An Integrated Multitiered Supply Chain Network Model of Competing Agricultural Firms and Processing Firms: The Case of Fresh Produce and Quality

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July 2021; revised April and June 2022

European Journal of Operational Research (2023), 307(1), pp 364-381.

Abstract: In this paper, we develop an integrated multitiered competitive agricultural supply chain network model in which agricultural firms and processing firms compete to sell their differentiated products. The focus here is on fresh produce and minimally processed such agricultural products, with quality also captured. The competition among agricultural firms and processing firms is studied through game theory, where the governing Cournot-Nash equilibrium conditions correspond to a variational inequality problem. The algorithm, at each iteration, yields explicit closed form expressions for the agricultural product path flows, the agricultural product shipments from agricultural firms to the processing firms, and the Lagrange multipliers. A numerical study consisting of several supply chain disruption scenarios demonstrates the applicability of our modeling framework.

Key words: OR in agriculture; supply chains; networks; agriculture; game theory; disruptions

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1. Introduction

Agricultural supply chains (ASCs) are very intricate local, regional, and global networks, creating pathways from farms to consumers, and encompassing the sets of activities such as: farming/production, processing, storage, transportation, and distribution (Yu and Nagurney (2013), Chandrasekaran and Raghuram (2014), Tsolakis et al. (2014), Sharma et al. (2020)). The dynamics in the agriculture industry are very complex; the connections between various stakeholders are intertwined, and all the players work towards providing food to the consumers, while maximizing profits under tight competition. The agricultural product industry includes small and large scale farms as well as global commercial food firms such as Tyson Foods, Dole, Cargill, etc., with the latter also involved in the processing of food, as in the form of cutting, canning, freezing, pasteurization, modified atmosphere packaging, etc. Minimally processed ready to eat fresh food such as chopped vegetables, fresh-cut fruits as well as frozen produce, which are the focus in this paper, have gained popularity in recent times due to the convenience of use and health benefits. The market share for minimally processed products such as fresh-cut produce is significant. According to research conducted by Global Market Insights Inc., the processed fruits and vegetables market is projected to cross USD 465 billion by 2027 (Globe Newswire (2021)). In the COVID-19 pandemic, with an increase in home cooking, sales of fresh food increased by 10 percent in 2020, whereas frozen food had an increase of 21 percent (Morisson (2021)). Processing can increase the shelf life of agricultural produce that is often highly perishable. Currently, it is evident that frozen alternatives are providing stiff competition to fresh or minimally processed agricultural products (Renner et al. (2021)).

However, when it comes to processed produce there can be concerns regarding the quality of the food. Hence, there have been numerous studies in the field of food science that aim to determine the quality loss or deterioration in agricultural produce under different storage conditions and subjected to different processing techniques (Labuza (1984), Aamir et al. (2013), Demiray and Tulek (2014), Goncalves et al. (2020)). Quality of fresh and processed agricultural products such as fruits and vegetables can be measured based on various attributes such as color change, texture softening, loss of nutrients such as Vitamin C, etc. Agricultural supply chains exhibit a fundamental difference from other supply chains, which is particularly prevalent with fresh produce and minimally processed products, in that the quality of agricultural products changes continuously from the point of production to the point of consumption (Sloof, Tijskens, and Wilkinson (1996), Lowe and Preckel (2004), Ahumada and Villalobos (2009), Blackburn and Scudder (2009), Akkerman, Farahani, and Grunow (2010), Aiello, La Scalia, and Micale (2012)). Quality decay of agricultural products along different stages of the supply chain varies by type of product and environmental conditions such as temperature maintained during storage and transportation (Lejarza and Baldea (2022)) as well as the duration. It has been discovered that the quality of fresh produce can be determined scientifically using chemical formulae, which include both time and temperature (Labuza (1982), Taoukis and Labuza (1989), Tijskens and Polderdijk (1996), Rong, Akkerman, and Grunow (2011)).

It is evident that there are several layers of complexity associated with managing agricultural supply chains. For example, agricultural supply chain networks worldwide are predominantly multitiered, underlined by the impact of competition among the various stakeholders including the agricultural firms and the processing firms (cf. Sharma et al. (2020) and the references therein). In this paper, we develop a multitiered competitive agricultural supply chain network model in which agricultural firms and processing firms compete to sell their products at the demand markets, which can include retailers and supermarkets. With the prevalence of multiple options for consumers in terms of fresh, minimally processed and frozen substitutes for agricultural produce that have different levels of quality, it is important to study how the agricultural firms and processing firms compete with each other based on their differentiated products at the demand markets. Hence, we develop in this paper such a model that also integrates the interactions and economic transactions between the supply chains of agricultural firms and processing firms. Below, we list the main contributions of our work to the literature on agricultural supply chains.

- We capture competition at two levels; at the first level, the agricultural firms compete and sell their harvested produce to processing firms and, at the second level, the agricultural and processing firms compete to sell their differentiated but substitutable products at the demand markets.

- We include in our model explicit functions to capture the quality decay of the harvested fresh produce products along the entire, multitiered supply chain since quality is an important factor in agricultural supply chain management.

- In our numerical study, we include supply chains of both fresh and minimally processed produce; specifically, that of carrots, that follow different kinetic functions of quality decay.

- Our numerical study reveals the impacts of various supply chain disruptions in agricultural supply chains. Our results suggest that agricultural firms achieve higher profits when they sell their agricultural products both at the demand markets and to the processing firms.

- We demonstrate quantitatively the impacts of the quality of agricultural products at the demand markets and how the level of quality affects the demand market prices of the products of the agricultural and the processing firms. The results reveal that, when there are temperature and time issues at different stages of the supply chain resulting in lower quality products, the demand market prices and the profits for the associated agricultural and processing firms decrease.

- Although there is a rich literature on optimization approaches to agriculture in the context of supply chains, the literature on the use of game theory for the modeling, analysis, and solution of supply chain network problems associated with agricultural products, including that of fresh produce, is much more limited.

This paper is organized as follows. In Section 2, we provide a literature review of agricultural supply chains, further emphasizing the novelty of our work, and noting papers relevant to our research. In Section 3, we provide preliminaries on quality deterioration. In Section 4, we present the integrated multitiered agricultural supply chain network of competing agricultural firms and processing firms. We state the governing Cournot-Nash equilibrium conditions and derive alternative variational inequality formulations. In Section 5, we present our results for a numerical study on carrot supply chains using the algorithm that is provided in the Appendix. We provide our managerial insights in Section 6, and we summarize and present our conclusions in Section 7.

2. Literature Review

In this section, we discuss the relevant literature on food supply chains focusing on the relevant topics of: i) perishability and quality, ii) competition, and iii) multitiered supply chain structures.

2.1 Perishability and Quality

Perishability and quality deterioration are major concerns in agricultural supply chain management. Given the growing interest in agricultural supply chains, there is an increasing amount of literature on this topic. In the early contributions, key emphasis was placed on perishability, particularly, with a focus on inventory management (Ghare and Schrader (1963), Nahmias (1982, 2011), and Silver, Pyke, and Peterson (1998)). Later on, additional agricultural supply chains studies considered more than a single supply chain network activity (see Zhang, Habenicht, and Spiess (2003), Widodo et al. (2006), and Kopanos, Puigjaner, and Georgiadis (2012)). Different methodologies such as mathematical programming, discrete event simulation, and game theory have been used in the existing literature to study agricultural supply chains while incorporating the issue of quality decay. For example, Rong, Akkerman, and Grunow (2011) constructed a mixed-integer linear programming model for production and distribution planning for food while integrating quality degradation.

More recently, Jonkman, Barbosa-Povoa, and Bloemhof (2019) used mixed-integer linear programming to design an agro-food supply chain model with quality constraints. They discussed two methods of including quality in a model: in the case that perishability is related to the decline of product quality due to environmental conditions (e.g. temperature, humidity) and time, the perishability of products can be modelled as a number of discrete quality categories, based on a quality decay function; alternatively, if the perishability is mainly related to time, it can be incorporated directly or indirectly in the form of shelf life constraints. Lejarza and Baldea (2021) also proposed a mixed integer programming model, which captured multiple quality attributes for agricultural product quality degradation in supply chains. Ferrer et al. (2008) presented an optimization model for the scheduling of wine grape harvesting, focusing on operational costs and quality. They discussed quality loss in relation to harvest time; that is, if the grapes for wine production are harvested earlier or later than the optimal date, it would lead to quality degradation and, eventually, loss of profit. Blackburn and Scudder (2009) proposed a hybrid supply chain design model for fresh produce which captured the product's marginal value of time that decreases according to quality deterioration. Van Der Vorst et al. (2009) developed a discrete event simulation tool that integrated quality decay and sustainability and implemented their model in a case study on pineapples. Gonzalez-Araya et al. (2015) presented a robust optimization model for planning of apple harvesting that minimizes labor cost, equipment use, and loss of fruit quality.

From the above mentioned papers, we see the different ways in which quality deterioration has been included in the modeling of agricultural supply chains. While there exists an extensive literature on quality and perishability in the context of agricultural supply chain management, the majority of these works, however, focuses on one or two stages in the supply chain and not on the entire network of supply chain activities with multiple stakeholders, as we do in this paper. In addition, we capture quality deterioration of agricultural products, both fresh and minimally processed ones, through explicit formulae, based on food science principles, and the values are not discrete, but are, rather, continuous, since the quality depends on time and temperature associated with the supply chain network economic activity.

2.2 Competition

The work of Yu and Nagurney (2013) is closely linked to this paper. Therein, the authors developed a network-based food supply chain model under oligopolistic competition with a focus on fresh produce and perishability. Their model captured perishability as the loss of yield through the introduction of arc multipliers. They further included discarding costs associated with the disposal of spoiled food products. In our paper, in contrast, we capture the "freshness" of the agricultural food products, in the form of fresh and minimally processed produce, through explicit quality deterioration functions for multiple tiers of decision-makers. Besik and Nagurney (2017) and Nagurney, Besik, and Yu (2018) also provided competitive supply chain network models for fresh produce incorporating the rate of quality deterioration with explicit mathematical formulae to include environmental parameters of temperature, humidity, etc. In contrast to these works, in our paper, the competitive network structure is significantly different as we have multiple distinct decision-makers, consisting of both agricultural firms and processing firms, with quality deterioration of products captured for each tier. As mentioned earlier, the agricultural firms produce and harvest their products, such as fruits and vegetables, and sell their fresh or minimally processed produce to the demand markets directly and to the processing firms. On the other hand, the processing firms maintain their supply chains and sell the minimally processed agricultural products at the demand markets.

2.3 Multitiered Supply Chain Structures

There exists a rich body of literature on competitive supply chain networks with multiple tiers of stakeholders. Yamada et al. (2011) studied a multitiered supply chain transport supernetwork and analyzed the behavior of manufacturers, wholesalers, and freight carriers. Furthermore, Van der Vorst, Beulens, and van Beek (2000) also studied multi-echelon food supply chains. The authors presented a study based on discrete event simulation where they evaluated alternative designs of the supply chain. In contrast to the above works, in our paper, we study the the equilibrium behavior in multitiered supply chain networks through a game theoretic approach and we also study the impacts of quality deterioration of fresh and minimally processed produce in agricultural supply chains.

Taghikhah et al. (2021) used a system dynamics based approach to simulate the behavior of farmers, food processors, retailers, and customers in wine supply chains. Nagurney, Dong, and Zhang (2002) conceptualized an equilibrium model for a competitive supply chain network with separate tiers for multiple manufacturers, retailers, and demand markets. They formulated and solved the multitiered supply chain network equilibrium problem as a variational inequality problem to obtain the equilibrium product flows and prices. Dong et al. (2005) presented a model with three tiers in their supply chain network to denote manufacturers, who can use one of several shipment alternatives to send the products to the distributors, who comprise the second tier, and, finally, to retailers, who are faced with stochastic demand. One of most relevant papers to our work is the paper by Li and Nagurney (2015). Therein, the authors modeled the behavior of two separate tiers of decision-makers in a generalized supply chain consisting of suppliers and manufacturing firms that procure components from the suppliers. The authors also provided a framework for tracking the quality of the product from the component level to the development of the final product and its distribution to the demand markets. While in Li and Nagurney (2015) quality is incorporated as a decision variable, in our paper, the quality of the food products at every stage of the supply chain is a parameter that is determined using a kinetic reaction equation since, in the case of agricultural produce, quality loss is a natural biological process. Additionally, in our paper, we look at agricultural firms selling their products at the demand markets and to the processing firms, with quality being tracked along the pathways. This is different from Li and Nagurney's (2015) modeling framework.

To the best of our knowledge, this is the first paper that constructs an integrated, multitiered agricultural supply chain network model by factoring in the agricultural product quality that enables the analysis of the complex interactions among agricultural firms (AFs) and their supply chains with those of processing firms (PFs) that compete noncooperatively. Furthermore, our research contributes to the agricultural supply chain management literature, since we study under-researched areas, according to Utomo, Onggo, and Elridge (2018), such as: competition, buyer-seller relations between agricultural and processing firms as well as the economic transactions at the demand markets.

3. Preliminaries on Quality Deterioration of Fresh and Processed Produce

In this section, we recall the kinetics associated with quality deterioration for both fresh and minimally processed produce. We recognize that it is not straightforward to provide a global definition of fresh produce quality due to its subjective nature, which varies across different cultures and nations. According to Kader (1997), quality of fresh produce can be defined over attributes such as color and appearance, flavor (taste and aroma), texture, and nutritional value. Furthermore, as biological products, fresh foods lose quality over time, defined by their quality attributes (Schouten et. al (2004), Singh and Anderson (2004)). In this paper, we define the quality of fresh and minimally processed produce, similar to that done by Besik and Nagurney (2017) and Nagurney, Yu, and Besik (2018), by using kinetic functions that capture the rate of quality deterioration over time for specific quality attributes.

Taoukis and Labuza (1989) define the rate of quality deterioration as a function of the microenvironment, gas composition, relative humidity, and temperature, with Labuza (1984) capturing the quality decay of a food attribute, q, over time t, through the differential equation:

$$\frac{\partial q}{\partial t} = -kq^n = -Ae^{(-E/RT)}q^n. \tag{1}$$

In (1), k is the reaction rate defined by the Arrhenius formula, $-Ae^{(-E/RT)}$, where A is a pre-exponential constant, T is the temperature, E is the activation energy, and R is the universal gas constant (Arrhenius (1889)). Moreover, n is the reaction order, which is a nonnegative integer and belongs to the set $\mathrm{Z}^* = \{0\} \cup \mathrm{Z}^+$. In general, the quality decay function of the food attribute can be expressed in terms of its reaction order. When the reaction order n is zero; that is, $\frac{\partial q}{\partial t} = k$, the quality decay rate of the food attribute q_t at time t can be expressed as the function (Tijskens, and

Polderdijk (1996)):

$$q_t = q_0 - kt. \tag{2}$$

In (2), is is assumed that the initial quality is known and given as q_0, and these type of quality functions are also referred to as *zero order quality decay* functions. Examples of fresh produce that follow a reaction order of zero for specific quality attributes include watermelons (Dermesonlouoglou et al. (2007)), and spinach (Aamir et al. (2013)). Furthermore, certain quality attributes of processed fresh produce such as frozen carrots (Gonsalves et al (2020)), frozen spinach and parsley (Iaccheri et al. (2021)), and fresh-cut pineapples (Benitez et al. (2012)) also follow (2).

Having the reaction order be 1, leads to an exponential function, which is observed commonly in food quality decay (Tijskens and Polderdijk (1996)). This type of quality decay is called *first order quality decay*. The quality q_t at time t should be written as a multiplication of the initial quality q_0 and the quality decay function, as shown in the following expression:

$$q_t = q_0 e^{-kt}. \tag{3}$$

Examples of fresh produce that follow a reaction order of 1 include raspberries, (Ochoa et al. (2001)), and asparagus (Aamir et al. (2013)). Additional examples of different types of quality decay functions for fresh produce, zero or first order, can be found in Besik and Nagurney (2017). There are also first order quality decay functions for certain quality attributes of processed fresh produce such as frozen green beans (Iaccheri et al. (2021)).

Next, we present the integrated multitiered agricultural supply chain network model, where we also define quality deterioration on a path for agricultural firms and processing firms by using (1), (2), and (3).

4. The Integrated Multitiered Agricultural Supply Chain Network Model

In this section, we present the integrated multitiered agricultural supply chain network model that captures the competitive network equilibrium behavior of agricultural firms and processing firms. Initial descriptions and assumptions are now enumerated in the following:

1. The notion of having a multitiered supply chain network model comes from the fact that we consider the behavior of agricultural firms and processing firms and their interactions.

