Multicommodity Trade, Tariffs, and Rerouting

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Abstract: The concept of "equilibrium" is fundamental to many disciplines, including economics and operations research. The theory of variational inequalities continues to provide a rich methodological framework for the formulation, analysis, and solution of a wide range of equilibrium problems with many recent applications, because of their relevance, including policies associated with the trade of commodities. In this paper, we add to the literature on multicommodity trade by constructing a spatial price equilibrium model with tariffs that allows for the investigation of rerouting of commodities to evade tariffs, as has been happening for several years now, along with "trade wars."

Key words: multicommodity trade, tariffs, networks, equilibrium problems, rerouting, variational inequalities

1. Introduction

Equilibrium is a central concept in many scientific disciplines, including economics and operations research. Its importance has been demonstrated in applications such as traffic network equilibrium problems (cf. Beckmann, McGuire and Winsten (1956), Braess (1968), Dafermos and Sparrow (1969), Smith (1979), Dafermos (1980), Bertsekas and Gafni (1982), Patriksson (1994), Nagurney (2000), Patriksson and Rockafellar (2003), Boyce, Mahmassani, and Nagurney (2005) and the references therein, and Braess, Nagurney, and Wakolbinger (2005)) and spatial price equilibrium problems (see Samuelson (1952), Takayama and Judge (1964, 1971), Florian and Los (1982), Dafermos and Nagurney (1984), Pang (1984), Friesz, Harker, and Tobin (1984), Harker (1985), Nagurney, Thore, and Pan (1996), Labys and Yang (1997), Nagurney (1999), Daniele (2004), Nagurney, Salarpour, and Dong (2022), Nagurney et al. (2024a, b), Hassani et al. (2025) and the references therein). Examples of other important equilibrium problems include: general economic equilibrium problems (cf. Debreu (1959), Scarf (1973), Todd (1976), Border (1985), Manne (1985), Dafermos (1990), Zhao and Dafermos (1991), Zhao and Nagurney (1993), Jofré, Rockafellar, and Wets (2007, 2017, 2023)), a variety of game theory (cf. Nash (1950, 1951)) and oligopolistic market equilibrium problems (see, e.g., Gabay and Moulin (1980), Dafermos and Nagurney (1987), Yu and Nagurney (2013), Besik and Nagurney (2017), Besik, Nagurney, and Dutta (2023)), and even problems of human migration (see Nagurney (1989), Nagurney, Pan, and Zhao (1992), Nagurney, Daniele, and Nagurney (2020), Nagurney and Daniele (2021), Nagurney, Daniele, and Cappello (2021)).

Equilibrium has also emerged as an important concept in numerous supply chain network problems (cf. Nagurney, Dong, and Zhang (2002) and the books by Nagurney (2006), Nagurney and Li (2016), Nagurney (2023) and the references therein). It is also emerging as a topic in disaster management and emergency response (see Nagurney, Salapour, and Daniele (2019), Nagurney et al. (2024b)). Equilibrium has, hence, been a topic driving methodological advances for both qualitative analysis of solutions in a plethora of applications as well as their computation.

Concomitantly, policies have emerged as powerful instruments to alter the behavior of various decision-makers in equilibrium problems, many of which may take on a network structure (see, for example, Nagurney (2021)). For example, tolls have been constructed so that, once applied, user-optimizing behavior on congested urban transportation networks becomes, at the same time, system-optimizing (see Dafermos and Sparrow (1971), Dafermos (1973), Nagurney (2025)). A variety of trade policy instruments include: tariffs (Nagurney, Nicholson, and Bishop (1996), Nagurney, Besik, and Dong (2019), Nagurney, Besik, and Li

(2019)), quotas (Nagurney, Besik, and Nagurney (2019), Nagurney, Salarpour, and Dong (2022)), with subsidies also garnering a lot of attention in terms of research and the real world due to impacts such as "trade wars." Subsidies have also been studied as a means to reduce food insecurity (see Nagurney et al. (2023), Nagurney (2024)).

The theory of variational inequalities (cf. Hartman and Stampacchia (1966), Kinderlehrer and Stampacchia (1980), Dafermos (1980), Rockafellar and Wets (1998), Nagurney and Zhang (1996), Nagurney (1999), Facchinei and Pang (2003)) is providing a rich framework for the formulation, analysis, and computation of solutions to a wide range of equilibrium problems, with some noted above. Being able to incorporate policies into equilibrium settings in economics and operations research enables the provision of insights for strategic decision-making and evaluation.

In this paper, we introduce a new multicommodity trade model based on spatial price equilibrium in which multicommodity tariffs can be imposed by destination countries on the commodities based on their country of origin. Through the introduction of path tariffs we are able to identify equilibrium conditions that allow for the rerouting of commodities in order to evade tariffs. The tariffs are unit tariffs (see Nagurney (2022), Nagurney, Salarpour, and Dong (2022), and Nagurney et al. (2023)) as opposed to ad valorem tariffs (cf. Nagurney, Nicholson, and Bishop (1996)) or tariff rate quotas, which are two-tiered tariffs (see Nagurney, Besik, and Dong (2019)). We allow for multiple trade routes; that is, paths, between country origin nodes and country destination nodes, and these paths can consist of one or more links. The links, in turn, in the case of rerouting of the commodity through countries to evade a tariff, can correspond to transportation links or repackaging/reprocessing links. For the sake of generality, we consider links to be transaction links, with associated costs.

Spatial price equilibrium models, introduced by Samuelson (1952) and Takayama and Judge (1964, 1971), are partial equilibrium models and assume perfect competition. As noted in Nagurney et al. (2023), variational inequality formulations of such problems relaxed previously imposed assumptions on the underlying supply price and demand price functions with unit transportation cost functions, typically, assumed to be fixed, that enabled optimization reformulations of the equilibrium conditions. Furthermore, the importance of having alternative routes between supply and demand markets, along with capturing congestion on transportation links, was recognized already in the models of Florian and Los (1982), Dafermos and Nagurney (1984), Friesz, Harker, and Tobin (1984), Harker (1985), Nagurney, Thore, and Pan (1996), and Daniele (2004)). Other variational inequality models of spatial price equilibrium problems have included product quality (Li, Nagurney, and Yu (2018)) and commodity perishability (see Nagurney and Aronson (1989), Nagurney (2022), Nagurney and Besik (2022), Nagurney, Pour, and Samadi (2024)). As noted in Nagurney, Li, and Nagurney (2014), spatial price equilibrium models have been widely applied to trade of agricultural products, including eggs, potatoes, cereal grains, soybeans, and dairy, as well as to minerals (coal and aluminum) and such commodities have also been subject to tariffs (cf. also Bradsher and Pierson (2025)). Hence, in our investigation in this paper on tariffs and rerouting, we utilize a multicommodity spatial price equilibrium framework.

