langle F(Y') - F(Y''), Y' - Y'' \rangle \geq 0, \quad \forall Y', Y'' \in \mathcal{K}. \tag{A.6}$$

Proof: Let $Y' = (x', q', \eta', \rho^{3\prime}), Y'' = (x'', q'', \eta'', \rho^{3\prime\prime})$ with $Y' \in \mathcal{K}$ and $Y'' \in \mathcal{K}$. Then inequality (A.6) can be seen in the following deduction:

$$\langle F(Y') - F(Y''), Y' - Y'' \rangle = \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P^i_j} \left[\frac{\partial \hat{C}_p(x')}{\partial x_p} - \frac{\partial \hat{C}_p(x'')}{\partial x_p} \right] \times [x'_p - x''_p]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[\frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q'_{jk})}{\partial q_{jk}} - \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q''_{jk})}{\partial q_{jk}} \right] \times [q'_{jk} - q''_{jk}]$$

$$+ \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[c_{jk}(q') - c_{jk}(q'') \right] \times [q'_{jk} - q''_{jk}]$$

$$+\sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [-d_{jk}(\rho^{3\prime}) + d_{jk}(\rho^{3\prime\prime})] \times [\rho^{3\prime}_{jk} - \rho^{3\prime\prime}_{jk}]$$

$$= (I) + (II) + (III) + (IV). \qquad (A.7)$$

Now, we can write (I) as:

$$(I) = \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P^i_j} \left[\frac{\partial \hat{C}_p(x')}{\partial x_p} - \frac{\partial \hat{C}_p(x'')}{\partial x_p} \right] \times [x'_p - x''_p]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \sum_{p \in P^i_j} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} \alpha_{ap} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \alpha_{ap} \right] \times [x'_p - x''_p]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times \sum_{p \in P^i_j} [x'_p \alpha_{ap} - x''_p \alpha_{ap}]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times [\sum_{p \in P^i_j} x'_p \alpha_{ap} - \sum_{p \in P^i_j} x''_p \alpha_{ap}]$$

$$= \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times [f'_a - f''_a]. \qquad (A.8)$$

Since the total link cost functions are convex, we have:

$$(I) = \sum_{i=1}^{I} \sum_{j=H_1}^{H_{n_H}} \left[\sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f')}{\partial f_a} - \frac{\partial \hat{c}_b(f'')}{\partial f_a} \right] \times [f'_a - f''_a] \geq 0. \qquad (A.9)$$

The convexity of the holding cost functions, $h_j(q_j)$ for all j yields

$$(II) = \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[\frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q'_{jk})}{\partial q_{jk}} - \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q''_{jk})}{\partial q_{jk}} \right] \times [q'_{jk} - q''_{jk}] \geq 0. \qquad (A.10)$$

Since c_{jk}, for all j,k, are assumed to be monotone increasing, and d_{jk}, for all j,k, are assumed to be monotone decreasing, we have that

$$(III) = \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} \left[c_{jk}(q') - c_{jk}(q'') \right] \times [q'_{jk} - q''_{jk}] \geq 0, \qquad (A.11)$$

and

$$(IV) = \sum_{j=H_1}^{H_{n_H}} \sum_{k=T_1}^{T_{n_T}} [-d_{jk}(\rho^{3\prime}) + d_{jk}(\rho^{3\prime\prime})] \times [\rho_{jk}^{3\prime} - \rho_{jk}^{3\prime\prime}] \geq 0. \tag{A.12}$$

Substituting (A.9) – (A.12) into the right-hand side of (A.7), we conclude that (A.7) is nonnegative. The proof is complete. □

Definition A.1: Lipschitz Continuity

The function that enters the variational inequality problem (21) is Lipschitz continuous if

$$\|F(Y') - F(Y'')\| \leq L\,\|Y' - Y''\| \quad \forall Y', Y'' \in \mathcal{K}, \tag{A.13}$$

where $L > 0$ is known as the Lipschitz constant.

We utilize the properties of monotonicity and Lipschitz continuity to establish the convergence of the algorithm in the following section.