2. In Section 4.1, we introduce definitions related to the quality of the products on paths in the supply chain network.

3. In Section 4.2, we study the behavior of agricultural firms, who compete noncooperatively to sell their differentiated products directly at demand markets, preserving their brand's value.

Agricultural firms also have the option to sell their unprocessed agricultural products to processing firms. In a way, we capture competition in both vertically integrated supply chain networks of agricultural firms as well as in multitiered ones.

4. In Section 4.3, we look at the behavior of processing firms who purchase unprocessed agricultural products from agricultural firms to sell at demand markets while competing noncooperatively with agricultural firms and other processing firms.

5. In Section 4.4, we investigate the integrated multitiered model with agricultural firms and processing firms, capturing the underlying interactions in the network system more holistically.

6. The integrated multitiered supply chain network model is appropriate for agricultural products that are in the category of minimally processed foods, which require minimal alteration during the processing operation. Some examples of minimally processed agricultural products include many fresh fruits, vegetables, whole grains, nuts, meats, and milk (Harvard School of Public Health (2021)).

In the multitiered agricultural supply chain network (cf. Figure 1) there are I competing agricultural firms, with a typical agricultural firm denoted by i; J competing processing firms, with a typical processing firm denoted by j, and a total of n_R demand markets, with a typical demand market denoted by k.

The supply chain network topology in Figure 1, denoted by G, includes the set of nodes N and the set of links L. Agricultural firm i's supply chain network topology; $i = 1, \ldots, I$, is denoted by G^i, that is, $G^i = [L^i, N^i]$. G^i contains the set of nodes, N^i, and set of links, L^i, that correspond to the supply chain network economic activities associated with agricultural firm i. Specifically, the supply chain network of agricultural firm i contains the operations: production/growing, processing, packaging, storage, and transportation, represented by the links in the supply chain network topology, depicted in Figure 1. The supply chain network of agricultural firm i; $i = 1, \ldots, I$, consists of n^i_G growing/harvesting facilities: $M^i_1, ..., M^i_{n^i_G}$; $n^i_{C,1}$ processing facilities: $C^i_{1,1}, ..., C^i_{n^i_{C,1}}$; $n^i_{C,2}$ packaging facilities: $C^i_{1,2}, ..., C^i_{n^i_{C,2}}$; n^i_S storage units: $S^i_1, ..., S^i_{n^i_S}$; and n^i_D distribution facilities: $D^i_1, ..., D^i_{n^i_D}$. For each agricultural firm i we have the link set, L^i, associated with its supply chain network operations down to the seventh tier representing the demand markets.

We let $L^i = L^i_1 \cup L^i_2$, where L^i_1 is the set of links associated with the production/growing links of agricultural firm i, whereas the link set L^i_2 contains all the remaining supply chain operations of agricultural firm i excluding the shipment links of agricultural firm i's agricultural products from its production/growing facilities to processing firms. It should be noticed from Figure 1 that, after the production/harvesting of the agricultural products, each agricultural firm i has the option to either keep its unprocessed agricultural products in its own supply chain, consisting of links in the set L_2^i, or sell its products to the processing firms. The shipment or transportation operation is depicted by the second set of links in Figure 1. If the agricultural firms sell their unprocessed agricultural products to processing firms, it is assumed that processing firms pay the shipment costs.

After receiving the unprocessed agricultural product shipments from the agricultural firms, each processing firm has its own supply chain network to further process, package, store, and transport the agricultural products to the demand markets. Each processing firm j; $j = 1, \ldots, J$, has a supply chain network topology $\hat{G}^j$, with set of nodes, $\hat{N}^j$, and the set of links, $\hat{L}^j$. The total number of processing firms in the supply chain network is J. The supply chain network of processing firms is comprised of $n^j_{PC,1}$ processing sites: $PC^j_{1,1}, ..., PC^j_{n^j_{PC,1}}$; $n^j_{PC,2}$ packaging sites: $PC^j_{1,2}, ..., PC^j_{n^j_{PC,2}}$; n^j_{PS} storage units: $PS^j_1, ..., PS^j_{n^j_{PS}}$; and n^j_{PD} distribution facilities: $PD^j_1, ..., PD^j_{n^j_{PD}}$. Note that, in this paper, we assume that processing firms do not own any agricultural firms to source their agricultural products from, but, rather, they depend on agricultural firms as suppliers.

We have the link set, $\hat{L}^j$, associated with the supply chain network operations of processing firm j including the shipment links from the agricultural firms' production sites, where $\hat{L}^j = \hat{L}^j_1 \cup \hat{L}^j_2$. Here, $\hat{L}^j_1$ is the set of links associated with the shipment of agricultural products from the agricultural firms' production sites to processing firm j, whereas the link set $\hat{L}^j_2$ contains all the remaining supply chain operations of processing firm j. Finally, it should be noted that the total set of nodes, $N = \bigcup_{i=1}^I N^i \bigcup_{j=1}^J \hat{N}^j$, and the total set of links $L = \bigcup_{i=1}^I L^i \bigcup_{j=1}^J \hat{L}^j$.

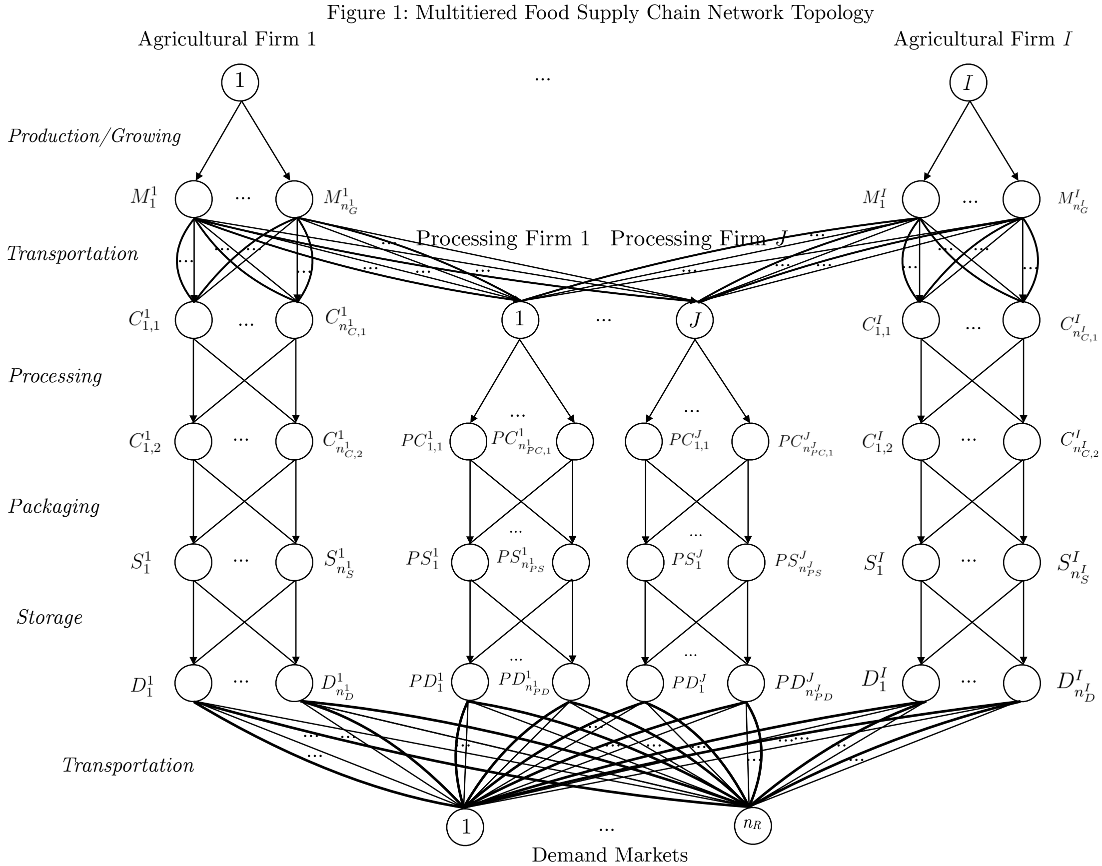
In this paper, we seek to determine the equilibrium amounts of agricultural product shipments from agricultural firms to the demand markets, from agricultural firms to processing firms, and from processing firms to the demand markets. The price that processing firms are willing to pay agricultural firms for their produce is determined endogenously. The agricultural firms and processing firms compete noncooperatively under a Cournot-Nash equilibrium in product shipments at demand markets. We provide some preliminary notation for the multitiered supply chain network equilibrium model in Table 1. The vectors are assumed to be column vectors. The equilibrium solution is denoted by "*".

Before we focus on the behavior of agricultural firms and the processing firms, we first present nonnegativity constraints and conservation of flow equations.

A path p_{AF} for an agricultural firm i consists of a sequence of links in $L^i = L^i_1 \cup L^i_2$ originating at the top tier node i and ending at a demand market node (bottom) node k in Figure 1. It should be noted that such a path does not include any shipment link of agricultural firm i to processing firms. For each path, p_{AF}, corresponding to an agricultural firm, the following nonnegativity condition must hold:

$$x_{p_{AF}} \geq 0, \quad \forall p_{AF} \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{4}$$

Figure 1: Multitiered Food Supply Chain Network Topology



We also let X_i^{AF}, denote the vector of path flows corresponding to agricultural firm i; $i = 1, \ldots, I$; that is:

$$X_i^{AF} = \{\{x_{p_{AF}}\} | p_{AF} \in P^i\} \in R_+^{n_{P^i}}, \tag{5}$$

where P^i denotes the set of all paths associated with agricultural firm i and n_{P^i} denotes the number of paths from agricultural firm i to the demand markets. Furthermore, X^{AF} is the vector of all path flows, that is, $X^{AF} \equiv \{\{X_i^{AF}\} | i = 1, \ldots, I\}$.

Table 1: Preliminary Notation for the Supply Chain Network Models

Notation	Definition
$x_{p_{AF}}$	The nonnegative flow of the agricultural product on a path p_{AF} sent from an agricultural firm i to a demand market k. Let P_k^i denote the set of all paths joining agricultural firm i with demand market k directly by agricultural firm i's set of links L^i, P denote the set of all paths from all agricultural firms to demand markets, and n_P is the number of such paths.
Q_{ij}	The nonnegative amount of agricultural firm i's product shipment from its production site to processing firm j through the link set $\hat{L}_1^j$.
ρ_{1ij}^*	The price that the processing firm j is willing to pay for the agricultural product shipment of agricultural firm i.
$x_{p_{PF}}$	The nonnegative flow of agricultural product on a path p_{PF} sent from a processing firm j to a demand market k. Let P_k^j denote the set of all paths joining processing firm j with demand market k directly by processing firm j's set of links $\hat{L}_2^j$, $\hat{P}$ denote the set of all paths from all processing firms to demand markets, and $n_{\hat{P}}$ is the number of such paths.
d_{ik}^{AF}	The demand for agricultural firm i's product at demand market k. We group all d_{ik}^{AF} elements into the vector $d^{AF} \in R_+^{In_R}$.
d_{jk}^{PF}	The demand for processing firm j's product at demand market k. We group all d_{jk} elements into the vector $d^{PF} \in R_+^{Jn_R}$.

The demand at the demand market k for the agricultural product of agricultural firm $i; i = 1, \ldots, I$, is given by:

$$d_{ik}^{AF} = \sum_{p_{AF} \in P_k^i} x_{p_{AF}}, \quad k = 1, \ldots, n_R. \tag{6}$$

The amount of product shipment, Q_{ij}, from an agricultural firm i to a processing firm j through shipment links in $\hat{L}_1^j$ must be nonnegative, that is:

$$Q_{ij} \geq 0, \quad i = 1, \ldots, I; \, j = 1, \ldots, J. \tag{7}$$

In addition, we group the product shipments for each agricultural firm i into the vector $Q_i \in R_+^J$. Similarly, we group the agricultural product shipments received by processing firm j into the vector $Q_j \in R_+^I$. We group the product shipments between the agricultural firms and processing firms into $IJ-$dimensional column vector $Q \in R_+^{IJ}$.

Similarly, we have the nonnegativity condition for the path flows from processing firms to the demand markets, where the path p_{PF} consists of a sequence of links that originates from the node corresponding to the processing firm $j; j = 1, \ldots, J$ and terminates in a demand market node:

$$x_{p_{PF}} \geq 0, \quad \forall p_{PF} \in P_k^j; \, j = 1, \ldots, J; \, k = 1, \ldots, n_R. \tag{8}$$

Furthermore, X_j^{PF} is the vector of path flows corresponding to processing firm j; $j = 1, \ldots, J$;

that is:

$$X_j^{PF} = \{\{x_{p_{PF}}\} | p_{PF} \in \hat{P}^j\} \in R_+^{n_{\hat{P}^j}}. \tag{9}$$

In (9), $\hat{P}^j$ denotes the set of all paths associated with processing firm j with $n_{\hat{P}^j}$ being the number of such paths from processing firm j to the demand markets. In addition, X^{PF} is the vector of all path flows corresponding to the processing firms, that is, $X^{PF} \equiv \{\{X_j^{PF}\} | j = 1, \ldots, J\}$.

Further, the demand at the demand market k for the agricultural product of processing firm j must satisy:

$$d_{jk}^{PF} = \sum_{p_{PF} \in P_k^j} x_{p_{PF}}, \quad j = 1, \ldots, J; \; k = 1, \ldots, n_R. \tag{10}$$

Next, we provide quality deterioration along the paths in the network for agricultural and processing firms. Then, we discuss the behavior of decision-makers in the multitiered supply chain network.

4.1 Quality Deterioration Along the Paths in the Network

We denote quality of a path from agricultural firms to demand markets, as $q_{p_{AF}}$, and the quality of a path from processing firms to demand markets is denoted by $q_{p_{PF}}$. Moreover, we let $\beta_{\bar{a}}$ denote the quality decay incurred on link $\bar{a}$, for $\bar{a} \in L^i \cup \hat{L}_j$, which is a factor that depends on the reaction order n, the reaction rate $k_{\bar{a}}$, and the time $t_{\bar{a}}$ on link $\bar{a}$, that is

$$\beta_{\bar{a}} \equiv \begin{cases} -k_{\bar{a}} t_{\bar{a}}, & \text{if } n = 0, \forall \bar{a} \in L, \\ \\ e^{-k_{\bar{a}} t_{\bar{a}}}, & \text{if } n \neq 0, \forall \bar{a} \in L. \end{cases} \tag{11}$$

Here, $k_{\bar{a}}$is the reaction constant related to the link $\bar{a}$. Since each link on a path can have different associated temperature conditions, the differentiation over the temperature of the links is essential. Thus, the reaction rate is described in the following equation for each link $\bar{a}$ by the Arrhenius formula with the same parameters as in (1), except that the temperature is now denoted for each link $\bar{a}$ as $T_{\bar{a}}$, where

$$k_{\bar{a}} = Ae^{(-E_A/RT_{\bar{a}})}. \tag{12}$$

Now, we can define the quality $q_{p_{AF}}$, over a path $p_{AF} \in P_k^i$, joining the origin agricultural firm node, i, with a destination node demand market, k, while incorporating the quality deterioration of the fresh produce as:

$$q_{p_{AF}} \equiv \begin{cases} q_{0i}^{AF} + \sum_{\bar{a} \in p_{AF}} \beta_{\bar{a}}, & \text{if } \; n = 0, \forall \bar{a} \in L^i, \; p_{AF} \in P_k^i, \forall i, k, \\ \\ q_{0i}^{AF} \prod_{\bar{a} \in p_{AF}} \beta_{\bar{a}}, & \text{if } \; n = 1, \forall \bar{a} \in L^i, p_{AF} \in P_k^i, \forall i, k. \end{cases} \tag{13}$$

In the above equation q_{0i}^{AF} represents the initial quality of the fresh produce, produced at the origin node i. Moreover, the quality deterioration of the fresh produce $q_{p_{PF}}$, over a path $p_{PF} \in P_k^j$, joining the origin processing firm, j, with a destination node demand market, k, can be defined as:

$$q_{p_{PF}} \equiv \begin{cases} q_{0j}^{PF} + \sum_{\bar{a} \in p_{PF}} \beta_{\bar{a}}, & \text{if} \;\; n = 0, \forall \bar{a} \in \hat{L}_2^j, \; p_{AF} \in P_k^j, \forall j, k, \\ q_{0j}^{PF} \prod_{\bar{a} \in p_{PF}} \beta_{\bar{a}}, & \text{if} \;\; n = 1, \forall \bar{a} \in \hat{L}_2^j, \; p_{AF} \in P_k^j, \forall j, k. \end{cases} \tag{14}$$

In the above equation q_{0j}^{PF} represents the average initial quality of the unprocessed fresh produce received by the processing firms after the unprocessed fresh produce are transported from all the agricultural firms that processing firm j conducts business with. Hence, q_{0j}^{PF} can be written as:

$$q_{0j}^{PF} \equiv \begin{cases} \dfrac{\sum_{i=1}^{I} \left(q_{0i}^{AF} + \sum_{\bar{a} \in L_1^i \cup \hat{L}_1^j} \beta_{\bar{a}} \right)}{I}, & \text{if} \;\; n = 0, \forall \bar{a} \in L_1^i \cup \hat{L}_1^j, \\ \dfrac{\sum_{i=1}^{I} \left(q_{0i}^{AF} \prod_{\bar{a} \in L_1^i \cup \hat{L}_1^j} \beta_{\bar{a}} \right)}{I}, & \text{if} \;\; n = 1, \forall \bar{a} \in L_1^i \cup \hat{L}_1^j. \end{cases} \tag{15}$$

Furthermore, when it comes to the demand markets, consumers respond not only to the quantity of products available but also to their average quality. Therefore, we denote, $\hat{q}_{ik}$, which is the average quality of the product at demand market k, associated with the fresh produce product of agricultural firm i, as:

$$\hat{q}_{ik} = \frac{\sum_{p_{AF} \in P_k^i} q_{p_{AF}} x_{p_{AF}}}{\sum_{p_{AF} \in P_k^i} x_{p_{AF}}}. \tag{16}$$

Here, $q_{p_{AF}}$ is the quality of fresh produce product quality which is product specific and given according to (13). We group the average product quality of all firms into the vector $\hat{q} \in R_+^{I \times n_R}$. Taking into account that all agricultural food firms and demand pairs that do not engage in business with each other are excluded, the denominator in (16) never equals zero.