The research reported in this paper is inspired by practice. Tariffs have a long history and have been considered to be protectionist measures by governments imposing them on imported products produced in other countries (see Holmes (2025)). They also can be viewed as a "tax" on the consumers purchasing products that have been subjected to tariffs (see Bednar (2025)). Furthermore, as is happening now, countries on whose products tariffs are imposed may respond with retaliatory tariffs leading to trade wars (for some background, see Nagurney, Besik, and Dong (2019), Boak, Wiseman, and Gillies (2025), Lu (2025)). Producing countries (and firms therein) have responded to tariffs in additional ways including through nearshoring; that is, moving production closer, or onshoring (producing within one's country), and, sometimes, even rerouting products through third countries. The latter, with alteration of the origin country through documentation and without substantive addition of value to the products, is not legal but, nevertheless, may happen and the economic trade-offs are worth quantifying and exploring. This topic has garnered recent interest and is being discussed in various studies (see Wen et al. (2025), Freund (2025), Perozo (2025)). However, as far as we are aware, this is the first rigorous mathematical modeling approach proposed for the investigation of rerouting of commodities for the purpose of tariff evasion. Such a model can also serve as the foundation for the exploration of interdiction mechanisms. For our related work on cybercrime and cybersecurity, see: Nagurney (2015), Nagurney, Daniele, and Shukla (2017), and Nagurney and Shukla (2017). For research on game theory and investments in security for high value freight, see Nagurney et al. (2018).

This paper is organized as follows. In Section 2, the multicommodity spatial price equilibrium model with tariffs and rerouting is proposed, with identification of the equilibrium conditions, and the variational inequality formulation, which is in path flows, constructed. Existence results, under reasonable conditions on the underlying economic functions, are also established. Several numerical examples are presented for illustrative purposes. In Section 3, the computational procedure is outlined, along with the closed form expressions for the multicommodity shipments at each of the two steps of the iterative process, along with conditions for convergence. In Section 4, we then implement the algorithm and apply it to compute solutions to expanded numerical examples, focusing on the export of tea. Section

5 summarizes the results in this paper and also provides suggestions for future research.

2. The Multicommodity Trade Spatial Price Equilibrium Model with Tariffs and Rerouting

Consider a network consisting of m origin nodes corresponding to distinct countries in which multiple homogeneous products (commodities) are produced at supply markets and with n destination nodes corresponding to countries with demand markets at which the products are consumed, with each also denoting a distinct country. There are K commodities, with a typical commodity denoted by k. Each origin node i is connected to a destination node j via one or more paths, assumed to be acyclic, with a typical path denoted by p. Each path represents a trade route and consists of one or more directed links that join nodes in the network. Intermediate nodes in the network, which are transshipment locations, also correspond to countries. However, such transshipment countries/nodes can be either those in which no avoidance of tariffs takes place for a specific commodity or one in which intentional misrepresentation does take place and, in effect, the commodity is repackaged/relabeled as having the new country of origin. Let P_{ij} denote the set of paths connecting the pair of origin/destination country nodes (i, j), with the set of all paths denoted by P. In our modeling framework, there may be multiple paths joining a pair of country origin/destination nodes. This feature provides for greater flexibility in modeling and allows for the investigation of alternative routes. The set P^i denotes all the paths from country i to the destination countries, and the set P_j denotes the set of paths from all origin countries to destination country j. The network is represented by the graph $G = [N, L]$, where N is the set of nodes in the network and L is the set of links. A typical link is denoted by a and can be either of two types: a transportation link or a processing link to evade a tariff. Transportation links can correspond to different modes of transportation. A depiction of a hypothetical spatial price network topology is given in Figure 1. As noted in the Introduction, we refer, for the sake of generality, to links as being "transaction" links.

The additional notation for the trade model is now presented. All vectors are column vectors. Let Q_p^k denote the flow of commodity k on path p and group all such flows into the vector $Q \in R_+^{Kn_P}$, where n_P denotes the number of paths in the network. The flow on a link a of commodity k is denoted by f_a^k and all the link flows are grouped into the vector $f \in R_+^{Kn_L}$, where n_L is the number of links in the network.

The Conservation of Flow Equations

The conservation of flow equations are now presented. We assume, due to capacity con-

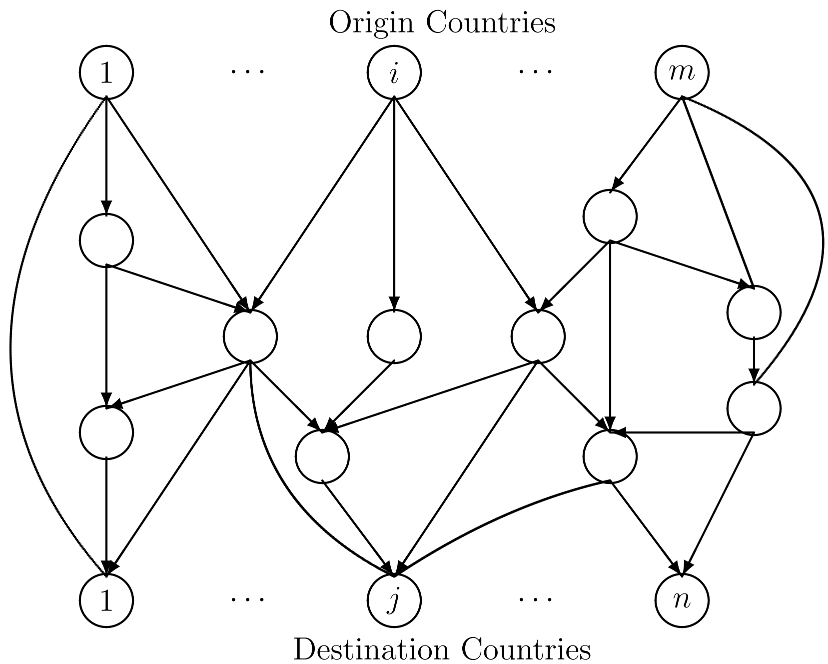

Figure 1: An Example of a Spatial Price Network Topology with Origin Countries and Destination Countries

straints, that each commodity path shipment Q_p^k is subject to a positive upper bound u_p^k for all commodities $k = 1, \ldots, K$ and all paths $p \in P$. We group the upper bounds into the vector $u \in R^{Kn_P}$.

All commodity path flows must be nonnegative and bounded; that is:

$$u_p^k \geq Q_p^k \geq 0, \quad k = 1, \ldots, K; \forall p \in P. \tag{1}$$

The flow on a link a of commodity k is equal to the sum of the path flows of the commodity k that use the link; that is:

$$f_a^k = \sum_{p \in P} Q_p^k \delta_{ap}, \quad k = 1, \ldots, K; \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and $\delta_{ap} = 0$, otherwise.

The supply of commodity k produced in country i, s_i^k, is equal to the shipments of the commodity from the country to all destination countries:

$$s_i^k = \sum_{p \in P^i} Q_p^k, \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{3}$$

The demand for commodity k in country j, d_j^k, in turn, is equal to the shipments of the commodity from all origin countries to that country:

$$d_j^k = \sum_{p \in P_j} Q_p^k, \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{4}$$

The supplies of the commodities are grouped into the vector $s \in R_+^{Km}$ and the demands into the vector $d \in R_+^{Kn}$.