The Algorithm

We recall here the algorithm that we utilize to solve variational inequality problem (21), which is in standard form, and, equivalently, (18), in our numerical examples in the next section. The algorithm is the modified projection method of Korpelevich (1977) and it is guaranteed to converge provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists).

The statement of the modified projection method is as follows, where τ is the iteration counter:

The Modified Projection Method

Step 0. Initialization

Initialize with $Y^0 \in \mathcal{K}$. Set $\tau = 1$ and select ψ, such that $0 < \psi \leq 1/L$, where L is the Lipschitz constant (see (A.13)).

Step 1: Computation

Compute $\bar{Y}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{Y}^\tau + \psi F(Y^{\tau-1}) - Y^{\tau-1}, Y - \bar{Y}^\tau \rangle \geq 0, \quad \forall Y \in \mathcal{K}. \tag{A.14a}$$

We now expand (A.14a), according to the details of (18), for our model. Compute

$(\bar{x}^\tau, \bar{q}^\tau, \bar{\eta}^\tau, \bar{\rho}^{3\tau}) \in K^4$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P_j^i}\left[\bar{x}_p^\tau + \psi\left(\frac{\partial \hat{C}_p(x^{\tau-1})}{\partial x_p} - \omega_i\gamma_{ij}\mu_p - \eta_j^{\tau-1}\mu_p\right) - x_p^{\tau-1}\right] \times [x_p - \bar{x}_p^\tau]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}\left[\bar{q}_{jk}^\tau + \psi\left(c_{jk}(q^{\tau-1}) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1})}{\partial q_{jk}} + \eta_j^{\tau-1} - \beta_j\theta_{jk} - \rho_{jk}^{3\ \tau-1}\right) - q_{jk}^{\tau-1}\right] \times [q_{jk} - \bar{q}_{jk}^\tau]$$

$$+\sum_{j=H_1}^{H_{n_H}}\left[\bar{\eta}_j^\tau + \psi\left(\sum_{i=1}^{I}\sum_{p\in P_j^i} x_p^{\tau-1}\mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1}\right) - \eta_j^{\tau-1}\right] \times [\eta_j - \bar{\eta}_j^\tau]$$

$$+\sum_{k=T_1}^{T_{n_T}}\left[\bar{\rho}_{jk}^{3\tau} + \psi\left(q_{jk}^{\tau-1} - d_{jk}(\rho^{3^{\tau-1}})\right) - \rho_{jk}^{3\ \tau-1}\right] \times [\rho_{jk}^3 - \bar{\rho}_{jk}^{3\tau}] \geq 0, \quad \forall (x, q, \eta, \rho^3) \in K^4. \quad (A.14b)$$

Step 2: Adaptation

Compute Y^τ by solving the variational inequality subproblem:

$$\langle Y^\tau + \psi F(\bar{Y}^\tau) - Y^{\tau-1}, Y - Y^\tau\rangle \geq 0, \quad \forall Y \in \mathcal{K}. \quad (A.15a)$$

We expand (36a) according to (18). Compute $(x^\tau, q^\tau, \eta^\tau, \rho^{3\tau}) \in K^4$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{I}\sum_{j=H_1}^{H_{n_H}}\sum_{p\in P_j^i}\left[x_p^\tau + \psi\left(\frac{\partial \hat{C}_p(\bar{x}^{\tau})}{\partial x_p} - \omega_i\gamma_{ij}\mu_p - \bar{\eta}_j^{\tau}\mu_p\right) - x_p^{\tau-1}\right] \times [x_p - x_p^\tau]$$

$$+\sum_{j=H_1}^{H_{n_H}}\sum_{k=T_1}^{T_{n_T}}\left[q_{jk}^\tau + \psi\left(c_{jk}(\bar{q}^{\tau}) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} \bar{q}_{jk}^{\tau})}{\partial q_{jk}} + \bar{\eta}_j^{\tau} - \beta_j\theta_{jk} - \bar{\rho}_{jk}^{3\tau}\right) - q_{jk}^{\tau-1}\right] \times [q_{jk} - q_{jk}^\tau]$$