Similar to (16), the average quality the average quality of the product at demand market k, denoted by $\bar{\hat{q}}_{jk}$, associated with the fresh produce product of processing firm j is given by the expression in the following as:

$$\bar{\hat{q}}_{jk} = \frac{\sum_{p_{PF} \in P_k^j} q_{p_{PF}} x_{p_{PF}}}{\sum_{p_{PF} \in P_k^j} x_{p_{PF}}}, \tag{17}$$

where according to (14) $q_{p_{PF}}$ is the quality of the specific fresh produce product. We group the average product quality of all firms into the vector $\bar{\hat{q}} \in R_+^{J \times n_R}$. Similar to (16), we exclude all processing firm and demand market pairs that do not conduct business with one another so that the denominator in (17) is never equal to zero.

We now discuss the behavior of the decision-makers in the multitiered supply chain network. We first describe the behavior of the agricultural firms, and then turn to the behavior of the processing firms.

4.2 Behavior of the Agricultural Firms and Their Supply Chain Network Equilibrium Conditions

The strategic variables for each agricultural firm i are: the flow of agricultural product on a path p_{AF} from the agricultural firm i to a demand market k through its vertically integrated supply chain network, $x_{p_{AF}}$, $\forall p_{AF} \in P^i_k$, and its product shipment to a processing firm j, Q_{ij}, $\forall j$.

Associated with each link a, $\forall a \in L^i_1$, is a total production/growing cost function $\hat{h}_a$ representing the cost for the activity, whereas, each remaining supply chain link b, $\forall b \in L^i_2$, has a total cost function $\hat{c}_b$. Let f_a denote the flow of agricultural product on link a, $\forall a \in L^i_1$. Then, the following conservation of flow equations must hold for each agricultural firm i; $i = 1, \ldots, I$:

$$f_a = \sum_{k=1}^{n_R} \sum_{p_{AF} \in P^i_k} x_{p_{AF}} \delta_{ap_{AF}} + \sum_{j=1}^{J} Q_{ij}, \quad \forall a \in L^i_1;\ i = 1, \ldots, I. \tag{18}$$

Here $\delta_{ap_{AF}}$ is equal to 1 if the link a is included in the path p_{AF}, and 0, otherwise. We group the link flows into the vector $f^1 \in R^{n_{L_1}}_+$, where L_1 denotes the set of production/growing links for all agricultural firms and n_{L_1} is the number of links in L_1. The total operational cost on link a, in general, is a function of all the flows in vector $f^1 \in R^{n_{L_1}}_+$, and we have that:

$$\hat{h}_a = \hat{h}_a(f^1), \quad \forall a \in L^i_1;\ i = 1, \ldots, I. \tag{19}$$

Furthermore, let f_b denote the flow of agricultural product on link b, $\forall b \in L^i_2$, in which the following conservation of flow equations must hold for each agricultural firm i; $i = 1, \ldots, I$:

$$f_b = \sum_{k=1}^{n_R} \sum_{p_{AF} \in P^i_k} x_{p_{AF}} \delta_{bp_{AF}}, \quad \forall b \in L^i_2;\ i = 1, \ldots, I, \tag{20}$$

where $\delta_{bp_{AF}}$ is equal to 1 if the link b is included in the path p_{AF}, and 0, otherwise. Furthermore, the operational cost functions associated with the remaining links in the supply chain of the agricultural firm i are as follows:

$$\hat{c}_b = \hat{c}_b(f^2), \quad \forall b \in L^i_2;\ i = 1, \ldots, I, \tag{21}$$

where, $\hat{c}_b$ is a function, in general, of all the flows in vector $f^2 \in R^{n_{L_2}}_+$. Here, L_2 is the set of links of all processing firms, excluding production/growing links and shipment links to processing firms, where n_{L_2} denotes the number of links in the set L_2.

The total cost on each link is assumed to be convex and continuously differentiable. The total cost incurred by an agricultural firm will be the sum of all the total costs on links operated by the agricultural firm.

The demand market price of food firm i's product at demand market k is denoted by ρ_{ik}^{AF}, where

$$\rho_{ik}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}), \quad i = 1, \dots, I;\ k = 1, \dots, n_R. \tag{22}$$

It should be noted that the price of an agricultural firm's product at a demand market may depend not only on the average quality and demand for its product, but also on the average quality and demand for other substitutable fresh foods from agricultural and processing firms at all the demand markets. The demand market price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

We also have a production capacity, CAP_i, for each agricultural firm $i; i = 1, \dots, I$, that is an upper bound on the sum of agricultural product shipments from agricultural firm i, which must satisfy the following:

$$CAP_i \geq \sum_{k=1}^{n_R} \sum_{p_{AF} \in P_k^i} x_{p_{AF}} + \sum_{j=1}^{J} Q_{ij}. \tag{23}$$

The profit function of agricultural firm i is the difference between its revenue and its total costs, where the total costs are the total operational costs over L^i. The profit/utility function of agricultural firm i, denoted by U_i^{AF}, is given by:

$$U_i^{AF} = \sum_{k=1}^{n_R} \rho_{ik}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}) d_{ik}^{AF} + \sum_{j=1}^{J} \rho_{1ij}^{*} Q_{ij} - \sum_{a \in L_1^i} \hat{h}_a(f^1) - \sum_{b \in L_2^i} \hat{c}_b(f^2). \tag{24a}$$

In light of (6) and (10) the demand price function $\hat{\rho}_{ik}^{AF}$ can be defined as $\hat{\rho}_{ik}(X^{AF}, X^{PF}, \hat{q}, \bar{\hat{q}}) \equiv \rho_{ik}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}); i = 1, \dots, I; k = 1, \dots, n_R$. Additionally, by using the conservation of flow equations (18), we can redefine the total operational cost function on link a, as $\bar{\hat{h}}_a(X^{AF}, Q) \equiv \hat{h}_a(f^1)$, $\forall a \in L_2^i$; $i = 1, \dots, I$. Similarly, with the conservation of flow equations (20), the operational cost functions of agricultural firm i can be redefined as $\bar{\hat{c}}_b(X^{AF}) \equiv \hat{c}_b(f^2)$,$\forall b \in L_2^i$; $i = 1, \dots, I$. Hence, we can rewrite (24a) as:

$$\hat{U}_i^{AF}(X^{AF}, Q, X^{PF})$$

$$= \sum_{k=1}^{n_R} \hat{\rho}_{ik}^{AF}(X^{AF}, X^{PF}, \hat{q}, \bar{\hat{q}}) \sum_{p_{AF} \in P_k^i} x_p^{AF} + \sum_{j=1}^{J} \rho_{1ij}^{*} Q_{ij} - \sum_{a \in L_1^i} \bar{\hat{h}}_a(X^{AF}, Q) - \sum_{b \in L_2^i} \bar{\hat{c}}_b(X^{AF}). \tag{24b}$$

The utility function $\hat{U}_i^{AF}(X^{AF}, Q, X^{PF})$ is assumed to be concave with respect to its strategic variables, X_i^{AF}, Q_i, and continuously differentiable.

The agricultural firms compete noncooperatively, where the governing equilibrium conditions are defined through the Nash (1950, 1951) equilibrium concept in the following.

Definition 1: A Cournot-Nash Equilibrium

Agricultural product path flows from agricultural firms to the demand markets, and agricultural product shipments from agricultural firms to processing firms $(X^{AF*}, Q^*) \in R_+^{n_P+IJ}$ *are said to constitute a Cournot-Nash equilibrium if for each agricultural firm i;* $i = 1, \ldots, I$,

$$\hat{U}_i^{AF}(X_i^{AF*}, \hat{X}_i^{AF*}, Q_i^*, \hat{Q}_i^*, X^{PF*}) \geq \hat{U}_i^{AF}(X_i^{AF}, \hat{X}_i^{AF*}, Q_i, \hat{Q}_i^*, X^{PF*}), \quad \forall (X^{AF}, Q) \in R_+^{n_P+IJ}, \tag{25}$$

where

$$\hat{X}_i^{AF*} \equiv (X_1^{AF*}, \ldots, X_{i-1}^{AF*}, X_{i+1}^{AF*}, \ldots, X_I^{AF*}) \quad and \quad \hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*).$$

According to (25), a Cournot-Nash equilibrium is established if no agricultural firm can unilaterally improve upon its profit by selecting an alternative vector of agricultural product path flows to demand markets, and product shipments from agricultural firms to processing firms.

4.1.1 Variational Inequality Formulation

We now derive the variational inequality formulation of the Cournot-Nash equilibrium (see e.g., Cournot (1938); Nash (1950, 1951); Gabay and Moulin (1980); Nagurney (1999, 2006)) in the following theorem.

Theorem 1: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions of Agricultural Firms

An agricultural product path flow from agricultural firms to the demand markets, and agricultural product shipment from agricultural firms to processing firm pattern, $(X^{AF*}, Q^*) \in R_+^{n_P+IJ}$, *is a Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$-\sum_{i=1}^{I} \langle \nabla_{X_i^{AF}} \hat{U}_i^{AF}(X^{AF*}, Q,^* X^{PF*}), X_i^{AF} - X_i^{AF*} \rangle - \sum_{i=1}^{I} \langle \nabla_{Q_i} \hat{U}_i^{AF}(X^{AF*}, Q^*, X^{PF*}), Q_i - Q_i^* \rangle \geq 0,$$

$$\forall (X^{AF}, Q) \in R_+^{n_P+IJ}. \tag{26}$$

Here, $\langle \cdot, \cdot \rangle$ *denotes the inner product in the corresponding Euclidean space where* $\nabla_{X_i^{AF}} \hat{U}_i^{AF}$ (X^{AF}, Q, X^{PF}) *denotes the gradient of* $\hat{U}_i^{AF}(X^{AF}, Q, X^{PF})$ *with respect to* X_i^{AF}, *and* $\nabla_{Q_i} \hat{U}_i^{AF}(X^{AF}, Q, X^{PF})$ *denotes the gradient of* $\hat{U}_i^{AF}(X^{AF}, Q, X^{PF})$ *with respect to* Q_i. *Variational inequality (26) in turn, is equivalent to the variational inequality that determines the vector of equilibrium agricultural* *product path flows, product shipments from agricultural firms to processing firms, and the Lagrange multipliers* $(X^{AF*}, Q^*, \lambda^*) \in R_+^{n_P+IJ+I}$*, such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\Bigg[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial x_{p_{AF}}}+\frac{\partial\hat{C}_{p_{AF}}(X^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}_{ik}^{AF}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})$$

$$-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_{il}^{AF}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x_{r_{AF}}^*+\lambda_i^*\Bigg]\times[x_{p_{AF}}-x_{p_{AF}}^*]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial Q_{ij}}-\rho_{1ij}^*+\lambda_i^*\right]\times[Q_{ij}-Q_{ij}^*]$$

$$\sum_{i=1}^{I}\left[CAP_i-\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}x_{p_{AF}}^*-\sum_{j=1}^{J}Q_{ij}^*\right]\times[\lambda_i-\lambda_i^*]\geq 0,\quad \forall(X^{AF},Q,\lambda)\in R_+^{n_P+IJ+I}. \tag{27}$$

Furthermore, for each p_{AF}*;* $p_{AF}\in P_k^i$*;* $i=1,\ldots,I$*;* $k=1,\ldots,n_R$*;* $l=1,\ldots,n_R$*,*

$$\frac{\partial\hat{C}_{p_{AF}}(X^{AF})}{\partial x_{p_{AF}}}\equiv\sum_{b\in L_2^i}\sum_{d\in L_2^i}\frac{\partial\hat{c}_d(f^2)}{\partial f_b}\delta_{bp_{AF}}\text{ , and }\frac{\partial\hat{\rho}_{il}^{AF}(X^{AF},X^{PF},\hat{q},\bar{\hat{q}})}{\partial x_{p_{AF}}}\equiv\frac{\partial\rho_{il}^{AF}(d^{AF},d^{PF},\hat{q},\bar{\hat{q}})}{\partial d_{ik}^{AF}}. \tag{28}$$

Proof: (26) follows from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). (27) then follows by using simple algebraic substitutions. For additional background on the variational inequality problem, please refer to the book by Nagurney (1999).

4.3 Behavior of the Processing Firms and Their Supply Chain Network Equilibrium Conditions

We now discuss the competitive behavior of the agricultural product processing firms. The agricultural product flow on a path, $x_{p_{PF}}$, from a processing firm j to a demand market k, $\forall p_{PF}\in P_k^j$, and the product shipments from an agricultural firm i to a processing firm j, Q_{ij}, are the strategic variables of a processing firm $j; j=1,\ldots,J$. Processing firms buy unprocessed agricultural products from agricultural firms to process, package, store, distribute, and sell their agricultural products at the demand markets through supply chain network operations defined in link set $\hat{L}^j$. The processing firms are responsible for the shipment of unprocessed agricultural products of the agricultural firms; hence, the shipment links, connecting production facilities and the processing firms in Figure 1, belong to the supply chain network of the processing firms.

Each processing firm j receives an amount Q_{ij} of unprocessed agricultural product shipments from agricultural firm i, where, after processing, each processing firms sells its products at the demand markets. The total amount of agricultural product shipments, by processing firm $j; j=$

$1, \ldots, J$, to the demand markets cannot exceed the total amount that it receives from its contracted agricultural firms. Therefore, the following condition must be satisfied for each $j; j = 1, \ldots, J$:

$$\sum_{i=1}^{I} Q_{ij} \geq \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}}. \tag{29}$$

We define the operational cost function related to shipment of agricultural products from the agricultural firms to processing firm j in the supply chain network as:

$$\hat{z}_b = \hat{z}_b(Q), \quad \forall b \in \hat{L}_1^j;\ j = 1, \ldots, J. \tag{30}$$

Furthermore, let f_e denote the flow of agricultural product on link e, $\forall e \in \hat{L}_2^j$. Then, the following conservation of flow equations must hold for each processing firm j; $j = 1, \ldots, , J$:

$$f_e = \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}} \delta_{e_{p_{PF}}}, \quad \forall e \in \hat{L}_2^j;\ j = 1, \ldots, J, \tag{31}$$

where $\delta_{e_{p_{PF}}}$ is equal to 1 if the link e is included in the path p_{PF}, and 0, otherwise. We group the link flows into the vector $f^3 \in R_+^{n_{\hat{L}_2}}$, where $n_{\hat{L}_2}$ denotes the number of links in $\hat{L}_2$.