The Tariffs

As mentioned in the Introduction, there are different types of tariffs. In order to be able to capture rerouting of commodities with the goal of avoiding tariffs, we introduce tariffs on paths, which can be commodity-specific. Let τ_p^k denote the unit tariff imposed on commodity k associated with trade route p and recall that a trade route is between a specific country origin / destination pair (i, j). Since tariffs are not imposed within a country on commodities produced and consumed there, if a path p joins origin and destination nodes within the same country, then $\tau_p^k = 0$ for all commodities k therein. Tariffs are nonnegative. A tariff will have a positive value if a country j imposes a tariff on a commodity k originating/produced in country i and the commodity is transported on a trade route / path that does not include a link that corresponds to masking the country of origin through repackaging and relabeling. In fact, for all such paths $p \in P_{ij}$, the tariff $\tau_p^k = \tau_{ij}^k$, for each commodity k; in other words, the unit tariff would be the same. For model consistency and formulation of the equilibrium conditions as a variational inequality problem, path tariffs on trade routes that reroute the commodity in order to evade a tariff payment are set to zero (except if there is a tariff on the country on the rerouting path, in which case that tariff would be applied). The packaging/processing link will be part of a rerouting path, if such an option exists, and will have an associated cost.

We assume in our model that the tariffs and the multicommodity supply price, demand price, and unit transaction cost functions that we elaborate upon below are all in a common currency and that they are all continuous.

The Multicommodity Functions for the Model

The supply price function for commodity k of country i is denoted by π_i^k. The supply price of a commodity k in country i can, in general, depend on the supply of the commodity in the country, and also on the supplies of the other commodities in the country as well as on the supplies of all the commodities in all the other countries; that is:

$$\pi_i^k = \pi_i^k(s), \quad k = 1, \dots, K; i = 1, \dots, m. \tag{5a}$$

We group the supply prices into the vector $\pi \in R^{Km}$.

In view of the conservation of flow equations (3), we may define new supply price functions $\tilde{\pi}_i^k$; $k = 1, \dots, K$; $i = 1, \dots, m$, such that

$$\tilde{\pi}_i^k(Q) \equiv \pi_i^k(s). \tag{5b}$$

The demand price functions can also be quite general and can depend not only on the demand for the commodity in the specific country, but also on the demands for the other commodities in the country plus on the demands for the commodities in other countries; thus:

$$\rho_j^k = \rho_j^k(d), \quad k = 1, \dots, K; j = 1, \dots, n, \tag{6a}$$

where ρ_j^k is the demand price function for commodity k in country j. We group the demand prices into the vector $\rho \in R^{Kn}$.

Through the use of conservation of flow equations (4), we may define demand price functions $\tilde{\rho}_j^k$; $k = 1, \dots, K$; $j = 1, \dots, n$, as:

$$\tilde{\rho}_j^k(Q) \equiv \rho_j^k(d). \tag{6b}$$

With each commodity k and each link $a \in L$, we associate a unit "transaction" cost c_a^k where

$$c_a^k = c_a^k(f), \quad k = 1, \dots, K; \forall a \in L. \tag{7a}$$

The unit transaction link cost functions may be of two types. The unit transaction cost may correspond to a unit transportation cost on a transportation link as the commodity is transported on a trade route from an origin country to a destination country or it may correspond to a unit processing cost on a link on a trade route to evade a tariff. Note that such a processing cost can expect to include a repackaging of the commodity as well as a relabeling and would occur (more likely) in a country that does not have a tariff imposed on it by the destination country.

We allow for the general situation that a transaction cost on a link associated with a commodity can depend on the entire vector of commodity link flows. Of course, in specific applications, the supply price, demand price, and unit transaction costs would be parametrized accordingly.

Due to the conservation of flow equations (2), we can define link unit transaction cost functions $\tilde{c}_a^k(Q)$, for all commodities and all links thus:

$$\tilde{c}_a^k(Q) \equiv c_a^k(f), \quad k = 1, \ldots, K; \forall a \in L. \tag{7b}$$

The unit transaction cost on a trade route / path p for commodity k, denoted by C_p^k, is equal to the sum of the unit transaction costs that make up the path; that is:

$$C_p^k(Q) = \sum_{a \in L} \tilde{c}_a^k(Q)\delta_{ap}, \quad k = 1, \ldots, K; \forall p \in P. \tag{8}$$

We group the unit transaction cost functions on the paths into the vector $C \in R^{Kn_P}$.

The multicommodity trade spatial price equilibrium conditions under tariffs and rerouting are now stated.

Definition 1: The Multicommodity Trade Spatial Price Equilibrium Conditions Under Tariffs and Rerouting

A multicommodity path trade flow pattern $Q^* \in \mathcal{K}$, where $\mathcal{K} \equiv \{Q | 0 \leq Q \leq u\}$, is a trade spatial price equilibrium pattern under tariffs and rerouting if the following conditions hold: For all pairs of country origin and destination nodes: (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and all paths $p \in P_{ij}$ as well as for all commodities k; $k = 1, \ldots, K$:

$$\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k \begin{cases} \leq \tilde{\rho}_j^k(Q^*), & \text{if} \quad Q_p^{k*} = u_p^k, \\ = \tilde{\rho}_j^k(Q^*), & \text{if} \quad 0 < Q_p^{k*} < u_p^k, \\ \geq \tilde{\rho}_j^k(Q^*), & \text{if} \quad Q_p^{k*} = 0. \end{cases} \tag{9}$$

According to equilibrium conditions (9), if there is a positive flow of a commodity between a pair of origin and destination countries on a path and the flow is not at its upper bound, then the supply price of the commodity at the origin country of the commodity plus the path unit transaction cost plus the tariff between the two countries on the specific path is equal to the demand price of the commodity at the destination country. On the other hand, the equilibrium commodity shipment between a pair of origin and destination countries will be equal to zero if the supply price of the commodity at the origin country plus the path unit transaction cost on the path (trade route) for the commodity plus the tariff on the commodity on the path is greater than or equal to the demand price consumers are willing to pay for the commodity at the destination country. If the equilibrium commodity shipment on a path is equal to its upper bound, then the commodity supply price at the origin country plus the path unit transaction cost on the path plus the tariff on the path for the commodity is less than or equal to the demand price for the commodity at the destination country.

The equilibrium conditions (9) capture, in a novel way, the possibility of evading a tariff. The consumers are responsive to the demand price that they pay for the commodity and, since the commodities are homogeneous, they are indifferent as to the country of origin. The demand market price has to cover the costs of production, transportation, and, in the case of evasion of the tariff, the repackaging/relabeling cost. In the above equilibrium conditions such costs are encumbered, although the commodity that is rerouted on a path to evade a tariff will have a "new" country of origin after labeling, and that path's tariff is set to zero (unless it is also tariffed).