$$+\sum_{j=H_1}^{H_{n_H}}\left[\eta^\tau + \psi\left(\sum_{i=1}^{I}\sum_{p\in P_j^i} \bar{x}_p^{\tau}\mu_p - \sum_{k=T_1}^{T_{n_T}} \bar{q}_{jk}^{\tau}\right) - \eta^{\tau-1}\right] \times [\eta_j - \eta_j^\tau]$$

$$+\sum_{k=T_1}^{T_{n_T}}\left[\rho_{jk}^{3\tau} + \psi\left(\bar{q}_{jk}^{\tau} - d_{jk}(\bar{\rho}^{3\tau})\right) - \rho_{jk}^{3\ \tau-1}\right] \times [\rho_{jk}^3 - \rho_{jk}^{3\tau}] \geq 0, \quad \forall (x, q, \eta, \rho^3) \in K^4. \quad (A.15b)$$

Step 3: Convergence Verification

If $|Y^\tau - Y^{\tau-1}| \leq \epsilon$, for $\epsilon > 0$, a pre-specified tolerance level, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

Specifically, for our model, if $|x_p^\tau - x_p^{\tau-1}| \leq \epsilon$, $|q_{jk}^\tau - q_{jk}^{\tau-1}| \leq \epsilon$, $|\eta_j^\tau - \eta_j^{\tau-1}| \leq \epsilon$, $|\rho^{3^\tau} - \rho^{3^{\tau-1}}| \leq \epsilon \quad \forall p \in P_j^i, i = 1, ..., I, j = H_1, ..., H_{n_H}, k = T_1, ..., T_{n_T}$ for $\epsilon > 0$, a pre-specified tolerance level, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

Explicit Formulae for the Modified Projection Method

The elegance of this algorithm applied to our blood supply chain network competition model in that at each iteration, closed form expressions are obtained for the variables, resulting in an easy to implement computational procedure. Below we provide the closed form expressions for the solutions of (A.14b).

The closed form expression for the blood path flows at iteration τ is: For each path $p \in P_j^i, \forall i, j$, compute:

$$\bar{x}_p^\tau = \max\left\{0, x_p^{\tau-1} - \psi\left(\frac{\partial \hat{C}_p(x^{\tau-1})}{\partial x_p} - \omega_i \gamma_{ij} \mu_p - \eta_j^{\tau-1} \mu_p\right)\right\}. \tag{A.16}$$

The amount of blood transfused, q_{jk}, $\forall j, k$, at iteration τ, is computed according to:

$$\bar{q}_{jk}^\tau = \max\left\{0, q_{jk}^{\tau-1} - \psi\left(c_{jk}(q^{\tau-1}) + \frac{\partial h_j(\sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1})}{\partial q_{jk}} + \eta_j^{\tau-1} - \beta_j \theta_{jk} - {\rho_{jk}^3}^{\tau-1}\right)\right\}. \tag{A.17}$$

The Lagrange multipliers, η_j, $j = H_1, \ldots, H_{n_H}$, are computed at iteration τ using the formula:

$$\bar{\eta}_j^\tau = \max\left\{0, \eta_j^{\tau-1} - \psi\left(\sum_{i=1}^{I} \sum_{p \in P_j^i} x_p^{\tau-1} \mu_p - \sum_{k=T_1}^{T_{n_T}} q_{jk}^{\tau-1}\right)\right\}. \tag{A.18}$$

Lastly, at iteration τ, the closed form expression for the demand prices, ρ_{jk}^3, $j = H_1, \ldots, H_{n_H}$; $k = T_1, \ldots, T_{n_T}$, is:

$$\bar{\rho}_{jk}^{3\tau} = \max\left\{0, {\rho^3}_{jk}^{\tau-1} - \psi\left(q_{jk}^{\tau-1} - d_{jk}(\rho^{3^{\tau-1}})\right)\right\}. \tag{A.19}$$

Analogous closed form expressions to those above can be easily obtained also for (A.15b).

Theorem A.2: Convergence

Assume that the function $F(Y)$ that enters the variational inequality (21) is monotone and Lipschitz continuous and that a solution exists. Then the modified projection method described above converges to the solution of the variational inequality (21); equivalently, (18).

Proof: This proof follows from Korpelevich (1977), since conditions for convergence are satisfied under te imposed assumptions. □