The remaining total operational cost functions for the supply chain operations of the processing firms, in general, are a function of all the flows in the vector $f^3 \in R_+^{n_{\hat{L}_2}}$. Hence, we have that:

$$\hat{c}_e = \hat{c}_e(f^3), \quad \forall e \in \hat{L}_2^j;\ j = 1, \ldots, J. \tag{32}$$

The demand price of processing firm j's product at demand market k is denoted by ρ_{jk}^{PF} and assume that

$$\rho_{jk}^{PF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}), \quad j = 1, \ldots, J;\ k = 1, \ldots, n_R. \tag{33}$$

Note that the price of food firm i's product at a particular demand market may depend not only on the demands for and the average quality of its product, but also on the demands for and the average quality of the other substitutable fresh foods from agricultural firms and processing firms at all the demand markets. These demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

The utility/profit, U_j^{PF}, of processing firm $j; j = 1, \ldots, J$, is the difference between its revenue and its costs, including the payouts to the AFs, where each processing firm j pays a price of ρ_{1ij}^* to agricultural firm i for its agricultural product shipment Q_{ij}.

Hence, the profit/utility function of processing firm j is given by:

$$U_j^{PF} = \sum_{k=1}^{n_R} \rho_{jk}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}}) d_{jk} - \sum_{i=1}^{I} \rho_{1ij}^* Q_{ij} - \sum_{b \in \hat{L}_1^j} \hat{z}_b(Q) - \sum_{e \in \hat{L}_2^j} \hat{c}_e(f^3). \tag{34a}$$

According to (6) and (10), the demand price function $\hat{\rho}_{jk}$ can be written as $\hat{\rho}_{jk}^{PF}(X^{PF}, X^{AF}, \hat{q}, \bar{\hat{q}}) \equiv \rho_{jk}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}}); j = 1, \ldots, J; k = 1, \ldots, n_R$. In addition, by using the conservation of flow equations in (28), we can define $\bar{\hat{c}}_e(X^{PF}) \equiv \hat{c}_e(f^2)$, $\forall e \in \hat{L}_2^j$; $j = 1, \ldots, J$. Hence, (34a) can be rewritten as:

$$\hat{U}_j^{PF}(X^{PF}, Q, X^{AF}) = \sum_{k=1}^{n_R} \hat{\rho}_{jk}^{PF}(X^{PF}, X^{AF}, \hat{q}, \bar{\hat{q}}) \sum_{p_{PF} \in P_k^j} x_{p_{PF}} - \sum_{i=1}^{I} \rho_{1ij}^* Q_{ij} - \sum_{b \in \hat{L}_1^j} \hat{z}_b(Q) - \sum_{e \in \hat{L}_2^j} \bar{\hat{c}}_e(X^{PF}). \tag{34b}$$

We assume that for each processing firm j; $j = 1, \ldots, J$, the utility function $\hat{U}_j^{PF}(X^{PF}, Q, X^{AF})$ is concave with respect to its strategic variables, X_j^{PF}, Q_j, and is continuously differentiable.

Definition 2: A Cournot-Nash Equilibrium of Processing Firms

Agricultural product path flows from processing firms to demand markets, and product shipments from agricultural firms to processing firms, $(X^{PF*}, Q^*) \in R_+^{n_{\hat{P}}+IJ}$, are said to constitute a Cournot-Nash equilibrium if for each processing firm j; $j = 1, \ldots, J$,

$$\hat{U}_j^{PF}(X_j^{PF*}, \hat{X}_j^{PF*}, Q_j^*, \hat{Q}_j^*, X^{AF*}) \geq \hat{U}_j^{PF}(X_j^{PF}, \hat{X}_j^{PF*}, Q_j, \hat{Q}_j^*, X^{AF*}), \quad \forall (X^{PF}, Q) \in R_+^{n_{\hat{P}}+IJ}, \tag{35}$$

where

$$\hat{X}_j^{PF*} \equiv (X_1^{PF*}, \ldots, X_{j-1}^{PF*}, X_{j+1}^{PF*}, \ldots, X_J^{PF*}) \quad \textit{and} \quad \hat{Q}_j^* \equiv (Q_1^*, \ldots, Q_{j-1}^*, Q_{j+1}^*, \ldots, Q_J^*).$$

(35) states that a Cournot-Nash equilibrium is established if no processing firm can unilaterally improve upon its profit by selecting an alternative vector of agricultural product path flows to demand markets, and agricultural product shipments from agricultural firms to processing firms.

4.2.1 Variational Inequality Formulation of Processing Firms

Let η_j be the Lagrange multiplier associated with constraint (29) for processing firm j with η denoting the J-dimensional vector of all the multipliers. We now state the following theorem.

Theorem 2: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions for Processing Firms

An agricultural product path flow from processing firms to the demand markets, and agricultural product shipment from agricultural firms to processing firm pattern, $(X^{PF*}, Q^*) \in R_+^{n_{\hat{P}}+IJ}$ is a Cournot-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{j=1}^{J} \langle \nabla_{X_j^{PF}} \hat{U}_j^{PF}(X^{PF*}, Q^*, X^{AF*}), X^{PF} - X_i^{PF*} \rangle - \sum_{j=1}^{J} \langle \nabla_{Q_j} \hat{U}_j^{PF}(X^{PF*}, Q^*, X^{AF*}), Q_j - Q_j^* \rangle \geq 0,$$

$$\forall (X^{PF}, Q) \in R_+^{n_{\hat{P}}+IJ}, \tag{36}$$

where $\nabla_{X_j^{PF}} \hat{U}_j^{PF}(X^{PF}, Q, X^{AF})$ *denotes the gradient of* $\hat{U}_j^{PF}(X^{PF}, Q, X^{AF})$ *with respect to* X_j^{PF}, *and* $\nabla_{Q_j} \hat{U}_j^{PF}$ (X^{PF}, Q, X^{AF}) *denotes the gradient of* $\hat{U}_j^{PF}(X^{PF}, Q, X^{AF})$ *with respect to* Q_j. *Furthermore, we can reformulate variational inequality (36) in terms of agricultural product path flows, product shipments from agricultural firms to processing firms, and the Lagrange multipliers as: determine the vectors of equilibrium agricultural product path flows, agricultural product shipments from agricultural firms to processing firms, and Lagrange multipliers* $(X^{PF*}, Q^*, \eta^*) \in R_+^{n_{\hat{P}}+IJ+J}$, *such that:*

$$\sum_{j=1}^{J}\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial \hat{C}_{p_{PF}}(X^{PF*})}{\partial x_{p_{PF}}} - \hat{\bar{\rho}}_{jk}^{PF}(X^{PF*}, X^{AF*}, \hat{q}, \bar{\hat{q}}) - \sum_{h=1}^{n_R}\frac{\partial \hat{\bar{\rho}}_{jh}^{PF}(X^{PF*}, X^{AF*}, \hat{q}, \bar{\hat{q}})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j} x^*_{s_{PF}}\right.$$
$$\left. +\eta_j^*\right] \times [x_{p_{PF}} - x^*_{p_{PF}}] + \sum_{j=1}^{J}\sum_{i=1}^{I}\left[\frac{\sum_{b\in \hat{L}_1^j}\partial \hat{z}_b(Q^*)}{\partial Q_{ij}} + \rho^*_{1ij} - \eta_j^*\right] \times [Q_{ij} - Q^*_{ij}]$$
$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I} Q^*_{ij} - \sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j} x^*_{p_{PF}}\right] \times [\eta_j - \eta_j^*] \geq 0, \quad \forall (X^{PF}, Q, \eta) \in R_+^{n_{\hat{P}}+IJ+J}. \tag{37}$$

Further, for each p_{PF}*;* $p_{PF} \in P_k^j$*;* $j = 1, \ldots, J$*;* $k = 1, \ldots, n_R$*;* $h = 1, \ldots, n_R$*:*

$$\frac{\partial \hat{C}_{p_{PF}}(X^{PF})}{\partial x_{p_{PF}}} \equiv \sum_{e\in \hat{L}_2^j}\sum_{g\in \hat{L}_2^j}\frac{\partial \hat{c}_g(f^3)}{\partial f_e}\delta_{ep_{PF}}, \ \ \textit{and} \ \ \frac{\partial \hat{\bar{\rho}}_{jh}^{PF}(X^{PF}, X^{AF}, \hat{q}, \bar{\hat{q}})}{\partial x_{p_{PF}}} \equiv \frac{\partial \rho_{jh}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}})}{\partial d_{jk}^{PF}}. \tag{38}$$

Proof: For a given processing firm j, (37) holds if and only if (see Bertsekas and Tsitsiklis (1989) page 287) the following holds:

$$\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial \hat{C}_{p_{PF}}(X^{PF*})}{\partial x_{p_{PF}}} - \hat{\bar{\rho}}_{jk}^{PF}(X^{PF*}, X^{AF*}, \hat{q}, \bar{\hat{q}}) - \sum_{h=1}^{n_R}\frac{\partial \hat{\bar{\rho}}_{jh}^{PF}(X^{PF*}, X^{AF*}, \hat{q}, \bar{\hat{q}})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j} x^*_{s_{PF}}\right.$$
$$\left. +\eta_j^*\right] \times [x_{p_{PF}} - x^*_{p_{PF}}] + \sum_{i=1}^{I}\left[\frac{\sum_{b\in \hat{L}_1^j}\partial \hat{z}_b(Q^*)}{\partial Q_{ij}} + \rho^*_{1ij} - \eta_j^*\right] \times [Q_{ij} - Q^*_{ij}]$$
$$+\left[\sum_{i=1}^{I} Q^*_{ij} - \sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j} x^*_{p_{PF}}\right] \times [\eta_j - \eta_j^*] \geq 0, \quad \forall (X^{PF}, Q, \eta) \in R_+^{n_{\hat{P}}+IJ+J}. \tag{39}$$

Variational inequality (39) holds for each processing firm j;$j = 1, \ldots, J$, and, hence, the summation of (39) yields variational inequality (37).

4.4 The Equilibrium Conditions for the Integrated Multitiered Supply Chain Network with Agricultural Firms and Processing Firms

In equilibrium, the Cournot-Nash conditions for all agricultural firms and processing firms must hold simultaneously, according to the definition below.

Definition 3: Integrated Multitiered Supply Chain Network with Agricultural Firms and Processing Firms

The equilibrium state of the integrated multitiered agricultural supply chain network with agricultural firms and processing firms is one where both variational inequalities (27) and (37) hold simultaneously.

Theorem 3: Variational Inequality Formulation of the Equilibrium Conditions for the Integrated Multitiered Supply Chain Network with Agricultural Firms and Processing Firms

The equilibrium conditions governing the integrated multitiered agricultural supply chain network model with agricultural firms and processing firms are equivalent to the solution of the variational inequality problem: determine $(X^{AF*}, Q^*, X^{PF*}, \lambda^*, \eta^*) \in R_+^{n_P+IJ+n_{\hat{P}}+I+J}$*, such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial x_{p_{AF}}}+\frac{\partial\hat{C}_{p_{AF}}(X^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}_{ik}^{AF}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})\right.$$

$$\left.-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_{il}^{AF}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x^*_{r_{AF}}+\lambda_i^*\right]\times[x_{p_{AF}}-x^*_{p_{AF}}]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial Q_{ij}}+\frac{\sum_{b\in\hat{L}_1^j}\partial\hat{z}_b(Q^*)}{\partial Q_{ij}}+\lambda_i^*-\eta_j^*\right]\times[Q_{ij}-Q^*_{ij}]$$

$$+\sum_{i=1}^{I}\left[CAP_i-\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}x^*_{p_{AF}}-\sum_{j=1}^{J}Q^*_{ij}\right]\times[\lambda_i-\lambda_i^*]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial\hat{C}_{p_{PF}}(X^{PF*})}{\partial x_{p_{PF}}}-\bar{\hat{\rho}}_{jk}^{PF}(X^{PF*},X^{AF*},\hat{q},\bar{\hat{q}})-\sum_{h=1}^{n_R}\frac{\partial\bar{\hat{\rho}}_{jh}^{PF}(X^{PF*},X^{AF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j}x^*_{s_{PF}}\right.$$

$$\left.+\eta_j^*\right]\times[x_{p_{PF}}-x^*_{p_{PF}}]+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}Q^*_{ij}-\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}x^*_{p_{PF}}\right]\times[\eta_j-\eta_j^*]\geq 0,$$

$$\forall(X^{AF},Q,X^{PF},\lambda,\eta)\in R_+^{n_P+IJ+n_{\hat{P}}+I+J}. \tag{40}$$

Proof: We first establish necessity, that the equilibrium conditions imply variational inequality (40). Observe that, indeed, the summation of (27), and (37), yields variational inequality (40), after algebraic simplification.

For sufficiency, we now establish the converse, that is, that a solution to variational inequality (40) satisfies the sum of inequalities (27) and (37), and is, therefore, an equilibrium according to Definition 3. To inequality (40), we add the term $-\rho^*_{1ij} + \rho^*_{1ij}$ in the second set of brackets preceding the multiplication sign. This terms does not change the value of the inequality since their value is equal to zero, with the resulting inequality of the form

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial x_{p_{AF}}}+\frac{\partial \hat{C}_{p_{AF}}(X^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}^{AF}_{ik}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})\right.$$
$$\left.-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}^{AF}_{il}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x^*_{r_{AF}}+\lambda_i^*\right]\times[x_{p_{AF}}-x^*_{p_{AF}}]$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial Q_{ij}}+\frac{\sum_{b\in\hat{L}_1^j}\partial\hat{z}_b(Q^*)}{\partial Q_{ij}}+\lambda_i^*-\eta_j^*-\rho^*_{1ij}+\rho^*_{1ij}\right]\times[Q_{ij}-Q^*_{ij}]$$
$$+\sum_{i=1}^{I}\left[CAP_i-\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}x^*_{p_{AF}}-\sum_{j=1}^{J}Q^*_{ij}\right]\times[\lambda_i-\lambda_i^*]$$
$$+\sum_{j=1}^{J}\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial\hat{C}_{p_{PF}}(X^{PF*})}{\partial x_{p_{PF}}}-\bar{\hat{\rho}}^{PF}_{jk}(X^{PF*},X^{AF*},\hat{q},\bar{\hat{q}})-\sum_{h=1}^{n_R}\frac{\partial\bar{\hat{\rho}}^{PF}_{jh}(X^{PF*},X^{AF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j}x^*_{s_{PF}}\right.$$
$$\left.+\eta_j^*\right]\times[x_{p_{PF}}-x^*_{p_{PF}}]+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}Q^*_{ij}-\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}x^*_{p_{PF}}\right]\times[\eta_j-\eta_j^*]\geq 0,$$
$$\forall(X^{AF},Q,X^{PF},\lambda,\eta)\in R_+^{n_P+IJ+n_{\hat{P}}+I+J}. \tag{41}$$

which, in turn, can be rewritten as

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial x_{p_{AF}}}+\frac{\partial \hat{C}_{p_{AF}}(X^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}^{AF}_{ik}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})\right.$$
$$\left.-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}^{AF}_{il}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x^*_{r_{AF}}+\lambda_i^*\right]\times[x_{p_{AF}}-x^*_{p_{AF}}]$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial Q_{ij}}-\rho^*_{1ij}+\lambda_i^*\right]\times[Q_{ij}-Q^*_{ij}]$$
$$+\sum_{i=1}^{I}\left[CAP_i-\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}x^*_{p_{AF}}-\sum_{j=1}^{J}Q^*_{ij}\right]\times[\lambda_i-\lambda_i^*]$$
$$\sum_{j=1}^{J}\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial\hat{C}_{p_{PF}}(X^{PF*})}{\partial x_{p_{PF}}}-\bar{\hat{\rho}}^{PF}_{jk}(X^{PF*},X^{AF*},\hat{q},\bar{\hat{q}})-\sum_{h=1}^{n_R}\frac{\partial\bar{\hat{\rho}}^{PF}_{jh}(X^{PF*},X^{AF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j}x^*_{s_{PF}}\right.$$

$$
+\eta_j^*\Bigg] \times [x_{p_{PF}} - x_{p_{PF}}^*] + \sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{b\in\hat{L}_1^j}\partial\hat{z}_b(Q^*)}{\partial Q_{ij}} + \rho_{1ij}^* - \eta_j^*\right] \times [Q_{ij} - Q_{ij}^*]
$$

$$
+\sum_{j=1}^{J}\left[\sum_{i=1}^{I} Q_{ij}^* - \sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j} x_{p_{PF}}^*\right]\times[\eta_j-\eta_j^*] \geq 0, \quad \forall (X^{AF}, Q, X^{PF}, \lambda, \eta) \in R_+^{n_P+IJ+n_{\hat{P}}+I+J}. \quad (42)
$$

But inequality (42) is equivalent to the product flow pattern satisfying the sum of (27) and (37). The proof is complete □.