Theorem 1: Variational Inequality Formulation of the Multicommodity Trade Spatial Price Equilibrium Conditions Under Tariffs and Rerouting

A multicommodity path trade flow pattern $Q^* \in \mathcal{K}$ is a multicommodity trade spatial price network equilibrium pattern under tariffs and rerouting, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p\in P_{ij}} \left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k - \tilde{\rho}_j^k(Q^*)\right] \times \left[Q_p^k - Q_p^{k*}\right] \geq 0, \quad \forall Q \in \mathcal{K}. \tag{10}$$

Proof:

First, we establish necessity; that is, we show that if $Q^* \in \mathcal{K}$ satisfies equilibrium conditions (9) then it also satisfies variational inequality (10).

Note that, according to equilibrium conditions (9), for a fixed commodity k, a fixed country pair (i, j), and path $p \in P_{ij}$, one must have that:

$$\left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k - \tilde{\rho}_j^k(Q^*)\right] \times \left[Q_p^k - Q_p^{k*}\right] \geq 0, \tag{11}$$

for any Q_p^k such that $0 \leq Q_p^k \leq u_p^k$.

Indeed, if $Q_p^{k*} = u_p^k$, then, according to (9),

$$\left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k - \tilde{\rho}_j^k(Q^*)\right] \leq 0. \tag{12}$$

Since $Q_p^k \leq u_p^k$, we also know that $(Q_p^k - Q_p^{k*}) \leq 0$ and, therefore, (11) holds.

On the other hand, if $0 < Q_p^{k*} < u_p^k$, then, according to (9):

$$\left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k - \tilde{\rho}_j^k(Q^*)\right] = 0, \tag{13}$$

and, consequently, (11) also holds.

Finally, if $Q_p^{k*} = 0$, then, according to equilibrium conditions (9):

$$\left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k - \tilde{\rho}_j^k(Q^*)\right] \geq 0, \tag{14}$$

and (11) also holds, since $Q_p^k \geq Q_p^{k*}$.

Since (11) holds for any commodity k, and any path p, summation of (11) over all commodities k, all paths $p \in P_{ij}$, and over all country pairs (i, j), yields:

$$\sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{p \in P_{ij}} \left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \tilde{\rho}_j^k(Q^*)\right] \times \left[Q_p^k - Q_p^{k*}\right] \geq 0, \quad \forall Q \in \mathcal{K}, \tag{15}$$

which is variational inequality (10). Necessity has been established.

We now turn to proving sufficiency. In particular, we show that, if $Q^* \in \mathcal{K}$ satisfies variational inequality (10), then it also satisfies equilibrium conditions (9).

Let $Q_q^g = Q_q^{g*}$, for all commodities g, $g \neq k$, and for all paths $q \in P_{hl}$, for all hl except for path $p \in P_{ij}$ and substitute the resultants into (10). Then, (10) simplifies to:

$$\left[\tilde{\pi}_i^k(Q^*) + C_p^k(Q^*) + \tau_p^k - \tilde{\rho}_j^k(Q^*)\right] \times \left[Q_p^k - Q_p^{k*}\right] \geq 0, \tag{16}$$

for all $0 \leq Q_p^k \leq u_p^k$. The equilibrium conditions (9) then follow for commodity k, this path $p \in P_{ij}$, and, hence, for all commodities k, and for all paths in P_{ij} plus for all paths in all other country origin/destination pairs. The proof is complete. $\Box$

Variational inequality (10) can be put into standard form (cf. Nagurney (1999)), VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

We define $X \equiv Q$, with $\mathcal{K}$ as previously defined, and $\mathcal{N} = Kn_P$. Also, $F(X)$ consists of the elements $F_p^k(X) \equiv \left[\tilde{\pi}_i^k(Q) + C_p^k(Q) + \tau_p^k - \tilde{\rho}_j^k(Q)\right]$, $\forall k$, $\forall i, j$, $\forall p \in P_{ij}$. It follows that variational inequality (10) can be put into standard form (17).

We now establish the existence of a solution to variational inequality (10).

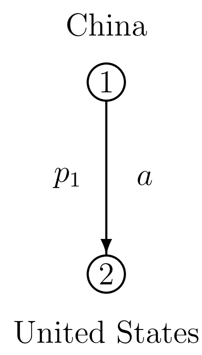

Figure 2: Network for Illustrative Example 1

Theorem 2: Existence

There exists a solution to variational inequality (10), with the solution corresponding to an equilibrium according to Definition 1.

Proof:

Given that the feasible set $\mathcal{K}$ is closed and bounded, i.e., compact, the existence of a solution to (10) is guaranteed by the standard theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)) since we have assumed that the supply price functions, the demand price functions, and the unit link transaction cost functions and, therefore, the path unit transaction cost functions are all continuous. $\square$

2.1 Illustrative Examples

We now present several examples for illustrative purposes.

Illustrative Example 1

We first consider an example without tariffs. There is a single commodity - that of tea, and, therefore, we suppress the superscript in the notation. As depicted in Figure 2, the origin country node 1 is China, a big producer of tea, and the destination country node 2 is the (western) United States. In order to construct these examples, we use data that is available online. The function data is for a ton of the commodity and the commodity shipments are also in tons, as is the supply and the demand. There is no capacity in the commodity shipment on path $p_1 = (a)$. The path corresponds to maritime transportation from a port in China to (western) United States.

The supply price function is:

$$\pi_1(s_1) = .3s_1 + 450$$

and the demand price function is:

$$\rho_2(d_2) = -d_2 + 15,500.$$

The unit transaction cost on path p_1, C_{p_1}, is:

$$C_{p_1}(Q_{p_1}) = .1Q_{p_1} + 3,150 = \tilde{c}_a(Q_{p_1}) = c_a(f_a) = .1f_a + 3,150.$$

Assuming that, in equilibrium, $Q^*_{p_1} > 0$, according to equilibrium conditions (9), we must have the following equation holding, after we have made use of converting both the supply and the demand to the commodity path flow:

$$\tilde{\pi}_1(Q^*_{p_1}) + C_{p_1}(Q^*_{p_1}) = \tilde{\rho}_2(Q^*_{p_1}) = .3Q^*_{p_1} + 450 + .1Q^*_{p_1} + 3,150 = -Q^*_{p_1} + 15,500. \tag{18}$$

Solution of the equation in (18), yields $Q^*_{p_1} = 8,500$ tons of tea, which is approximately the value exported from China to the United States in 2023 (see Bolton (2024)). Hence, also, $s^*_1 = d^*_2 = 8,500$. With this equilibrium solution, the supply price for a ton of tea (we do not distinguish types nor quality levels here), $\pi_1(s^*_1)$ is 3,000, the unit transaction cost, which, here is mainly the transportation cost per ton, $C_{p_1}(Q^*_{p_1})$, is 4,000, and the demand price $\rho_2(d^*_2)$ is 7,000. These prices and costs are commensurate with those in practice.