We now put variational inequality (40) into standard form (Nagurney (1999)) for the multitiered agricultural supply chain network equilibrium model with a agricultural firms and processing firms, define $X \equiv (X^{AF}, Q, X^{PF}, \lambda, \eta)$ and $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$, where

$$
F^1(X) = \Bigg[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF}, Q)}{\partial x_{p_{AF}}} + \frac{\partial\hat{C}_{p_{AF}}(X^{AF})}{\partial x_{p_{AF}}} - \hat{\rho}_{ik}^{AF}(X^{AF}, X^{PF}, \hat{q}, \bar{\hat{q}})
$$

$$
-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_{il}^{AF}(X^{AF}, X^{PF}, \hat{q}, \bar{\hat{q}})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i} x_{r_{AF}} + \lambda_i; \quad (43)
$$

$$
\forall p_{AF} \in P_k^i; i = 1, \ldots, I; k = 1, \ldots, n_R\Bigg],
$$

$$
F^2(X) = \left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF}, Q)}{\partial Q_{ij}} + \frac{\sum_{b\in\hat{L}_1^j}\partial\hat{z}_b(Q)}{\partial Q_{ij}} + \lambda_i - \eta_j; i = 1, \ldots, I; j = 1, \ldots, J\right], \quad (44)
$$

$$
F^3(X) = \left[CAP_i - \sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i} x_{p_{AF}} - \sum_{j=1}^{J} Q_{ij}; i = 1, \ldots, I\right], \quad (45)
$$

$$
F^4(X) = \Bigg[\frac{\partial\hat{C}_{p_{PF}}(X^{PF})}{\partial x_{p_{PF}}} - \bar{\hat{\rho}}_{jk}^{PF}(X^{PF}, X^{AF}, \hat{q}, \bar{\hat{q}}) - \sum_{h=1}^{n_R}\frac{\partial\bar{\hat{\rho}}_{jh}^{PF}(X^{PF}, X^{AF}, \hat{q}, \bar{\hat{q}})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j} x_{s_{PF}} + \eta_j;
$$

$$
\forall p_{PF} \in P_k^j; j = 1, \ldots, J; k = 1, \ldots, n_R\Bigg], \quad (46)
$$

$$
F^5(X) = \left[\sum_{i=1}^{I} Q_{ij} - \sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j} x_{p_{PF}}; j = 1, \ldots, J\right]. \quad (47)
$$

We note that a simple example for illustrative purposes of our modeling framework is provided in the Appendix.

Next, we provide a numerical study with multiple examples that we solve using the Euler method algorithm. The detailed statement of the algorithm and explicit closed form expressions can be found in The Algorithm for the Numerical Study section of the Appendix.

5. Numerical Study

In this section, we present a numerical study, focusing on carrot supply chains where agricultural firms sell fresh carrots and processing firms sell frozen carrots at demand markets. It is assumed that these two products, fresh carrots and frozen carrots, are substitutable at demand markets. The supply chain network economic activities are production/growing, processing, packaging, storage, and transportation. The numerical study consists of five different scenarios. Our goals for constructing these scenarios are to first test our modeling framework presented in Section 4 and to also show the impacts of various supply chain disruptions on the profitability of the agricultural firms and the processing firms as well as on the quality of their agricultural products.

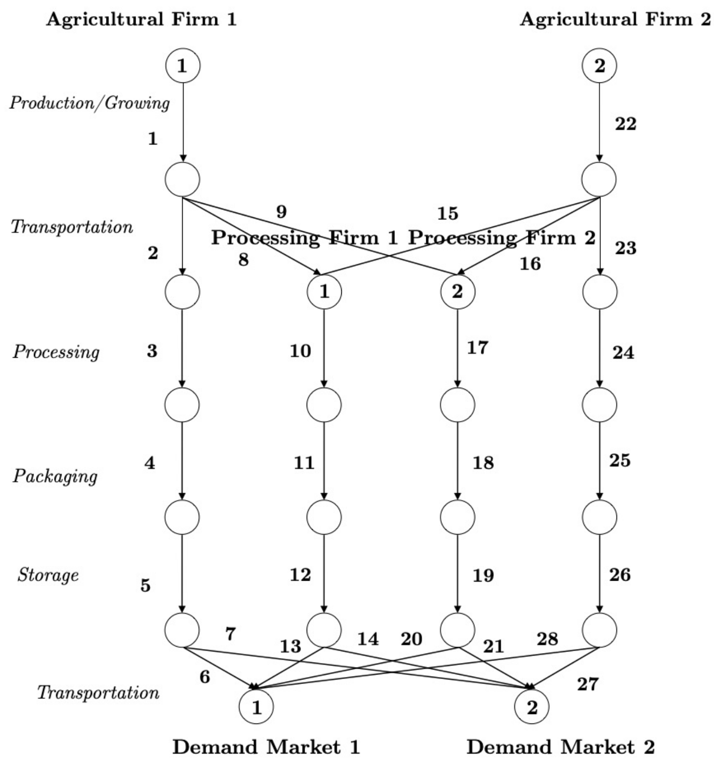
The algorithm that we use for the computation of the solution to the integrated multitiered supply chain network model with competing agricultural firms and processing firms is the Euler method. We provide a detailed statement of the algorithm and explicit closed form expressions in the Appendix section. The Euler method for the integrated multitiered agricultural supply chain problem (cf. (37)) is implemented in Python and the sequence $\{\alpha^t\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-6}, that is, the Euler method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. At this convergence tolerance the variational inequality (40) is satisfied with excellent accuracy. The code in Python is executed on a Macbook Pro laptop with a 2.0 GHz Quad-core Intel Core i5 processor and 16GB 3733 MHz LPDDR4X memory.

5.1 Scenario 1 Baseline Example - Without Any Supply Chain Disruptions

The first scenario is a baseline case for the supply chain network. The supply chain network topology of Scenario 1 is depicted in Figure 2. There are two agricultural firms: Agricultural Firm 1 and Agricultural Firm 2. We also have two processing firms: Processing Firm 1 and Processing Firm 2 and two demand markets: Demand Market 1 and Demand Market 2. The processing firms purchase fresh unprocessed carrots from the agricultural firms, since they do not own any farms that produce carrots. Both of the agricultural firms have their own supply chains and have the means to sell their products at demand markets. We assume that the processing firms sell frozen carrots, whereas the agricultural firms sell fresh carrots at demand markets.

The paths p_{1AF} and p_{2AF} belong to Agricultural Firm 1, where $p_{1AF} = (1, 2, 3, 4, 5, 6)$ and $p_{2AF} = (1, 2, 3, 4, 5, 7)$. We also have the paths p_{3AF} and p_{4AF} of Agricultural Firm 2, in which $p_{3AF} = (22, 23, 24, 25, 26, 27)$ and $p_{4AF} = (22, 23, 24, 25, 26, 28)$. The paths associated with the processing firms, Processing Firm 1 and Processing Firm 2, are: p_{1PF}, p_{2PF}, p_{3PF} and p_{4PF}; hence, we have $p_{1PF} = (10, 11, 12, 13)$, $p_{2PF} = (10, 11, 12, 14)$, $p_{3PF} = (17, 18, 19, 20)$, and $p_{4PF} = (17, 18, 19, 21)$. The quality parameter of the produce considered in this study is the overall quality

Figure 2: Multitiered Supply Chain Network Topology for Scenario 1



including color change of fresh and frozen carrots at demand markets. According to Gonsalves et al. (2020), the color change of frozen carrots follows a zero order quality decay function. Also according to Chapter 9 and Table 9.2 of the Handbook of Food Engineering Practice (Valentas, Rotstein, and Singh (1997)) by Taoukis, Labuza, and Saguy (1997), the overall color loss quality of fresh produce follows a first order quality decay function. Hence, we define the color change quality parameter of fresh carrots through a first order quality decay function. We make use of the statement in Labuza (1984), "...data from studies at several high temperatures can be used to project the shelf life at lower temperature, subject, of course, to the errors in evaluating k (reaction rate)..." to gather parameters for the Arrhenius equation defined for the reaction rate in (12) for fresh and frozen carrot color change quality decay. The parameters in (12) are retrieved from Gonsalves et al. (2020) and Demiray and Tulek (2015).

The demand market price functions, quality decay parameters, link and path quality decay values, total production, operational cost functions of agricultural firms as well as the total shipment and operational cost functions are provided in the Appendix. The production capacities, $CAP_1 = 150,000$, and $CAP_2 = 150,000$. The initial quality levels of the agricultural firms are: $q_{01}^{AF} = 1.00$ and $q_{02}^{AF} = 0.95$, whereas the initial quality levels of the processing firms are calculated as: $q_{01}^{PF} =$

0.964 and $q_{02}^{PF} = 0.969$. The quality values of the product flows on the paths are:

$$q_{p_{1AF}} = \hat{q}_{11} = 0.934, \quad q_{p_{2AF}} = \hat{q}_{12} = 0.927, \quad q_{p_{3AF}} = \hat{q}_{21} = 0.940, \quad q_{p_{4AF}} = \hat{q}_{22} = 0.920,$$

$$q_{p_{1PF}} = \bar{\hat{q}}_{11} = 0.924, \quad q_{p_{2PF}} = \bar{\hat{q}}_{12} = 0.907, \quad q_{p_{3PF}} = \bar{\hat{q}}_{21} = 0.865, \quad q_{p_{4PF}} = \bar{\hat{q}}_{22} = 0.840.$$

The average quality values at the demand markets are the same as the quality values on the paths due to the supply chain network topology of this example.

The computed equilibrium agricultural product path flows, the equilibrium product shipments from agricultural firms to processing firms, and the equilibrium Lagrange multipliers are reported in Table 2.

Table 2: Computed Equilibrium Agricultural Product Path Flows, Equilibrium Agricultural Product Shipments, Equilibrium Lagrange Multipliers, and for Scenario 1

Notation	**Equilibrium Solution**
$x^*_{p_{1AF}}$	4.977
$x^*_{p_{2AF}}$	18.433
$x^*_{p_{3AF}}$	16.960
$x^*_{p_{4AF}}$	0.835
$x^*_{p_{1PF}}$	172.039
$x^*_{p_{2PF}}$	149.703
$x^*_{p_{3PF}}$	160.243
$x^*_{p_{4PF}}$	168.827
Q^*_{11}	256.245
Q^*_{12}	148.607
Q^*_{21}	65.494
Q^*_{22}	180.472
λ^*_1	0.00
λ^*_2	0.00
η^*_1	0.241
η^*_2	0.232

The demand market prices of the agricultural firms and of the processing firms, in dollars per pound of carrots, are:

Agricultural Firm 1: $\rho_{11}^{AF} = 3.798, \quad \rho_{12}^{AF} = 3.839,$

Agricultural Firm 2: $\rho_{21}^{AF} = 3.708, \quad \rho_{22}^{AF} = 3.746,$

Processing Firm 1: $\rho_{11}^{PF} = 2.843, \quad \rho_{12}^{PF} = 2.913,$

Processing Firm 2: $\rho_{21}^{PF} = 2.639, \quad \rho_{22}^{PF} = 2.663.$

We observe that the demand market prices are lower for the processing firms' frozen carrots than the agricultural firms' fresh carrots. Also, we have that: $\rho^*_{111} = 0.181$, $\rho^*_{112} = 0.181$, $\rho^*_{121} = 0.205$, and $\rho^*_{122} = 0.205$. Recall that the price ρ^*_{1ij}, for $i = 1, 2$ and $j = 1, 2$ is the price that processing firm j is willing to pay for the agricultural product, which here is carrots, from agricultural firm i.

The profits of the agricultural firms and processing firms are:

$$\hat{U}_1^{AF}(X^{AF*}, Q^*, X^{PF*}) = 64.422, \quad \hat{U}_2^{AF}(X^{AF*}, Q^*, X^{PF*}) = 33.783,$$

$$\hat{U}_1^{PF}(X^{PF*}, Q^*, X^{AF*}) = 464.519, \quad \hat{U}_2^{PF}(X^{PF*}, Q^*, X^{AF*}) = 440.945.$$

The highest profit is enjoyed by Processing Firm 1 in this baseline example scenario. In contrast, Agricultural Firm 2 has the lowest profit. Also, notice that both processing firms enjoy higher profits than the agricultural firms.

5.2 Scenario 2 - Supply Chain Disruption at Agricultural Firm 1

In the Scenario 2 example, we reduce the production capacity of Agricultural Firm 1 to 100.00 and report our results in Table 3. The production capacity can be affected by a disruption due to a disaster or by labor shortages.

Table 3: Computed Equilibrium Agricultural Product Path Flows, Equilibrium Agricultural Product Shipments, and Equilibrium Lagrange Multipliers for Scenario 2

Notation	**Equilibrium Solution**
$x^*_{p_{1AF}}$	3.778
$x^*_{p_{2AF}}$	17.507
$x^*_{p_{3AF}}$	16.465
$x^*_{p_{4AF}}$	0.415
$x^*_{p_{1PF}}$	161.797
$x^*_{p_{2PF}}$	140.920
$x^*_{p_{3PF}}$	152.141
$x^*_{p_{4PF}}$	160.422
Q^*_{11}	78.718
Q^*_{12}	0.000
Q^*_{21}	224.007
Q^*_{22}	312.563
λ^*_1	0.342
λ^*_2	0.000
η^*_1	0.416
η^*_2	0.383

The demand market prices of the agricultural firms and of the processing firms are now:

$$\textbf{Agricultural Firm 1: } \rho_{11}^{AF} = 3.803, \quad \rho_{12}^{AF} = 3.843,$$

$$\textbf{Agricultural Firm 2: } \rho_{21}^{AF} = 3.712, \quad \rho_{22}^{AF} = 3.749,$$

$$\textbf{Processing Firm 1: } \rho_{11}^{PF} = 2.873, \quad \rho_{12}^{PF} = 2.941,$$

$$\textbf{Processing Firm 2: } \rho_{21}^{PF} = 2.667, \quad \rho_{22}^{PF} = 2.690.$$

We see that the demand market prices increase for all agricultural and processing firms in the supply chain network from their values in Scenario 1.

Furthermore, we have that: $\rho_{111}^* = 0.392$, $\rho_{112}^* = 0.392$, $\rho_{121}^* = 0.321$, and $\rho_{122}^* = 0.321$. Notice that the agricultural firms' prices for their unprocessed fresh carrots increase from their values in Scenario 1 when production capacity is reduced for Agricultural Firm 1. The profits of the agricultural firms and processing firms are:

$$\hat{U}_1^{AF}(X^{AF*}, Q^*, X^{PF*}) = 59.330, \quad \hat{U}_2^{AF}(X^{AF*}, Q^*, X^{PF*}) = 79.172,$$

$$\hat{U}_1^{PF}(X^{PF*}, Q^*, X^{AF*}) = 415.082, \quad \hat{U}_2^{PF}(X^{PF*}, Q^*, X^{AF*}) = 400.939.$$

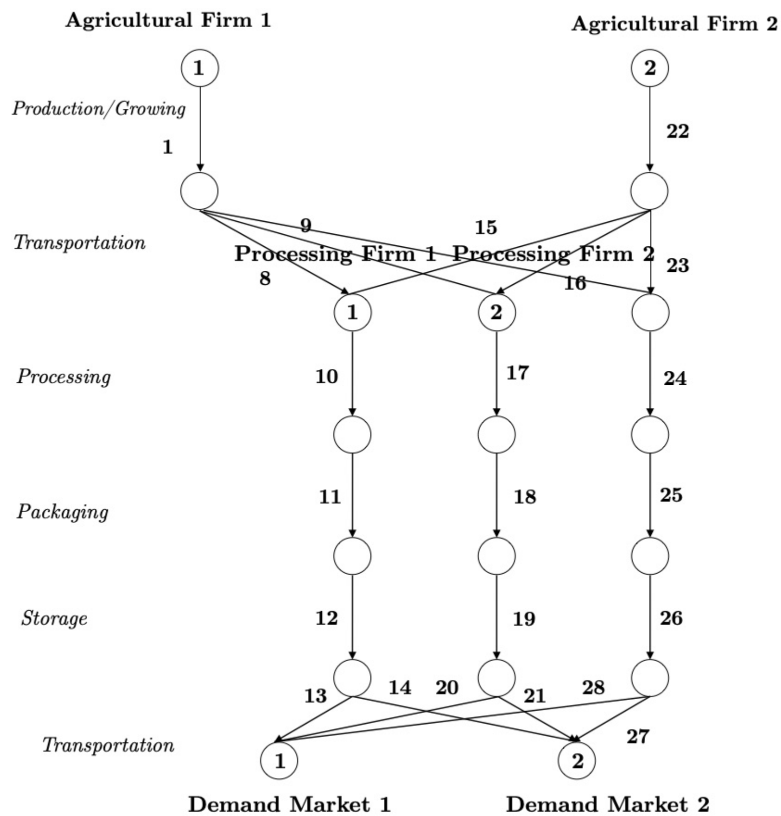
With the production capacity reduction, we observe that Agricultural Firm 1's profit decreases, as expected, whereas the profit of Agricultural Firm 2 increases as it takes advantage of the production capacity reduction of Agricultural Firm 1. Our results for Scenario 2 show that when there is a production capacity reduction, even for one agricultural firm, demand market prices and profits of all the players in the supply chain network may be affected. The highest profit is, again, enjoyed by Processing Firm 1.