With the above data, one can easily calculate the minimum path tariff τ_{p_1} such that there would be zero trade in tea between China and the United States. Indeed, given that the sum of the fixed cost associated with production plus that of transaction is equal to $450 + 3,150 = 3,600$ and that the demand price function intercept is equal to: 15,500, we conclude that the minimum such tariff would have a value of $15,500 - 3,600 = 11,900$, with the associated equilibrium commodity shipment being $Q^*_{p_1} = 0.00$ and, therefore, also, $s^*_1 = d^*_2 = 0.00$. Note that such a tariff would be approximately 75% of the original demand price per ton of the tea without a tariff.

Illustrative Example 2

In the second and third illustrative examples, we consider the addition of a competitor to China, in terms of tea production and trade - that of Vietnam, as depicted in Figure 3. There is a single path connecting Vietnam to the (western) United States, path p_2, consisting of

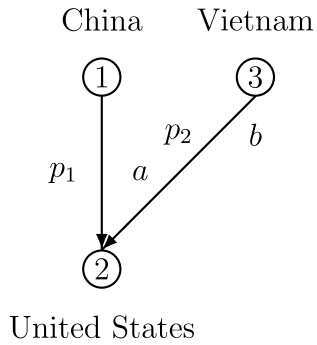

Figure 3: Network for Illustrative Examples 2 and 3

link b, which corresponds to a maritime link. In Illustrative Example 2 there is no assigned tariff, whereas in Illustrative Example 3 there is a tariff on tea from China to the United States.

Illustrative Example 2 has the same data as that in Illustrative Example 1 except that we add the data for Vietnam, which is as follows:

$$\pi_3(s_3) = .25s_3 + 500,$$

and the unit transaction cost function on path p_2 is:

$$C_{p_2}(Q_{p_2}) = .1Q_{p_2} + 2,900.$$

There is no capacity on the commodity shipment on path p_2.

The demand price function for tea in the United States remains as in Illustrative Example 1. Note that the model that we constructed assumes homogeneous commodities so that consumers at the destination countries are indifferent as to the country of origin of the commodity.

Assuming that $Q^*_{p_1} > 0$ and $Q^*_{p_2} > 0$, since there are no tariffs, equilibrium conditions (9) reduce to the solution of the following system of equations:

$$1.4Q^*_{p_1} + Q^*_{p_2} = 11,900$$

$$Q^*_{p_1} + 1.35Q^*_{p_2} = 12,100,$$

with solution: $Q^*_{p_1} = 4,455.10$ and $Q^*_{p_2} = 5,662.90$. The incurred supply prices at the equilibrium are: $\pi_1(s^*_1) = 1,786.50$ and $\pi_3(s^*_3) = 1,915.70$. The unit path transaction costs are: $C_{p_1}(Q^*_{p_1}) = 3,595.50$ and $C_{p_2}(Q^*_{p_2}) = 3,466.30$. The demand price $\rho_2(d^*_2)$ is now: $5,382.00$. With increased competition, the demand price of tea in the United States decreases from $7,000.00$ to $5,382.00$. The supply price for a ton of tea in China has now also decreased from $3,000.00$ to $1,786.50$. With increased competition, China now exports a much reduced volume of tea to the United States at $4,455.10$ tons, whereas in Illustrative Example 1 it exported $8,500$ tons. Vietnam exports a greater amount of tea than China does in this example.

Illustrative Example 3

This example has the same data as that in Illustrative Example 2 except that we now investigate the impact of a tariff placed on Chinese tea by the United States of $\tau_{p_1} = 1,345.50$, which is 25% of the demand market price without a tariff, as in Illustrative Example 2.

Applying equilibrium conditions (9), under the assumption of both commodity path shipments being positive at the equilibrium, yields the following system of equations:

$$1.4Q^*_{p_1} + Q^*_{p_2} = 10,554.50$$

$$Q^*_{p_1} + 1.35Q^*_{p_2} = 12,100.00,$$

with solution: $Q^*_{p_1} = 2,414.10$ and $Q^*_{p_2} = 7,174.70$.

Under the tariff, the shipment of tea from China to the United States, $Q^*_{p_1}$, drops precipitously from $4,455.10$, as in Illustrative Example 2 without a tariff, to $2,414.10$, under the tariff.

The incurred supply prices at the equilibrium are: $\pi_1(s^*_1) = 1,174.10$ and $\pi_3(s^*_3) = 2,293.70$. The unit path transaction costs are: $C_{p_1}(Q^*_{p_1}) = 3,391.40$ and $C_{p_2}(Q^*_{p_2}) = 3,617.50$. The equilibrium demand price $\rho_2(d^*_2)$ is now: $5,911.20$. The price for tea in the United States increases for consumers under the tariff. China suffers in terms of the supply price dropping from $1,786.50$ to $1,174.10$. The tea farmers in Vietnam, on the other hand, enjoy an increase in supply price for their tea with $\pi_3(s^*_3)$ now equal to $2,293.70$, whereas in Illustrative Example 2: $\pi_3(s^*_3) = 1,915.70$ at the equilibrium. Furthermore, Vietnam now exports about $1,500$ more tons of their tea to the United States in the case of the tariff on China's tea than it did without that tariff.

Illustrative Example 4

This example builds on Illustrative Example 3 to include a route to evade the tariff, as depicted in Figure 4. Specifically, there is now a new route p_3 joining China with the United States consisting of links: (c, d, e). Link d represents repackaging of the tea in Vietnam, with relabeling identifying the origin being Vietnam (and not China). Links c and e are transportation links.

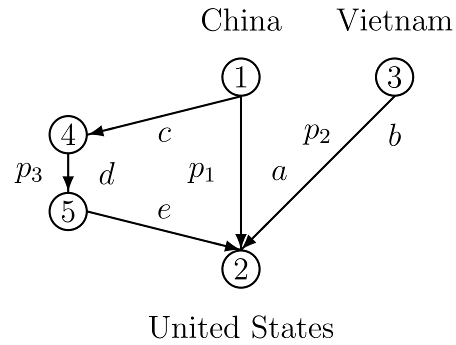

Figure 4: Network for Illustrative Example 4

The data remain as in Illustrative Example 3 but with the following data for the transportation links c and e and the processing link d. Since $f_c = f_d = f_e = Q_{p_3}$ we can write:

$$\tilde{c}_c(Q_{p_3}) = .1Q_{p_3} + 500, \quad \tilde{c}_d(Q_{p_3}) = .1Q_{p_3} + 100, \quad \tilde{c}_e(Q_{p_3}) = .1Q_{p_3} + 2,800,$$

and, therefore:

$$C_{p_3}(Q_{p_3}) = \tilde{c}_c(Q_{p_3}) + \tilde{c}_d(Q_{p_3}) + \tilde{c}_e(Q_{p_3}) = .3Q_{p_3} + 3,400.$$

Assuming, as we have done in the above examples, that the commodity shipments are all positive in equilibrium, according to equilibrium conditions (9), we now obtain, after algebraic simplification, the following system of equations:

$$1.4Q^*_{p_1} + 1.3Q^*_{p_2} + Q^*_{p_3} = 10,554.50$$

$$Q^*_{p_1} + 1.35Q^*_{p_2} + Q^*_{p_3} = 12,100$$

$$1.3Q^*_{p_1} + Q^*_{p_2} + 1.6Q^*_{p_3} = 11,650.$$

The solution of the above system yields:

$$Q^*_{p_1} = -2,936.70, \quad Q^*_{p_2} = 7,401.10, \quad Q^*_{p_3} = 5,038.60.$$

Clearly, with $Q^*_{p_1}$ being negative, this is not a feasible solution. We, therefore, can set $Q^*_{p_1} = 0$, and, with substitution into the second and third equations above (since the first one corresponds to path p_1 and is no longer an equality), we obtain:

$$1.35Q^*_{p_2} + Q^*_{p_3} = 12,100$$

$$Q^*_{p_2} + 1.6Q^*_{p_3} = 11,650,$$

with solution:

$$Q^*_{p_2} = 6,646.60, \quad Q^*_{p_3} = 3,127.20.$$

The incurred supply prices at the equilibrium are: $\pi_1(s^*_1) = 1,388.10$ and $\pi_3(s^*_3) = 2,161.60$. The unit path transaction costs are: $C_{p_1}(Q^*_{p_1}) = 3,150.00$, $C_{p_2}(Q^*_{p_2}) = 3,564.70$, and $C_{p_3}(Q^*_{p_3}) = 4,338.10$. Recall that the path tariff on path p_1, $\tau_{p_1} = 1,345.50$. The equilibrium demand price $\rho_2(d^*_2)$ is now: $5,726.30$.

Through rerouting, China's export of tea, albeit, illegal, increases from 2,414.10 tons to 3,127.20 tons. The demand price in the United States, in turn, decreases from $5,911.20$ to $5,726.30$. Consumers benefit in terms of a greater volume of tea imported to the United States than in Illustrative Example 3 and at a lower price. Vietnam now exports more tea than it did in Illustrative Example 2 but less than in Illustrative Example 3 when China's exports of tea encumbered a tariff to the United States. Hence, the illegal rerouting hurts Vietnam in terms of both the supply market price that farmers get and the amount of tea exported. This example demonstrates that decision-makers may wish to reduce/eliminate rerouting (with the goal of evading tariffs) through their country.

3. The Computational Method

In this Section, for completeness and easy reference, we recall the algorithm, the modified projection method of Korpelevich (1977), that we apply in Section 4 to solve larger-scale numerical examples. In addition, we present the explicit formulae for the computation of the multicommodity path flows at each iteration of the algorithm. The modified projection method has been applied to solve a plethora of equilibrium problems formulated as variational inequality problems (cf. Nagurney (1999, 2006, 2023)). An advantage of this algorithm is that it converges under reasonable assumptions on the function $F(X)$ that enters the variational inequality (17) and it does not require, for example, strong monotonicity of $F(X)$ for convergence as needed by the projection method of Dafermos (1980). The modified projection method is guaranteed to converge if the function $F(X)$ that enters the variational inequality problem (17) is monotone and Lipschitz continuous and that a solution exists. We know that a solution exists for our model because of Theorem 2.

Recall that F is said to be monotone if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{18}$$

Furthermore, F is Lipschitz continuous, if there exists an $\eta > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq \eta \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{19}$$

It is not unreasonable to assume that the supply price functions $\tilde{\pi}(Q)$ will be monotone increasing in Q; the demand price functions $\tilde{\rho}(Q)$ will be monotone decreasing in Q, and that the path cost functions $C(Q)$ will be monotone increasing in Q and, also, that these functions are Lipschitz continuous. Additional discussion of this algorithm, when applied to path-based network equilibrium problems and spatial price equilibrium problems can be found in Nagurney et al. (2023). In particular, as noted therein, the modified projection method, as recognized by Solodov and Tseng (1996) is a "very practical method." Furthermore, as is clear below, it is easy to implement for the solution of the multicommodity trade model with tariffs and rerouting.

In the statement of the algorithm, t denotes an iteration counter.

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $t = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{20}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{21}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

Due to the simple structure of the feasible set $\mathcal{K}$ underlying the multicommodity trade spatial price equilibrium model with tariffs and rerouting, which consists of box type constraints, as was also the case in the model of Nagurney et al. (2023), the solution of each of the subproblems in (20) and (21) can be obtained via closed form expressions for the multicommodity path flows as follows:

Explicit Formulae at Iteration t for the Multicommodity Path Flows in Step 1

The modified projection method results in the following closed form expressions for (20) for the multicommodity flows in Step 1 for the solution of variational inequality (10):

$$\bar{Q}_p^{kt} = \max\{0, \min\{u_p^k, Q_p^{kt-1} + \beta(\tilde{\rho}_j^k(Q^t) - \tilde{\pi}_i^k(Q^t) - C_p^k(Q^t) - \tau_p^k)\}\},$$

$$k = 1, \ldots, K, \forall p \in P. \tag{22}$$

Explicit Formulae at Iteration t for the Multicommodity Path Flows in Step 2

The modified projection method results in the following closed form expressions for (20) for the multicommodity flows in Step 2 for the solution of variational inequality (10):

$$Q_p^{kt} = \max\{0, \min\{u_p^k, Q_p^{kt-1} + \beta(\tilde{\rho}_j^k(\bar{Q}^t) - \tilde{\pi}_i^k(\bar{Q}^t) - C_p^k(\bar{Q}^t) - \tau_p^k)\}\},$$

$$k = 1, \ldots, K, \forall p \in P. \tag{23}$$

Note that, from the multicommodity equilibrium path flows, we can easily compute the multicommodity equilibrium supplies, demands, and also the link flows.

4. Algorithmically Solved Numerical Examples

In this Section, the modified projection method is applied to compute solutions to larger-scale examples. The algorithm was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst was used for the computations. The modified projection method was deemed to have converged if the absolute difference between each successive path flow iterate was less than equal to 10^{-3}.

Example 5: Addition of a New Demand Market - Japan

Example 5 has the same data as that in Illustrative Example 4 but now we have an added demand market - that of Japan, which imports much of its tea from China. In addition, we assume that tea has become more popular in the US and, therefore, the demand price function in the US is now: $\rho_2(d_2) = -.1d_2 + 15,500$. This change enables us to better represent impacts of various issues on a product with rising popularity. The network topology

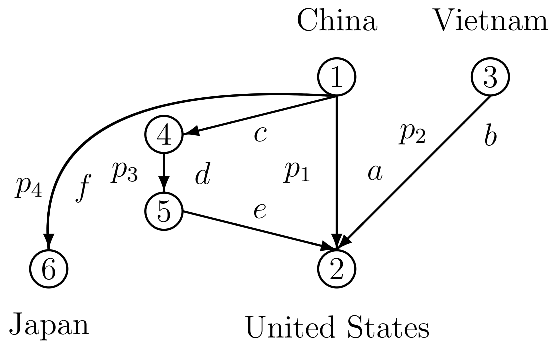

Figure 5: Network Topology for Examples 5, 6, and 8

for Example 5 is given in Figure 5. In Examples 5 through 8 we do not impose upper bounds on the commodity shipments.