5.3 Scenario 3 Example - Removal of Paths from Agricultural Firm 1 to Demand Markets

Here, we construct another scenario, in which Agricultural Firm 1 sells its unprocessed fresh carrots only to the processing firms. The supply chain network topology for the example of Scenario 3 is depicted in Figure 3, where all the operational links except the production link of Agricultural Firm 1 are removed. This means the paths in the supply chain network from Agricultural Firm 1 to Demand Market 1, p_{1AF}, and Demand Market 2, p_{2AF}, are no longer available. The total production, total operational, and total shipment cost functions associated with the supply chain network topology in Figure 3 are the same as in Scenario 1.

Table 4 reports the equilibrium solutions. Quality deterioration on the existing paths and the average quality levels are the same as reported in Scenario 1.

Figure 3: Multitiered Supply Chain Network Topology for Scenario 2



The agricultural product path flows from Agricultural Firm 1 to Demand Market 1 and to Demand Market 2, $x^*_{p_{1AF}}$ and $x^*_{p_{2AF}}$, respectively, are 0.000. The demand market prices of the agricultural firms and the processing firms, in dollars, at the demand markets, are:

$$\textbf{Agricultural Firm 2: } \rho^{AF}_{21} = 3.715, \quad \rho^{AF}_{22} = 3.762,$$

$$\textbf{Processing Firm 1: } \rho^{PF}_{11} = 2.849, \quad \rho^{PF}_{12} = 2.926,$$

$$\textbf{Processing Firm 2: } \rho^{PF}_{21} = 2.644, \quad \rho^{PF}_{22} = 2.675.$$

In this example, Agricultural Firm 1 can't sell its fresh carrots at the demand markets. The demand market prices for Agricultural Firm 2, Processing Firm 1, and Processing Firm 2 increase significantly from their values in Scenario 1 and Scenario 2. We observe this demand market price increase due to reduced competition.

Note that: $\rho^*_{111} = 0.176$, $\rho^*_{112} = 0.176$, $\rho^*_{121} = 0.202$, and $\rho^*_{122} = 0.202$. We observe a small decrease in the agricultural firms' prices for their unprocessed fresh carrots from their values in

Table 4: Computed Equilibrium Agricultural Product Path Flows, Equilibrium Agricultural Product Shipments, and Equilibrium Lagrange Multipliers for Scenario 3

Notation	**Equilibrium Solution**
$x^*_{p_{3AF}}$	16.083
$x^*_{p_{4AF}}$	1.820
$x^*_{p_{1PF}}$	172.167
$x^*_{p_{2PF}}$	151.225
$x^*_{p_{3PF}}$	160.276
$x^*_{p_{4PF}}$	170.560
Q^*_{11}	261.627
Q^*_{12}	153.481
Q^*_{21}	61.763
Q^*_{22}	177.365
λ^*_1	0.000
λ^*_2	0.000
η^*_1	0.237
η^*_2	0.229

Scenario 1. The profits of the agricultural firms and processing firms are:

$$\hat{U}_1^{AF}(X^{AF*}, Q^*, X^{PF*}) = 34.463, \quad \hat{U}_2^{AF}(X^{AF*}, Q^*, X^{PF*}) = 33.234,$$

$$\hat{U}_1^{PF}(X^{PF*}, Q^*, X^{AF*}) = 469.420, \quad \hat{U}_2^{PF}(X^{PF*}, Q^*, X^{AF*}) = 445.842.$$

Similar to Scenario 1 and Scenario 2, Processing Firm 1 enjoys the highest profit in this scenario. Agricultural Firm 1's profits decrease from its value in Scenario 1 when there is a supply chain disruption. On the other hand, processing firms enjoy more profits from their values in Scenario 1.

5.4 Scenario 4 Example - Quality issues for Agricultural Firm 1 and Processing Firm 2

In this scenario, we are building off of Scenario 3 and its supply chain network topology in Figure 3, but we focus on the impact of quality of the agricultural products. We assume that Agricultural Firm 1's products are affected by supply chain disruptions, with the initial quality of Agricultural Firm 1, q_{01}^{AF}, decreasing to 0.7. The initial quality of Agricultural Firm 2 is the same as in Scenario 1. In addition, we assume that Processing Firm 2 also runs into issues in terms of the time and temperature of its supply chain operations. We increase the temperature of the processing activity of Processing Firm 1 to 30 degrees Celsius (303.15 Kelvin) and the time of processing to 72 hours from the values reported in the Appendix Table 3 for Scenario 1. Furthermore, we modify the transportation time from Processing Firm 2 to Demand Market 2 to 360 hours instead of 72 hours in Appendix Table 3. Both agricultural firms' production capacities are the same as in Scenario 1. The equilibrium solutions are reported in Table 5.

The initial quality of the processing firms are: $q_{01}^{PF} = 0.817$ and $q_{02}^{PF} = 0.821$. Since the time and temperature data are changed, the quality deterioration values associated with the product path flows are now:

$$q_{p_{3AF}} = \hat{q}_{21} = 0.940, \quad q_{p_{4AF}} = \hat{q}_{22} = 0.920,$$

$$q_{p_{1PF}} = \bar{\bar{q}}_{11} = 0.776, \quad q_{p_{2PF}} = \bar{\bar{q}}_{12} = 0.760, \quad q_{p_{3PF}} = \bar{\bar{q}}_{12} = 0.599, \quad q_{p_{4PF}} = \bar{\bar{q}}_{22} = 0.406.$$

Table 5: Computed Equilibrium Agricultural Product Path Flows, Equilibrium Agricultural Product Shipments, and Equilibrium Lagrange Multipliers for Scenario 4

Notation	**Equilibrium Solution**
$x^*_{p_{3AF}}$	6.324
$x^*_{p_{4AF}}$	15.532
$x^*_{p_{1PF}}$	168.753
$x^*_{p_{2PF}}$	174.742
$x^*_{p_{3PF}}$	164.713
$x^*_{p_{4PF}}$	135.631
Q^*_{11}	272.685
Q^*_{12}	140.966
Q^*_{21}	70.808
Q^*_{22}	159.388
λ^*_1	0.000
λ^*_2	0.000
η^*_1	0.239
η^*_2	0.223

The demand market prices of the agricultural firms and the processing firms, in dollars, are now:

Agricultural Firm 2: $\rho_{21}^{AF} = 4.149, \quad \rho_{22}^{AF} = 4.316,$

Processing Firm 1: $\rho_{11}^{PF} = 2.949, \quad \rho_{12}^{PF} = 3.158,$

Processing Firm 2: $\rho_{21}^{PF} = 2.504, \quad \rho_{22}^{PF} = 2.378.$

We observe an increase in the demand market prices for Agricultural Firm 2 from its values in Scenario 3, where it did not experience any quality issues. However, Processing Firm 2's demand market prices fall from their values in Scenario 3 due to the quality issues that they are facing.

We now have that: $\rho^*_{111} = 0.175$, $\rho^*_{112} = 0.175$, $\rho^*_{121} = 0.200$, and $\rho^*_{122} = 0.200$. Our results for the prices that agricultural firms charge for their unprocessed fresh carrots show an overall small decrease from their values in Scenario 1. The profits of the agricultural firms and processing firms are now:

$$\hat{U}_1^{AF}(X^{AF*}, Q^*, X^{PF*}) = 34.221, \quad \hat{U}_2^{AF}(X^{AF*}, Q^*, X^{PF*}) = 42.455,$$

$$\hat{U}_1^{PF}(X^{PF*}, Q^*, X^{AF*}) = 531.126, \quad \hat{U}_2^{PF}(X^{PF*}, Q^*, X^{AF*}) = 368.623.$$

Similar to the previous scenarios, Processing Firm 1 achieves the highest profit in this scenario, whereas the profits of Agricultural Firm 1 and Processing Firm 2 decrease from their respective values in Scenario 3. These changes are as a consequence of the changes in quality.

5.5 Scenario 5 Example - Removal of Paths from Agricultural Firm 1 and Agricultural Firm 2 to Demand Markets

In this scenario example, we have a new supply chain network topology. Now, both agricultural firms, Agricultural Firm 1 and Agricultural Firm 2, sell their unprocessed carrots only to the processing firms: Processing Firm 1 and Processing Firm 2. This scenario is designed to reveal the impacts of supply chain disruptions associated with the transportation, processing, packaging, and storage links of all the agricultural firms in the integrated multitiered agricultural supply chain network. Our goal here is to explore the possible consequences of the Covid-19 pandemic, such as the shift in demand markets. We use the same total production, total operational, and total shipment cost functions as in the Appendix for the supply chain network economic activities in Figure 4. The supply chain network topology in Figure 4 is different from that in Figure 3 since, in this scenario, we also remove all the supply chain network economic activities except for the production associated with Agricultural Firm 2. For this scenario, we use the same path quality values as in Scenario 1 for the agricultural and processing firms.

The equilibrium agricultural product path flows, equilibrium agricultural product shipments from agricultural firms to processing firms, and the equilibrium Lagrange multipliers are given in Table 6.

The demand market prices of the processing firms at the demand markets are:

Processing Firm 1: $\rho_{11}^{PF} = 2.860, \quad \rho_{12}^{PF} = 2.929,$

Processing Firm 2: $\rho_{21}^{PF} = 2.655, \quad \rho_{22}^{PF} = 2.678.$

We report that: $\rho_{111}^* = 0.173$, $\rho_{112}^* = 0.173$, $\rho_{121}^* = 0.198$, and $\rho_{122}^* = 0.198$. The prices that agricultural firms charge for their unprocessed fresh carrots are slightly lower than their values in Scenario 1. The profits of the agricultural firms and the processing firms are:

$$\hat{U}_1^{AF}(X^{AF*}, Q^*, X^{PF*}) = 33.610, \quad \hat{U}_2^{AF}(X^{AF*}, Q^*, X^{PF*}) = 12.201,$$

Table 6: Computed Equilibrium Agricultural Product Path Flows, Equilibrium Agricultural Product Shipments, and Equilibrium Lagrange Multipliers for Scenario 5

Notation	**Equilibrium Solution**
$x^*_{p_{1PF}}$	173.791
$x^*_{p_{2PF}}$	150.873
$x^*_{p_{3PF}}$	162.023
$x^*_{p_{4PF}}$	170.259
Q^*_{11}	259.230
Q^*_{12}	150.711
Q^*_{21}	65.435
Q^*_{22}	181.562
λ^*_1	0.00
λ^*_2	0.00
η^*_1	0.218
η^*_2	0.246

$$\hat{U}_1^{PF}(X^{PF*}, Q^*, X^{AF*}) = 473.012, \quad \hat{U}_2^{PF}(X^{PF*}, Q^*, X^{AF*}) = 449.582.$$

Note that when the agricultural firms in this scenario face a decrease in profits from their values in Scenario 1.

6. Managerial Insights

In this section, we discuss some of the interesting results from our numerical study in Section 5. In Figures 5, 6, and 7, we summarize our results in bar charts. Figure 5 shows the profits of the agricultural and processing firms across various scenarios, whereas Figures 6 and 7 illustrate the demand market prices in the various scenarios in Section 5 for agricultural firms and processing firms. We refer to agricultural firms, processing firms, and demand markets as “AF”, “PF”, and “DM”, respectively. In Figure 6, we display the demand market prices of agricultural firms for the first four scenarios in Section 5 since, in Scenario 5, the agricultural firms only sell their fresh produce to the processing firms.

Firstly, notice, from Figure 5, that there is a substantial decrease in Agricultural Firm 1’s profits from Scenario 1 to Scenario 2 due to the production capacity reduction. This result is expected, since the production capacity reduction directly affects profitability; however, what is interesting is that we also observe an increase in Agricultural Firm 2’s profit in Scenario 2 from its value in Scenario 1 when Agricultural Firm 1’s production capacity is lower. This can be explained by the fact that Agricultural Firm 2 takes advantage of Agricultural Firm 1’s lack of production. Furthermore, when the paths of Agricultural Firm 1 to the demand markets in Scenario 3 are no

Figure 5: Profits of Agricultural and Processing Firms Under Various Scenarios

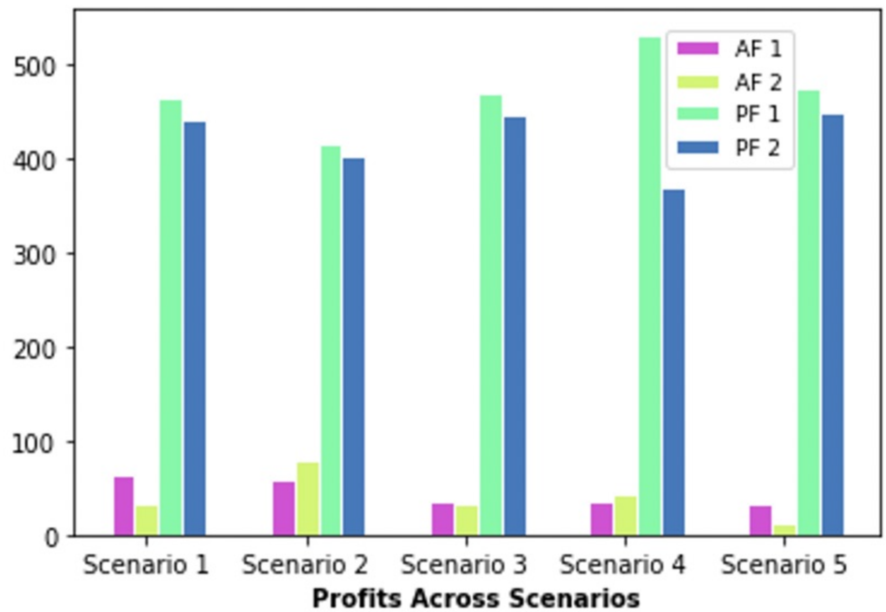


Figure 6: Demand Market Prices of Agricultural Firms Under Various Scenarios

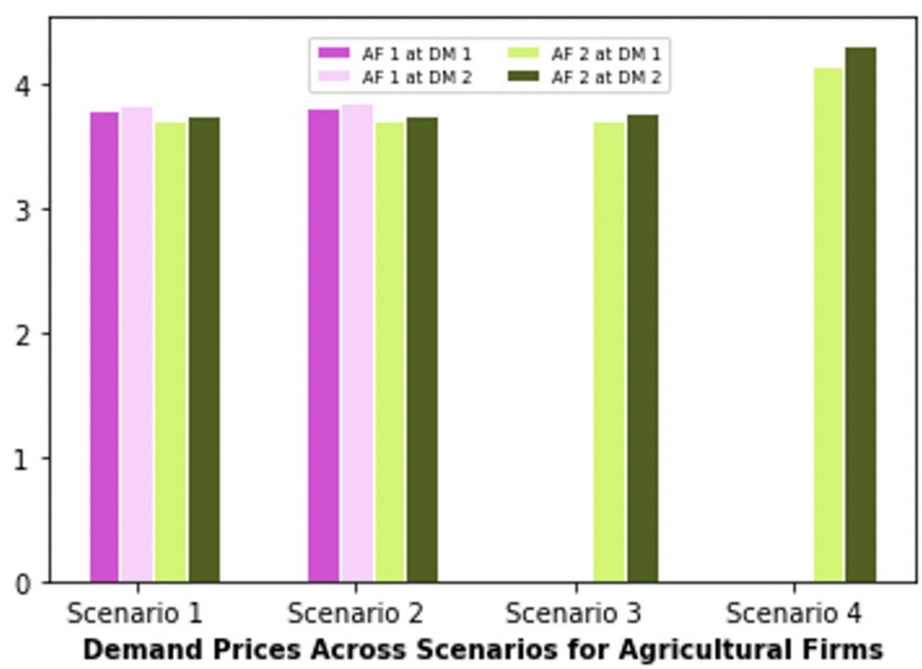
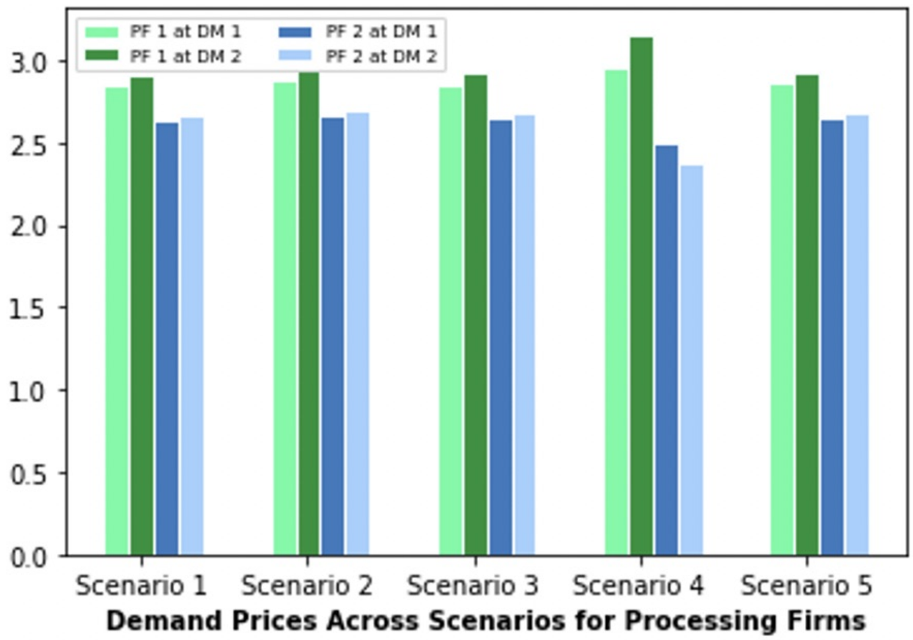


Figure 7: Demand Market Prices of Processing Firms Under Various Scenarios



longer available, we observe a profit decrease for Agricultural Firm 1. A similar result is obtained for Agricultural Firm 1 in Scenario 5. When Agricultural Firm 2 can only sell its unprocessed agricultural products to the processing firms, it makes a lower profit than it made in Scenario 1. Additionally, notice that, from Scenario 3 to Scenario 4, there is a decrease in the profits of Agricultural Firm 1 and Processing Firm 2 due to the quality problems that these two firms face in Scenario 4. This result shows the importance of quality preservation in the fresh food industry for profitability.