The unit transaction cost function for link f, which makes up path p_4 and joins China with Japan via maritime transport is as follows:

$$c_f(f_f) = .1f_f + 1,200.$$

Hence, we have that

$$C_{p_4}(Q_{p_4}) = \tilde{c}_f(Q_{p_4}) = .1Q_{p_4} + 1,200.$$

The demand price function for tea (in tons) in Japan is:

$$\rho_6(d) = -.1d_6 + 10,000.00.$$

The computed commodity equilibrium path flow pattern is:

$$Q^*_{p_1} = 5,865.82, \quad Q^*_{p_2} = 24,339.38, \quad Q^*_{p_3} = 5,606.88, \quad Q^*_{p_4} = 9,816.37.$$

The equilibrium supplies are:

$$s^*_1 = 21,289.07, \quad s^*_3 = 24,339.38.$$

The supply market prices are:

$$\pi_1(s^*_1) = 6,836.72, \quad \pi_3(s^*_3) = 6,584.84.$$

The equilibrium demands are:

$$d^*_2 = 35,812.03, \quad d^*_6 = 9,816.37.$$

The demand market prices are:

$$\rho_2(d_2^*) = 11,918.79, \quad \rho_6(d_6^*) = 9,018.36.$$

The unit transaction costs on the paths are:

$$C_{p_1}(Q_{p_1}^*) + \tau_{p_1} = 5,082.08, \quad C_{p_2}(Q_{p_2}^*) = 5,333.94, \quad C_{p_3}(Q_{p_3}^*) = 5,082.06,$$

and

$$C_{p_4}(Q_{p_4}^*) = 2,181.64.$$

With another demand market, that of Japan, and the rising popularity of tea in the US, China dramatically increases its production of tea from 3,127.20 to 21,289.07 tons. Vietnam also increases its production of tea (although it is not trading with Japan in this example), from 6,646.60 to 24,339.38. The supply price also increases not only in China but also in Vietnam for tea. The amount of commodity shipment of tea on the "illegal" trade route p_3 increases from 3,127.20 to 5,606.88. The demand market price for tea in the US more than doubles. The commodity shipment of tea from Vietnam to the US is more than 3 times that in Example 4. Japan enjoys almost 10,000 tons of tea from China at a demand market price of 9,018.36, which is lower than that in the United States but reasonable, given the shorter distance of Japan to China as compared to that of the U.S. These results are interesting and show that, in unveiling an additional demand market, a country can gain.

Example 6: Example 5 with Tariff Removed

Example 6 has the same data as that in Example 5 except that now we remove the tariff on tea from China to the United States. Nevertheless, we keep the path through Vietnam with reprocessing to see whether it could be "efficient" even without a tariff imposed on another trade route.

The modified projection method now yields the following equilibrium commodity path flow pattern:

$$Q_{p_1}^* = 12,003.98, \quad Q_{p_2}^* = 23,517.35, \quad Q_{p_3}^* = 3,168.06, \quad Q_{p_4}^* = 7,596.79.$$

The equilibrium supplies are now:

$$s_1^* = 22,768.83, \quad s_3^* = 23,517.35.$$

The supply market prices are:

$$\pi_1(s_1^*) = 7,280.65, \quad \pi_3(s_3^*) = 6,379.34.$$

The equilibrium demands are now:

$$d_2^* = 38,689.38, \quad d_6^* = 7,596.79$$

and the demand market prices are now:

$$\rho_2(d_2^*) = 11,631.06, \quad \rho_6(d_6^*) = 9,240.32.$$

The unit transaction costs on the paths are:

$$C_{p_1}(Q_{p_1}^*) = 4,350.40, \quad C_{p_2}(Q_{p_2}^*) = 5,251.73, \quad C_{p_3}(Q_{p_3}^*) = 4,350.42,$$

and

$$C_{p_4}(Q_{p_4}^*) = 1,959.68.$$

From the results we can see that, as compared to the values in Example 5, the supply price for tea in China increases from 6,836.72 to 7,280.65, but decreases in Vietnam, from 6,584.84 to 6,379.34. The production output (supply) of tea in China increases from 21,289.07 to 22,768.83, whereas that in Vietnam decreases, from 24,339.38 to 23,517.35; again, demonstrating effects of competition. The farmers in China, hence, gain with the removal of the tariff in terms of increased production and a higher supply price. The flow on path p_3 decreases from 5,606.88 to 3,168.06; however, it still has a positive flow, suggesting that transporting from China through Vietnam even with some possible encumbrance of additional transaction cost and even without tariffs on tea from China to the US can be economically efficient. The demand price for tea in the US decreases from 11,918.79 to 11,631.06 but increases in Japan (from 9,018.36 to 9,240.32).

Example 7: Example 6 with Tariff Evasion Route Removed

Example 7 has the same data as that in Example 6 except that now we explore the following scenario: the trade route, p_3, is no longer available and, therefore, the network for this example is that depicted in Figure 5 but with path p_3 consisting of links c, d, and e removed. The modified projection method converges to the following equilibrium solution.

$$Q_{p_1}^* = 14,108.29, \quad Q_{p_2}^* = 23,753.69, \quad Q_{p_4}^* = 8,235.00.$$

The equilibrium supplies are now:

$$s_1^* = 22,343.29, \quad s_3^* = 23,753.69.$$

The supply market prices are:

$$\pi_1(s_1^*) = 7,152.99, \quad \pi_3(s_3^*) = 6,438.42.$$

The equilibrium demands are now:

$$d_2^* = 37,861.98, \quad d_6^* = 8,235.00$$

and the demand market prices are now:

$$\rho_2(d_2^*) = 11,713.80, \quad \rho_6(d_6^*) = 9,176.50.$$

The unit transaction costs on the paths are:

$$C_{p_1}(Q_{p_1}^*) = 4,560.83, \quad C_{p_2}(Q_{p_2}^*) = 5,275.37,$$

and

$$C_{p_4}(Q_{p_4}^*) = 2,023.50.$$

The production output (supply) of tea in China decreases, as compared to the value in Example 6, from $22,768.83$ to $22,343.29$ but increases in Vietnam from $23,517.35$ to $23,753.69$. The demand market price of tea increases in the US from $11,631.06$ to $11,713.80$ but decreases in Japan (by a small amount). This example suggests that having additional routes for trade can be beneficial, which makes sense. This was also noted, in the context of agricultural trade from Ukraine in wartime in Hassani et al. (2025). Indeed, all path transaction costs to the US have now increased, as has that from China to Japan.

Example 8: Example 5 with Tariff Added on Tea from Vietnam to the US

With tariffs now dominating the news, especially in terms of the United States imposing them on goods from other countries (cf. Yousif (2025)), Example 8 has the same data as that in Example 5, where there was a tariff on path p_1 on tea from China to the United States and also a path for tariff evasion - path p_3, except that now, in Example 8, we investigate the impact of an additional tariff on the tea from Vietnam to the United States on path p_2 such that $\tau_{p_2} = 2,000$, which is, approximately, 20% of the demand market price of tea in the United States at the equilibrium solution for Example 5. Note that, with the evasion path going through Vietnam, route p_3 also encumbers a tariff $\tau_{p_3} = 2,000$.

The modified projection method converges to the following equilibrium solution.

The computed commodity equilibrium path flow pattern is:

$$Q_{p_1}^* = 10,712.89, \quad Q_{p_2}^* = 19,940.26, \quad Q_{p_3}^* = 556.04, \quad Q_{p_4}^* = 9,938.66.$$

The equilibrium supplies are:

$$s_1^* = 21,207.59, \quad s_3^* = 19,940.26.$$

The supply market prices are:

$$\pi_1(s_1^*) = 6,812.28, \quad \pi_3(s_3^*) = 5,485.06.$$

The equilibrium demands are:

$$d_2^* = 31,209.19, \quad d_6^* = 9,938.66.$$

The demand market prices are now:

$$\rho_2(d_2^*) = 12,379.08, \quad \rho_6(d_6^*) = 9,006.13.$$

The unit transaction costs on the paths are:

$$C_{p_1}(Q_{p_1}^*) + \tau_{p_1} = 5,566.81, \quad C_{p_2}(Q_{p_2}^*) + \tau_{p_2} = 6,894.03, \quad C_{p_3}(Q_{p_3}^*) + \tau_{p_3} = 5,566.81,$$

and

$$C_{p_4}(Q_{p_4}^*) = 2,193.87.$$

The shipment of tea from Vietnam to the United States now drops from $24,339.38$ tons, as in Example 5, to $19,940.26$ tons, under the added tariff on tea from Vietnam, with the supply (production output of tea) dropping accordingly. However, and this is quite interesting, by "punishing" Vietnam with a tariff, the commodity shipment of tea from China now increases on its tariffed path, path p_1, from $5,865.82$ tons, as in Example 5, to $10,712.89$ tons. The shipment on the tariff evasion path through Vietnam, path p_3, now decreases from $5,606.88$ tons to 556.04 tons, a precipitous drop of about 90%. Nevertheless, China's production of tea drops minimally as does its supply price for tea. Japan gains in terms of an increased volume of tea imported and at a lower price than in Example 5. This example also shows the importance of a supply market country, such as China, having demand markets in countries that are not tariffing products from your country. The price of tea in the United States from China increases, as compared to that in Example 5, demonstrating, again, the negative impacts of tariffs on consumers in terms of price increases of commodities.

This example highlights what is happening in the real world now because of a trade war, with the new US administration imposing massive tariffs. Also, the example illustrates what may happen as alternative routes to the United States for Chinese goods are "severed" as reported in Bradsher (2025). In particular, such routes for evasion are getting renewed attention with even additional tariffs (as we explore here) being placed on transshipment countries.

The above examples are stylized; nevertheless, they demonstrate the kinds of exercises that can be conducted in order to ascertain impacts of tariffs, the possibilities of tariff evasion through rerouting, and also transshipment of commodities through other countries.

5. Summary and Suggestions for Future Research

Mathematical modeling, and the accompanying theory, applications, and algorithms are essential for the evaluation of policies that affect multicommodity trade through the quantification of prices, on the production and on the consumption sides, and the commodity shipments between countries. Governments have been increasingly imposing policies, in the form of tariffs, on commodities produced in different countries leading to what are known as "trade wars." Some, in turn, are trying to identify alternative routes in order to evade tariffs and the associated costs.

In this paper, we add to the literature on the modeling of multicommodity trade in a spatial equilibrium framework in the presence of tariffs and rerouting, with the latter for the purpose of evading tariffs. The equilibrium model consists of countries that are supply markets for commodities as well as countries that are demand markets for them. Each pair of supply and demand market countries can be connected by one or more paths that correspond to trade routes. Each trade route, in turn, can consist of one or more links that can correspond to a mode of transportation or, in the case of a link in a country to evade a tariff - a processing/relabeling link to mask the country of origin. The equilibrium conditions, which include the unit path tariffs, are formulated as a variational inequality problem in path flows and existence results established. We note that evading tariffs through rerouting, with the purposes of relabeling, is not legal but it is happening. Having a computable framework enables the investigation of possible interdiction mechanisms by authorities.

We present both illustrative numerical examples as well as algorithmically solved ones, which expand on the former examples. The application is that of trade of tea from China.

The solutions reveal the impacts of a tariff on tea from China to the United States, of rerouting, and also of the addition of a demand market, and a further tariff on Vietnam, a country for possible transshipment as well as rerouting to evade the original tariff. The results reinforce, in part, what we have found in the context of agricultural trade from Ukraine in wartime as in Hassani et al. (2025) and Nagurney et al. (2024a, b) - that having alternative trade routes is beneficial. The results also reinforce that consumers in the country imposing a tariff lose in terms of the demand price, which becomes higher, and the volume of commodity shipment, which becomes lower, as do producers in the country whose product is tariffed, with the supply market price decreasing. The results reveal the effects of additional tariffs on transshipment countries, where rerouting may be taking place.

As we mentioned in the Introduction, we believe that this is the first rigorous investigation of an equilibrium framework that captures tariffs and rerouting for the evasion of tariffs. There are many opportunities for additional research. First, one can expand the number of supply countries and demand countries as well as the number of commodities. One could also investigate the effects of retaliatory tariffs; in other words, after a country A imposes a tariff on a product or products of country B, country B retaliates by adding (or increasing) tariffs on products from country A. One can also investigate additional countries through which rerouting might take place in order to evade tariffs. In addition, it would be very interesting to study tariffs in the context of multitiered supply chains in which there are multiple suppliers, as modeled in the book by Nagurney and Li (2016). Tariffs can be imposed not only on commodities and finished products but, also, on inputs into production processes. Furthermore, as is the case on automobile manufacturing in North America, components may criss-cross national boundaries several times as they are incorporated into higher level parts of a vehicle (cf. Domonoske (2025)). It would also be worthwhile to investigate opportunities for on-shoring and which products might be profitable to produce in-house. Finally, the model in this paper is a perfectly competitive one - developing an imperfectly competitive model, as in the case of an oligopoly, using game theory, and exploring tariffs with rerouting would also be timely.

Acknowledgments

This paper is dedicated to Professor R. Tyrrell "Terry" Rockafellar in honor of his 90th birthday with gratitude for his immense contributions to convex and variational analysis.

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