From Figure 6 and Figure 7 we see the impact of competition on demand market prices that consumers have to pay. With increased competition from agricultural firms, the processing firms have to keep their prices lower. The lowest demand market prices for the processing firms, hence, are observed in Scenario 1 where they face competition from the agricultural firms. The demand market prices charged by the agricultural firms are higher than that of processing firms which aligns with what we observe in reality where fresh produce from local farms have a higher price than their frozen alternatives from food processing companies such as Dole and Del Monte. Additionally, in Figure 6 we see how the demand market prices of Agricultural Firm 2 increase significantly in Scenarios 3 and 4 in the absence of competition from Agricultural Firm 1. Further, from Figure 7, it is evident how reduced quality can affect demand market prices. Notice the drop in the demand market prices associated with Processing Firm 2 in Scenario 4 as compared to Scenario 3,

Figure 8: Initial Quality Levels of Agricultural and Processing Firms in Scenario 1 (S1) and Scenario 4 (S4)

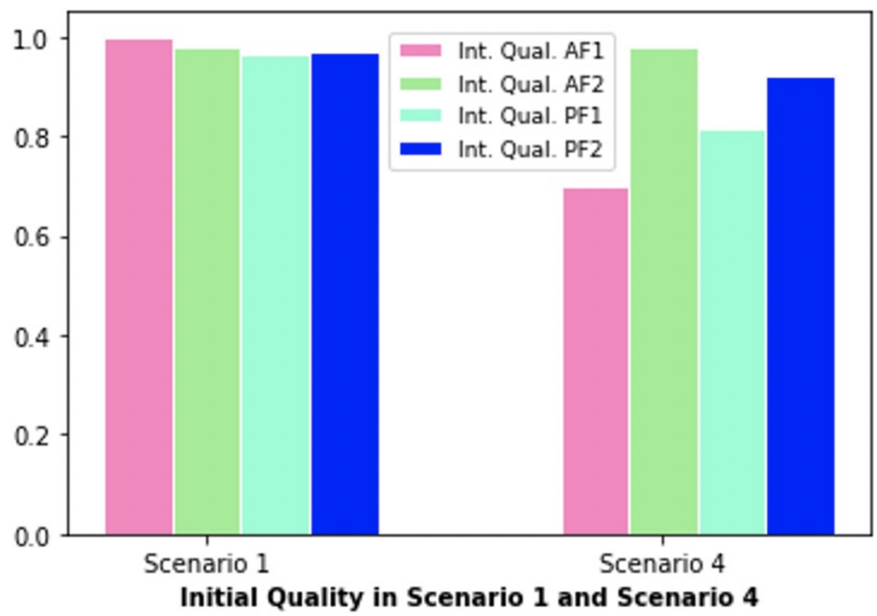
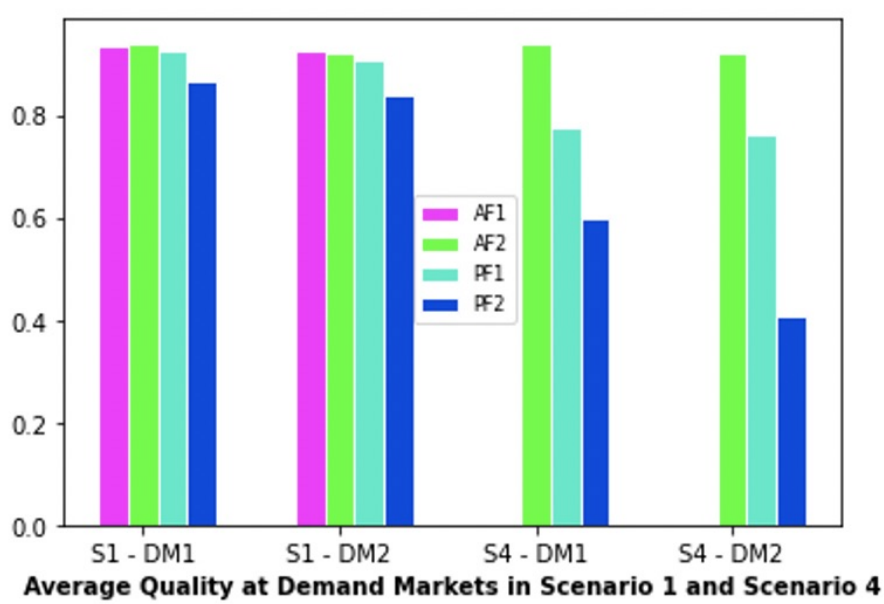


Figure 9: Average Quality Levels of Agricultural and Processing Firms at Demand Markets in Scenario 1 (S1) and 4 (S4)



even though there is less competition. These findings emphasize the need for competition and the importance of quality of the agricultural products at the demand markets from both agricultural firms' as well as consumers' perspectives. Further, from Figure 9, it is evident how the decrease in initial quality of the fresh produce product from Agricultural Firm 1 significantly reduces the average quality levels at the demand markets of the frozen carrot products from the processing firms.

In Figures 8 and 9, we present the different initial and average quality levels of the products observed in Scenarios 1 and 4. In Scenarios 2 and 3, the quality levels remain the same as in Scenario 1. In Scenario 1, note that the initial quality level of Agricultural Firm 1 is the highest. Agricultural Firm 1 ends up with the highest average quality at the demand markets as the quality loss along the path is minimal. This finding emphasizes the fact that it is crucial to ensure that the produce is harvested at the right time and temperature and also to maintain the correct temperature and timeliness of operations through the entire supply chain. In keeping with reality, we see, in Figure 9, that the average quality levels of the processed substitutes from Processing Firms 1 and 2 sold at the demand markets are lower than those of the agricultural firms. Further, from the columns for Scenario 4 in Figures 8 and 9 it is evident that the decrease in the initial quality level of the produce from Agricultural Firm 1 has a significant impact on the initial and subsequent average quality levels of the products from the processing firms. In Scenario 4, in addition to obtaining lower quality produce from Agricultural Firm 1, Processing Firm 2 is not able to maintain the required temperature during processing and faces longer processing and transportation times. This results in Processing Firm 2 having significantly lower average quality at Demand Market 2 than its competitors. This should be of major concern for a processing firm and highlights the complexity and importance of efficient supply chain management of agricultural products and all of the network paths.

7. Summary and Conclusions

It is important to introduce and study an integrated multitiered agricultural supply chain network model because of the interactions among stakeholders in such complex, critical supply chain networks to economies and societies alike. With the Covid-19 pandemic, the fragility of agricultural supply chains became evident and prompted the need for reforms in policies and supply chain practices. Analytical results obtained from studies on agricultural supply chains can provide valuable insights on the market economy. In this paper, we develop the first integrated multitiered agricultural supply chain network model in which agricultural firms and processing firms compete noncooperatively to sell their differentiated products at the demand markets. The focus here is on fresh produce, which can then be processed minimally, as well as its quality. While multitiered supply chain models have been studied in previous works, an extensive literature review did not return any paper modeling the competitive behavior of various tiers of stakeholders in an agricultural supply chain. Here, we formulate the competition among agricultural firms and processing firms at the demand markets using game theory, with the governing equilibrium being that of Cournot-Nash. Moreover, we present variational inequality formulations of the equilibrium conditions and use an algorithmic scheme that yields closed-form expressions at each iteration in order to obtain the product flows to demand markets, the shipments from agricultural firms to processing firms, and the Lagrange multipliers associated with important constraints in the mathematical model. We test our modeling framework through a numerical study based on the fresh produce agricultural product of carrots and analyze several disruption scenarios, including several ones related to the Covid-19 pandemic.

The results obtained from the numerical study align with what we would expect to see in reality and, yet, yield valuable insights. For example, we see that the demand market prices of the processing firms are lower than that of agricultural firms since larger processing firms such as Naturipe or Dole Food can utilize economies of scale and offer lower prices. We look at the effects of supply chain disruptions associated with the transportation, processing, packaging, and storage links of all the agricultural firms in the integrated multitiered agricultural supply chain network. Our findings show that, when agricultural firms sell their produce only to the processing firms and do not compete at the demand markets, their profits decrease. These findings highlight the trade-off for agricultural firms between maintaining their own supply chain networks, maintaining their brand, and competing at the demand markets versus selling their produce entirely to processing firms. We also study and report the effects of reduced production capacity, reduced competition, and decreases in quality of products on profits and demand market prices in various simulated scenarios.

An understanding of the interactions among different stakeholders in agricultural supply chain networks can assist in making the agricultural supply chains become more efficient. In addition, a fundamental framework constructed in this paper can also serve as the basis to finding solutions, via appropriate extensions, to issues such as: food waste, food insecurity, and more equitable food distribution (Orgut et al. (2018), Chen et al. (2019), Principato et al. (2020), Vargas-Lopez et al. (2021), Granillo-Macias (2021)). Such extensions can aid in making additional positive societal impact.

Acknowledgments

The authors are grateful to the three anonymous reviewers and to the Editor for their constructive comments and suggestions on two earlier versions of this paper.

The authors dedicate this paper to freedom-loving people around the globe with special acknowledgment of those fighting for the freedom of Ukrainians and the world.

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An Integrated Multitiered Model of Competing Agricultural Firms and Processing Firms

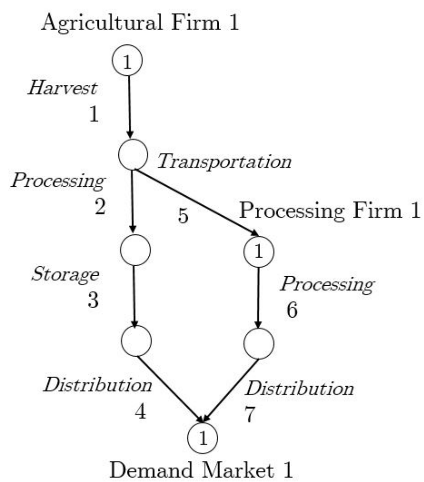
Appendix

In this Appendix, we first provide an illustrative example. We then outline the algorithm that is used in our numerical study, along with the closed form expressions induced by it for the model variables. In addition, we present the data used for the examples in our numerical study.

Simple Illustrative Example

We now present an example to illustrate the multitiered supply chain network model. The integrated multitiered agricultural supply chain network topology is depicted in Figure A.1.

Figure A.1: Supply Chain Network Topology of the Simple Illustrative Example



In this example, there is a single agricultural firm, Agricultural Firm 1, and a single processing firm, Processing Firm 1, competing in a duopolistic manner to sell their agricultural products, which are substitutable. We assume that the product is pineapple, which is minimally processed to produce fresh cut pineapples. There is a single demand market, represented by Demand Market 1. The strategic variables, $x_{p_{1AF}}$, Q_{11}, and $x_{p_{1PF}}$, represent the amount of path flow from Agricultural Firm 1 to Demand Market 1, from Agricultural Firm 1 to Processing Firm 1, and from Processing Firm 1 to Demand Market 1, respectively. Since the supply chain network topology is a simple one, there are two paths: p_{1AF} and p_{1PF}, consisting of links associated with supply chain operations of Agricultural Firm 1 and Processing Firm 1. The link sets of Agricultural Firm 1 are $L_1^1 = (1)$ and $L_2^1 = (2, 3, 4)$, whereas Processing Firm 1 has the link set $\hat{L}_1^1 = (5)$. and $\hat{L}_2^1 = (6, 7)$. Hence, we have path $p_{1AF} = (1, 2, 3, 4)$, and path $p_{1PF} = (6, 7)$. Due to simplicity, we also have that the link flows: $f_1 = x_{p_1AF} + Q_{11}$, $f_2 = f_3 = f_4 = x_{p_1AF}$, $f_5 = Q_{11}$, and $f_6 = f_7 = x_{p_1PF}$.

The total production/growing cost function of Agricultural Firm 1 in the link set L_1^1 is:

$$\bar{\hat{h}}_1(X^{AF}, Q) = (x_{p_1AF})^2 + x_{p_1AF} + Q_{11}^2 + Q_{11}.$$

The total operational cost functions related to processing, storage and distribution links of Agricultural Firm 1 in the link set L_2^1 are:

$$\bar{\hat{c}}_2(X^{AF}) = \bar{\hat{c}}_3(X^{AF}) = \bar{\hat{c}}_4(X^{AF}) = (x_{p_1AF})^2 + x_{p_1AF}.$$

The total operational cost function related to the supply chain operations of Processing Firm 1 for the shipment of unprocessed agricultural product from Agricultural Firm 1 through the transportation links in its supply chain, belonging to link set $\hat{L}_1^1$ is:

$$\bar{\hat{z}}_7(Q) = Q_{11}^2 + Q_{11}.$$

The remaining total operational cost functions associated with Processing Firm 1's supply chain links in the link set $\hat{L}_2^1$ are:

$$\bar{\hat{c}}_6(X^{PF}) = \bar{\hat{c}}_7(X^{PF}) = (x_{p_1PF})^2 + x_{p_1PF}.$$

We assume that the capacity of production at Agricultural Firm 1 is 5.00. The quality attribute considered in this example is texture loss. According to Benítez et al. (2012) texture loss of fresh cut pineapples during storage follows zero order kinetic function.The activation energy and pre-exponential factor were reported to be -9.55kJ/mol and 3.13E-3 1/hour respectively. The universal gas constant is known to be 8.314J$mol^{-1}K^{-1}$. Table A.1 displays the $\beta_{\bar{a}}$ values. We note that the Arrhenius equation includes different units for different orders of quality decay, where the zero order quality decay has the units of quality/time period. For this example, the quality decay parameters are consistent with the Arrhenius equation units for zero order quality decay where $\beta_{\bar{a}}$ has the units of quality/hour. We followed the units provided in Benitez et al. (2012). The initial quality at Agricultural Firm 1, q_{01}^{AF} is assumed to be 1.000. Using these values, we obtain the parameters reported in Table A.1. The initial quality for Processing Firm 1 is: $q_{01}^{PF} = 0.847$.

The demand price functions are:

$$\hat{\rho}_{11}^{AF}(X^{AF}, X^{PF}, \hat{q}, \bar{\hat{q}}) = 35 - x_{p_1AF} - 0.5x_{p_1PF} + 2q_{p_1AF} - q_{p_1PF},$$

$$\hat{\rho}_{11}^{PF}(X^{PF}, X^{AF}, \hat{q}, \bar{\hat{q}}) = 30 - x_{p_1PF} - 0.05x_{p_1AF} + 2q_{p_1PF} - 1.5q_{p_1AF}.$$

By using the $\beta_{\bar{a}}$ values in Table A.1, (13), and (14), we have $q_{p_1AF} = 0.217$ and $q_{p_1PF} = 0.733$. Note that here the quality for the agricultural firm is much lower than that of the processing firm

Table A.1: Parameters for the Calculation of Quality Decay

Operations	Link a	Time (hours)	Temperature (Kelvin)	Reaction Rate (k_a)	$\beta_{\bar{a}}$
Production/Harvest	1	5	298.15	0.147	0.050
Processing	2	5	298.15	0.147	0.050
Storage	3	48 (2 days)	279.15	0.192	0.619
Distribution	4	5	298.15	0.192	0.064
Transportation	5	8	279.15	0.192	0.103
Processing	6	5	298.15	0.147	0.050
Distribution	7	5	279.15	0.192	0.064

due to the fact that we have more links on the path from the agricultural firm to the demand market.

It should be noted that, for the simple example, we assume that the Agricultural Firm 1's production capacity constraint is tight. Hence, we have that: $x^*_{p_{1AF}} + Q^*_{11} = 5.00$. Using variational inequality (40), we obtain the following linear equations:

$$10x^*_{p_{1AF}} + 0.5x^*_{p_{1PF}} + \lambda^*_1 = 30.70,$$

$$4Q^*_{11} + \lambda^*_1 - \eta^*_1 = -2.00.$$

$$0.05x^*_{p_{1AF}} + 6x^*_{p_{1PF}} + \eta^*_1 = 29.14,$$

$$Q^*_{11} - x^*_{p_{1PF}} = 0.00,$$

$$x^*_{p_{1AF}} + Q^*_{11} = 5.00,$$

with the solution:

$$x^*_{p_{1AF}} = 2.63, \quad x^*_{p_{1PF}} = Q^*_{11} = 2.37, \quad \lambda^*_1 = 3.26, \quad \eta^*_1 = 14.76.$$

The demand prices are: $\rho^{AF}_{11} = 30.89$ and $\rho^{PF}_{11} = 28.63$. From variational inequality (39) we also obtain that: $\rho^*_{111} = 9$. Hence, the Agricultural Firm 1 enjoys a profit (in dollars) of $\hat{U}^{AF}_1(X^{AF*}, Q^*, X^{PF*}) = 56.38$, whereas Processing Firm 1 has a profit of $\hat{U}^{PF}_1(X^{PF*}, Q^*, X^{AF*}) = 22.58$. In this simple example, when the Lagrange multiplier associated with (29), η^*_1, is positive, $x^*_{p_{1PF}} = Q^*_{11}$.

The Algorithm for the Numerical Study

The algorithm that we use in Section 5 for the computation of the solution to the integrated multitiered supply chain network model with competing agricultural firms and processing firms is the Euler method. This method is a discrete-time algorithm that captures the dynamics of the integrated multitiered model and is one of the algorithms induced by the general iterative scheme of

Dupuis and Nagurney(1993). Specifically, at an iteration t of the Euler method (see also Nagurney and Zhang (1996)), one computes:

$$X^{t+1} = P_{\mathcal{K}}(X^t - \alpha^t F(X^t)), \tag{A.1}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (40).

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, the sequence $\{\alpha^t\}$ must satisfy: $\sum_{t=0}^{\infty} \alpha^t = \infty$, $\alpha^t > 0$, $\alpha^t \to 0$, as $t \to \infty$. Specific conditions for convergence of this scheme as well as solutions of a plethora of network oligopoly problems, see Nagurney, Dupuis, and Zhang (1994), Nagurney and Zhang (1996), Nagurney (2010), Nagurney and Yu (2012), and Masoumi, Yu, and Nagurney (2012).

Explicit Formulae for the Euler Method Applied to the Solution of the Integrated Multitiered Supply Chain Network Model

The explicit formulae are now given. The closed form expressions for the agricultural product path flows from agricultural firms to the demand markets at iteration $t+1$ are: For each path $p_{AF} \in P^i_k$ compute:

$$x^{t+1}_{p_{AF}} = \max\{0, x^t_{p_{AF}} + \alpha^t(\hat{\rho}^{AF}_{ik}(X^{AF^t}, X^{PF^t}, \hat{q}^t, \bar{\hat{q}}^t) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}^{AF}_{il}(X^{AF^t}, X^{PF^t}, \hat{q}^t, \bar{\hat{q}}^t)}{\partial x_{p_{AF}}} \sum_{r_{AF} \in P^i_k} x^t_{r_{AF}} -$$

$$\frac{\sum_{a \in L^i_1} \partial \bar{\hat{h}}_a(X^{AF^t}, Q^t)}{\partial x_{p_{AF}}} - \frac{\partial \hat{C}_{p_{AF}}(X^{AF^t})}{\partial x_{p_{AF}}} - \lambda^t_i)\}, \; \forall p_{AF} \in P^i_k; \; i = 1, \ldots, I; \; k = 1, \ldots, n_R. \tag{A.2}$$

The closed form expressions for the agricultural product shipments from the agricultural firms to the processing firms at iteration t are: For each product shipment from the production facility of agricultural firm i to the processing firm j compute:

$$Q^{t+1}_{ij} = \max\{0, Q^t_{ij} + \alpha^t(\eta_j - \frac{\sum_{a \in L^i_1} \partial \bar{\hat{h}}_a(X^{AF^t}, Q^t)}{\partial Q_{ij}} - \frac{\sum_{b \in \hat{L}^j_2} \partial \bar{\hat{z}}_b(Q^t)}{\partial Q_{ij}} - \lambda^t_i)\},$$

$$\forall i, j; \; i = 1, \ldots, I; \; j = 1, \ldots, J. \tag{A.3}$$

The closed form expressions for the Lagrange multipliers associated with constraint (23) are:

$$\lambda^{t+1}_i = \max\{0, \lambda^t_i + \alpha^t(\sum_{j=1}^{J} Q^t_{ij} + \sum_{k=1}^{n_R} \sum_{p_{AF} \in P^i_k} x^t_{p_{AF}} - CAP_i)\}, \forall i; \; i = 1, \ldots, I. \tag{A.4}$$

We also have the following closed form expressions for each product path flow from the processing firms to the demand markets $p_{PF} \in P_k^j$:

$$x_{p_{PF}}^{t+1} = \max\{0, x_{p_{PF}}^t + \alpha^t(\hat{\rho}_{jk}^{PF}(X^{PF^t}, X^{AF^t}, \hat{q}^t, \bar{\hat{q}}^t) + \sum_{h=1}^{n_R} \frac{\partial \bar{\hat{\rho}}_{jh}^{PF}(X^{PF^t}, X^{AF^t}, \hat{q}^t, \bar{\hat{q}}^t)}{\partial x_{p_{PF}}} \sum_{s_{PF} \in P_k^j} x_{s_{PF}}^t - \frac{\partial \hat{C}_{p_{PF}}(X^{PF^t})}{\partial x_{p_{PF}}}$$

$$-\eta_j^t)\}, \ \forall p_{PF} \in P_k^j; \ j = 1, \ldots, J; \ k = 1, \ldots, n_R. \qquad (A.5)$$

Finally, the closed form expressions for the Lagrange multipliers associated with constraint (29) are:

$$\eta_j^{t+1} = \max\{0, \eta_j^t + \alpha^t(\sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}}^t - \sum_{i=1}^{I} Q_{ij}^t)\}, \ \forall j; \ j = 1, \ldots, J. \qquad (A.6)$$

Numerical Study Data

We now report the data that are used in the examples in our numerical study. Recall that the the fresh produce considered is carrots with the minimal processing consisting of freezing the carrots. The parameters for the calculation of quality decay in Scenario 1 are reported in Tables A.2 and A.3.

Table A.2: Parameters for the Calculation of Quality Decay in Scenario 1

Arrhenius Parameters	**Fresh Carrots - First Order**	**Frozen Carrots - Zero Order**
Activation Energy (E_a)	44.33	35.59
Pre-exponential factor, A	94.32 (sec)	60240 (days)
Universal gas constant, R	8.314	8.314

The demand price functions of the agricultural firms and the processing firms in the numerical study are given below. The production cost functions, the total operational cost functions, and the total shipment cost functions, in turn, are given in Table A.4.

Agricultural Firm 1:

$$\rho_{11}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}) = -0.002 d_{11}^{AF} - 0.001 d_{21}^{AF} - 0.0001 d_{11}^{PF} - 0.0001 d_{21}^{PF} + 5 + 1.7 q_{p_{1AF}} - q_{p_{3AF}} - q_{p_{1PF}} - q_{p_{3PF}},$$

$$\rho_{12}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}) = -0.002 d_{12}^{AF} - 0.001 d_{22}^{AF} - 0.0001 d_{12}^{PF} - 0.0001 d_{22}^{PF} + 5 + 1.7 q_{p_{2AF}} - q_{p_{4AF}} - q_{p_{2PF}} - q_{p_{4PF}},$$

Agricultural Firm 2:

$$\rho_{21}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}) = -0.002 d_{21}^{AF} - 0.001 d_{11}^{AF} - 0.0001 d_{11}^{PF} - 0.0001 d_{21}^{PF} + 5 + 1.6 q_{p_{3AF}} - q_{p_{1AF}} - q_{p_{1PF}} - q_{p_{3PF}},$$

$$\rho_{22}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}) = -0.002 d_{22}^{AF} - 0.001 d_{12}^{AF} - 0.0001 d_{12}^{PF} - 0.0001 d_{22}^{PF} + 5 + 1.6 q_{p_{4AF}} - q_{p_{2AF}} - q_{p_{2PF}} - q_{p_{4PF}},$$

Table A.3: Parameters for the Calculation of Quality Decay

Operations	**Links**	**Time (sec)**	**Time (hrs)**	**Time (days)**	**Temperature (Kelvin)**	**Reaction Rate (k_a)**	$\beta_{\bar{a}}$
Production/Harvest	1	21600	6	0.250	285.15	7.1E-07	0.985
Transporation	2	3600	1	0.042	285.15	7.1E-07	0.997
Processing	3	7200	2	0.083	278.15	4.5E-07	0.997
Packaging	4	3600	1	0.042	278.15	4.5E-07	0.998
Storage	5	25200	7	0.292	274.15	3.4E-07	0.992
Transporation DM 1	6	72000	20	0.833	280.15	5.1E-07	0.964
Transporation DM 2	7	86400	24	1.000	280.15	5.1E-07	0.957
Ship AF 1 -PF 1	8	3600	1	0.042	283.15	6.3E-07	0.998
Ship AF1 - PF 2	9	3600	1	0.042	283.15	6.3E-07	0.998
Processing PF 1	10	86400	24	1.000	280.15	1.4E-02	0.014
Packaging PF 1	11	25200	7	0.292	280.15	1.4E-02	0.004
Storage PF 1	12	36000	10	0.417	283	1.6E-02	0.007
Transportation PF 1 - DM 1	13	86400	24	1.000	283.15	1.6E-02	0.016
Transportation PF 1 - DM 2	14	172800	48	2.000	283.15	1.6E-02	0.033
Ship AF2 - PF 1	15	36000	10	0.417	280.15	5.1E-07	0.982
Ship AF2 -PF 2	16	18000	5	0.208	280.15	5.1E-07	0.991
Processing PF 2	17	86400	24	1.000	280.15	1.4E-02	0.014
Packaging PF 2	18	25200	7	0.292	280.15	1.4E-02	0.004
Storage PF 2	19	432000	120	5.000	280.15	1.4E-02	0.070
Transportation PF 2 - DM 1	20	86400	24	1.000	283.15	1.6E-02	0.016
Transportation PF 2 - DM 2	21	259200	72	3.000	280.15	1.4E-02	0.042
Production/Harvest	22	21600	6	0.250	285.15	7.1E-07	0.985
Transporation	23	3600	1	0.042	285.15	7.1E-07	0.997
Processing	24	7200	2	0.083	285.15	7.1E-07	0.995
Packaging	25	3600	1	0.042	285.15	7.1E-07	0.997
Storage	26	25200	7	0.292	283.15	6.3E-07	0.984
Transporation DM 1	27	36000	10	0.417	280.15	5.1E-07	0.982
Transporation DM 2	28	43200	12	0.500	280.15	5.1E-07	0.978

Processing Firm 1:

$$\rho_{11}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}}) = -0.002d_{11}^{PF} - 0.001d_{21}^{PF} - 0.001d_{11}^{AF} - 0.001d_{21}^{AF} + 5 + 1.2q_{p_{1PF}} - q_{p_{1AF}} - q_{p_{3AF}} - q_{p_{3PF}},$$

$$\rho_{12}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}}) = -0.002d_{12}^{PF} - 0.001d_{22}^{PF} - 0.001d_{12}^{AF} - 0.001d_{22}^{AF} + 5 + 1.2q_{p_{2PF}} - q_{p_{2AF}} - q_{p_{4AF}} - q_{p_{4PF}},$$

Processing Firm 2:

$$\rho_{21}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}}) = -0.002d_{21}^{PF} - 0.001d_{11}^{PF} - 0.001d_{11}^{AF} - 0.001d_{21}^{AF} + 5 + 1.1q_{p_{3PF}} - q_{p_{1AF}} - q_{p_{3AF}} - q_{p_{1PF}},$$

$$\rho_{22}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}}) = -0.003d_{22}^{PF} - 0.001d_{12}^{PF} - 0.001d_{12}^{AF} - 0.001d_{22}^{AF} + 5 + 1.1q_{p_{4PF}} - q_{p_{2AF}} - q_{p_{4AF}} - q_{p_{1PF}}.$$

Table A.4: Total Production Cost Functions, Total Operational Cost Functions, and Total Shipment Cost Functions

Operations	**Links**	$\hat{h}_a(f^1)$	$\hat{c}_b(f^2)$	$\hat{z}_b(Q)$	$\hat{c}_e(f^3)$
production	1	$0.0002f_1^2 + 0.01f_1$	-	-	-
transportation	2	-	$0.003f_2^2 + 0.2f_2$	-	-
processing	3	-	$0.015f_3^2 + 0.3f_3$	-	-
packaging	4	-	$0.015f_4^2 + 0.3f_4$	-	-
storage	5	-	$0.015f_5^2 + 0.3f_5$	-	-
transportation	6	-	$0.001f_6^2 + 0.25f_6$	-	-
transportation	7	-	$0.002f_7^2 + 0.2f_7$	-	-
transportation	8	-	-	$0.0001Q_{11}^2 + 0.001Q_{11}$	-
transportation	9	-	-	$0.0002Q_{12}^2 + 0.002Q_{12}$	-
processing	10	-	-	-	$0.0008f_{10}^2 + 0.02f_{10}$
packaging	11	-	-	-	$0.0005f_{11}^2 + 0.03f_{11}$
storage	12	-	-	-	$0.0005f_{12}^2 + 0.03f_{12}$
transportation	13	-	-	-	$0.001f_{13}^2 + 0.04_{13}$
transportation	14	-	-	-	$0.0015f_{14}^2 + 0.05f_{14}$
transportation	15	-	-	$0.0002Q_{21}^2 + 0.002Q_{21}$	-
transportation	16	-	-	$0.0001Q_{22}^2 + 0.001Q_{22}$	-
processing	17	-	-	-	$0.0005f_{17}^2 + 0.01f_{17}$
packaging	18	-	-	-	$0.0005f_{18}^2 + 0.03f_{18}$
storage	19	-	-	-	$0.0015f_{19}^2 + 0.03f_{19}$
transportation	20	-	-	-	$0.001f_{20}^2 + 0.04f_{20}$
transportation	21	-	-	-	$0.001f_{21}^2 + 0.03f_{21}$
production	22	$0.0002f_{22}^2 + 0.1f_{22}$	-	-	-
transportation	23	-	$0.005f_{23}^2 + 0.3f_{23}$	-	-
processing	24	-	$0.025f_{24}^2 + 0.3f_{24}$	-	-
packaging	25	-	$0.015f_{25}^2 + 0.3f_{25}$	-	-
storage	26	-	$0.015f_{26}^2 + 0.3f_{26}$	-	-
transportation	27	-	$0.002f_{27}^2 + 0.2f_{27}$	-	-
transportation	28	-	$0.001f_{28}^2 + 0.1f_{28}$	-	-